SUPERSYMMETRY AND QCD

Michael Klasen
LPSC Grenoble
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- Why SUSY? What is SUSY? How is SUSY broken?
- The Feynman rules of SUSY-QCD
- QCD effects in SUSY and vice versa
- SUSY particle production at hadron, $e^+e^-$, and $\gamma\gamma$ colliders
- Virtual loop diagrams/calculations
- Real emission diagrams/calculations
- SUSY effects in $\alpha_s$, PDFs, and FFs
- Summary
Why Supersymmetry?

- The Standard Model is successful, but it has many deficiencies:
  - Gravity
  - Hierarchy of $m_h \ll m_{P1}$
  - Electroweak symmetry breaking
  - Unification of the coupling constants
  - Cold dark matter in the universe

- Supersymmetry is a theoretically attractive extension:
  - SUSY is the only non-trivial extension of the Poincaré group
  - SUSY unifies fermions and bosons, matter and forces
  - SUSY as a local symmetry includes gravity [= supergravity]
  - SUSY appears naturally in string theories
  - SUSY stabilizes the mass of the Higgs boson
  - SUSY can break the electroweak symmetry radiatively
  - SUSY can explain the unification of couplings and $\sin^2 \theta_W$

- Minimal Supersymmetric Standard Model (MSSM):
  - N=1 SUSY generators: One superpartner for each SM particle
  - Two Higgs doublets to give mass to up- and down-type quarks
  - Strongly interacting gluino: $\tilde{g}$, squarks: $\tilde{q}_{L,R}, \tilde{t}_{1,2}, \tilde{b}_{1,2}$
  - Weakly interacting gauginos: $\tilde{\chi}^0_{1-4}, \tilde{\chi}^\pm_{1,2}$, sleptons: $\tilde{l}_{L,R}, \tilde{\nu}_L$
  - Renormalizability, $B - L$ conservation $\rightarrow R$-parity conserved
  - SUSY particles must be produced in pairs, LSP is stable
Only non-trivial extension of the Poincaré group

Generated by an operator $Q$ and $Q^\dagger$ [= anticommuting spinors]:

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0 \quad ; \quad \{Q, Q^\dagger\} = P^\mu$$

Transforms as a Lorentz vector $\rightarrow$ SUSY = space-time symmetry

Chiral fermions: $Q|\phi\rangle = |\psi\rangle$ ; $Q|\psi\rangle = |\phi\rangle$

$m = 0 : \psi=$fermion(2), $\phi=$comp.scalar(2)

$m \neq 0 : \psi=$fermion(4), $\phi=$comp.scalar(2), $F=$aux.comp.scalar(2)

Gauge bosons: $Q|A\rangle = |\lambda\rangle$ ; $Q|\lambda\rangle = |A\rangle$

$m = 0 : A=$vector boson(2), $\lambda=$fermion(2)

$m \neq 0 : A=$vector boson(3), $\lambda=$fermion(4), $D=$aux.real scalar(1)

General SUSY Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{SUSY-gauge}}$$

$$\mathcal{L}_{\text{chiral}} = -(D^\mu \phi_i^*)(D_{\mu} \phi_i) - \bar{\psi}_i \gamma^\mu \psi_i + F_i^* F_i$$

$$- \frac{1}{2} W_{ij} \psi_i \psi_j + W^i F_i + (c.c.)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu
u}^a F^{\mu
u}_a - \lambda^a \gamma^\mu \lambda\lambda + \frac{1}{2} D^a D_a$$

$$\mathcal{L}_{\text{SUSY-gauge}} = g_a (\phi^* T^a \phi) D_a - \sqrt{2} g_a [(\phi^* T^a \psi) \lambda_a + \lambda^a (\psi^* T^a \phi)]$$

Superpotential: $W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$

$M^{ij} = \text{Fermion mass matrix}$

$y^{ijk} = \text{Yukawa interactions}$

$W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$

$W^i = \frac{\partial W}{\partial \phi_i} = - F^* i$ [eq. of motion]

$D^a = - g_a (\phi^* T^a \phi)$ [eq. of motion]
How is Supersymmetry Broken?

- No SUSY particles observed → SUSY masses, beyond exp. reach
- Soft SUSY breaking Lagrangian in the MSSM:

\[
\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} \right) 
- \bar{Q}^\dagger m_Q^2 \tilde{Q} - \bar{L}^\dagger m_L^2 \tilde{L} - \bar{u} m_u^2 \tilde{u} + \tilde{d} m_d^2 \tilde{d} - \bar{\tilde{e}} m_\tilde{e}^2 \tilde{\tilde{e}} 
- \left( \bar{u} a_u \tilde{Q} H_u - \bar{d} a_d \tilde{Q} H_d - \bar{\tilde{e}} a_e \tilde{L} H_d \right) 
- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - b H_u H_d + (c.c.)
\]

- \( M_{3,2,1} \) = gluino, wino, bino masses [with complex phases]
- \( m_{Q,L,\ldots}^2 \) = squark and slepton masses [3 \( \times \) 3 matrices]
- \( a_{u,d,e} \) = trilinear couplings [complex 3 \( \times \) 3 matrices]
- Only scalars and gauginos get mass, not their superpartners
- These masses do not reintroduce quadratic divergences
- MSSM has 124 (105 SUSY + 19 SM) free parameters!

- Low-energy (\( m_Z \)) constraints:
  - Conservation of \( L_e, L_\mu, L_\tau \), and \( C P \), no FCNC, EDM
  - Generation universality, diagonal mass matrices

- High-energy (\( m_{P_{1.}} \)) constraints:
  - Depend on different SUSY breaking models
  - Parameters must be evolved down to \( M_Z \) with RGE’s
  - Gaugino mass relation:

\[
\frac{M_1(Q)}{\alpha_1(Q)} = \frac{M_2(Q)}{\alpha_2(Q)} = \frac{M_3(Q)}{\alpha_3(Q)} = \frac{m_{1/2}(M_X)}{\alpha_{\text{GUT}}(M_X)}
\]

- Radiative electroweak symmetry breaking
How is Supersymmetry Broken?

- Spontaneous breaking: $Q|0\rangle \neq 0; Q^\dagger |0\rangle \neq 0; \langle 0|H|0\rangle \sim \langle 0|V|0\rangle \neq 0$

- Scalar potential: $V = F_i^* F^i + \frac{1}{2} D_\alpha D^\alpha = W_i^* W^i + \frac{1}{2} g^2_\alpha (\phi^* T^\alpha \phi)^2$

- Fayet-Iliopoulos mechanism: $V = \frac{1}{2} D^2 - \kappa D + g D q_i \phi^*_i \phi^i$

- O’Raifeartaigh mechanism: $V = F_i^* F^i$

- Gravity-mediated models: $\mathcal{L} = \frac{-F_X}{m_{Pl}} \frac{f_\alpha}{2} \lambda_\alpha \lambda^{\alpha} - \frac{F_X F^*_X}{m^2_{Pl}} k^i_j \phi^*_i \phi^j + \ldots$
  
  $m_{1/2} = f \frac{\langle F_X \rangle}{m_{Pl}}, m_0 = \sqrt{\kappa} \frac{\langle F_X \rangle}{m_{Pl}}, A_0 = \alpha \frac{\langle F_X \rangle}{m_{Pl}}, B_0 = \beta \frac{\langle F_X \rangle}{m_{Pl}}, \text{sgn}(\mu)$

  - Auxiliary chiral field $F_X$ from non-renormalizable SUGRA

- Gauge-mediated models: Ordinary gauge interactions

  $M_i = \frac{\alpha_i}{4\pi} \Lambda, \quad m^2_\phi = 2\Lambda^2 \left(\frac{\alpha_i}{4\pi}\right)^2 C_i$

  - Auxiliary chiral field $S$ and chiral messenger fields

  - Typically one messenger generation in SU(5)

  - Messenger scale: $\Lambda \in [40; 150]$ TeV

- Anomaly-mediated models: [Giudice, Rattazzi; Randall, Sundrum]

  $M_i = \frac{b_i \alpha_i}{4\pi} m_{3/2}$

  - Gravity supermultiplet $\rightarrow$ Super-Weyl-Anomaly

  - Gravitino mass $m_{3/2} \in [30; 60]$ TeV
Low energy SUSY particle masses

- Universal boundary conditions at high energies ($m_{P_1}$)
- Renormalization group equations predict physical masses ($m_Z$)
- Loop corrections to masses, couplings [BPMZ, NPB 491 (1997) 3]
- Programs: SUSPECT, SOFTSUSY, SUSYGEN; ISAJET, SPYTHIA
- Mass Spectrum in a Typical SUGRA Scenario:

![Mass Spectrum Diagram]

- $m_{1/2}$
- $m_0$
- $M_1$
- $M_2$
- $M_3$
- $\tilde{g}$
- $\tilde{q}$
- $\tilde{b}_1$
- $\tilde{\tau}_1$
- $\tilde{\chi}^0_1$
- $\tilde{\chi}^+_1$
- $\tilde{\chi}^0_2$
- $\tilde{\chi}^+_2$
- $\tilde{\chi}^0_3$
- $\tilde{\chi}^+_3$
- $\mu$
- $A_0$
- $\tan \beta = 4$
- $\text{sgn } \mu = +$

GUT Scale | Weak Scale | Neutralinos | Charginos | Squarks/Gluinos
THE FEYNMAN RULES OF SUSY-QCD

- Standard references:
  - H.E. Haber, G.L. Kane, Phys. Rep. 117 (1985) 75

- More recent compilations:

- Treatment of Majorana fermions (such as gluinos):
  - Avoid explicit charge conjugation matrices
  - Fix reference order for spinors, fermion flow for fermion chains
  - Multiply with permutation parity of the spinors

\[ iS(p) \]
\[ iS(-p) \]
\[ iS(p) \]

Figure 2.2: The Feynman rules for fermion propagators with orientation (thin arrows). The momentum \( p \) flows from left to right.

\[ \overline{\tau}(p, s) \]
\[ \nu(p, s) \]
\[ \nu(p, s) \]
\[ \overline{\tau}(p, s) \]
QCD EFFECTS IN SUPERSYMMETRY

- **Standard Model particle decays:**
  - $t \rightarrow \tilde{t}\tilde{\chi}$ [Mrenna, Yuan, PLB 367 (1996) 188]

- **Standard Model particle production:**
  - $p\bar{p} \rightarrow b\bar{b}$: Light $\tilde{b}$, $\tilde{g}$ [Berger et al., PRL 86 (2001) 4231]
  - $p\bar{p} \rightarrow t\bar{t}$ [Alam et al., PRD 55 (1997) 1307; Sullivan, PRD 56 (1997) 451]

- **SUSY particle decays (LEP, TESLA, Tevatron, LHC searches):**
  - $\tilde{g} \rightarrow g\tilde{\chi}$ (1-loop) [Baer, Tata, Woodside, PRD 42 (1990) 1568]
  - $\tilde{\chi} \rightarrow q\bar{q}$ [Berge, Klasen, to be published]
  - $\tilde{q} \rightarrow q\tilde{\chi}, \tilde{q}W/Z/H$ [Bartl et al., PLB 386 (1996) 175; 419 (1998) 243; PRD 59 (1999) 115007]
  - $\tilde{q} \rightarrow q\tilde{g}, \tilde{g} \rightarrow q\bar{q}$ [Beenakker et al., PLB 378 (1996) 159; ZPC 75 (1997) 349]
  - $H \rightarrow q\bar{q}', \tilde{q}\tilde{q}'$ [Bartl et al., PLB 373 (1996) 117; 378 (1996) 167; 402 (1997) 303]

- **SUSY particle production (LEP, TESLA, Tevatron, LHC searches):**
  - $e^+ e^- \rightarrow q\bar{q}$ [Bartl, Eberl, Majerotto, NPB 472 (1996) 481]
  - $e^+ e^- \rightarrow \tilde{g}\tilde{g}$ (1-loop) [Kileng, Osland, ZPC 66 (1995) 503; Berge, Klasen, to be published]
  - $p\bar{p} \rightarrow \tilde{\chi}\tilde{\chi}, \tilde{\ell}\bar{\ell}$ [Baer, Harris, Reno, PRD 57 (1998) 5871; Beenakker et al., PRL 83 (1999) 3780]
  - $p\bar{p} \rightarrow \tilde{q}\tilde{q}, \tilde{g}\tilde{g}, \tilde{\chi}\tilde{q}, \tilde{\chi}\tilde{g}$ [Berger, Klasen, Tait, PLB 459 (1999) 165; PRD 62 (2000) 095014; Beenakker et al., NPB 492 (1997) 51; 515 (1998) 3]

- **SUSY particle scattering ($\rightarrow$ dark matter searches, cosmic rays):**
  - $\tilde{\chi} N \rightarrow \tilde{\chi} X$ [Djouadi, Drees, PLB 484 (2000) 183]
  - $\tilde{q}, \tilde{g}$ parton densities [Kounnas et al., NPB 211 (1983) 216; 214(1983)317; Corianò, 627(2002)66]
  - $\tilde{q}, \tilde{g}$ fragmentation functions [Corianò, Faraggi, PRD 65 (2002) 075001]
- QCD without fermions (pure Yang-Mills theory):
  - Assume fermions are gluinos, so QCD → SUSY-QCD
  - Useful for multi-gluon scattering amplitudes

- Supersymmetric Ward identities:
  - In exact SUSY $Q|0\rangle = 0 \rightarrow [Q, \phi_i] = 0$ in helicity amplitudes
  - Useful relations for helicity amplitudes:
    \[
    A_n^{SUSY}(1^\pm, 2^+, 3^+, \ldots, n^+) = 0
    \]
    \[
    A_n^{SUSY}(1^-, 2^-, 3^+, 4^+, \ldots, n^+) = \left( \frac{\langle 12 \rangle}{\langle 13 \rangle} \right)^2 |h_P| \times A_n^{SUSY}(1^-, 2^-, \phi^+, \ldots, n^+)
    \]
  - $h_P$ = helicity $(0, \frac{1}{2}, 1), \langle j|l \rangle = \bar{u}_-(k_j)u_+(k_l)$

- One-loop amplitudes via unitarity
  - Absorptive parts of loop amplitudes: integrate lower amplitudes
  - Simplify tree amplitudes before integration
  - Tree amplitudes possess “effective” supersymmetry
  - On-shell conditions for intermediate particles
  - Polynomial ambiguities only for masses, not for massless QCD

- Application: $gg \rightarrow gg$ at two loops

- Cross sections depend only on physical squark and gluino masses
- Mixing is important for \( \tilde{t} \) – (a2) and (c) do not occur \( [f_{t/P} = 0] \)
- Off-diagonal squark pairs can be produced from quark pairs (c)
- Squarks can be produced in association with gluinos (f)
- LO: Dawson, Eichten, Quigg, PRD 31 (1985) 1581
- NLO: Beenakker et al., NPB 492 (1997) 51; 515 (1998) 3
- Gauginos and sleptons:

- Neutralino production: $Z$-exchange in s-channel
- Chargino production: also $\gamma$-exchange in s-channel
- Slepton production: only $s$-channel, $\gamma$- and $Z$-exchange, like Drell-Yan
- Associated production of $\tilde{g}\tilde{X}$: only $t$- and $u$-channel, $\tilde{q}_{L,R}$-exchange
- LO: Dawson, Eichten, Quigg, PRD 31 (1985) 1581
  Baer, Karatas, Tata, PRD 42 (1990) 2259
  
  $$ \frac{d\sigma}{dt}(q\bar{q} \to \tilde{\chi}_i\tilde{\chi}_j) = \frac{\pi}{s^2} \frac{N_C}{4N_C^2} \left[ A_s [(t - m_i^2)(t - m_j^2) + (u - m_i^2)(u - m_j^2)] + 2A'_s m_i m_j s \right] $$
  $$ + A_t \frac{(t - m_i^2)(t - m_j^2)}{(t - \frac{m^2}{q})^2} + A_u \frac{(u - m_i^2)(u - m_j^2)}{(u - \frac{m^2}{q})^2} $$
  $$ + A_{st} (t - m_i^2)(t - m_j^2) + A'_{st} m_i m_j s $$
  $$ + \frac{A_{su}(u - m_i^2)(u - m_j^2) + A'_{su} m_i m_j s}{s(u - \frac{m^2}{q})} + A_{tu} \frac{m_i m_j s}{(t - \frac{m^2}{q})(u - \frac{m^2}{q})} $$

- $s, t, u$ are partonic Mandelstam variables, $m_i$ are physical masses,
  $A_s, A_t, A_u, A_{st}, A_{su}, A_{tu}$ contain electroweak/strong couplings

- NLO: Baer, Harris, Reno, PRD 57 (1998) 5871;
  Beenakker et al., PRL 83 (1999) 3780;
• Self-energy corrections (factorize the LO cross section):

\[ q = q + g = q + g + q \]

\[ \bar{q} = \bar{q} + g = \bar{q} + g + \bar{q} \]

• Vertex corrections (factorize the LO amplitude):

\[ q \bar{q} q = q \bar{q} q + q \bar{q} q + q \bar{q} q + q \bar{q} q \]

\[ q \bar{q} \chi = q \bar{q} \chi + q \bar{q} \chi \]

• Box diagrams (factorize the LO amplitude):

Boxes:

\[ = \]

• Additional Feynman rules:
  – Colored parts of LO diagrams: SM/SUSY particle exchanges
  – Factor (-1) for loop diagrams with a closed fermion line
  – Factor 1/2 for loop diagrams with identical particles
  – Need interference of loop and LO diagrams → only real part
VIRTUAL LOOP CALCULATIONS

- 1- to 4-point tensor loop integrals (loop four-momentum $l$):

\[ A_0 = (2\pi\mu)^{4-n} \int \frac{d^n l}{i\pi^2} \frac{1}{D_1}, \]
\[ B_{0,\mu,\mu\nu} = (2\pi\mu)^{4-n} \int \frac{d^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu\}}{D_1 D_2}, \]
\[ C_{0,\mu,\mu\nu,\mu\nu\rho} = (2\pi\mu)^{4-n} \int \frac{d^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu, l_\mu l_\nu l_\rho\}}{D_1 D_2 D_3}, \]
\[ D_{0,\mu,\mu\nu,\mu\nu\rho} = (2\pi\mu)^{4-n} \int \frac{d^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu, l_\mu l_\nu l_\rho\}}{D_1 D_2 D_3 D_4}. \]

- Denominators:

\[ D_1 = l^2 - m_1^2 + i\eta, \]
\[ D_2 = (l + p_1)^2 - m_2^2 + i\eta, \]
\[ D_3 = (l + p_1 + p_2)^2 - m_3^2 + i\eta, \]
\[ D_4 = (l + p_1 + p_2 + p_3)^2 - m_4^2 + i\eta. \]

- Variables:

\[ p_1, \ldots, p_3 = \text{external particle momenta} \]
\[ m_1, \ldots, m_4 = \text{internal particle masses} \]

- Reduction to scalar integrals
  
  - Based on Lorentz invariance
  
  - UV divergences: $|l| \to \infty$ in $A_0, B_0$
  
  - IR divergences: $|l| \to 0$ and coll. splittings in $B_0, C_0, D_0$

- Numerical evaluation of tensor integrals
VIRTUAL LOOP CALCULATIONS

- **Dimensional regularization:**
  - Dirac traces and loop integrals in $n$ dimensions
  - $\gamma_5$ anti-commutes in 4 dimensions, commutes in $n - 4$
  - Breaks SUSY: $g$ has n-2 degrees of freedom, but $\tilde{g}$ has 2

- **Dimensional reduction:**
  - Dirac traces in 4 dimensions, loop integrals in $n$ dimensions
  - $\gamma_5$ anti-commutes in all (4) dimensions
  - Manifestly supersymmetric: $g$ and $\tilde{g}$ have 2 degrees of freedom

- **Evaluation of scalar integrals:**
  - Feynman parameters
    [t Hooft, Veltman, NPB 153 (1979) 365]
  - Cutkosky cutting, dispersion integral
    [t Hooft, Veltman, New York, NY, 1973]
  - Analytical continuation of logarithms $\rightarrow$ large $\pi^2$ terms

- **Renormalization:**
  - Heavy particle masses: on-shell scheme
  - Couplings: $\overline{\text{MS}}$ scheme
  - Finite renormalization to restore supersymmetry in $\overline{\text{MS}}$

$$\tilde{g} = g \left[ 1 + \frac{g^2}{32 \pi^2} \left( \frac{4}{3} N_C - C_F \right) \right]$$

[Martin, Vaughn, PLB 318 (1993) 331]
- **Gluons:**

- **Quarks / Antiquarks:**
REAL EMISSION CALCULATIONS

- **Phase space slicing method (heavy quarks):** [Beenakker et al., PRD 40 (1989) 54]
  - Simplification of matrix elements in soft/collinear limit
  - Analytical integration up to cut-off $\Delta$
  - Numerical integration above cut-off $\Delta$
  - Numerical cancellation of cut-off dependence

- **Subtraction method (massless QCD):** [Catani et al., NPB 485 (1997) 291; 627 (2002) 189]
  - Construct counter terms form dipole form of parton splittings
  - Add integrated counter terms to virtual corrections
  - Subtract unintegrated counter terms from real corrections
  - Point-by-point cancellation of singularities
REAL EMISSION CALCULATIONS

- Treatment of IR singularities:
  - KLN-cancellation between real and virtual corrections
  - Factorization of collinear divergences into $\overline{\text{MS}}$ parton densities
  - On-shell particle decays (intermediate squarks):

  ![Diagram of parton decays](image)

  * Assoc. production $p\bar{p} \rightarrow \tilde{q}\tilde{\chi}$, subsequent decay $\tilde{q} \rightarrow q\tilde{\chi}$
  * To avoid double counting, one must subtract

  $$\frac{d\sigma}{dM^2} = \sigma(gq \rightarrow \tilde{q}\tilde{\chi}_i) \text{BR}(\tilde{q} \rightarrow q\tilde{\chi}_j) \frac{m_q \Gamma_{\tilde{q}}/\pi}{(M^2 - m_{\tilde{q}}^2)^2 + m_q^2 \Gamma_{\tilde{q}}^2}.$$

- Implementation in flexible Monte Carlo (FORTRAN,C++) programs:
  - Partonic scaling functions: $\hat{\sigma}_{ij}(\hat{s})$
  - Total hadronic cross sections: $\sigma(m^2)$
  - $K$-factors: $\sigma^{\text{NLO}} / \sigma^{\text{LO}}$
  - Distributions: $d\sigma/dE_T, d\sigma/d\eta$

- Implementation in event generators (ISAJET, SPYTHIA, HERWIG):
  - SUSY mass spectra, LO scattering matrix elements, $K$-factors
  - Parton showering, (s)particle decays, hadronization
  - Detector simulation
- Tevatron:

\[ \sigma_{\text{tot}}[\text{pb}]: \text{pp} \rightarrow \bar{g}g, \bar{q}q, t\bar{t}_1, \tilde{\chi}_2^{0}\tilde{\chi}_1^{+}, \tilde{\chi}_L\tilde{\chi}_L \]

- LHC:

\[ \sigma_{\text{tot}}[\text{pb}]: \text{pp} \rightarrow \bar{g}g, \bar{q}q, t\bar{t}_1, \tilde{\chi}_2^{0}\tilde{\chi}_1^{+}, \nu\bar{\nu}, \tilde{\chi}_2^{0}\tilde{g} \]
• $e^+e^- \rightarrow \tilde{g}\tilde{g}$:

$g_\gamma/Z^0\tilde{g}\tilde{g} = 0 \rightarrow$ 1-loop, UV-finite ($C$-functions), IR-finite ($m_\tilde{q} \neq 0$)

- Majorana fermions $\rightarrow P$ axial vector coupling
- Photon exchange cancels for $m_{\tilde{u}}_L = m_{\tilde{u}}_R; m_{\tilde{d}}_L = m_{\tilde{d}}_R$
- $Z^0$ exchange cancels for $m_w = m_d; m_{\tilde{u}}_L = m_{\tilde{u}}_R = m_{\tilde{d}}_L = m_{\tilde{d}}_R$
- $m_t \gg m_b, m_{\tilde{t}}_2 \gg m_{\tilde{t}}_1 \rightarrow$ largest contribution

• $\gamma\gamma \rightarrow \tilde{g}\tilde{g}$:

$g_\gamma\tilde{g}\tilde{g} = 0 \rightarrow$ 1-loop, UV-finite ($C, D$-functions), IR-finite ($m_\tilde{q} \neq 0$)

- Depends on physical ($m_\tilde{q}, m_\tilde{u}, m_\tilde{d}$), ($e_\tilde{q}, e_\tilde{d}$)
- No cancellations, single squark exchange dominates
• $e^+e^- \rightarrow \tilde{g}\tilde{g}$:

$e^-e^+ (P-80+60) \rightarrow \tilde{g}\tilde{g}$ Annihilation

- $m_{\tilde{g}} = 200$ GeV
- $m_{\tilde{g}} = 300$ GeV
- $m_{\tilde{g}} = 500$ GeV

$M_S = 325$ GeV

• $\gamma\gamma \rightarrow \tilde{g}\tilde{g}$:

$\gamma\gamma \rightarrow \tilde{g}\tilde{g}$ with Laserbackscattering

- $M_S = 250$ GeV
- $M_S = 500$ GeV
- $M_S = 700$ GeV

$M_S = 300$ GeV

$M_S = 500$ GeV

$M_S = 700$ GeV

$M_S = 800$ GeV
• Usually heavy particles ($t, \bar{q}, \bar{g}$) are decoupled, since $m^2 \gg Q^2$

• Strong coupling constant: [Antoniadis et al., PLB 262 (1991) 109; Jezabek, Kühn, 301 (1993) 121]
  [Machacek, Vaughn, NPB 222 (1983) 83]

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0} \frac{1}{\ln(Q^2/\Lambda^2)} \left( 1 - \frac{\beta_1}{\beta_0} \frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} + O\left( \frac{1}{\ln^2(Q^2/\Lambda^2)} \right) \right)$$

$$\beta_0^{\text{SM}} = \frac{11}{3} C_A - \frac{4}{3} T_R n_f, \quad \beta_1^{\text{SM}} = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_R n_f - 4 C_F T_R n_f$$

$$\beta_0^{\text{SUSY}} = \beta_0^{\text{SM}} - \frac{2}{3} C_A n_g - \frac{2}{3} T_R n_{\bar{q}}, \quad \beta_1^{\text{SUSY}} = \beta_1^{\text{SM}} - 16 C_A n_g - \frac{4}{3} C_A T_R n_{\bar{q}} - 8 C_F T_R n_{\bar{q}}$$

• Non-singlet evolution equations: [Kounnas, Ross, NPB 214 (1983) 317]

$$Q^2 \frac{d}{dQ^2} q_V(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \left( P_{qq} \otimes q_V + P_{q\bar{q}} \otimes \bar{q}_V \right)$$

$$Q^2 \frac{d}{dQ^2} \bar{q}_V(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \left( P_{q\bar{q}} \otimes q_V + P_{\bar{q}q} \otimes \bar{q}_V \right),$$

• Singlet evolution equations: [Kounnas, Ross, NPB 214 (1983) 317]

$$Q^2 \frac{d}{dQ^2} \begin{bmatrix} G(x, Q^2) \\ \lambda(x, Q^2) \\ q^+(x, Q^2) \\ \bar{q}^+(x, Q^2) \end{bmatrix} = \begin{bmatrix} P_{GG} & P_{G\lambda} & P_{Gq} & P_{G\bar{q}} \\ P_{\lambda G} & P_{\lambda\lambda} & P_{\lambda q} & P_{\lambda\bar{q}} \\ P_{qG} & P_{q\lambda} & P_{qq} & P_{q\bar{q}} \\ P_{sG} & P_{s\lambda} & P_{s\bar{q}} & P_{s\bar{q}} \end{bmatrix} \otimes \begin{bmatrix} G(x, Q^2) \\ \lambda(x, Q^2) \\ q^+(x, Q^2) \\ \bar{q}^+(x, Q^2) \end{bmatrix}.$$}

• SUSY relations among splitting functions:

$$P_{gg} + P_{\lambda g} = P_{g\lambda} + P_{\lambda\lambda}, \quad P_{qq} + P_{sg} = P_{q\lambda} + P_{s\lambda}, \quad P_{gq} + P_{\lambda q} = P_{gs} + P_{\lambda s}, \quad P_{qq} + P_{sq} = P_{qs} + P_{ss}$$

• SUSY sum rules (momentum and baryon number conservation):

$$1 = \int_0^1 x \, dx \, \left( x G(x) + x \lambda(x) + x q^+(x) + x \bar{q}^+(x) \right)$$

$$3 = \int_0^1 dx \left( q^-(x) + q^-(x) \right)$$
P_{GG} = 2C_A \left[ \frac{1 + x^2}{(1 - x)_+} + \frac{1 + (1 - x)^2}{x} - (x^2 + (1 - x)^2) \right] + [3C_A - T_R] \delta(1 - x)

P_{AG} = 2C_A \left[ x^2 + (1 - x)^2 \right]

P_{qG} = 2T_R \left[ x^2 + (1 - x)^2 \right]

P_{sG} = 2T_R \left[ 1 - \left( x^2 + (1 - x)^2 \right) \right]

P_{G\lambda} = 2C_A \left[ \frac{1 + (1 - x)^2}{x} \right]

P_{\lambda \lambda} = 2C_A \left[ \frac{1 + x^2}{(1 - x)_+} \right] + (3C_A - T_R) \delta(1 - x)

P_{q\lambda} = 2T_R [1 - x]

P_{s\lambda} = 2T_R [x]

P_{Gq} = 2C_F \left[ \frac{1 + (1 - x)^2}{x} \right]

P_{\lambda q} = 2C_F (1 - x)

P_{qq} = 2C_F \left[ \frac{1 + x^2}{(1 - x)_+} \right] + 2C_F \delta(1 - x)

P_{sq} = 2C_F [x]

P_{Gs} = 2C_F \left[ \frac{1 + (1 - x)^2}{x} - x \right]

P_{\lambda s} = 2C_F [1]

P_{qs} = 2C_F [1]

P_{ss} = 2C_F \left[ \frac{1 + x^2}{(1 - x)_+} - (1 - x) \right] + 2C_F \delta(1 - x)
- $m_{\tilde{g}} = 30 \text{ GeV}, Q = 100 \text{ GeV}$

- $m_{\tilde{g}} = 100 \text{ GeV}, Q = 10^5 \text{ GeV}$
SUMMARY

What is SUSY and why is it interesting?

- Unifies fermions and bosons, matter and forces, couplings
- Can include gravity, appears in string theories
- Stabilizes Higgs mass, can break electroweak symmetry
- MSSM has one superpartner for each SM particle, 2HDM
- SUSY-breaking introduces soft masses (and phases)

The Feynman rules of SUSY-QCD

- Fermion direction for Majorana fermions
- Yukawa couplings contain $\gamma_5$
- Dimensional regularization vs. dimensional reduction
- Treatment of intermediate unstable particles

QCD effects in SUSY and vice versa

- SUSY Ward identities for QCD helicity amplitudes
- SUSY effects in SM (bottom) production and decay
- SUSY particle production at hadron, $e^+e^-$, and $\gamma\gamma$ colliders
- Higher order SUSY-QCD corrections
- SUSY effects in $\alpha_s$, PDFs, and FFs