# The stochastic Laplacian Heaviside Method in twisted mass Lattice QCD - Part 2
# $\eta$ mass and $\pi$-$\pi$ scattering

Christian Jost
C. Helmes, B. Knippschild, C. Urbach, M. Werner

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# Outline

# Hadron Spectrum in QCD

- Most states in hadron spectrum non-stable resonances
- Scattering amplitude from scattering lengths at low energies
- Non perturbative method: Lattice QCD
- Easiest case: $\pi^+$-$\pi^+$-scattering at $p = 0$

# Scattering at low Energies

- At low energies the precise details of potentials are not important for scattering
- In the partial wave expansion of the scattering process, only the lowest partial waves contribute, here only $s$-wave
- The scattering phase-shift $\delta_s$ can be related to the scattering length $a_s$ via

$$\lim_{k \to 0} k \cot(\delta_s(k)) = -\frac{1}{a_s}$$

# Scattering Length on a Lattice

- Lüscher[1]: two particles in a box cause a shift in the energy, due to interaction
- Energy shift $\delta E$ is related to the scattering length of the particles

$$\delta E = (E - 2m) = -\frac{4\pi a_0}{mL^3}\left\{1 + c_1\frac{A_0}{L} + c_2\frac{a_0^2}{L^2}\right\} + \mathcal{O}(L^{-6})$$

- Obtain energies $E_n$ from correlation functions

$$C_1^{\text{2-pt}}(t) = \left\langle \mathcal{O}_1 \overline{\mathcal{O}}_1 \right\rangle(t) = \sum_n \left|\langle 0|\mathcal{O}_n\rangle\right|^2 e^{-E_n t}$$

$$\operatorname{acosh}\left(\frac{C_1^{\text{2-pt}}(t+1) + C_1^{\text{2-pt}}(t-1)}{2C_1^{\text{2-pt}}(t)}\right) = E_{\text{eff}}(t)$$

[1] M.Lüscher, Comm. Math. Phys. Volume 105, Number 2 (1986), 153-188.

# $\pi$-$\pi$ scattering

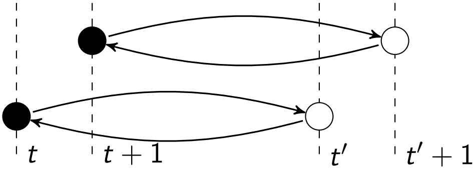
- The easiest possible scattering to calculate is $\pi$-$\pi$ scattering with the pions at rest
- Contributions with $I = 2$ ($\pi^+$-$\pi^+$ scattering) and $I = 0$ ($\pi^0$-$\pi^0$ and $\pi^+$-$\pi^-$ scattering)
- Investigated $I = 2$, since it has no disconnected contributions
- In total, 4 diagrams contribute

# Fierz Rearrangement

- For two particles on the same timeslice, Fierz rearrangement has to be taken into account[2]

$$\sum_{\vec{x},\vec{y},a} (\bar{q}(\vec{x},t)^a \Gamma q(\vec{y},t)^a), \ \sum_{\vec{x}',\vec{y}',b} (\bar{q}(\vec{x}',t)^b \Gamma q(\vec{y}',t)^b)$$
$$\rightarrow (\bar{q}(\vec{x},t)^a, q(\vec{y}',t)^b), \ (\bar{q}(\vec{x}',t)^a, q(\vec{y},t)^b)$$

- To avoid this, place the operators on different timeslices




[2]M. Fukugita et al, Phys. Rev. D 52 (1995) 3003

# Thermal States

- Total time extent $T$, Partition function $Z = \mathrm{tr}(e^{-HT})$

$$C_{\pi\pi}(t) = \frac{1}{Z} \sum_{m,n} |\langle n|\mathcal{O}|m\rangle|^2 \, e^{-(E_m+E_n)T/2} \cosh\left((E_m - E_n)(t - T/2)\right)$$

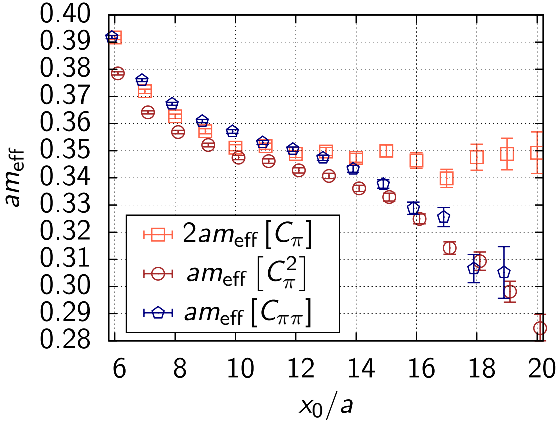
- For finite $T$ contributions from $\langle n| = \langle \pi^+|$ and $\langle m| = \langle \pi^-|$

$$\frac{1}{Z} \left|\langle \pi^+|\mathcal{O}_{\pi\pi}|\pi^-\rangle\right|^2 e^{-m_\pi T}$$

- Comparable to standard contribution at $t = T/2$

$$\frac{1}{Z} \left|\langle \pi^+\pi^+|\mathcal{O}_{\pi\pi}|0\rangle\right|^2 e^{-E_{\pi\pi}^{I=2} T/2} \cosh(E_{\pi\pi}^{I=2}(t - T/2))$$

# Thermal States



# Removal of Thermal states[3]

- taking the ratio

$$\frac{C_{\pi\pi}(t)}{C_{\pi}^{2}(t)} \propto \exp(-\delta E_{\pi\pi}^{I=2} t)$$

$\rightarrow$ thermal states do not cancel in the ratio

- use derivative method

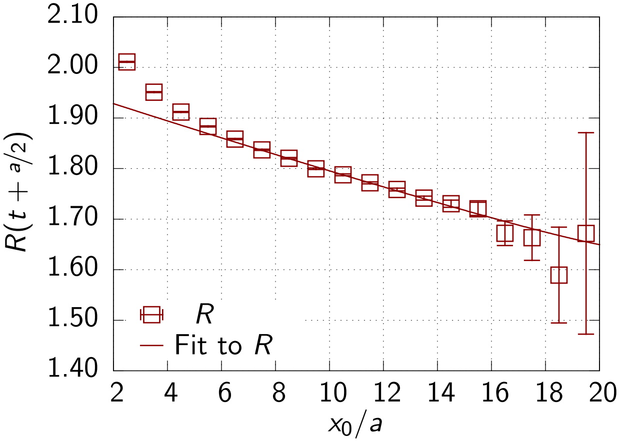
$$\begin{aligned} R(t + {}^{1}\!/_{2}) &= \frac{C_{\pi\pi}(t) - C_{\pi\pi}(t+1)}{C_{\pi}^{2}(t) - C_{\pi}^{2}(t+1)} \\ &= A\left(\cosh(\delta E_{\pi\pi}^{I=2} t') + \sinh(\delta E_{\pi\pi}^{I=2} t') \coth(2 m_{\pi} t')\right) \end{aligned}$$

with $t' = t + \frac{1}{2} - \frac{T}{2}$

- extract $\delta E_{\pi\pi}^{I=2}$ by fitting

[3] X. Feng et al, arxiv:0909.3255 [hep-lat]

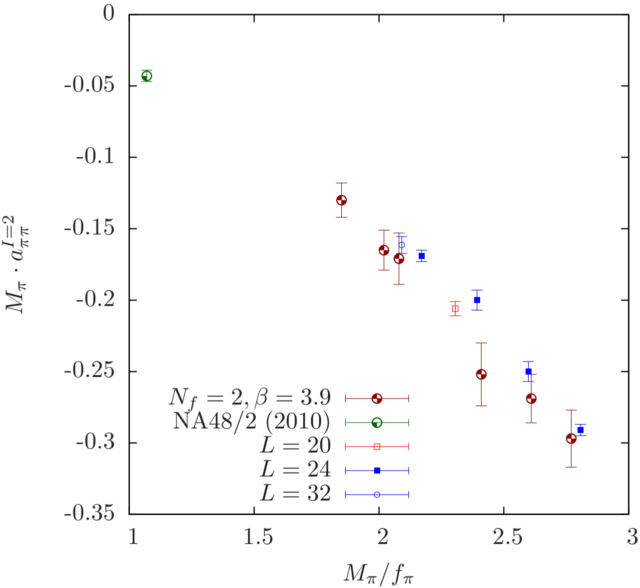
# Removal of Thermal States



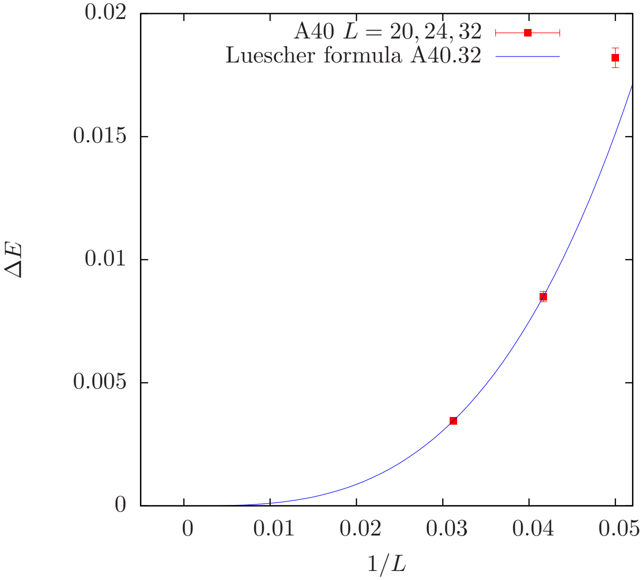
# Overview over Ensembles

| name | $L_s$ | $L_t$ | $r_0$ | $m_\pi$ | $f_\pi$ | # conf |
|---|---|---|---|---|---|---|
| A40.20 | 20 | 48 | 5.231 | 0.14927 | 0.06198 | 150 |
| A40.24 | 24 | 48 | 5.231 | 0.14492 | 0.06568 | 202 |
| A40.32 | 32 | 64 | 5.231 | 0.14142 | 0.06791 | 50 |
| A60.24 | 24 | 48 | 5.231 | 0.17275 | 0.07169 | 97 |
| A80.24 | 24 | 48 | 5.231 | 0.19875 | 0.07623 | 100 |
| A100.24 | 24 | 48 | 5.231 | 0.22293 | 0.07926 | 202 |

# overall data



# Volume Effects on the A40 Ensembles



# Summary

- Analysed the $\eta$ and $\eta'$ on more configurations
- Investigated $\pi^+$-$\pi^+$ scattering and extracted the scattering length
- Good agreement between our new data and older data
- Next order of Lüschers formula has to be investigated

# Future Tasks

- Implementing momentum operators
- Investigation of the $\rho$ meson
- Investigation of $D$-$D^{\star}$ systems
- Investigation of scalar mesons
- advance to larger lattices

Thank you