Julius-Maximilians-
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Supersymmetric RGEs with Threshold Effects

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Outline

Introduction

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- Solves hierarchy problem, gauge unification

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name		spin 0	spin 1/2	Q. N.
Squarks, Quarks	Q	$(\tilde{u}_L\, \tilde{d}_L)$	$(u_L\, d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(3 families)	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
Sleptons, Leptons	L	$(\tilde{\nu}\, \tilde{e}_L)$	$(\nu\, e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(3 families)	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\bar{\mathbf{1}}, \mathbf{1}, 1)$
Higgs, Higgsinos	H_u	$(H_u^+\, H_u^0)$	$(\tilde{H}_u^+\, \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
	H_d	$(H_d^0\, H_d^-)$	$(\tilde{H}_d^0\, \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

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Winos, W Bosons	$\tilde{W}^{\pm}\, \tilde{W}^0$	$W^{\pm}\, W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
Bino, B Boson	$\tilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

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- The particles in the previous tables are the so called gauge eigenstates
- This particles mix to the mass eigenstates, e.g. neutralinos, charginos, light and heavy Higgs

Lagrangian

- The MSSM superpotential is

$$W = -\mathbf{Y_e} L \bar{e} H_d - \mathbf{Y_d} Q \bar{d} H_d + \mathbf{Y_u} Q \bar{u} H_u + \mu H_d H_u$$

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$$\mathcal{L} = -\sqrt{2} g (\phi^* T^a \Psi) \lambda^a - \sqrt{2} g \lambda^{\dagger a} (\Psi^\dagger T^a \phi).$$

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Relations between trilinear couplings

- Higgs-fermion-fermion- and Higgsino-fermion-sfermion-couplings have same strength
- Sfermion-fermion-gaugino couplings are proportional to the gauge couplings

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F- and D-Terms

The quartic scalar couplings in SUSY are proportional to the square of gauge and Yukawa couplings:

$$\lambda = c_1 Y^2 + c_2 g^2$$

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- Suppression of flavour changing and CP-violation has to be explained

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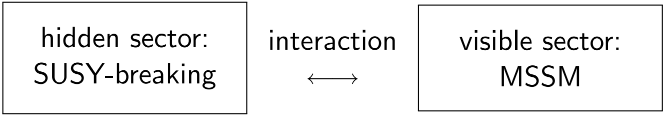

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- The nine SPS-Points describe common SUSY scenarios

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- Connection between μ and your physical parameters are described by the Renormalization Group (RG)

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- Can be calculated from the renormalization constants Z

SUSY RGEs

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  $\rightarrow$ Step Beta Approach: Use step functions $\Theta_x = \Theta(\mu^2 - m_x^2)$

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Example: Mass of the up-Squark:

$$\begin{aligned}\frac{d}{d\ln\mu}m^2_{\tilde U} \quad = \quad & \frac{1}{16\pi^2}\Bigg(4Y_u^2(m^2_{\tilde U}\Theta_{\tilde H_u}+m^2_{\tilde Q}\Theta_{\tilde Q}+(m^2_{H_u}+\mu^2)\Theta_{H_u}+ \\ & A_u^2\Theta_{H_u\tilde Q}+\mu^2(\Theta_{H_d\tilde Q}-2\Theta_{\tilde H_u}))- \\ & -\frac{32}{3}g_3^2M_3^2\Theta_{\tilde g}-\frac{32}{15}g_1^2M_1^2\Theta_{\tilde B}-\frac{4}{5}S- \\ & -\left(\frac{4}{3}g_3^2(\Theta_{\tilde U}-\Theta_{\tilde g})+\frac{16}{15}g_1^2(\Theta_{\tilde U}-\Theta_{\tilde B})\right)m^2_{\tilde U}\Bigg)\end{aligned}$$

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That's right in SUSY, but what happens below the thresholds?

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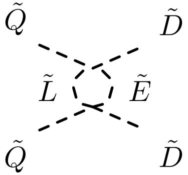

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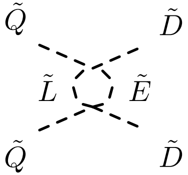

$\rightarrow$ SUSY-Thresholds!

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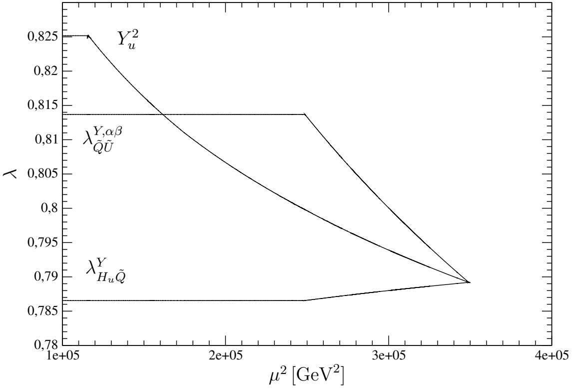

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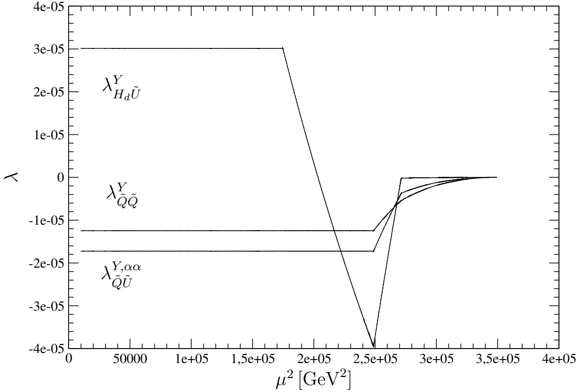

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- Convention: The new couplings are named by the scalar particle involved, e.g. $Y_{u,\tilde{Q}}$
- Also the trilinear couplings proportional to μY are different
- Also the relation

$$h_i = A_i Y_i$$

  can't always be right

Fermion-Sfermion-Gaugino Couplings

- The β-functions for gauge- or Gaugino-sfermion-fermion-coupling are

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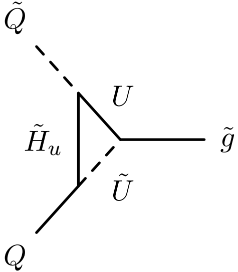

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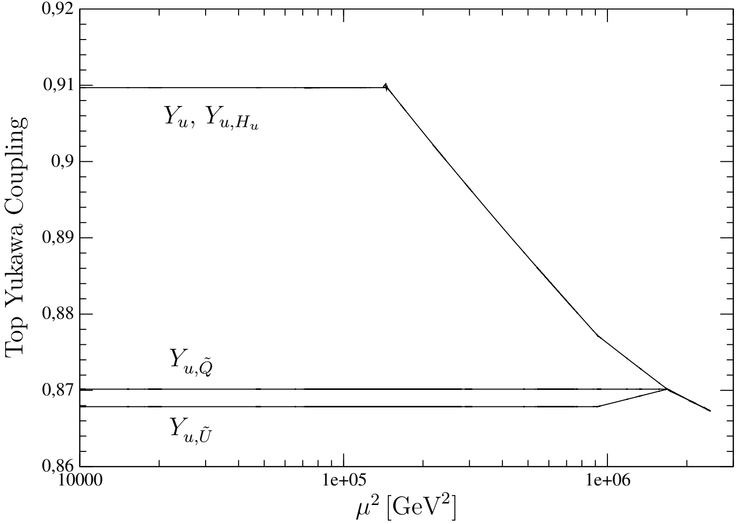

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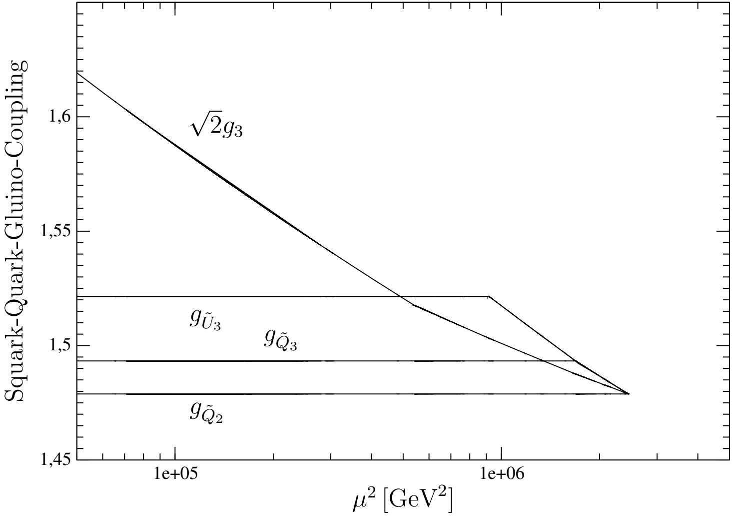
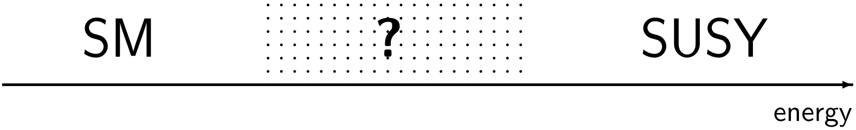


Summary



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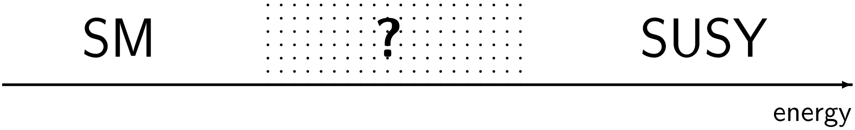

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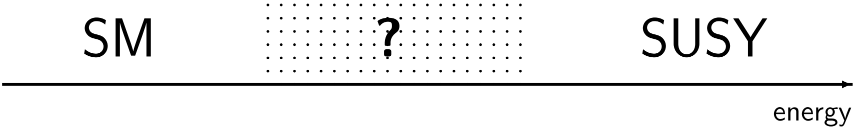
> Below the mass of the heaviest SUSY particle some SUSY relations can be wrong

Summary



SUSY and SM only set the boundary conditions, but between both you have to consider many different effective theories

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New set of RGEs: 82 instead of only 21 coupled equations

Operator Product Expansion

Local Operators and Wilson Coefficients

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- The coefficients fullfill also an RGE: $\frac{dC(\mu)}{d\ln\mu} = \gamma C(\mu)$

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- Anomaly Mediated SUSY Breaking:
  The gauginos, esp. the gluino, much heavier than the scalars
  $\longrightarrow$ dimension 5 operators
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$\longrightarrow$ All effective operators and the running of the Wilson coefficients for these scenarios were calculated

Using

The effective operators and the Wilson coefficients could be used for, e.g.

- Heavy gluino: Production of SUSY particles

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$\longrightarrow$ Contributions to SUSY masses and couplings

Results

Masses: General Results

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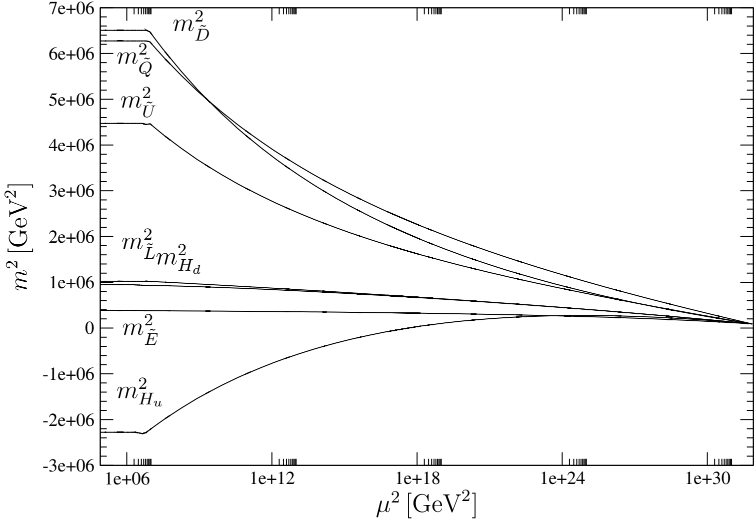
- Different mass hierarchies: Can be used to favour/disfavour high energy theory

Running Scalar Masses: mSugra (SPS 2) I



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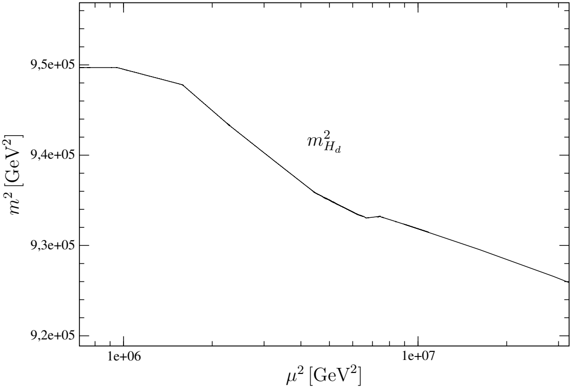
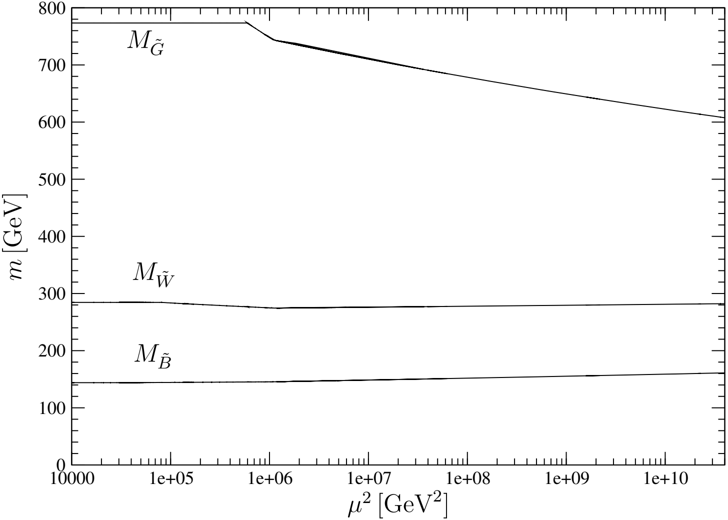


Running Gaugino Masses: GMSB (SPS 8)



Effects of exact Decoupling

In many calculations all SUSY particles are integrated out at one scale

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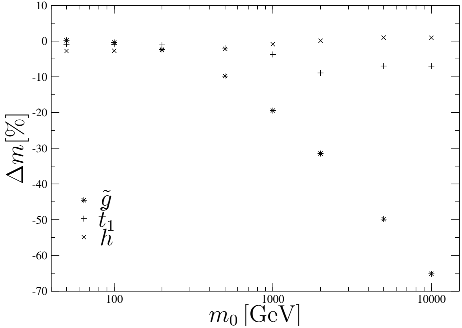

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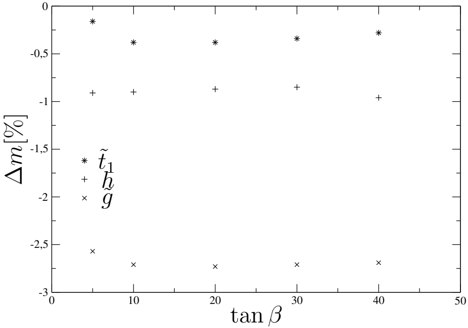

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- Multi scale decoupling changes the calculated masses of squarks 0.5% - 2.0%
- Changes are sensitive to m_0 and $M_{\frac{1}{2}}$, not so to $\tan\beta$
- Example (SPS 2): light Higgs mass changes about 4.1%

Effect of decoupling will be measurable at the LHC!

Effects of independent Couplings and Wilson Coefficients

SPS 2	one scale	mulit scale	'exact couplings'
$m_{\tilde{g}}$ [GeV]	795.58	883.95	883.92
$m_{\tilde{t}_1}$ [GeV]	971.33	987.33	987.52
m_{h^0} [GeV]	115.38	120.33	120.32

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Effects of independent couplings will be measurable at the ILC

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- The dependence of the masses on the energy is described by RGEs
- The running of the masses is fixed by the high energy limit you use as input
- Below a mass threshold you get a new, effective theory by integrating out the heavy particles
- The effect of integrating out every particle by its mass instead of integrating out all particles at one could be measurable at the LHC for some scenarios
- The effects of independent couplings and effective operators are small and won't be measurable at the LHC

Backup

Superfields

- Chiral Superfields ($D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot\alpha}\Theta^{*\dot\alpha}\partial_\mu$)

$$D_\alpha \Phi(x,\Theta,\bar\Theta) = 0 \;\rightarrow\; \Phi = A(x) + \sqrt{2}\Theta\Psi(x) + \Theta^2 F(x)$$

- Vector Superfields in Wess-Zumino-gauge:

$$V = V^\dagger \;\rightarrow\; V = -\Theta\sigma^\mu\Theta^* A_\mu + i\Theta^2\bar\Theta\bar\lambda - i\bar\Theta^2\Theta\lambda + \frac{1}{2}\Theta^2\bar\Theta^2 D$$

- Field Strength $W_\alpha = \bar D^2 D_\alpha V = \lambda_\alpha(x) + \Theta\sigma^{\mu\nu}F_{\mu\nu} + \Theta_\alpha D$
- Lagrangian: $\mathcal{L}_{\text{kin}} = \int d^4\Theta \sum_i \Phi_i^\dagger e^V \Phi_i$,
  $\mathcal{L}_{\text{W}} = \int d^2\Theta W(\Phi) + h.c.$, $\mathcal{L}_{\text{gauge}} = \frac{1}{g^{(i)2}} \int d^2\Theta W_\alpha^{(i)2}$

F- and D-Terms

- Relevant Terms of the Lagrangian:

$$\mathcal{L}_{F,D} = F^* F + \frac{\partial W}{\partial \phi^i} F + \frac{1}{2g} D^a D^a + g(\phi^* T^a \phi) D^a \quad (2)$$

- Euler Lagrange Formulas:

$$\frac{d}{dt} \frac{\partial \mathcal{L}_{F,D}}{\partial F} = 0, \; \frac{d}{dt} \frac{\partial \mathcal{L}_{F,D}}{\partial D} = 0 \quad (3)$$

- Equations of Motion:

$$D^a = g^2 (\Phi^* T^a \Phi) \quad (4)$$

$$F = \frac{\partial W}{\partial \Phi^i} = \frac{1}{2} y^{ijk} \phi_j \phi_k \quad (5)$$

Derivation of the β-function

- Dimensional Reduction: $g_0 = Zg\mu^\epsilon$
- β-function: $\frac{dg(\mu)}{d\ln\mu} = \beta(g(\mu),\epsilon)$
  $\rightarrow\ \beta(g,\epsilon) = g_0\mu\frac{d}{d\mu}(\mu^{-\epsilon}Z^{-1}) = -\epsilon g - g\mu\frac{1}{Z}\frac{dZ}{d\mu}$
- 4 dimensions: $\beta(g) = -g\frac{1}{Z}\frac{dZ}{\ln\mu}$
- Expand Z: $Z = 1 + \sum_k \frac{1}{\epsilon^k}Z_k$
- $\frac{\mu}{Z}dZd\mu(1 + \frac{Z_1}{\epsilon} + \frac{Z_2}{\epsilon^2} + \dots) = \mu\frac{dZ}{d\mu} = \mu\frac{dg}{d\mu}\frac{dZ}{dg} =$
  $\frac{1}{\epsilon}\beta(g,\epsilon)\left(\frac{dZ_1}{dg} + \frac{1}{\epsilon}\frac{dZ_2}{dg} + \dots\right)$
- coefficient comparison: $\beta(g) = 2g^3\frac{dZ_1}{dg^2}$

anomalous Dimensions

- Scalar wave function:
  - Counter term: $\delta Z = -\text{Re}\frac{\partial \Pi_S^{\text{un}}(m^2)}{\partial k^2}\Big|_{k^2=m^2} = c\frac{1}{\epsilon}$
  - anomalous dimension:
    $\gamma_{S,ab} = -c = \frac{1}{16\pi^2}\left(\text{Tr}(\mathbf{Y_a}\mathbf{Y_b}^\dagger) - 2g^2 C(S)\delta_{ab}\right)$
  - Mass renormalization:
    $\delta m^2 = \text{Re}\Pi_S^{\text{un}}(m_S^2)$, $Z_m = 1 + \frac{\delta m_S^2}{m_S^2}$, $\gamma_m = -2g^2\frac{dZ_m}{dg^2}$
- Fermion wave function:
  - Counter term $\delta Z = -\text{Re}\Pi_F^{\text{un}}(m_f^2)$
  - anomalous dimension: same as scalar (Superfields!)
  - Mass renormalization: $\delta m_F = \frac{1}{2} m_F \text{Re}\left(\Pi_F^{\text{un},F} + 2\Pi_F^{\text{un},S}\right)$
- Vector wave function:
  - Counter term: analog scalar
  - anomalous dimension:
    $\gamma_V = -\frac{1}{16\pi^2} g^2\left(\frac{11}{3}C(G) - \frac{2}{3}S(F) - \frac{1}{3}S(S)\right)$

Beta functions without thresholds

- Gauge coupling: $\beta_g = g\gamma_V$
- Yukawa coupling: $\beta_Y = \frac{1}{16\pi^2}\Big(\frac{1}{2}(\mathbf{Y}_2^\dagger(F)\mathbf{Y_a} + \mathbf{Y_a}\mathbf{Y}_2(F)) +$
$+2\mathbf{Y_b}\mathbf{Y_a}^\dagger\mathbf{Y_b} + 2\mathbf{Y_b}\mathrm{Tr}(\mathbf{Y_b}^\dagger\mathbf{Y_a}) - 6g^2C_2(F)\mathbf{Y_a}\Big)$

Method with dummy fields

- Superpotential:

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j. \tag{6}$$

- Quartic Couplings:

$$\lambda^{kl}_{ij} = Y_{ijm} Y^{klm} + g^2 (T^{Ak}_i T^{Al}_j + T^{Ak}_j T^{Al}_i) \tag{7}$$

- Softbreaking parameters:

$$\mathcal{L}_{SB} = -\frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k - \frac{1}{2} b^{ij} \phi_i \phi_j - \frac{1}{2} (m^2)^j_i \phi^{*i} \phi_j - \frac{1}{2} M \lambda \lambda + h.c.. \tag{8}$$

- Dummy fields:

$$M^{ij} \Psi_i \Psi_j = \phi_d Y^{ij}_d \Psi_i \Psi_j \tag{9}$$

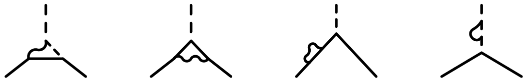
$$(m^2)^j_i \phi^{*i} \phi_j = \phi_{d_1} \phi_{d_2} \lambda^j_{d_1 d_2 i} \phi^{*i} \phi_j \tag{10}$$

$$h^{ijk} \phi_i \phi_j \phi_k = \phi_d \lambda^{ijk}_d \phi_i \phi_j \phi_k \tag{11}$$

Example: SM Yukawa Coupling

Steps to get the 1-Loop RGE in $\mathcal{O}(g^2)$

- Calculate the diagrams and their symmetric counterparts



- Extract the renormalization constants

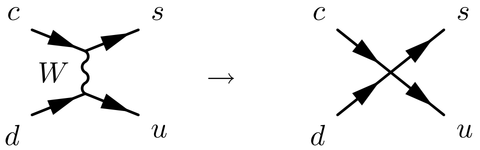
$$\Psi_L^{\text{un}} = \sqrt{Z_{\Psi_L}}\Psi_L^{\text{ren}}, \ \ldots \ , Y_{\text{un}}\bar{\Psi}_L^{\text{un}}\Psi_R^{\text{un}}\Phi^{\text{un}} = Z_{\text{coup}} Y_{\text{ren}}\bar{\Psi}_L^{\text{ren}}\Psi_R^{\text{ren}}\Phi^{\text{ren}}$$

- Renormalization: $Y_{\text{un}} = Z_Y Y_{\text{ren}} = \frac{Z_{\text{coup}}}{\sqrt{Z_{\Psi_L}}\sqrt{Z_{\Psi_R}}\sqrt{Z_\Phi}} Y_{\text{ren}}$
- The 1-Loop β-function is $\beta_Y = \frac{d}{d\left(\frac{1}{\epsilon}\right)} Z_Y Y_{\text{ren}}$
- The result is $\beta_Y = \left(Z_{\text{coup}}^{(1)} - \frac{1}{2}\left(Z_{\Psi_L}^{(1)} + Z_{\Psi_R}^{(1)} + Z_\Phi^{(1)}\right)\right) Y_{\text{ren}}$

Famous Example

Fermi’s theory of electroweak interaction:

- Let’s consider $c \to s u \bar{d}$, for $M_W^2 \gg k^2$



- effective 4-fermion-interaction

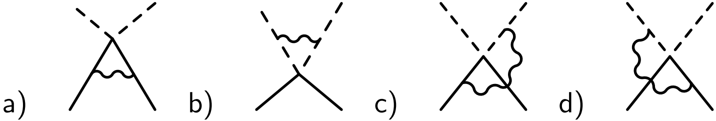
$$Q = \bar{s}\gamma_\mu(1-\gamma_5)c\,\bar{u}\gamma_\mu(1-\gamma_5)d$$

- C is the effective coupling constant for the 4-fermion-vertex b
- At 1-loop-level two operators possible:
  $Q_1 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}, Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}$
- The full and effective amplitude is

$$A_{\rm full} = -\frac{G_F}{\cdots} V_{cs}^* V_{ud} \frac{M_W^2}{\cdots} (\bar{s}c)_{V-A}(\bar{u}d)_{V-A} \quad (13)$$

5d Operators for heavy Gluon

- Tree-Level-Amplitude:
  $\mathcal{M}_{\text{Tree}} = A_{\text{Tree}} K_{\text{Tree}}$ with $A_{\text{Tree}} = \frac{1}{2}, K_{\text{Tree}} = 2g_3^2$.
- One Loop diagrams



- 1-Loop contributions: $\mathcal{M} = \Delta \sum_i K_i \cdot A_i,\ \Delta = \frac{2}{\epsilon}$
- Amplitudes A_i: $A_1 = -\frac{1}{8\pi^2}(\bar{v}(1-\gamma_5)u)$,
  $A_2 = \frac{1}{32\pi^2}(\bar{v}(1-\gamma_5)u)$, $A_3 = \frac{1}{32\pi^2}(\bar{v}(1-\gamma_5)u)$

$$t_L^\alpha \, \tilde{t}_L^{*\beta} \to \bar{t}_R^\gamma \, \tilde{t}_R^\delta$$

- Coefficients K_i:

Graph	K		A
a)	$K_1 =$	$\frac{1}{27} g_3^2 g'^2 (3\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta})$	A_1
	$K_3 =$	$g_3^4 (\frac{5}{9}\delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{1}{3}\delta_{\alpha\delta}\delta_{\gamma\beta})$	A_1
b)	$K_1 =$	$\frac{1}{27} g_3^2 g'^2 (3\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta})$	A_2
	$K_3 =$	$-g_3^4 (\frac{5}{9}\delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{1}{3}\delta_{\alpha\delta}\delta_{\gamma\beta})$	A_2
c)	$K_1 =$	$-\frac{2}{9} g'^2 g_3^2 (\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta})$	A_3
	$K_3 =$	$-2 g_3^4 (\frac{7}{12}\delta_{\alpha\delta}\delta_{\beta\gamma} + \frac{1}{36}\delta_{\alpha\beta}\delta_{\gamma\delta})$	A_3
d)	$K_1 =$	$-\frac{2}{9} g'^2 g_3^2 (\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta})$	A_3
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- Operators:

$$S_1 = \left(\bar{t}^\beta (1-\gamma_5) \tilde{t}_R^\beta\right) \left(\tilde{t}_L^\alpha t^\alpha\right), \; S_2 = \left(\bar{t}^\alpha (1-\gamma_5) \tilde{t}_R^\beta\right) \left(\tilde{t}_L^\beta t^\alpha\right)$$

Anomalous Dimensions

- Renormalization constant

$$Z = 1+\frac{2}{\epsilon\pi}\begin{pmatrix} -\frac{1}{72}\left(3\cdot\frac{3}{5}\cdot\alpha_1+26\alpha_3\right) & -\frac{1}{216}\left(5\cdot\frac{3}{5}\cdot\alpha_1+18\alpha_3\right) \\ -\frac{1}{216}\left(5\cdot\frac{3}{5}\cdot\alpha_1+18\alpha_3\right) & -\frac{1}{72}\left(3\cdot\frac{3}{5}\cdot\alpha_1+26\alpha_3\right) \end{pmatrix}$$

- Diagonalization: $S_{12,\pm} = \frac{S_1 \pm S_2}{2}$
- anomalous dimensions:

γ	$S_{12,+}$	$S_{12,-}$
γ_3	$\frac{8}{9\pi}\alpha_3$	$\frac{5}{9\pi}\alpha_3$
γ_1	$\frac{7}{54\pi}\cdot\frac{3}{5}\cdot\alpha_1$	$\frac{1}{27\pi}\cdot\frac{3}{5}\cdot\alpha_1$

Fermi's Theory: 1-Loop-Corrections

- There are two operators with different colour structures $S_1 = (\overline{s}_\alpha c_\beta)_{V-A}(\overline{u}_\beta d_\alpha)_{V-A}, S_2 = (\overline{s}_\alpha c_\alpha)_{V-A}(\overline{u}_\beta d_\beta)_{V-A}$
- Colour algebra: $T^a_{\alpha\beta}T^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$
- Mixing of Operators at 1-Loop-Level: $Q_i = c_{i,1}S_1 + c_{i,2}S_2$
- Renormalization constant: 2×2-matrix in (Q_1, Q_2)
- Diagonalization: $Q_\pm = \frac{Q_2 \pm Q_1}{2}, C_\pm = C_2 \pm C_1$
- Renormalization constants: $Z_\pm = 1 + \frac{\alpha_3}{4\pi}\frac{1}{\epsilon}\left(\mp 3\frac{N \pm 1}{N}\right)$
- anomalous Dimensions: $\gamma_\pm(g) = \frac{1}{Z_\pm}\frac{dZ_\pm}{d\ln\mu} = \frac{\alpha^2}{4\pi}(\pm 6\frac{N \mp 1}{N})$
- Running: $\frac{dC_\pm(\mu)}{d\ln\mu} = \gamma_\pm(g)C_\pm(\mu) \rightarrow C_{1,2}(\mu) = \frac{1}{2}(C_+(\mu) \mp C_-(\mu))$

Scalar quartic couplings

The gluino is integrated out:

- $\frac{d}{dt}Y_u^2 \sim Y^2 - Y\frac{32}{3}g_3^2\left(\frac{1}{4}(6-2\Theta_{\tilde{g}})\right)$
  $\rightarrow$ After integration out the gluino: $\sim Y^2 - Y\frac{48}{3}g_3^2$
- $\frac{d}{dt}\lambda^Y_{\tilde{Q}\tilde{U}} \sim Y^2 - Y\frac{32}{3}g_3^2\left(\frac{1}{32}(-32\Theta_{\tilde{g}}+64)\right)$
  $\rightarrow$ After integration out the gluino: $\sim Y^2 - Y\frac{64}{3}g_3^2$
  $\rightarrow$ Decreases more for larger μ^2
- $\frac{d}{dt}\lambda^Y_{H_u\tilde{U}} \sim Y^2 - Y\frac{32}{3}g_3^2\left(\frac{1}{2}(1+\Theta_{\tilde{g}})\right)$
  $\rightarrow$ After integration out the gluino: $\sim Y^2 - Y\frac{16}{3}g_3^2$
  $\rightarrow$ Contributions of strong interaction small: Effect of Yukawa couplings dominates
  $\rightarrow$ Increases for larger μ^2

Scalar quartic couplings II

- The gluino is integrated out: No gluino tresholds for all three couplings
  $\rightarrow$ Indirect effect, because other quartic couplings don't cancel exactly
- $\tilde{D}$ is integrated out: Big effect on $\lambda^{Y}_{H_d \tilde{U}}$
- $\tilde{Q}$ is integrated out: Evolution of couplings with $\tilde{Q}$ as outer particle stops

Independent Yukawa couplings

$\tilde{Q}$ is integrated out:

- Y_{u,H_u}: Integrating out the $\tilde{Q}$ changes wave function renormalizaton of U
  $\rightarrow$ larger decreasing for larger μ^2 because of gauge couplings
- $Y_{u,\tilde{U}}$: Integrating out the $\tilde{Q}$ changes wave function renormalization of $\tilde{H}_u$ $\rightarrow$ Reduces contributions of Yukawa couplings
  But gluino exchange betwenn Q and $\tilde{U}$ not longer possible $\rightarrow$ Dominating effect $\rightarrow$ Contributions of Yukawa coupling bigger than contributions of gauge couplings $\rightarrow$ Increasing for larger μ^2
- $Y_{u,\tilde{Q}}$: constant, because an external field is integrated out

Integrating out $\tilde{U}$:

- $Y_{u,\tilde{U}}$: Stops
- Y_{u,H_u}: Small bend $\rightarrow$ larger decreasing

Independent gauge couplings

- Anomalous dimenions of gluon and gluino:

$$\gamma_g = -7 + 2\Theta_{\tilde{G}} + \frac{1}{6}\sum_{i=1}^{N_g}\left(2\Theta_{\tilde{Q}_i} + \Theta_{\tilde{D}_i} + \Theta_{\tilde{U}_i}\right) \quad (15)$$

$$\gamma_{\tilde{g}} = -3\left(3\Theta_{\tilde{g}} - \frac{1}{6}\sum_{i=1}^{N_g}\left(2\Theta_{\tilde{Q}_i} + \Theta_{\tilde{U}_i} + \Theta_{\tilde{D}_i}\right)\right) \quad (16)$$

- Integrating out a squark changes the value of -3:
  - Gluon: $-3\frac{1}{6}$
  - Gluino: $-3\frac{1}{2}$

$\rightarrow$ larger slope of the gaugino coupling

GUT Masses

- mSugra:

$$m^2_{\tilde{Q}} = \cdots = m^2_0, \quad M_{\tilde{g}} = \cdots = M_{\frac{1}{2}}$$

- AMSB :

$$m^2_i = -\frac{1}{4}\frac{d\gamma_i}{d\ln\mu} m^2_{\frac{3}{2}}, \quad M_\lambda = \frac{\beta(g^2)}{2g^2} m_{\frac{3}{2}}$$

- GMSB:

$$\begin{aligned} m^2_i(M_{mess}) &= 2N_{mess}\Lambda^2 \sum_a C_a \left(\frac{g^2_a}{16\pi^2}\right)^2 \\ M_a(M_{mess}) &= N_{mess}\Lambda \frac{g^2}{16\pi^2} \end{aligned}$$

Running Masses: mSugra (SPS 2) III