Precise Determination of the Electric and Magnetic Form Factors of the Proton

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Saclay, France, November 25, 2010
Outline

1. Introduction I: The size of the proton from the Lamb shift in muonic hydrogen and electron scattering
2. Introduction II: Electric and magnetic form factors of the Proton
3. The Mainz high-precision p(e,e’)p measurement
   - Design considerations
   - Covered kinematical region
4. Results
   - Analysis technique
   - Cross section results
   - Checks: Rosenbluth and model dependence
5. Conclusion and Outlook
6. Discussion of the Lamb shift / electron scattering discrepancy

Form Factors of the Proton
Introduction I: The size of the proton

Nature 466, 213-216 (8 July 2010)

Form Factors of the Proton
Cross section and form factors for elastic e-p scattering

The cross section:

\[
\frac{\frac{d\sigma}{d\Omega}}{\frac{d\sigma}{d\Omega}}_{\text{Mott}} = \frac{1}{\varepsilon (1 + \tau)} \left[ \varepsilon G_E^2 \left( Q^2 \right) + \tau G_M^2 \left( Q^2 \right) \right]
\]

with:

\[
\tau = \frac{Q^2}{4 m_p^2}, \quad \varepsilon = \left( 1 + 2 \left( 1 + \tau \right) \tan^2 \frac{\theta_e}{2} \right)^{-1}
\]

Fourier-transform of \( G_E, G_M \rightarrow \) spatial distribution (Breit frame)

\[
\left\langle r_E^2 \right\rangle = -6 \hbar^2 \left. \frac{d G_E}{d Q^2} \right|_{Q^2=0}, \quad \left\langle r_M^2 \right\rangle = -6 \hbar^2 \left. \frac{d (G_M/\mu_p)}{d Q^2} \right|_{Q^2=0}
\]
Overview of different proton charge-radius results

Filled dots: Results from new measurements.
Hollow dots: Reanalysis of existing data.
Overview of different proton charge-radius results

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Form Factors of the Proton

Discrepancy of existing values for proton electric radius:

- 0.809(11) fm: standard dipole at HEPL (Hand et al. 1963)
- 0.862(12) fm: low $Q^2$ at Mainz (Simon et al. 1979)
- 0.847(09) fm: dispersion relation (Mergell et al. 1996)
- 0.890(14) fm: Hydrogen Lamb shift (Udem et al. 1997)

MAMI-A: 180 MeV fixed
The Mainz Microtron MAMI

- **MAMI-A**: 180 MeV fixed
- **MAMI-B**: 855 MeV, 15 MeV steps

Form Factors of the Proton
The Mainz Microtron MAMI

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- MAMI-C: 1.5 GeV, 15 MeV steps
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The Mainz high-precision p(e,e’)p measurement: Three spectrometer facility of the A1 collaboration
Design goal: High precision

- Statistical precision: 20 min beam time for <0.1%
Design goal: High precision through redundancy

- Statistical precision: 20 min beam time for <0.1%
- Control of luminosity and systematic errors:
  Measure all quantities in as many ways as possible:

  - Beam current: Foerster probe (usual way) → measures down to extremely low currents for small θ
  - Luminosity: current × density × target length ⇒ third magnetic spectrometer as monitor

Overlapping acceptance
Where possible: Measure at the same scattering angle with two spectrometers
Statistical precision: 20 min beam time for <0.1%

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Measured settings and future (high $Q^2$) expansion

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{1}{\varepsilon (1 + \tau)} \left[ \varepsilon G_E^2 \left( Q^2 \right) + \tau G_M^2 \left( Q^2 \right) \right] \]

Form Factors of the Proton
Measured settings and future (high $Q^2$) expansion

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{1}{\epsilon (1 + \tau)} \left[ \epsilon G_E^2 \left( Q^2 \right) + \tau G_M^2 \left( Q^2 \right) \right]
\]

- $\sim 90$ GByte on disc
- $\sim 1400$ Settings
- $> 10^9$ events.
Background

Form Factors of the Proton
Background

Liquid Hydrogen

Form Factors of the Proton
Background

Liquid Hydrogen

Havar foil

Form Factors of the Proton
Data $\Leftrightarrow$ Simulation matching

Simulation:
- Model for energy loss and small angle scattering
- Input: momentum-, angular-, vertex resolution

Form Factors of the Proton
Feynman graphs of leading and next to leading order for elastic scattering

All graphs are taken into account:

- vacuum polarization (v1): e, (µ, τ)
- electron vertex correction
- Coulomb distortion (two photon exchange)
- real photon emission
Description of the radiative tail

Form Factors of the Proton
Cross sections

Form Factors of the Proton
Cross sections / standard dipole

Form Factors of the Proton
How to extract the form factors?

Two methods:
1. Classical Rosenbluth separation
2. "Super-Rosenbluth separation": Fit of form factor models directly to the measured cross sections
   Feasible due to fast computers.
   All data at all $Q^2$ and $\varepsilon$ values contribute to the fit, i.e. full kinematical region used, no projection (to specific $Q^2$) needed.
   Easy fixing of normalization.
   Model dependence?
   For radii extraction: Needs a fit anyway!
   Classical Rosenbluth: Extracted $G_E$ and $G_M$ highly correlated! $\Rightarrow$ Error propagation very involved.
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Classical Rosenbluth: Extracted $G_E$ and $G_M$ highly correlated!
$$\Rightarrow$$ Error propagation very involved.
Models: Dipols

Dipole (different $b$ for $G_E$ and $G_M$):

\[ G_D\left(Q^2, b\right) = \frac{1}{\left(1 + \frac{Q^2}{b}\right)^2} \]


\[ G_{DD}\left(Q^2, a, b_1, b_2\right) = aG_D\left(Q^2, b_1\right) + (1 - a) G_D\left(Q^2, b_2\right) \]
Models: Polynomial

Polynomial

\[ G_P\left(Q^2, a_1, \ldots, a_n\right) = 1 + \sum_{i=1}^{n} a_i Q^{2 \cdot i} \]

Polynomial + standard Dipole

\[ G_{PAD}\left(Q^2, a_1, \ldots, a_n\right) = G_D\left(Q^2, 0.71\right) + \sum_{i=1}^{n} a_i Q^{2 \cdot i} \]

Polynomial × standard Dipole

\[ G_{PMD}\left(Q^2, a_1, \ldots, a_n\right) = G_D\left(Q^2, 0.71\right) \cdot \left(1 + \sum_{i=1}^{n} a_i Q^{2 \cdot i}\right) \]
Models: Splines

Uniform cubic splines

\[ \text{spline} \left( Q^2, a_1, \ldots, a_n \right) \]

Spline:

\[ G_{\text{Spline}} \left( Q^2, a_1, \ldots, a_n \right) = 1 + Q^2 \cdot \text{spline} \left( Q^2 \right) \]

Spline \times \text{standard Dipole}

\[ G_{\text{SMD}} \left( Q^2, a_1, \ldots, a_n \right) = G_D \left( Q^2, 0.71 \right) \cdot \left( 1 + Q^2 \cdot \text{spline} \left( Q^2 \right) \right) \]
Also:
- Friedrich / Walcher phenomenological ansatz
- extended Gari-Krümpelmann (VMD), Lomon et al.
- Arrington type:

\[
\frac{P^N}{P^{N+2}}
\]
Cross sections / standard dipole

Form Factors of the Proton
Cross sections + spline fit

Form Factors of the Proton
Form Factors of the Proton
Jan C. Bernauer et al., “High-precision determination of the electric and magnetic form factors of the proton”, arXiv:1007.5076
The electric rms radius - extracted by different models

Form Factors of the Proton
Conclusion – Part I

- High precision e-p scattering data from MAMI.
  e-print: arXiv:1007.5076 accepted by PRL
- $Q^2$ range from 0.003 to 1 (GeV/c)$^2$.
- Consistent data set.
- “Super-Rosenbluth” fit to determine form factors and radii.
- The charge and magnetic rms radii are determined as

\[
\begin{align*}
\langle r_e \rangle &= 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{model}} \pm 0.004_{\text{group}} \text{ fm}, \\
\langle r_m \rangle &= 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{model}} \pm 0.002_{\text{group}} \text{ fm}.
\end{align*}
\]

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Form Factors of the Proton
Discussion of the Lamb shift / electron scattering discrepancy

The following tables are taken from the ’QED supplement’ published in *Nature* 466, 213-216 (8 July 2010). ’All known radius-independent contributions’ and ’all relevant radius-dependent contributions’ to the Lamb shift in $\mu p$ from different authors are listed.
2S – 2P splitting in muonic hydrogen

\[ \Delta E_{\text{LS}} = 206.06 \text{ meV} \]

\[ \Delta E_{\text{FS}} = 8.35 \text{ meV} \]

\[ \Delta E_{\text{HFS}}^{2S} = 22.81 \text{ meV} \]

\[ \Delta E_{\text{HFS}}^{2P_{3/2}} = 3.39 \text{ meV} \]

Form Factors of the Proton
2S – 2P splitting in muonic hydrogen

$\Delta E_{\rm LS} = 206.06$ meV

finite size effect: \[ \sqrt{\langle r^2 \rangle} \]
proton radius: \[ \langle r^3 \rangle \]
3rd Zemach moment: \[ \langle r \rangle_{(2)} \]

$\Delta E_{\rm 2S}^{\rm HFS} = 22.81$ meV \{ includes Zemach radius: \[ \langle r \rangle_{(2)} \rightarrow -0.145 \text{ meV} \]

$\Delta E_{\rm FS} = 8.35$ meV

$\Delta E_{\rm HFS}^{2P_{3/2}} = 3.39$ meV

Form Factors of the Proton
## Discussion of the Lamb shift / electron scattering discrepancy

<table>
<thead>
<tr>
<th>#</th>
<th>Contribution</th>
<th>Ref.</th>
<th>Our selection</th>
<th>Pachucki(^1)(^-)(^3)</th>
<th>Borie(^5)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Value</td>
<td>Unc.</td>
<td>Value</td>
<td>Unc.</td>
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<tr>
<td>1</td>
<td>NR One loop electron VP</td>
<td>1,2</td>
<td>205.0282</td>
<td>0.0169</td>
<td>205.0282</td>
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<tr>
<td>2</td>
<td>Relativistic correction (corrected)</td>
<td>1–3,5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Relativistic one loop VP</td>
<td>5</td>
<td>1.5081</td>
<td>1.5079</td>
<td>1.5081</td>
<td></td>
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<tr>
<td>4</td>
<td>NR two-loop electron VP</td>
<td>5,14</td>
<td>1.509</td>
<td>1.509</td>
<td>1.509</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Polarization insertion in two Coulomb lines</td>
<td>1,2,5</td>
<td>0.1509</td>
<td>0.1509</td>
<td>0.1509</td>
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<tr>
<td>6</td>
<td>NR three-loop electron VP</td>
<td>11</td>
<td>0.00529</td>
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<tr>
<td>7</td>
<td>Polarization insertion in two and three Coulomb lines (corrected)</td>
<td>11,12</td>
<td>0.00223</td>
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<td></td>
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<tr>
<td>8</td>
<td>Three-loop VP (total, uncorrected)</td>
<td>5,15,16</td>
<td>–0.00103</td>
<td>0.0076</td>
<td>0.00760</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Wichmann-Kroll</td>
<td>6</td>
<td>0.00135</td>
<td>0.00135</td>
<td>–0.00103</td>
<td>0.00015</td>
</tr>
<tr>
<td>10</td>
<td>Light by light electron loop contribution (Virtual Delbrück scattering)</td>
<td>6</td>
<td>0.00135</td>
<td>0.00135</td>
<td>–0.00103</td>
<td>0.00015</td>
</tr>
<tr>
<td>11</td>
<td>Radiative photon and electron polarization in the Coulomb line (a(Za)^4)</td>
<td></td>
<td>–0.00500</td>
<td>0.0010</td>
<td>–0.00600</td>
<td>0.00100</td>
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<tr>
<td>12</td>
<td>Electron loop in the radiative photon of order (a(Za)^4)</td>
<td>17–19</td>
<td>–0.00150</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>13</td>
<td>Mixed electron and muon loops</td>
<td>20</td>
<td>0.00007</td>
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<td></td>
<td></td>
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<tr>
<td>14</td>
<td>Hadronic polarization (a(Za)^4 m_r)</td>
<td>21–23</td>
<td>0.01077</td>
<td>0.00038</td>
<td>0.01130</td>
<td>0.00030</td>
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<td>15</td>
<td>Hadronic polarization (a(Za)^4 m_r)</td>
<td>22,23</td>
<td>0.000047</td>
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<td>16</td>
<td>Hadronic polarization in the radiative photon (a(Za)^4 m_r)</td>
<td>22,23</td>
<td>–0.000015</td>
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<tr>
<td>17</td>
<td>Recoil contribution</td>
<td>24</td>
<td>0.05750</td>
<td>0.0575</td>
<td>0.05750</td>
<td>0.05750</td>
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<tr>
<td>18</td>
<td>Recoil finite size</td>
<td>5</td>
<td>0.01300</td>
<td>0.0013</td>
<td>0.01300</td>
<td>0.00130</td>
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<tr>
<td>19</td>
<td>Recoil correction to VP</td>
<td>5</td>
<td>–0.00410</td>
<td>–0.00410</td>
<td>–0.00410</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Radiative corrections of order (a(Za)^5 m_r)</td>
<td>2,7</td>
<td>–0.66770</td>
<td>–0.66770</td>
<td>–0.66770</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Muon Lamb shift 4th order</td>
<td>5</td>
<td>–0.00169</td>
<td>–0.00169</td>
<td>–0.00169</td>
<td></td>
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<tr>
<td>22</td>
<td>Recoil corrections of order (a(Za)^5 m_r)</td>
<td>2,5,7</td>
<td>–0.04497</td>
<td>–0.0450</td>
<td>–0.04497</td>
<td></td>
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<tr>
<td>23</td>
<td>Recoil of order (a^6)</td>
<td>2</td>
<td>0.000300</td>
<td>0.000300</td>
<td></td>
<td></td>
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<tr>
<td>24</td>
<td>Radiative recoil corrections of order (a(Za)^5 m_r)</td>
<td>1,2,7</td>
<td>–0.00960</td>
<td>–0.00990</td>
<td>–0.00960</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Nuclear structure correction of order (a(Za)^5) (Proton polarizability)</td>
<td>2,5,22,25</td>
<td>0.015</td>
<td>0.012</td>
<td>0.015</td>
<td>0.004</td>
</tr>
<tr>
<td>26</td>
<td>Polarization operator induced correction to nuclear polarization (a(Za)^5 m_r)</td>
<td>23</td>
<td>0.00019</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>27</td>
<td>Radiative photon induced correction to nuclear polarization (a(Za)^5 m_r)</td>
<td>23</td>
<td>–0.00001</td>
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<td></td>
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</tr>
</tbody>
</table>

**Form Factors of the Proton**
\[ \Delta E = 209.9779(49) - 5.2262 \, r_p^2 + 0.0347 \, r_p^3 \]

Values are in meV and radii in fm.

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<th>Borie(^5)</th>
</tr>
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<tr>
<td>Leading nuclear size contribution</td>
<td></td>
<td>-5.19745 (&lt; r_p^2 &gt;)</td>
<td>-5.1974</td>
<td>-5.1971</td>
</tr>
<tr>
<td>Radiative corrections to nuclear finite size effect</td>
<td>(^2,26)</td>
<td>-0.0275 (&lt; r_p^2 &gt;)</td>
<td>-0.0282</td>
<td>-0.0273</td>
</tr>
<tr>
<td>Nuclear size correction of order ((Z\alpha)^6) (&lt; r_p^2 &gt;)</td>
<td>(^1,27-29)</td>
<td>-0.001243 (&lt; r_p^2 &gt;)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ( &lt; r_p^2 &gt;) contribution</td>
<td></td>
<td>-5.22619 (&lt; r_p^2 &gt;)</td>
<td>-5.2256</td>
<td>-5.2244</td>
</tr>
<tr>
<td>Nuclear size correction of order ((Z\alpha)^5)</td>
<td>(^1,2)</td>
<td>0.0347 (&lt; r_p^3 &gt;)</td>
<td>0.0363</td>
<td>0.0347</td>
</tr>
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</table>

Form Factors of the Proton
Zemach-Moments:


\[< r^3 >_{(2)} = \int_0^\infty \frac{dq}{q^4} \left( G_E^2(q^2) - 1 + q^2 < r^2 >_p / 3 \right) \]

\[< r^3 >_{(2)} = 2.27 \text{ fm}^3 \quad \rightarrow \quad r_p = 0.84184(67) \text{ fm} \]
Zemach-Moments:


\[
\langle r^3 \rangle_{(2)} = \int_0^\infty \frac{dq}{q^4} \left( G^2_E(q^2) - 1 + q^2 \langle r^2 \rangle_p / 3 \right)
\]

\[
\langle r^3 \rangle_{(2)} = 2.27 \text{ fm}^3 \quad \rightarrow \quad r_p = 0.84184(67) \text{ fm}
\]

\[
\langle r^3 \rangle_{(2)} = 2.85(8) \text{ fm}^3 \quad \rightarrow \quad r_p = 0.84245(67) \text{ fm}
\]

2S – 2P splitting in muonic hydrogen

\[ \Delta E_{2P_{3/2}} = 3.39 \text{ meV} \]

\[ \Delta E_{FS} = 8.35 \text{ meV} \]

\[ \Delta E_{LS} = 206.06 \text{ meV} \]

\[ \Delta E_{2S_{HFS}} = 22.81 \text{ meV} \]

includes Zemach radius:
\[ < r > (2) \rightarrow -0.145 \text{ meV} \]

finite size effect:
proton radius: \( \sqrt{< r^2 >} \)
3rd Zemach moment: \( < r^3 > (2) \)
De Rújula’s toy model


- **Sum of “single pole” and “dipole”**

\[
\rho_{\text{Proton}}(r) = \frac{1}{D} \left[ \frac{M^4 e^{-Mr} \cos^2(\theta)}{4\pi r} + \frac{m^5 e^{-mr} \sin^2(\theta)}{8\pi} \right]
\]

\[D \equiv M^2 \cos^2(\theta) + m^2 \sin^2(\theta)\]

using \( M = 0.750 \text{ GeV}/c^2, \ m = 0.020 \text{ GeV}/c^2, \) and \( \sin^2(\theta) = 0.3 \)

and

\[\rho(2)(r) = \int d^3 r_2 \rho_{\text{charge}}(|\vec{r} - \vec{r}_2|) \rho_{\text{charge}}(r_2)\]

we get the **third Zemach moment**:

\[\langle r^3 \rangle_{(2)} = \int d^3 r \ r^3 \rho(2)(r) = 36.2 \text{ fm}^3\]
We put $\langle r^3 \rangle_{(2)} = 36.2 \text{ fm}^3$ in the Lamb shift formula:

$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] =$$

$$\left(209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\text{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\text{fm}^3}\right) \text{ meV}$$

and get $r_p = 0.878 \text{ fm}$
We put $\langle r^3 \rangle_{(2)} = 36.2 \text{ fm}^3$ in the Lamb shift formular:

$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \left( 209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\text{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\text{fm}^3} \right) \text{meV}$$

and get $r_p = 0.878 \text{ fm}$

problem solved
De Rújula’s toy model – is excluded by experiment

- De Rújula’s toy model
- standard dipole
- Bernauer-Arrington fit assembly
High precision form factors from MAMI provide constraints for the charge distribution of the proton.

Standard dipole approximation is not sufficient for correction of the muonic hydrogen Lamb shift.


Supported by the “Deutsche Forschungsgemeinschaft (DFG)” with a “Sonderforschungsbereich (SFB443)”.
Jan Bernauer joined the OLYMPUS Experiment @ DESY/Hamburg (Spokesperson: Richard Milner, MIT)

OLYMPUS: Determine the effect of two-photon exchange in elastic lepton-proton scattering by precisely measuring the ratio of positron-proton to electron-proton elastic unpolarized cross sections.

low $Q^2$ extension: ISR @ MAMI
Outlook: Initial state radiation

Form Factors of the Proton
Outlook: Initial state radiation

Form Factors of the Proton
Discussion of the Lamb shift / electron scattering discrepancy

The following tables are taken from the ’QED supplement’ published in *Nature* 466, 213-216 (8 July 2010). ’All known radius-independent contributions’ and ’all relevant radius-dependent contributions’ to the Lamb shift in $\mu$ from different authors are listed.
## Discussion of the Lamb shift / electron scattering discrepancy

<table>
<thead>
<tr>
<th>#</th>
<th>Contribution</th>
<th>Ref.</th>
<th>Our selection</th>
<th>Pachucki¹⁻³</th>
<th>Borie⁴⁻⁵</th>
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<tbody>
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<td>NR One loop electron VP</td>
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<td>3</td>
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<td>Polarization insertion in two and three Coulomb lines (corrected)</td>
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<td>8</td>
<td>Three-loop VP (total, uncorrected)</td>
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<td>0.0076</td>
<td>0.00761</td>
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<tr>
<td>9</td>
<td>Wichmann-Kroll</td>
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### Form Factors of the Proton
## Discussion of the Lamb shift / electron scattering discrepancy

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<th>Contribution</th>
<th>Ref.</th>
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<th>Pachucki$^{1-3}$</th>
<th>Borie$^{5}$</th>
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Form Factors of the Proton
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<td></td>
<td><strong>206.0573</strong></td>
<td><strong>0.0045</strong></td>
</tr>
</tbody>
</table>
Discussion of the Lamb shift / electron scattering discrepancy

\[ \Delta E = 209.9779(49) - 5.2262 r_P^2 + 0.0347 r_P^3 \]

Values are in meV and radii in fm.

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<th>Borie$^5$</th>
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<td>$^2$6</td>
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<td>$-5.1974$</td>
<td>$-5.1971$</td>
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<td>Radiative corrections to nuclear finite size effect</td>
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<td>$-0.0275$ $\langle r_P^2 \rangle$</td>
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<td>Nuclear size correction of order $(Z\alpha)^6 \langle r_P^2 \rangle$</td>
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<td>$-5.2244$</td>
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## Data taking periods

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<th>August 2006</th>
<th>November 2006</th>
<th>May 2007</th>
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<tr>
<td><strong>Duration</strong></td>
<td>10 days</td>
<td>11 days</td>
<td>17 days</td>
</tr>
<tr>
<td><strong>without setup/calibration</strong></td>
<td>8 days</td>
<td>9 days</td>
<td>11 days</td>
</tr>
<tr>
<td><strong>Energies</strong></td>
<td>575, 850 MeV</td>
<td>180, 720 MeV</td>
<td>315, 450 MeV</td>
</tr>
<tr>
<td><strong>Setting changes</strong></td>
<td>152</td>
<td>173</td>
<td>202</td>
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<tr>
<td><strong>data taking time</strong></td>
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<td>5.3 days</td>
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<tr>
<td><strong>Average time per setting</strong></td>
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<td>35 min</td>
<td>38 min</td>
</tr>
<tr>
<td><strong>Overhead per setting</strong></td>
<td>44 min</td>
<td>40 min</td>
<td>40 min</td>
</tr>
</tbody>
</table>

- Overhead includes angle changes, momentum changes and down times.
- Average time for "angle only" setting changes: 10 min.
Form factor results
Cross sections: 315 MeV

Form Factors of the Proton
Cross sections: 450 MeV

Form Factors of the Proton
Cross sections: 585 MeV

Form Factors of the Proton
Cross sections: 720 MeV

Form Factors of the Proton
Cross sections: 855 MeV

Form Factors of the Proton
Rosenbluth separation

\[ Q^2 = 0.15 \text{ (GeV}/c)^2 \]

Form Factors of the Proton
Form Factors of the Proton

... more ...
Form Factors of the Proton
... and even more

Form Factors of the Proton
Comparison: Rosenbluth vs. Spline fit

Form Factors of the Proton
Check extraction of radii with Monte-Carlo data:

- Monte-Carlo data from given parametrization (known radii!)
- Error distribution of this simulated data according to errors from real data
- Fit with different models

Assumption: ±5% normalization error (per spectrometer/energy)
## Model dependence: Charge Radius

### Analysis of simulated pseudo data

<table>
<thead>
<tr>
<th>Input</th>
<th>Analysis</th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Dipole</td>
<td>Dbl-D.</td>
<td>Poly.</td>
<td>P.+D.</td>
<td>P.×D.</td>
<td>Spline</td>
<td>S.×D.</td>
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<td>0±1</td>
<td>0±3</td>
<td>0±3</td>
<td>0±4</td>
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<tr>
<td>Arrington 07</td>
<td>878</td>
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<td>3±3</td>
<td>−3±3</td>
<td>−2±3</td>
<td>−1±4</td>
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<td>Arr. 03 (P)</td>
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<td>1±3</td>
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<td>FW</td>
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<td>−1±3</td>
<td>−1±3</td>
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Form Factors of the Proton
Form Factors of the Proton
The graph shows the ratio of the magnetic form factor $\mu_p G_M/G_{std. dipole}$ as a function of $Q^2 = [(GeV/c)^2]$. The data points represent various experimental results: Price et al., Berger et al., Hanson et al., Borkowski et al., and Janssens et al., along with their associated uncertainties. The lines represent different fits: spline fit, statistical errors, experimental systematic errors, theoretical systematic errors, and a spline fit with error bands. The graph is labeled 'Spline fit + error band'.
The cryogenic target system

Form Factors of the Proton
## Model dependence: Magnetic Radius

### Analysis of simulated pseudo data

<table>
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<tr>
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<th>P.+D.</th>
<th>P.×D.</th>
<th>Spline</th>
<th>S.×D.</th>
<th>F./W.</th>
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<td>0±1</td>
<td>−1±7</td>
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<td>Arrington 07</td>
<td></td>
<td>−55±1</td>
<td>4±4</td>
<td>−5±6</td>
<td>−4±6</td>
<td>−1±9</td>
<td>2±13</td>
<td>0±17</td>
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*Arrington 03 (P)*

### Form Factors of the Proton