THE TRANSIENT FISSION MATRIX APPROACH USING SERPENT

-

APPLICATION TO FLATTOP EXPERIMENT AND MSFR COUPLING

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SERPENT AND MULTIPHYSICS WORKSHOP : GRENOBLE - 26-27 FEB 2015

SUMMARY

INTRODUCTION: OBJECTIVE OF THE CURRENT DEVELOPMENTS

I. TRANSIENT FISSION MATRIX
   - PRESENTATION
   - TFM KINETIC EQUATIONS
   - KINETIC PARAMETERS CALCULATION
   - TFM SIMPLIFIED KINETIC EQUATIONS

II. APPLICATION CASES
   - FLATTOP EXPERIMENT
   - MSFR TRANSIENT CALCULATION

INTRODUCTION: OBJECTIVE OF THE CURRENT DEVELOPMENTS

Context:

- Need to perform transient calculations for the MSFR
  - → neutronics / thermal-hydraulics coupling

Objectives:

- with a high precision of the T&H modeling (flow distribution, precursor transport, …)
  - → CFD code (OpenFOAM)
- with a high precision of the neutronics modeling
  - → Monte Carlo code (MCNP and SERPENT) …
- … with a low computational cost (need to perform many cases)
  - → Diffusion? Improved point kinetics? … something else?

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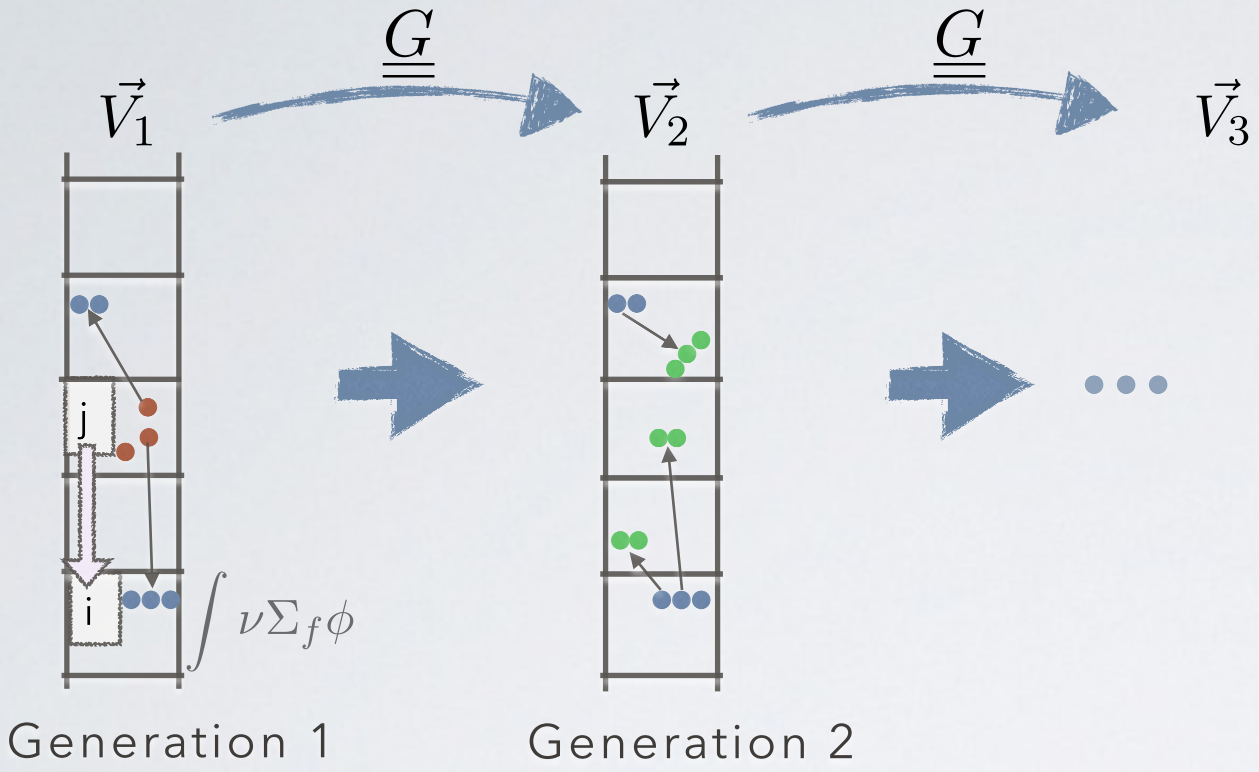
I. Transient Fission Matrix: Presentation



Overall principle:
Characterize the system response
on one generation (Green function)

step 1: Matrix element *ij*: volume *i* neutron production probability induced by an incoming source neutron injected in *j*

↓ TFM

step 2: Discretization of this matrix through time to get the temporal response

↓

step 3: Interpolation (Doppler & density feedback effects)

With $S(\boldsymbol{r}, t)$ the prompt source neutron distribution rate at time t in $\boldsymbol{r}$

With $G_{\chi_p\nu_p}(t'-t, \boldsymbol{r'}, \boldsymbol{r})$ the continuous operator associated to the transient fission matrix: the probability that a neutron created in $\boldsymbol{r'}, t'$ induces a new neutron in $\boldsymbol{r}, t$

prompt emission spectrum
prompt production

The kinetics of a prompt neutron population is given by:

$$S(\boldsymbol{r}, t) = \Big| G_{\chi_p\nu_p}(t'-t, \boldsymbol{r'}, \boldsymbol{r}) \Big| S(\boldsymbol{r'}, t') \Big\rangle = \iint_{t'<t, \boldsymbol{r'}\in\mathcal{R}} G_{\chi_p\nu_p}(t'-t, \boldsymbol{r'}, \boldsymbol{r}) \cdot S(\boldsymbol{r'}, t')\, \mathrm{d}\boldsymbol{r'}\, \mathrm{d}t'$$

I. TRANSIENT FISSION MATRIX: KINETIC EQUATIONS

AND WITH THE DELAYED NEUTRON PRECURSORS:

- Precursor family *f*

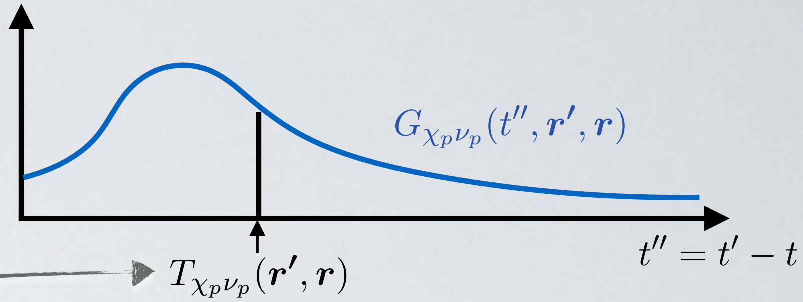
$$\frac{dP_f}{dt}(t,\boldsymbol{r}) \quad = \quad \underset{\substack{\uparrow \\ \text{family ratio}}}{\frac{\beta_f}{\beta_0}} \left[\overset{\text{delayed} \;\longleftarrow}{\Big| G_{\chi_p \nu_d}(t-t',\boldsymbol{r'},\boldsymbol{r}) \Big| S(t',\boldsymbol{r'}) \Big\rangle} + \overset{\text{delayed} \;\longleftarrow}{\Big| G_{\chi_d \nu_d}(t-t',\boldsymbol{r'},\boldsymbol{r}) \Big| \sum_f \lambda_f P_f(t',\boldsymbol{r'}) \Big\rangle} \right] - \underset{\substack{\uparrow \\ \text{decay constant}}}{\lambda_f} P_f$$

- Prompt source neutron distribution rate

$$S(t,\boldsymbol{r}) \quad = \quad \overset{\text{prompt} \;\longleftarrow}{\Big| G_{\chi_p \nu_p}(t-t',\boldsymbol{r'},\boldsymbol{r}) \Big| S(t',\boldsymbol{r'}) \Big\rangle} + \overset{\text{prompt} \;\longleftarrow}{\Big| G_{\chi_d \nu_p}(t-t',\boldsymbol{r'},\boldsymbol{r}) \Big| \sum_f \lambda_f P_f(t',\boldsymbol{r'}) \Big\rangle}$$

I. TRANSIENT FISSION MATRIX: KINETIC PARAMETERS CALCULATION

EFFECTIVE LIFE TIME l_{eff} CALCULATION:



We need the average time response:
directly computed in the SERPENT code

$$T_{\chi_p\nu_p}(\boldsymbol{r'},\boldsymbol{r}) = \frac{\displaystyle\int_{t''>0} G_{\chi_p\nu_p}(t'',\boldsymbol{r'},\boldsymbol{r})\cdot t''\,\mathrm{d}t''}{\displaystyle\int_{t''>0} G_{\chi_p\nu_p}(t'',\boldsymbol{r'},\boldsymbol{r})\,\mathrm{d}t''}$$

With the total response through time:
the classic FM operator

$$\widetilde{G}_{\chi_p\nu_p}(\boldsymbol{r'},\boldsymbol{r}) = \int_{-\infty}^{t} G_{\chi_p\nu_p}(t-t',\boldsymbol{r'},\boldsymbol{r})dt'$$

The adjoint operator and its Eigenvector
the neutron goes backward in generation = importance!

$$\widetilde{G}^{adj}_{\chi_p\nu_p} \longrightarrow N_p^*(\boldsymbol{r})$$

Finally:

$$l_{eff} = \frac{\displaystyle\iint_{\boldsymbol{r'}\in\mathcal{R},\boldsymbol{r}\in\mathcal{R}} N_p^*(\boldsymbol{r})\left[T_{\chi_p\nu_p}(\boldsymbol{r'},\boldsymbol{r}).\widetilde{G}_{\chi_p\nu_p}(\boldsymbol{r'},\boldsymbol{r})\right] N_p(\boldsymbol{r'})\,\mathrm{d}\boldsymbol{r'}\,\mathrm{d}\boldsymbol{r}}{\displaystyle\iint_{\boldsymbol{r'}\in\mathcal{R},\boldsymbol{r}\in\mathcal{R}} N_p^*(\boldsymbol{r})\widetilde{G}_{\chi_p\nu_p}(\boldsymbol{r'},\boldsymbol{r})N_p(\boldsymbol{r'})\,\mathrm{d}\boldsymbol{r'}\,\mathrm{d}\boldsymbol{r}}$$

And its discretized version:

$$l_{eff} = \frac{\displaystyle\sum_{\mathcal{R}} \boldsymbol{N_p^*}\left(\underline{\underline{T_{\chi_p\nu_p}}}\cdot\underline{\underline{\widetilde{G}_{\chi_p\nu_p}}}\right)\boldsymbol{N}_p}{\displaystyle\sum_{\mathcal{R}} \boldsymbol{N_p^*}\,\underline{\underline{\widetilde{G}_{\chi_p\nu_p}}}\boldsymbol{N}_p}$$

aimed average time × neutron production per incoming neutron × neutron population

importance weighting

produced neutron

I. TRANSIENT FISSION MATRIX: KINETIC PARAMETERS CALCULATION

EFFECTIVE FRACTION OF DELAYED NEUTRON β_{eff} CALCULATION:

We create the prompt and delay matrix operator:
+ Eigenvalue & Eigenvector

$$\underline{\underline{\widetilde{G}_{all}}} = \begin{pmatrix} \underline{\underline{\widetilde{G}_{\chi_p\nu_p}}} & \underline{\underline{\widetilde{G}_{\chi_d\nu_p}}} \\ \underline{\underline{\widetilde{G}_{\chi_p\nu_d}}} & \underline{\underline{\widetilde{G}_{\chi_d\nu_d}}} \end{pmatrix} \longrightarrow k_{eff} \quad \& \quad \boldsymbol{N} = (\boldsymbol{N}_p \; \boldsymbol{N}_d)$$

Its importance:
transpose matrix and Eigenvector

$$\underline{\underline{\widetilde{G}_{all}^{adj}}} \longrightarrow \boldsymbol{N}^* = \left(\boldsymbol{N}_p^* \; \boldsymbol{N}_d^*\right)$$

Finally, we can calculate the physical and effective fractions of delayed neutrons:

$$\beta_0 = \frac{\overset{\text{delayed}}{\sum \boldsymbol{N}_d}}{\underset{\text{total}}{\sum \boldsymbol{N}}} = \frac{k_{eff}.\sum(\boldsymbol{N}_d)}{k_{eff}.\sum(\boldsymbol{N})} = \frac{\sum\left(\underline{\underline{G_{\chi_p\nu_d}}}\boldsymbol{N}_p + \underline{\underline{G_{\chi_d\nu_d}}}\boldsymbol{N}_d\right)}{\sum\left(\underline{\underline{G_{all}}}\boldsymbol{N}\right)}$$

$$\boxed{\beta_{eff} = \frac{\boldsymbol{N}_d^*\boldsymbol{N}_d}{\boldsymbol{N}^*\boldsymbol{N}}} = \frac{\boldsymbol{N}_d^*\left(\underline{\underline{G_{\chi_p\nu_d}}}\boldsymbol{N}_p + \underline{\underline{G_{\chi_d\nu_d}}}\boldsymbol{N}_d\right)}{\boldsymbol{N}^*\underline{\underline{G_{all}}}\boldsymbol{N}}$$

importance
weighting

$$\updownarrow$$

classic formulation: $$\beta_{eff} = \frac{\int \psi^*\chi_d\nu_d\Sigma_f\psi \;\; \mathrm{d}E\,\mathrm{d}\Omega\,\mathrm{d}E'\,\mathrm{d}\Omega'\,\mathrm{d}\boldsymbol{r}}{\int \psi^*\chi\nu\Sigma_f\psi \;\; \mathrm{d}E\,\mathrm{d}\Omega\,\mathrm{d}E'\,\mathrm{d}\Omega'\,\mathrm{d}\boldsymbol{r}}$$

I. TRANSIENT FISSION MATRIX: TFM SIMPLIFIED KINETIC EQUATIONS

OVERALL PRINCIPE:

Replace the neutron production rate $S(\boldsymbol{r},t)$ by a neutron population $N(t,\boldsymbol{r})$ associated to a time constant l_{eff}:
note: can not model phenomenas with a shorter time constant

$$\begin{tikzcd}
 & & \Big|\widetilde{G}_{\chi_p\nu_p}(\boldsymbol{r}',\boldsymbol{r})\Big|N(t,\boldsymbol{r}')\dfrac{\mathrm{d}t}{l_{eff}}\Big\rangle & \text{new prompt neutron} \\
\text{prompt neutron disappearance during dt} & N(t,\boldsymbol{r})\dfrac{\mathrm{d}t}{l_{eff}} \arrow[ur] \arrow[dr] & & \\
 & & \Big|\widetilde{G}_{\chi_p\nu_d}(\boldsymbol{r}',\boldsymbol{r})\Big|N(t,\boldsymbol{r}')\dfrac{\mathrm{d}t}{l_{eff}}\Big\rangle & \text{new precursors}
\end{tikzcd}$$

Estimated using SERPENT (easy TFM implementation)

$$\begin{tikzcd}
 & & \Big|\widetilde{G}_{\chi_d\nu_p}(\boldsymbol{r}',\boldsymbol{r})\Big|\sum_f\lambda_f P_f(t,\boldsymbol{r}')\,\mathrm{d}t\Big\rangle & \text{new prompt neutron} \\
\text{precursor disappearance during dt} & \sum_f\lambda_f P_f(t,\boldsymbol{r})\,\mathrm{d}t \arrow[ur] \arrow[dr] & & \\
 & & \Big|\widetilde{G}_{\chi_d\nu_d}(\boldsymbol{r}',\boldsymbol{r})\Big|\sum_f\lambda_f P_f(t,\boldsymbol{r}')\,\mathrm{d}t\Big\rangle & \text{new precursors}
\end{tikzcd}$$

NEW SET OF EQUATIONS:

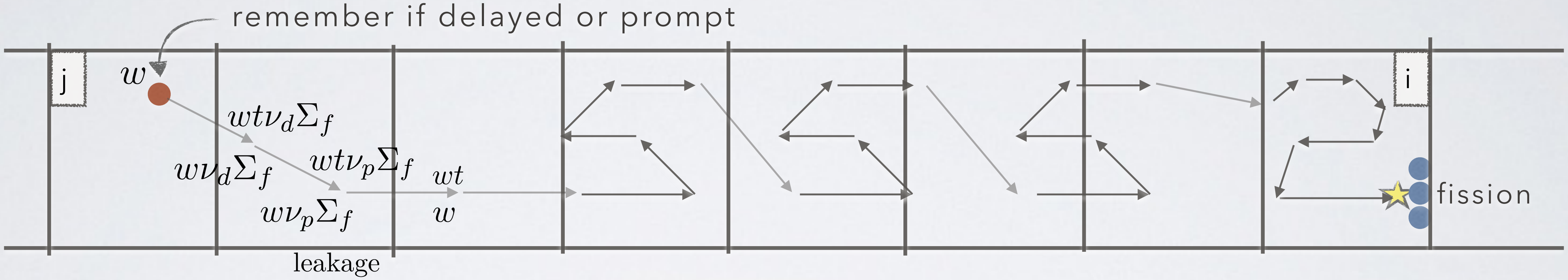
$$\begin{aligned}
\frac{dP_f}{dt}(t,\boldsymbol{r}) &= \frac{\beta_f}{\beta_0}\left[\frac{1}{l_{eff}}\Big|\widetilde{G}_{\chi_p\nu_d}(\boldsymbol{r}',\boldsymbol{r})\Big|N(t,\boldsymbol{r}')\Big\rangle + \Big|\widetilde{G}_{\chi_d\nu_d}(\boldsymbol{r}',\boldsymbol{r})\Big|\sum_f\lambda_f P_f(t,\boldsymbol{r}')\Big\rangle\right] - \lambda_f P_f(t,\boldsymbol{r}) \\
\frac{dN}{dt}(t,\boldsymbol{r}) &= \frac{1}{l_{eff}}\Big|\widetilde{G}_{\chi_p\nu_p}(\boldsymbol{r}',\boldsymbol{r})\Big|N(t,\boldsymbol{r}')\Big\rangle + \Big|\widetilde{G}_{\chi_d\nu_p}(\boldsymbol{r}',\boldsymbol{r})\Big|\sum_f\lambda_f P_f(t,\boldsymbol{r}')\Big\rangle - \frac{1}{l_{eff}}N(t,\boldsymbol{r})
\end{aligned}$$

this simplified formulation only requires simple matrix-vector products (instead of series of matrix vector previously)

I. TRANSIENT FISSION MATRIX: TFM IMPLEMENTATION

SERPENT ESTIMATION OF THE MATRICES:

During a classical critical calculation:



Simple explicite implementation: summing the neutron production of the fission events normalized by the neutron creation amount (prompt and delayed).
Trouble: extremely slow convergence

Better implicite implementation (*this work*): integration of the fission neutron production and absorption at each interaction (« delta tracking on »)
Advantage: much more events per neutron history, improved statistics

Advantages of the matrices estimation in a critical calculation:

- Utilisation of the correct emission spectrum
- Utilisation of the correct source neutron distribution inside the elementary volume (j)

I. TRANSIENT FISSION MATRIX: FISSION MATRIX INTERPOLATION

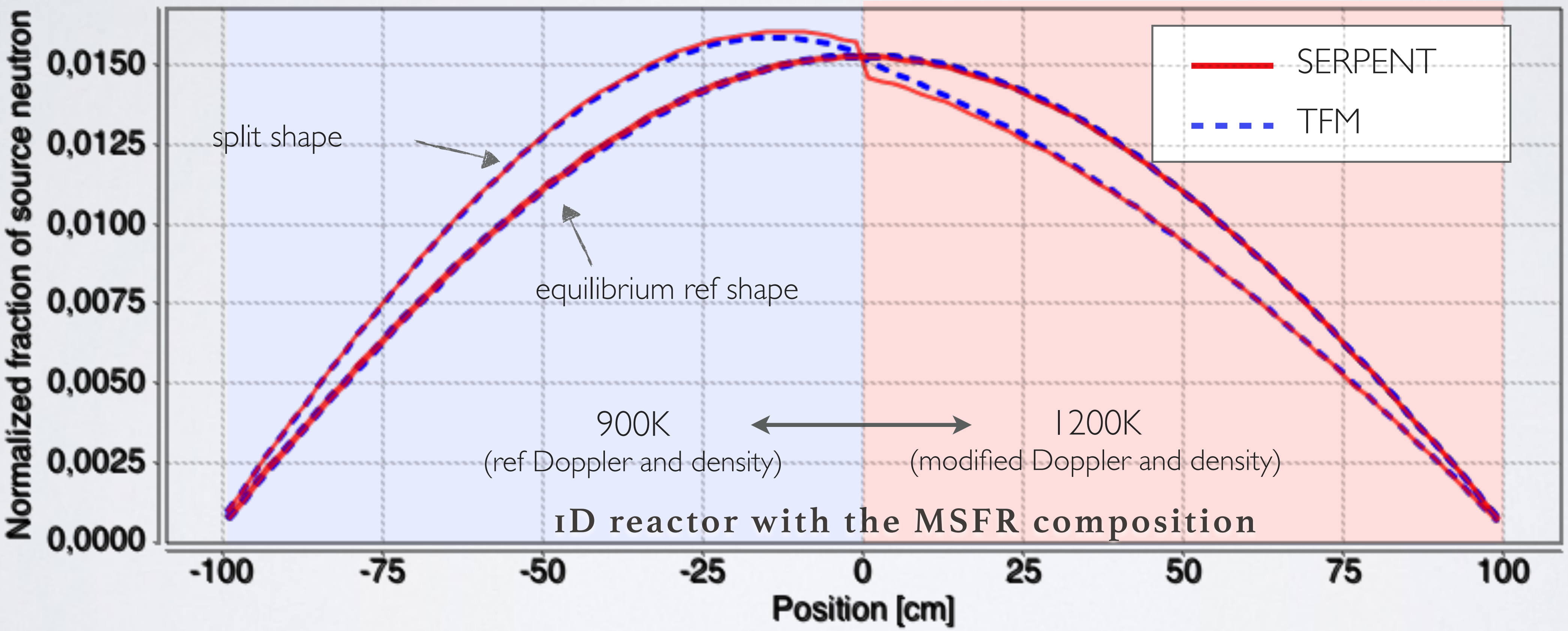
AND FOR TRANSIENT CALCULATIONS?

Matrix interpolation!

$$\widetilde{G}(\boldsymbol{r'},\boldsymbol{r}) = \widetilde{G}_{ref}(\boldsymbol{r'},\boldsymbol{r}) + (T(\boldsymbol{r'}) - T_{ref}) \cdot \Delta_\rho \widetilde{G}(\boldsymbol{r'},\boldsymbol{r}) + \log \frac{T(\boldsymbol{r'})}{T_{ref}} \cdot \Delta_{Doppler} \widetilde{G}(\boldsymbol{r'},\boldsymbol{r})$$

linear dependency

logarithmic dependency



- good modeling of the neutron shift
- good prediction of the multiplication factor variation (~1-2% error on 1000pcm)

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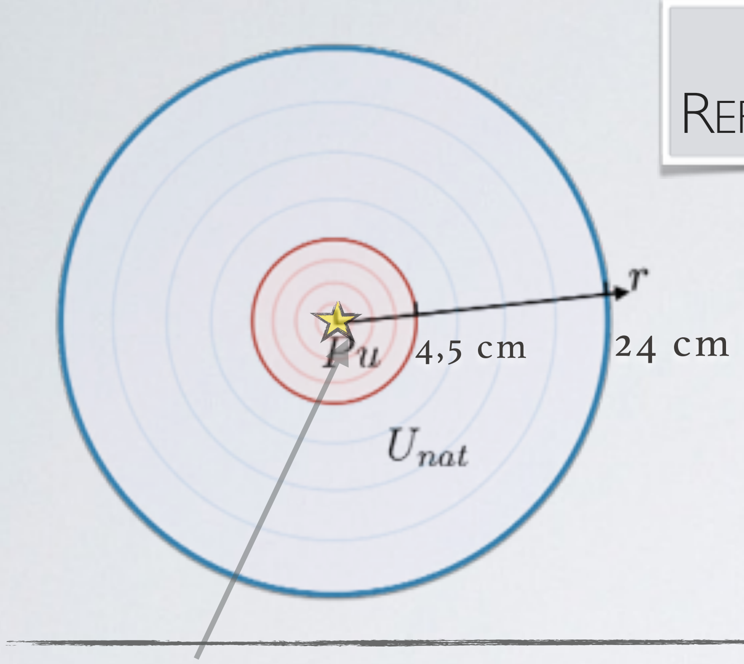
II. APPLICATION CASES: FLATTOP EXPERIMENT

BENCH CASE



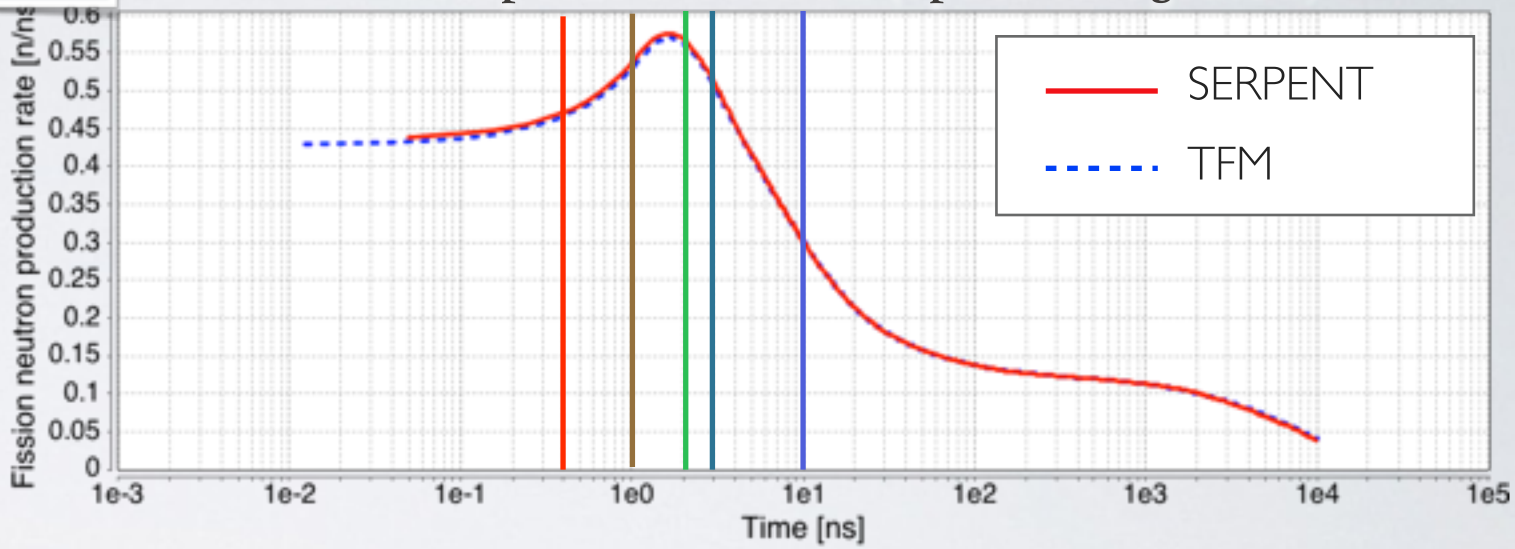
FLATTOP EXPERIMENT
REFERENCE CODE: SERPENT

Flattop subjected to a neutron burst release

TFM EQUATION & IMPLEMENTATION TO CHECK:

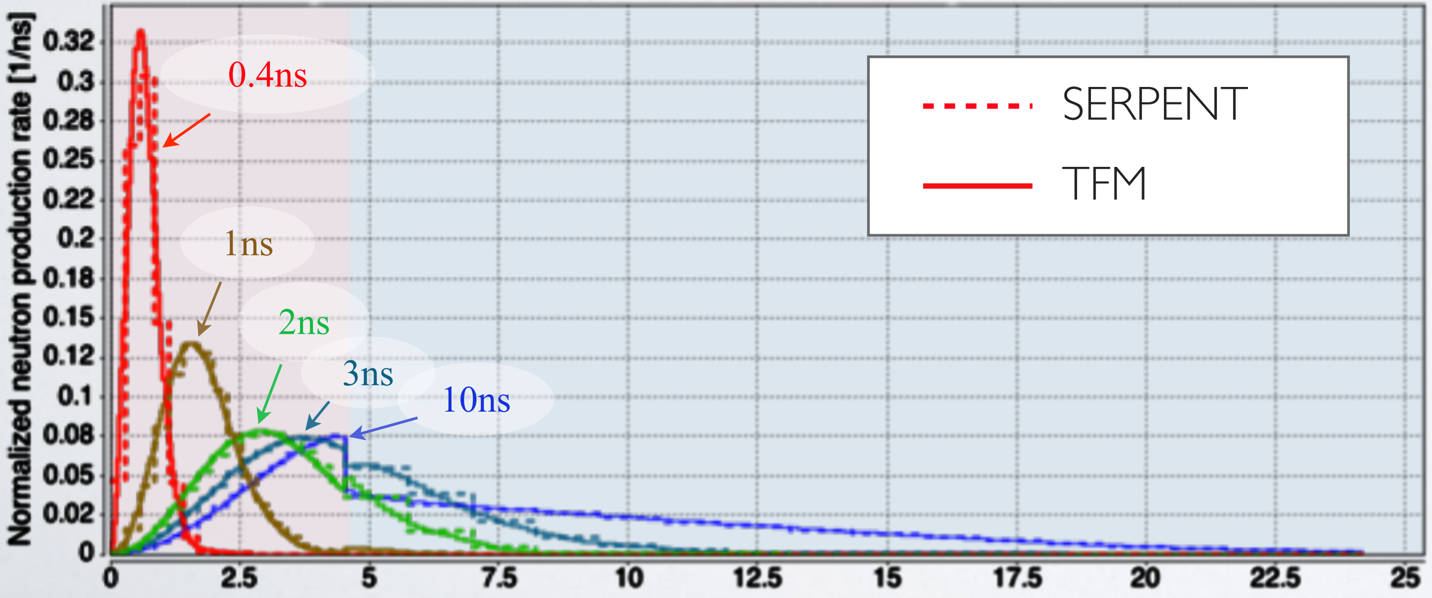
$$S(\boldsymbol{r},t) = \Big|G_{\chi_p\nu_p}(t'-t,\boldsymbol{r'},\boldsymbol{r})\Big|S(\boldsymbol{r'},t')\Big\rangle$$

Temporal evolution (space integrated)



- Same evolution behavior of the neutron population through time

Spatial evolution (time sampled)



- Good agreement of the spatial neutron propagation

🟥🟫 The neutron burst is limited to the Pu area: $k_p >> 1$

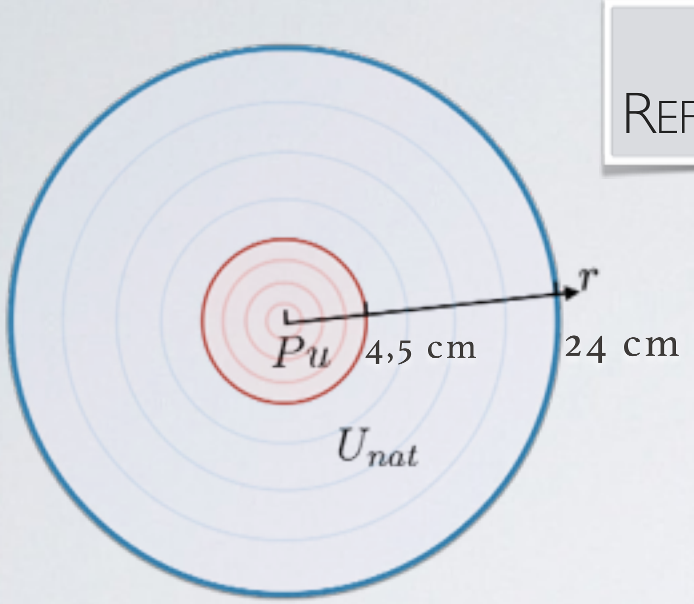
🟩🟦 The neutron burst reaches the U_{nat} area: $k_p << 1$

The neutron distribution tends to the equilibrium's one: $k_p \sim 0.997$

APPLICATION CASES: FLATTOP EXPERIMENT

BENCH CASE

FLATTOP EXPERIMENT
REFERENCE CODE : SERPENT



Flattop kinetics calculated parameters:

- effective fraction of delayed neutron:

$$\beta_{eff}$$

- effective generation time:

$$\Lambda_{eff} = \frac{l_{eff}}{k_p}$$

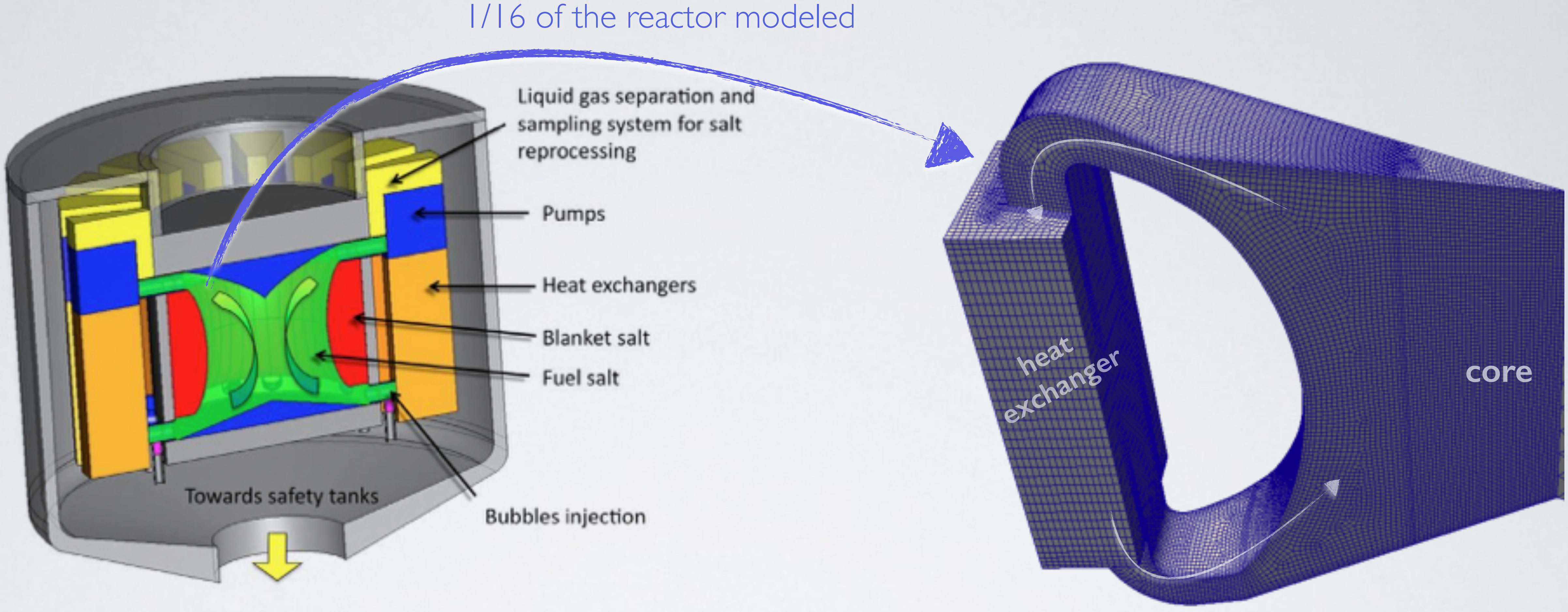
Experimental observable:

$$\alpha_{Rossi} = -\frac{\beta_{eff}}{\Lambda_{eff}}$$

method	β_{eff}	Λ_{eff}	α_{Rossi}
TFM (this work)	$275 \pm 4\ pcm$	13.351 ± 0.03 ns	$0.206 \pm 0.004\ \mu s^{-1}$
SERPENT IFP	$274 \pm 2\ pcm$	13.24 ± 0.02 ns	$0.207 \pm 0.002\ \mu s^{-1}$
Experiment	-	-	$0.214 \pm 0.005\ \mu s^{-1}$

- good agreement between TFM and SERPENT…
- … and with the experimental measurements!

II. APPLICATION CASES: MOLTEN SALT FAST REACTOR (MSFR)

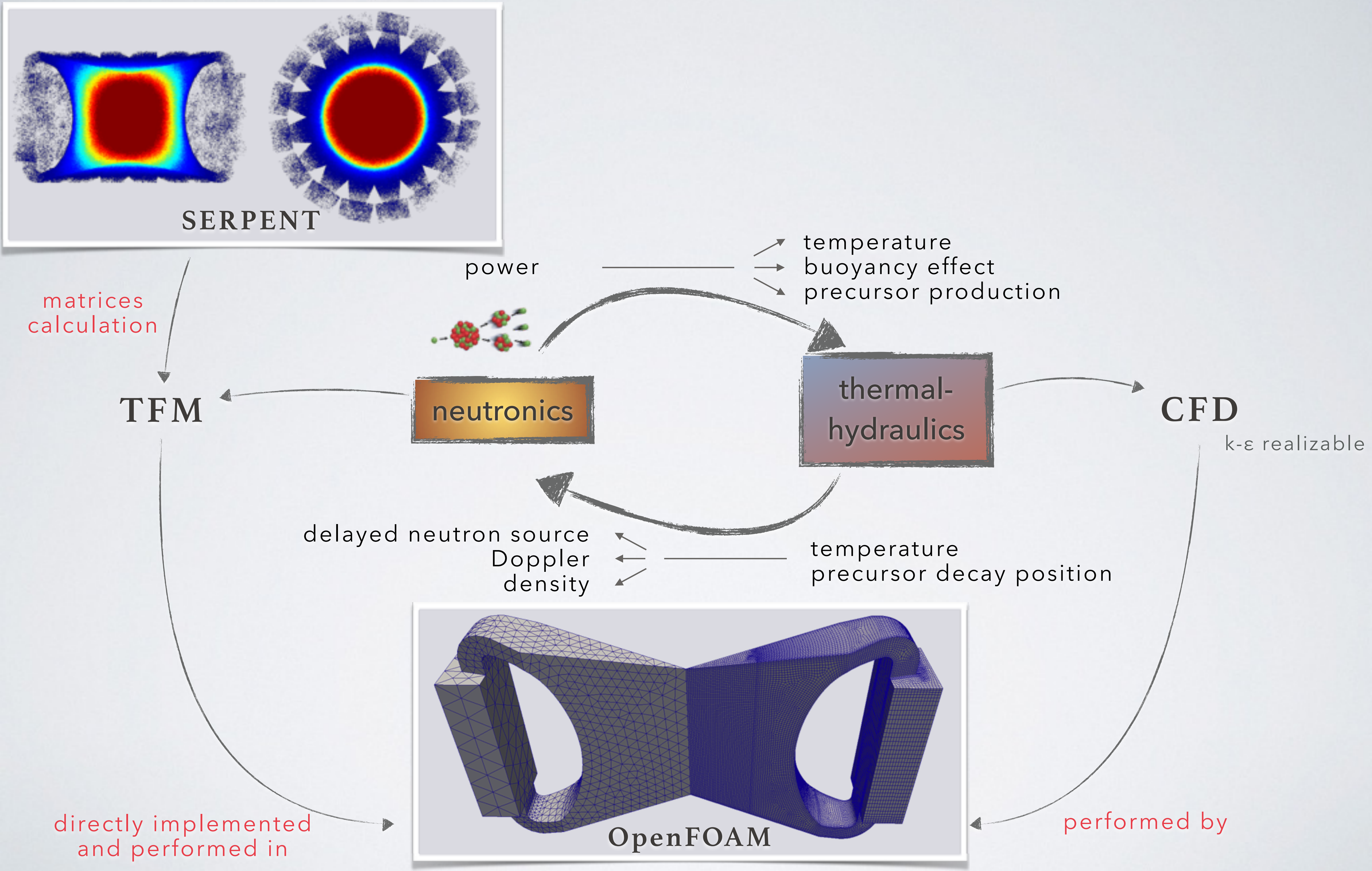


1/16 of the reactor modeled

- Liquid fuel (precursor motion)
- Fuel = coolant
- Fast neutron spectrum
- Circulation time ~ 4 s
- Reynolds in core: ~ 500000

- Molten Salt : LiF - (Th/^{233}U)F_4
  density: 4 x water
  viscosity: 2 x water (oil ~ 1000x water)
  low pressure
  mean fuel temperature ~ 900 K

II. APPLICATION CASES: MOLTEN SALT FAST REACTOR (MSFR)



SERPENT

matrices calculation

TFM

power

temperature
buoyancy effect
precursor production

neutronics

thermal-hydraulics

CFD

k-ε realizable

delayed neutron source
Doppler
density

temperature
precursor decay position

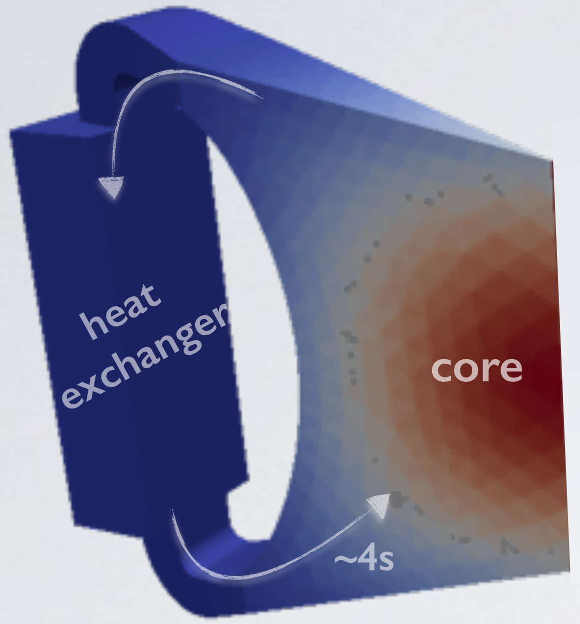
directly implemented and performed in

OpenFOAM

performed by

II. APPLICATION CASES: MSFR

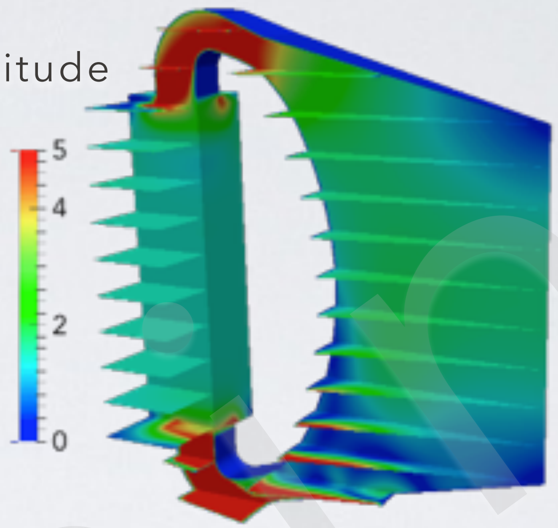
Over cooling transient - 1/16 reactor



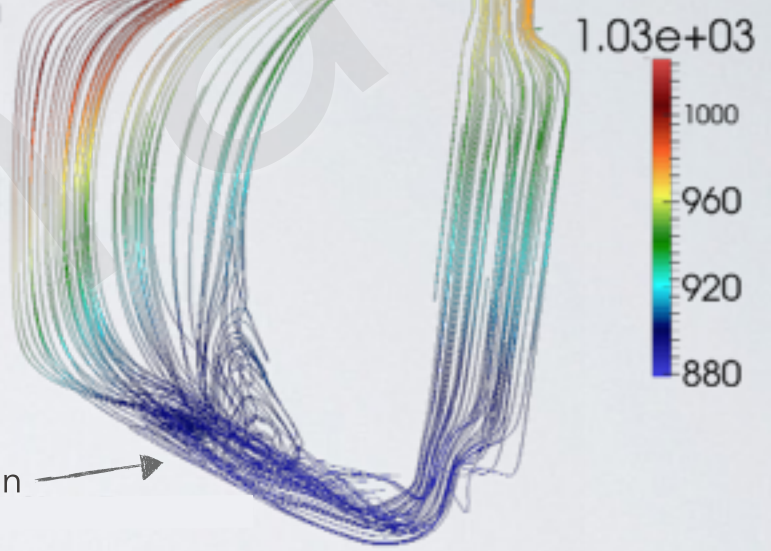
normalized power distribution calculated by the fission matrix

INITIAL CONDITIONS
(STEADY STATE)

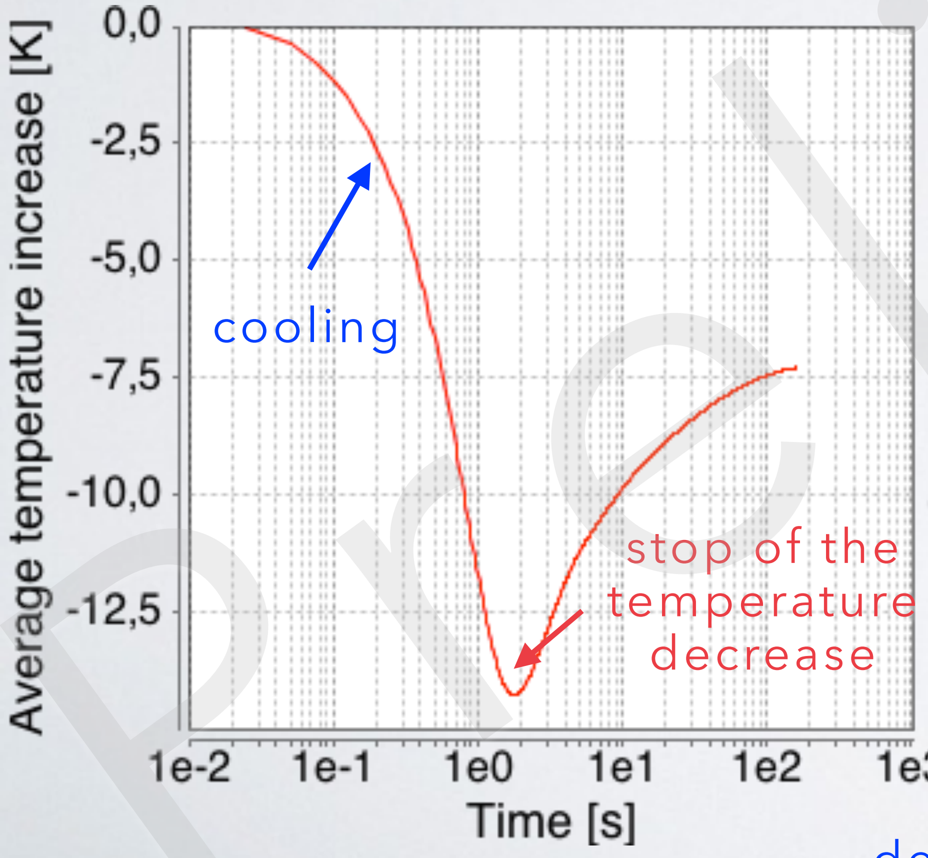
velocity magnitude
m/s

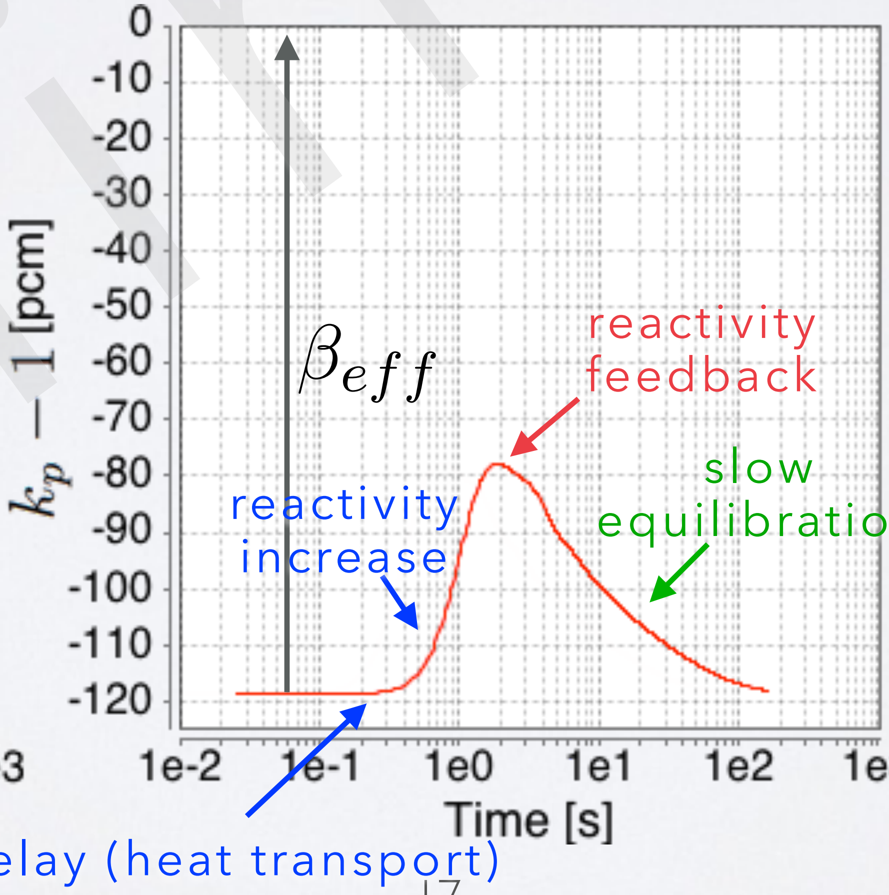


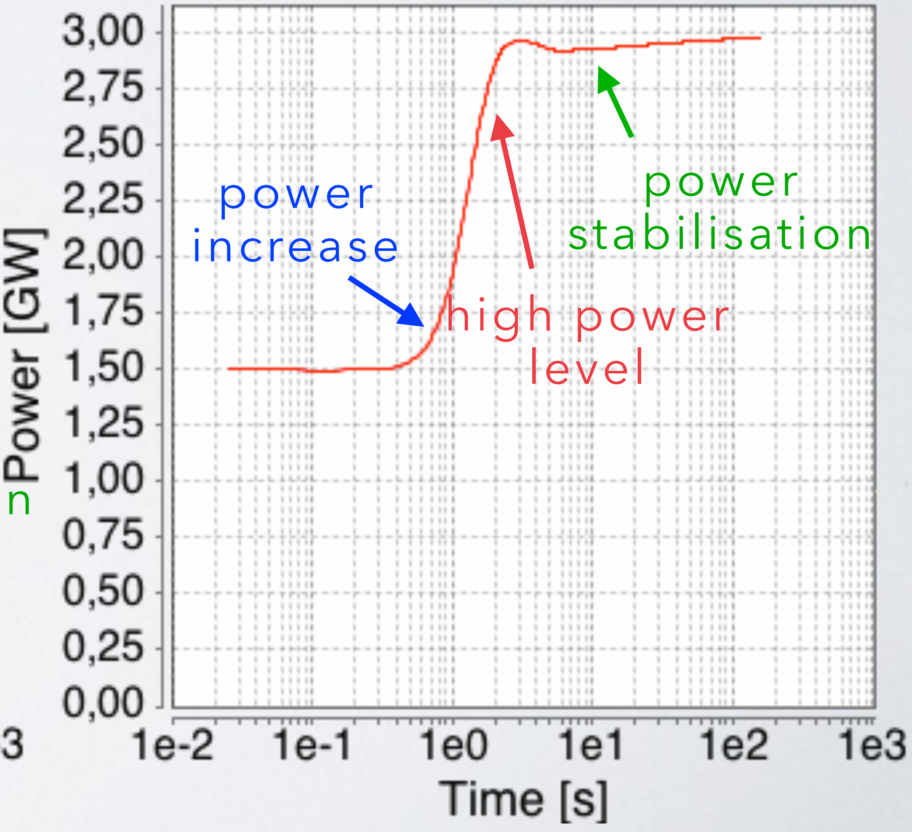
stream line - temperature
K



recirculation

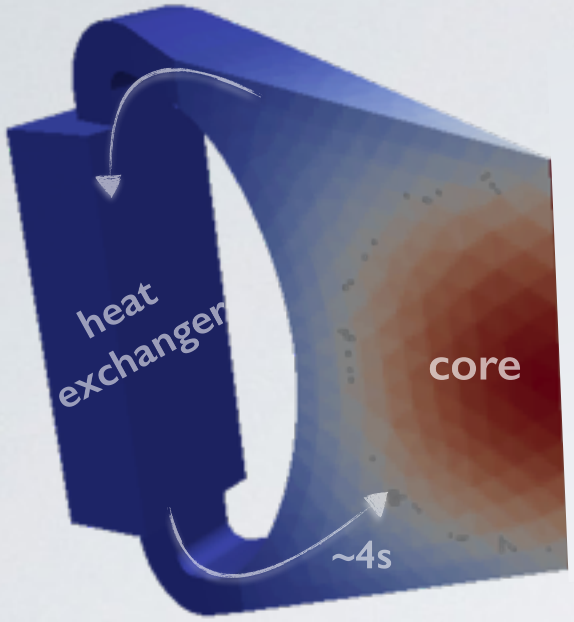






II. APPLICATION CASES: MSFR

Over cooling transient - 1/16 reactor



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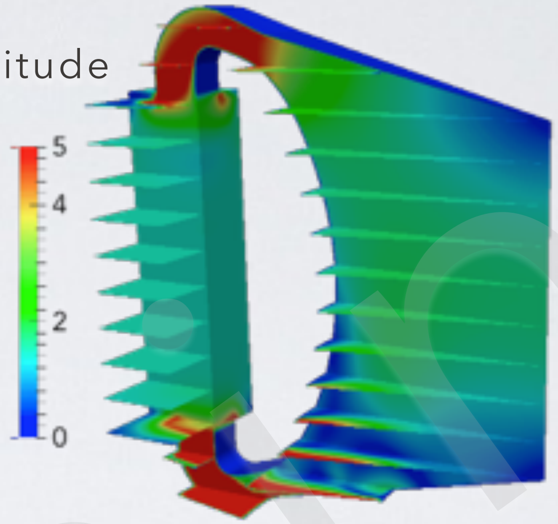
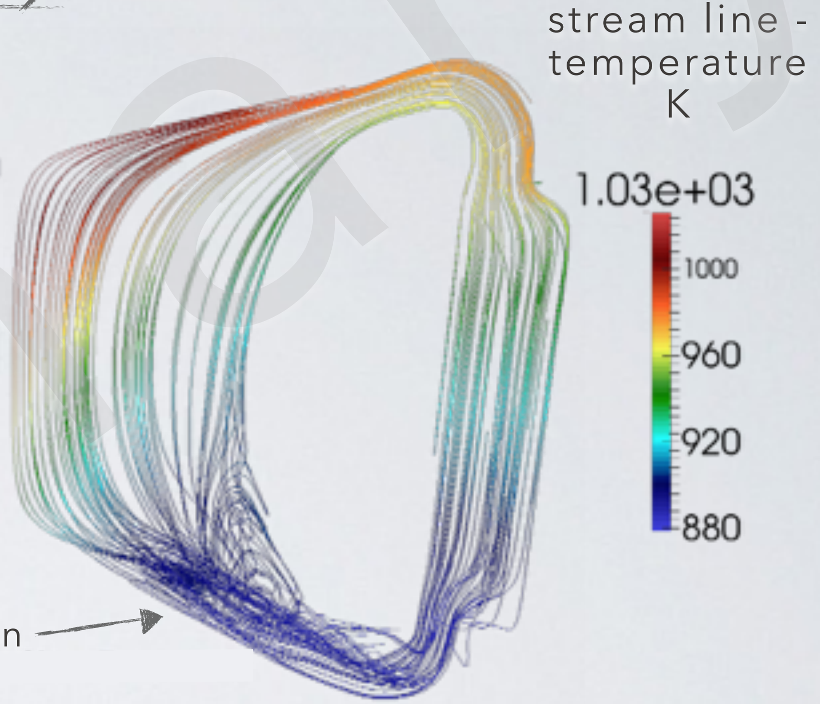
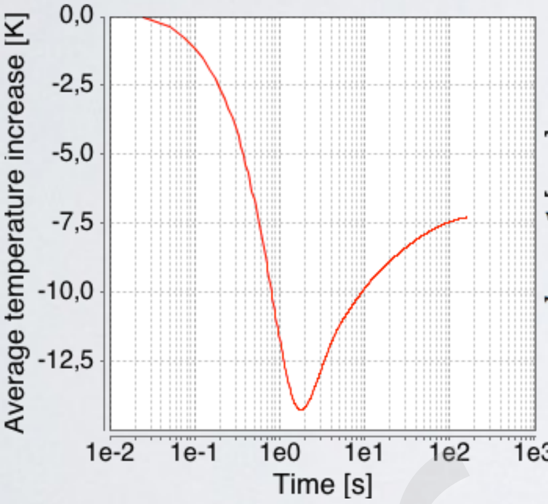
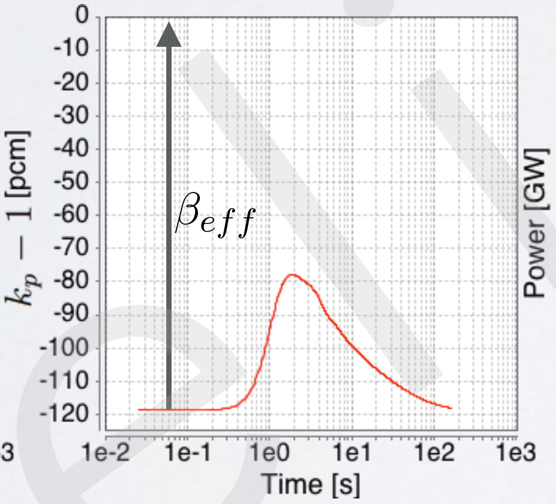
velocity magnitude m/s

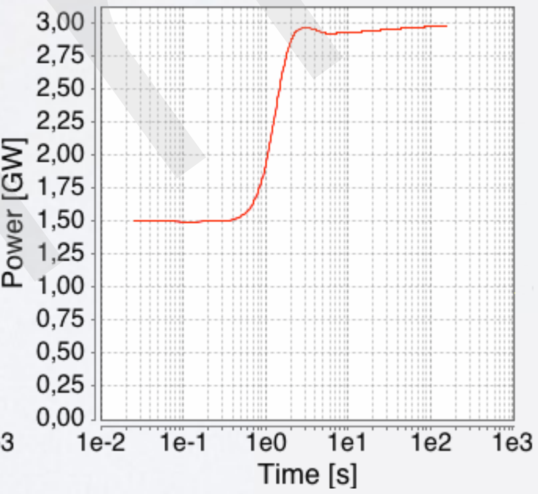


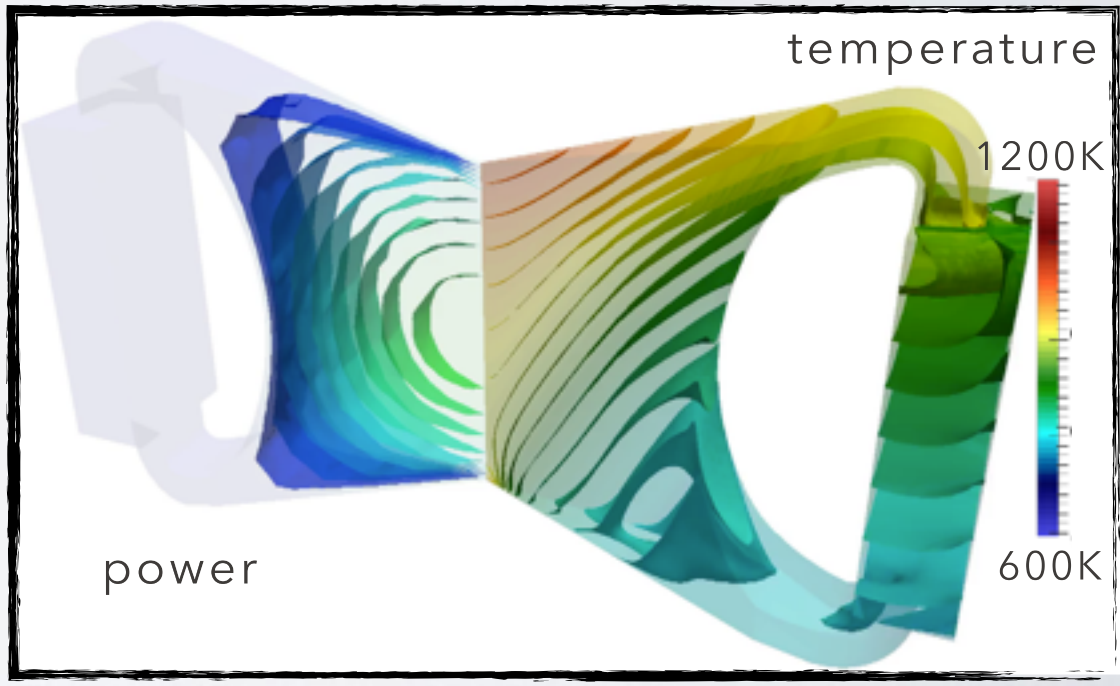


recirculation









- TFM-OpenFOAM coupling operational
- high precision
- low computational cost ~few hours on a personal laptop
- need for a coupling benchmark! quantitative check

III.

CONCLUSION

Conclusions

- Implementation of the transient fission matrices in SERPENT
- Good results for the kinetics parameters calculation
- TFM-OpenFOAM coupling operational: high precision & low computational cost
- Full coupling benchmark ⟶ Manuele Aufiero's talk

Future work

- Ongoing transient calculations on the MSFR
- Available / to apply for other reactor
  SERPENT - transient fission matrices calculations
  OpenFOAM - TFM module

Thanks for your attention!

More discussions: tomorrow morning!