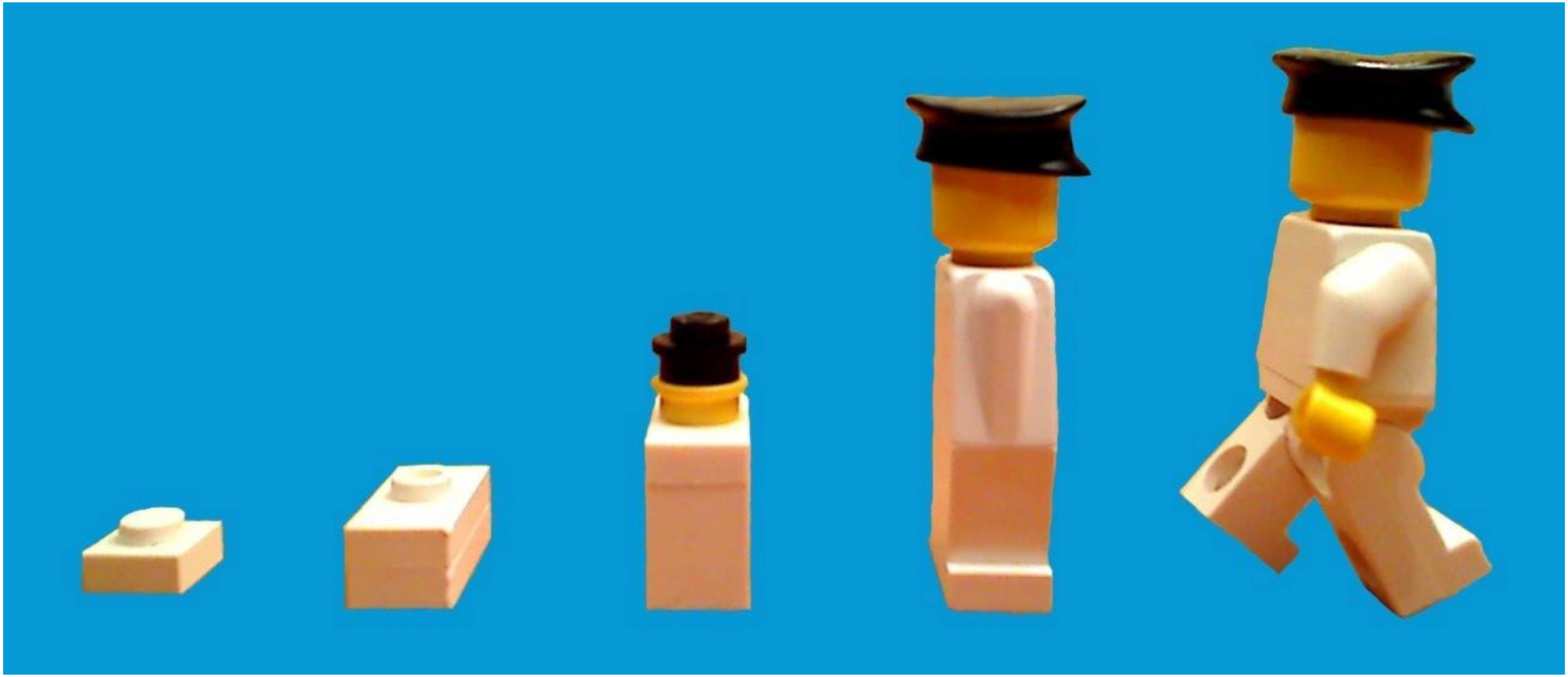
First tests of a minimalistic "Nodal Drift Method" for spatial kinetics

and possible links with Monte Carlo based production of Diffusion Data

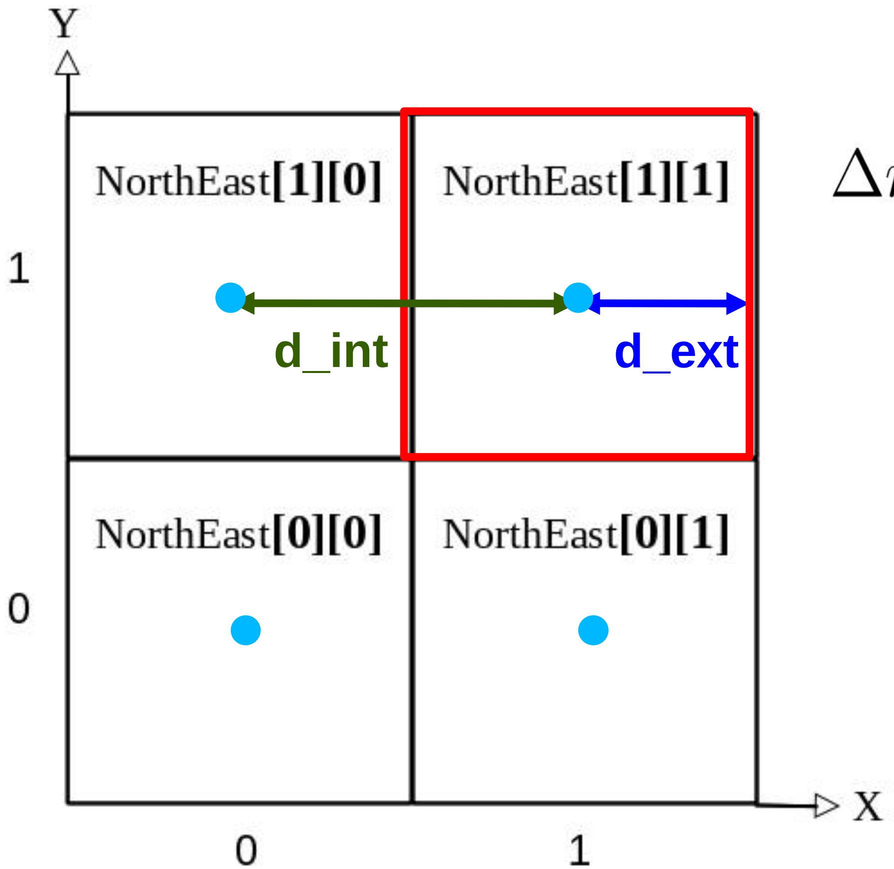
Objective : **missing link** between point kinetics (way too simple and thus wrong)
and industrial spatial kinetics (often unusable for academics)

Constraint : **simplest possible model** (for easy N/TH coupling and written distribution)

Contents : Neutron Diffusion Theory, Lumped Parameter Model & **Nodal Drift Method**



P. Prévot, A. Nuttin, N. Capellan, LPSC / Reactor Physics / MAP team



$$\Delta n_{NE[1][1]} = \frac{1}{d_{int}^2}\left[\left(n_{NE[0][1]} - n_{NE[1][1]}\right) + \left(n_{NE[1][0]} - n_{NE[1][1]}\right)\right] + \frac{1}{d_{ext}^2}\left[-2\; n_{NE[1][1]}\right]$$

- directly from central difference formula
- with an asymptotic solution (finer mesh)

$$\frac{dn_1}{dt} = D_1 v_1\, \Delta n_1 - \Sigma_{a1}\, n_1 v_1 - \Sigma_{12}\, n_1 v_1 + (1-\beta)\, \nu\Sigma_{f2}\, n_2 v_2 + \sum_g \lambda_g p_g$$

$$\frac{dn_2}{dt} = D_2 v_2\, \Delta n_2 - \Sigma_{a2}\, n_2 v_2 + \Sigma_{12}\, n_1 v_1$$

$$\frac{dp_g}{dt} = \beta_g\, \nu\Sigma_{f2}\, n_2 v_2 - \lambda_g p_g$$

2-g Diffusion Theory

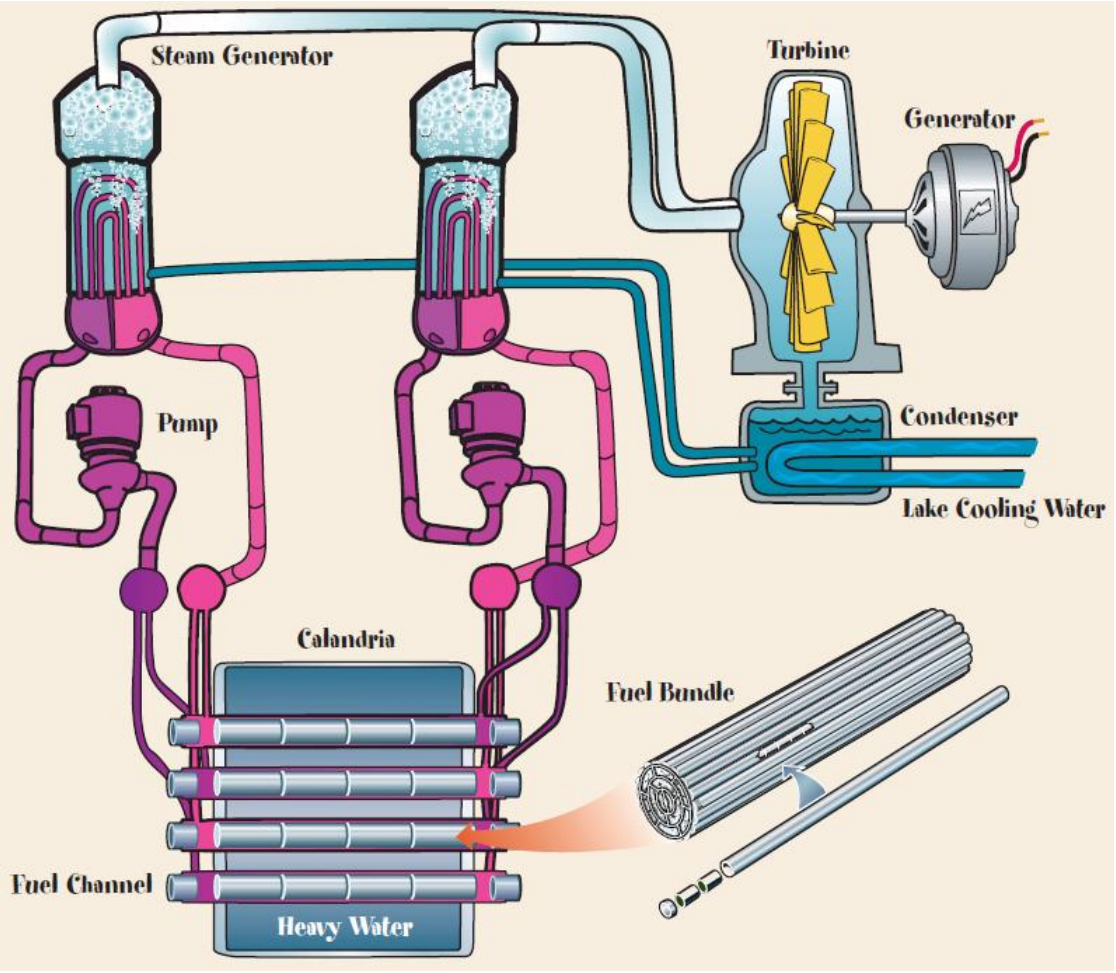
First 2D Benchmark CANDU transient

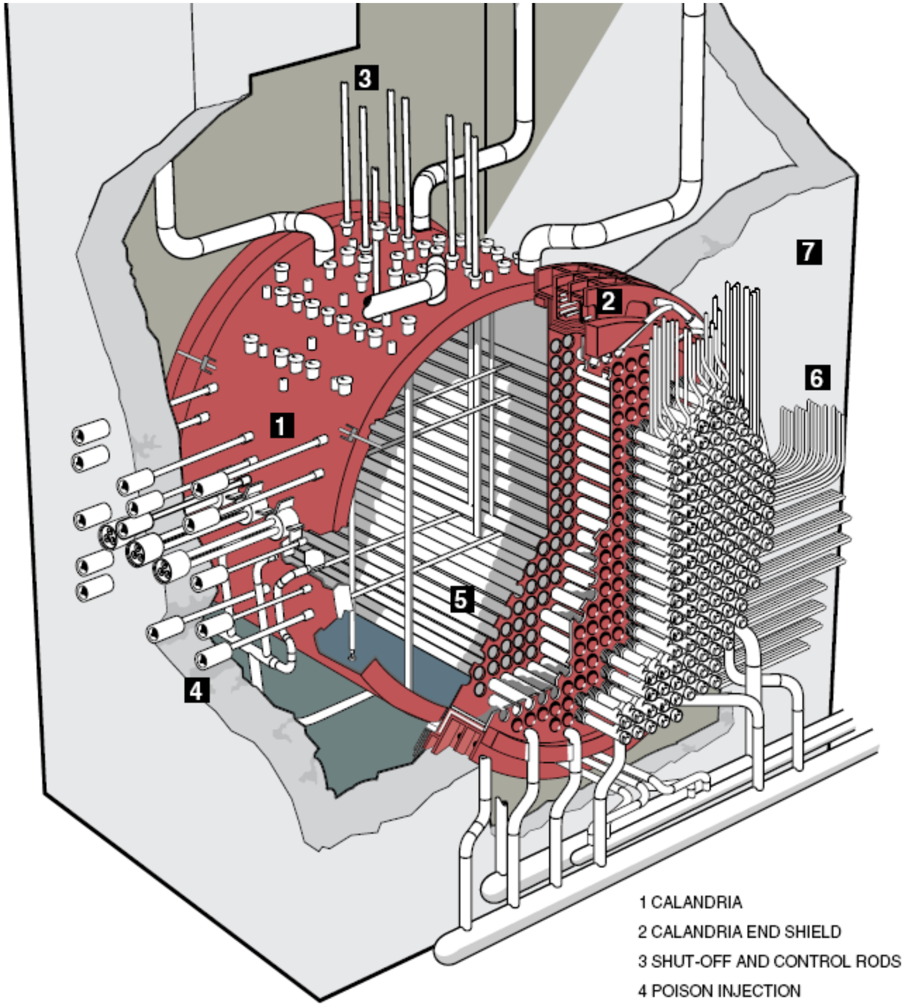
Validation on the typical LOCA (asymmetric scram) of a 2D CANDU-like core

in **non-coupled** mode, described by **time-dependent diffusion data** in two parts :

1) uniform part = void reactivity insertion ... during first s via *Core nuf2 +0.5%*

2) x-broken part = W/E-asymmetric scram ... until t = 2.9 s via *West abs2 +12.5%*





NUCLEAR SCIENCE AND ENGINEERING: 64, 95-105 (1977)

CANDU Reactor Kinetics Benchmark Activity

F. N. McDonnell, A. P. Baudouin, P. M. Garvey, and J. C. Luxat

Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories
Chalk River, Ontario, Canada

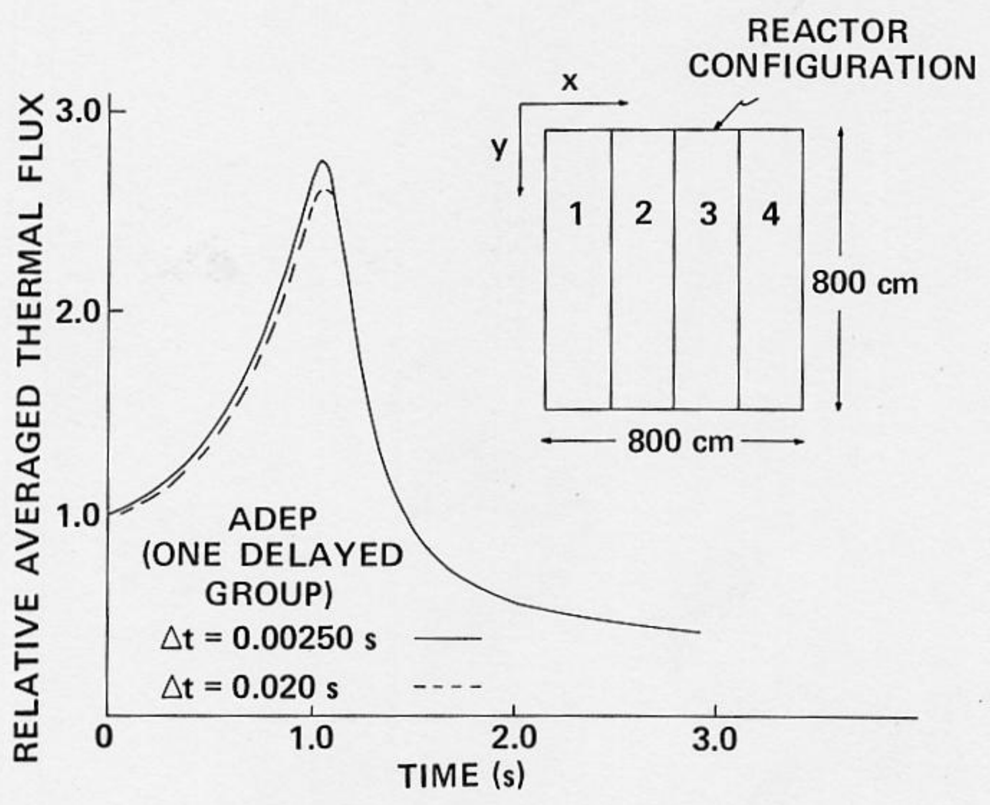


Fig. 1. Reactor configuration and time-dependent thermal flux, problem I.

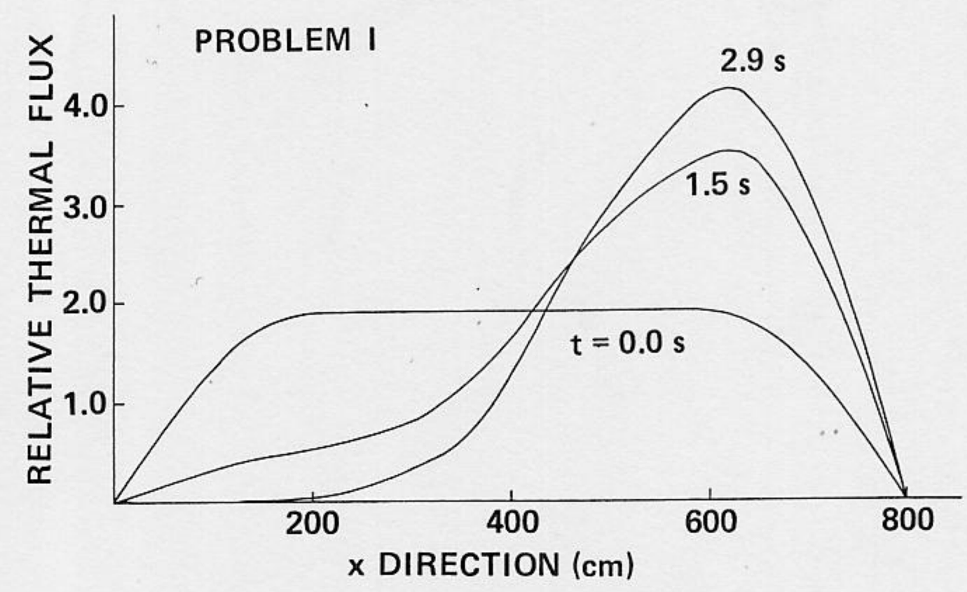
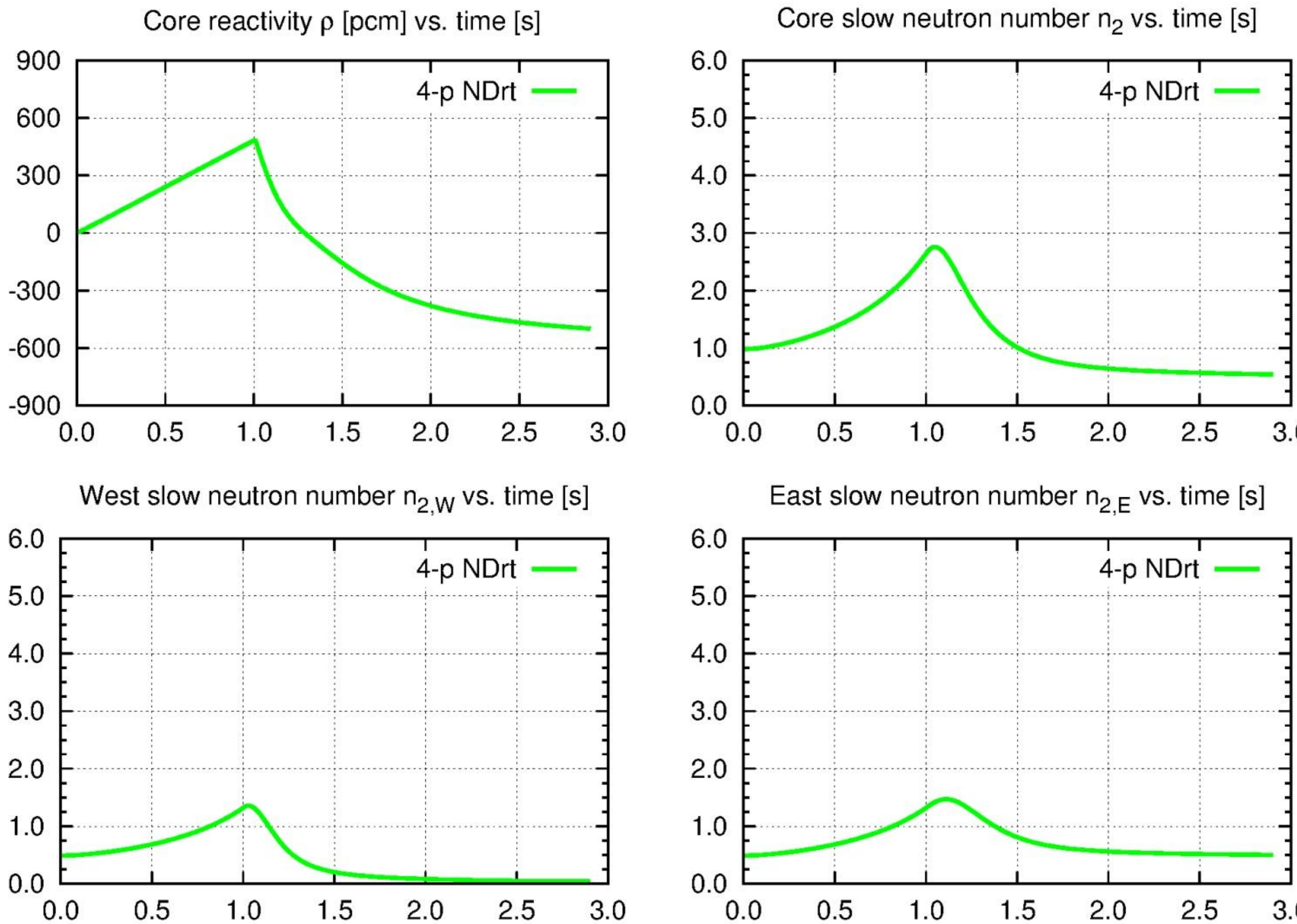
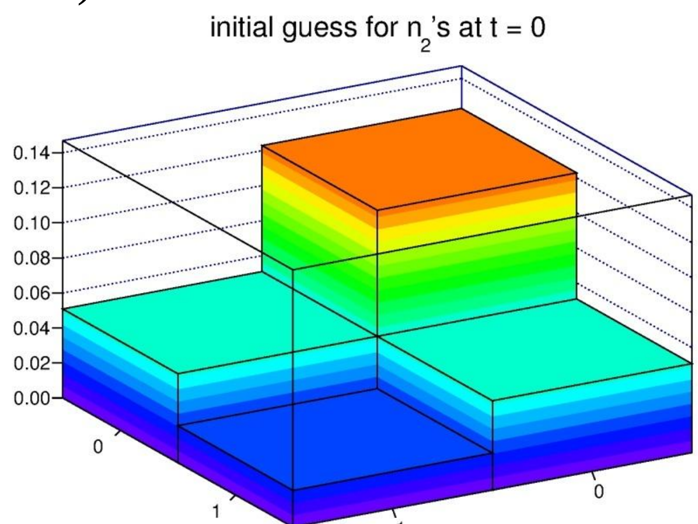


Fig. 2. Time-dependent thermal flux shape, CERKIN reference solution.

```
nuttin@lpsc3008x[cxx] drift
2-g 4-p (2D) Nodal Drift Method
... few nodes connected via drift rates
****************************************
Initialization of the solution vector :
OK, BOT has converged (at t = 0.8640s)
and reconverged after renormalization
with criticality correction = 1.00182
****************************************
Diffusion definition of the transient :
Core nuf2 +0.5% from 0 to 1s
West abs2 +12.5% from 1 to 2.9s
East abs2 constant until the end
****************************************
Computation of full spatial transient :
From t = 0s (Keff = 1)
n_tot = 1
n2_tot = 0.982357
n2_E_tot = 0.491178
... up to t = 2.9s (Keff = 0.99504)
n_tot = 0.55165
n2_tot = 0.54181
n2_E_tot = 0.50081
```

BOT convergence : iterative calc. without precursors and with constant Diffusion Data (Criticality Correction)





```
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n2_tot = 0.54181
n2_E_tot = 0.50081
```
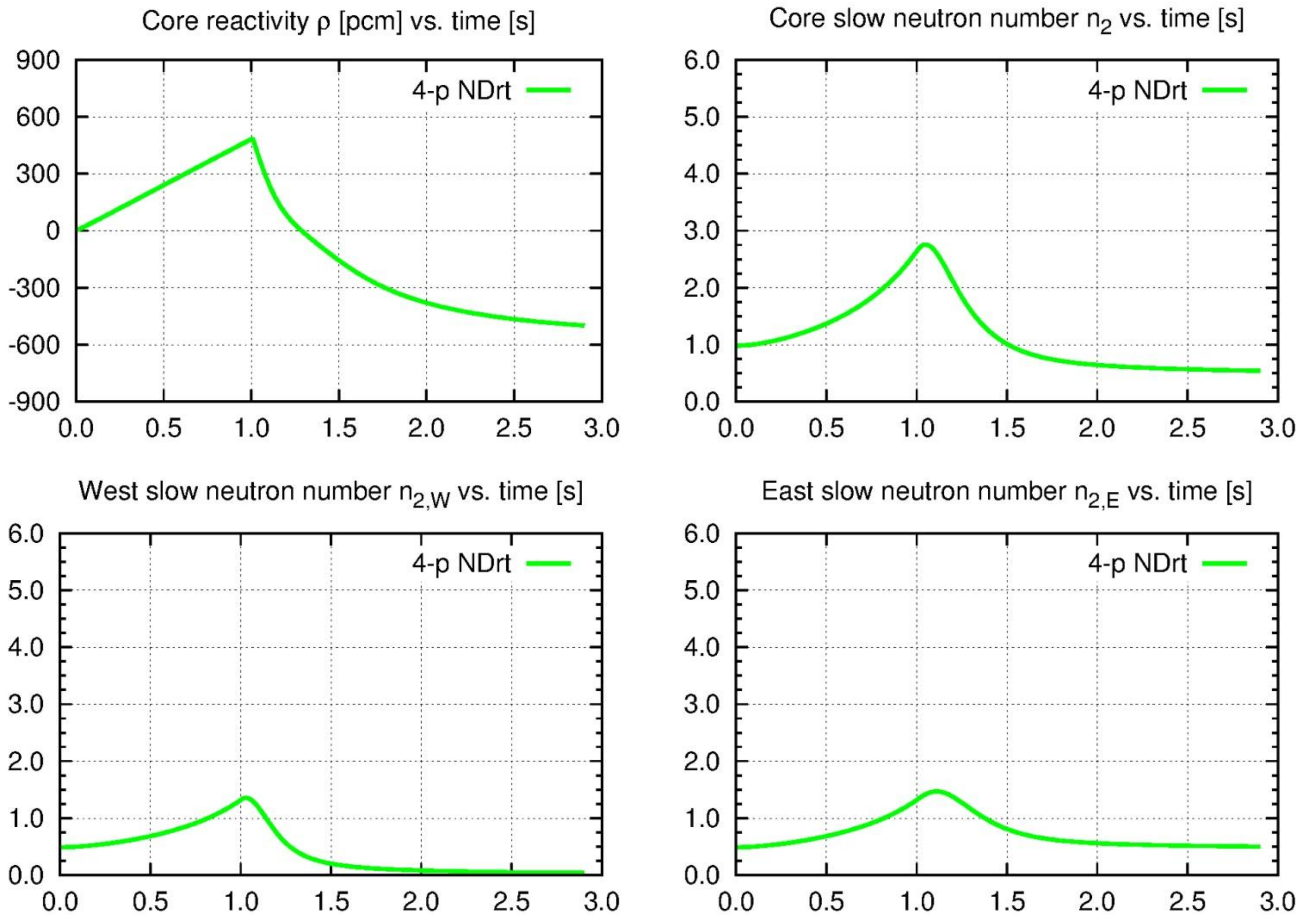
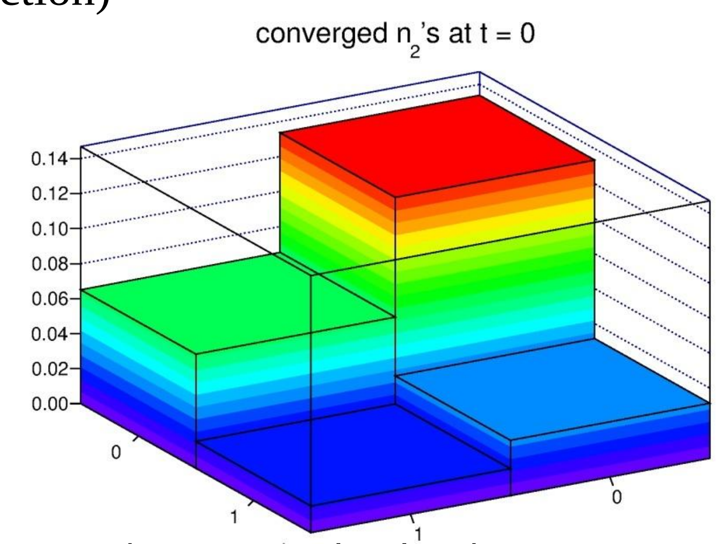
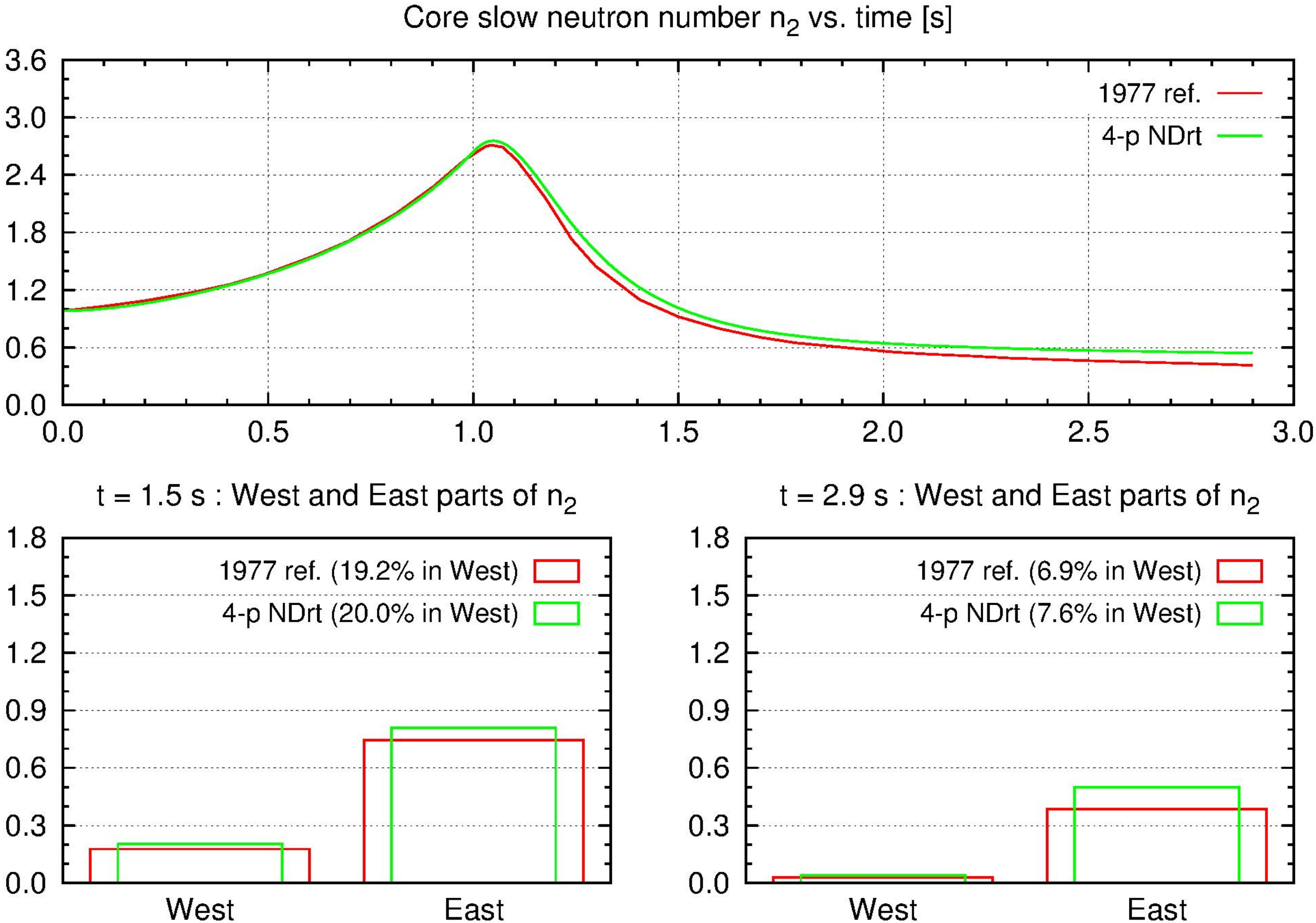
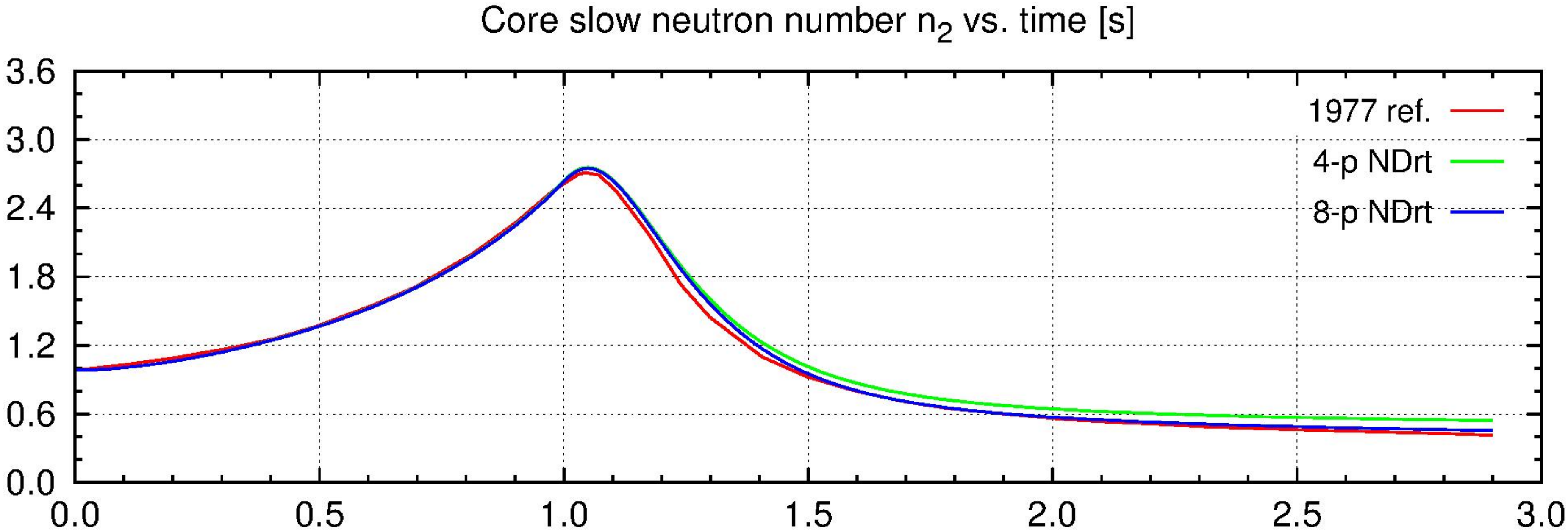



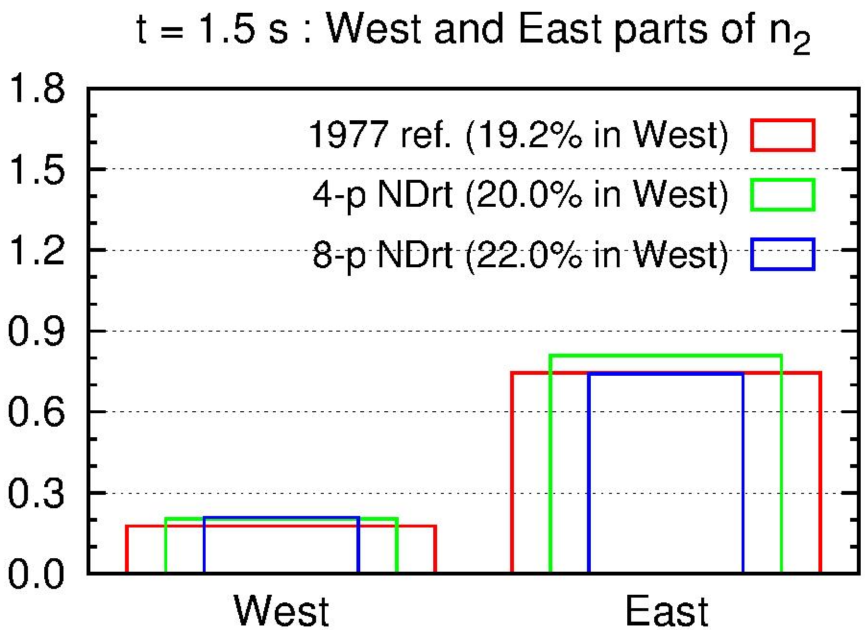
BOT convergence : iterative calc. without precursors and with constant Diffusion Data (Criticality Correction)

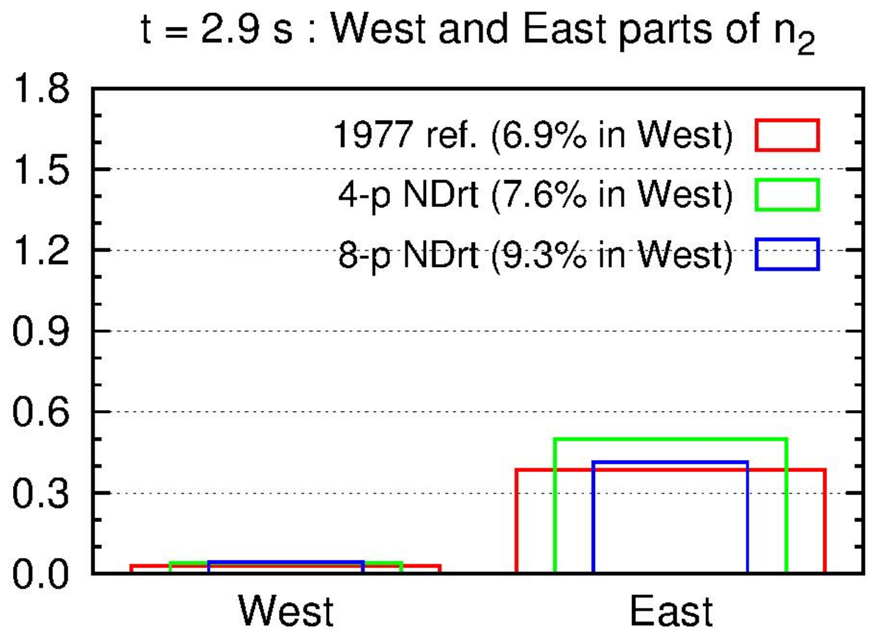




Core slow neutron number n2 vs. time [s]
1977 ref.
4-p NDrt
3.6
3.0
2.4
1.8
1.2
0.6
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t = 1.5 s : West and East parts of n2
1977 ref. (19.2% in West)
4-p NDrt (20.0% in West)
West
East
t = 2.9 s : West and East parts of n2
1977 ref. (6.9% in West)
4-p NDrt (7.6% in West)
1.8
1.5
1.2
0.9
0.6
0.3
0.0
West
East


Core slow neutron number n_2 vs. time [s]
3.6
3.0
2.4
1.8
1.2
0.6
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1977 ref.
4-p NDrt
8-p NDrt


t = 1.5 s : West and East parts of n_2
1.8
1.5
1.2
0.9
0.6
0.3
0.0
1977 ref. (19.2% in West)
4-p NDrt (20.0% in West)
8-p NDrt (22.0% in West)
West
East

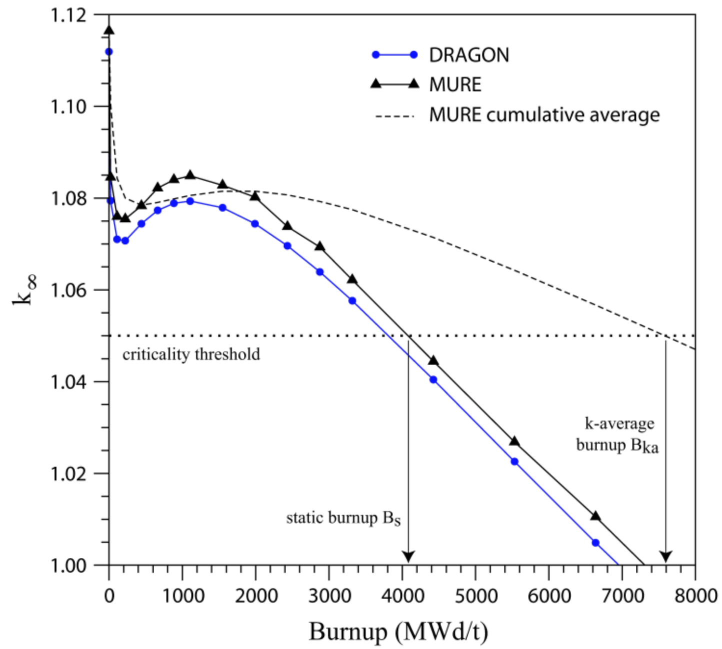

t = 2.9 s : West and East parts of n_2
1.8
1.5
1.2
0.9
0.6
0.3
0.0
1977 ref. (6.9% in West)
4-p NDrt (7.6% in West)
8-p NDrt (9.3% in West)
West
East

MCNP calculation of core-equivalent diffusion data for LOCA in CANDU6 NU

Middle Of Cycle (MOC) stands for the core refueling equilibrium (at ~ 4 GWd/t for CANDU NU →)

General rule is conservation of total reaction rates (ΦV-weighting)

Like **P_1 Diffusion Coefficients** from transport cross-sections (with A-dependent correction)



CANDU6 assembly

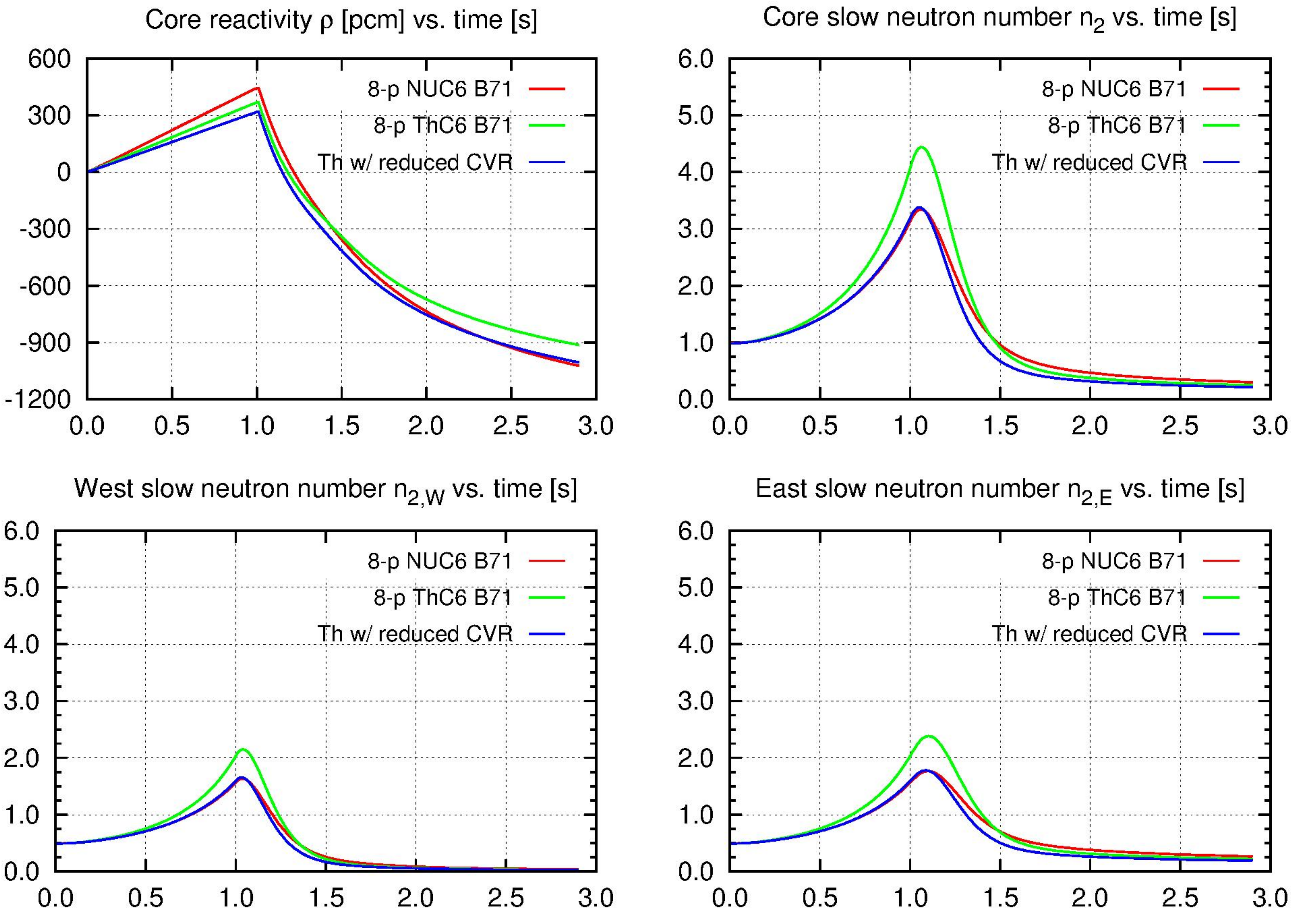
	Group 1 (above 0.625 eV)					Group 2			
	D_1 [cm]	v_1 [10^6 cm.s^{-1}]	Σ_{a1} [10^{-3} cm^{-1}]	$\upsilon\Sigma_{f1}$ [10^{-3} cm^{-1}]	Σ_{12} [10^{-2} cm^{-1}]	D_2 [cm]	v_2 [10^5 cm.s^{-1}]	Σ_{a2} [10^{-3} cm^{-1}]	$\upsilon\Sigma_{f2}$ [10^{-3} cm^{-1}]
MBGL 1977 & 2013	1.20	7.16	1.00	-	0.90	0.90	2.86	4.00	4.53
CANDU6 NU	1.22	8.32	2.10	0.91	1.02	1.07	2.87	4.11	4.75
CANDU6 Th	1.24	8.38	2.06	1.46	1.02	1.06	2.89	4.96	5.39

Table 2 : 2-group initial diffusion data before criticalization (from MCNP5 w/ B-VII.1 assembly calculations @MOC for CANDU6)

Other methods to test for D's : Serpent's $\mathbf{B_1}$ (leakage correction) and $\mathbf{L^2}$ (diffusion area), $\mathbf{G_1}$ (G_3 ?)

Recent inspiring works : *E. Fridman & J. Leppänen (PHYSOR 2012), B. Herman et al. (M&C 2013), ...*

Application to a global safety assessment of Th/^{233}U CANDU, compared on LOCA to ^{nat}U CANDU



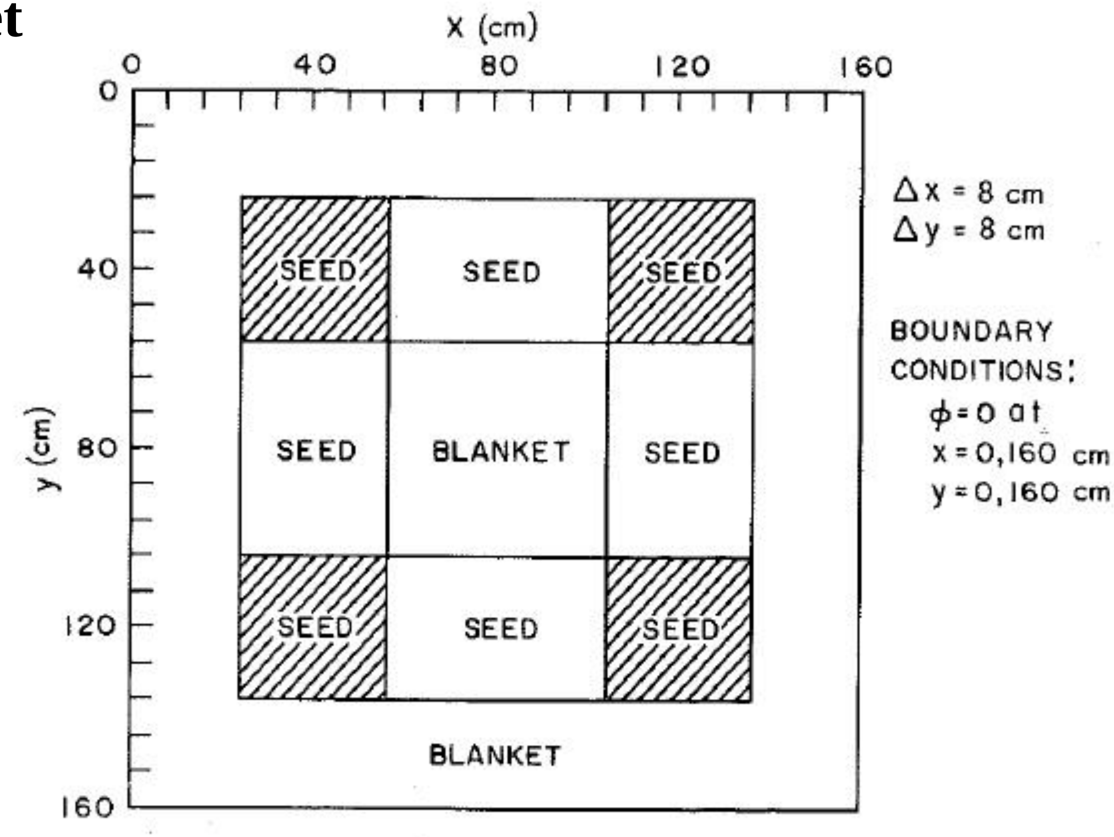
Towards LWR studies and more sophisticated benchmarks

benchmarks
Towards LWR studies and more sophisticated

TWIGL 2D ***uncoupled*** seed-blanket

ramp REA :

in the 4 square seed regions
***XS_abs2 decreases* linearly**
down to 0.1465 cm^{-1} at t = 0.2 s



CRITICAL PARAMETERS

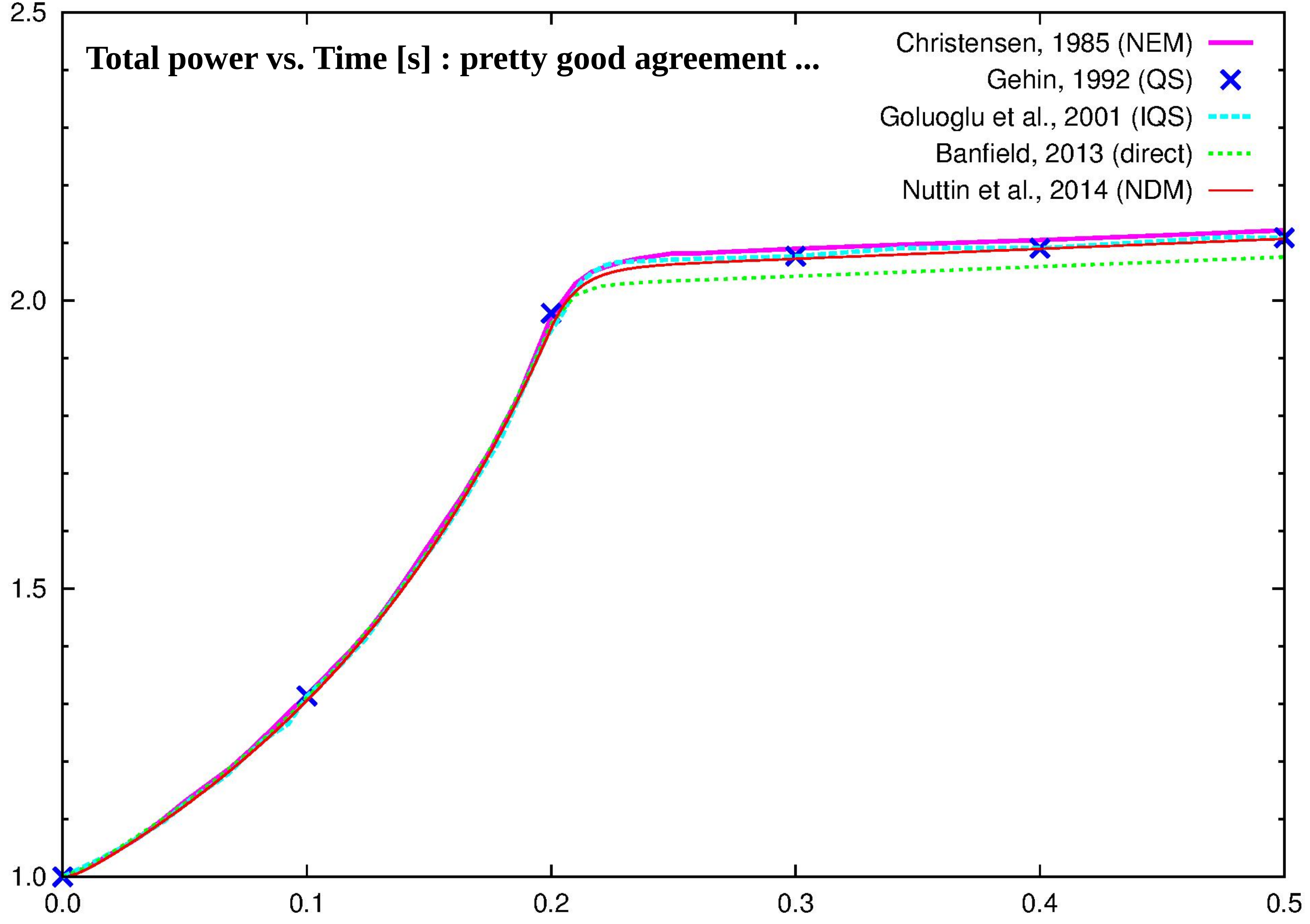
	D_1 (cm)	D_2 (cm)	Σ_{a_1} (cm^{-1})	Σ_{a_2} (cm^{-1})	Σ_{r_1} (cm^{-1})	$\nu\Sigma_{f_1}$ (cm^{-1})	$\nu\Sigma_{f_2}$ (cm^{-1})
SEED	1.4	0.4	0.01	0.15	0.01	$0.007/\lambda_0$	$0.2/\lambda_0$
BLANKET	1.3	0.5	0.008	0.05	0.01	$0.003/\lambda_0$	$0.06/\lambda_0$

Criticality Correction → λ_0 = 0.914193

INVERSE-NEUTRON SPEEDS: $1/v_1 = 1 \times 10^{-7}$ sec/cm
$1/v_2 = 5 \times 10^{-6}$ sec/cm

DELAYED-NEUTRON PARAMETERS: $\lambda = 0.08$ SEC^{-1}
$\beta = 0.0075$

Well-known benchmark, used a lot since its first definition by Hageman and Yasinsky (Nucl. Sc. Eng., 1969) for the TWIGL code (Bettis Lab)


Total power vs. Time [s] : pretty good agreement ...
Christensen, 1985 (NEM)
Gehin, 1992 (QS)
Goluoglu et al., 2001 (IQS)
Banfield, 2013 (direct)
Nuttin et al., 2014 (NDM)
2.5
2.0
1.5
1.0
0.0
0.1
0.2
0.3
0.4
0.5

... but important discrepancies on local powers

(20-p calc., relative errors in % with Christensen, 1985)

y

seed

-45.45	-45.38	-45.34	-45.49	-45.56	-45.53	-46.42	-47.85	-51.41	-71.03
-9.51	-9.61	-9.53	-9.56	-9.76	-10.27	-11.50	-14.47	-21.04	-51.41
4.07	4.03	4.06	3.91	3.66	3.07	0.94	-5.43	-14.47	-47.85
-2.30	-2.32	-2.34	-2.38	-2.59	-3.07	-7.38	0.94	-11.50	-46.42
2.65	2.66	2.61	2.54	2.31	1.83	-3.07	3.07	-10.27	-45.53
3.30	3.25	3.09	3.10	2.80	2.31	-2.59	3.66	-9.76	-45.56
-1.03	-1.12	-0.10	2.54	3.10	2.54	-2.38	3.91	-9.56	-45.49
7.40	7.28	11.87	-0.10	3.09	2.61	-2.34	4.06	-9.53	-45.34
1.36	1.40	7.28	-1.12	3.25	2.66	-2.32	4.03	-9.61	-45.38
1.34	1.36	7.40	-1.03	3.30	2.65	-2.30	4.07	-9.51	-45.45

x

Nodal powers for North-East core quarter without any correction

... but important discrepancies on local powers (20-p calc., relative errors in % with Christensen, 1985)

y

seed

-45.45	-45.38	-45.34	-45.49	-45.56	-45.53	-46.42	-47.85	-51.41	-71.03
-9.51	-9.61	-9.53	-9.56	-9.76	-10.27	-11.50	-14.47	-21.04	-51.41
4.07	4.03	4.06	3.91	3.66	3.07	0.94	-5.43	-14.47	-47.85
-2.30	-2.32	-2.34	-2.38	-2.59	-3.07	-7.38	0.94	-11.50	-46.42
2.65	2.66	2.61	2.54	2.31	1.83	-3.07	3.07	-10.27	-45.53
3.30	3.25	3.09	3.10	2.80	2.31	-2.59	3.66	-9.76	-45.56
-1.03	-1.12	-0.10	2.54	3.10	2.54	-2.38	3.91	-9.56	-45.49
7.40	7.28	11.87	-0.10	3.09	2.61	-2.34	4.06	-9.53	-45.34
1.36	1.40	7.28	-1.12	3.25	2.66	-2.32	4.03	-9.61	-45.38
1.34	1.36	7.40	-1.03	3.30	2.65	-2.30	4.07	-9.51	-45.45

x

Nodal powers for North-East core quarter without any correction

y

12.51	12.68	12.88	12.81	13.19	14.31	14.56	15.91	19.17	31.91
1.37	1.32	1.55	1.79	2.07	2.41	2.77	2.96	4.57	19.17
6.65	6.69	6.88	7.03	7.31	7.63	7.13	3.75	2.96	15.91
-3.11	-3.06	-2.91	-2.67	-2.40	-2.07	-4.93	7.13	2.77	14.56
0.27	0.35	0.49	0.71	0.98	1.32	-2.07	7.63	2.41	14.31
0.09	0.12	0.14	0.46	0.66	0.98	-2.40	7.31	2.07	13.19
-4.58	-4.59	-3.43	-0.57	0.46	0.71	-2.67	7.03	1.79	12.81
3.25	3.24	7.85	-3.43	0.14	0.49	-2.91	6.88	1.55	12.88
-2.74	-2.61	3.24	-4.59	0.12	0.35	-3.06	6.69	1.32	12.68
-2.87	-2.74	3.25	-4.58	0.09	0.27	-3.11	6.65	1.37	12.51

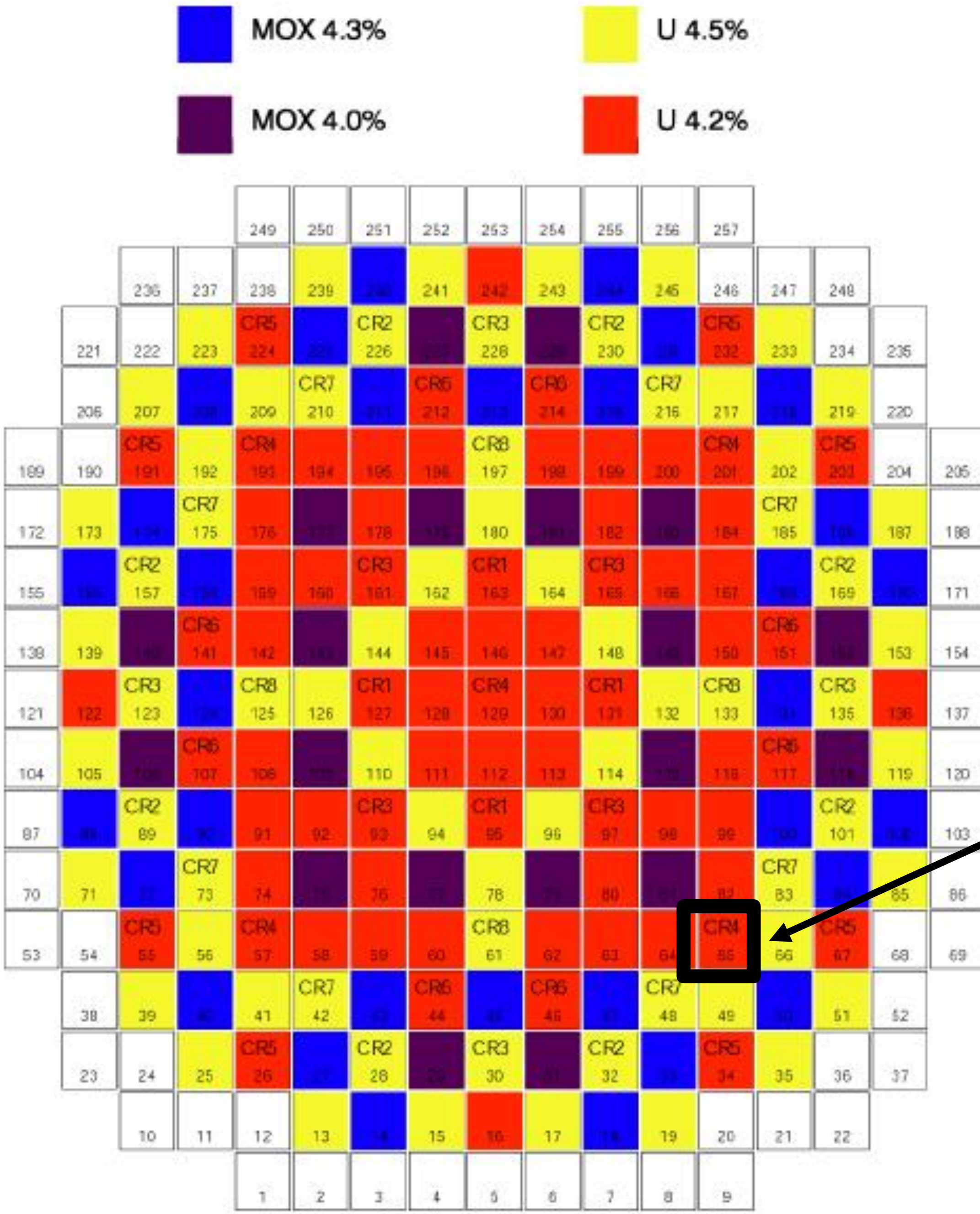
x

Same with an extrapolation distance for outer nodes (about 2 cm)

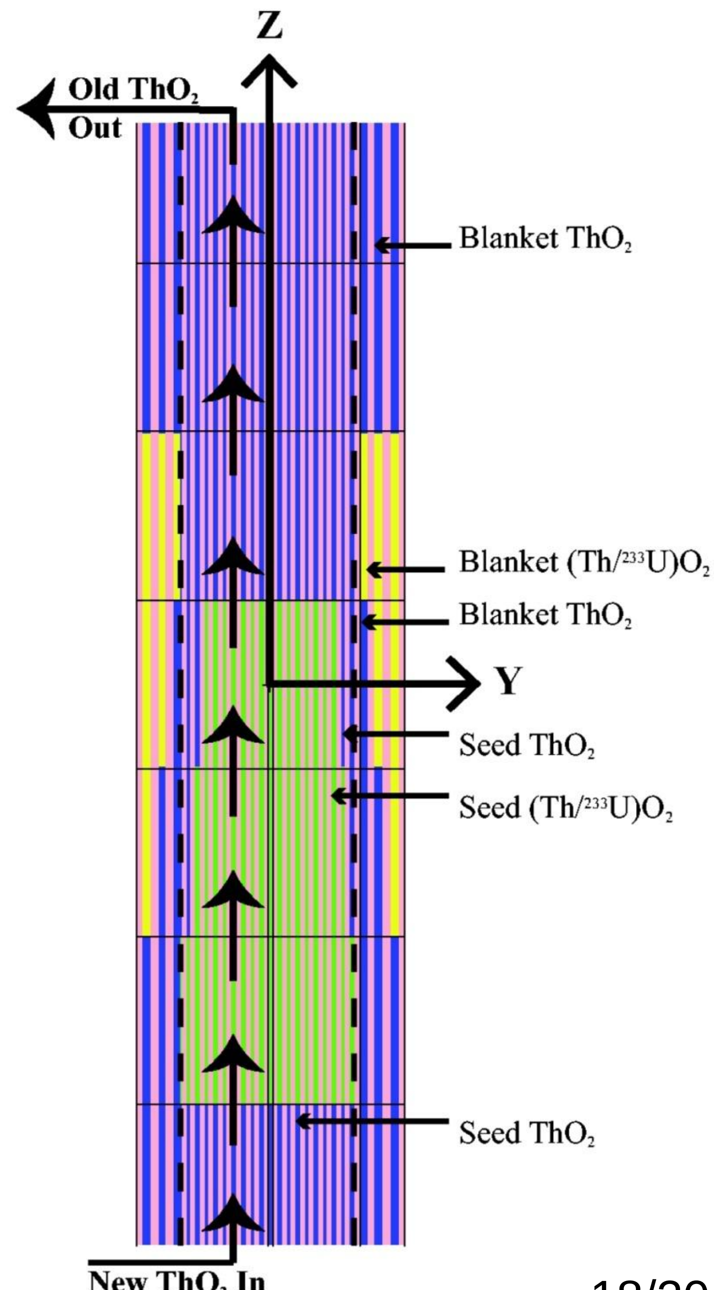
Coupled LWR benchmarks

Purdue full-core REA benchmark (Kozlowski and Downar, 2003)
. and then HCWR

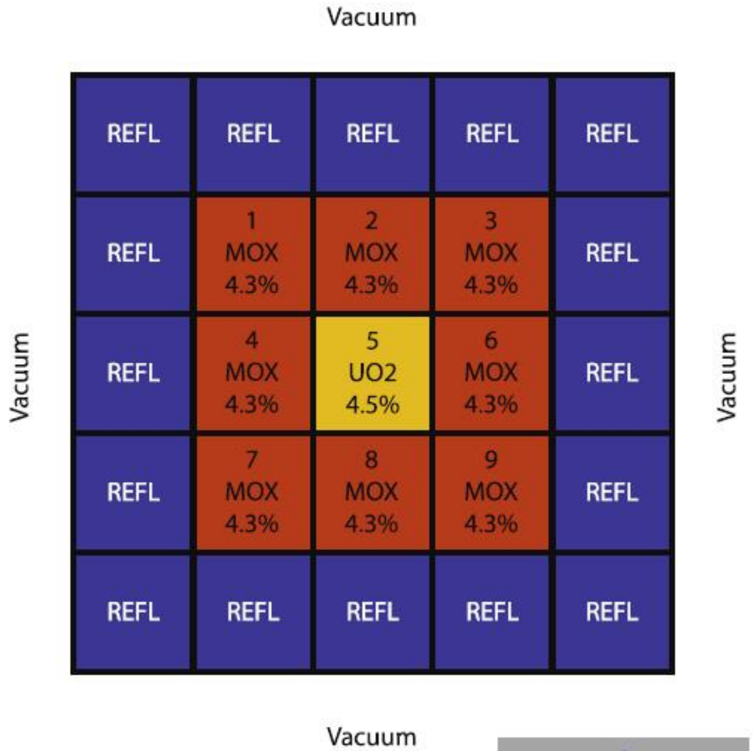
Pierre Prévot's Master Degree (then PhD thesis 2015-18 @LPSC)

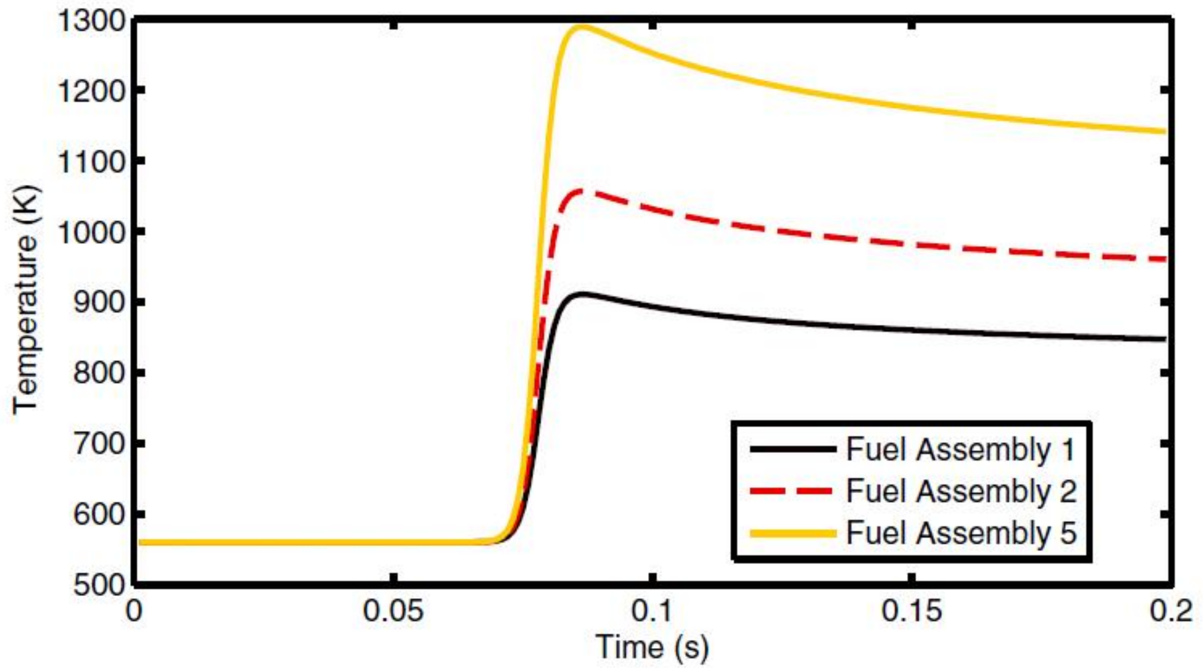


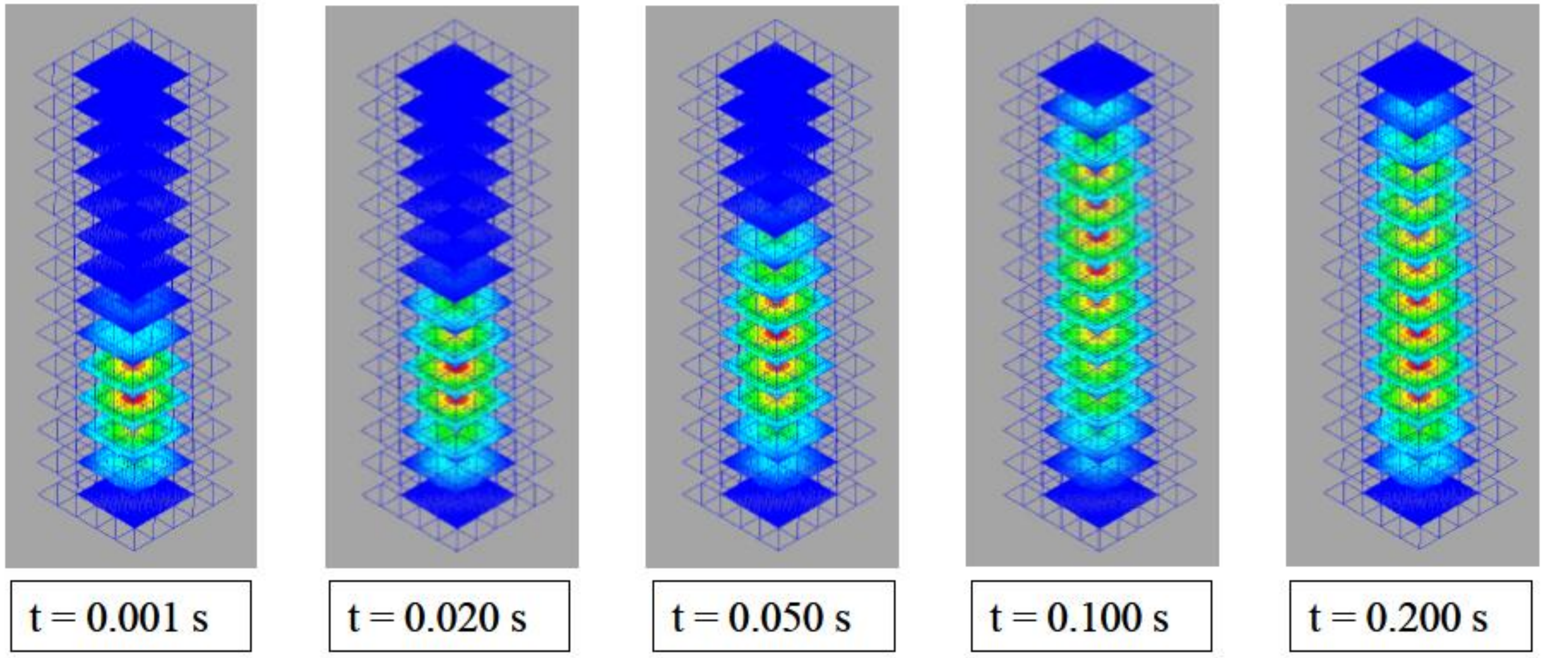
ejected rod (in 0.1 s) from HZP



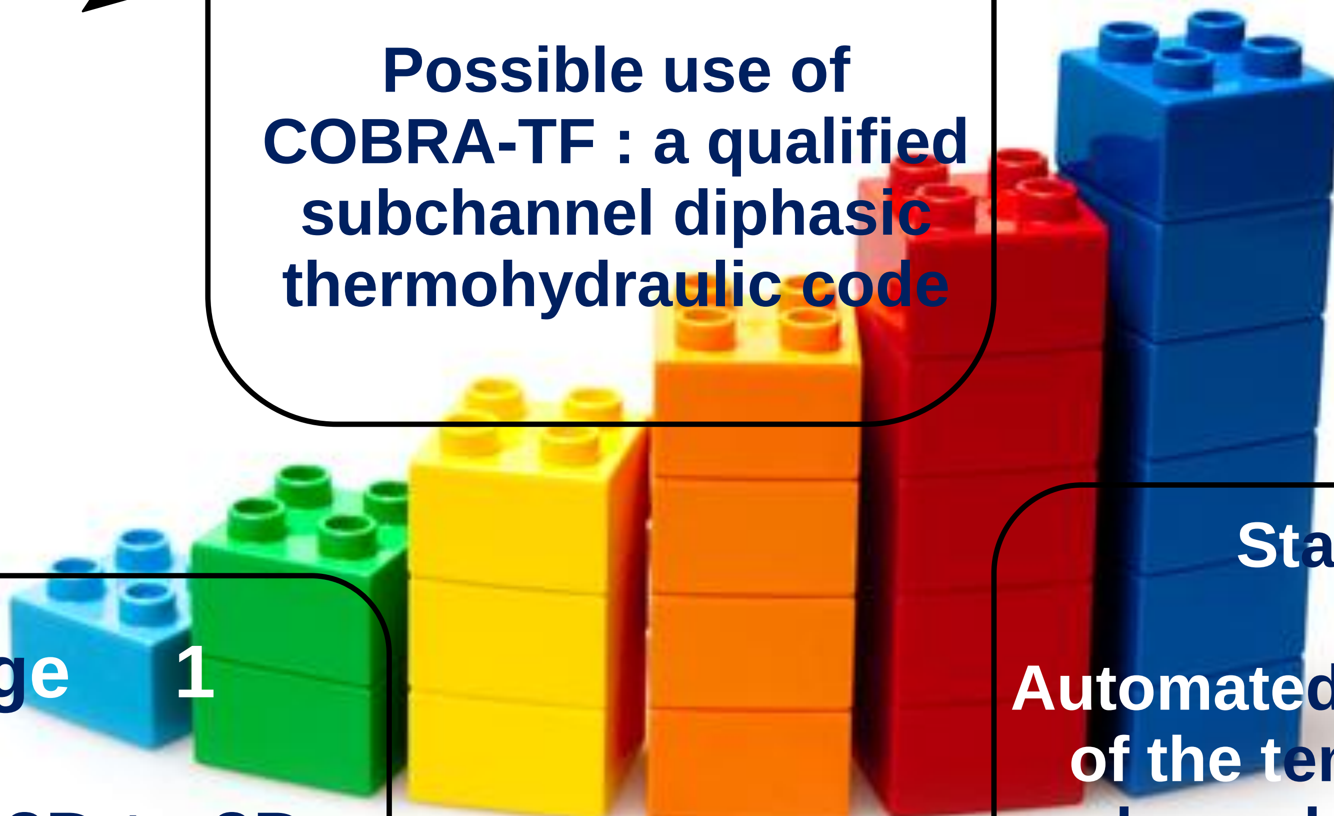
NURISP mini-core REA benchmark (Kliem et al., FZD, 2011) : first coupling







Next implementations in the Nodal Drift Method



Stage 2

Improvement of the thermohydaulic model

Possible use of COBRA-TF : a qualified subchannel diphasic thermohydraulic code

Stage 1

From 2D to 3D

Stage 3

Automated generation of the temperature dependent cross sections

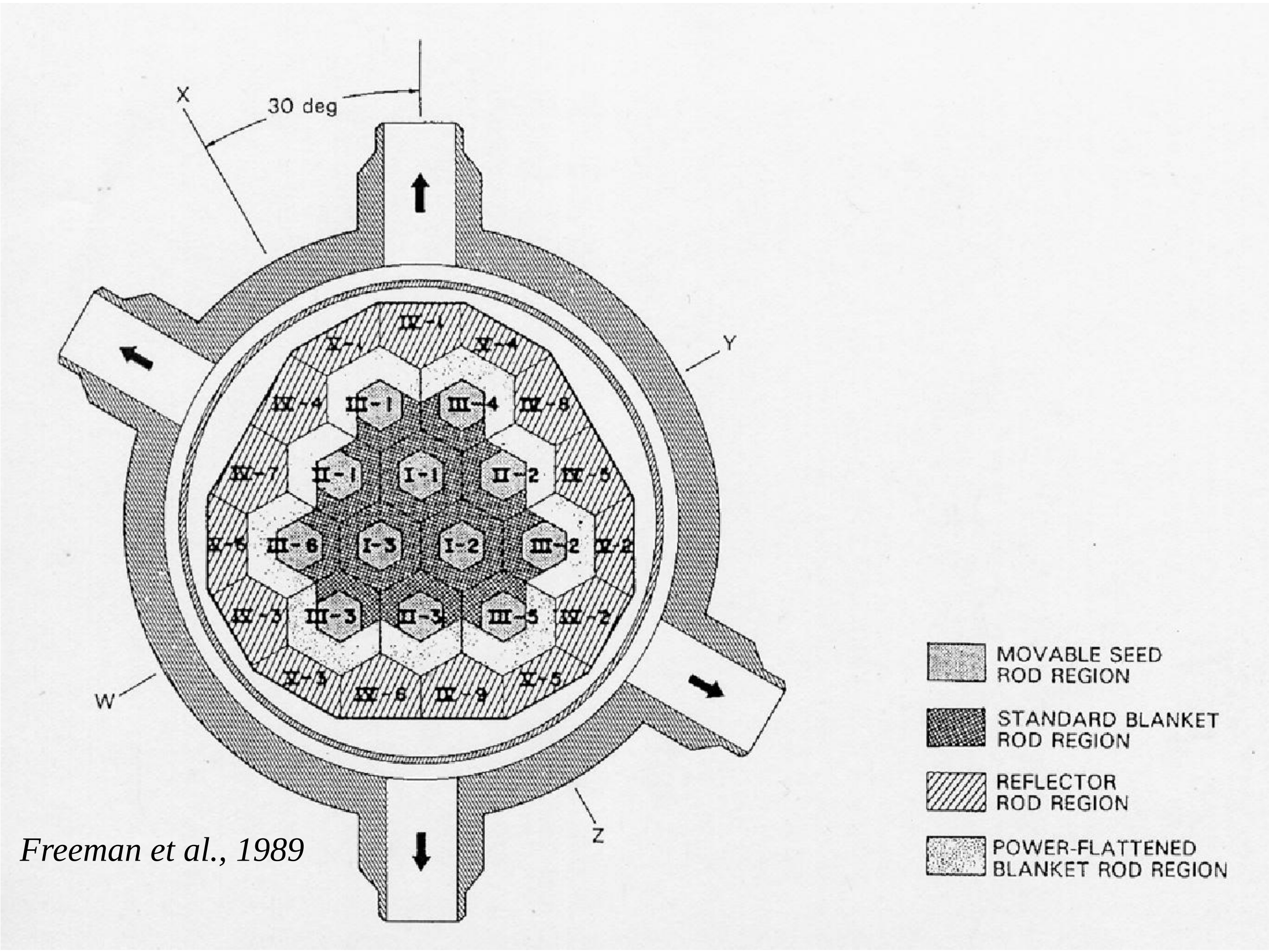
Strongly sub-moderated LWR with derated power : the LWBR case

- Light Water Breeder ("Proof of Breeding") Reactor (60 MWe, 1977-82, Shippingport)

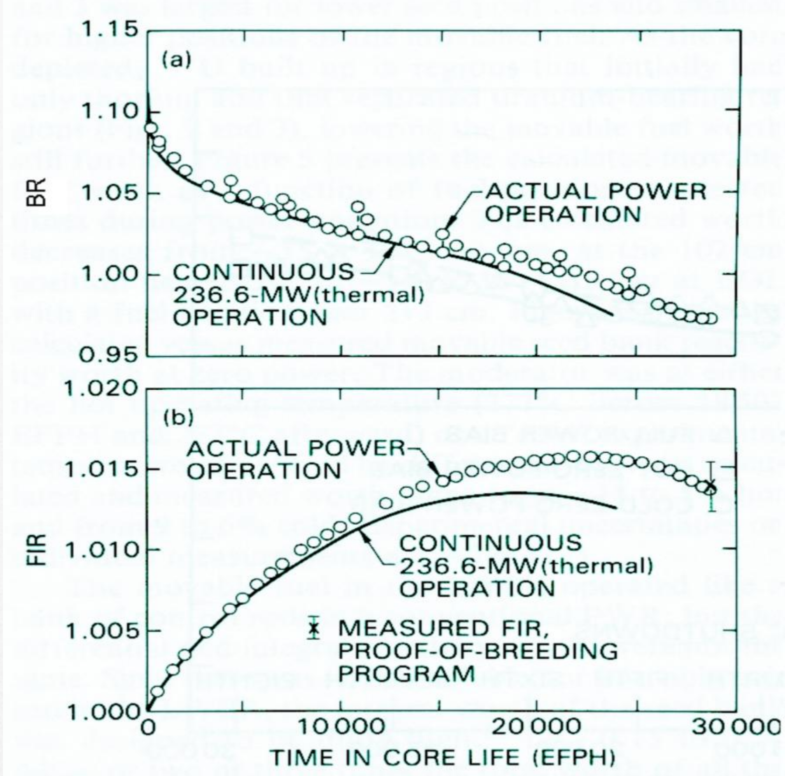
 Experimental result : breeding with Th/^{233}U oxide in a PWR H_2O-cooled vessel

- Main features : **global MR of 0.5** (very heterogeneous seed/blanket core)

 and **reduced vol. power** (30 W/cm^3 on the entire core)

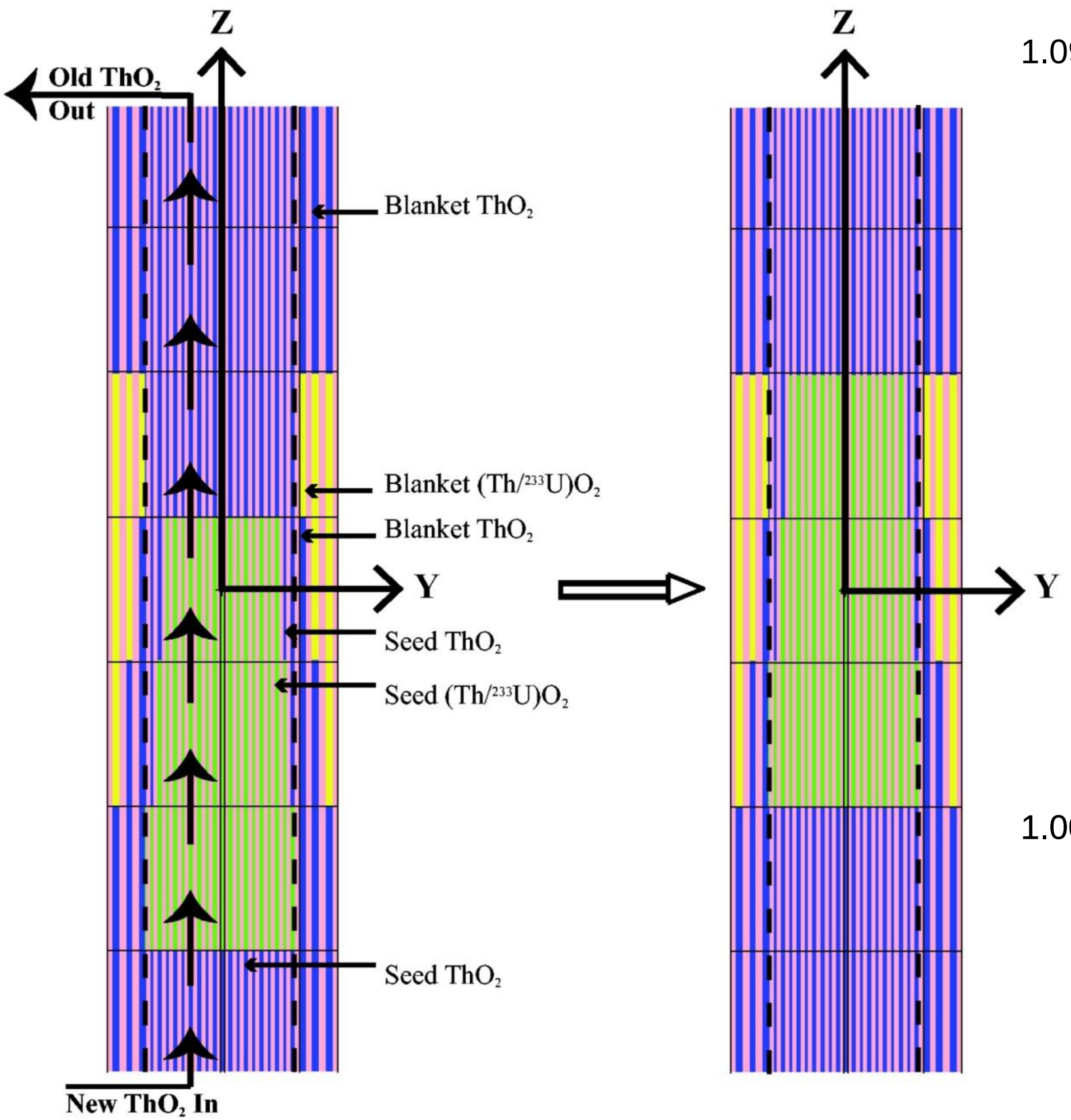


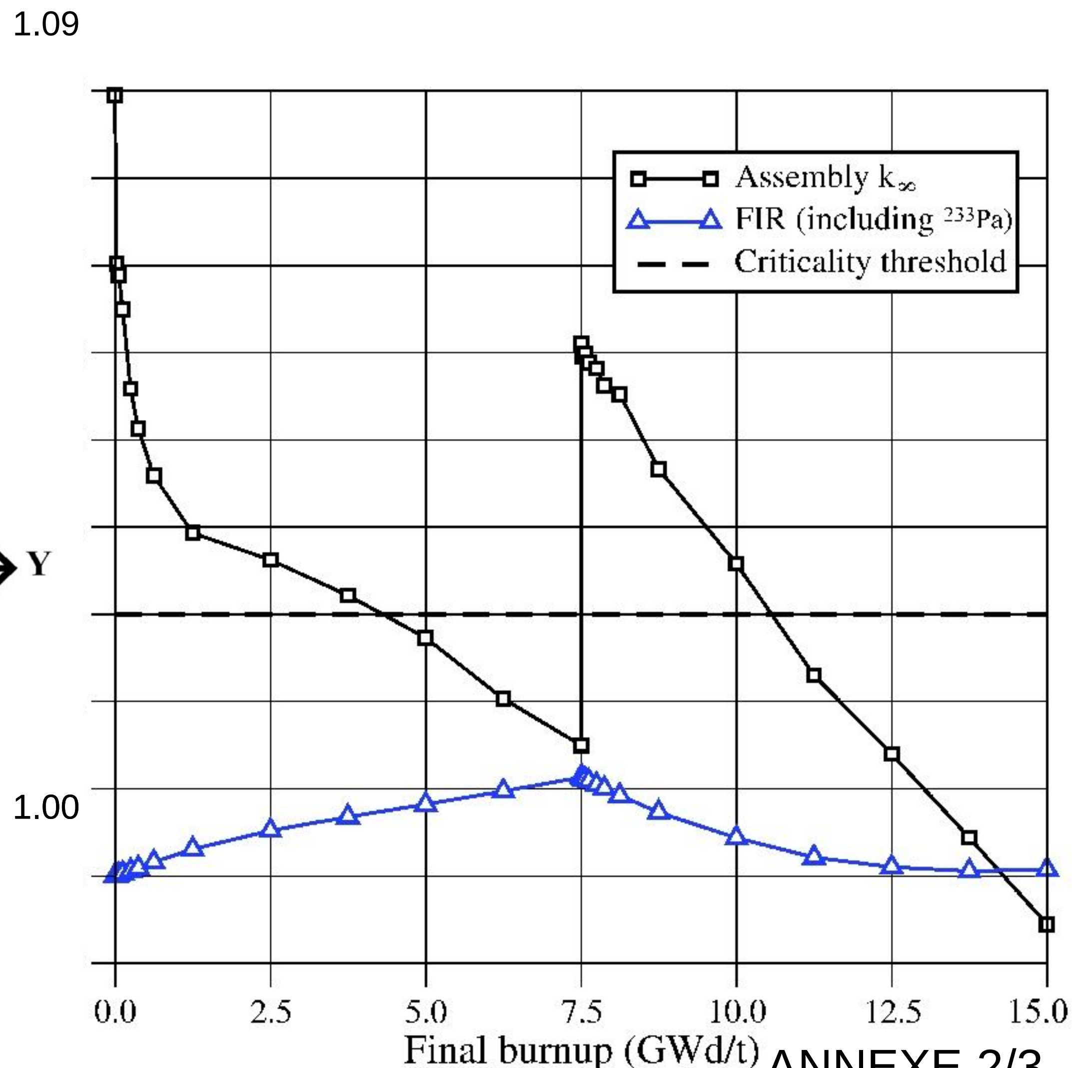
Freeman et al., 1989



Simple MCNP/MURE model of a LWBR-like core (before extrapolation)

- Seed/Blanket mirrored assembly (20 MWth, enrich. ~ 3/1 wt%, MR ~ 0.7/0.4)
- Reactivity managed by withdrawal of central Seed => **FIR = 1.01 at 15 GWd/t**
- Extrapolation within a PWR envelope => **total power limited to 300 MWe**





NUCLEAR SCIENCE AND ENGINEERING: 22, 171-181 (1965)

Some Numerical Experiments Concerning Space-Time Reactor Kinetics Behavior

J. B. Yasinsky and A. F. Henry

Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, West Mifflin, Pennsylvania 15122

b) The Effective D/L Method

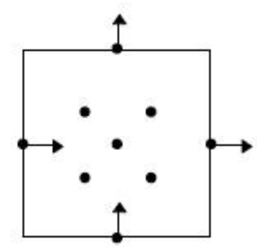
This method is similar to a course finite difference approach. It consists of integrating the diffusion equations over each node and defining average parameters and average fluxes that are located at the center of each node. The net current between the i'th and the j'th node is then approximated by

$$j_{ij} = -\frac{\widetilde{D}_{ij}}{L_{ij}} (\phi_j - \phi_i) S_{ij} , \qquad (13)$$

where S_{ij} is the area of the i,j interface, L_{ij} is the distance from the center of the i'th node to the center of the j'th node, and $\widetilde{D}_{ij}$ is an effective diffusion coefficient.

The Nodal Expansion Method (NEM) with quadratic leakage [5][6] is used to solve (1) To approximate the neutron flux, fourth order polynomials in (x,y) space and second order polynomials in the z-axis are needed. There are nine unknowns per node to determine as presented in Fig. 1:

Figure 1. Fourth degre nodal element with interface currents



B. Akherraz et al.

=> Nodal Drift Method
= unique coupling coefficient
= Nodal Expansion "at zeroth order"
= only unknowns are lumped fluxes