Nuclear Parton Distributions

(concentrating on `nCTEQ`)

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Outline

Introduction

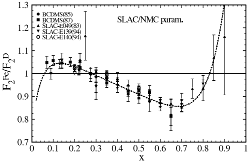
Global Analysis

Summary of available nuclear PDFs

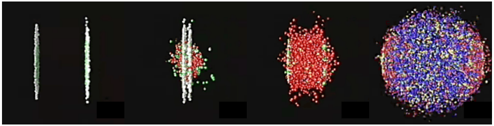
`nCTEQ` results

Motivations: Why do we need nuclear PDFs?

- What are PDFs of bound protons/neutrons?



- Heavy ion collisions in LHC and RHIC



Motivations: differentiate flavors in proton PDFs

- To differentiate flavors we need to probe different linear combinations of PDFs

charged lepton DIS

$$F_2^{l^\pm} \sim \left(\tfrac{1}{3}\right)^2 [d+s] + \left(\tfrac{2}{3}\right)^2 [u+c]$$

neutrino DIS

$$F_2^{\nu} \sim \left[d+s+\bar{u}+\bar{c}\right]$$
$$F_2^{\bar{\nu}} \sim \left[\bar{d}+\bar{s}+u+c\right]$$

$$F_3^{\nu} \sim 2\left[d+s-\bar{u}-\bar{c}\right]$$
$$F_3^{\bar{\nu}} \sim 2\left[u+c-\bar{d}-\bar{s}\right]$$

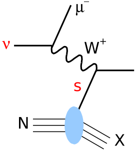
- Neutrino data gives access to different flavor combinations.
- Can be done only with heavy nuclear targets.
- Depend on nuclear corrections.

Motivations: proton Strange PDF

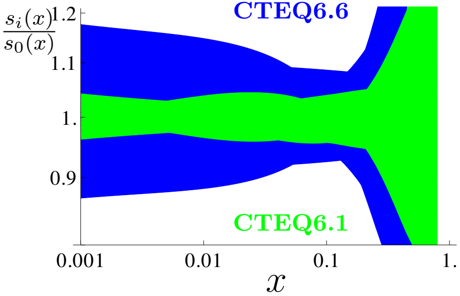
Before CTEQ6.6 proton PDFs it was assumed
P. Nadolsky et al. PRD 78, 013004 (2008), arXiv:0802.0007

$$s(x) = \bar{s}(x) \sim \kappa \, \frac{\bar{u}(x) + \bar{d}(x)}{2}, \qquad \kappa = \frac{1}{2}$$

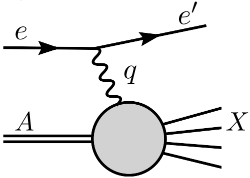
- Underestimating s PDF uncertainty, as $\bar{u}$, $\bar{d}$ are much better constrained.
- Neutrino-nucleon dimuon data (CCFR, NuTeV)



allowed to fit s PDF independently of $\bar{u}$, $\bar{d}$ sea.



Variables: DIS of nuclear target $eA \to e'X$



- DIS variables in case on nucleons

  in nucleus $\begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2\,p_A \cdot q} \end{cases}$

  - p^A – nucleus momentum
  - $x_A \in (0, 1)$ – analog of Bjorken variable (fraction of the nucleus momentum carried by a nucleon)

- Analogue variables for partons:

  - $p_N = \frac{p_A}{A}$ – *average* nucleon momentum
  - $x_N \equiv \frac{Q^2}{2\,p_N \cdot q} = A\,x_A$ – parton momentum fraction with respect to the avarage nucleon momentum p_N
  - $x_N \in (0, A)$ – parton can carry more than the average nucleon momentum p_N.

Assumptions entering the nuclear PDF analysis

1. **Factorization** & DGLAP evolution
   - allow for definition of **universal PDFs**
   - make the formalism **predictive**
   - needed even if it is broken
2. Isospin symmetry $\begin{cases} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{cases}$
3. The *bound proton* PDFs have the *same evolution equations* and sum rules as the free proton PDFs *provided we neglect any contributions from the region* $x > 1$ (which is expected to have negligible contribution [PRC 73, 045206 (2006), `arXiv:hep-ph/0509241`])

Then observables $\mathcal{O}^A$ can be calculated as:

$$\mathcal{O}^A = Z\,\mathcal{O}^{p/A} + (A - Z)\,\mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

Schematics of Global Analysis

1. Parametrize PDFs at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f(x, Q_0) = f(x; a_0, a_1, \dots) = a_0 x^{a_1} (1-x)^{a_2} P(x; a_3, \dots)$$

2. Use DGLAP equation to evolve $f(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
3. Define and minimize appropriate χ^2 function (with respect to parameters $a_0, a_1, \dots$)

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$

$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}}\right)^2$$

(by default $w_n = 1$)

Uncertainties in global analysis

- **Experimental errors** (included in PDFs error analysis)
- **Theoretical uncertainties** (not included)
- **“Details”** of Global Fits
  (e.g. parametrization, HF schemes; hard to estimate not included)

<u>Propagating experimental errors to PDFs:</u>

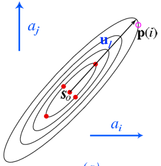
- Hessian Method
  - Eigenvector PDFs
  - Quadratic approximation
  - Simple computation of correlations
- Lagrange Multipliers
- Monte Carlo Methods

Hessian method

[JHEP 07 (2002) 012, arXiv:hep-ph/0201195]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$,

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0) \left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right)_0$$



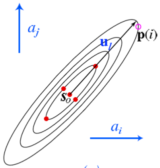
(a)
Original parameter basis

Hessian method

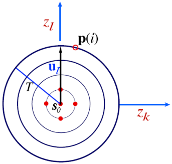
[JHEP 07 (2002) 012, `arXiv:hep-ph/0201195`]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2\chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i z_i^2$$



(a)
Original parameter basis



(b)
Orthonormal eigenvector basis

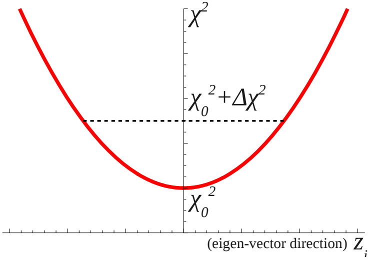
Hessian method

[JHEP 07 (2002) 012, arXiv:hep-ph/0201195]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i z_i^2$$

- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1-σ uncertainty),
  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$



Hessian method

[JHEP 07 (2002) 012, `arXiv:hep-ph/0201195`]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

  $$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i z_i^2$$

- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1-σ uncertainty),
  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$
- Construct error PDFs corresponding to each eigenvector direction:

  $$f_i^{\pm} = f(\{z_i\}) = f(0, \dots, z_i = \pm\sqrt{\Delta\chi^2}, \dots, 0)$$
  $$z_i = \pm\sqrt{\Delta\chi^2}$$

- Calculate errors of observable X:

  $$\Delta X = \frac{1}{2}\sqrt{\sum_i \left[X(f_i^+) - X(f_i^-)\right]^2}$$

Differences with the free-proton PDFs

- Theoretical status of Factorization
- Parametrization – more parameters
- Different data sets – much less data:
  - Less data $\rightarrow$ less constraining power $\rightarrow$ more assumptions (fixing) about a_i parameters
  - **Assumptions replace uncertainities!**

Outline

Available nuclear PDFs

- **Multiplicative nuclear correction factors**

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A)\; f_i^{free\ proton}(x_N, \mu_0)$$

  - Hirai, Kumano, Nagai [PRC 76, 065207 (2007), `arXiv:0709.3038`]
  - Eskola, Paukkunen, Salgado [JHEP 04 (2009) 065, `arXiv:0902.4154`]
  - de Florian, Sassot, Stratmann, Zurita [PRD 85, 074028 (2012), `arXiv:1112.6324`]

- **Native nuclear PDFs**
  - `nCTEQ` [PRD 80, 094004 (2009), `arXiv:0907.2357`]

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0)$$

$$f_i(x_N, A = 1, \mu_0) \equiv f_i^{free\ proton}(x_N, \mu_0)$$

HKN framework [PRC 76, 065207 (2007), `arXiv:0709.3038`]

- LO & NLO PDFs with errors
- Error PDFs produced with *Hessian method*
- Parametrization (Q_0=1GeV)

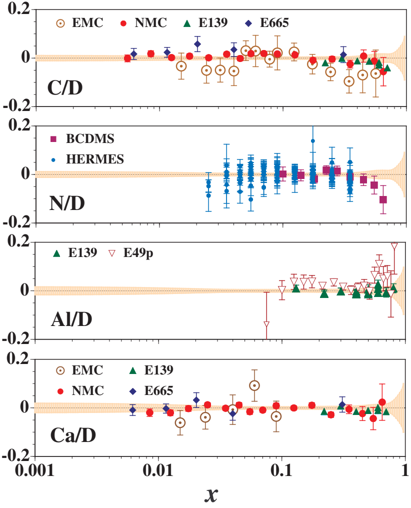
$$f_i^{p/A}(x_N, Q_0) = R_i^A(x_N, Q_0)\, f_i^p(x_N, Q_0), \qquad i = u_v, d_v, g, \mathrm{sea}$$

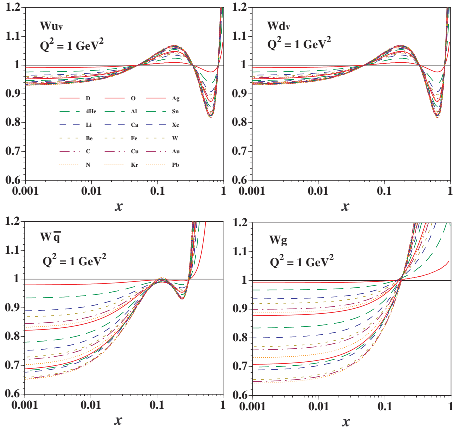
$$R_i(x, Q_0, A) = 1 + \left(1 - \tfrac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

- MRST 1998 free proton baseline
- Neglects $x_N > 1$
- Data: DIS & DY

HKN framework [PRC 76, 065207 (2007), arXiv:0709.3038]

- NLO fit: $\chi^2/dof = 1.21$





EPS09 framework [JHEP 04 (2009) 065, arXiv:0902.4154]

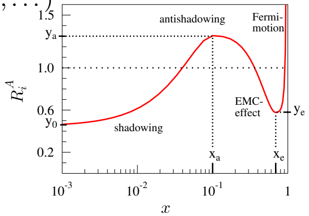
- LO & NLO PDFs with errors
- Error PDFs produced with *Hessian method*
- Parametrization (Q_0=1.3GeV)

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A, Z)\, f_i(x_N, \mu_0), \qquad i = \text{valence}, s, g$$

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

A-dependence of fitting parameters ($d_i = a_i, b_i, \ldots$)

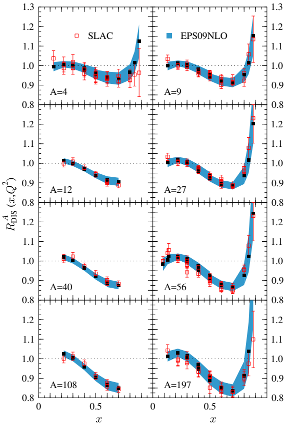
$$d_i^A = d_i^{A_{ref}} \left(\frac{A}{A_{ref}} \right)^{p_{d_i}}$$

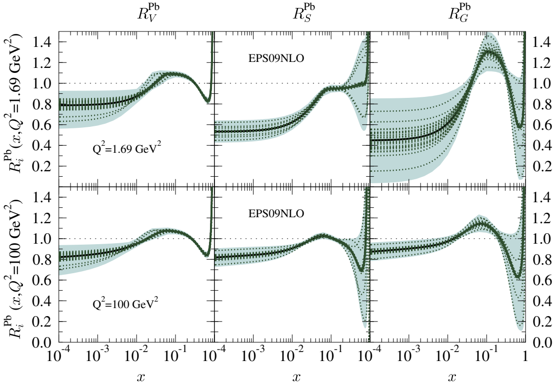


- CTEQ6.1M free proton baseline
- Neglects $x_N > 1$
- Data: DIS, DY, π^0 @ RHIC

EPS09 framework [JHEP 04 (2009) 065, `arXiv:0902.4154`]

- NLO fit: $\chi^2/dof = 0.79$





DSSZ framework [PRD 85, 074028 (2012), arXiv:1112.6324]

- NLO PDFs with errors
- Error PDFs produced with *Hessian method*
- Parametrization (Q_0=1GeV)

$$f_i^{p/A}(x_N, Q_0) = R_i^A(x_N, Q_0)\, f_i^p(x_N, Q_0), \qquad i = \text{valence}, s, g$$

$$R_v^A(x, Q_0) = \epsilon_1 x^{\alpha_1}(1-x)^{\beta_1}\left[1 + \epsilon_2(1-x)^{\beta_2}\right]\left[1 + a_v(1-x)^{\beta_3}\right]$$
$$R_s^A(x, Q_0) = R_v^A(x, Q_0)\frac{\epsilon_s}{\epsilon_1}\frac{1 + a_s x^{\alpha_s}}{1 + a_s}$$
$$R_g^A(x, Q_0) = R_v^A(x, Q_0)\frac{\epsilon_g}{\epsilon_1}\frac{1 + a_g x^{\alpha_g}}{1 + a_g}$$

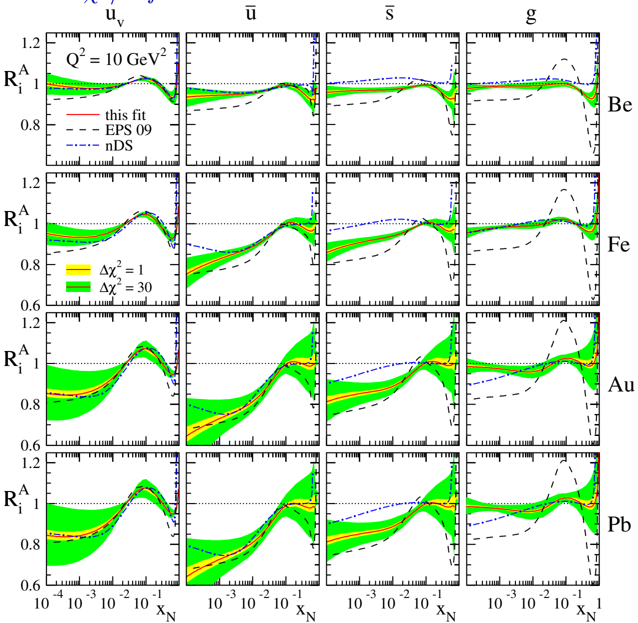
A-dependence of fitting parameters ($\xi = \alpha_v, \alpha_s, \dots$)

$$\xi = \gamma_\xi + \lambda_\xi A^{\delta_\xi}$$

- MSTW 2008 free proton baseline
- Neglects $x_N > 1$
- Data: DIS, DY, $\pi^{0\pm}$ @ RHIC, Neutrino DIS

DSSZ framework [PRD 85, 074028 (2012), `arXiv:1112.6324`]

- NLO fit: $\chi^2/dof = 0.99$



nCTEQ framework [PRD 80, 094004 (2009), arXiv:0907.2357]

- Functional form of the bound proton PDF same as for the free proton (CTEQ6M, x restricted to $0 < x < 1$)

$$xf_i^{p/A}(x,Q_0) = c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}, \qquad i = u_v, d_v, g, \ldots$$

$$\bar{d}(x,Q_0)/\bar{u}(x,Q_0) = c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

- A-dependent fit parameters (reduces to free proton for $A = 1$)

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right), \quad k = \{1, \ldots, 5\}$$

- PDFs for nucleus (A, Z)

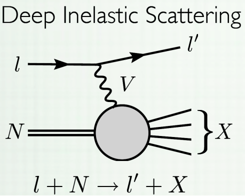
$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

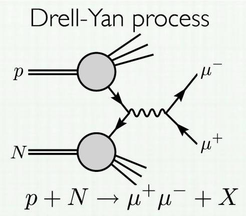
(bound neutron PDF $f_i^{n/A}$ by isospin symmetry)

Data sets

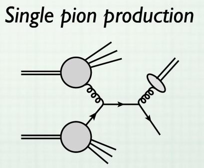
- NC DIS & DY

  CERN BCDMS & EMC & NMC
  N = (D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W)
  FNAL E-665
  N = (D, C, Ca, Pb, Xe)
  DESY Hermes
  N = (D, He, N, Kr)
  SLAC E-139 & E-049
  N = (D, Ag, Al, Au, Be,C, Ca, Fe, He)
  FNAL E-772 & E-886
  N = (D, C, Ca, Fe,W)





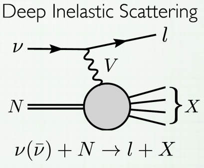
- Single pion production (not yet included)



RHIC - PHENIX & STAR

N = Au

- Neutrino (to be included later)



CHORUS CCFR & NuTeV

N = Pb N = Fe

nCTEQ fits

Fit properties:

- fit @NLO
- $Q_0 = 1.3\text{GeV}$
- using ACOT heavy quark scheme
- kinematical cuts: $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
- 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- 16 free parameters
  - 7 gluon
  - 7 valence
  - 2 sea
- $\chi^2/\text{dof} = 0.87$

Error analysis:

- use Hessian method

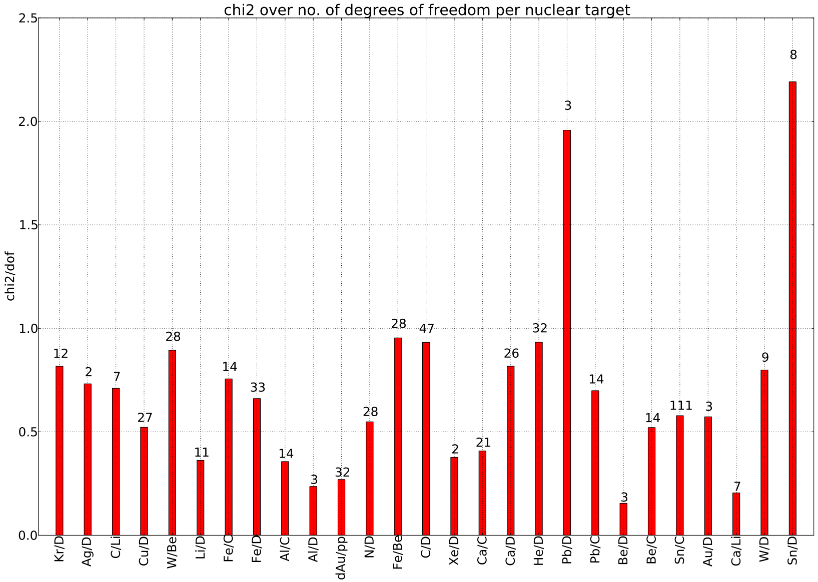
$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- eigenvalues span 10 orders of magnitude $\rightarrow$ require numerical precision
- use noise reducing derivatives

nCTEQ RESULTS
(preliminary)

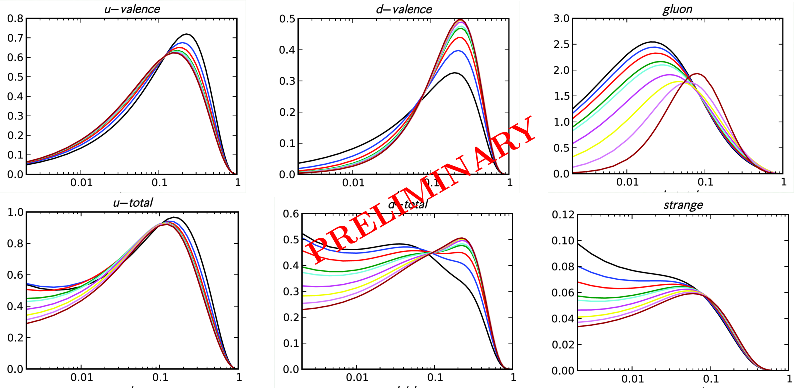
nCTEQ results



nCTEQ results

A-dependence of bound proton PDFs

$$x\, f_i^A(x,Q) \qquad A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$$



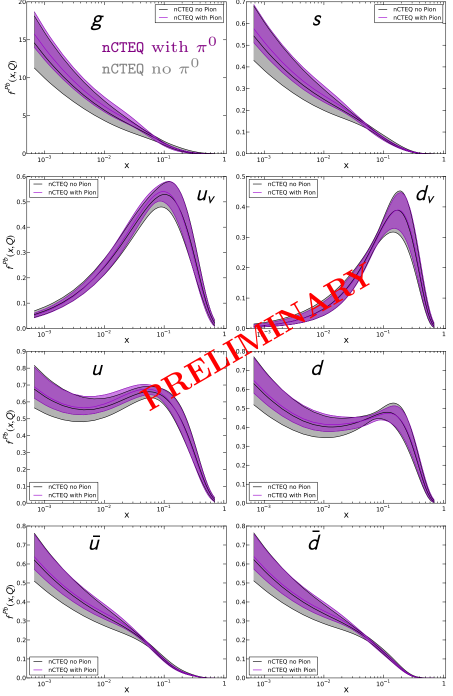
nCTEQ results

Nuclear PDFs ($Q = 10\mathrm{GeV}$)

$$xf_i^{Pb}(x,Q)$$

Compare nCTEQ fits:

- with π^0 data (violet)
- without π^0 data (gray)



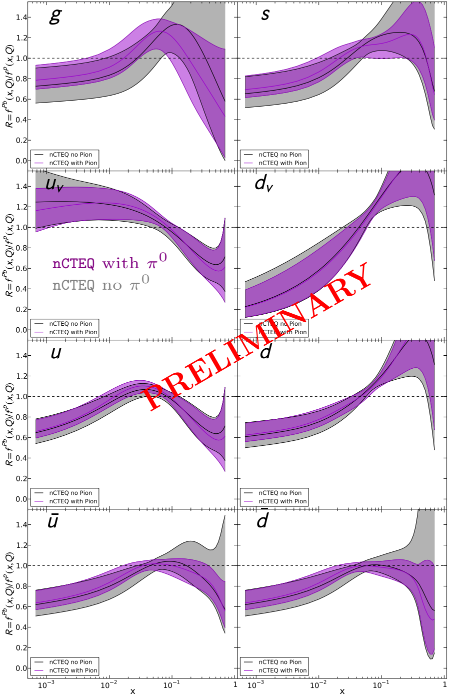
nCTEQ results

Nuclear correction factors ($Q = 10\text{GeV}$)

$$R_i(Pb) = \frac{f_i^{Pb}(x,Q)}{f_i^{p}(x,Q)}$$

Compare `nCTEQ` fits:

- with π^0 data (violet)
- without π^0 data (gray)

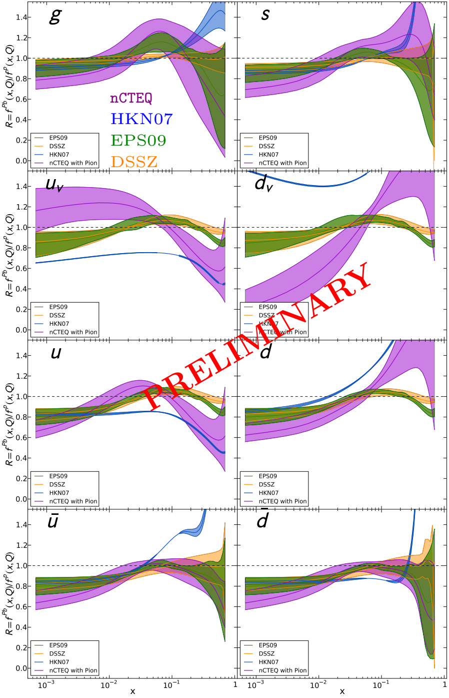


nCTEQ results

Nuclear correction factors
$(Q = 10\text{GeV})$

$$R_i(Pb) = \frac{f_i^{Pb}(x,Q)}{f_i^p(x,Q)}$$

- different solution for d-valence & u-valence compared to EPS09 & DSSZ
- sea quark nuclear correction factors similar to EPS09
- nuclear correction factors depend largely on underlying proton baseline

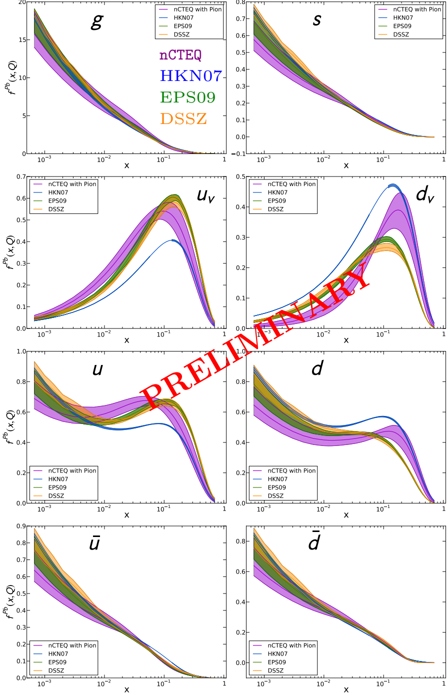


nCTEQ results

Nuclear PDFs ($Q = 10\text{GeV}$)

$$x\, f_i^{Pb}(x, Q)$$

- nCTEQ d-valence & u-valence solution between HKN07 & EPS09
- nCTEQ features larger uncertainties than previous nPDFs
- better agreement between different groups (nPDFs don't depend on proton baseline)



nCTEQ vs. EPS09

nCTEQ

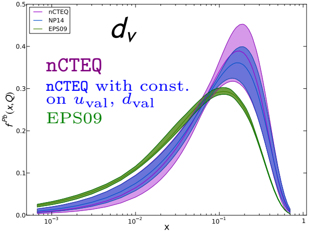
$$xu_v^{p/A}(Q_0) = x^{c_1^u}(1-x)^{c_2^u} e^{c_3^u x}(1+e^{c_4^u}x)^{c_5^u}$$
$$xd_v^{p/A}(Q_0) = x^{c_1^d}(1-x)^{c_2^d} e^{c_3^d x}(1+e^{c_4^d}x)^{c_5^d}$$

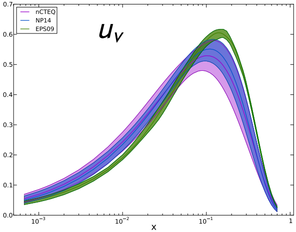
$$c_k^{u_v} = c_{k,0}^{u_v} + c_{k,1}^{u_v}\left(1 - A^{-c_{k,2}^{u_v}}\right)$$
$$c_k^{d_v} = c_{k,0}^{d_v} + c_{k,1}^{d_v}\left(1 - A^{-c_{k,2}^{d_v}}\right)$$

EPS09

$$u_v^{p/A}(Q_0) = R_v(x, A, Z)\, u_v(x, Q_0)$$
$$d_v^{p/A}(Q_0) = R_v(x, A, Z)\, d_v(x, Q_0)$$

$$R_v = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \le x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1 \end{cases}$$





we set:

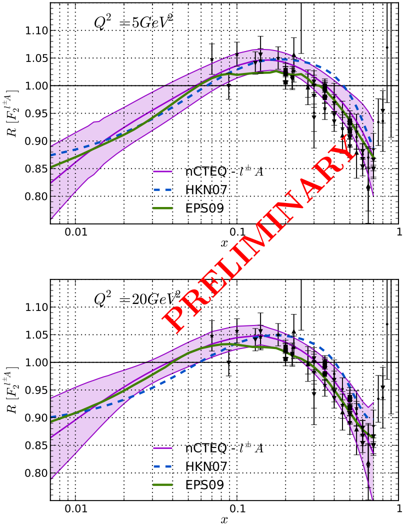
$$\begin{cases} c_1^{d_v} = c_1^{u_v} \\ c_2^{d_v} = c_2^{u_v} \end{cases}$$

nCTEQ results: F_2 ratios

Structure function ratio

$$R = \frac{F_2^{Fe}(x,Q)}{F_2^{D}(x,Q)}$$

- good data description
- despite different u-valence & d-valence ratios are similar to EPS09

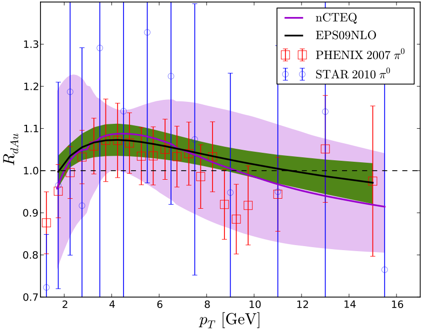


nCTEQ results: π^0 production

Pion production, ratio

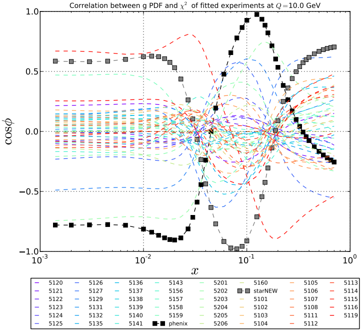
$$R^{\pi}_{\mathrm{dAu}} = \frac{\frac{1}{2A} d^2\sigma^{\mathrm{dAu}}_{\pi}/dp_T dy}{d^2\sigma^{\mathrm{pp}}_{\pi}/dp_T dy}$$

- good data description, however big experimental uncertainities do not allow for strong constraints on PDFs
- despite different u-valence & d-valence ratios are similar to EPS09



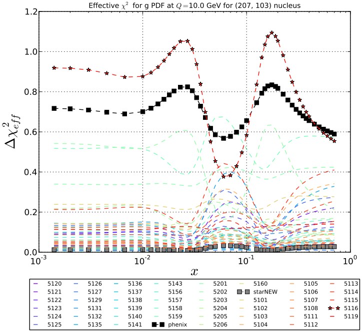
nCTEQ results: gluon and π^0 production

Cosine of the correlation angle:



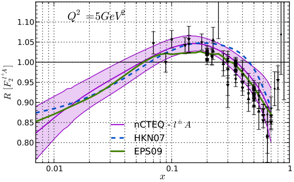
$$\cos\phi(X,Y) = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{4\Delta X \Delta Y}$$

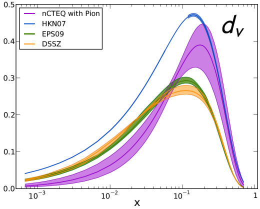
Effective χ^2 change: $\Delta\chi^2_{eff}$



Summary

- We have updated the `nCTEQ` error PDFs (still preliminary).
- `nCTEQ` PDFs features larger uncertainties than the other groups but they are still underestimated.





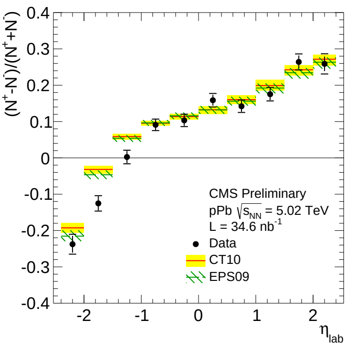
- To have reliable estimate of nuclear corrections we need more data (LHC *lead* run can help).
- Nuclear component important not only for heavy ion collisions, but also for the free-proton analysis.

Summary

Plans for future:

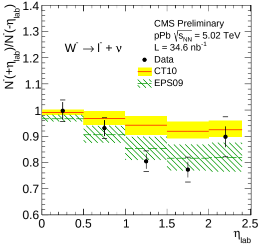
- Official release of current analysis
- Among other things analyse LHC data, e.g. [CMS PAS HIN-13-007]

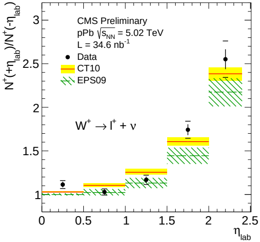
Charge asymmetry



- Sensitive to $\frac{u}{d}$ ratio should help to distinguish modification for u and d

Forward/backward asymmetry



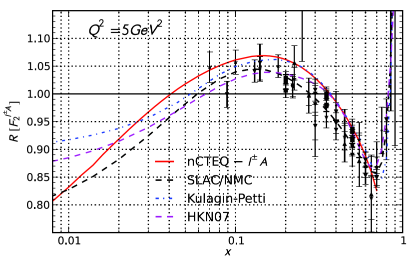


Summary

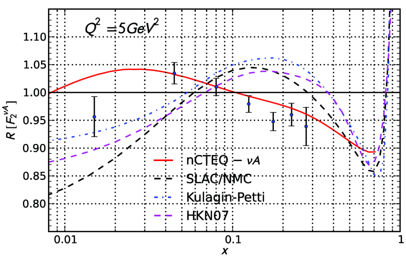
PRL106, 122301 (2011), `arXiv:1012.0286`
PRD80, 094004 (2009), `arXiv:0907.2357`

Plans for future:

- Reanalyze neutron nuclear corrections (does the universality holds?)



$l^\pm A$ DIS and DY data, $\chi^2/\text{dof} = 0.89$



νA DIS data only, $\chi^2/\text{dof} = 1.33$

Nuclear CTEQ parton distribution functions - Hepforge - Google Chrome

Nuclear CTEQ parton di ×

ncteq.hepforge.org

nCTEQ is hosted by Hepforge, IPPP Durham

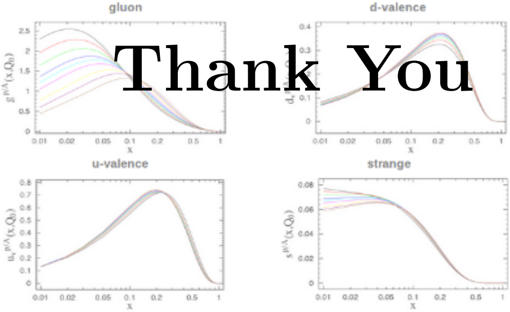
nCTEQ

nuclear parton distribution functions

- Home
- PDF grids & code
- Papers & Talks
- Subversion
- Tracker
- Wiki

nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). More details on the framework and the first results can be found in arXiv:09072357 [hep-ph].
The effects of the nuclear environement on the parton densities can be shown as modified parton densities

Thank You



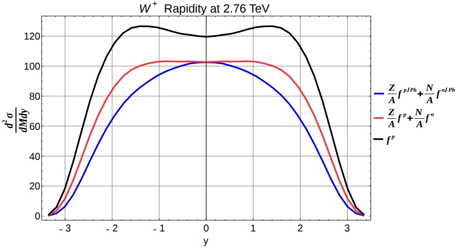
where all black curves stand for free proton PDF and red, green, blue, cyan, pink, yellow, magenta and brown curves show PDF in protons bound in nuclei - from deuterium (red) to lead (brown).

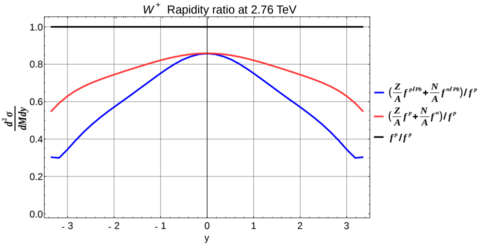
An alternative way how effects of nuclear environement can be displayed is in ratios of Deep Inelastic Scattering (DIS) structure functions e.g. ratios of of the structure function F_2 for a neutral current DIS as in the figure below on the left or ratios of of the same structure function F_2 but for a charged current DIS.

BACKUP SLIDES

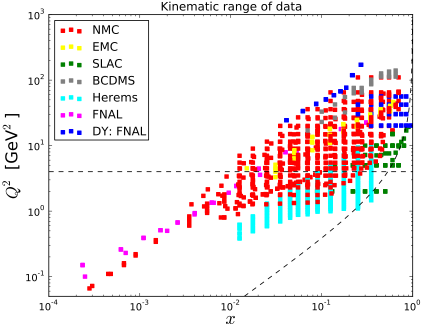
Distinguish between isospin and actual nuclear effetcs

- *Isospin effects* – modifications connected purly to the proton/neutron (u/d) content of the nucleus
- *Nuclear effects* – modifications due to the interactions with the nuclear medium





Status of nuclear PDFs



Problems:

- Unrealiable error estimates
- Problem with flavor differentation
- Not enough data...

- New input from LHC welcome:
  - W/Z production can give important constranints
  - other processes which?
    - ? Dijets [CMS-PAS-HIN-13-001] $\rightarrow$ gluon, $x \sim (0.006, 0.3)$ [arXiv:1308.6733]
    - ? Jets [ATLAS-CONF-2014-025, ATLAS-CONF-2014-024]
    - ? $\gamma + Q \rightarrow$ gluon [arXiv:1012.1178]
    - ? isolated γ [arXiv:1201.3093] $\rightarrow$ gluon
    - ? charged-particle production [ATLAS-CONF-2013-107]

Correlation cosine

$$\cos\phi(X,Y) = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{4\Delta X \Delta Y}$$

$$\vec{\nabla}X \cdot \vec{\nabla}Y = \frac{1}{2}\sum_{i_{pdf}} \left(X_{i_{pdf}}^{(+)} - X_{i_{pdf}}^{(-)}\right)\left(Y_{i_{pdf}}^{(+)} - Y_{i_{pdf}}^{(-)}\right)$$

$$\Delta X = \sqrt{\sum_{i_{pdf}} \left(X_{i_{pdf}}^{(+)} - X_{i_{pdf}}^{(-)}\right)^2}$$

In the considered case:

$$\cos\phi(g(x,Q), \chi^2(i_{exp})) = \sum_{i_{pdf}} \frac{\left(g_{i_{pdf}}^{(+)}(x) - g_{i_{pdf}}^{(-)}(x)\right)\left(\chi_{i_{pdf}}^{2\,(+)}(i_{exp}) - \chi_{i_{pdf}}^{2\,(-)}(i_{exp})\right)}{\sqrt{\sum_{i'_{pdf}}\left(g_{i'_{pdf}}^{(+)}(x) - g_{i'_{pdf}}^{(-)}(x)\right)^2}\sqrt{\sum_{i''_{pdf}}\left(\chi_{i''_{pdf}}^{2\,(+)}(i_{exp}) - \chi_{i''_{pdf}}^{2\,(-)}(i_{exp})\right)^2}}$$

Another measure of correlations: Effective χ^2 change

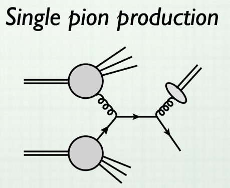
$$\Delta\chi^2_{eff}(i_{exp}, X) = \sum_{i_{pdf}} \frac{1}{2}\left(\left|\chi^{2\,(+)}_{i_{pdf}}(i_{exp}) - \chi^{2\,(0)}_{i_{pdf}}(i_{exp})\right| + \left|\chi^{2\,(-)}_{i_{pdf}}(i_{exp}) - \chi^{2\,(0)}_{i_{pdf}}(i_{exp})\right|\right)$$

$$\times \left(\frac{X^{(+)}_{i_{pdf}} - X^{(-)}_{i_{pdf}}}{\sqrt{\sum_{i'_{pdf}}\left(X^{(+)}_{i'_{pdf}} - X^{(-)}_{i'_{pdf}}\right)^2}}\right)^2$$

Data sets: Single pion production

RHIC - PHENIX & STAR

(N = Au)



PHENIX Collaboration:

[Phys.Rev.Lett. 98 (2007) 172302, `nucl-ex/0610036`]

STAR Collaboration:

[Phys.Rev. C81 (2010) 064904, `arXiv:0912.3838`]

- Theory calculation:

  P. Aurenche, M. Fontannaz, J.-Ph. Guillet, B. A. Kniehl, M. Werlen

  [Eur. Phys. J. C13, 347-355, (2000), `arXiv:hep-ph/9910252`]

- Fragmentation functions:

  J. Binnewies, Bernd A. Kniehl, G. Kramer

  [Z. Phys. C65 (1995) 471-480, `arXiv:hep-ph/9407347`]