Introduction to Loop Quantum Cosmology

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$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R$$

Einstein-Hilbert action

$$\downarrow$$

Reformulation to Ashtecar variabels

$$H_G = \frac{1}{16\pi G}\int d^3x\left(NC + N^a C_a + \lambda^i G_i\right)$$

Grav. Hamiltoinan in Ashtekar variables

$$\downarrow$$

Symetry reduce

$$H_G = -N\frac{3}{8\pi G\,\gamma^2}\,c^2\sqrt{p}$$

Homogenius and isotropic Grav. Hamiltonian

$$\downarrow$$

Holonomy correction

$$H_G = -N\frac{3\,p^{3/2}}{8\pi G\,\gamma^2\Delta}\sin^2\left(\sqrt{\frac{\Delta}{p}}\;c\right)$$

LQC effective Grav. Hamiltonian

$$\downarrow$$

Reformulation to observable variables

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1-\frac{\rho}{\rho_c}\right)$$

Modified Friedman equation

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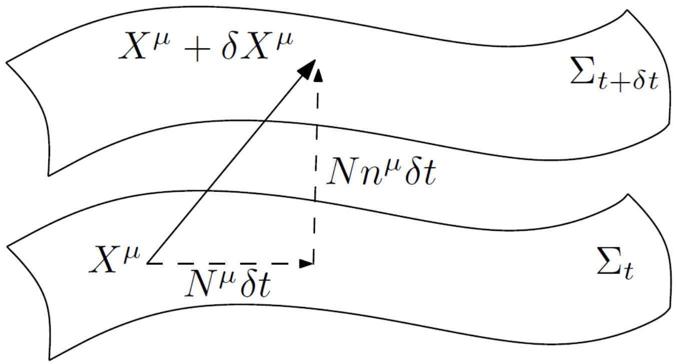
ADM formalism: $ds^2 = (-N^2 + N_a N^a)dt^2 + 2\, N_a\, dx^a\, dt + q_{ab} dx^a dx^b$

Ashtecar variables:

$$E^a{}_i\, E^b{}_j\, \delta^{ij} = \det(q)\, q^{ab}$$

$$A_a{}^i = \epsilon^{ijk}\Gamma_{ajk} + \gamma K_a{}^i$$



$$H_G = \frac{1}{16\pi G}\int d^3x\big(NC(A,E) + N^a C_a(A,E) + \lambda^i G_i(A,E)\big)$$

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$$ds^2 = (-N^2 + N_a N^a)dt^2 + 2\, N_a\, dx^a\, dt + q_{ab} dx^a dx^b$$

$$ds^2 = -N^2\, dt^2 + a(t)\delta_{ab} dx^a dx^b$$

$$N_a = 0 \qquad A_a{}^i = c\, \delta_a{}^i \qquad E^a{}_i = p\, \delta^a{}_i$$

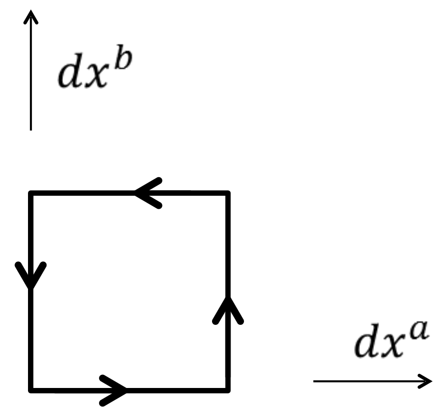
$$H_G = -N \frac{3}{8\pi G\, \gamma^2}\, c^2 \sqrt{p} \qquad p = a^2 \qquad c = \dot{a}$$

Holonomy correction

Holonomy: $h_l = e^{-i\int_l dx\, A}$

$\cancel{c^2}$ NO!

$e^{\pm i\lambda c}$, $\sin(\lambda c)$, $\cos(\lambda c)$ OK



$$c^2 \leftarrow F_{ab}{}^{i}(A) = -2 \lim_{\boxdot \to 0} Tr\left[\frac{h_{\boxdot ab} - 1}{Area_{\boxdot}} \tau^i\right]$$

$$c^2 \to \frac{p}{\Delta} \sin^2\left(\sqrt{\frac{\Delta}{p}}\; c\right) \quad \Rightarrow \quad H_G = -N \frac{3\, p^{3/2}}{8\pi G\, \gamma^2 \Delta} \sin^2\left(\sqrt{\frac{\Delta}{p}}\; c\right)$$

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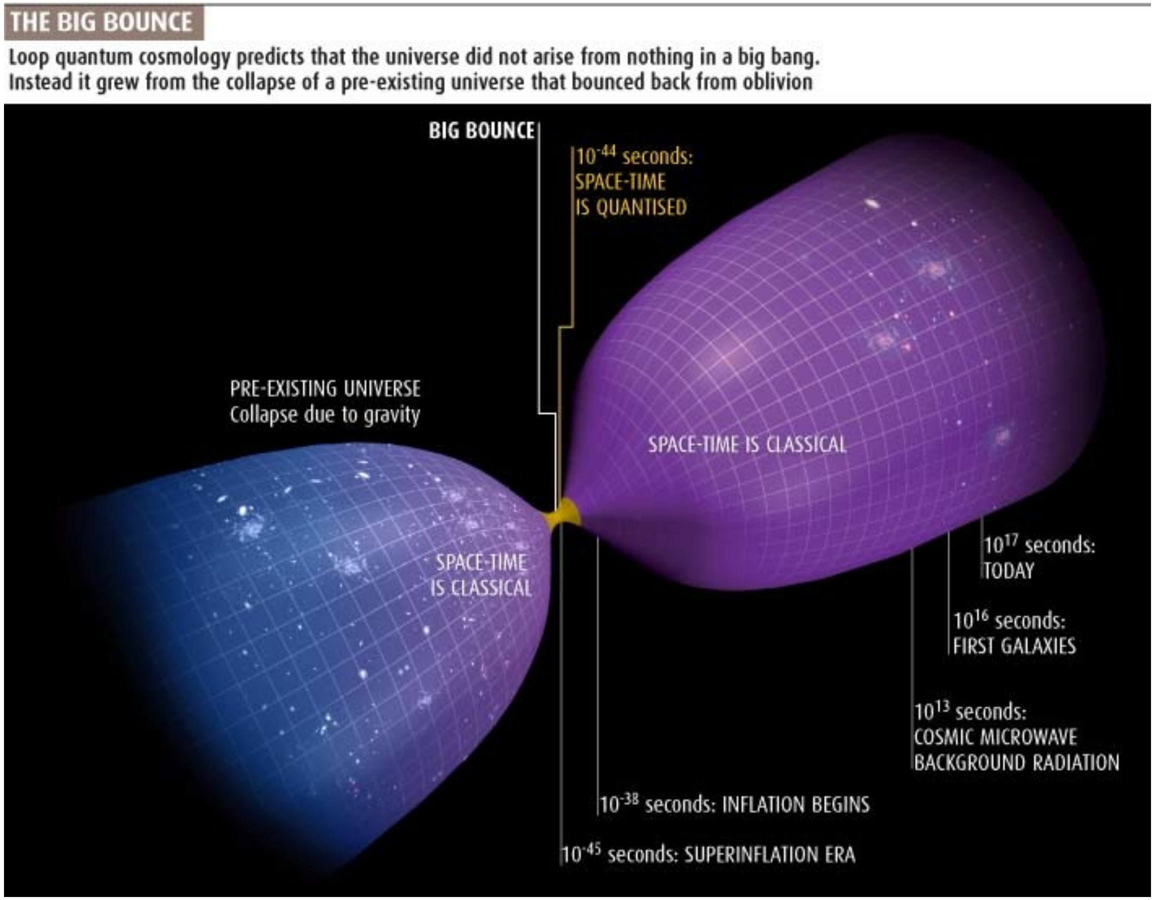
Modified Friedman equation

Reformulation to observable variables

$$H_{tot} = H_G + H_m = -N \frac{3\, p^{\frac{3}{2}}}{8\pi G\, \gamma^2 \Delta} \sin^2\left(\sqrt{\frac{\Delta}{p}}\ c\right) + N\, p^{\frac{3}{2}} \rho = N C_{tot}$$

$$C_{tot} = 0$$



Modified Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right)$$

Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho$$

Generalize to anisotropic universe

Bianchi I: $ds^2 = -N^2 dt^2 + a_1(t)dx^2 + a_2(t)dy^2 + a_3(t)dz^2$

$$A_a{}^i = c_i\,\delta_a{}^i \qquad E^a{}_i = p_i\,\delta^a{}_i$$

$$H_G = -N\frac{3\sqrt{p_1p_2p_3}}{8\pi G\,\gamma^2\Delta}\left[\sin^2\left(\sqrt{\frac{\Delta\,p_1}{p_2p_3}}\;c_1\right)\sin^2\left(\sqrt{\frac{\Delta\,p_2}{p_1p_3}}\;c_2\right)+\right.$$

$$\left.+\sin^2\left(\sqrt{\frac{\Delta\,p_2}{p_1p_3}}\;c_2\right)\sin^2\left(\sqrt{\frac{\Delta\,p_3}{p_2p_1}}\;c_3\right)+\sin^2\left(\sqrt{\frac{\Delta\,p_3}{p_2p_1}}\;c_3\right)\sin^2\left(\sqrt{\frac{\Delta\,p_1}{p_2p_3}}\;c_1\right)\right]$$

$$\left(\frac{\dot a}{a}\right)^2 = {\sigma_Q}^2 + \frac{8\pi G}{3}\rho - \gamma^2\Delta\left(\frac{3}{2}{\sigma_Q}^2 - \frac{8\pi G}{3}\rho\right)^2$$

Questions

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