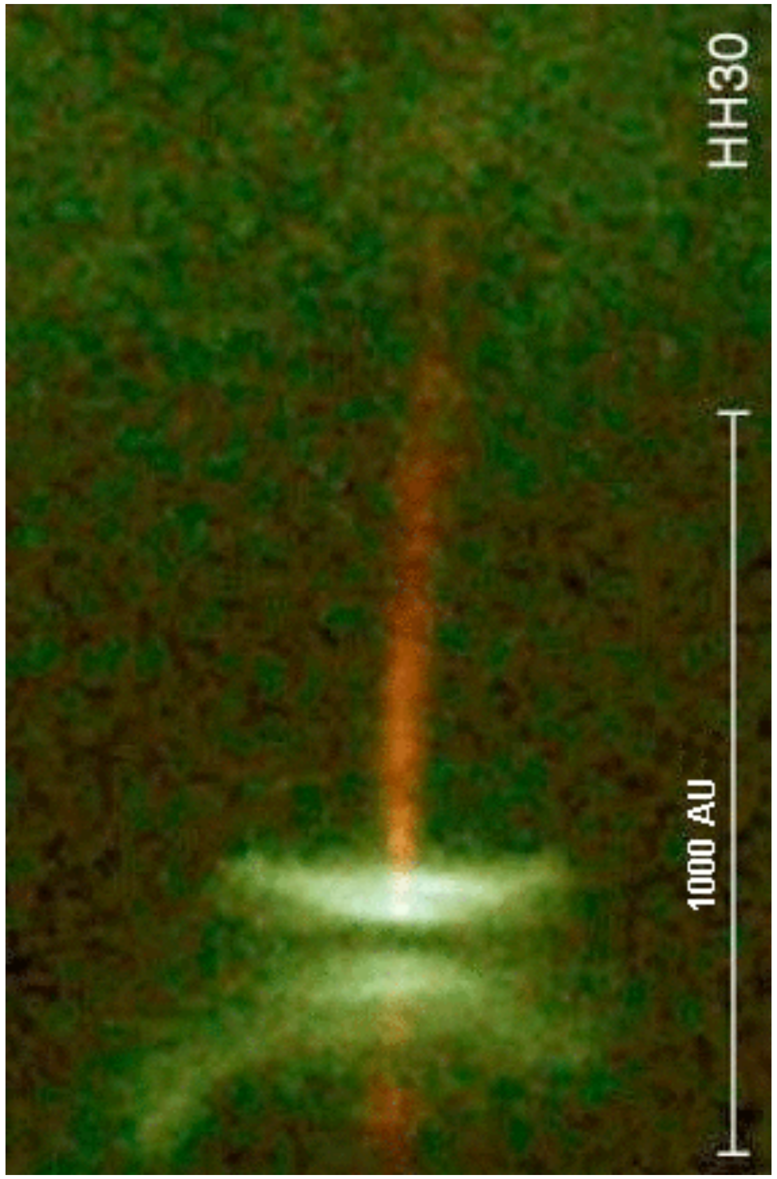
MHD turbulence and planet formation

Geoffroy Lesur (IPAG, Grenoble, France)

Protoplanetary discs



Credit: C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Artist view

- Size: 10^{11}-10^{15} cm (0.1-100 AU)
- Temperature: 10-10^3 K

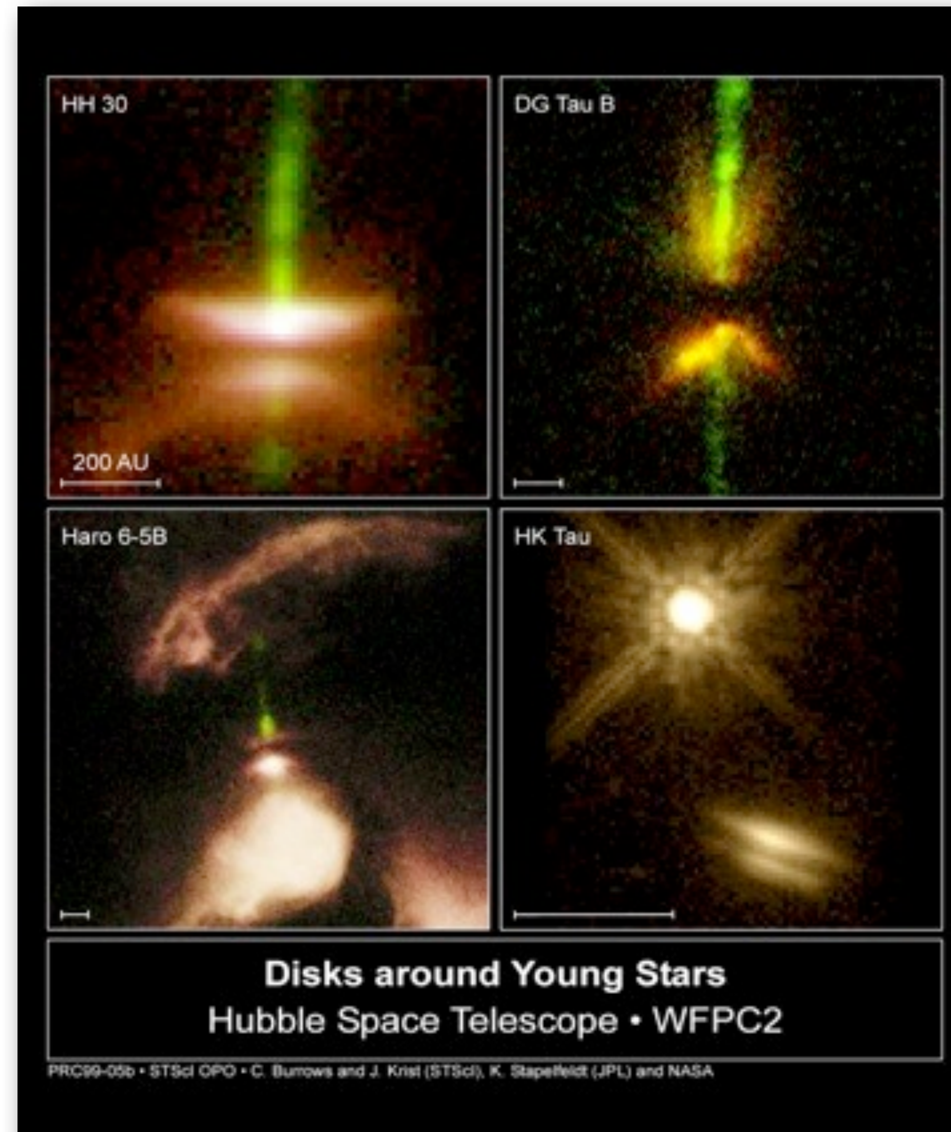
An accretion problem...

- Gas can fall on the central object only if it looses angular momentum.
- One needs a way to transport angular momentum outward to have accretion:
  «angular momentum transport problem»

First idea: molecular viscosity

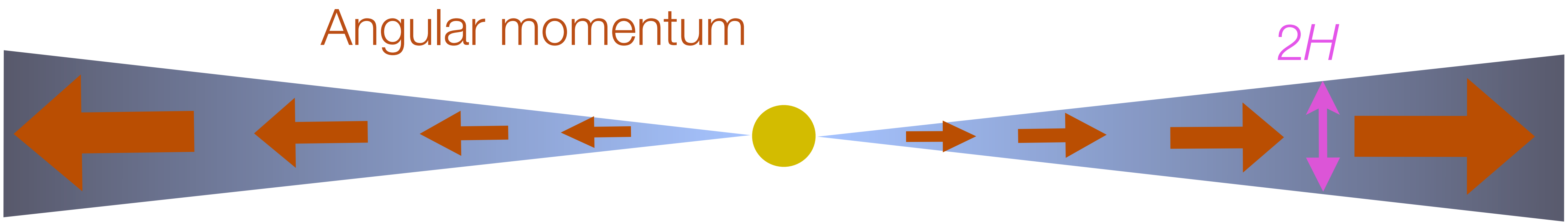
- Theoretical accretion rate due to viscous transport is very small compared to observational constrains

→ Other ways to extract angular momentum in discs?



Credit: C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Angular momentum transport processes
I- turbulent transport



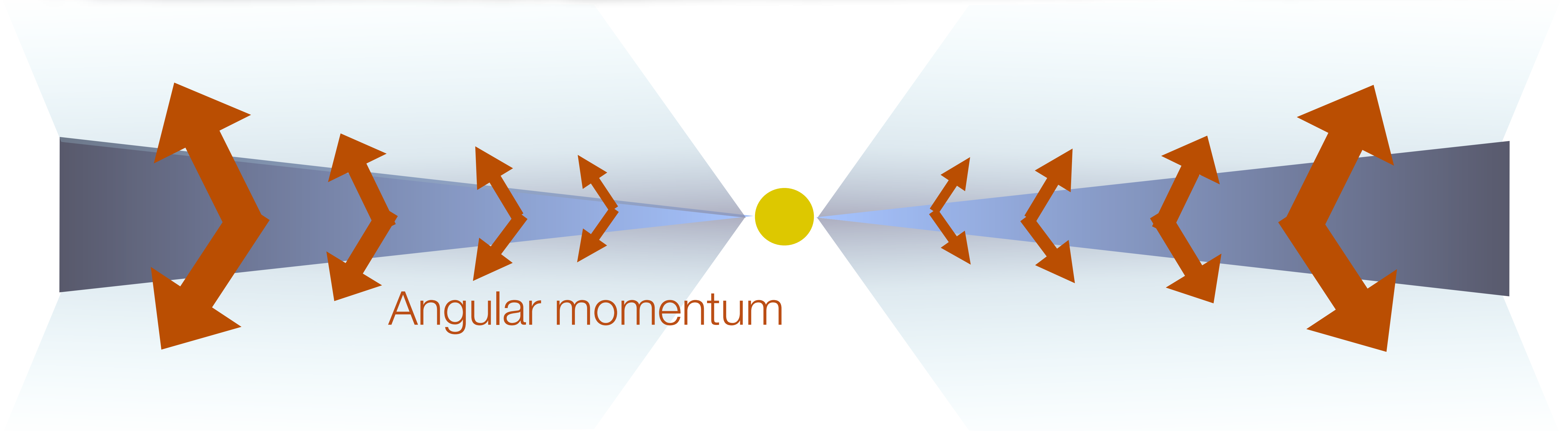
- Transport angular momentum in the bulk of the disc
- Suggested by Shakura & Sunyaev (1973)
- Turbulence leads to enhanced transport («mixing length theory»)
- Turbulent viscosity

$$\nu_t = \alpha c_s H$$

«turbulent transport» «sound speed» «1/2 disc thickness»

$$10^{-3} < \alpha < 10^{-2}$$

Angular momentum transport processes
II- disc wind



- Angular momentum *extracted* from the disc by a magnetic wind
  [Blandford & Payne 1982, MNRAS, 199, 883]
- Magnetic field exerts a torque on the disc surface which generates accretion (not described by α−disc!)

Origin of turbulence in discs
Instabilities

Local instabilities:

- Magnetorotational instability (MRI): shear driven instability but requires an ionised plasma (Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991)
- Subcritical shear instability: probably not efficient enough, if exists (Lesur & Longaretti 2005, Ji+ 2006)
- Baroclinic instabilities: Transport due to waves. Driven by the disk radial entropy profile
- Gravitational instabilities: only for massive & cold enough disk
- Rossby wave instability: requires a local maximum of vortensity (Lovelace et. al 1999)
- Vertical convection: Requires a heat source in the midplane (Cabot 1996, Lesur & Ogilvie 2010)

Global instabilities:

- Papaloizou & Pringle instability: density wave reflection on the inner edge (Papaloizou & Pringle 1985)
- Accretion-ejection instability: spiral Alfvén wave reflection on the inner edge (Tagger & Pellat 1999)

Ideal MHD equations
Derivation

- Magnetic fields create a force on the flow: the Lorentz force

$$\partial_t \rho + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u} = 0,$$
$$\partial_t \rho \boldsymbol{u} + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u}\boldsymbol{u} = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \rho \boldsymbol{g},$$

Lorentz force

- The evolution of the field is dictated by Maxwell-Faraday equation

$$\partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}$$

- To close the system, we introduce Ohm's law in the co-moving frame for a perfect conductor

$$\boldsymbol{E}_{\mathrm{cm}} = \eta \boldsymbol{J} = 0$$

($U = RI$ with $R = 0$)

- So the electric field in the Laboratory frame is:

$$\boldsymbol{E} = -\boldsymbol{u} \times \boldsymbol{B} + \boldsymbol{E}_{\mathrm{cm}}$$

Ideal MHD equations

Consequences

- Set of ideal MHD equations

$$\partial_t \rho + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u} = 0,$$
$$\partial_t \rho \boldsymbol{u} + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u}\boldsymbol{u} = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \rho \boldsymbol{g},$$
$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B})$$

- The Lorentz force can be decomposed into

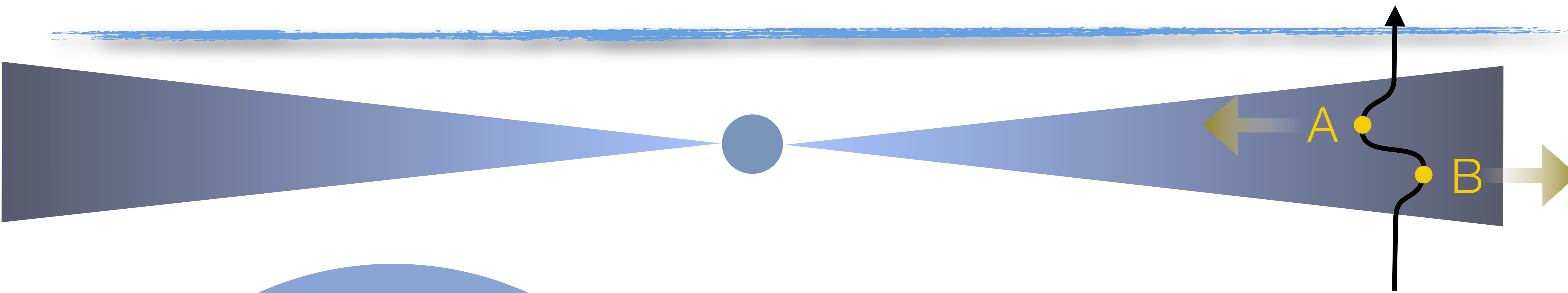
$$\boldsymbol{J} \times \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} - \boldsymbol{\nabla} \frac{\boldsymbol{B}^2}{2}$$

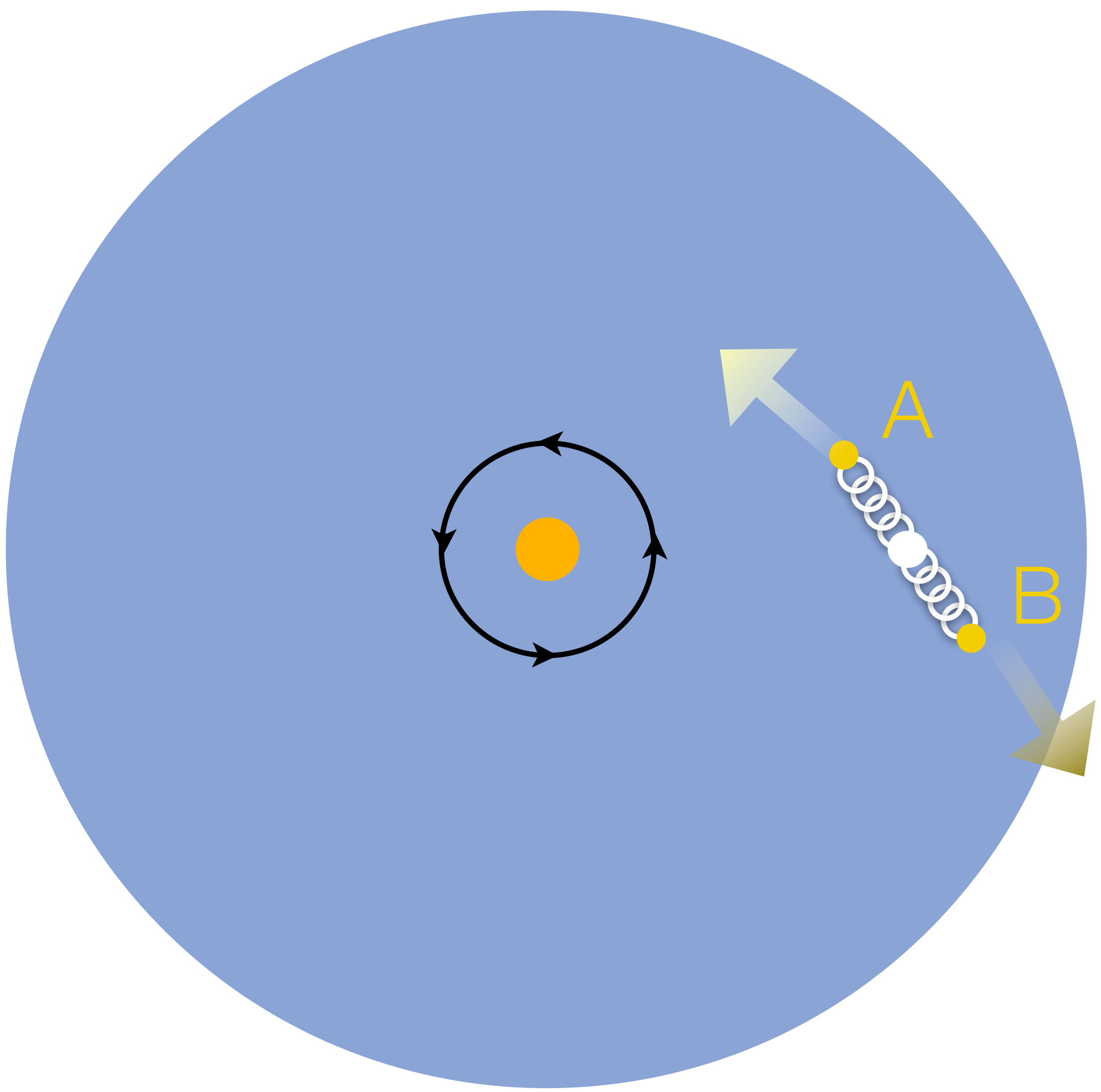
«magnetic tension» «magnetic pressure»

- Alfvén waves are magnetised waves driven by magnetic tension

$$V_A = \frac{B}{\sqrt{\rho}}$$

Origin of turbulence in discs
The Magnetorotational instability (MRI)

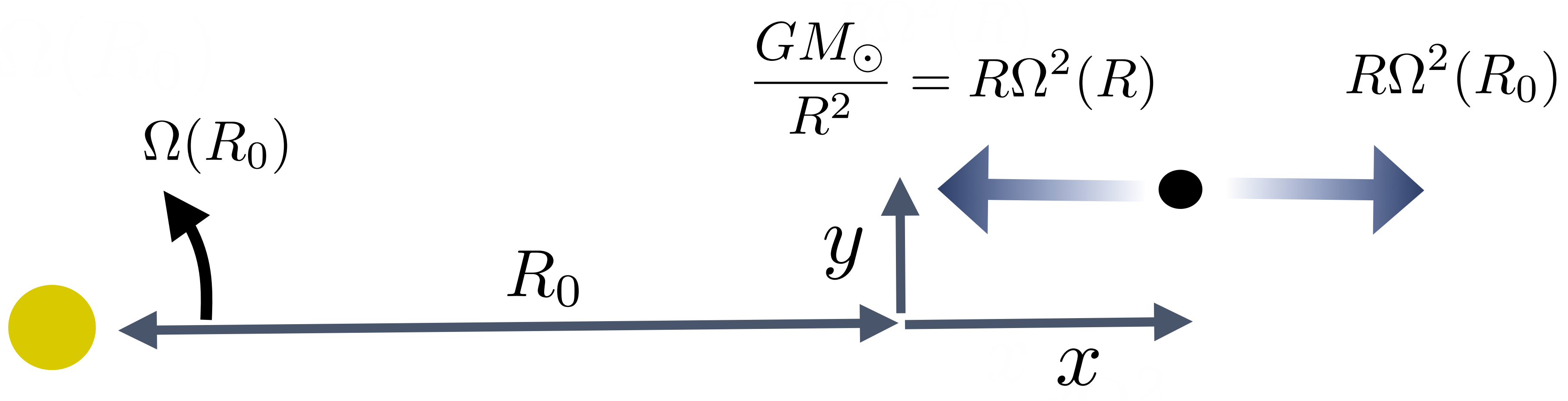




[Balbus, & Hawley (1991)]
[Balbus (2003)]

Ideal MHD instability, modified (possibly suppressed) by nonideal effects

Stability analysis
Hydrodynamic case



Effective (tidal) radial acceleration: $-x\frac{d\Omega^2}{d\ln R}\boldsymbol{e_x}$

Resulting equation of motion for a fluid particle:

$$\begin{aligned}\ddot{x} - 2\Omega\dot{y} &= -\frac{d\Omega^2}{d\ln R}x \\ \ddot{y} + 2\Omega\dot{x} &= 0\end{aligned}$$

→ Epicyclic oscillations at frequency $\kappa = \left(4\Omega^2 + \frac{d\Omega^2}{d\ln R}\right)^{1/2}$

Stability analysis

Magnetised case

Induction equation for a small displacement $\boldsymbol{\xi}$

and a spatial dependence $\propto \exp(ikz)$:

$$\delta \boldsymbol{B} = i(\boldsymbol{k} \cdot \boldsymbol{B}_{z_0})\boldsymbol{\xi}$$

The magnetic tension force is then

$$\frac{\boldsymbol{B}_{z_0} \cdot \boldsymbol{\nabla} \boldsymbol{B}}{\rho} = \frac{i(\boldsymbol{k} \cdot \boldsymbol{B}_{z_0})}{\rho} \delta \boldsymbol{B} = -(\boldsymbol{k} \cdot \boldsymbol{v_A})^2 \boldsymbol{\xi}$$

Resulting equation of motion for a fluid particle:

$$\begin{aligned} \ddot{x} - 2\Omega \dot{y} &= -\left(\frac{d\Omega^2}{d \ln R} + (\boldsymbol{k} \cdot \boldsymbol{v_A})^2\right) x \\ \ddot{y} + 2\Omega \dot{x} &= -(\boldsymbol{k} \cdot \boldsymbol{v_A})^2 y \end{aligned}$$

Stability analysis

Dispersion relation

Introduce:

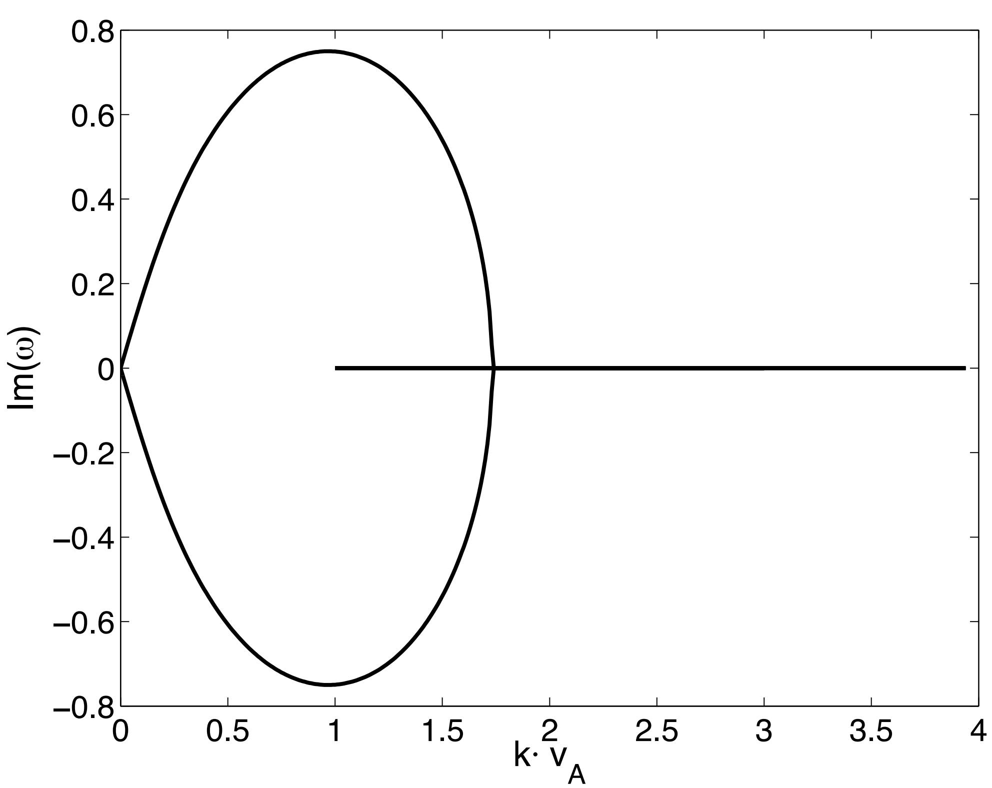
$$x = x_0 \exp(i\omega t)$$
$$y = y_0 \exp(i\omega t)$$

➡ Dispersion relation

$$\omega^4 - \omega^2[\kappa^2 + 2(\boldsymbol{k}\cdot\boldsymbol{v_A})^2] + (\boldsymbol{k}\cdot\boldsymbol{v_A})^2\left[(\boldsymbol{k}\cdot\boldsymbol{v_A})^2 + \frac{d\Omega^2}{d\ln R}\right] = 0$$

Stabilizing Destabilizing

[Balbus & Hawley 1991]



➡ The MRI is a «weak field» instability

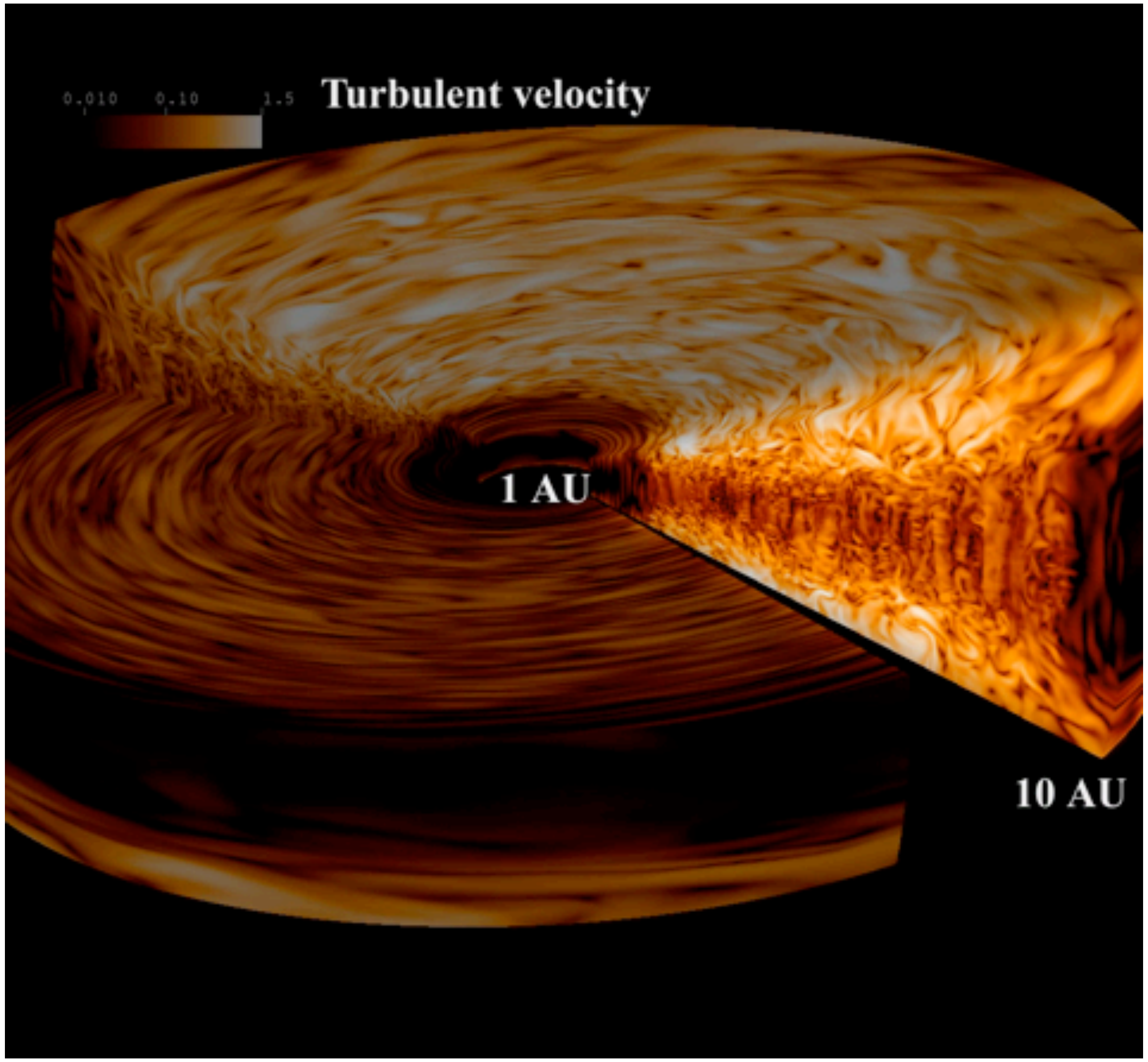
$$(\boldsymbol{k}\cdot\boldsymbol{v_A})^2 < -\frac{d\Omega^2}{d\ln R}$$

Nonlinear evolution: ideal MHD case

Global simulations in the ideal MHD limit are consistent with observational constraints

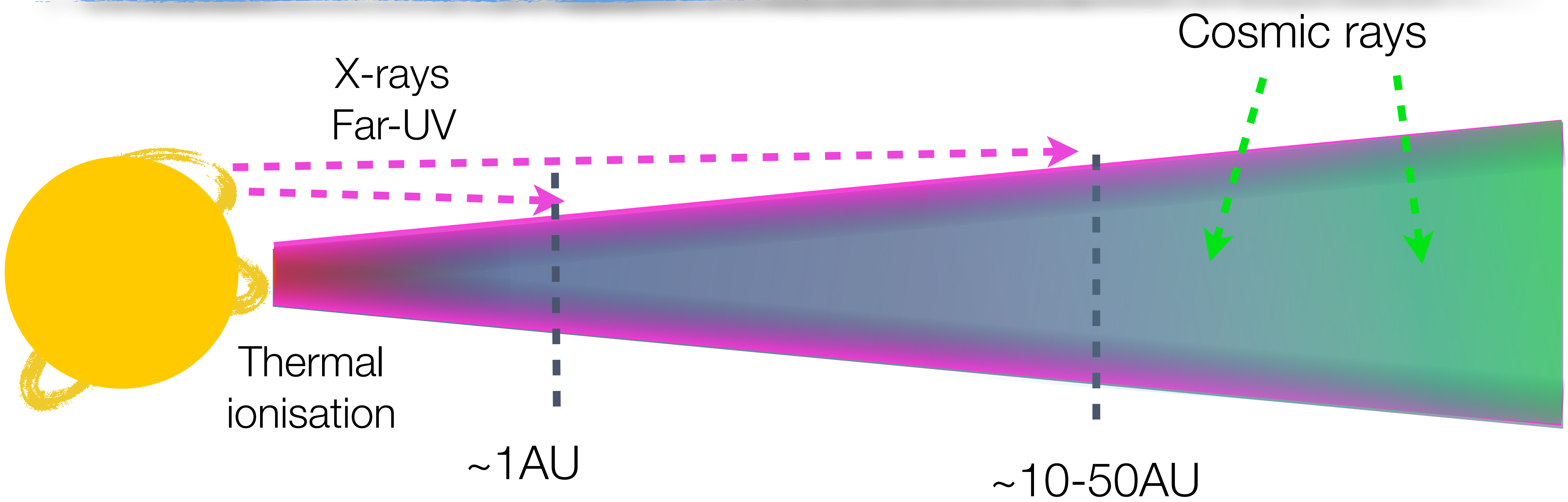
$$\alpha \sim 10^{-3}\text{—}10^{-2}$$

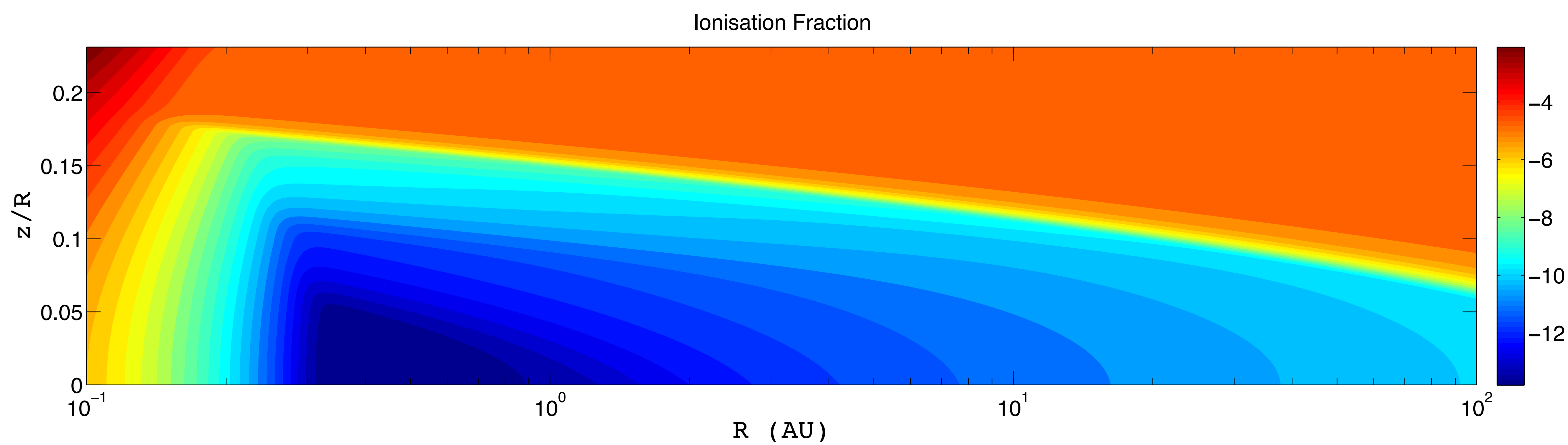
[Hawley+ (1995) ; Fromang & Nelson (2006) ; Sorathia+ (2012)]



[Flock+ 2011]

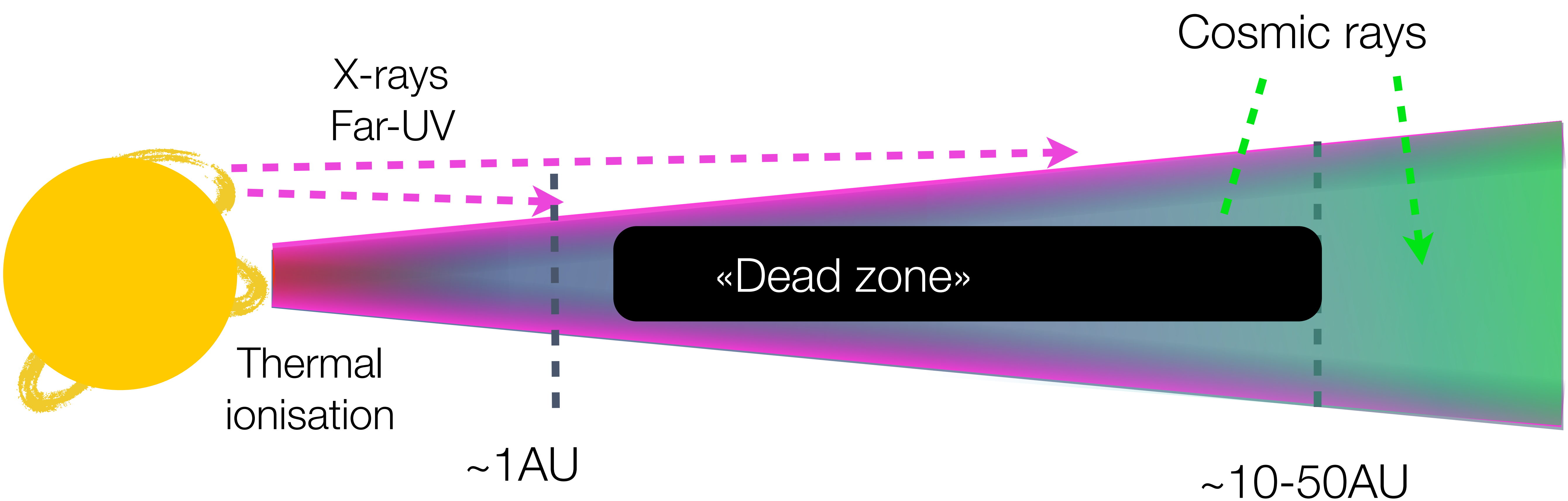
Ionisation sources in protoplanetary discs





Protoplanetary disc plasmas are dominated by neutrals

Dead zone in protoplanetary discs



- How large is the dead zone?
- What's happening inside the dead zone?

Ions and neutrals dynamics

- Equation of motion for the neutrals:

$$\rho \frac{\partial v}{\partial t} + (\rho \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla} P + \boldsymbol{f}_{ni}$$

- With a drag force:

$$\boldsymbol{f}_{ni} = \gamma \rho \rho_i (\boldsymbol{v}_i - \boldsymbol{v})$$

- For ions (assuming large collision frequency):

$$0 = \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{f}_{in}$$

- Resulting equation of motion & drift speed

$$\rho \frac{\partial v}{\partial t} + (\rho \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla} P + \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B}$$

$$\boldsymbol{v}_i - \boldsymbol{v} = \frac{1}{c \gamma \rho \rho_i} \boldsymbol{J} \times \boldsymbol{B}$$

Electrons dynamics

- Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{v_e} \times \boldsymbol{B} - \frac{4\pi\eta}{c}\boldsymbol{J} \right)$$

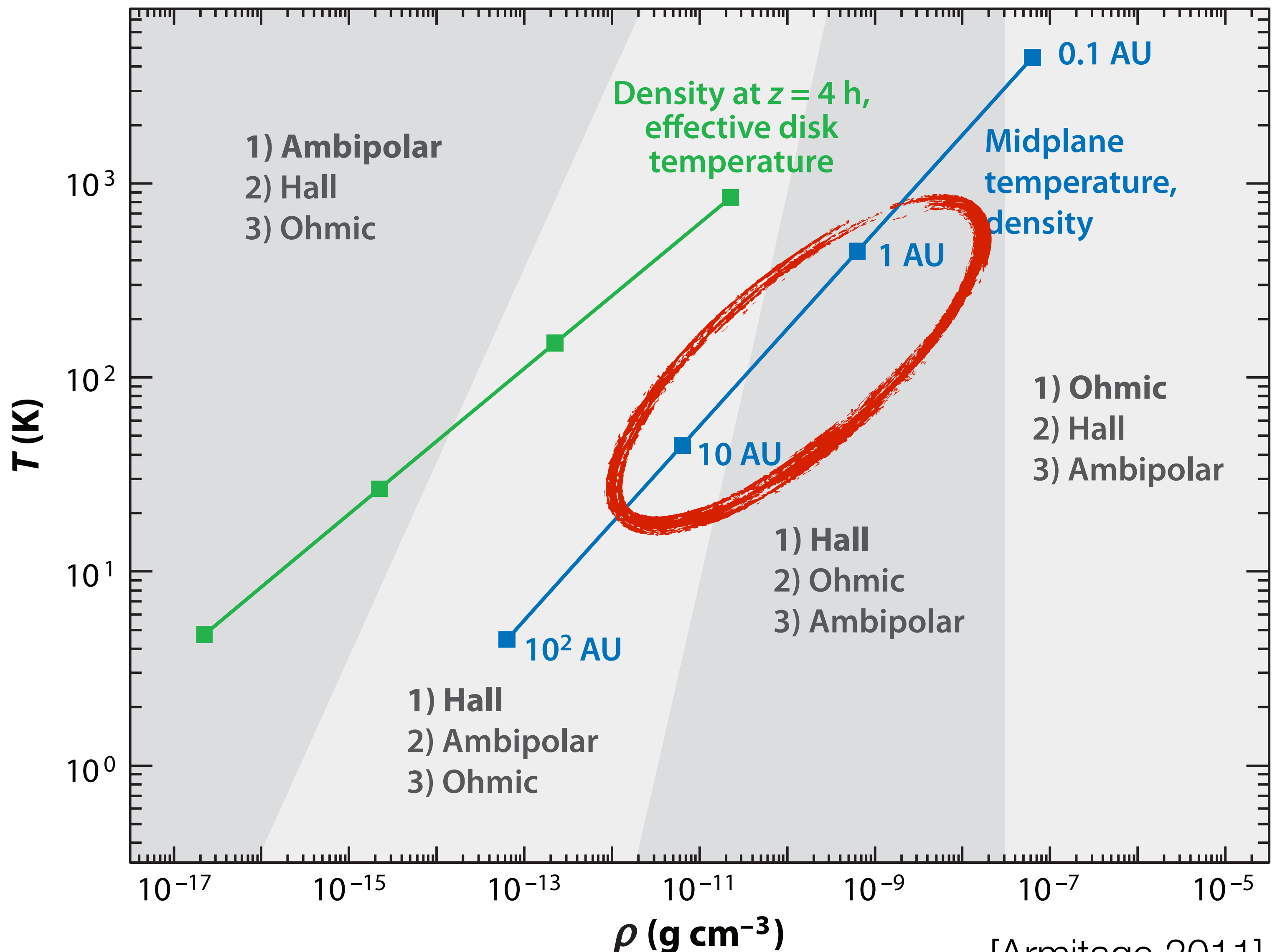
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{B} + \underbrace{(\boldsymbol{v_i} - \boldsymbol{v})}_{\frac{1}{c\gamma\rho\rho_i}\boldsymbol{J} \times \boldsymbol{B}} \times \boldsymbol{B} + \underbrace{(\boldsymbol{v_e} - \boldsymbol{v_i})}_{-\frac{\boldsymbol{J}}{en_e}} \times \boldsymbol{B} - \frac{4\pi\eta}{c}\boldsymbol{J} \right)$$

Ambipolar diffusion

Hall effect

Ohmic diffusion

Non-ideal protoplanetary discs



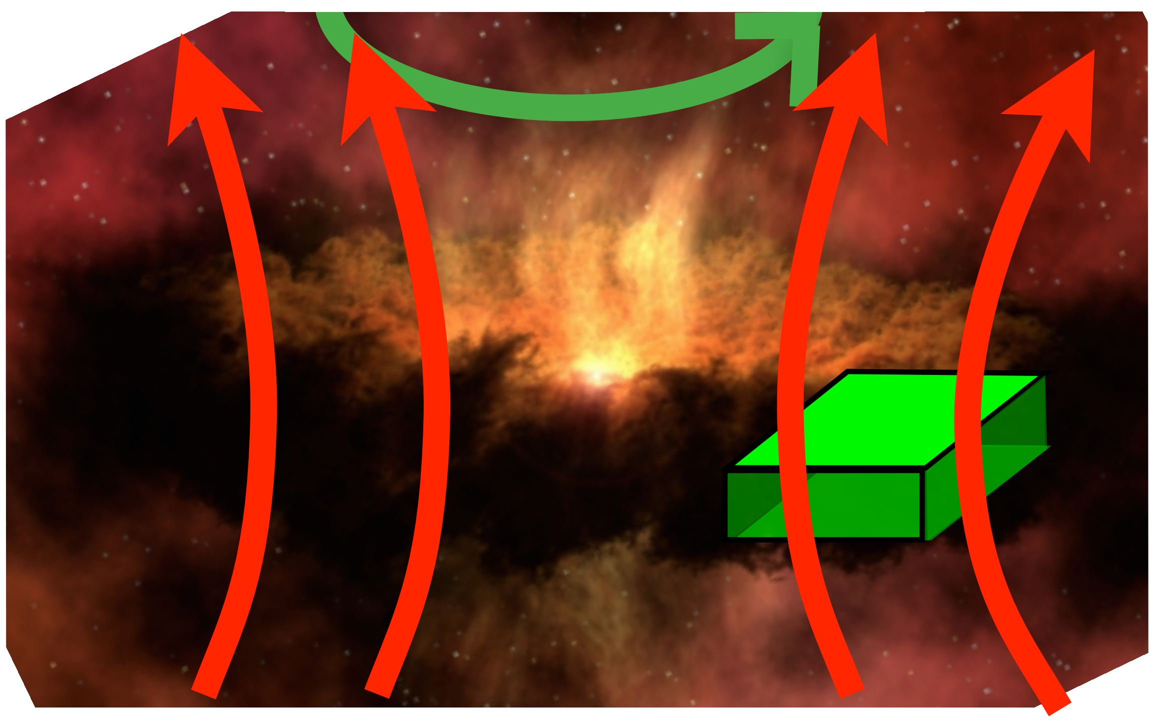
[Armitage 2011]

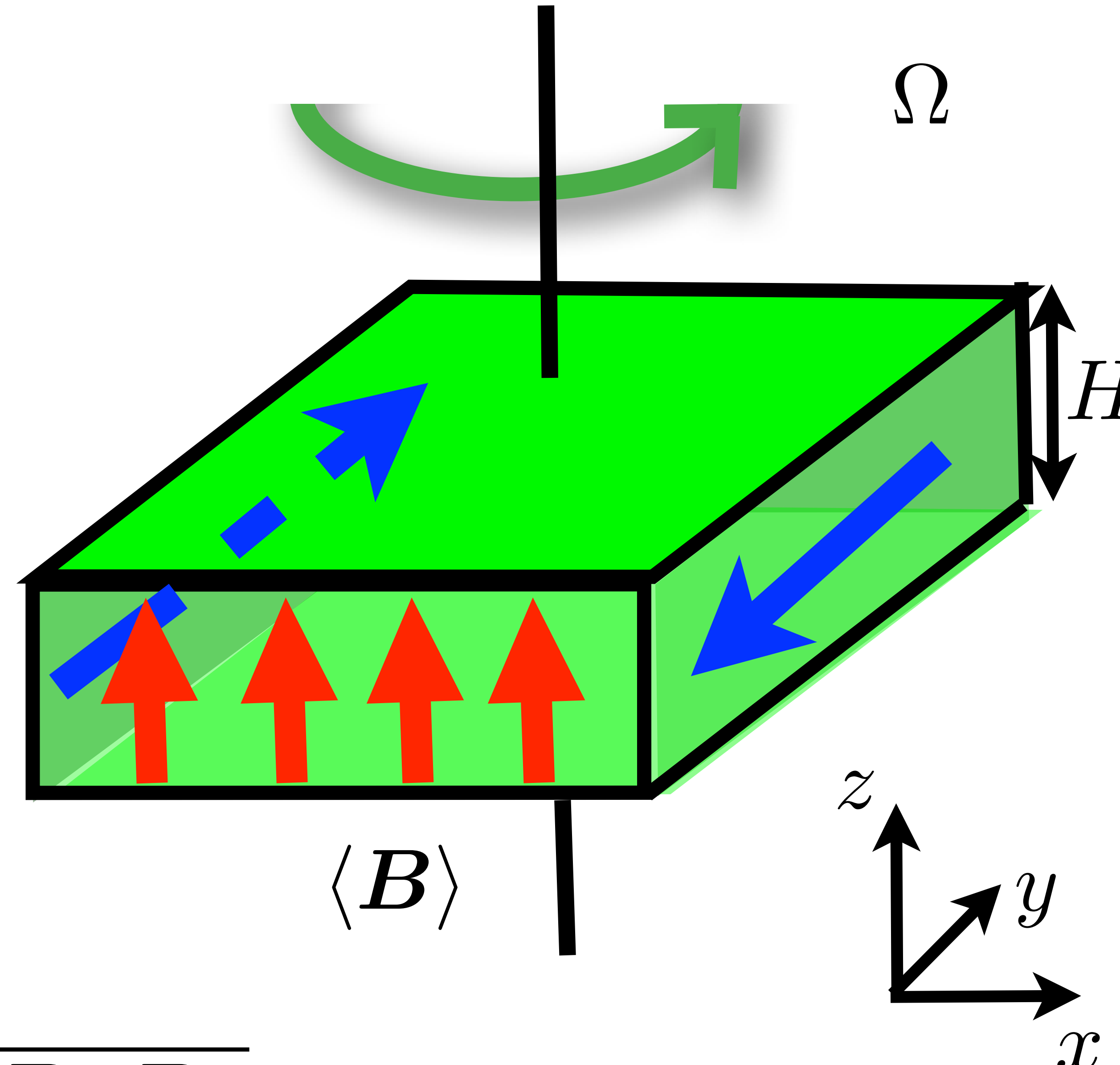
NB: strongly depends on grain size and metallically

Hall effect dominates in most of the disc midplane

Ambipolar diffusion dominates in the upper layer

The shearing box model

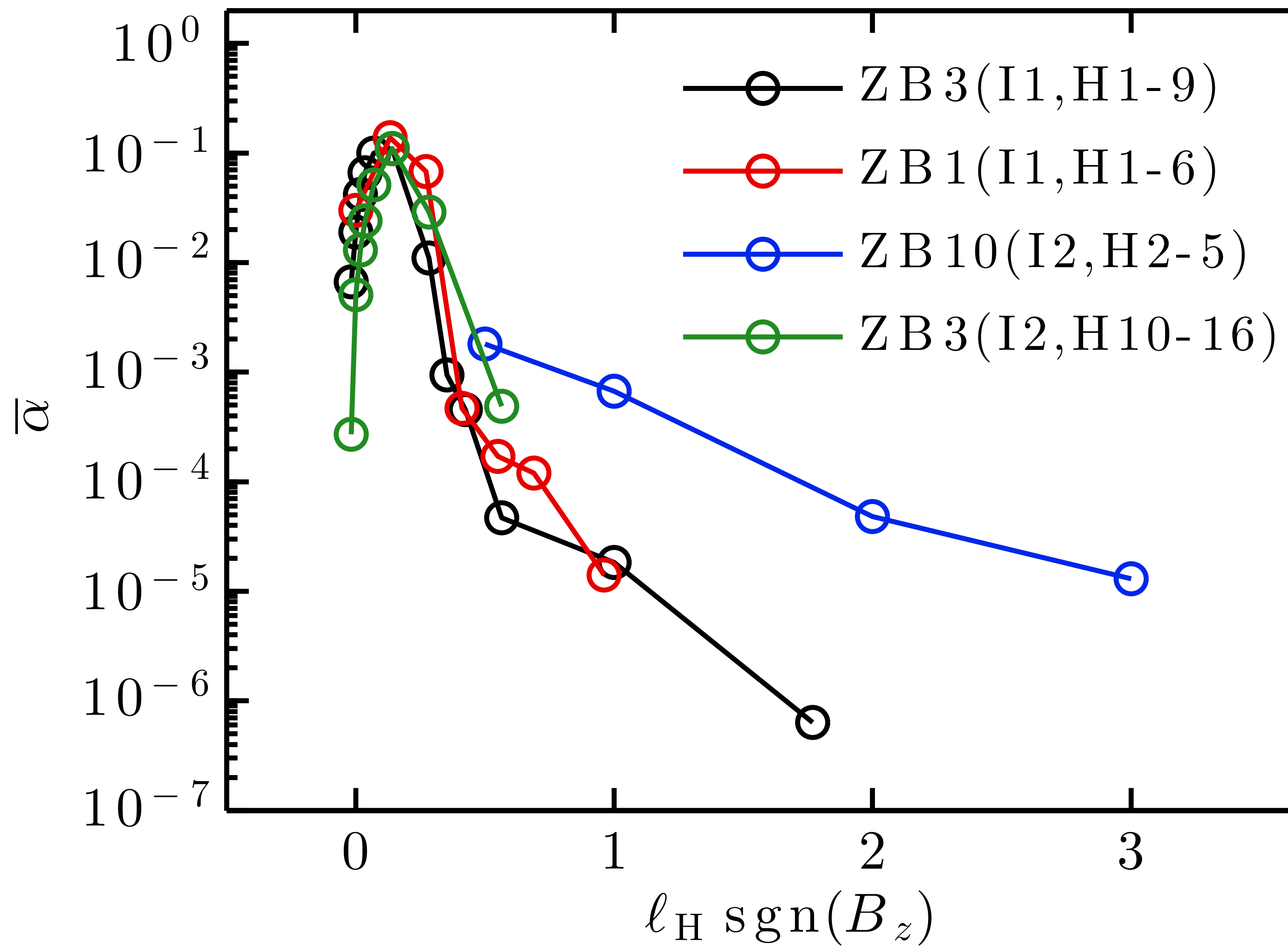




$$\alpha = \frac{\overline{\rho v_x v_y - B_x B_y}}{\overline{\rho}\Omega^2 H^2}$$

Hall-MRI: turbulent viscosity

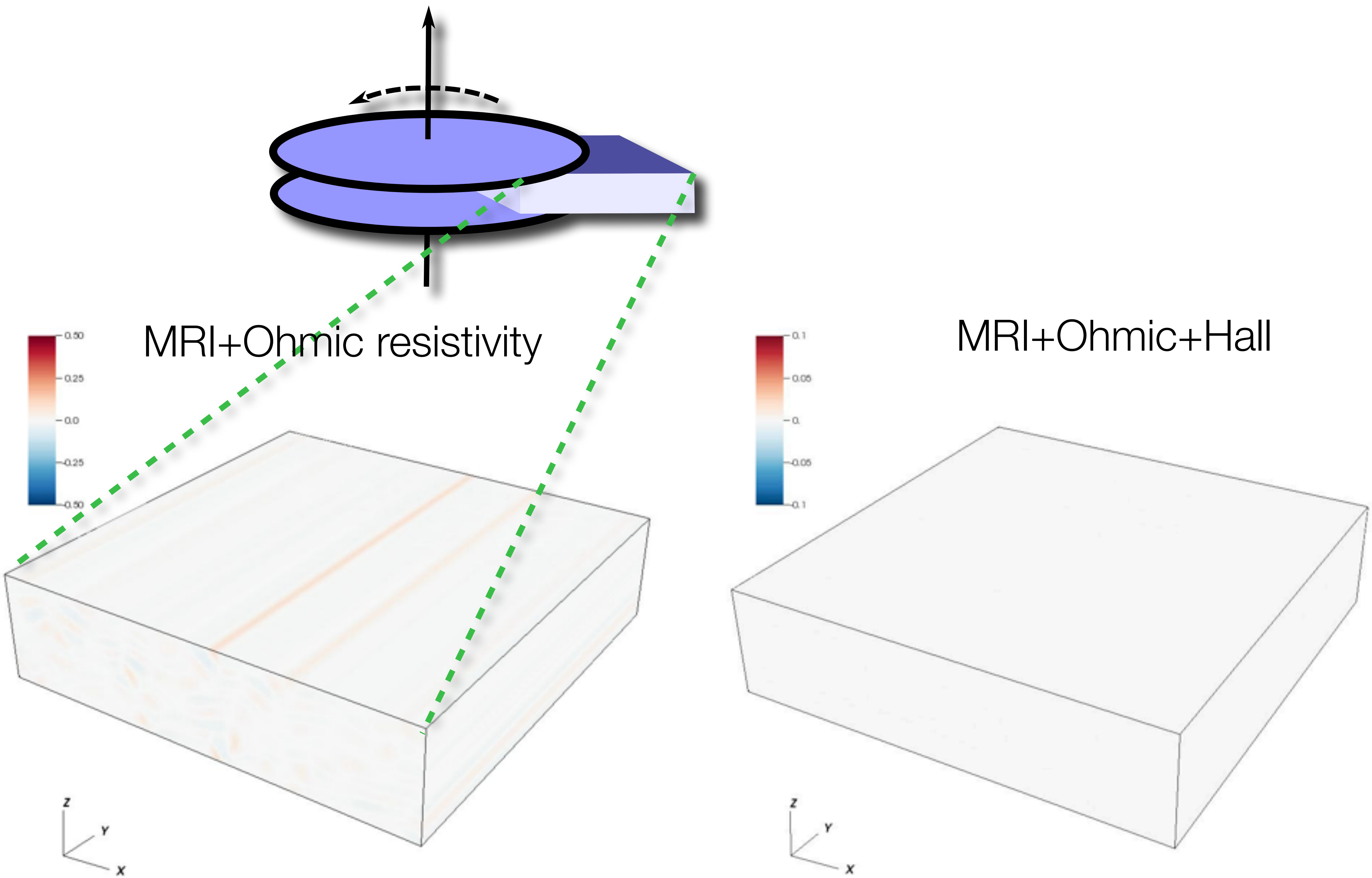
Varying field strength and Ohmic resistivity



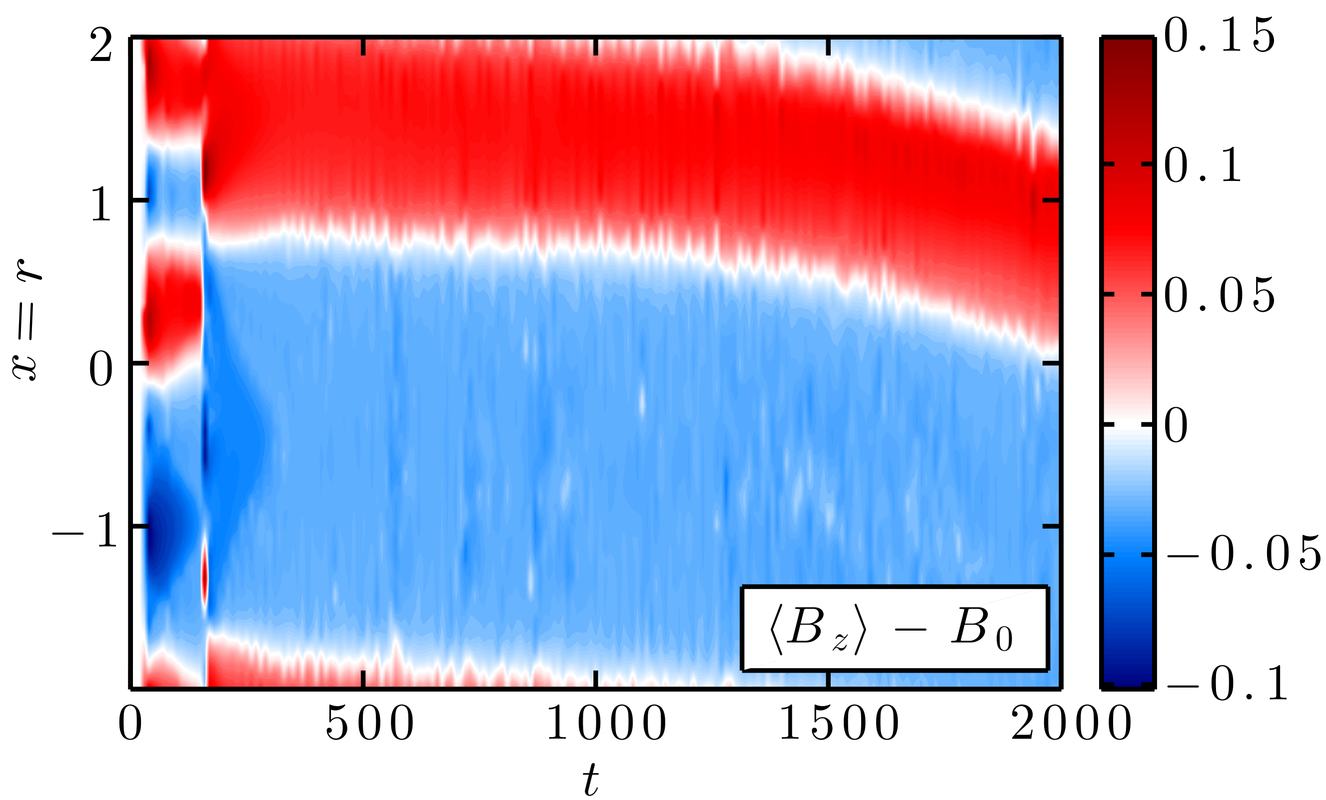
Transport is controlled by

$$\ell_{\rm H} \equiv \left(\frac{m_{\rm i}c^2}{4\pi e^2 n_{\rm i}}\right)^{1/2}\left(\frac{\rho}{\rho_{\rm i}}\right)^{1/2}$$

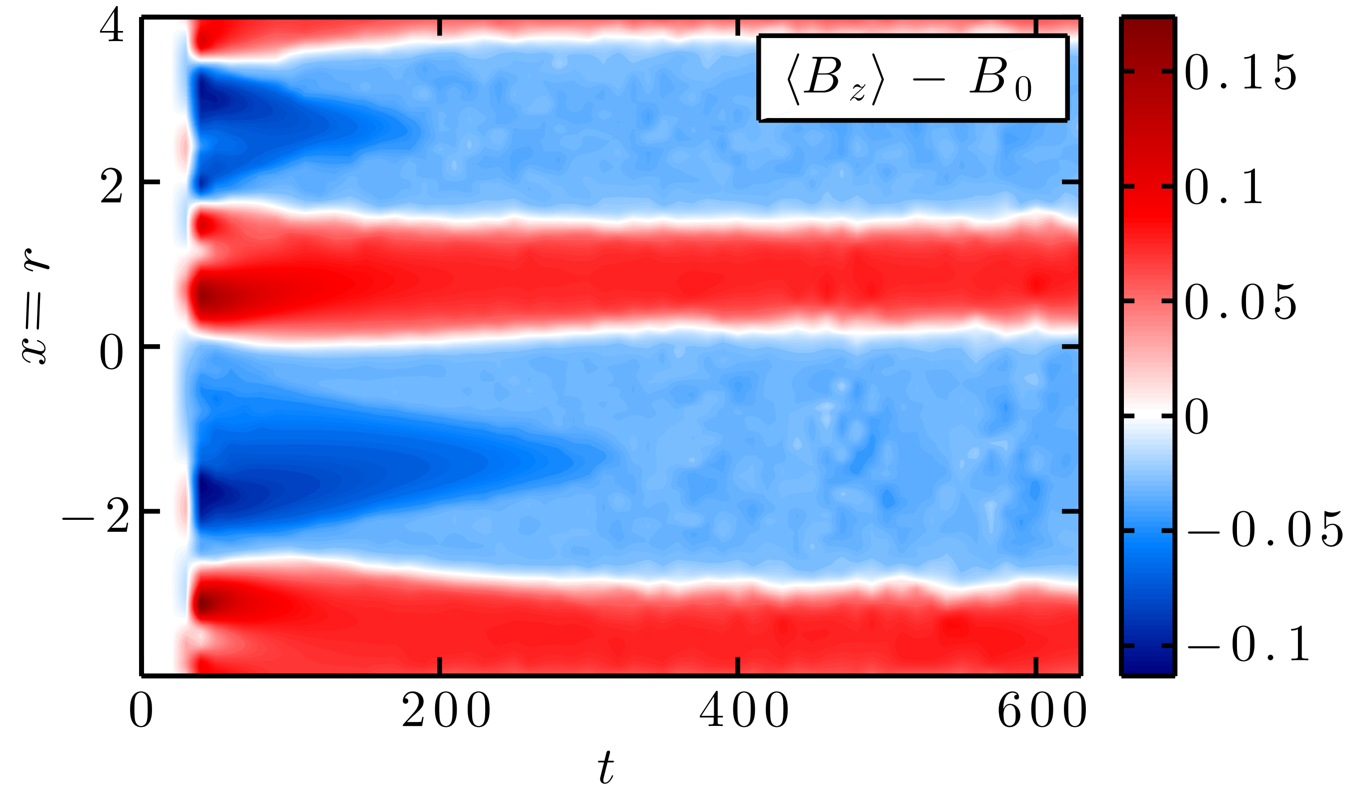
Hall-MRI animation: Bz



Zonal field structures in Hall-dominated discs



Box twice larger in *x*-*y*



→ Self Organisation!

Conservation laws in Hall-MHD

Induction

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{B} - \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e} \right)$$

Vorticity

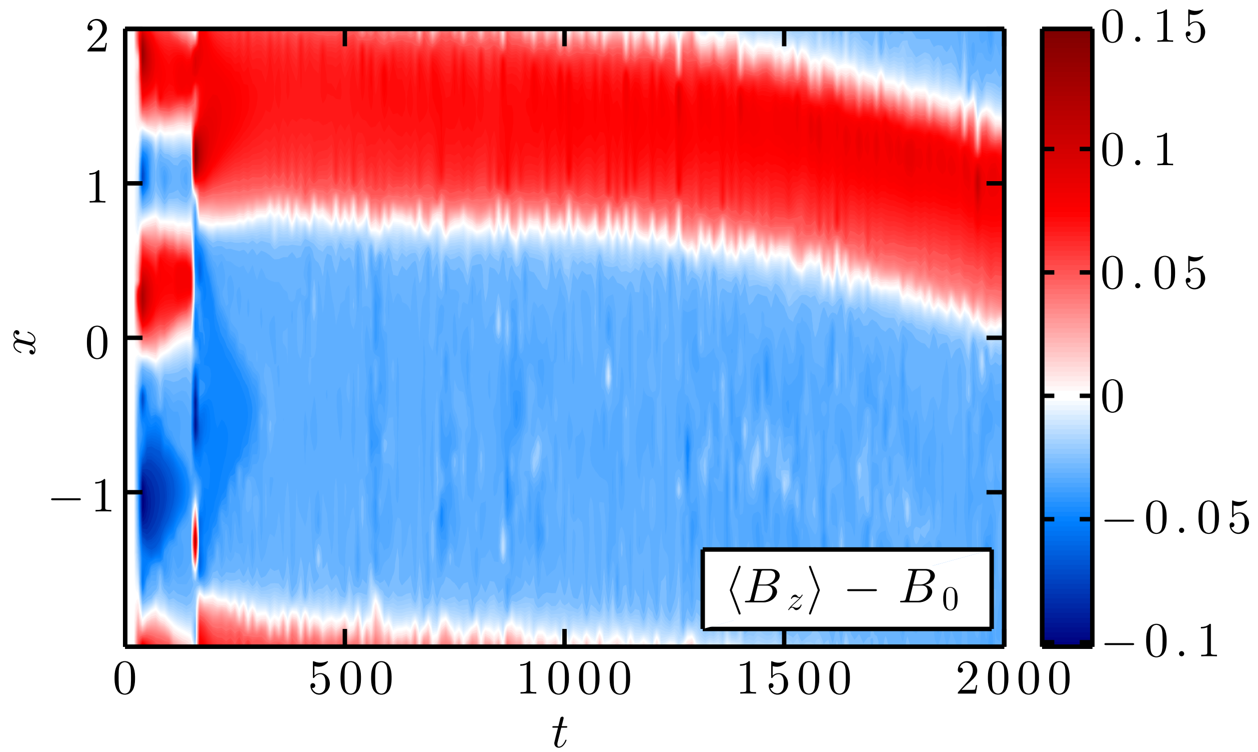
$$\partial_t \boldsymbol{\omega} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{\omega} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c \rho} \right)$$

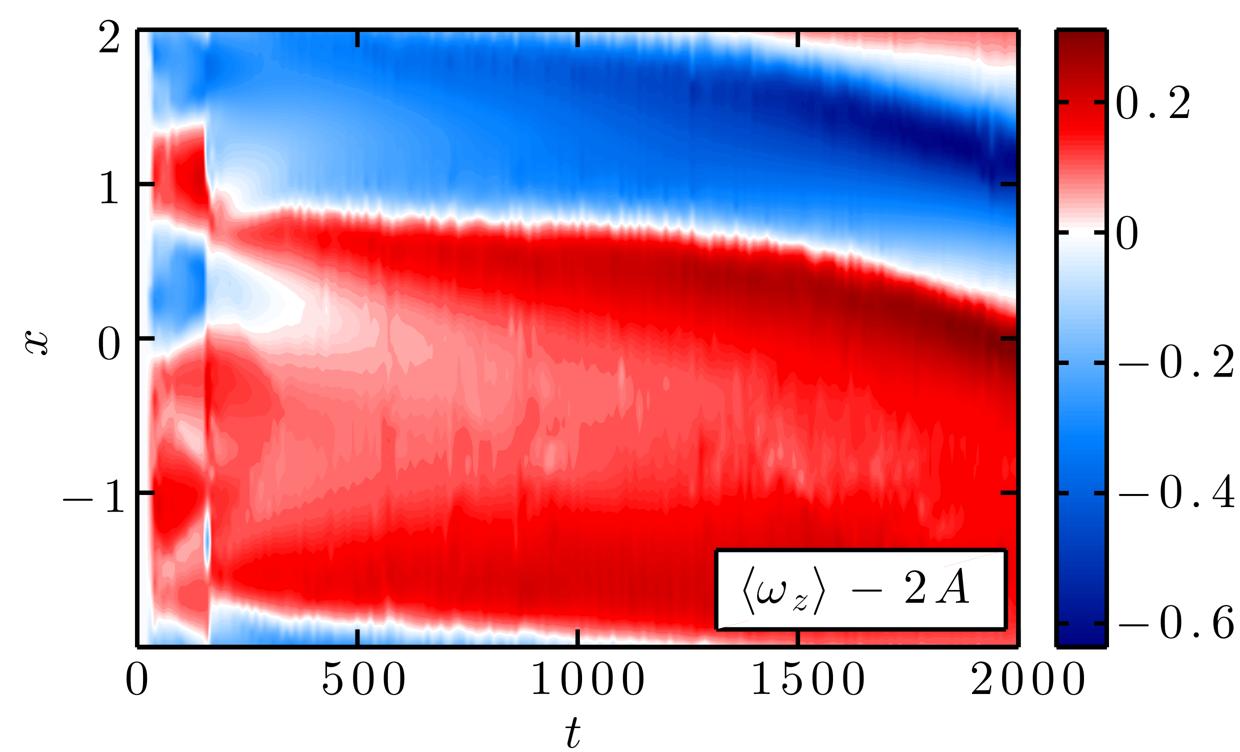
$$\boldsymbol{\omega}_C = \boldsymbol{\omega} + \frac{e \boldsymbol{B} n_e}{\rho c}$$

$$\partial_t \boldsymbol{\omega}_C = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{\omega}_C \right)$$

- Canonical vorticity behaves like magnetic field in ideal MHD
- Field line redistribution implies a redistribution of vorticity in the flow

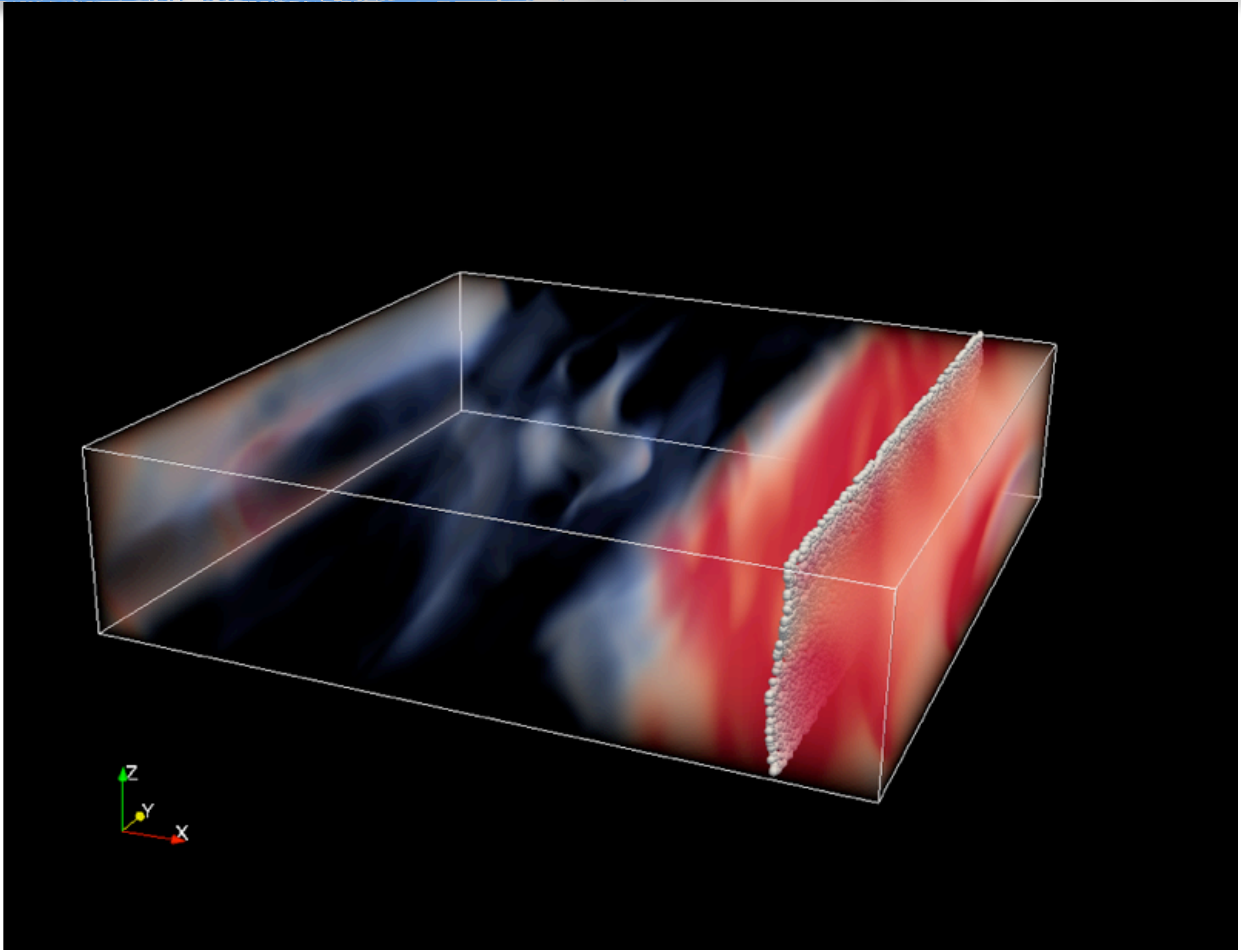
Strong zonal flows in PP discs





- long-lived zonal flows are associated to Hall-MRI
- Good for planet formation?

The Cherry on the cake



Conclusions

- Non ideal MHD effects are essential in PP discs
- Hall MRI reorganises the magnetic field creating large scale structures
- Potentially a strong impact on planet formation (dust agregation)