Pricing dilemma in social systems

or why don't successful restaurants raise prices ?

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why successful sellers do not increase their prices ?

"... why many successful restaurants do not raise prices even with persistent excess demand ? " [Becker (1991)]

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other examples :

- bestseller books and music
- theater plays or films
- sporting events

⋮

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⋮

common feature :

- fashionable ("bandwagon") goods
  $\Rightarrow$ importance of social interactions

why successful sellers do not increase their prices ?

explanation : modeling the demand and the offer

plan

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- demand of a good by a population of interacting heterogeneous agents (customers)

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- demand of a good by a population of interacting heterogeneous agents (customers)
- optimal supply by a monopolist informed of the characteristics of the customers' population

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- pricing strategies

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- demand of a good by a population of interacting heterogeneous agents (customers)
- optimal supply by a monopolist informed of the characteristics of the customers' population
- pricing strategies
- experimental tests

the demand

Schelling, Föllmer, Granovetter, Durlauf (since 1971)
generic properties → Gordon et al. (2009)

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customers' model with social interactions

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customers' model with social interactions

- N potential customers (single good)
- P : unitary price (monopolistic pricing)
- $N\eta$: number of buyers (η : fraction of buyers)

the demand

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customers' model with social interactions

- N potential customers (single good)
- P : unitary price (monopolistic pricing)
- $N\eta$: number of buyers (η : fraction of buyers)

- idiosyncratic reservation prices H_i ($i = 1, 2, \ldots, N$)
- mutual influence with strength $J > 0$
  $\Rightarrow$ the "value" of the good for individual i increases with η

$$H_i + J\eta \Rightarrow \text{"bandwagon good"}$$

the demand

basic assumptions

the demand

basic assumptions

the reservation prices H_i are distributed $(1 \leq i \leq N)$
$\mathcal{P}(H_i) \rightarrow$ mean H, variance σ

convenient normalization

$p = \frac{P}{\sigma}$; $h_i = \frac{H_i}{\sigma}$; $j = \frac{J}{\sigma}$

$$\Rightarrow \quad h_i = h + x_i$$

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the "value" of the good for individual i in adimensional units is :

$$h + x_i + j\eta$$

with pdf $f(x_i)$ of zero mean and unit variance

the demand

basic assumptions

utility or payoff = value - price

- when buying : $u_i = (h + x_i + j\eta) - p$
- when not buying : $u_i = 0$

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agent's *i* rational decision is :

- to buy if $u_i > 0$
- not to buy if $u_i < 0$

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- not to buy if $u_i < 0$
- decision : $s_i = \text{sign}(u_i)$

$\sim$ Ising model with quenched disorder x_i (RFIM)

the demand

equilibria

underlying energy ⇒ fixed points

$$\mathcal{P} \text{ of buying} = \mathcal{P}(h + x_i + j\eta - p > 0) = \mathcal{P}(x_i > p - h - j\eta)$$

$$\eta = \int_z^\infty f(x)dx \qquad with \qquad z \equiv p - h - j\eta$$

the demand

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method

- invert $\eta(z) : z = \Gamma(\eta)$ (Γ is a monotonic decreasing function)
- define $\mathcal{D}(\eta, j) \equiv \Gamma(\eta) + j\eta$
- solve $p - h = \mathcal{D}(\eta; j)$

the demand

equilibria

underlying energy $\Rightarrow$ fixed points

- for illustration $\rightarrow$ logistic $\mathcal{P}(h)$

$$\mathcal{P}(h) = \frac{1}{1 + e^{-2\beta h}}$$

$$f(x) \propto \frac{1}{cosh^2(\beta x)}$$

the demand

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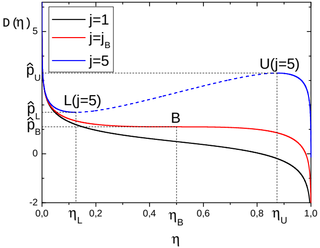
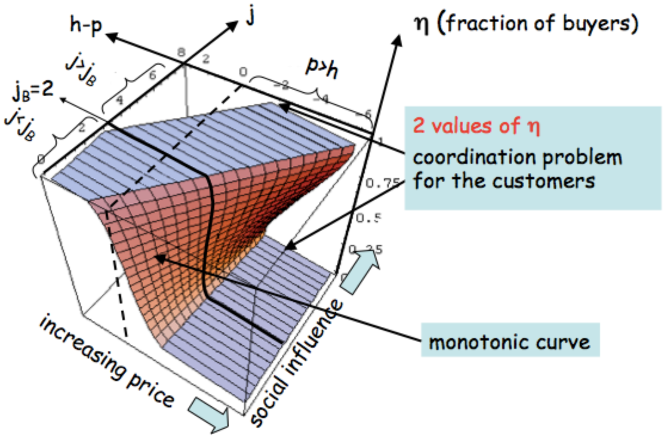


$j > j_B \Rightarrow \mathcal{D}(\eta, j)$ not monotonic

phase diagram of the demand



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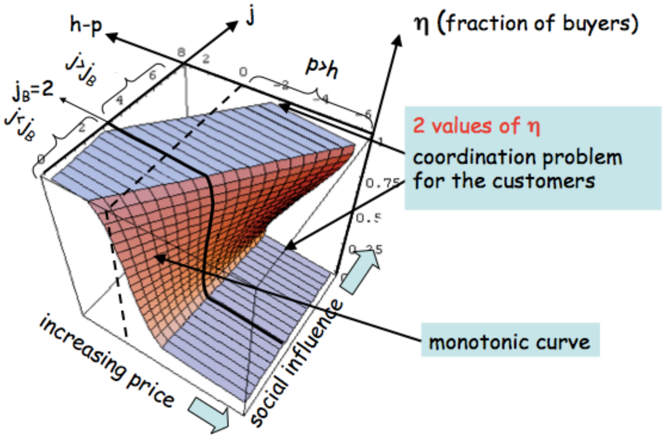
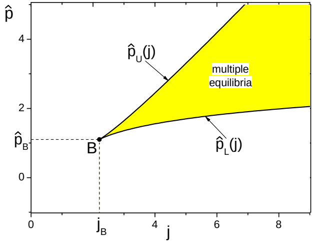


$j_B \approx 2$ for most distributions

phase diagram of the demand

$\hat{p} \equiv p - h$ vs. j



$j_B = 2.2, \quad \hat{p}_B = 1.1 > 0 \Rightarrow p_B > h_B$!

phase diagram of the demand

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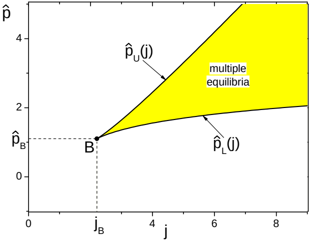
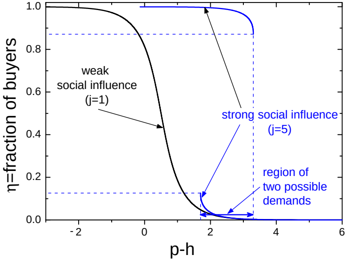


η vs. $(p - h)$



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phase diagram of the demand

generic properties (Gordon et al. 2012)

phase diagram of the demand

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- for j (social interaction) large enough ($j > j_B$)
  $\Rightarrow$ multiple demand equilibria for a given price

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- number of possible coexistent equilibria :
  $1 +$ number of modes of $f(x)$

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- number of possible coexistent equilibria :
  $1 +$ number of modes of $f(x)$
- with contrarians : no energy function (Gonçalves et al. in progress)
  - fixed points are reached through oscillations
  - if enough contrarians $\Rightarrow$ cycles

phase diagram of the demand

back to Becker (1991)

"... why many successful restaurants do not raise prices even with persistent excess demand ? "

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Becker's intuition :
social interactions ⇒ non-monotonic demand curves

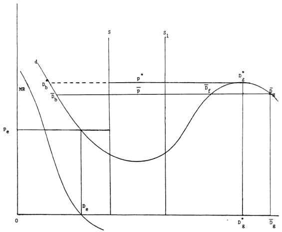

Fig. 2

phase diagram of the demand

back to Becker (1991)

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Becker's intuition :
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mathematical model :
allows to explore all the possibilities and the seller's optimal strategy

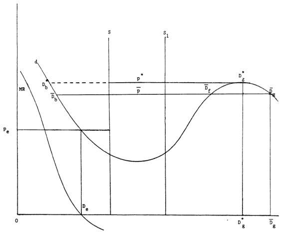
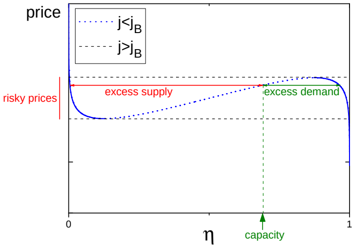


FIG. 2



the supply

optimal pricing strategies (Gordon et al. 2013)

the model

- single seller (monopole)
- profit $= N\eta\,(p - c) \equiv N\pi$
- assumption : cost=0 (origin of the monetary values)

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profit optimization $\Rightarrow$ optimal price p

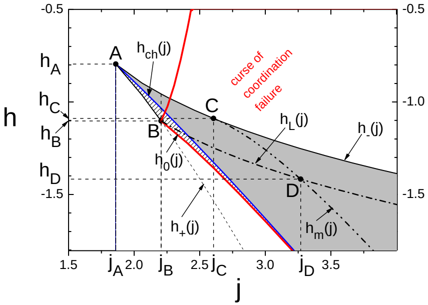
- maximize $\pi(\eta, p) \equiv p\,\eta$
  under the condition $\eta = \eta^c(p - h)$ (from customer's model)
- extremum : $\partial\pi/\partial p = 0$
- maximum : $\partial^2\pi/\partial p^2 < 0$

optimal supply

phase diagram at optimal price p

optimal supply

phase diagram at optimal price *p*



optimal supply

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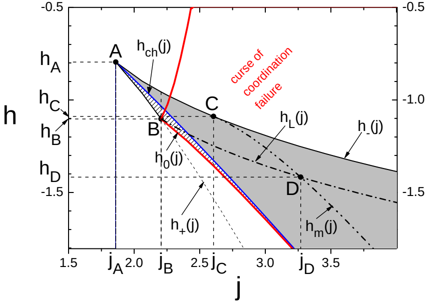


two relative max of Π

between $h_-(j)$ and $h_+(j)$

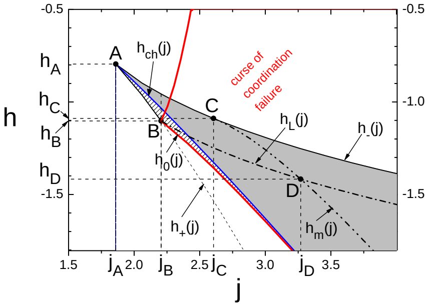
- strategy change at $h = h_{ch}(j)$

$h \leq h_{ch}(j)$ low-η, high p
risk dominant

$h \geq h_{ch}(j)$ high-η, low p
Pareto optimal

optimal supply

phase diagram at optimal price p



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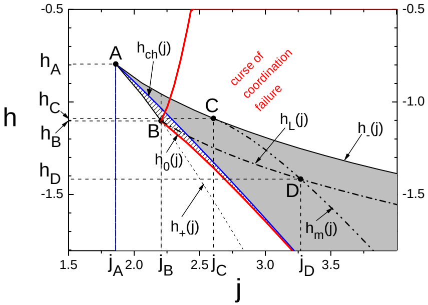
boundaries

- $h_0(j)$: boundary of high-η maximum
  $p < 0$ if taget is high-η and $h < h_0(j)$

- $h_L(j)$ and $h_m(j)$: lines where low-η
  maxima change characteristics

optimal supply

phase diagram at optimal price p

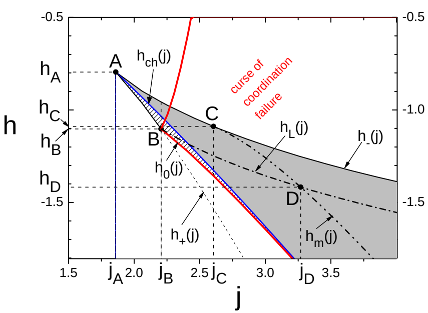


$j < j_A$

- unique optimum

optimal supply

phase diagram at optimal price p



$j < j_A$

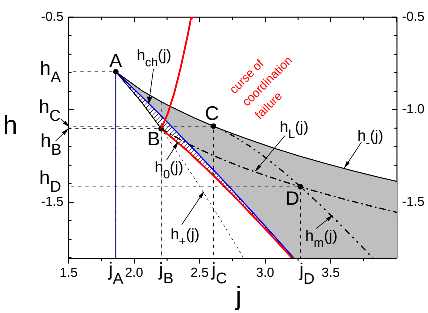
- unique optimum

$j_A < j < j_B$

- customers : single equilibrium
- the seller selects the number of buyers : p drives the market

optimal supply

phase diagram at optimal price *p*



$j < j_A$

- unique optimum

$j_A < j < j_B$

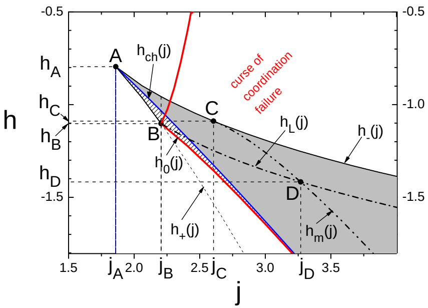
- customers : single equilibrium
- the seller selects the number of buyers : *p* drives the market

$j > j_B$

- customers : multiple equilibria for *h* between the red lines
- risk of coordination failure if the target is high-η

optimal supply

phase diagram at optimal price p

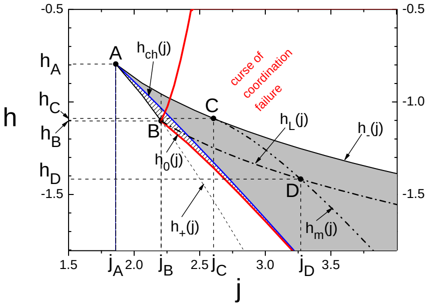


if target is low-η

$(h < h_{-}(j))$

optimal supply

phase diagram at optimal price p



if target is low-η

$(h < h_-(j))$

$j_B < j < j_C$

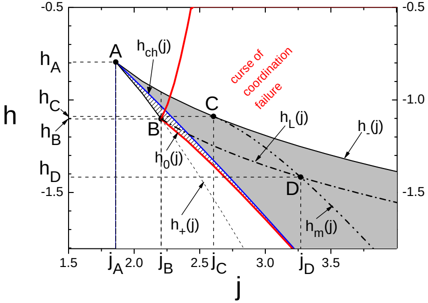
- single relative max of Π

$j > j_C$

- two relative max of Π (on the low-η manifold)

optimal supply

phase diagram at optimal price p

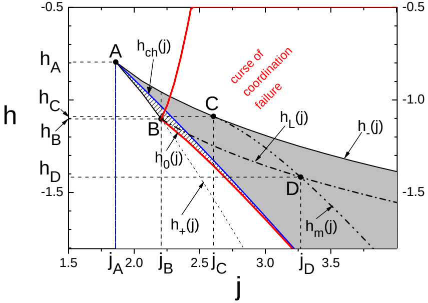


if target is high-η

(optimal for $h > h_{ch}(j)$)

optimal supply

phase diagram at optimal price p



if target is high-η

(optimal for $h > h_{ch}(j)$)

$j > j_B$

- requires customers coordination
- very large region of uncertainty
- coordination failure (empty restaurant)

$\Rightarrow$ profit much lower than expected

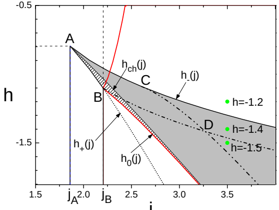
optimal pricing strategies

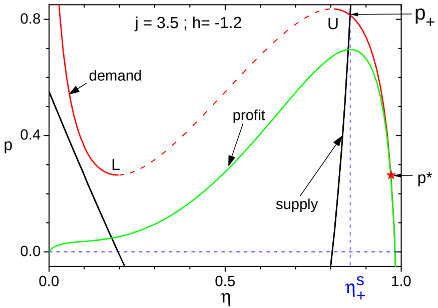
targeting the high-η customers equilibrium

optimal pricing strategies

targeting the high-η customers equilibrium

$j = 3.5, \; h = -1.2$



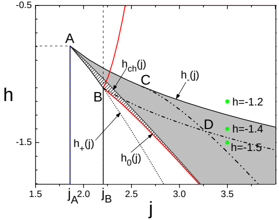


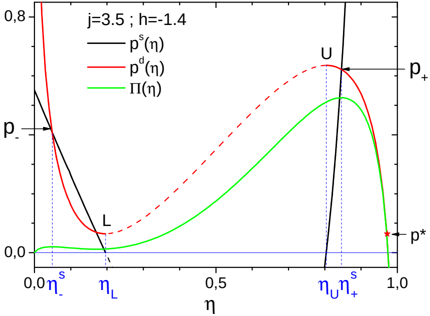
- optimal strategy, $p_+ = 0.81, \eta_+ = 0.86 \Rightarrow \Pi = 0.70$
- without coordination : $\eta = 0.03 \Rightarrow \Pi = 0.0248163$
- pricing strategy : start with $p^* \ll p_+, \eta^* = 0.97 \Rightarrow \Pi^* = 0.26$ and increase p

optimal pricing strategies

targeting the high-η customers equilibrium

$j = 3.5,\ h = -1.4$





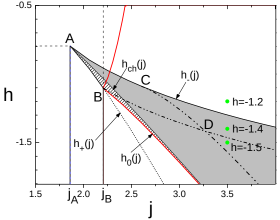
- optimal strategy, $p_+ = 0.62, \eta_+ = 0.85 \Rightarrow \Pi = 0.53$
- without coordination : $\eta = 0.03 \Rightarrow \Pi = 0.019$
- pricing strategy targeting high-η : start with $p^* = 0.064, \eta^* = 0.97 \Rightarrow \Pi^* = 0.06$

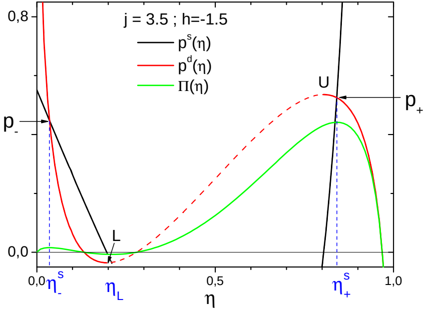
- targeting the low-η sub-optimum, $p = 0.41, \eta = 0.05 \Rightarrow \Pi = 0.02$
- if coordination, $p = 0.41, \eta = 0.93 \Rightarrow \Pi = 0.38$

optimal pricing strategies

targeting the high-η customers equilibrium

$j = 3.5,\ h = -1.5$





- optimal strategy, $p_+ = 0.53, \eta_+ = 0.84 \Rightarrow \Pi = 0.44$
- without coordination : $\eta = 0.03 \Rightarrow \Pi = 0.0156$
- pricing strategy targeting high-η would require negative p^*)

- sub-optimum, $p = 0.45, \eta = 0.04 \Rightarrow \Pi = 0.0158$
- if coordination, $p = 0.45, \eta = 0.90 \Rightarrow \Pi = 0.40$

experimental results

unpredictability of collective outcomes due to social interactions

- Schelling's dying seminar setting in experimental economics (Semeshenko et al., 2010)
  - coordination depends strongly on available information
  - complete information (number of buyers) favors coordination

experimental results

unpredictability of collective outcomes due to social interactions

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  - complete information (number of buyers) favors coordination

- why it is difficult to predict success in cultural markets ? (Salganik et al 2006, 2009)
  - information about the others' choices increases the popularity of the most popular songs

conclusion

social interactions

- demand curves are non-monotonic
- optimal supply is unpredictable (for some large range of parameters)
    - empty vs overcrowded restaurants
    - success vs flop of cultural products
- possible pricing strategies (under complete information)

conclusion

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to be done :

- supply with incomplete information
- pricing with learning customers
- networks
- competing sellers
- ....

Thank you !

References :

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