1st Serpent and Multiphysics Workshop

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POLITECNICO DI MILANO

Reduced order modelling for dynamics and control of Lead-cooled Fast Reactor

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NUCLEAR REACTORS GROUP

Issue: need to improve the plant availability enhancing the performance for Nuclear Power Plants (NPPs) + a more and more ability to follow grid demands

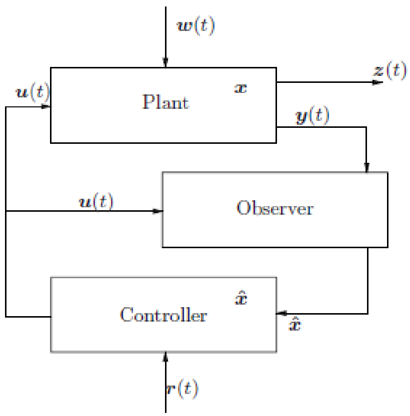
Control of nuclear reactor plays a fundamental role

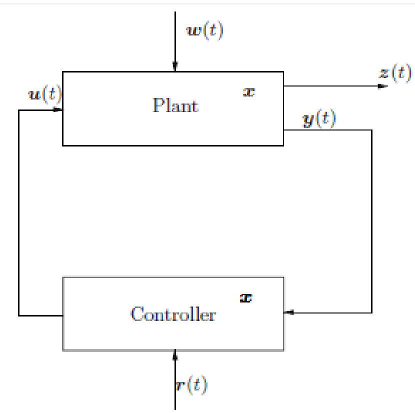
Classic control techniques	Innovative control techniques
Example: Proportional-Integral-Derivative controller	**Example:** Optimal control
Require a **simple knowledge of the system**, i.e., total thermal power or average temperature	**Additional information** regarding spatial distribution of neutron flux and temperature may be necessary
Zero dimensional or **lumped parameter** approach is sufficient	Accurate and **detailed spatial models** are required

Developing specific simulation tools allows improving the **control system design**

A simulation tool for control purposes has to fulfil some typical desiderata

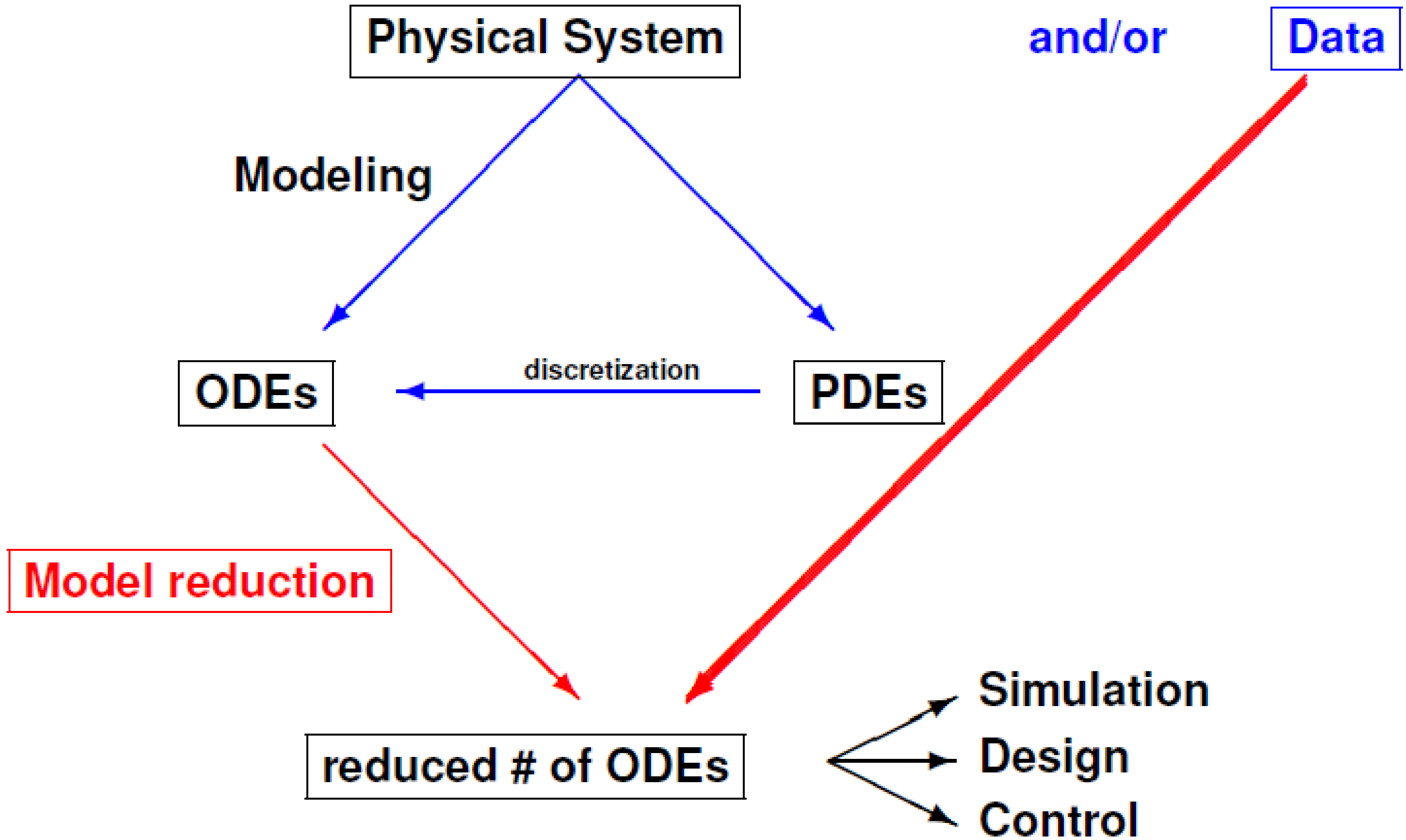
Fast-running, low order, ODE (Ordinary Differential Equation) based, easy to linearize, comprehensive behaviour of the system, possibility to couple with the control system simulator





How to enhance the level of detail and accuracy of the simulation tool in order to employ **advanced control strategies** without a strong computational burden?

Reduced order modelling approach



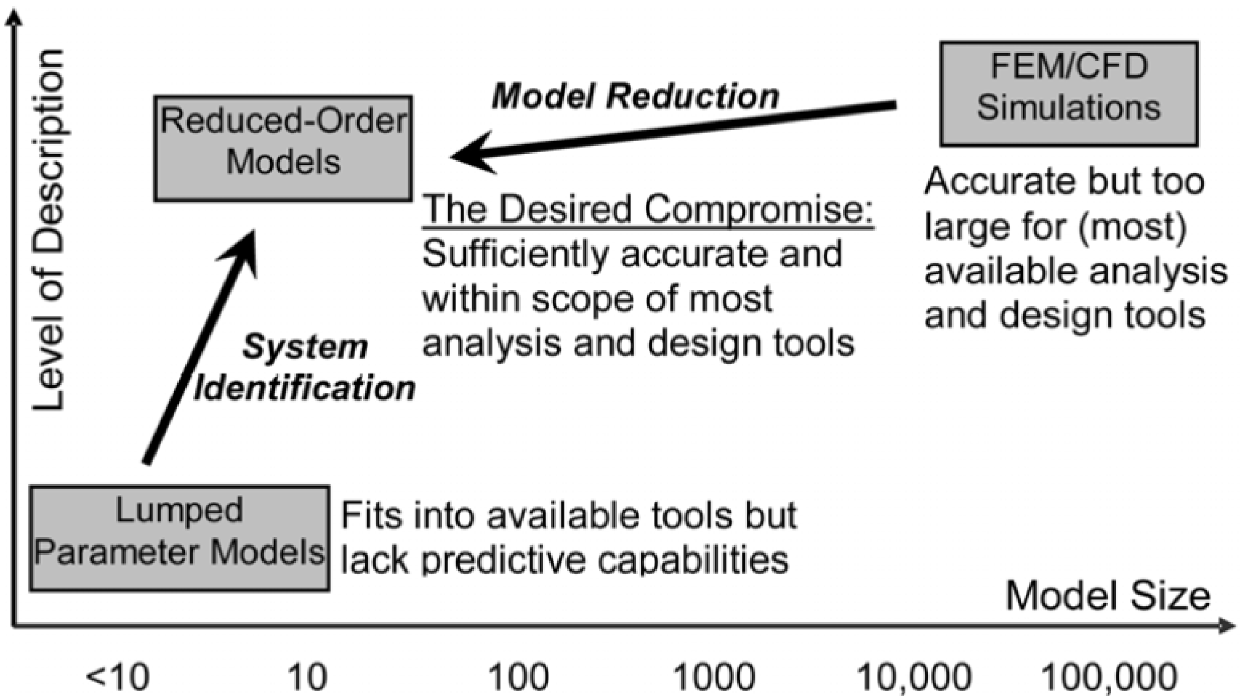
Courtesy of T. Antoulas (Rice university), Model reduction of large-scale systems, YY Fest, Kyoto University, 29 - 31 March 2010

Reduced order modelling · Requirements

Typical requirements for a Reduced Order Model (ROM):

. Input-output relationships have to be preserved
. Guarantee the stability of the ROM
. Computationally efficient

❯ Compromise between accuracy and efficiency



Shapiro, B., 2003, "Creating Compact Models for Electronic Systems: An Overview and Suggested Use of Existing Model Reduction and Experimental System Identification Tools", IEEE Transactions on Components, Packaging, and Manufacturing Technology - Part A, **26** (1), pp. 165-172.

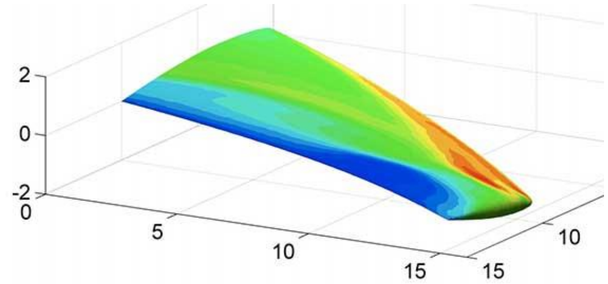
Reduced order modelling · Applications

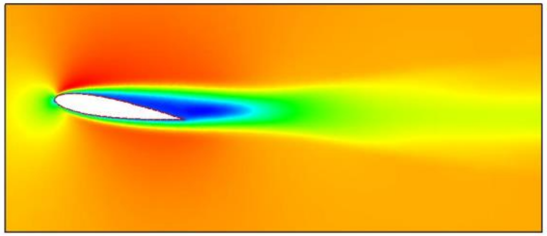
Main applications [1]:

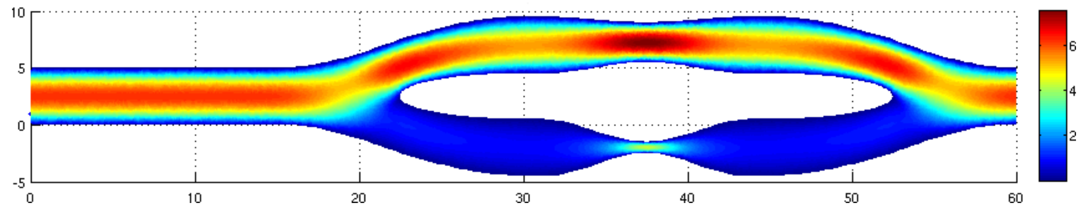
. Real time context (parameter estimation, control, inverse problem)
. Many-query context (design optimization, multi-model simulation)
. Uncertainty quantification

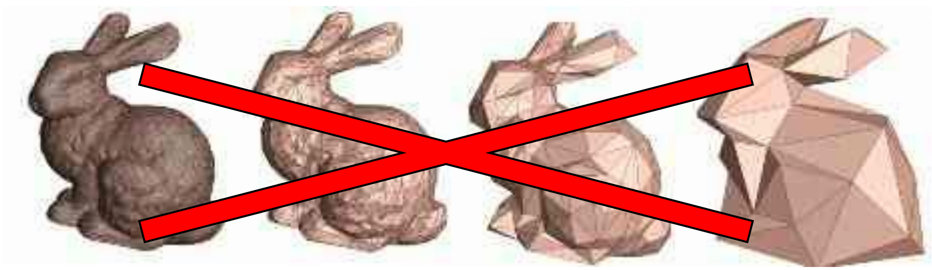
Fields:

. CFD
. Solid Mechanics
. Fluid Mechanics
. Heat Transfer
. Multiphysics
. Nuclear (Neutronics)
. Etc…









Many ways to obtain a reduced order model [2]:

. Modelling assumption (i.e., RANS)

. Projection framework (Galerkin but not only)
Eigenvectors, POD, PGD, CVT, Reduced Basis, balanced truncation, Kyrlov subspaces.

Problem-dependent

$$\underbrace{U_q^T}_{\text{qxN}} \underbrace{V_q}_{\text{Nxq}} \frac{d\hat{x}}{dt} = \underbrace{\underbrace{U_q^T}_{\text{qxN}} \underbrace{A}_{\text{NxN}} \underbrace{V_q}_{\text{Nxq}}}_{} \hat{x} + U_q^T b u$$

$$\frac{d\hat{x}}{dt} = \underbrace{\boxed{\hat{A}}}_{\text{qxq}} \hat{x}(t) + \hat{b} u(t)$$

Courtesy of L. Daniel (MIT), Introduction to Compact Dynamical Modeling,

. System identification / Data fitting
Polynomial response surface, Kriging modelling

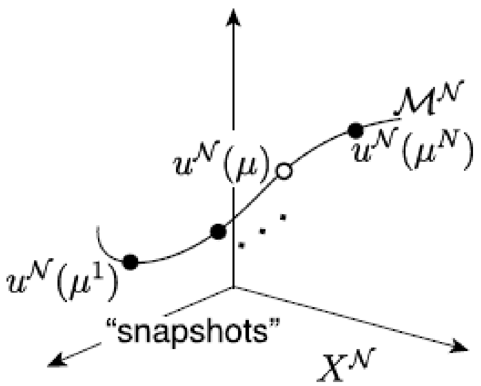
Problem-transparent

Reduced order model based on projection (ROM-P)

1) Collect few high-fidelity PDE solutions (*snapshots*)

2) Calculate the basis where project the governing equations

3) (Galerkin) projection



Taken form Rozza et al., Reduced Basis Approximation and a Posteriori Error estimation for Affinely Parametrized Elliptic Coercive PDEs. Arch Comput Methods Eng (2008) 15: 229–275

Five pillars of the ROM-P [3]

. “Never try to reduce the irreducible”

. “If it is not in the snapshots, it is not in the ROM”

. “Exploit the known structure of the solutions”

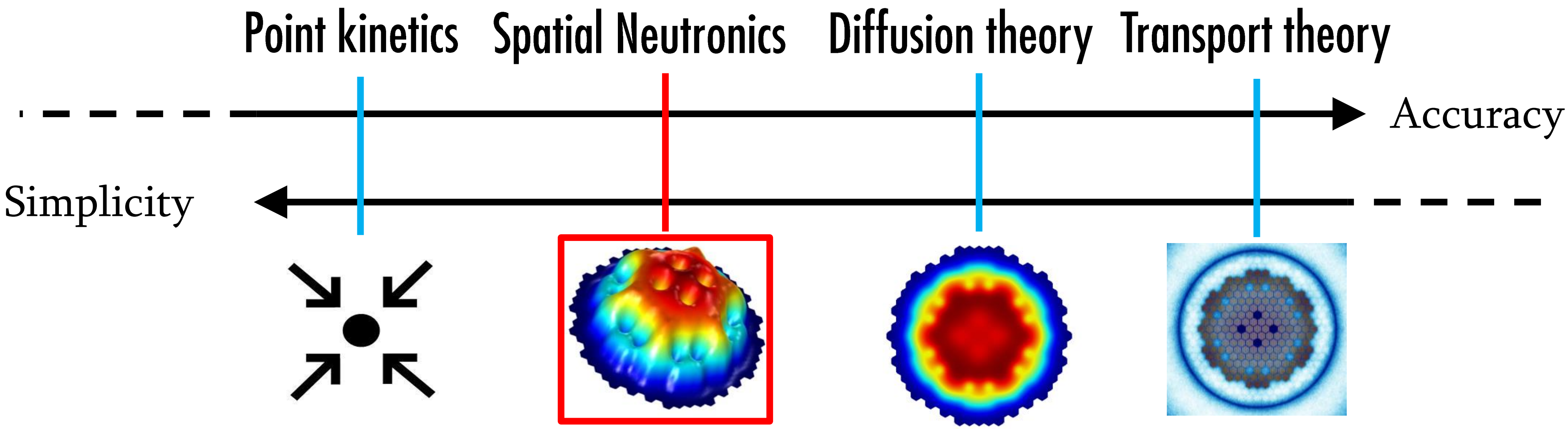
. “Exploit the known physics of the problem”

. “*Divide et impera* whenever possible”

Spatial neutronics ROM · Rationale

Development of spatial neutronics modelling for dynamics and control purposes

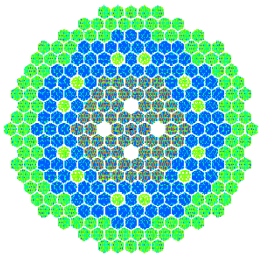
The accuracy vs. the computational requirements can be tuned in according to the own goal (dynamics or control)

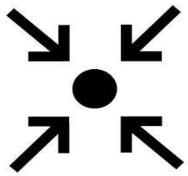


Spatial neutronics ROM · Rationale

How to improve the modelling accuracy…

Neutronics usually modeled by point-wise kinetics





0-D description of the main physics of a nuclear reactor

$$\begin{cases} \dfrac{dn}{dt} = \dfrac{\rho - \beta}{\Lambda} n + \displaystyle\sum_{i=1}^{8} \lambda_i c_i + q \\ \dfrac{dc_i}{dt} = \dfrac{\beta_i}{\Lambda} n - \lambda_i c_i \qquad i = 1, \dots, 8 \end{cases}$$

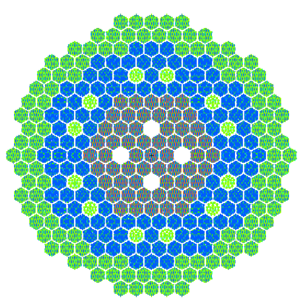
$$\rho(t) = \rho_0 + 1.1 \cdot K_D \left(ln \frac{T_f^{\,eff}(t)}{T_{f0}^{\,eff}} \right) + \alpha_Z \left(T_f(t) - T_{f0} \right)$$

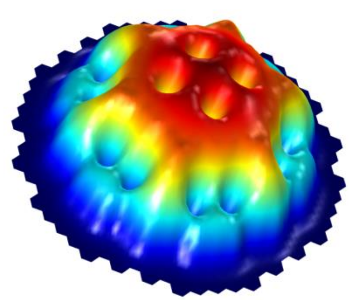
. No info on spatial distribution of the flux

. Simplified evaluation of the temperature feedbacks (constant coefficient)

Spatial Neutronics ROM

taking into account the reactivity and flux spatial distribution





From a Full Order Model to a Reduced order model

Transform the partial differential equations (PDEs) of the neutron diffusion into a set of ODEs involving only the time-dependent coefficient:

$$\underline{\underline{V}}^{-1}\frac{\partial \underline{\phi}}{\partial t} = \nabla \cdot \left(\underline{\underline{D}}\nabla \underline{\phi}\right) - \underline{\underline{\Sigma_a}}\,\underline{\phi} - \underline{\underline{\Sigma_s}}\,\underline{\phi} + (1-\beta)\underline{\chi_p}\underline{F}^T\underline{\phi} + \sum_j \lambda_j \underline{\chi_d} C_j$$

$$\frac{\partial C_j}{\partial t} = -\lambda_j C_j + \beta_j \underline{F}^T\underline{\phi} \qquad\qquad j = 1 \div 8$$

Ansatz or God's bounty theorem*

$$\underline{\phi}(\boldsymbol{r}, t) \cong \sum_{i=1}^{N} \underline{\underline{\psi_i}}(\boldsymbol{r}) \cdot \underline{n_i}(t)$$

Time-dependent coefficients

Spatial basis where the flux is projected

* P. Ravetto. Politecnico di Torino.

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1) The Ansatz has to be substituted in the Diffusion equation
2) Multiplying by the test functions $\underline{\underline{\xi}}_i$ and integrating (inner product)

$$\sum_{m=1}^{N} \underline{\underline{RV}}_{im} \cdot \dot{\underline{n}}_m = \sum_{m=1}^{N} \left(-\underline{L}_{im} - \underline{\underline{\delta L}}_{im} + (1-\beta) \cdot \left(\underline{M}_{im} + \underline{\underline{\delta M}}_{im} \right) \right) \cdot \underline{n}_m + \sum_{j=1}^{8} \lambda_j \underline{c}_{ij}$$

$$\dot{\underline{c}}_{ij} = \beta_j \underline{\underline{X}} \left[\sum_{m=1}^{N} \left(\underline{M}_{im} + \underline{\underline{\delta M}}_{im} \right) \cdot \underline{n}_m \right] - \lambda_j \underline{c}_{ij} \quad j = 1 \div 8$$

$$\underline{\underline{RV}}_{im} = \int \underline{\underline{\xi}}_i \cdot \underline{\underline{V}}^{-1} \cdot \underline{\underline{\psi}}_m d\Omega \quad \underline{L}_{im} = \int \underline{\underline{\xi}}_i \cdot \left(-\nabla \cdot \underline{\underline{D}} \nabla + \underline{\underline{\Sigma_a}} + \underline{\underline{\Sigma_s}} \right) \underline{\underline{\psi}}_m d\Omega$$

$$\underline{\underline{\delta L}}_{im} = \int \underline{\underline{\xi}}_i \cdot \delta \left(-\nabla \cdot \underline{\underline{D}} \nabla + \underline{\underline{\Sigma_a}} + \underline{\underline{\Sigma_s}} \right) \underline{\underline{\psi}}_m d\Omega \quad \underline{M}_{im} = \int \underline{\underline{\xi}}_i \cdot \left(\underline{\chi_p F^T} \right) \underline{\underline{\psi}}_m d\Omega$$

$$\underline{\underline{\delta M}}_{im} = \int \underline{\underline{\xi}}_i \cdot \delta \left(\underline{\chi_p F^T} \right) \underline{\underline{\psi}}_m d\Omega \qquad \underline{c}_{ij} = \int \underline{\underline{\xi}}_i \cdot \underline{\chi_d} \cdot C_j \, d\Omega$$

$$\underline{\underline{X}} = \begin{bmatrix} \chi_d^1 / \chi_p^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \chi_d^7 / \chi_p^7 \end{bmatrix}$$

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Affine representation

$$\underline{\underline{\delta L}}_{im}(T) = \sum_z \int \underline{\underline{\xi}}_i \cdot \left[-\nabla \cdot \left(\underline{\underline{D}}(T_z)\nabla\right) + \underline{\underline{\Sigma}}_a(T_z) + \underline{\underline{\Sigma}}_s(T_z)\right] \underline{\underline{\psi}}_m d\Omega_z =$$

$$= \sum_z \left[\underline{\underline{D}}(T_z) \int \nabla\underline{\underline{\xi}}_i \cdot \nabla\underline{\underline{\psi}}_m \, d\Omega_z + \underline{\underline{\Sigma}}_a(T_z) \int \underline{\underline{\xi}}_i \cdot \underline{\underline{\psi}}_m \, d\Omega_z + \underline{\underline{\Sigma}}_s(T_z) \int \underline{\underline{\xi}}_i \cdot \underline{\underline{\psi}}_m \, d\Omega_z\right] +$$

$$+\gamma_r \int_{\partial\Omega_r} \underline{\underline{\xi}}_i \cdot \underline{\underline{\psi}}_m dS_r + \gamma_a \int_{\partial\Omega_a} \underline{\underline{\xi}}_i \cdot \underline{\underline{\psi}}_m dS_a$$

. # of spatial basis N

. Affine representation

. Selection of the spatial basis $\underline{\underline{\psi}}_i$

. Selection of the test functions $\underline{\underline{\xi}}_i$

They have an impact in terms of efficiency, accuracy and computational time

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Modal Method (MM)

Spatial basis choice: eigenfunctions associated to the neutron diffusion equation

$$\left(-\nabla \cdot \underline{\underline{D}}\nabla + \underline{\underline{\Sigma_a}} + \underline{\underline{\Sigma_s}}\right)\underline{\psi_i} = \lambda_i^* \underline{\chi_p}\,\underline{F}^T \underline{\psi_i} \quad \Rightarrow \quad L\underline{\psi_i} = \lambda_i^* M \underline{\psi_i}$$

Proper Orthogonal Decomposition (POD)

POD optimal basis that minimizes $$\int \left[\underline{\phi}(\boldsymbol{r},t) - \sum_{i=1}^{N} \underline{\underline{\psi_i}}(\boldsymbol{r}) \cdot \underline{n_i}(t)\right]^2 dt$$

Procedure:

1. "Snapshot" computation — *Representative of the simulated conditions*
2. Spatial basis calculation — *Singular Value Decomposition*

$$S = U\Sigma V \qquad \begin{cases} S \ snapshot\ matrix \\ U \ basis\ matrix \\ \Sigma \ singular\ values\ matrix \end{cases} \qquad info = \frac{\sum_{i=1}^{l} \sigma_i}{\sum_{i=1}^{n} \sigma_i}$$

3. Projection of spatial basis — *Set of ODEs (reduced order model)*

Which test functions can be selected? [4]

Galerkin projection

The same functions which constitute the spatial basis $\underline{\underline{\xi_i}} = \underline{\underline{\psi_i}}$

Petrov-Galerkin projection

The test functions can be different from the spatial basis modes

Due to the specific meaning in the neutronics field (and for other interesting properites), a possible candidate are the adjoint functions of the spatial basis!

$$\underline{\underline{\xi_i}} = \underline{\underline{\psi_i}}^{\dagger}$$

		Test function	
		Function of spatial basis	Adjoint functions
Method for the spatial basis	MM	Case A	Case B
	POD	Case C	Case D

Spatial neutronics ROM · Procedure

Generation of group constants (neutron cross-sections)
Monte Carlo simulation (MC) **SERPENT**

↓

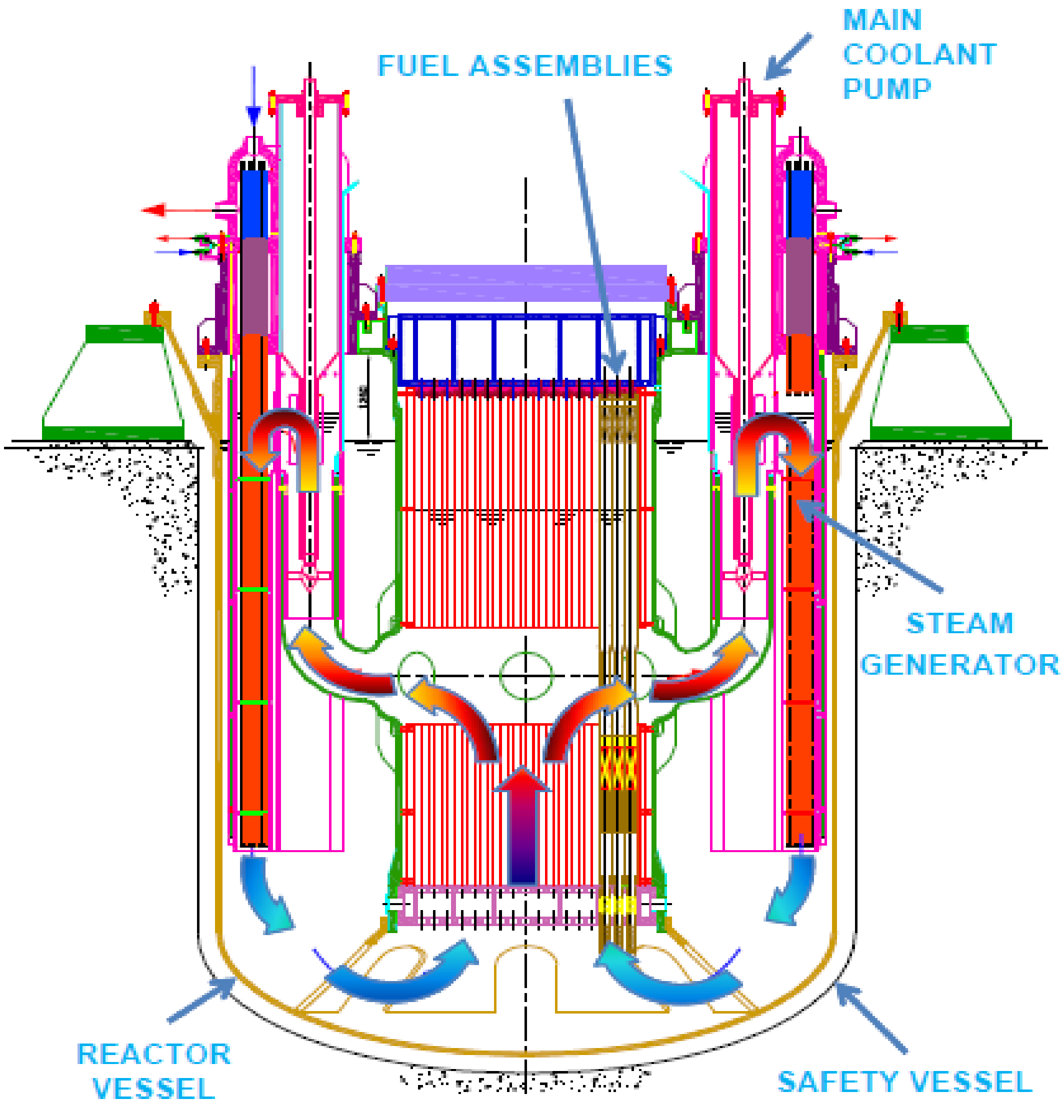
Neutronics (Diffusion equation) + Spatial basis calculation
Finite Element software **COMSOL**

↓

CONTROL-ORIENTED MODEL
Spatial neutronics model + Reactivity evaluation
ODE solver **MATLAB**

OFFLINE procedure: the computation is made only once

ONLINE calculation: ODE set can be run as many times as required



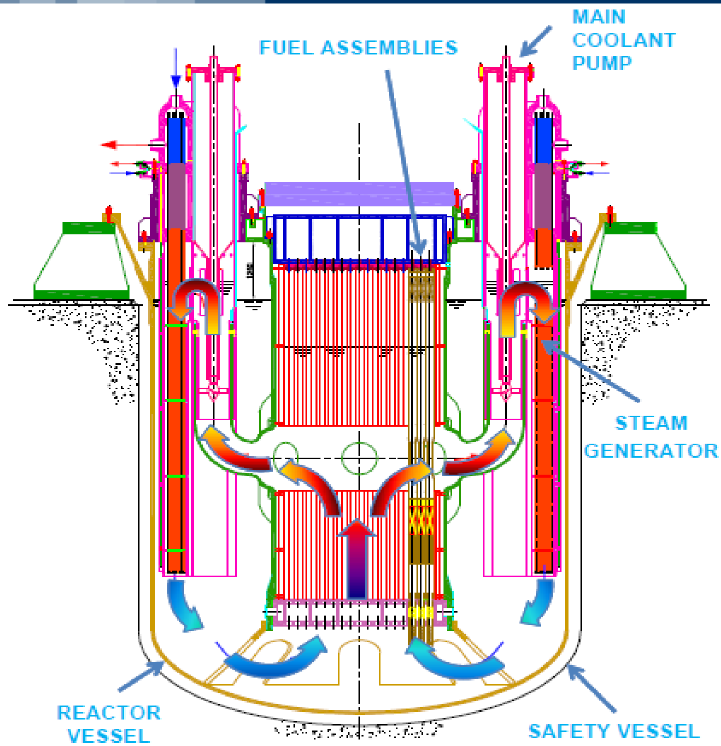
Advanced Lead Fast Reactor European Demonstrator

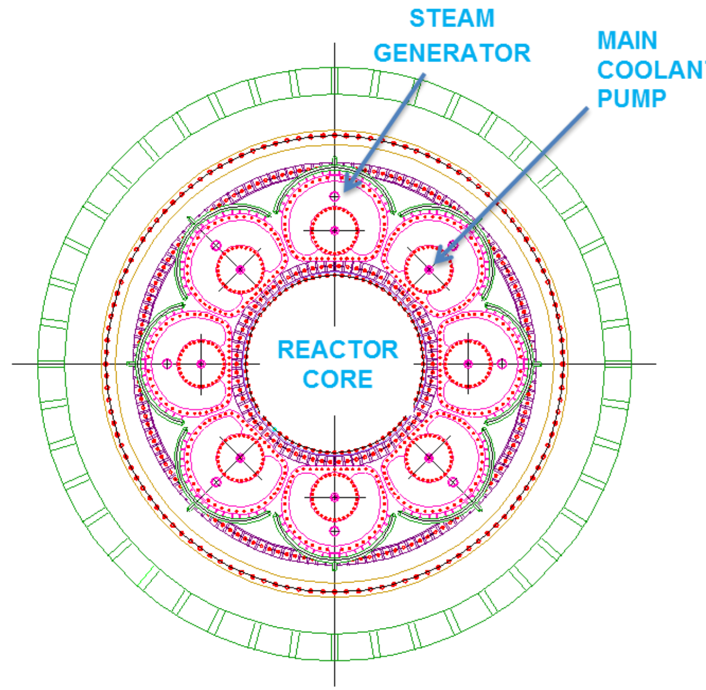
. 300 MW_{th} – 125 MW_e
. Pool reactor – 8 loops
. Scaled-down demonstrator for ELFR (European Lead Fast Reactor)

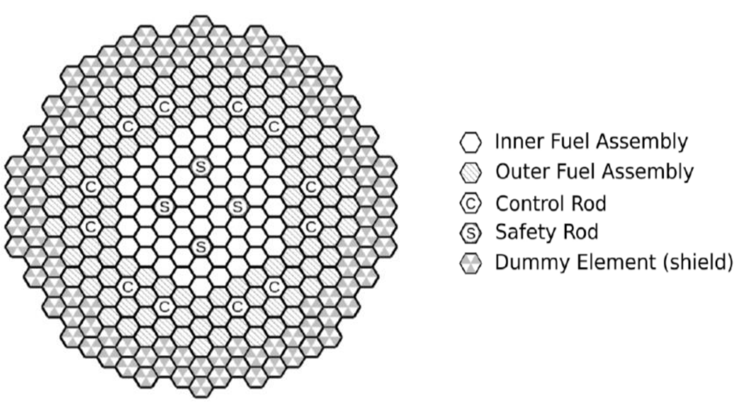
Gen-IV framework

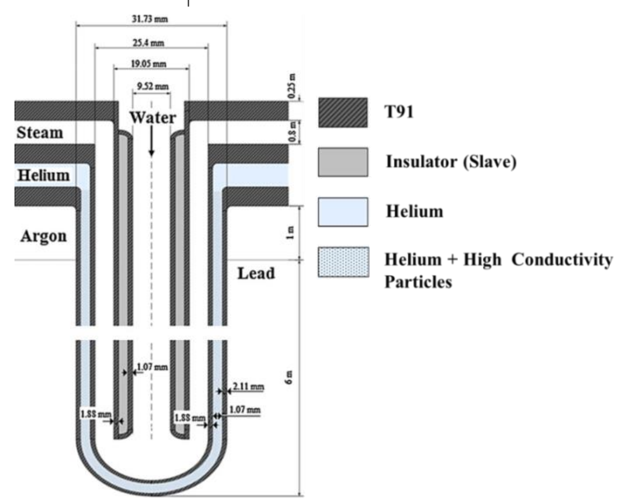
LFR embodies Gen-IV key concepts of economics, sustainability, safety & reliability, proliferation resistance and physical protection

LFR · ALFRED


FUEL ASSEMBLIES
MAIN COOLANT PUMP
STEAM GENERATOR
REACTOR VESSEL
SAFETY VESSEL


STEAM GENERATOR
MAIN COOLANT PUMP
REACTOR CORE


Inner Fuel Assembly
Outer Fuel Assembly
Control Rod
Safety Rod
Dummy Element (shield)


31.73 mm
25.4 mm
19.05 mm
9.52 mm
Water
Steam
Helium
Argon
Lead
T91
Insulator (Slave)
Helium
Helium + High Conductivity Particles

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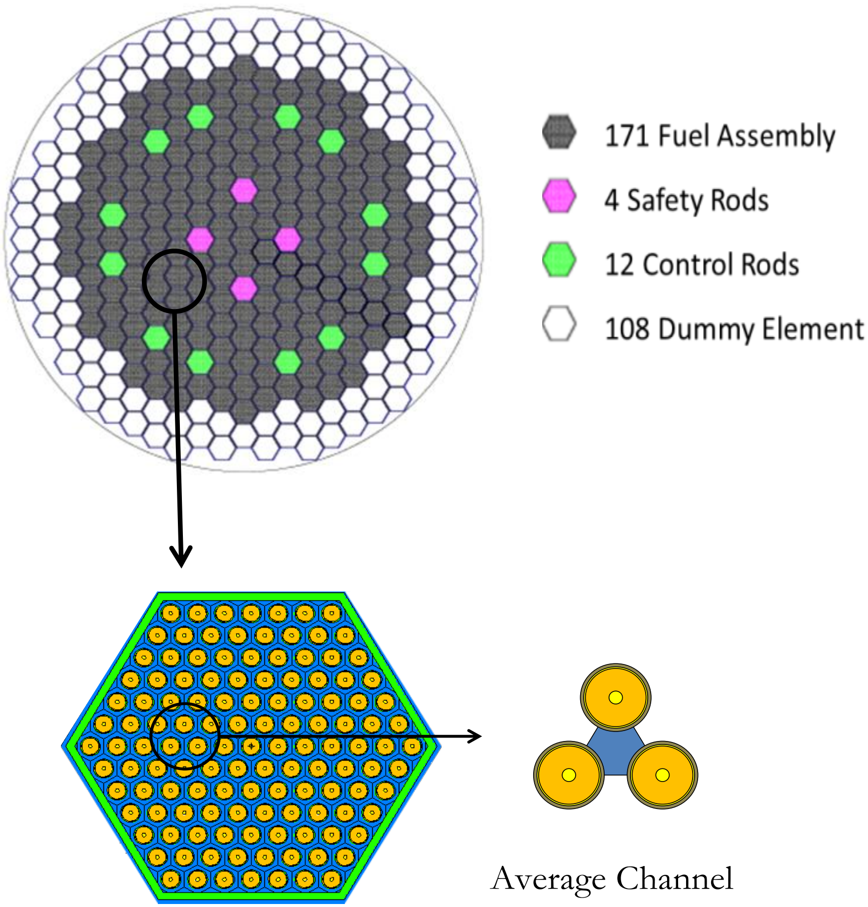
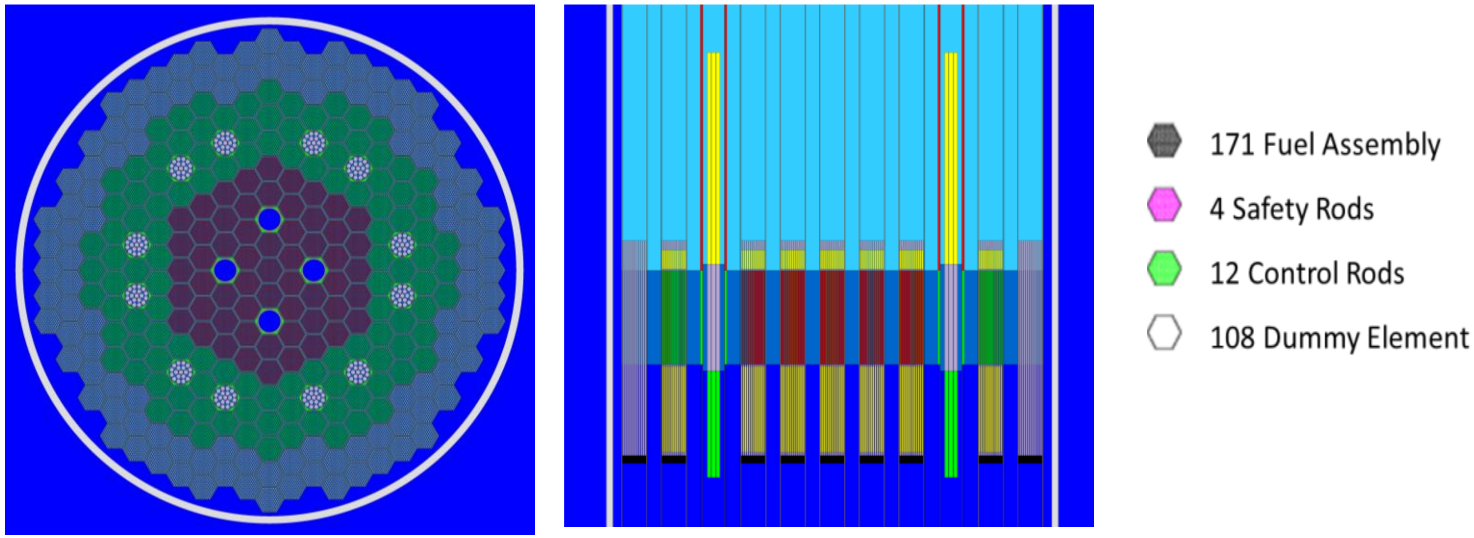
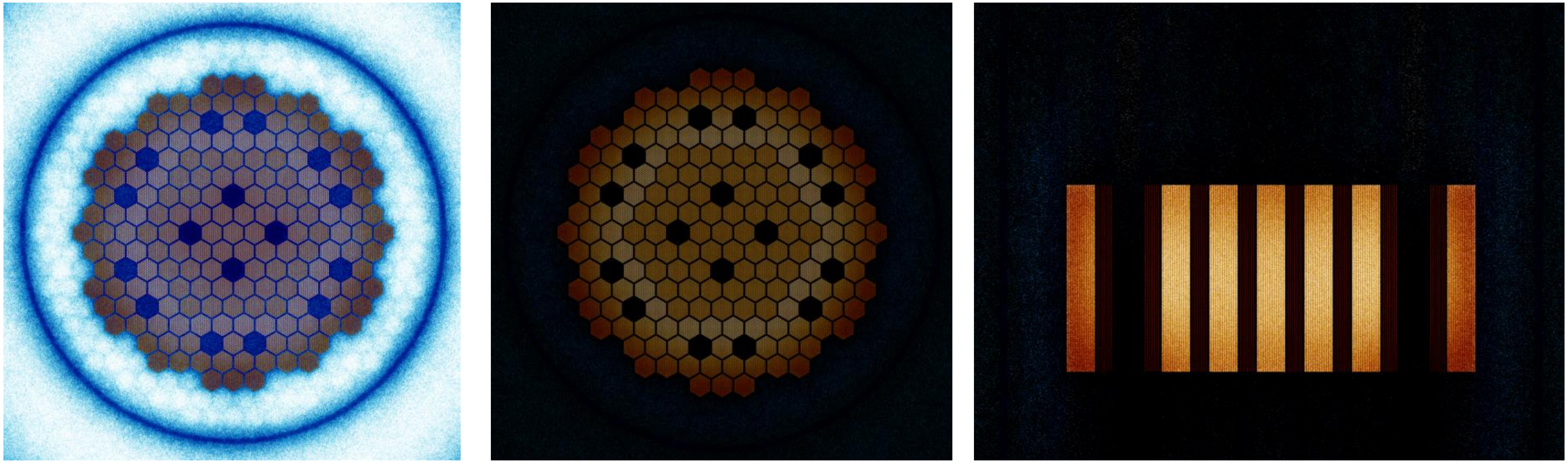


Fuel pin design parameter	Value
Average pin power, kW	≈ 13.7
Coolant inlet temperature, °C	400
Average coolant outlet temperature, °C	480
Average coolant velocity, m s^{-1}	≈ 1.4
Fuel type	MOX
Average enrichment as Pu/(Pu+U), wt%	25.77
Cladding	Ti-15-15
Fill gas	He
Active length, mm	600
Cladding outer diameter, mm	10.5
Cladding inner diameter, mm	9.3
Fuel pellet outer diameter, mm	9
Fuel pellet inner diameter, mm	2
Pin pitch, mm	13.86

Spatial neutronics ROM · SERPENT model

SERPENT model of the ALFRED reactor





Spatial neutronics ROM · SERPENT model

```
lat 1000 2 0.0 0.0 23 23 17.1
99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99
99   99   99   99   99   99   99   99   99   99   99   99   99   99  506  506  506  506  506   99   99   99   99
99   99   99   99   99   99   99   99   99   99   99   99  506  506  506  506  506  506  506  506   99   99   99
99   99   99   99   99   99   99   99   99   99  506  506  506  506 3020 3010 3000  506  506  506  506   99   99
99   99   99   99   99   99   99   99  506  506  506 2690 2680 2670 2660 2650 2640 2630 2620  506  506  506   99
99   99   99   99   99   99   99  506  506  506 2700 2400 2390 7040 2380 7030 2370 2360 2610  506  506  506   99
99   99   99   99   99   99  506  506 3030 2710 2410 2110 2100 2090 2080 2070 2060 2350 2600 2990  506  506   99
99   99   99   99   99  506  506 3040 2720 7050 2120 1420 1410 1400 1390 1380 2050 7020 2590 2980  506  506   99
99   99   99   99  506  506 3050 2730 2420 2130 1430 1220 1210 1200 1190 1370 2040 2340 2580 2970  506  506   99
99   99   99   99  506  506 2740 7060 2140 1440 1230 1100 8020 1090 1180 1360 2030 7010 2570  506  506   99   99
99   99   99  506  506 2750 2430 2150 1450 1240 1110 1040 1030 1080 1170 1350 2020 2330 2560  506  506   99   99
99   99  506  506 2760 2440 2160 1460 1250 8030 1050 1010 1020 8010 1160 1340 2010 2320 2550  506  506   99   99
99   99  506  506 2770 2450 2170 1470 1260 1120 1060 1070 1150 1330 1570 2300 2310 2960  506  506   99   99   99
99   99  506  506 2780 7070 2180 1480 1270 1130 8040 1140 1320 1560 2290 7120 2950  506  506   99   99   99   99
99  506  506 3060 2790 2460 2190 1490 1280 1290 1300 1310 1550 2280 2540 2940 3140  506  506   99   99   99   99
99  506  506 3070 2800 7080 2200 1500 1510 1520 1530 1540 2270 7110 2930 3130  506  506   99   99   99   99   99
99  506  506 3080 2810 2470 2210 2220 2230 2240 2250 2260 2530 2920 3120  506  506   99   99   99   99   99   99
99  506  506  506 2820 2480 2490 7090 2500 7100 2510 2520 2910  506  506  506   99   99   99   99   99   99   99
99  506  506  506 2830 2840 2850 2860 2870 2880 2890 2900  506  506  506   99   99   99   99   99   99   99   99
99   99  506  506  506  506 3090 3100 3110  506  506  506  506   99   99   99   99   99   99   99   99   99   99
99   99   99  506  506  506  506  506  506  506  506   99   99   99   99   99   99   99   99   99   99   99   99
99   99   99   99  506  506  506  506  506   99   99   99   99   99   99   99   99   99   99   99   99   99   99
99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99   99
```

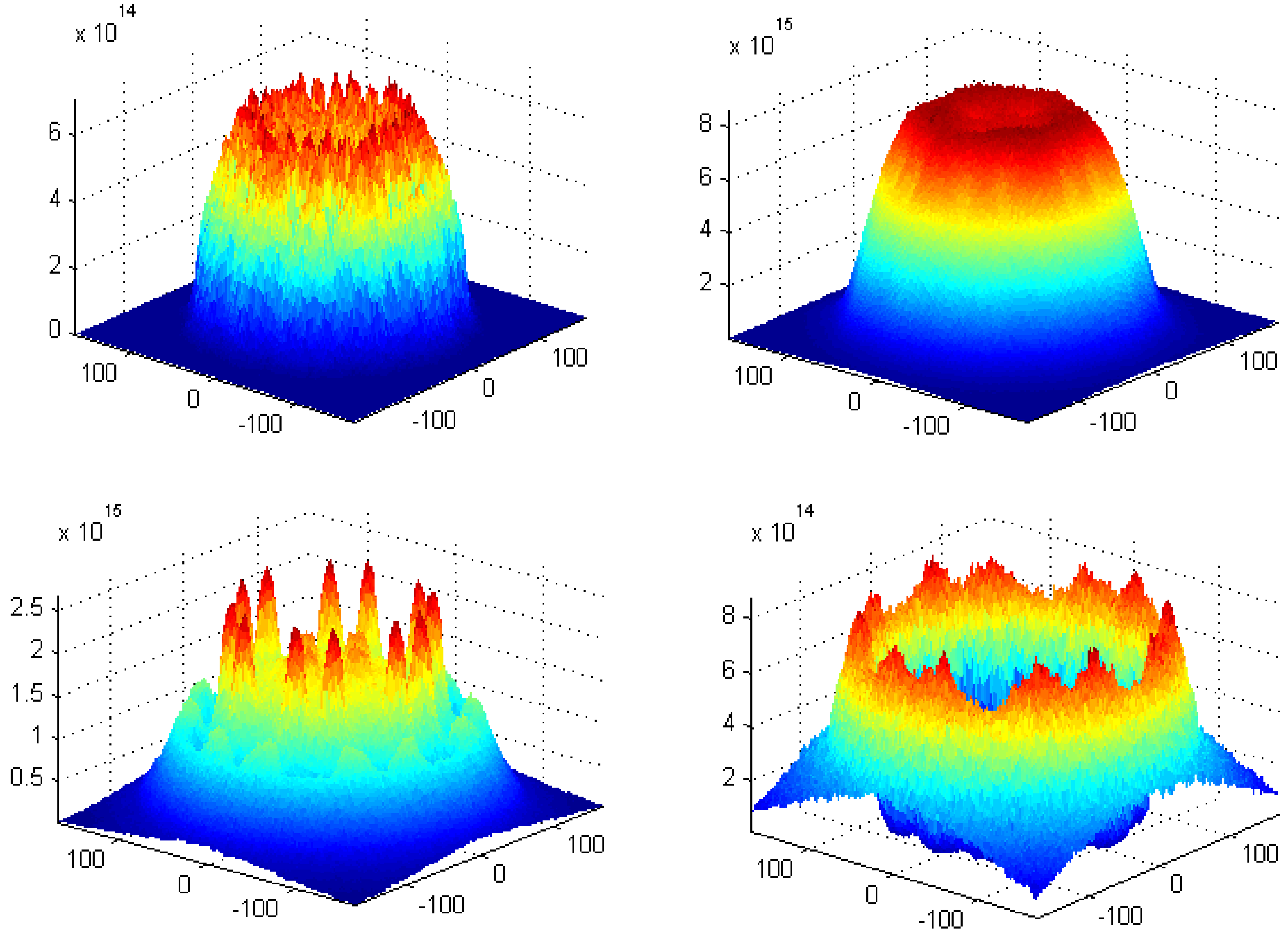
```
The geometry consists of 6 levels:

Level 0 size: max 6 zones
Level 1 size: max 529 zones
Level 2 size: max 3 zones
Level 3 size: max 3 zones
Level 4 size: max 225 zones
Level 5 size: max 36 zones
```

Each FA has been modelled separately in order to calculate the group constants

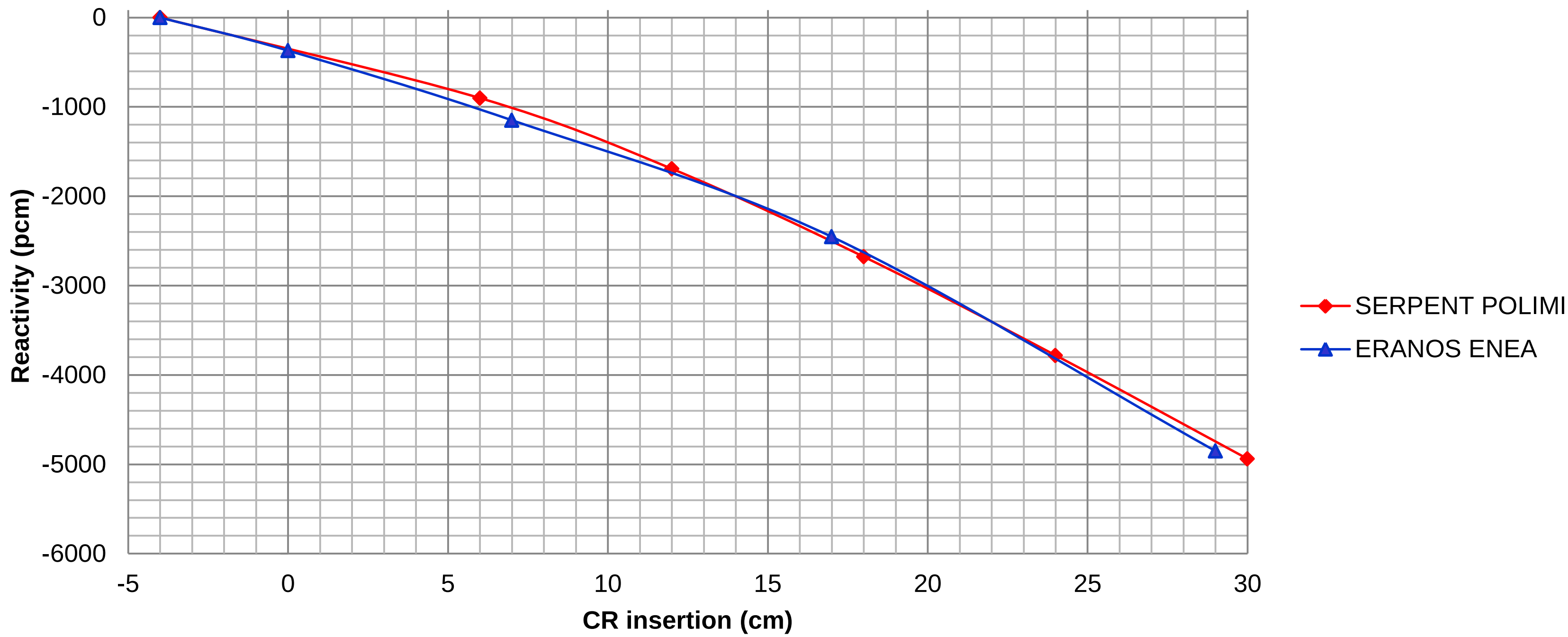
$$\underline{\underline{V}}^{-1} \quad \underline{\underline{D}} \quad \underline{\underline{\Sigma_a}} \quad \underline{\underline{\Sigma_s}} \quad \underline{\chi_p} \quad \underline{F}^T \quad \underline{\chi_d} \quad \beta_j \quad \lambda_j$$

Group	Upper boundary	Lower boundary
1	20.00 MeV	2.23 MeV
2	2.23 MeV	0.82 MeV
3	0.82 MeV	0.30 MeV
4	0.30 MeV	67.38 keV
5	67.38 keV	15.03 keV
6	15.03 keV	0.75 keV
7	0.75 keV	0.00 keV

Spatial neutronics ROM · SERPENT results



Spatial neutronics ROM · SERPENT results



	SERPENT	Uncertainty (σ)	ERANOS
Doppler constant (pcm)	-549	18	-555
Lead expansion coefficient (pcm/K)	-0.327[1]	0.019	-0.271[2]
Axial fuel expansion (pcm/K)	-0.152	0.006	-0.148
Axial cladding expansion (pcm/K)	0.044	0.006	0.037
Grid expansion (pcm/K)	-0.780	0.007	-0.789
Axial wrapper expansion (pcm/K)	0.036	0.006	0.022

[1]Calculated considering all the lead inside the inner vessel. [2]Calculated for the whole height of the fissile subassemblies.

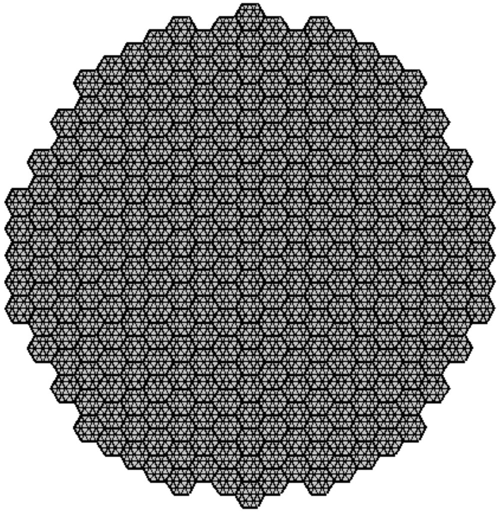
Diffusion model of the ALFRED reactor – Finite Element

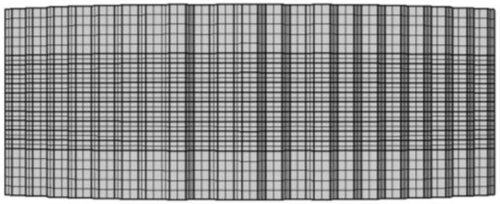
$$\underline{\underline{V}}^{-1}\frac{\partial\underline{\phi}}{\partial t} = \nabla\cdot\left(\underline{\underline{D}}\nabla\underline{\phi}\right) - \underline{\underline{\Sigma_a}}\,\underline{\phi} - \underline{\underline{\Sigma_s}}\,\underline{\phi} + (1-\beta)\underline{\chi_p}\underline{F}^T\underline{\phi} + \sum_j \lambda_j \underline{\chi_d} C_j$$

$$\frac{\partial C_j}{\partial t} = -\lambda_j C_j + \beta_j \underline{F}^T\underline{\phi} \qquad\qquad j = 1 \div 8$$

- Radial and axial albedo boundary conditions

$$\boldsymbol{n}\cdot\left(D_g\nabla\phi_g\right) = -\gamma_a\phi_g \qquad \boldsymbol{n}\cdot\left(D_g\nabla\phi_g\right) = -\gamma_r\phi_g$$



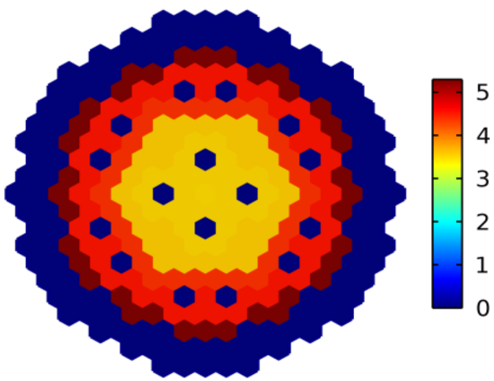


- Temperature dependence of the cross sections (from SERPENT) for each coarse zone and slice

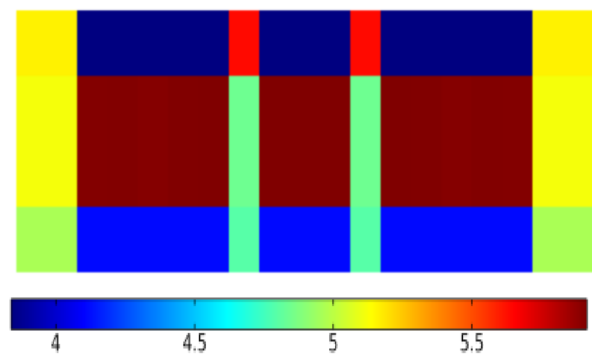
5 for the inner fuel, 5 for the outer fuel, 4 for the SRs, 5 for the CRs and 3 for the dummy elements.

$$\Sigma(T) = \Sigma_0 + \alpha \cdot log\left(\frac{T}{T_0}\right)$$

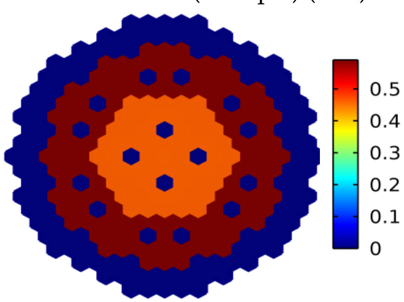
Production XS (Group 7) (1/m)



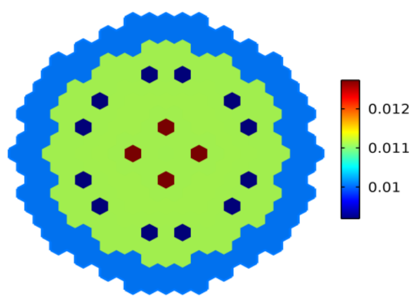
Removal XS (Group 1) (1/m)



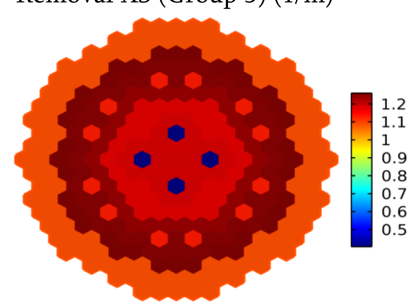
Production XS (Group 4) (1/m)



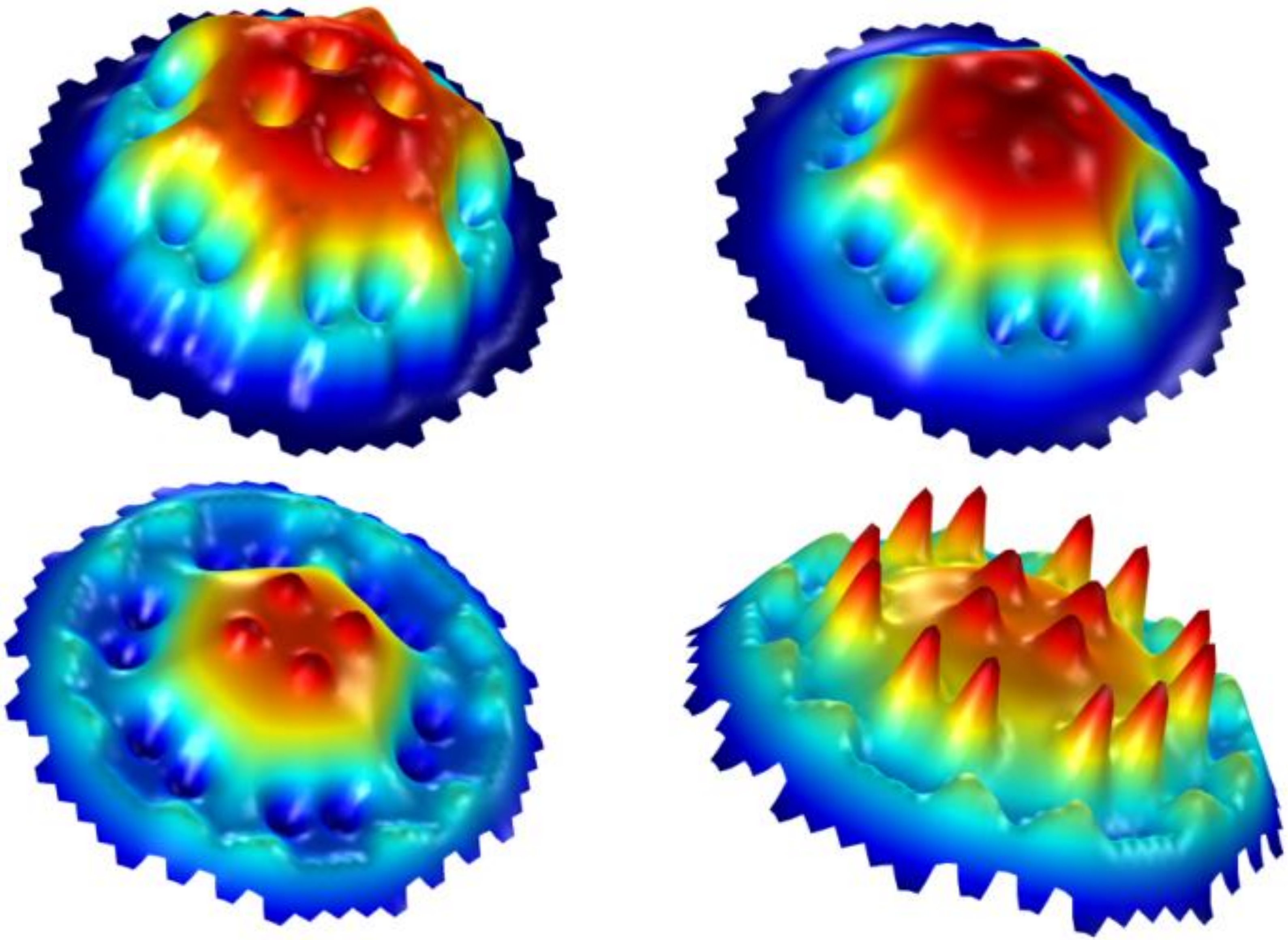
Diffusion coeff. (Group 4) (1/m)



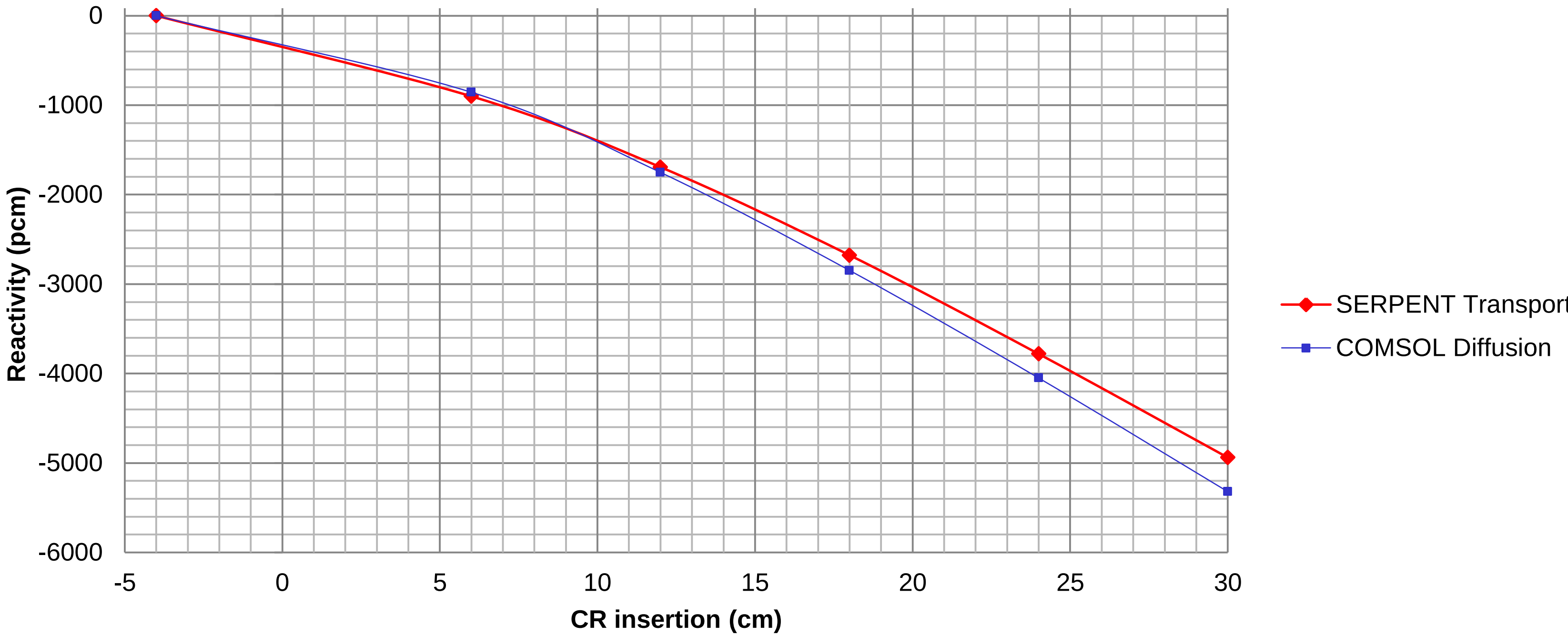
Removal XS (Group 5) (1/m)



Spatial neutronics ROM · Diffusion equation



Spatial neutronics ROM · Diffusion equation results



CR insertion (pcm)	SERPENT		COMSOL		Error	
	k	Δρ (pcm)	k	Δρ (pcm)	k	Δρ(pcm)
-4	1.07391	0	1.07387	0	7.39%	-
6	1.06362	-901	1.06409	-856	6.41%	-5.00%
12	1.05474	-1692	1.05405	-1752	5.40%	3.50%
18	1.04390	-2677	1.04201	-2848	4.20%	6.37%
24	1.03201	-3781	1.02911	-4050	2.91%	7.13%
30	1.01982	-4939	1.01584	-5320	1.58%	7.72%

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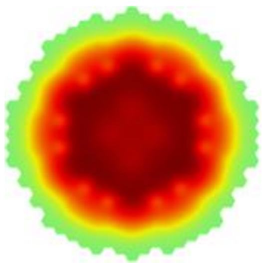
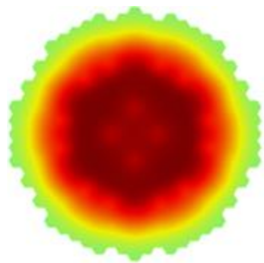
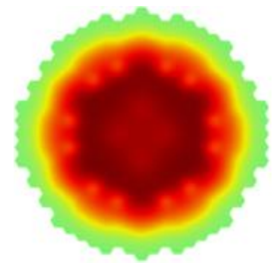
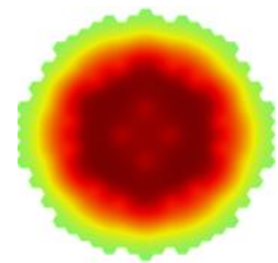
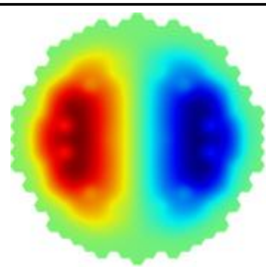
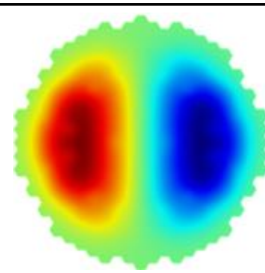
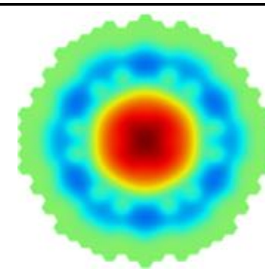
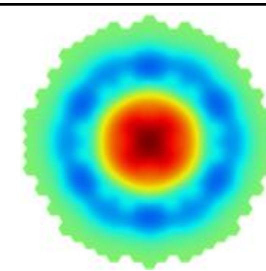
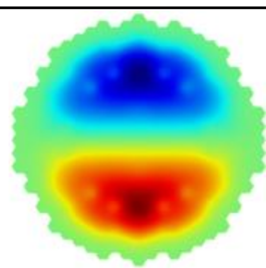
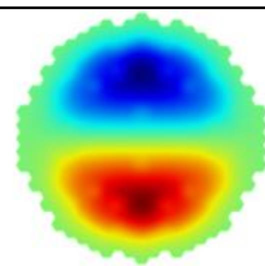
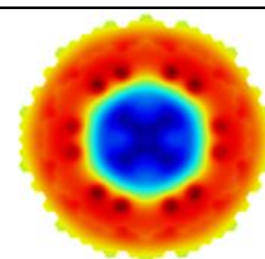
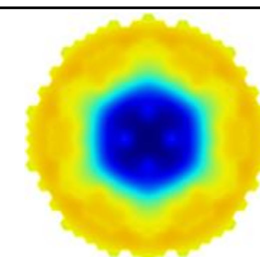
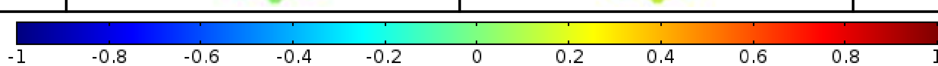
The snapshots have been collected from the eigenfunctions related to different configurations of the ALFRED core (CR position and characteristic temperature).

#	$\underline{\underline{\psi}}_i$ for Case A,B $\underrightarrow{\xi}_i$ for Case A	$\underrightarrow{\xi}_i$ for Case B	$\underline{\underline{\psi}}_i$ for Case C,D $\underrightarrow{\xi}_i$ for Case C	$\underrightarrow{\xi}_i$ for Case D
1				
2				
3				



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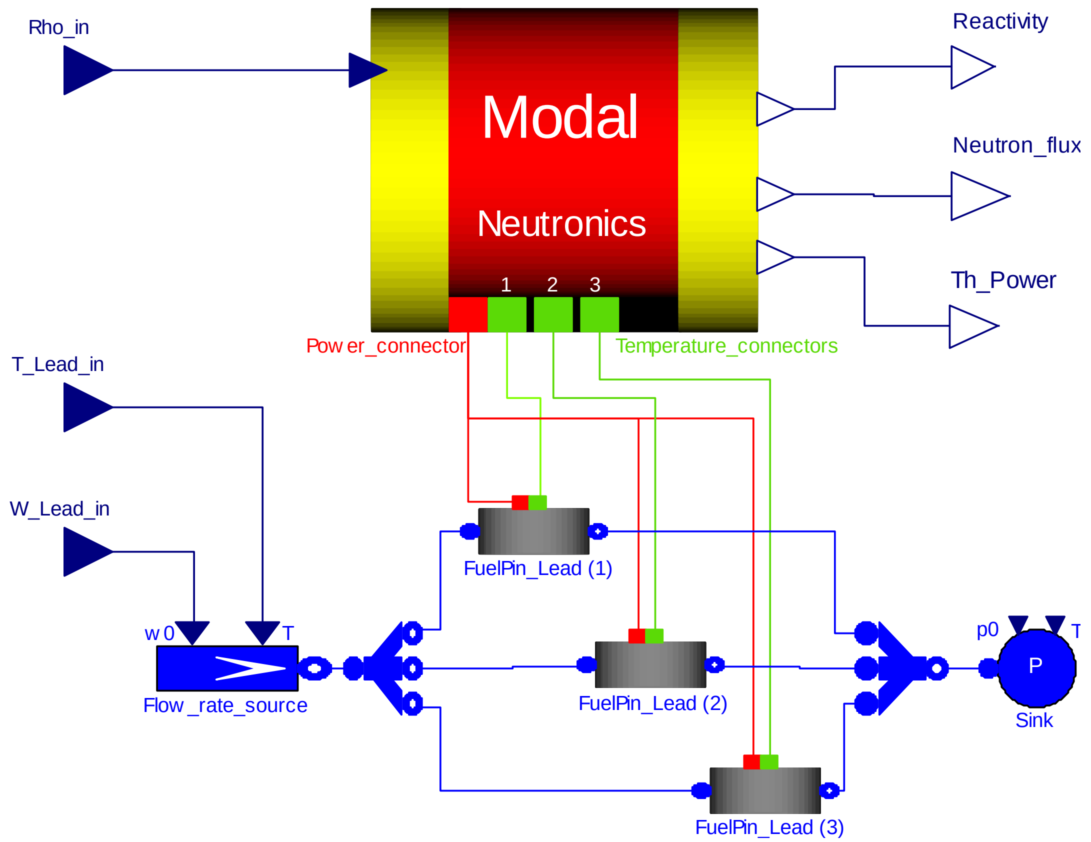
Implementation of the ODE system in a control simulator environment [5]

$$\dot{\underline{n}} = \underline{\underline{iRV}} \cdot \left[\left(\underline{\underline{A}}_{1,np} + \underline{\underline{A}}_{1,p} \right) \underline{n} + \underline{\underline{A}}_{2} \underline{c} \right]$$
$$\dot{\underline{c}} = \left(\underline{\underline{A}}_{3,np} + \underline{\underline{A}}_{3,p} \right) \underline{n} - \underline{\underline{A}}_{4} \underline{c}$$



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Validation of Modal approach (code-to-code comparison) in simple geometry – 3 Pin case test

Reactivity comparison

Neutronics modeling approach	Reactivity inserted (pcm)		
	Case 1	Case 2	Case 3
Multi-group diffusion PDEs (reference)	78.4	-25.5	1267.6
Spatial Neutronics ROM (SN-ROM)	78.8	-25.6	1268.9
Point Kinetics (PK)	81	-18.9	1363.8
SN-ROM/PK error	0.51 % / 3.3 %	0.39 % / 25 %	0.10 % / 7.6 %

From stationary case:

1. Uniform temperature decrease, $\Delta T_{f1} = \Delta T_{f2} = \Delta T_{f3} = \Delta T_{l} = -50$ K;
2. Temperature enhancement in a single pin and in the 5th axial slice, i.e., $\Delta T_{f1} = +400$ K, $\Delta T_{f2} = +300$ K, $\Delta T_{f3} = +200$ K, $\Delta T_{l} = +100$ K;
3. Shutdown scenario: all the temperature are set equal to the inlet lead temperature, i.e., T=673.15 K.

Good agreement between the reference model and the SN-ROM

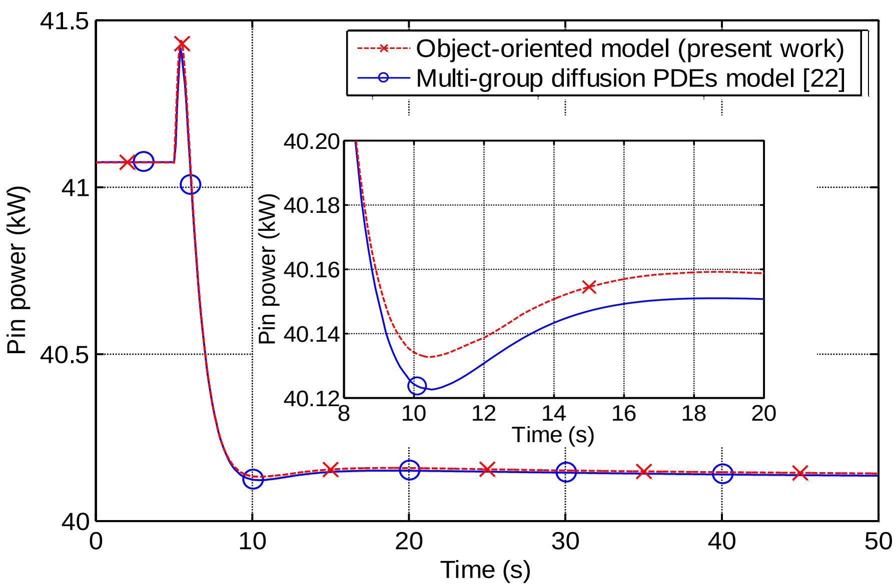
PK not accurate if perturbation is localized or not uniform

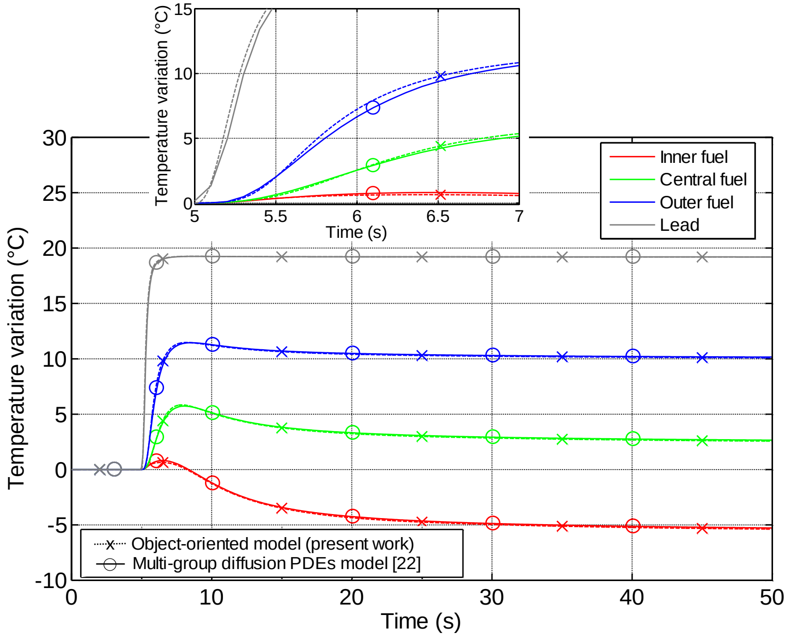
Validation of Modal approach (code-to-code comparison) in simple geometry – 3 Pin case test

Transient comparison – Increase of the inlet lead temperature (20°C)

Neutronics modeling approach	Computational time			
Multi-group diffusion PDEs (reference)	40 h			
Spatial Neutronics ROM	N=1	N=3	**N=5**	N=10
	10 s	23 s	**45 s**	143s

The SN-ROM run with a laptop (2.20 GHz, 8 GB RAM). The neutron diffusion PDE solved with a workstation (8 x 2.8 GHz, 64 GB RAM).

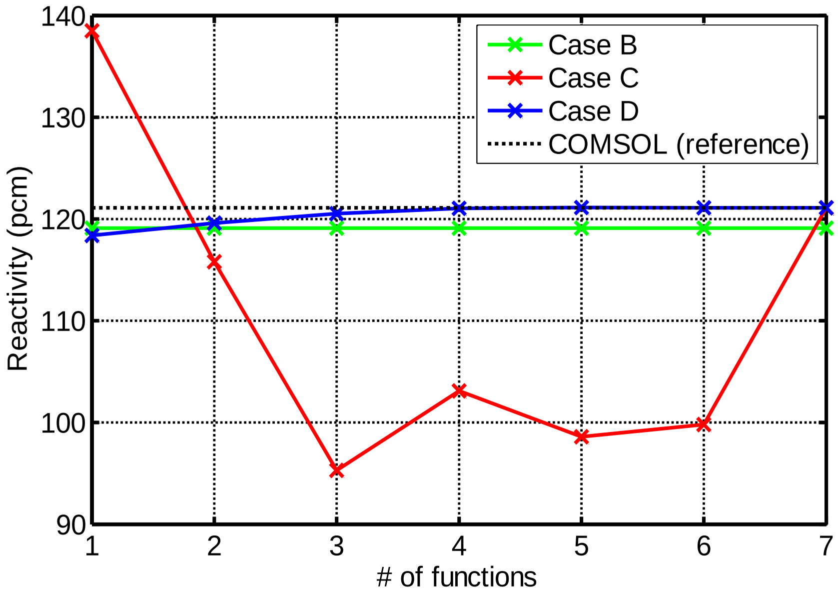


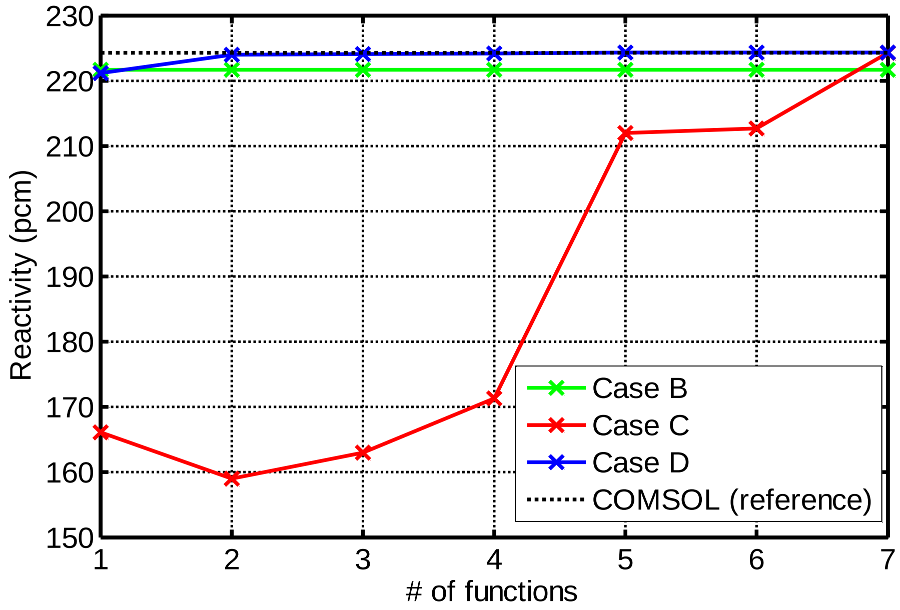


Spatial neutronics ROM · Doppler effect

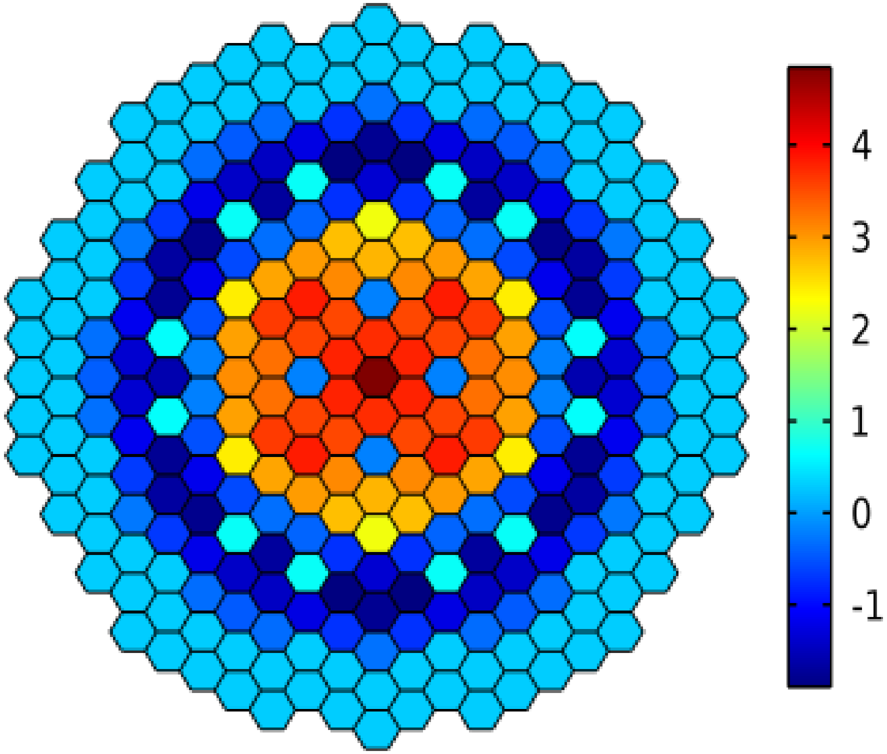
	SERPENT (transport)	COMSOL (diffusion)	Error (%)
Inner zone	128.1±7.5	121.1	5.5
Outer zone	206.7±7.5	224.3	8.5

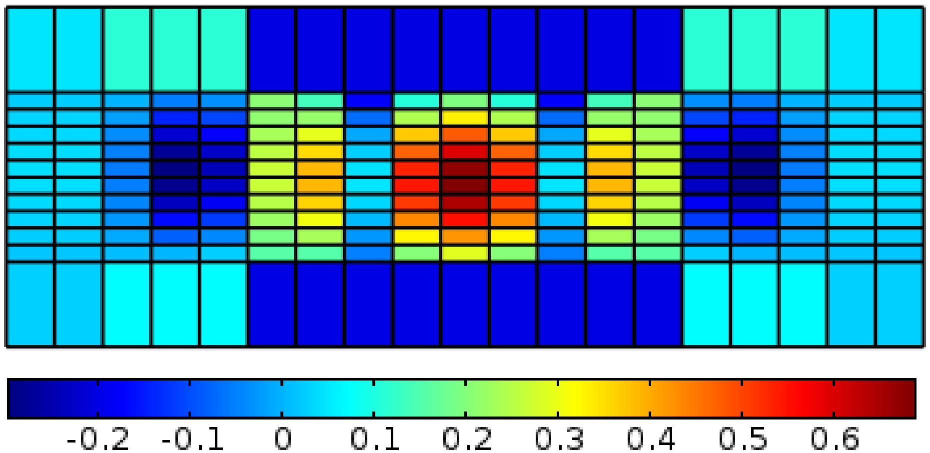
	Case A (N=5)	Case B (N=5)	Case C (N=5)	Case C (N=7)	Case D (N=5)	COMSOL (reference)
Inner zone	46.7	119.1	98.6	121.1	121.1	121.1
Outer zone	73.8	221.7	212.0	224.3	224.3	224.3





Spatial neutronics ROM · Doppler effect



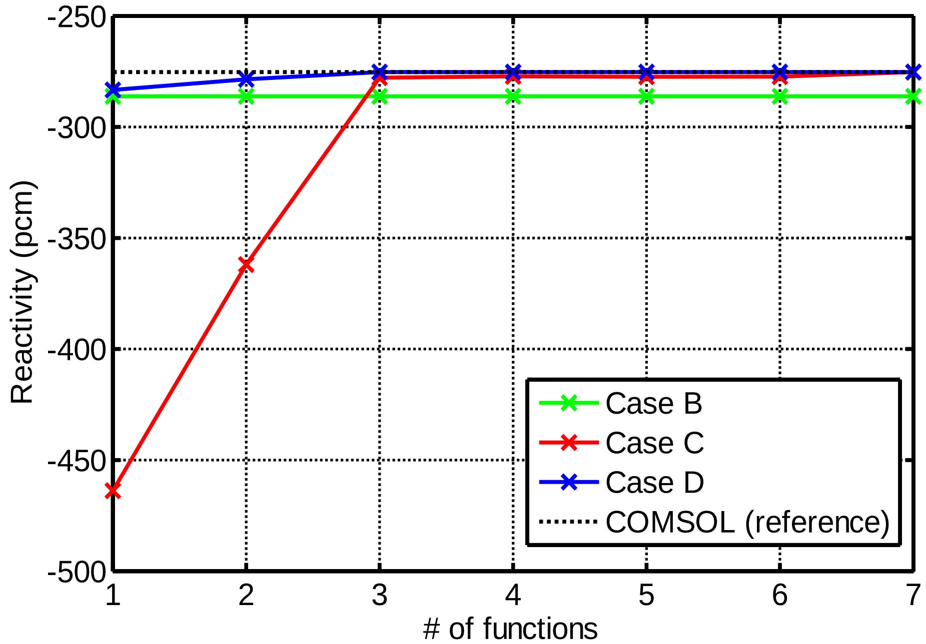


Doppler effect. Spatial reactivity (pcm) variation in each assembly in the active zone and along the axial direction.

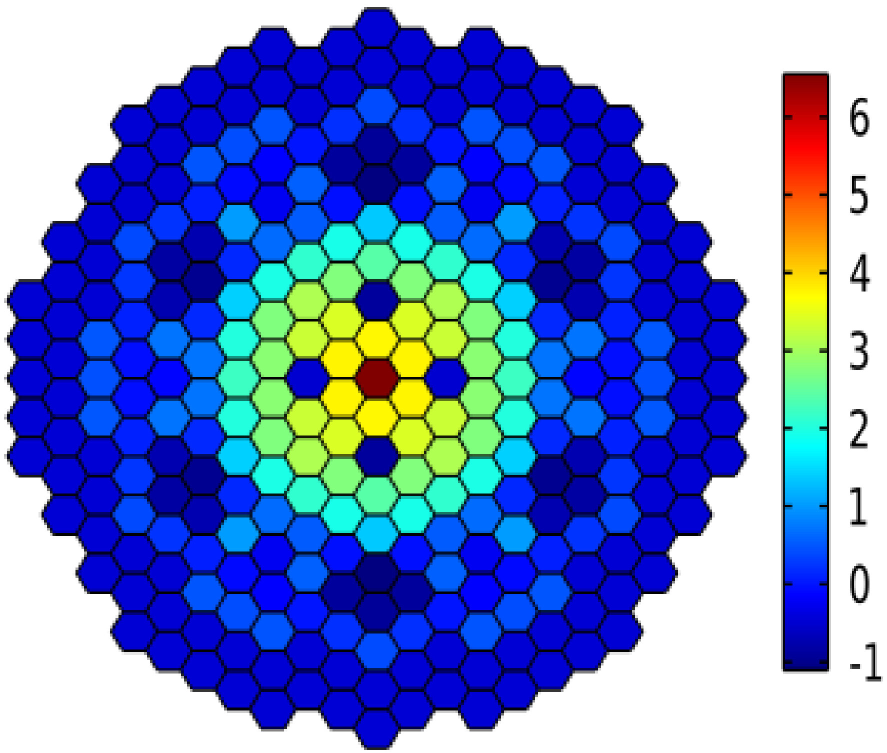
Spatial neutronics ROM · Lead density effect

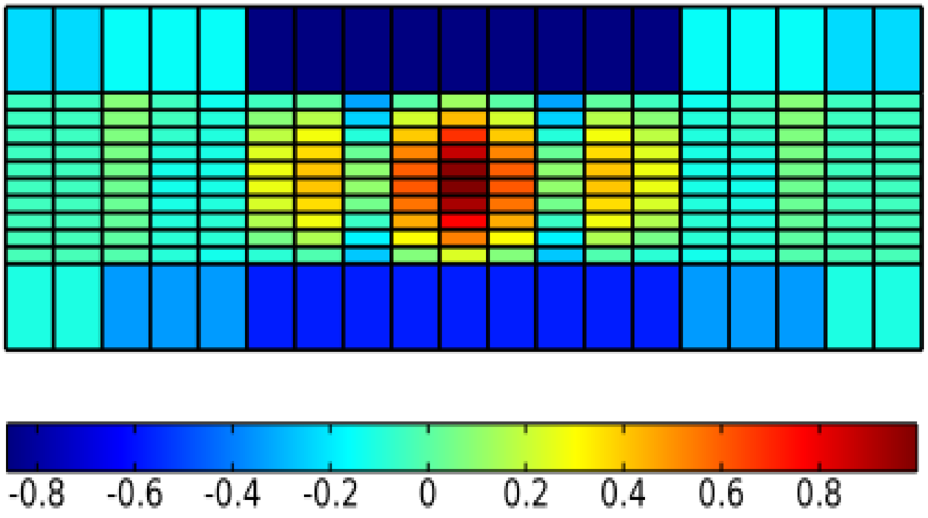
	SERPENT (transport)	COMSOL (diffusion)	Error (%)
Lead density	-262±7.5	-275.3	5.21

Case A (N=5)	Case B (N=5)	Case C (N=5)	Case C (N=7)	Case D (N=5)	COMSOL (reference)
-559.1	-286.2	-277.4	-275.3	-275.3	-275.3



Spatial neutronics ROM · Lead density effect

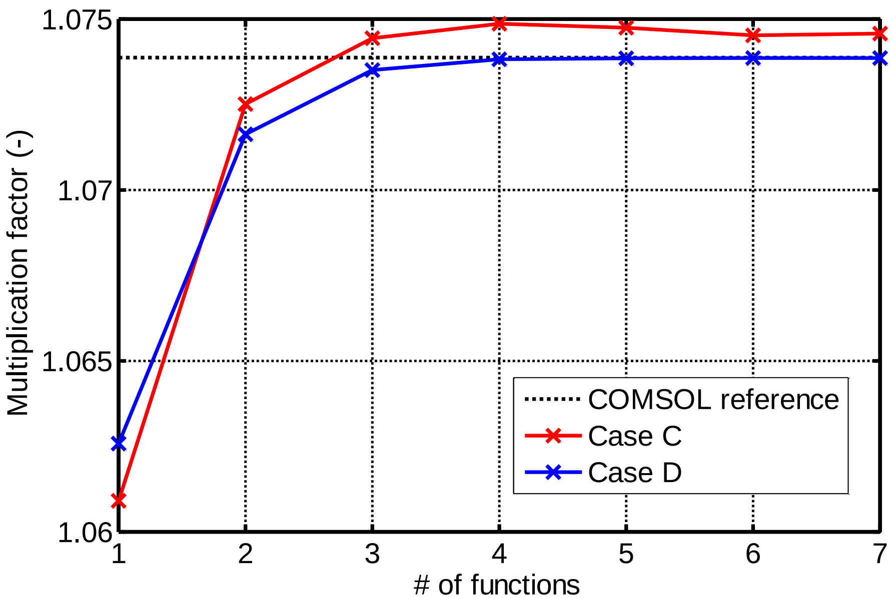


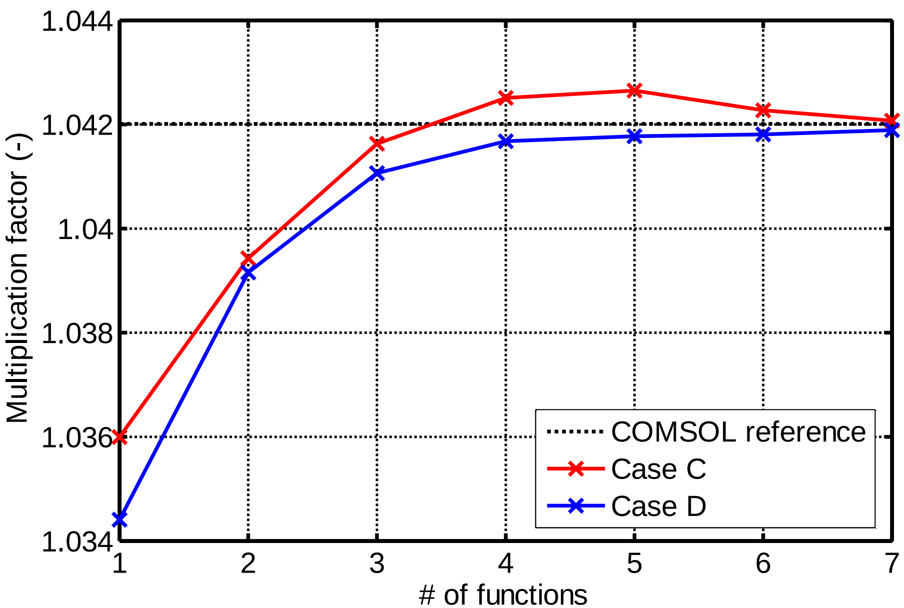


Lead density effect. Spatial reactivity (pcm) variation in each assembly in the active zone along the axial direction.

Spatial neutronics ROM · CR movement

CR position (cm)	COMSOL (reference)	Case C (N=7)		Case D (N=7)	
		k	Δρ (pcm)	k	Δρ (pcm)
-4	1.073873	1.07458	65.80	1.073864	-0.82
6	1.064094	1.06447	34.93	1.064055	-3.63
12	1.054045	1.05485	76.10	1.053916	-12.29
18	1.042010	1.04207	6.00	1.041890	-11.52
24	1.029115	1.02980	66.11	1.029048	-6.52
30	1.015837	1.01624	39.35	1.015817	-1.93





The best results are obtained if adjoint is used for the test functions

WHY?

- Thermal feedbacks

$$\underline{\underline{\delta L}}_{im}(d,T) = \int \underline{\underline{\psi}}_i^{\dagger} \cdot \delta L(d,T) \underline{\underline{\psi}}_m d\Omega = \sum_{gz} \int \underline{\underline{\psi}}_i^{\dagger} \cdot \left[-\nabla \cdot \left(\underline{\underline{D}}(d_{gz}, T_{gz}) \nabla \right) + \underline{\underline{\Sigma_a}}(d_{gz}, T_{gz}) + \underline{\underline{\Sigma_s}}(d_{gz}, T_{gz}) \right] \underline{\underline{\psi}}_m d\Omega_{gz} =$$

$$= \sum_{gz} \left[\underline{\underline{D}}(d_{gz}, T_{gz}) \int \nabla \underline{\underline{\psi}}_i^{\dagger} \cdot \nabla \underline{\underline{\psi}}_m \, d\Omega_{gz} + \underline{\underline{\Sigma_a}}(d_{gz}, T_{gz}) \int \underline{\underline{\psi}}_i^{\dagger} \cdot \underline{\underline{\psi}}_m \, d\Omega_{gz} + \underline{\underline{\Sigma_s}}(d_{gz}, T_{gz}) \int \underline{\underline{\psi}}_i^{\dagger} \cdot \underline{\underline{\psi}}_m \, d\Omega_{gz} \right] +$$

$$+ \gamma_r \int_{\partial\Omega_r} \underline{\underline{\psi}}_i^{\dagger} \cdot \underline{\underline{\psi}}_m dS_r + \gamma_a \int_{\partial\Omega_a} \underline{\underline{\psi}}_i^{\dagger} \cdot \underline{\underline{\psi}}_m dS_a$$

- Reactivity: formulation alternative to Inverse Method

$$\rho(t) = \frac{\int d\Omega \phi^{\dagger} \left[\mathcal{L}\phi + \left[(1-\beta)\chi_p + \beta\chi_d \right] \mathcal{F}\phi \right]}{\int d\Omega \phi^{\dagger} \left[\left[(1-\beta)\chi_p + \beta\chi_d \right] \mathcal{F}\phi \right]} \quad \rangle \quad \rho(t) = \frac{\underline{n}^{T} \cdot \left[\underline{\underline{A}}_{1,np} + \underline{\underline{A}}_{1,p} + \beta \underline{\underline{A}}_{md} \right] \cdot \underline{n}}{\underline{n}^{T} \cdot \left[(1-\beta) \underline{\underline{A}}_{mp} + \beta \underline{\underline{A}}_{md} \right] \cdot \underline{n}}$$

$$\hat{H}\phi = (\hat{L} - \lambda\hat{M})\phi = 0$$

. Perturbation theory

$\hat{H}\phi = 0 \qquad \hat{H}^\dagger\phi^\dagger = 0 \qquad \langle f|\hat{H}g\rangle = \langle \hat{H}^\dagger f|g\rangle$ Adjoint definition

$\hat{H}' = \hat{H} + d\hat{L}$

$\phi \cong \sum_{i=1}^{N} \psi \cdot a(t)$ Ansatz $\qquad \phi' = \phi + d\varepsilon$

$d\varepsilon$ is the error introduced by the truncation

$$\begin{aligned}
\langle \phi^\dagger|\hat{H}'\phi'\rangle &= \langle \phi^\dagger|(\hat{H} + d\hat{L})(\phi + d\varepsilon\,)\rangle = \\
&= \langle \phi^\dagger|\hat{H}\phi\rangle + \langle \phi^\dagger|\hat{H}d\varepsilon\rangle + \langle \phi^\dagger|d\hat{L}\phi\rangle + \langle \phi^\dagger|d\hat{L}d\varepsilon\rangle = \\
&= 0 + \langle \hat{H}^\dagger\phi^\dagger|d\varepsilon\rangle + \langle \phi^\dagger|d\hat{L}\phi\rangle + \langle \phi^\dagger|d\hat{L}d\varepsilon\rangle = \\
&= 0 + 0 + \langle \phi^\dagger|d\hat{L}\phi\rangle + \langle \phi^\dagger|d\hat{L}d\varepsilon\rangle =
\end{aligned}$$

First order perturbative term $\qquad$ Second error term

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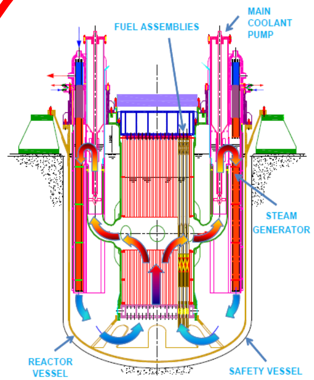
How to improve the modelling accuracy…

Coolant pool usually represented by 0D/1D models



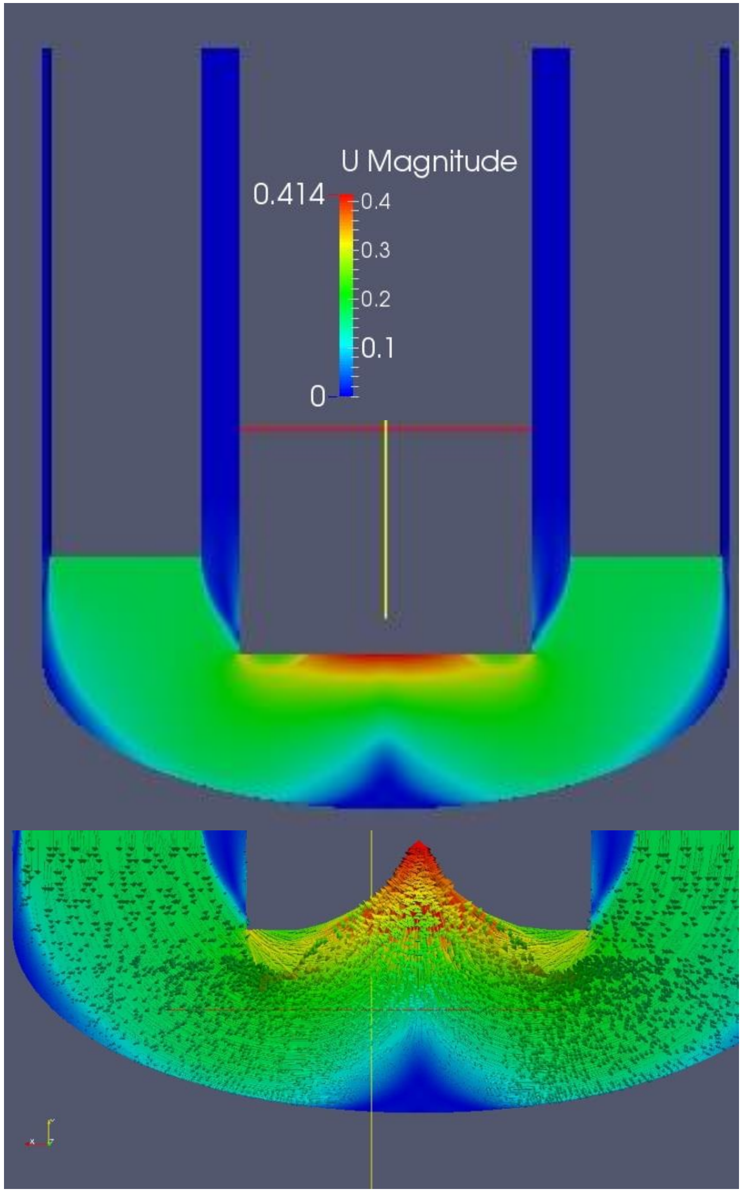
The local heterogeneity **cannot be represented** by a classic Thermal-Hydraulic (T-H) model

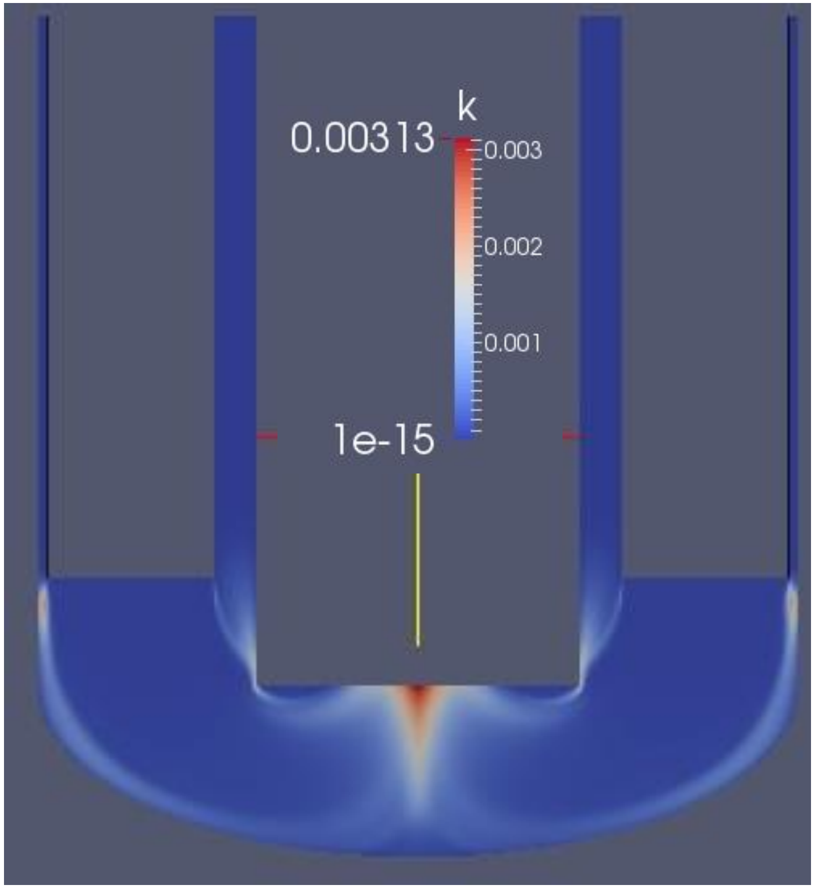
. No info on spatial distribution of the flow, turbulence mixing and 3D effects
. No possibility of exploiting all the capacities of advanced control schemes (i.e., the oxygen control is crucial for LFR)

Spatial coolant pool model

From a Computational Fluid Dynamics (CFD) model of the pool to a reduced model able to catch the main T-H dynamics

Reduce order modelling · Further development





Collaboration with
Gianluigi Rozza (SISSA, Italy)

Reduced order modelling · Conclusion

- Innovative techniques to improve NPP availability and to follow grid demands
- Development of specific simulation tools allows improving **control system design**

Need to develop models that are both accurate and fast-running

Reduced order modelling approach – ROM-Projection framework

Spatial Neutronics Model

- ✓ Good agreement w/ reference model for reactivity and transient behavior w/o an excessive computational cost
- ✓ Preliminary results on the entire core models (Serpent, Comsol) indicate satisfactory agreement w/ literature data
- ✓ Best results when considering the adjoint flux as test function

Spatial Coolant Pool Model

- ✓ Model under development

ALFRED (LFR concept) as reference reactor for the present general purpose approach

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3. Toni Lassila, Andrea Manzoni, Alfio Quarteroni and Gianluigi Rozza. *Model Order Reduction in Fluid Dynamics: Challenges and Perspectives*. In Reduced Order Methods for Modeling and Computational Reduction. MS&A - Modeling, Simulation and Applications, **9**, 235-273 (2014).
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Thanks for your kind attention

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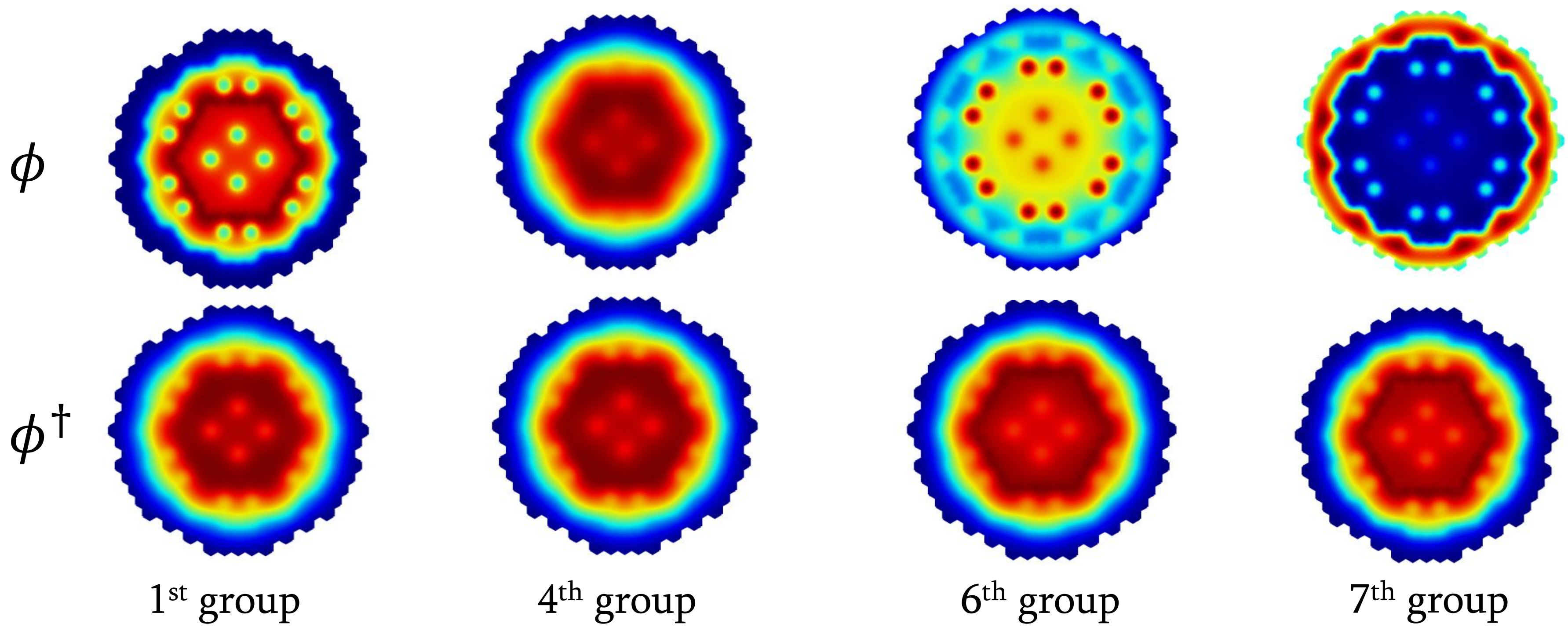
ϕ
$\phi^\dagger$
1st group
4th group
6th group
7th group