Renormalization constants for $N_\mathrm{f} = 2+1+1$ twisted mass QCD

Savvas Zafeiropoulos

Laboratoire de Physique Corpusculaire
Université Blaise Pascal, CNRS/IN2P3

May 26-28 2014
LPSC Grenoble

Acknowledgements

To my collaborators: Benoit Blossier, Mariane Brinet, Pierre Guichon, Vincent Morenas, Olivier Péne, Pepe Rodríguez-Quintero and to Tassos Vladikas for fruitful discussions

Outline

- Motivation
- RI'-MOM scheme-generalities
- RCs
- Conclusions and Outlook

Bare vs Renormalized

- Lattice formalism is bare QFT
- One computes bare matrix elements of operators at fixed cutoff
- Must renormalize to obtain continuum Physics
- $O_R = Z_O O_b$
- Renormalization can be done perturbatively or non-perturbatively

Bare vs Renormalized

- Lattice formalism is bare QFT
- One computes bare matrix elements of operators at fixed cutoff
- Must renormalize to obtain continuum Physics
- $O_R = Z_O O_b$
- Renormalization can be done perturbatively or non-perturbatively

Bare vs Renormalized

- Lattice formalism is bare QFT
- One computes bare matrix elements of operators at fixed cutoff
- Must renormalize to obtain continuum Physics
- $O_R = Z_O O_b$
- Renormalization can be done perturbatively or non-perturbatively

Bare vs Renormalized

- Lattice formalism is bare QFT
- One computes bare matrix elements of operators at fixed cutoff
- Must renormalize to obtain continuum Physics
- $O_R = Z_O O_b$
- Renormalization can be done perturbatively or non-perturbatively

Lattice PT

- Lattice PT-notorious for its bad convergence
- MILC collaboration found that m_s was raised by 14% once its renormalization constant (RC) known in 1-loop PT was calculated @ 2-loops.
- Göckeler et al found that m_s was raised by 24% once its RC known in 1-loop PT was calculated non-perturbatively.

Lattice PT

- Lattice PT-notorious for its bad convergence
- MILC collaboration found that m_s was raised by 14% once its renormalization constant (RC) known in 1-loop PT was calculated @ 2-loops.
- Göckeler et al found that m_s was raised by 24% once its RC known in 1-loop PT was calculated non-perturbatively.

Lattice PT

- Lattice PT-notorious for its bad convergence
- MILC collaboration found that m_s was raised by 14% once its renormalization constant (RC) known in 1-loop PT was calculated @ 2-loops.
- Göckeler et al found that m_s was raised by 24% once its RC known in 1-loop PT was calculated non-perturbatively.

Non-Perturbative Renormalization

- RI-MOM scheme Martinelli et al (1995)
- Work on the calculation of the RCs by many groups many of them belonging to the ETMC

  Göckeler et al (1998), Constantinou et al (2009-2012), Dimopoulos et al (2011), Blossier et al (2011)
- Schrödinger Functional scheme Lüscher et al (1996)

Non-Perturbative Renormalization

- RI-MOM scheme Martinelli et al (1995)
- Work on the calculation of the RCs by many groups many of them belonging to the ETMC

  Göckeler et al (1998), Constantinou et al (2009-2012), Dimopoulos et al (2011), Blossier et al (2011)
- Schrödinger Functional scheme Lüscher et al (1996)

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures

- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12} \mathrm{tr}\left[P_O \Lambda_O(p, p)\right]$
- $\Gamma_O(p)_R = \lim_{a \to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2} \mathrm{tr}\left[S_{bare}^{-1}(p) \not{p}\right]$

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures

- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12} \mathrm{tr}\left[P_O \Lambda_O(p,p)\right]$
- $\Gamma_O(p)_R = \lim_{a \to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2} \mathrm{tr}\left[S_{bare}^{-1}(p) \not{p}\right]$

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures
- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12} \mathrm{tr}\left[P_O \Lambda_O(p,p)\right]$
- $\Gamma_O(p)_R = \lim_{a\to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2} \mathrm{tr}\left[S_{bare}^{-1}(p) \not{p}\right]$

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures

- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2)\rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12}\mathrm{tr}\left[P_O \Lambda_O(p, p)\right]$
- $\Gamma_O(p)_R = \lim_{a\to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2}\mathrm{tr}\left[S_{bare}^{-1}(p)\not{p}\right]$

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures

- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12} \mathrm{tr}\left[P_O \Lambda_O(p, p)\right]$
- $\Gamma_O(p)_R = \lim_{a \to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2} \mathrm{tr}\left[S_{bare}^{-1}(p) \not{p}\right]$

RI-MOM

- focus on local fermion bilinears $O_\Gamma = \bar{\psi}(x)\Gamma\psi(x)$

  Vladikas Les Houches lectures

- Γ can be any Dirac structure and can even potentially contain covariant derivatives
- inserting O_Γ in the fermion 2-pt function
- $G_O = \langle u(x_1) O_\Gamma \bar{d}(x_2) \rangle$
- the amputated Green's function
- $\Lambda_O(p_1, p_2) = S_u^{-1}(P_1) G_O(p_1, p_2) S_d^{-1}(p_2)$
- $\Gamma_O(p) = \frac{1}{12} \mathrm{tr}\left[P_O \Lambda_O(p, p)\right]$
- $\Gamma_O(p)_R = \lim_{a \to 0} Z_q^{-1} Z_O \Gamma_O(p)$
- $Z_q(\mu^2 = p^2) = -\frac{i}{12p^2} \mathrm{tr}\left[S_{bare}^{-1}(p) \not{p}\right]$

RI-MOM

- impose that the amputated Green's function in the chiral limit @ a large Euclidean scale $p^2 = \mu^2$ is equal to its tree level value
- $\Gamma_O(p)_R(\mu, g_R, m_R = 0) = \lim_{a \to 0} [Z_q^{-1}(a\mu, g_0) Z_O(a\mu, g_0) \Gamma_O(p, g_0, m)]_{p=\mu^2, m \to 0}$

Window of applicability of RI-MOM

- $\Lambda_{QCD} \ll \mu \ll \frac{\pi}{a}$
- first inequality ensures the possibility of matching with some perturbative scheme $\overline{\text{MS}}$ and protects from Goldstone pole contaminations
- second inequality ensures small cutoff effects

Conversion to $\overline{\text{MS}}$

- make connection with phenomenological calculations and experiments
- need to convert to $\overline{\text{MS}}$ with factors $Z_q^{\overline{\text{MS}}} = C_q^{-1}\, Z_q^{RI'-MOM}$ and $Z_{\mathcal{O}}^{\overline{\text{MS}}} = C_{\mathcal{O}}^{-1}\, Z_{\mathcal{O}}^{RI'-MOM}$
- experiments usually provide results in $\overline{\text{MS}}$ at a reference scale $\mu = 2$ GeV
- evolve $\overline{\text{MS}}$ RCs $Z_{\mathcal{O}}^{\overline{\text{MS}}}$ using the scale dependence predicted by the RG equation van Ritbergen et al (1997), Vermaseren et al (1997), Chetyrkin (1997), Göckeler et al (1998)

$$R_{\mathcal{O}(\mu,\mu_0)} := \frac{Z_{\mathcal{O}(\mu)}}{Z_{\mathcal{O}(\mu_0)}} = \exp\left\{-\int_{\bar{g}(\mu_0^2)}^{\bar{g}(\mu^2)} dg \frac{\gamma(g)}{\beta(g)}\right\}$$

β is the usual QCD-beta function, γ the anomalous dimension of operator $\mathcal{O}$ and $\bar{g}(\mu^2)$ the running coupling

Conversion to $\overline{\text{MS}}$

- make connection with phenomenological calculations and experiments
- need to convert to $\overline{\text{MS}}$ with factors $Z_q^{\overline{\text{MS}}} = C_q^{-1}\, Z_q^{RI'-MOM}$ and $Z_{\mathcal{O}}^{\overline{\text{MS}}} = C_{\mathcal{O}}^{-1}\, Z_{\mathcal{O}}^{RI'-MOM}$
- experiments usually provide results in $\overline{\text{MS}}$ at a reference scale $\mu = 2$ GeV
- evolve $\overline{\text{MS}}$ RCs $Z_{\mathcal{O}}^{\overline{\text{MS}}}$ using the scale dependence predicted by the RG equation van Ritbergen et al (1997), Vermaseren et al (1997), Chetyrkin (1997), Göckeler et al (1998)

$$R_{\mathcal{O}(\mu,\mu_0)} := \frac{Z_{\mathcal{O}(\mu)}}{Z_{\mathcal{O}(\mu_0)}} = \exp\left\{-\int_{\bar{g}(\mu_0^2)}^{\bar{g}(\mu^2)} dg \frac{\gamma(g)}{\beta(g)}\right\}$$

β is the usual QCD-beta function, γ the anomalous dimension of operator $\mathcal{O}$ and $\bar{g}(\mu^2)$ the running coupling

Simulation setup

- $S = S_{Iwa}^{YM} + a^4 \sum_{x,f} \bar{\chi}_f \left(\gamma \cdot \nabla - \frac{a}{2} \nabla \cdot \nabla + m_0 + i r_f \mu_f \gamma_5 \right) \chi_f$
- to achieve the benefits of the TM formulation one needs to work at maximal twist $\theta = \pi/2$ Frezzotti and Rossi (2003-2004)
- automatic $\mathcal{O}(a)$ improvement
- for $N_\mathrm{f} = 4$ maximal twist (tuning m_{PCAC} to zero was a highly non trivial task at the time these configurations where produced

Simulation setup

- $S = S_{Iwa}^{YM} + a^4 \sum_{x,f} \bar{\chi}_f \left(\gamma \cdot \nabla - \frac{a}{2} \nabla \cdot \nabla + m_0 + i r_f \mu_f \gamma_5 \right) \chi_f$
- to achieve the benefits of the TM formulation one needs to work at maximal twist $\theta = \pi/2$ Frezzotti and Rossi (2003-2004)
- automatic $\mathcal{O}(a)$ improvement
- for $N_f = 4$ maximal twist (tuning m_{PCAC} to zero was a highly non trivial task at the time these configurations where produced

Simulation setup

- $S = S_{Iwa}^{YM} + a^4 \sum_{x,f} \bar{\chi}_f \left(\gamma \cdot \nabla - \frac{a}{2} \nabla \cdot \nabla + m_0 + i r_f \mu_f \gamma_5 \right) \chi_f$
- to achieve the benefits of the TM formulation one needs to work at maximal twist $\theta = \pi/2$ Frezzotti and Rossi (2003-2004)
- automatic $\mathcal{O}(a)$ improvement
- for $N_\mathrm{f} = 4$ maximal twist (tuning m_{PCAC} to zero was a highly non trivial task at the time these configurations where produced

Simulation setup

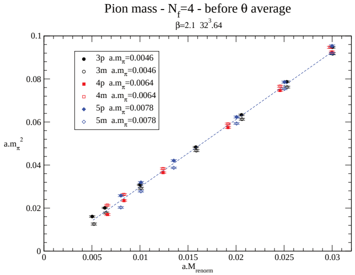
- $S = S_{Iwa}^{YM} + a^4 \sum_{x,f} \bar{\chi}_f \left(\gamma \cdot \nabla - \frac{a}{2} \nabla \cdot \nabla + m_0 + i r_f \mu_f \gamma_5 \right) \chi_f$
- to achieve the benefits of the TM formulation one needs to work at maximal twist $\theta = \pi/2$ Frezzotti and Rossi (2003-2004)
- automatic $\mathcal{O}(a)$ improvement
- for $N_\mathrm{f} = 4$ maximal twist (tuning m_{PCAC} to zero was a highly non trivial task at the time these configurations where produced

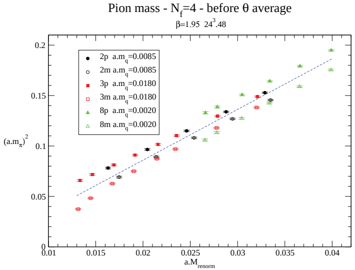
Ensembles

ensemble	κ	am_{PCAC}	$a\mu$ ($a\mu_{sea}$ in bold)	confs #
$\beta = 2.10$ - $32^3.64$				
3p	0.156017	+0.00559(14)	0.0025, **0.0046**, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297	250
3m	0.156209	-0.00585(08)	0.0025, **0.0046**, 0.0090, 0.0152, 0.0201, 0.0249, 0.0297	250
4p	0.155983	+0.00685(12)	0.0039, **0.0064**, 0.0112, 0.0184, 0.0240, 0.0295	210
4m	0.156250	-0.00682(13)	0.0039, **0.0064**, 0.0112, 0.0184, 0.0240, 0.0295	210
5p	0.155949	+0.00823(08)	0.0048, **0.0078**, 0.0119, 0.0190, 0.0242, 0.0293	220
5m	0.156291	-0.00821(11)	0.0048, **0.0078**, 0.0119, 0.0190, 0.0242, 0.0293	220
$\beta = 1.95$ - $24^3.48$				
2p	0.160826	+0.01906(24)	**0.0085**, 0.0150, 0.0203, 0.0252, 0.0298	290
2m	0.161229	-0.02091(16)	**0.0085**, 0.0150, 0.0203, 0.0252, 0.0298	290
3p	0.160826	+0.01632(21)	0.0060, 0.0085, 0.0120, 0.0150, **0.0180**, 0.0203, 0.0252, 0.0298	310
3m	0.161229	-0.01602(20)	0.0060, 0.0085, 0.0120, 0.0150, **0.0180**, 0.0203, 0.0252, 0.0298	310
8p	0.160524	+0.03634(14)	**0.0020**, 0.0085, 0.0150, 0.0203, 0.0252, 0.0298	310
8m	0.161585	-0.03627(11)	**0.0020**, 0.0085, 0.0150, 0.0203, 0.0252, 0.0298	310
$\beta = 1.90$ - $24^3.48$				
1p	0.162876	+0.0275(04)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	450
1m	0.163206	-0.0273(02)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	450
4p	0.162689	+0.0398(01)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	370
4m	0.163476	-0.0390(01)	0.0060, **0.0080**, 0.0120, 0.0170, 0.0210, 0.0260	370

$N_f = 4$ ensembles used in our analysis
The lattice spacing values are respectively $a = 0.062$ fm for $\beta = 2.10$, $a = 0.078$ fm for $\beta = 1.95$ and $a = 0.086$ fm for $\beta = 1.90$

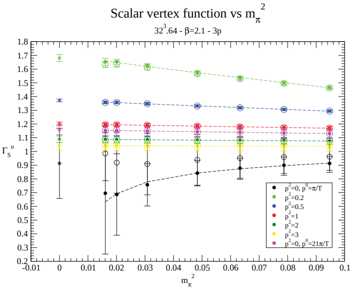
Pion Masses

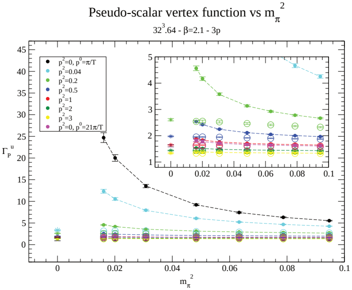




Pion mass for each ensemble, before θ average. The x-axis is the renormalized quark mass $M_{renorm} = \sqrt{(Z_A m_{PCAC})^2 + m_q^2}$ and the y-axis is the pion mass squared. The difference between m/p ensembles illustrates the consequence of non maximal twist and $\mathcal{O}(a)$ effects. The result of the straight line fit using pion mass values computed after θ average is shown in dashed blue curve.

Vertex Functions - The effect of the Goldstone pole subtraction





u scalar (LHS) and pseudo-scalar (RHS) vertex functions versus pion mass squared (in lattice unit) for ensemble $3p$ for several values of $a^2\vec{p}^{\,2}$. (Full-) empty circles correspond to (un-)subtracted values while $*$ to the chiral extrapolation, ($a.p^0 = \frac{\pi}{T}$ for all curves except the magenta one, for which $a.p^0 = \frac{21\pi}{T}$).

Pion Pole Contamination

- Correlation functions of the pseudoscalar operator have pion pole contamination
- need to be addressed carefully
- ansatz for the amputated pseudoscalar vertex
  $\Gamma_P = a_P + b_P m_\pi^2 + \frac{c_P}{m_\pi^2}$
- $\Gamma_P^{sub} = \Gamma_P - \frac{c_P}{m_\pi^2}$

Pion Pole Contamination

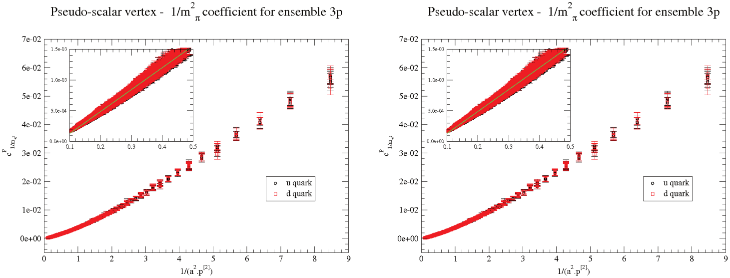
- Correlation functions of the pseudoscalar operator have pion pole contamination
- need to be addressed carefully
- ansatz for the amputated pseudoscalar vertex $\Gamma_P = a_P + b_P m_\pi^2 + \frac{c_P}{m_\pi^2}$
- $\Gamma_P^{sub} = \Gamma_P - \frac{c_P}{m_\pi^2}$

Pion Pole Contamination

- Correlation functions of the pseudoscalar operator have pion pole contamination
- need to be addressed carefully
- ansatz for the amputated pseudoscalar vertex $\Gamma_P = a_P + b_P m_\pi^2 + \frac{c_P}{m_\pi^2}$
- $\Gamma_P^{sub} = \Gamma_P - \frac{c_P}{m_\pi^2}$

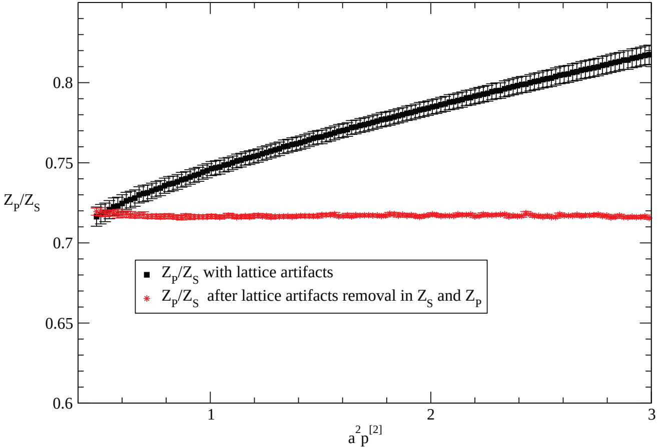
Pion Pole Contamination

- Correlation functions of the pseudoscalar operator have pion pole contamination
- need to be addressed carefully
- ansatz for the amputated pseudoscalar vertex
  $\Gamma_P = a_P + b_P m_\pi^2 + \frac{c_P}{m_\pi^2}$
- $\Gamma_P^{sub} = \Gamma_P - \frac{c_P}{m_\pi^2}$



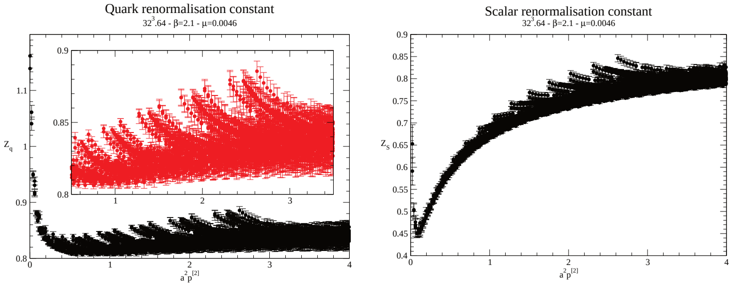
Coefficient of the $1/m_\pi^2$ term (LHS) (varies as $1/p^2$ @ large p^2) and of the m_π^2 term (RHS) in the chiral fit, $C(p^2)$ and $B(p^2)$ respectively as a function of $1/(a^2p^2)$, for ensemble $3p$. The green line is for eye guidance mainly and represents a linear fit at large p^2

Z_P/Z_S



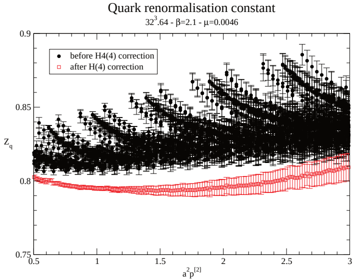
Z_P/Z_S for ensemble $3mp$ ($\beta = 2.10$, $\mu = 0.0046$, volume $32^3.64$). Lattice artifacts have been removed separately from Z_S and Z_P. The ratio of these two RCs is compatible with a constant over the whole a^2p^2 interval and $Z_P/Z_S = 0.717(3)$.

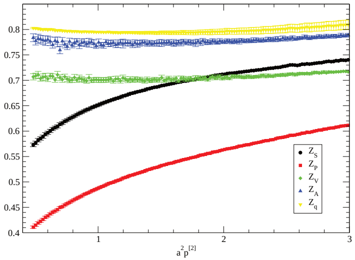
Z_q and Z_S after $H(3)$ corrections



Quark renormalization constant (LHS) and scalar renormalization constant (RHS.) as a function of $a^2p^{[2]}$. Both exhibit the typical "fishbone" structure induced by the breaking of the $O(4)$ rotational symmetry of the Euclidian space-time by the lattice discretization, into the hypercubic group $H(4)$.

RCs after $H(4)$ corrections





LHS: Effect of hypercubic corrections on quark renormalization constant, as a function of $a^2p^{[2]}$. RHS: renormalization constants as a function of $a^2p^{[2]}$, after removing $H(4)$ artifacts.

Correcting for artifacts

- hypercubic artifacts that respect $\mathrm{H}(4)$ but not $\mathrm{O}(4)$
- artifacts that respect $\mathrm{O}(4)$ will be treated NP by introducing corrections to the running
- egalitarian method (does not rely on the selection of diagonal momenta which have small $\mathrm{H}(4)$ artifacts like the method of democratic cuts Boucaud et al (2003), de Soto et al (2007)
- keeps maximum amount of info- allows for the testing of the running of RCs

Correcting for artifacts

- hypercubic artifacts that respect $\mathrm{H}(4)$ but not $\mathrm{O}(4)$
- artifacts that respect $\mathrm{O}(4)$ will be treated NP by introducing corrections to the running
- egalitarian method (does not rely on the selection of diagonal momenta which have small $\mathrm{H}(4)$ artifacts like the method of democratic cuts Boucaud et al (2003), de Soto et al (2007)
- keeps maximum amount of info- allows for the testing of the running of RCs

Correcting for artifacts

- hypercubic artifacts that respect $\mathrm{H}(4)$ but not $\mathrm{O}(4)$
- artifacts that respect $\mathrm{O}(4)$ will be treated NP by introducing corrections to the running
- egalitarian method (does not rely on the selection of diagonal momenta which have small $\mathrm{H}(4)$ artifacts like the method of democratic cuts Boucaud et al (2003), de Soto et al (2007)
- keeps maximum amount of info- allows for the testing of the running of RCs

Correcting for artifacts

- we define the $\mathrm{H}\,(4)$ invariants
- $p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4, \qquad p^{[6]} = \sum_{\mu=1}^{4} p_\mu^6, \qquad p^{[8]} = \sum_{\mu=1}^{4} p_\mu^8$
- Expand the RC already averaged over the cubic orbits around $p^{[4]} = 0$
- $Z_{latt}(a^2p^2, a^4p^{[4]}, a^6p^{[6]}, ap_4, a^2\Lambda_{QCD}) = Z_{hypcorrected}(a^2p^2, ap_4, a^2\Lambda_{QCD}) + R(a^2p^2, a^2\Lambda_{QCD})\frac{a^2p^{[4]}}{p^2} + \dots$
- $R(a^2p^2, a^2\Lambda_{QCD}) = \frac{dZ_{latt}(a^2p^2,0,0,0,a^2\Lambda_{QCD})}{d\epsilon}|_{\epsilon=0} = c_{a2p4} + c_{a4p4}a^2p^2$

Correcting for artifacts

- we define the $\mathrm{H}(4)$ invariants
- $p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4, \qquad p^{[6]} = \sum_{\mu=1}^{4} p_\mu^6, \qquad p^{[8]} = \sum_{\mu=1}^{4} p_\mu^8$
- Expand the RC already averaged over the cubic orbits around $p^{[4]} = 0$
- $Z_{latt}(a^2p^2, a^4p^{[4]}, a^6p^{[6]}, ap_4, a^2\Lambda_{QCD}) = Z_{hypcorrected}(a^2p^2, ap_4, a^2\Lambda_{QCD}) + R(a^2p^2, a^2\Lambda_{QCD})\frac{a^2p^{[4]}}{p^2} + \ldots$
- $R(a^2p^2, a^2\Lambda_{QCD}) = \frac{dZ_{latt}(a^2p^2,0,0,0,a^2\Lambda_{QCD})}{d\epsilon}|_{\epsilon=0} = c_{a2p4} + c_{a4p4}a^2p^2$

Correcting for artifacts

- we define the $\mathrm{H}(4)$ invariants
- $p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4, \qquad p^{[6]} = \sum_{\mu=1}^{4} p_\mu^6, \qquad p^{[8]} = \sum_{\mu=1}^{4} p_\mu^8$
- Expand the RC already averaged over the cubic orbits around $p^{[4]} = 0$
- $Z_{latt}(a^2p^2, a^4p^{[4]}, a^6p^{[6]}, ap_4, a^2\Lambda_{QCD}) = Z_{hypcorrected}(a^2p^2, ap_4, a^2\Lambda_{QCD}) + R(a^2p^2, a^2\Lambda_{QCD})\frac{a^2p^{[4]}}{p^2} + \ldots$
- $R(a^2p^2, a^2\Lambda_{QCD}) = \frac{dZ_{latt}(a^2p^2,0,0,0,a^2\Lambda_{QCD})}{d\epsilon}|_{\epsilon=0} = c_{a2p4} + c_{a4p4}a^2p^2$

Running of Z_q

- consider for the running of Z_q Blossier et al (2010)
-

$$
\begin{aligned}
Z_q^{hyp-corr}(a^2p^2) &= Z_q^{pert\,RI'}(\mu^2)\, c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu)) \\
&\times \left(1 + \frac{\langle A^2\rangle_{\mu^2}}{32p^2} \frac{c_{2Z_q}^{\overline{\mathrm{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))} \frac{c_{2Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{2Z_q}^{\overline{\mathrm{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}\right) \\
&+ c_{a2p2}\, a^2\, p^2 + c_{a4p4}\, (a^2p^2)^2
\end{aligned}
$$

- coefficients $c_{0Z_q}^{RI'}$ and $c_{2Z_q}^{\overline{\mathrm{MS}}}$ known from PT
- the running formula contains lattice artifact terms $\propto a^2p^2$ and $\propto (a^2p^2)^2$, not yet removed.
- need to determine, $Z_q^{pert\,RI'}(\mu^2)$, $\langle A^2\rangle_{\mu^2}$, c_{a2p2} and c_{a4p4}

Running of Z_q

- consider for the running of Z_q Blossier et al (2010)
-

$$\begin{aligned} Z_q^{hyp-corr}(a^2p^2) &= Z_q^{pert\,RI'}(\mu^2)\, c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu)) \\ &\times \left(1 + \frac{\langle A^2\rangle_{\mu^2}}{32p^2} \frac{c_{2Z_q}^{\overline{\text{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))} \frac{c_{2Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{2Z_q}^{\overline{\text{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}\right) \\ &+ c_{a2p2}\, a^2\, p^2 + c_{a4p4}\, (a^2p^2)^2 \end{aligned}$$

- coefficients $c_{0Z_q}^{RI'}$ and $c_{2Z_q}^{\overline{\text{MS}}}$ known from PT
- the running formula contains lattice artifact terms $\propto a^2p^2$ and $\propto (a^2p^2)^2$, not yet removed.
- need to determine, $Z_q^{pert\,RI'}(\mu^2)$, $\langle A^2\rangle_{\mu^2}$, c_{a2p2} and c_{a4p4}

Running of Z_q

- consider for the running of Z_q Blossier et al (2010)
-

$$
\begin{aligned}
Z_q^{hyp-corr}(a^2p^2) &= Z_q^{pert\,RI'}(\mu^2)\, c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu)) \\
&\times \left(1 + \frac{\langle A^2\rangle_{\mu^2}}{32p^2}\frac{c_{2Z_q}^{\overline{\mathsf{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}\frac{c_{2Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{2Z_q}^{\overline{\mathsf{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}\right) \\
&+ c_{a2p2}\, a^2\, p^2 + c_{a4p4}\, (a^2p^2)^2
\end{aligned}
$$

- coefficients $c_{0Z_q}^{RI'}$ and $c_{2Z_q}^{\overline{\mathsf{MS}}}$ known from PT
- the running formula contains lattice artifact terms $\propto a^2p^2$ and $\propto (a^2p^2)^2$, not yet removed.
- need to determine, $Z_q^{pert\,RI'}(\mu^2)$, $\langle A^2\rangle_{\mu^2}$, c_{a2p2} and c_{a4p4}

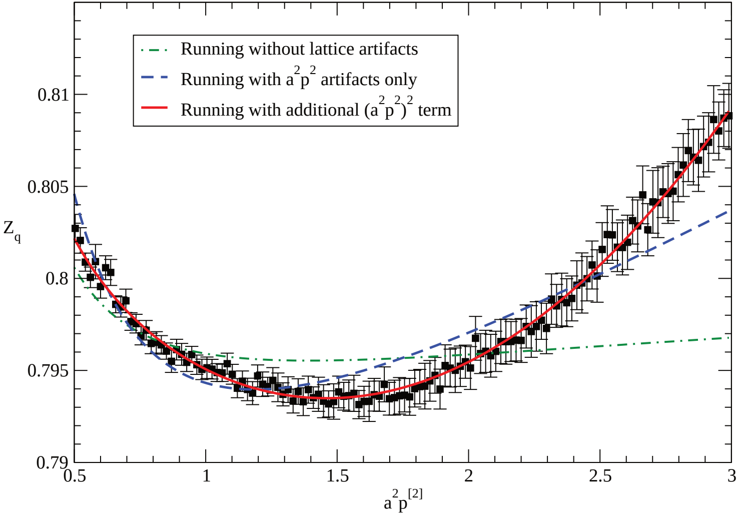
Running of Z_q

- consider for the running of Z_q Blossier et al (2010)
-

$$
\begin{aligned}
Z_q^{hyp-corr}(a^2p^2) &= Z_q^{pert\,RI'}(\mu^2)\, c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu)) \\
&\times \left(1+\frac{\langle A^2\rangle_{\mu^2}}{32p^2}\frac{c_{2Z_q}^{\overline{\mathsf{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{0Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}\frac{c_{2Z_q}^{RI'}(\frac{p^2}{\mu^2},\alpha(\mu))}{c_{2Z_q}^{\overline{\mathsf{MS}}}(\frac{p^2}{\mu^2},\alpha(\mu))}\right) \\
&+ c_{a2p2}\, a^2\, p^2 + c_{a4p4}\,(a^2p^2)^2
\end{aligned}
$$

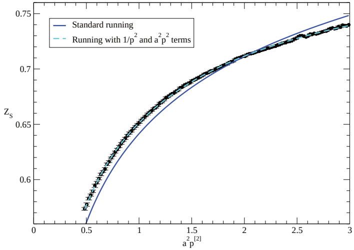
- coefficients $c_{0Z_q}^{RI'}$ and $c_{2Z_q}^{\overline{\mathsf{MS}}}$ known from PT
- the running formula contains lattice artifact terms $\propto a^2p^2$ and $\propto (a^2p^2)^2$, not yet removed.
- need to determine, $Z_q^{pert\,RI'}(\mu^2)$, $\langle A^2\rangle_{\mu^2}$, c_{a2p2} and c_{a4p4}

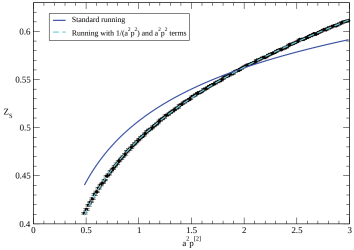
Running of Z_q



Running of Z_q for ensemble $3mp$ ($\beta = 2.10$, $\mu = 0.0046$, volume $32^3.64$) using different fitting formulae.

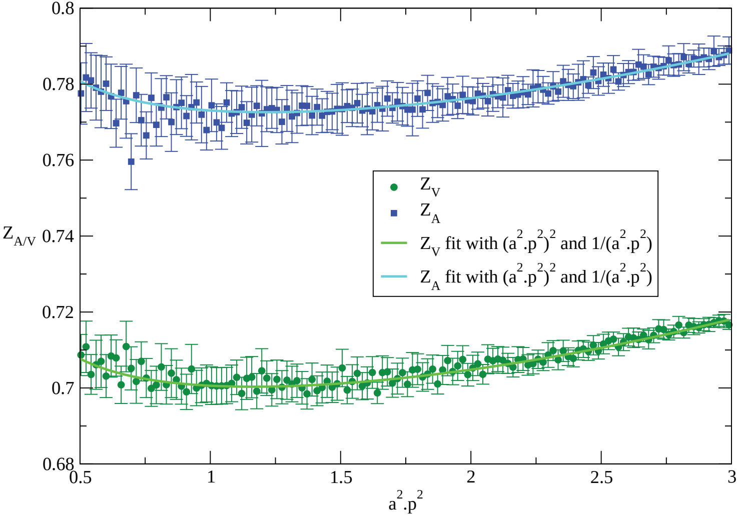
Running of Z_S and Z_P





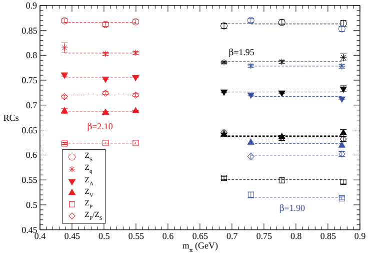
LHS: running of Z_S for ensemble $3mp$ ($\beta = 2.10$, $\mu = 0.0046$, volume $32^3.64$). The standard running formula is represented in solid blue line, the dashed cyan curve includes an $1/a^2p^2$ and an a^2p^2 term. This latter fit leads to $Z_S(10$ GeV$) = 0.869(4)$. RHS: Running of Z_P with the standard running expression Chetyrkin et al (1999) (solid blue curve), and adding an $1/a^2p^2$ and an a^2p^2 terms (dashed cyan curve). The modified running gives $Z_P(10$ GeV$) = 0.623(2)$.

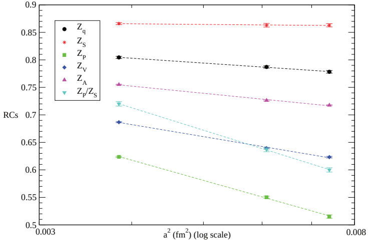
Z_V and Z_A



Fits of the residual a^2p^2 dependence of Z_V and Z_A for ensemble $3mp$ ($\beta = 2.10$, $\mu = 0.0046$, volume $32^3.64$)

Chiral extrapolation and lattice spacing dependence





LHS: $N_f = 4$ local RCs dependence with the pion mass. The straight dashed lines are constant fits for each β values. The red points correspond to $\beta = 2.10$, the black ones to $\beta = 1.95$, and the blue ones to $\beta = 1.90$.
RHS: RCs after chiral extrapolation, vs $\log a^2$. All RCs follow a linear dependence with $\log a^2$ to a very high accuracy.

Results in $\overline{\text{MS}}$

β	Z_q	Z_S	Z_P	Z_V	Z_A	Z_P/Z_S
1.90	0.767(3)	0.910(3)	0.543(3)	0.623(2)	0.717(1)	0.600(4)
1.95	0.775(2)	0.903(4)	0.576(2)	0.639(2)	0.726(2)	0.637(4)
2.10	0.791(2)	0.887(2)	0.639(1)	0.687(1)	0.755(1)	0.720(4)

converted our RI'-MOM results at 10 GeV to $\overline{\text{MS}}$ values at a reference scale of 2 GeV leads to the final RCs

Conclusions and Outlook

- Provided NP results for the RCs of $N_\mathrm{f} = 2 + 1 + 1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

Conclusions and Outlook

- Provided NP results for the RCs of $N_\mathrm{f} = 2 + 1 + 1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

Conclusions and Outlook

- Provided NP results for the RCs of $N_{\rm f} = 2 + 1 + 1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

Conclusions and Outlook

- Provided NP results for the RCs of $N_{\mathrm{f}} = 2+1+1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

Conclusions and Outlook

- Provided NP results for the RCs of $N_\mathrm{f} = 2+1+1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

Conclusions and Outlook

- Provided NP results for the RCs of $N_\mathrm{f} = 2 + 1 + 1$ Twisted Mass QCD
- Hypercubic artifacts were taken correctly into account by the "egalitarian" method
- Complete the analysis of twist-2 operators
- Extend our work to the new ensembles of ETMC with the large volumes $48^3 \times 96$ and masses @ the physical point
- Check the effect of Gribov copies
- Perform the analysis using the RI-SMOM scheme

for upcoming results ...
Thank you for your attention!