

# Precision Cosmology with Cosmic Voids

LPSC Seminar

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*CPPPM & IAP (France)*



Grenoble  
20/10/2015

Credit: Millennium simulation

# Outline

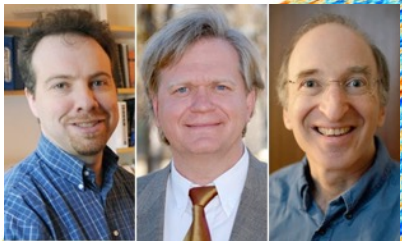
- ▶ Introduction
- ▶ Voids as tools for Cosmology
- ▶ Finding voids and measuring the expansion
- ▶ Can we access to the real space information?
- ▶ Can we master peculiar velocities on voids?
- ▶ What can we expect from the future?



# The standard cosmological model

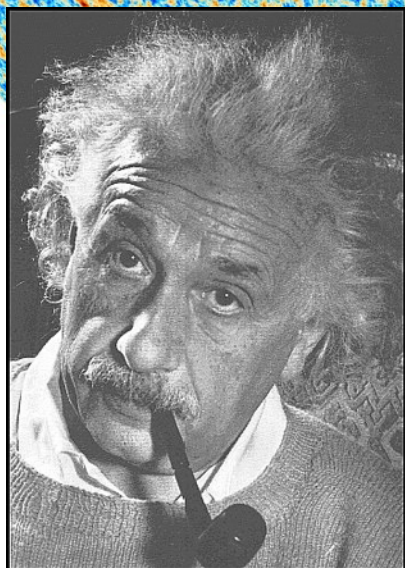
$\Lambda$ CDM

Physics Nobel  
Prize 2011  
Perlmutter,  
Riess, Schmidt



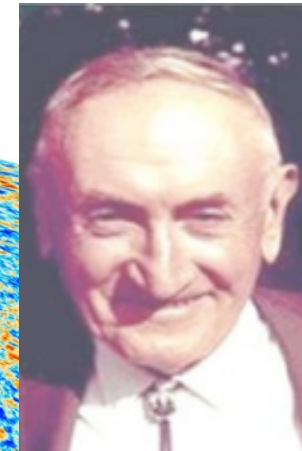
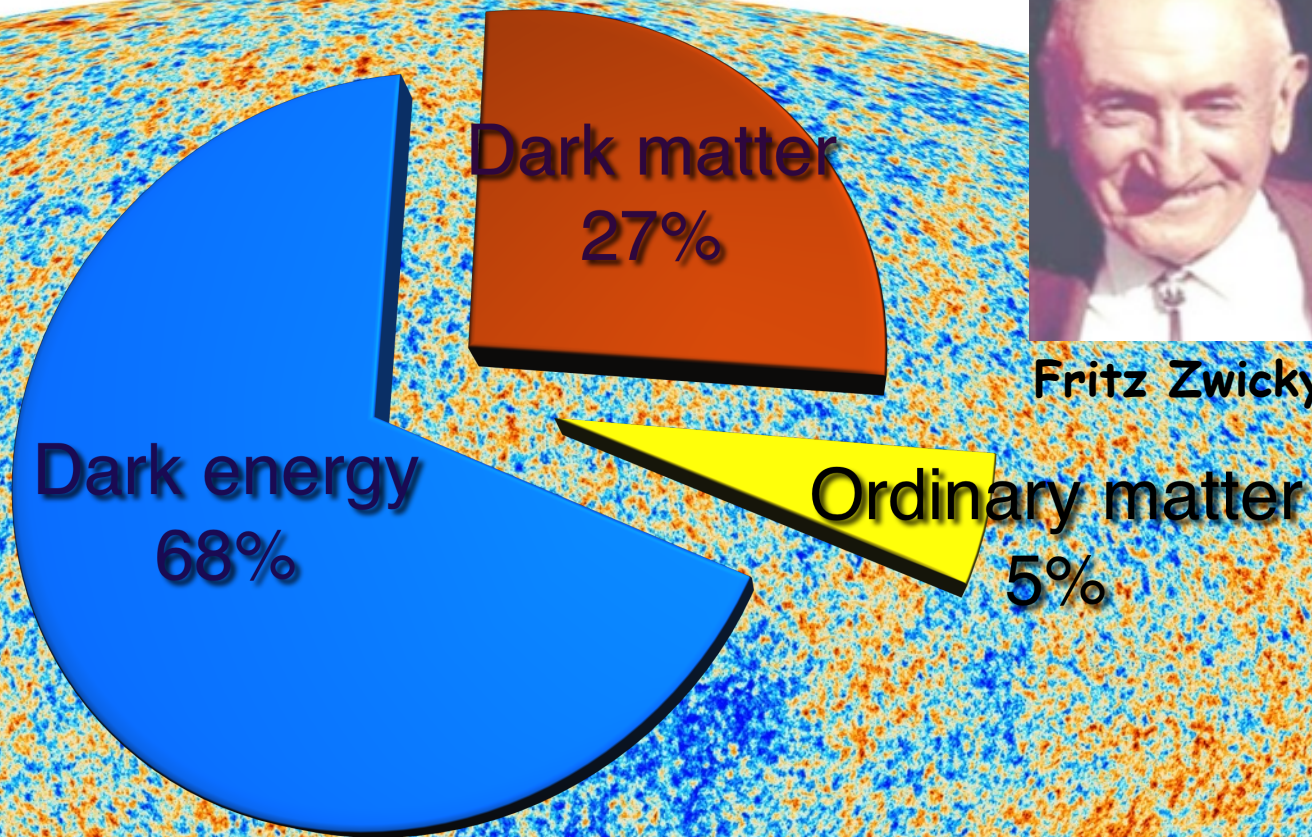
Supernova I a

CMB-Planck collaboration



Albert Einstein

$\Lambda$

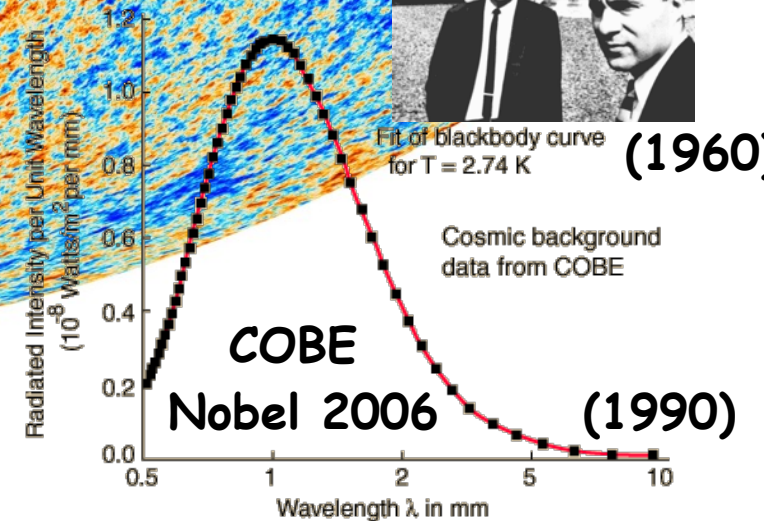


Fritz Zwicky



Vera Rubin

Penzias & Wilson  
Nobel 1978

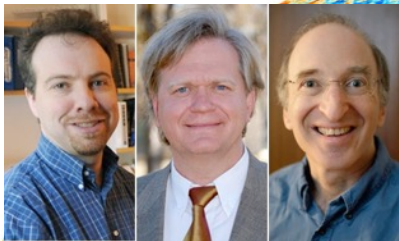




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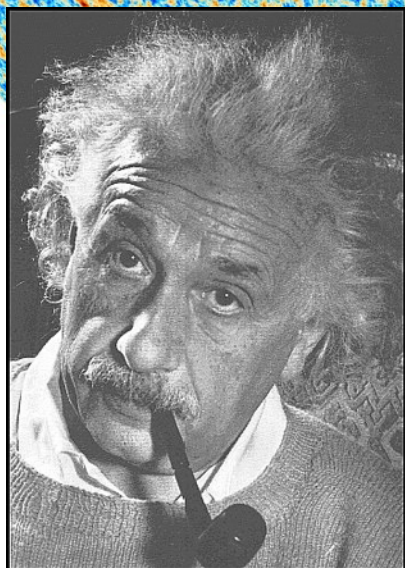
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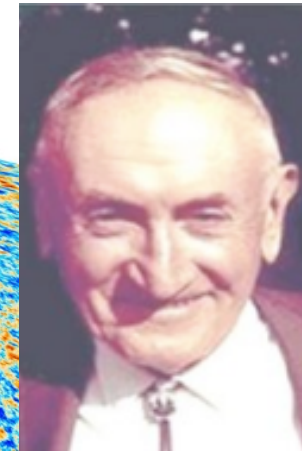
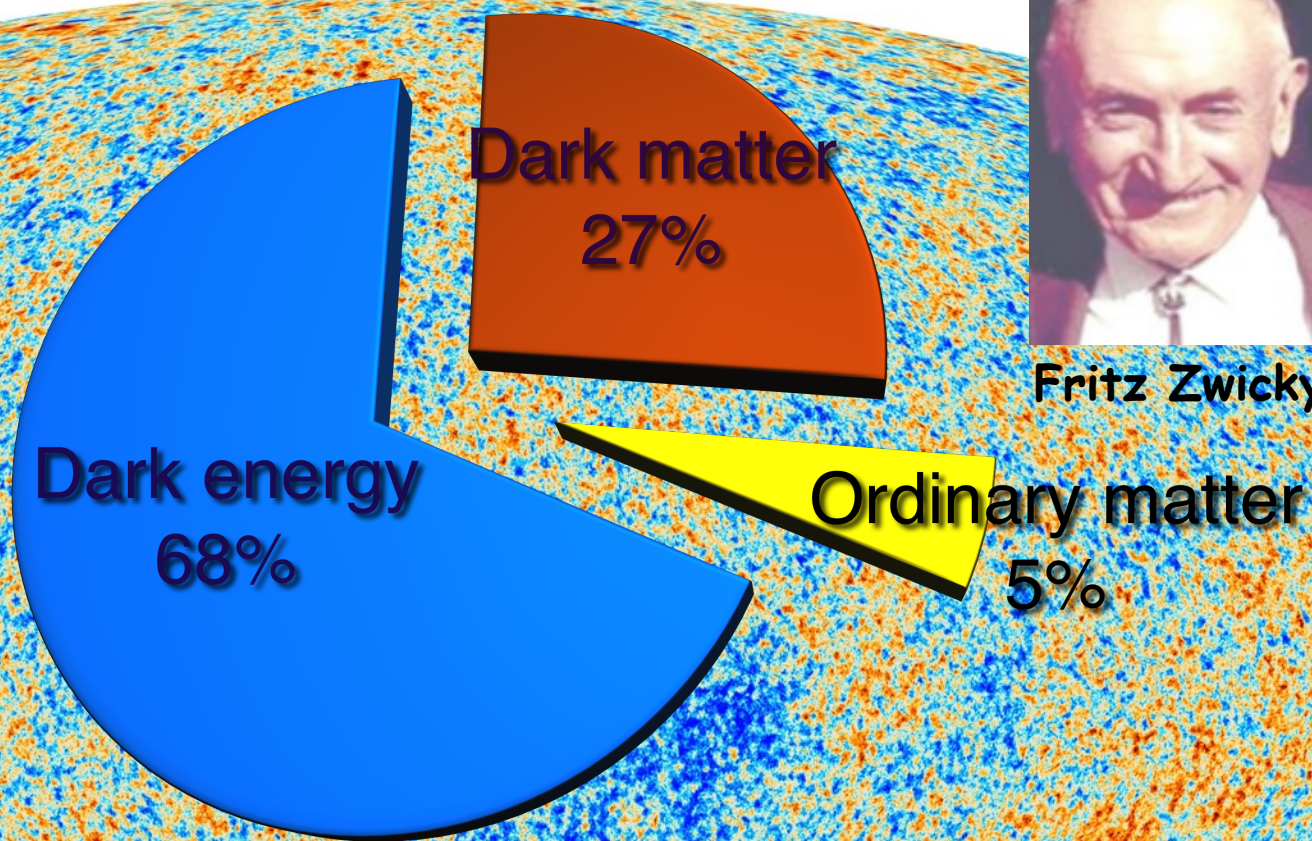


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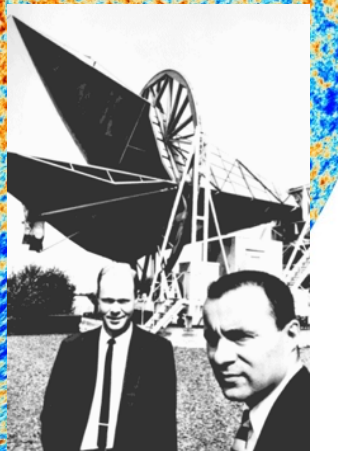


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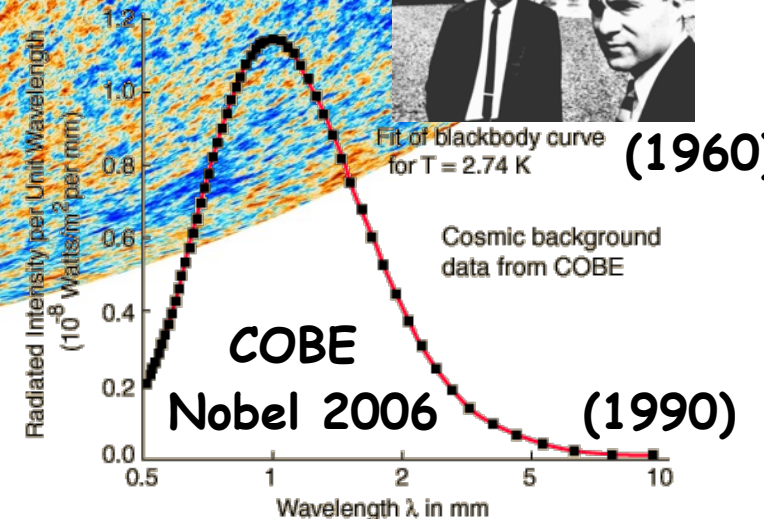


Vera Rubin

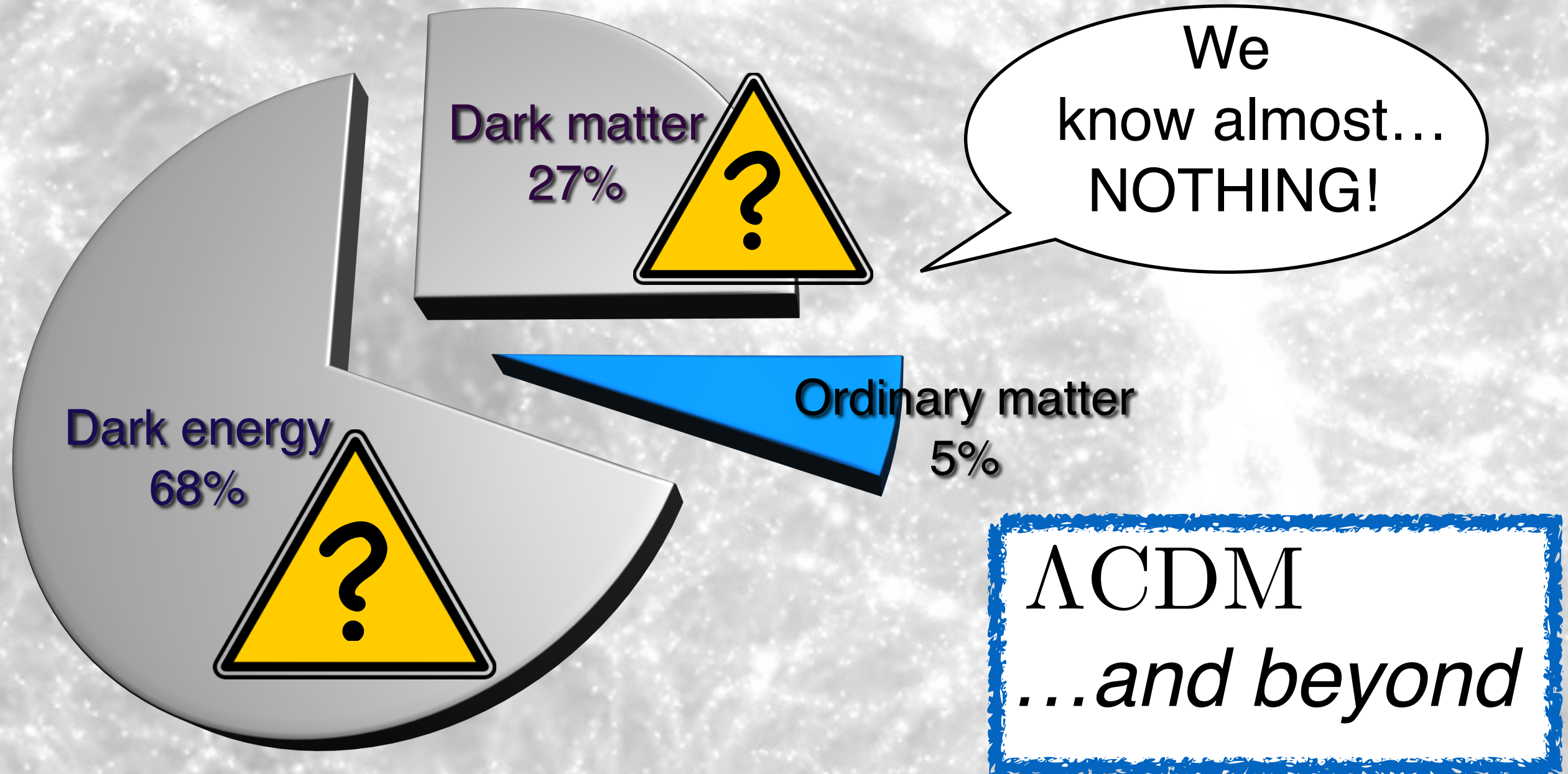
Penzias & Wilson  
Nobel 1978



$\Lambda$  We have a great  
understanding of our  
Universe!







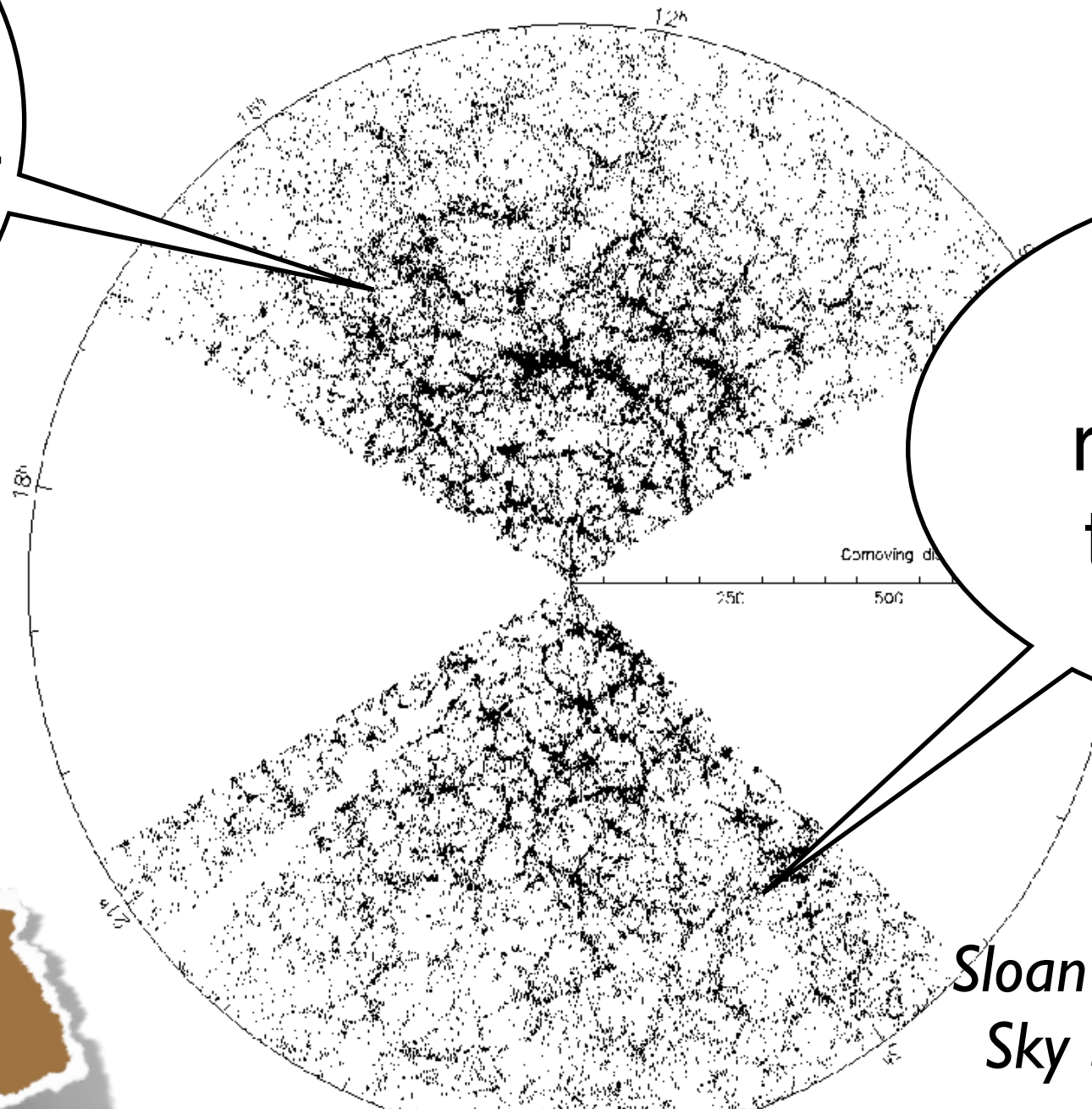
The study of large scale structures is a powerful tool to understand the composition of the universe.



# The cosmic web

complex  
filamentary  
supercluster  
structures

emptier  
(not empty!)  
regions from 10  
to 100 Mpc/h:  
**VOIDS**



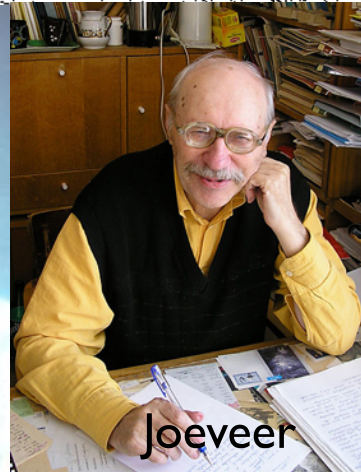
1977



Gregory



Thompson



Joeveer



Einasto



Tago



# Are voids there?

1977



Credit: Thompson and  
Gregory 1977



Credit: Jaan Einasto private collection

Peebles

Abell

Longair

Einasto

# Are voids there?

1977



Credit: Thompson and Gregory 1977



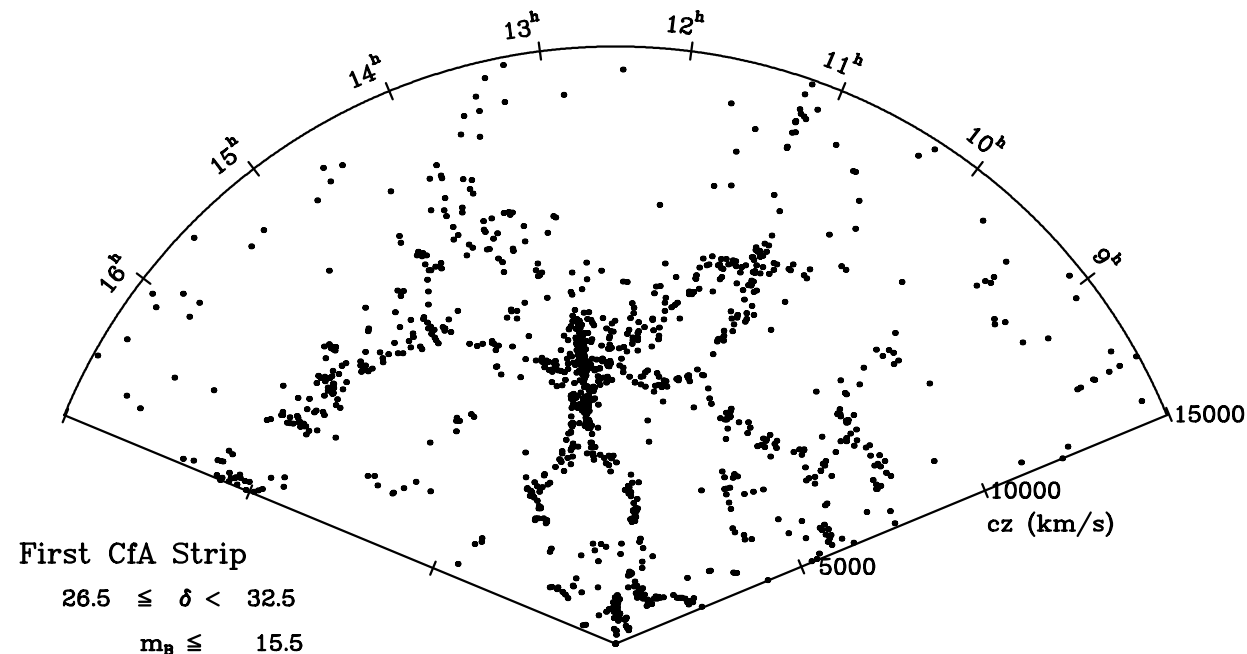
Credit: Jaan Einasto private collection

Peebles

Abell

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1986

Credit: de Lapparent et al. 1986

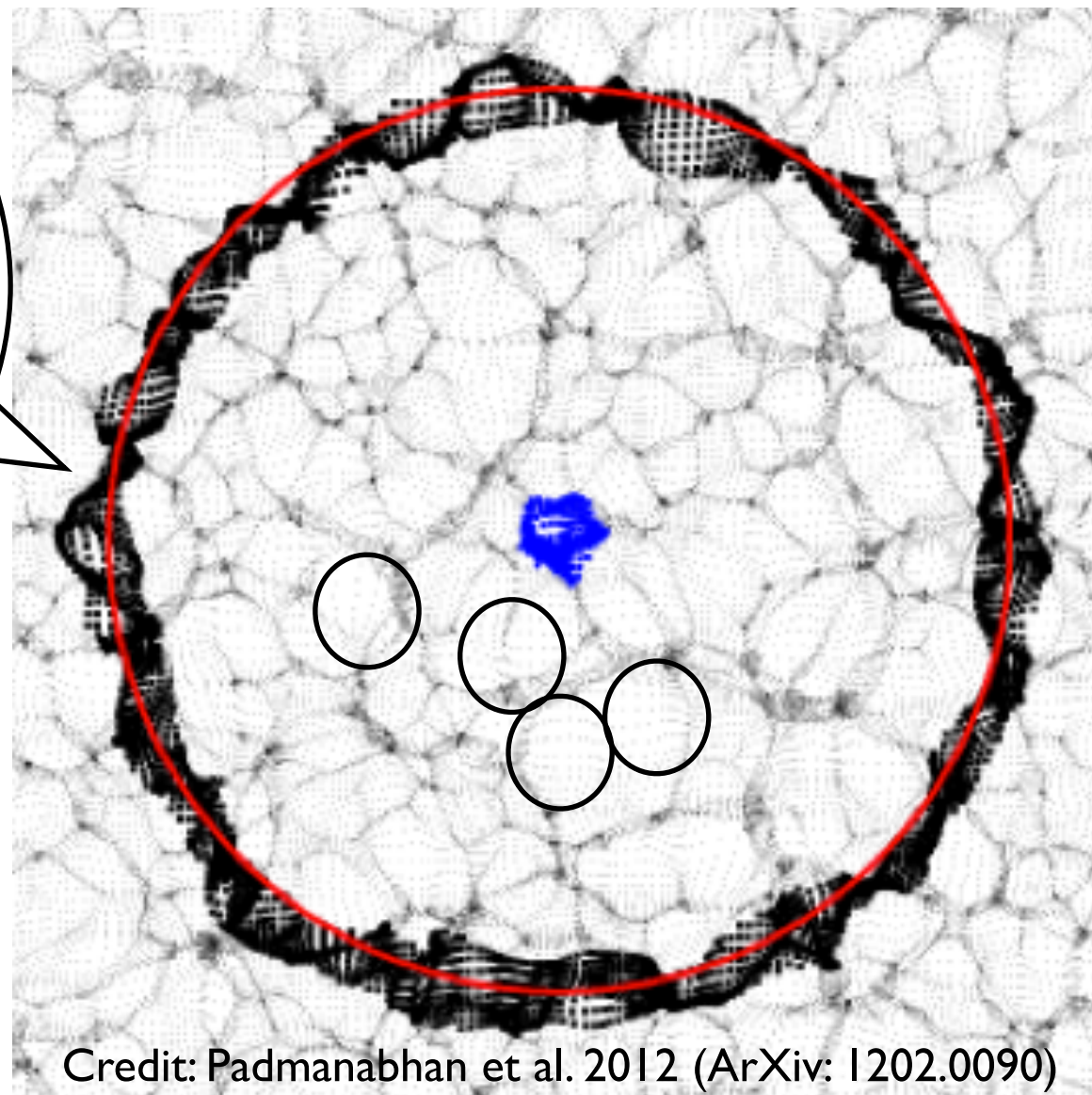


Can we use voids  
to get cosmological  
information?

# In voids matter is missing=> Dark Energy

BAO compared to void

Small to intermediate scale distribution is potentially more sensitive: more scales!



Fundamental physics: voids are more likely to be sensitive to diffuse components (e.g.  $v$ ).

**If Dark Energy exists**, cosmic voids are a new tool to constrain it.



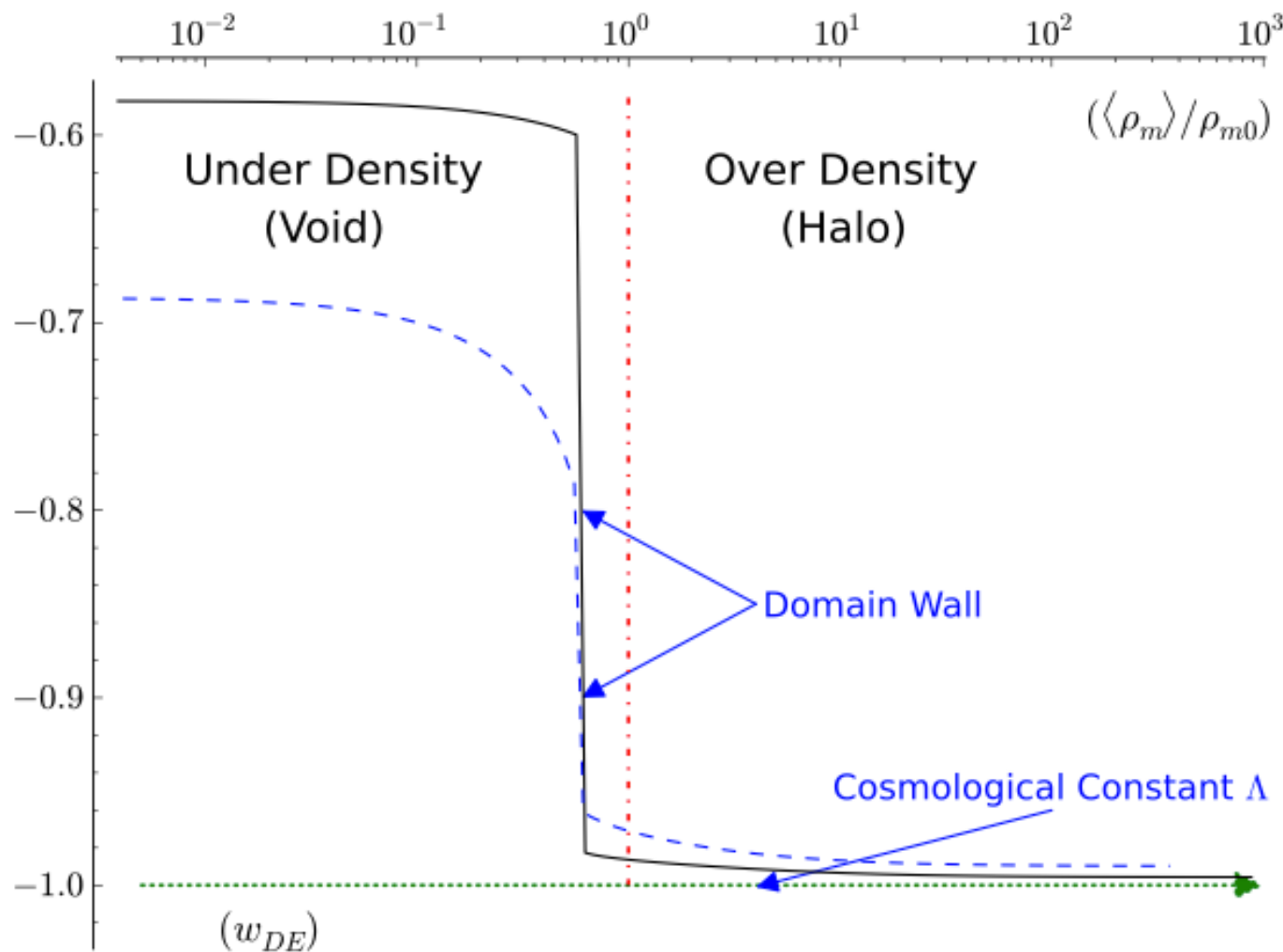
If not  $\Lambda$ CDM ? Do we need to modify gravity?

# If not $\Lambda$ CDM ? Do we need to modify gravity?

MASSIVE GRAVITY MODELS  
Graviton could become massive,  
which would introduce a new  
scalar field.



The equation of state  
could be DENSITY and  
SCALE DEPENDENT



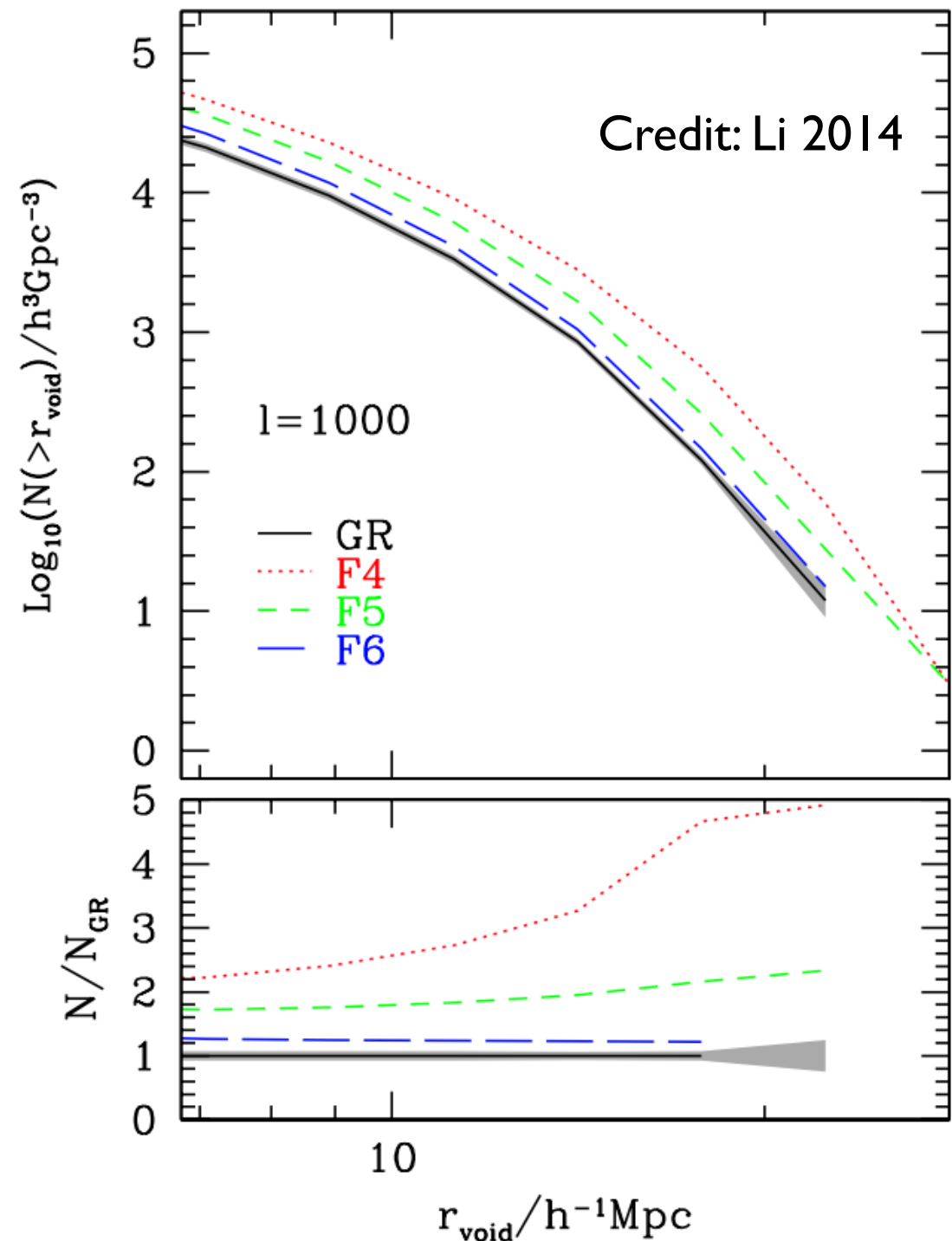
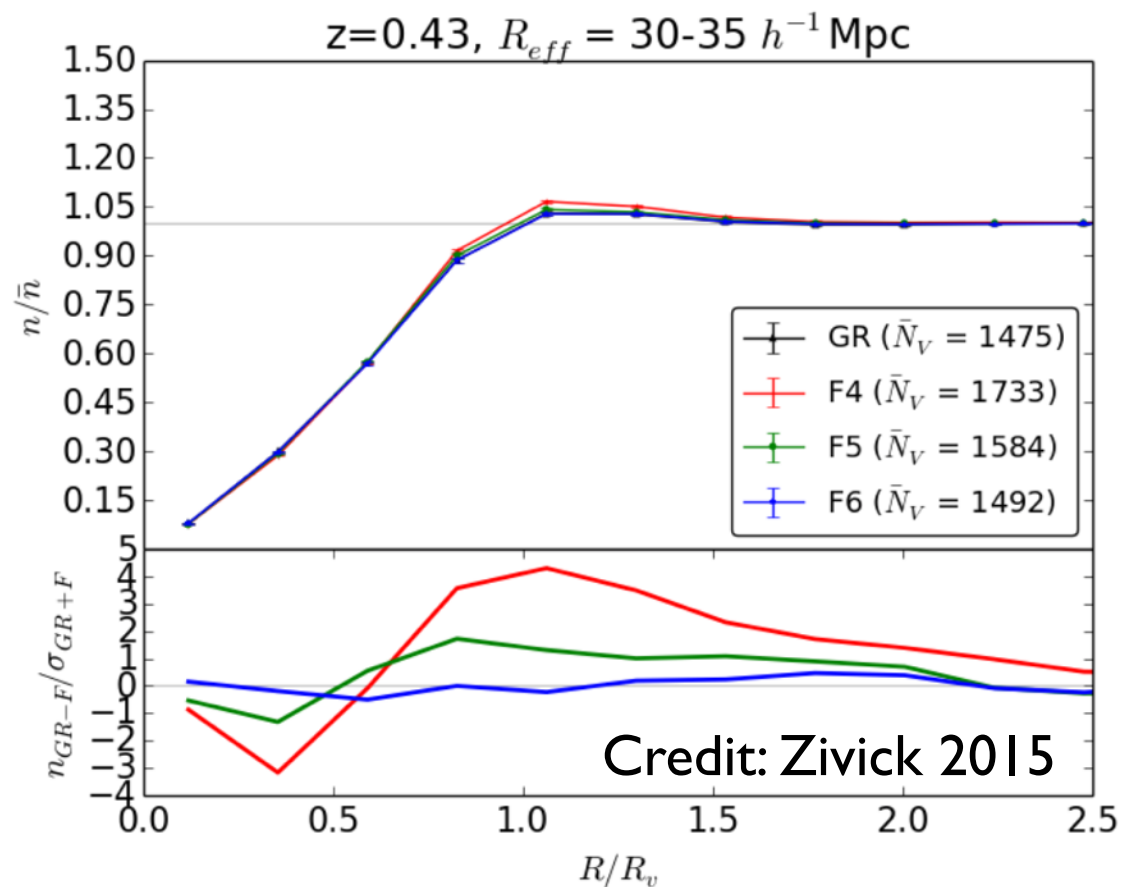
In lower density zones the effect of  
Modified Gravity should be different!

► Growth rate

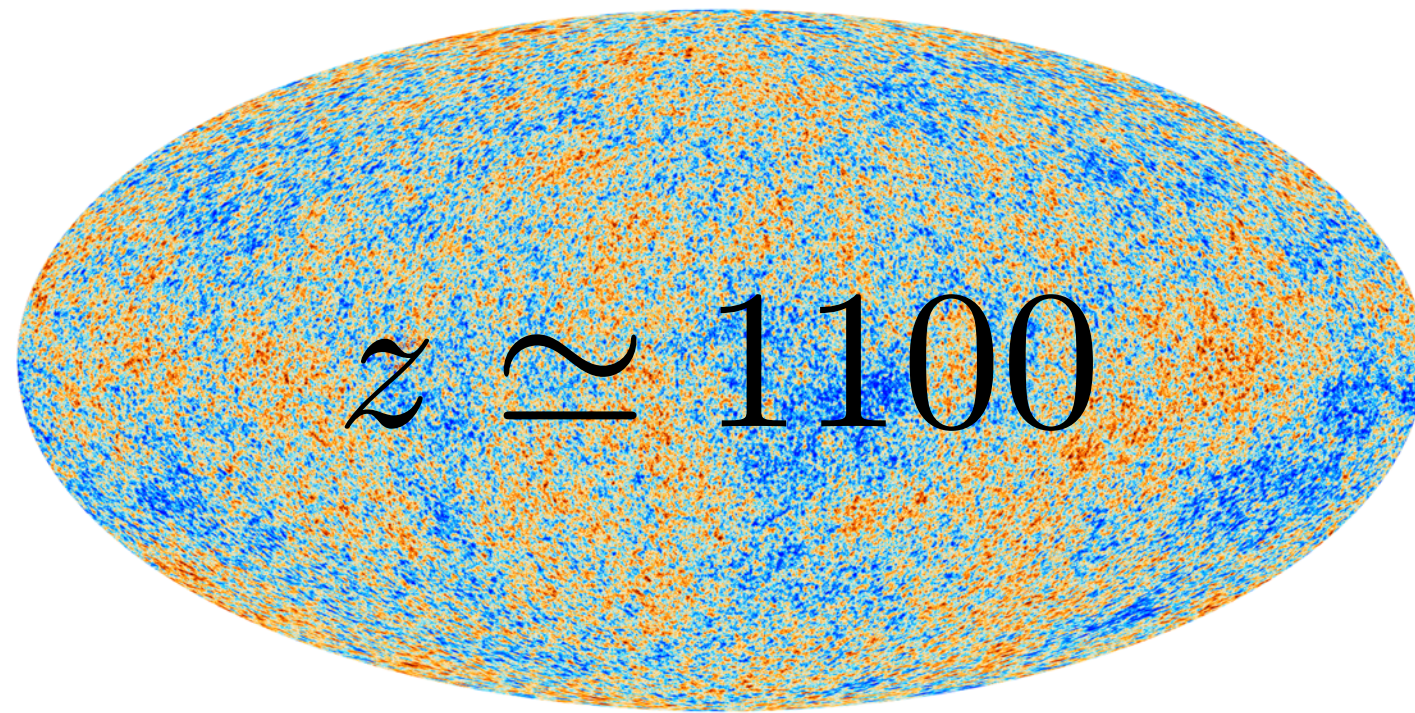


# Modified gravity, the LSS might be different

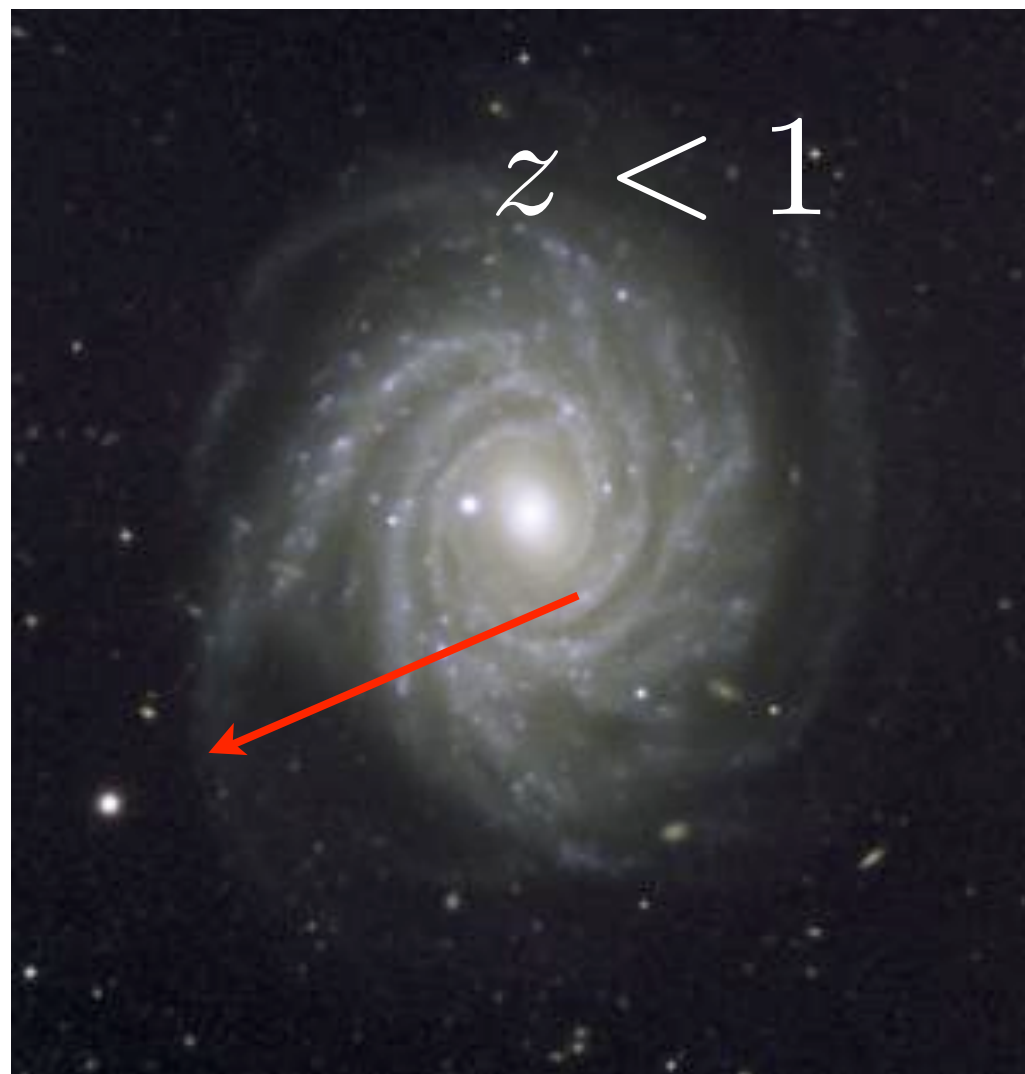
- Density profiles
- Void abundances



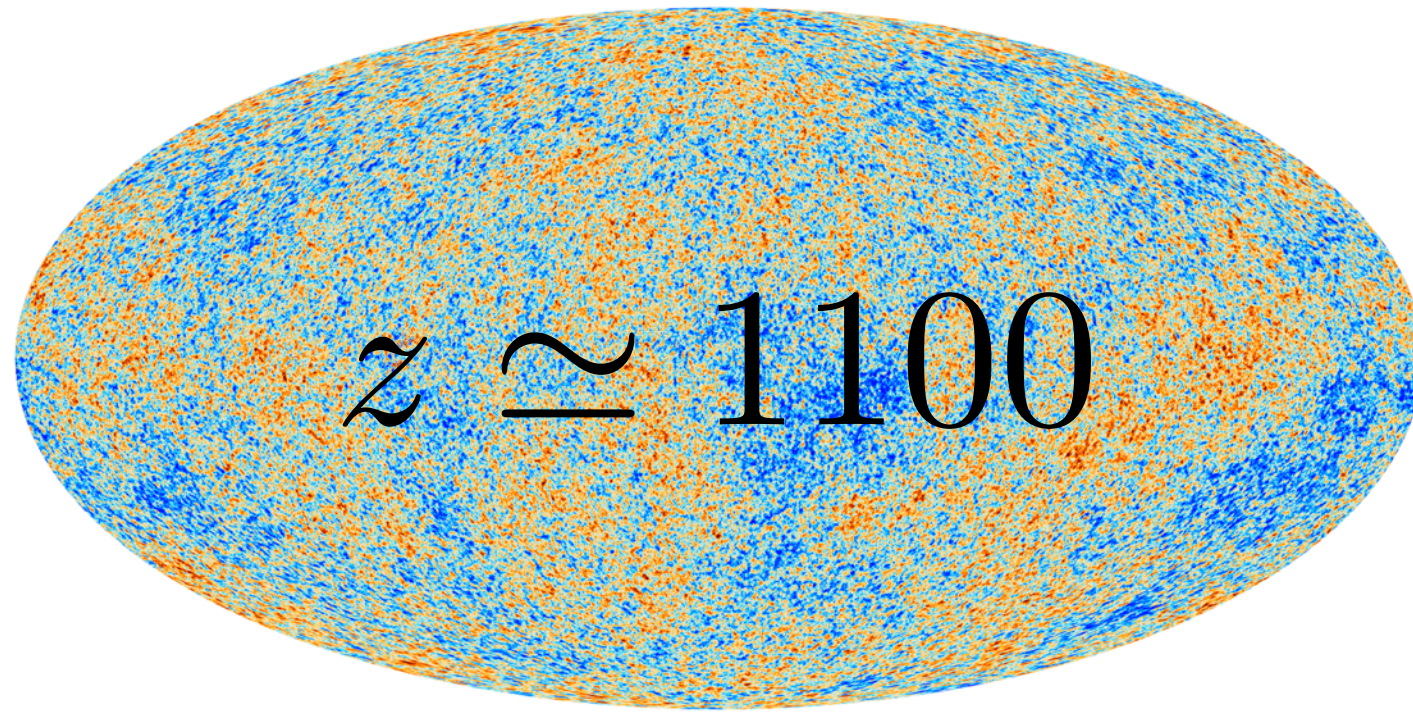
Analyze void properties to constrain cosmological models



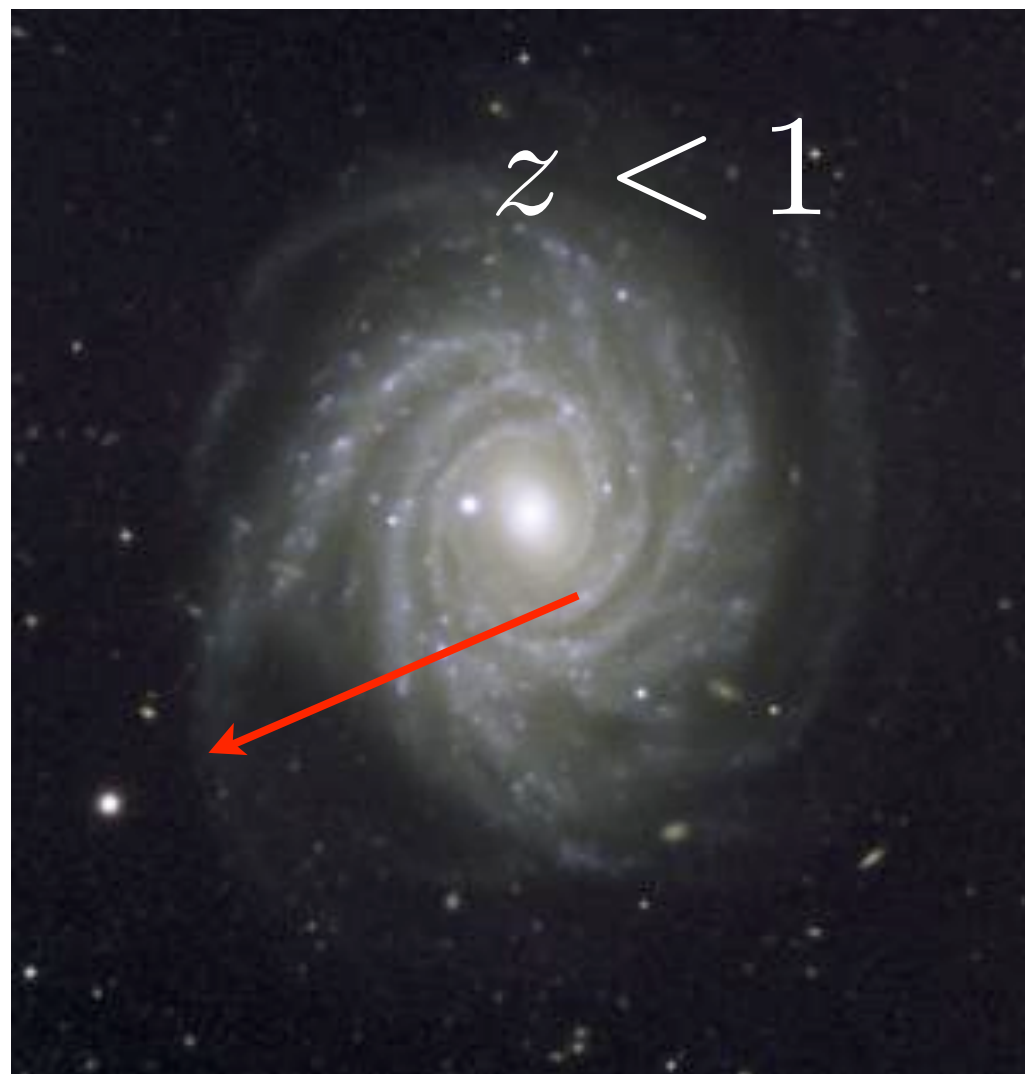
CMB anchor + late  
time measure of the  
expansion is the way  
to constrain dark  
energy







CMB anchor + late time measure of the expansion is the way to constrain dark energy



Voids (LSS) can give a measure at late time



# Studying voids gives a window on dark energy

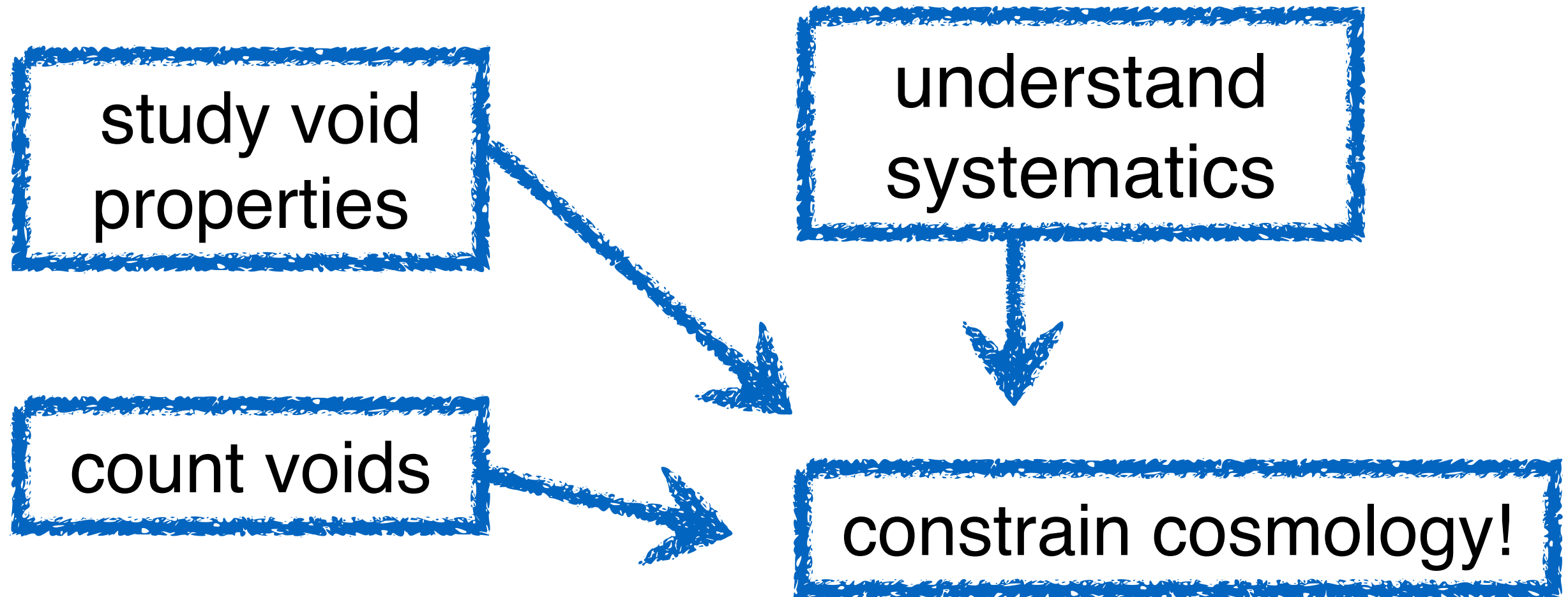
study void  
properties

understand  
systematics

count voids

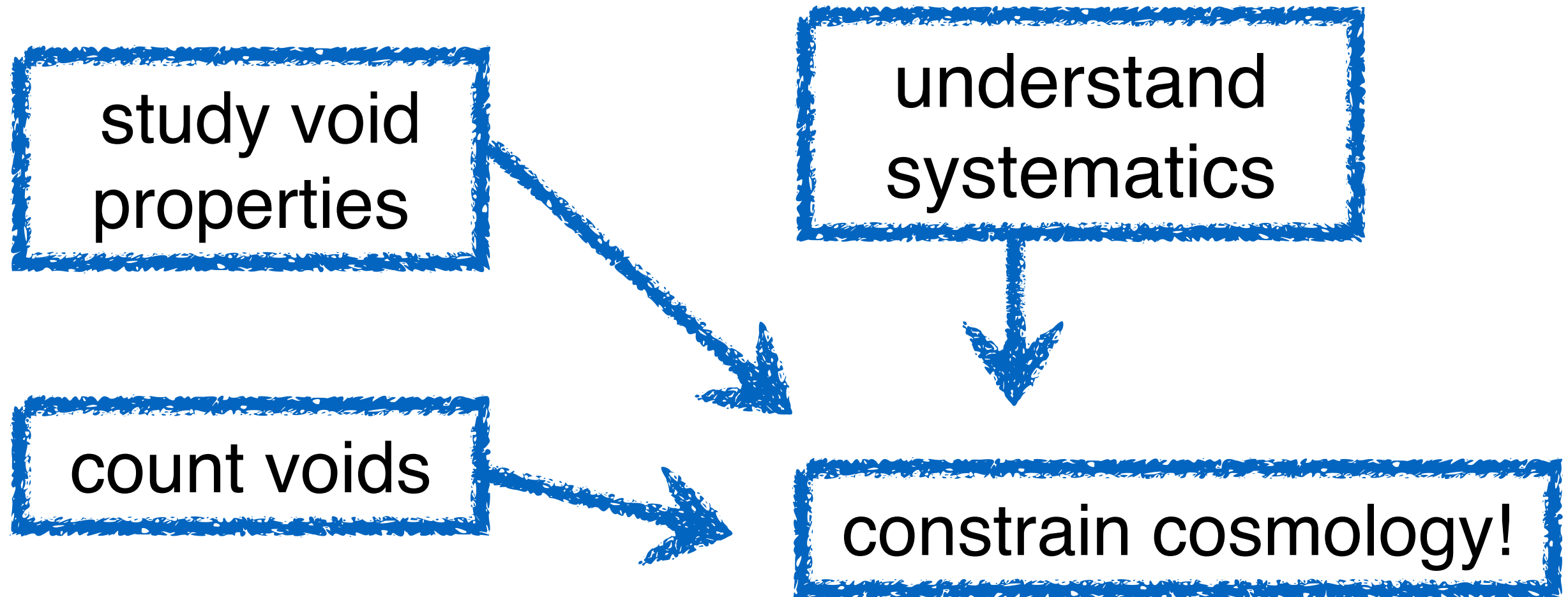


# Studying voids gives a window on dark energy





# Studying voids gives a window on dark energy



But first we need to ***find voids***

# Void IDentification and Examination

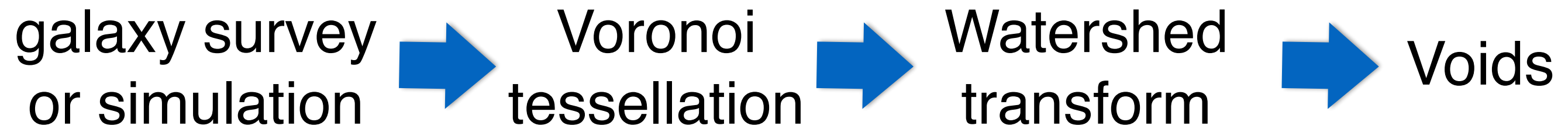
*Based on Zobov (Neyrinck 2008)*





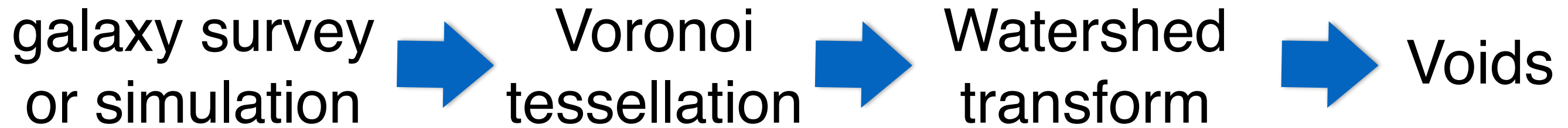
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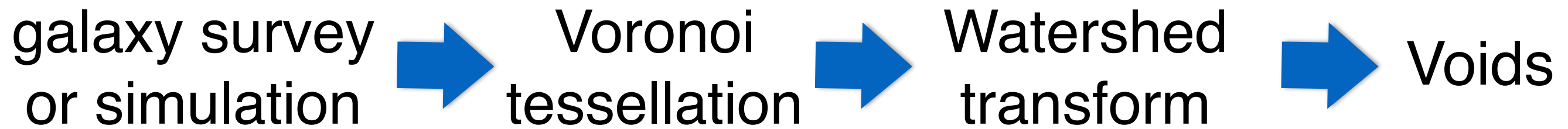
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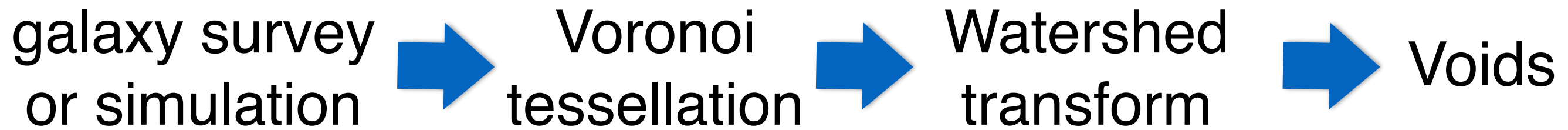


Tracer



# Void IDentification and Examination

*Based on Zobov (Neyrinck 2008)*



Tracer

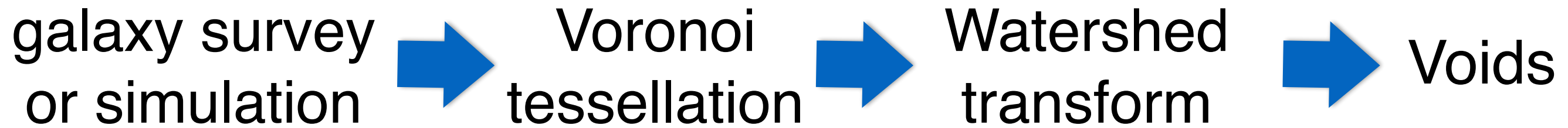
All points closer to the tracer than to any other point

$$\rho_{local} = \frac{1}{V_{cell}}$$




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*Based on Zobov (Neyrinck 2008)*



Tracer

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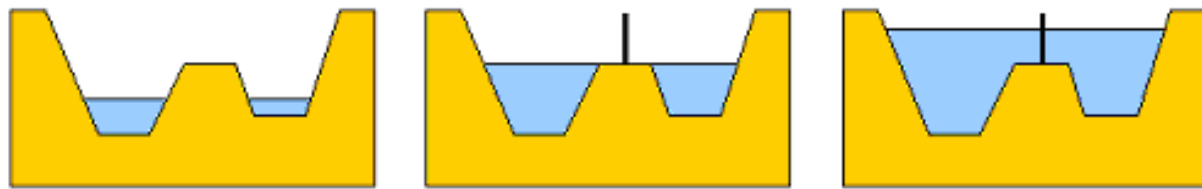

Cells merged into basins, which center is the cell only surrounded by higher density cells (local minima).

Icke & Van de Weygaert (1987)

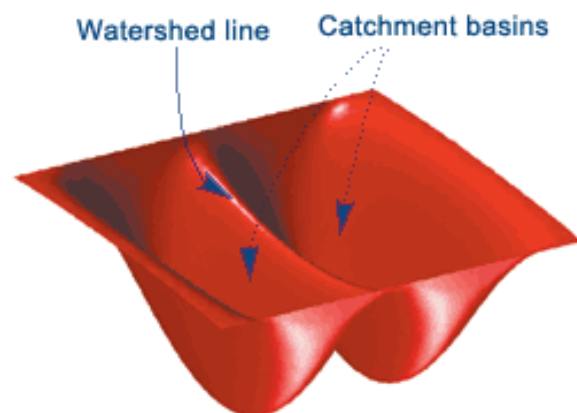
# Void Identification and Examination

*Based on Zobov (Neyrinck 2008)*

galaxy survey  
or simulation → Voronoi  
tessellation → Watershed  
transform → Voids



Each basin is a sub-void.  
Basins are merged in one  
void if, the border with lower  
density is common.

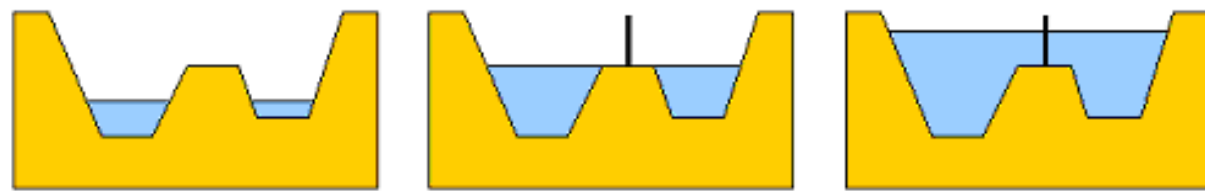




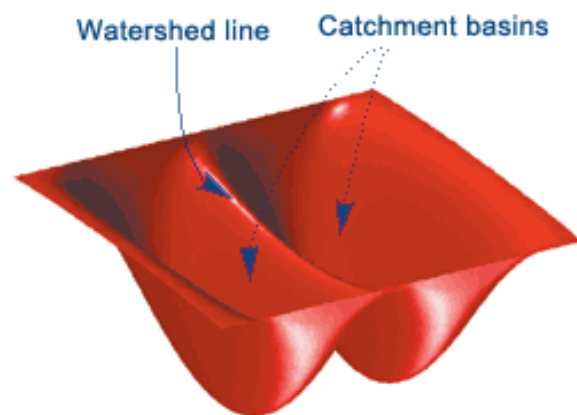
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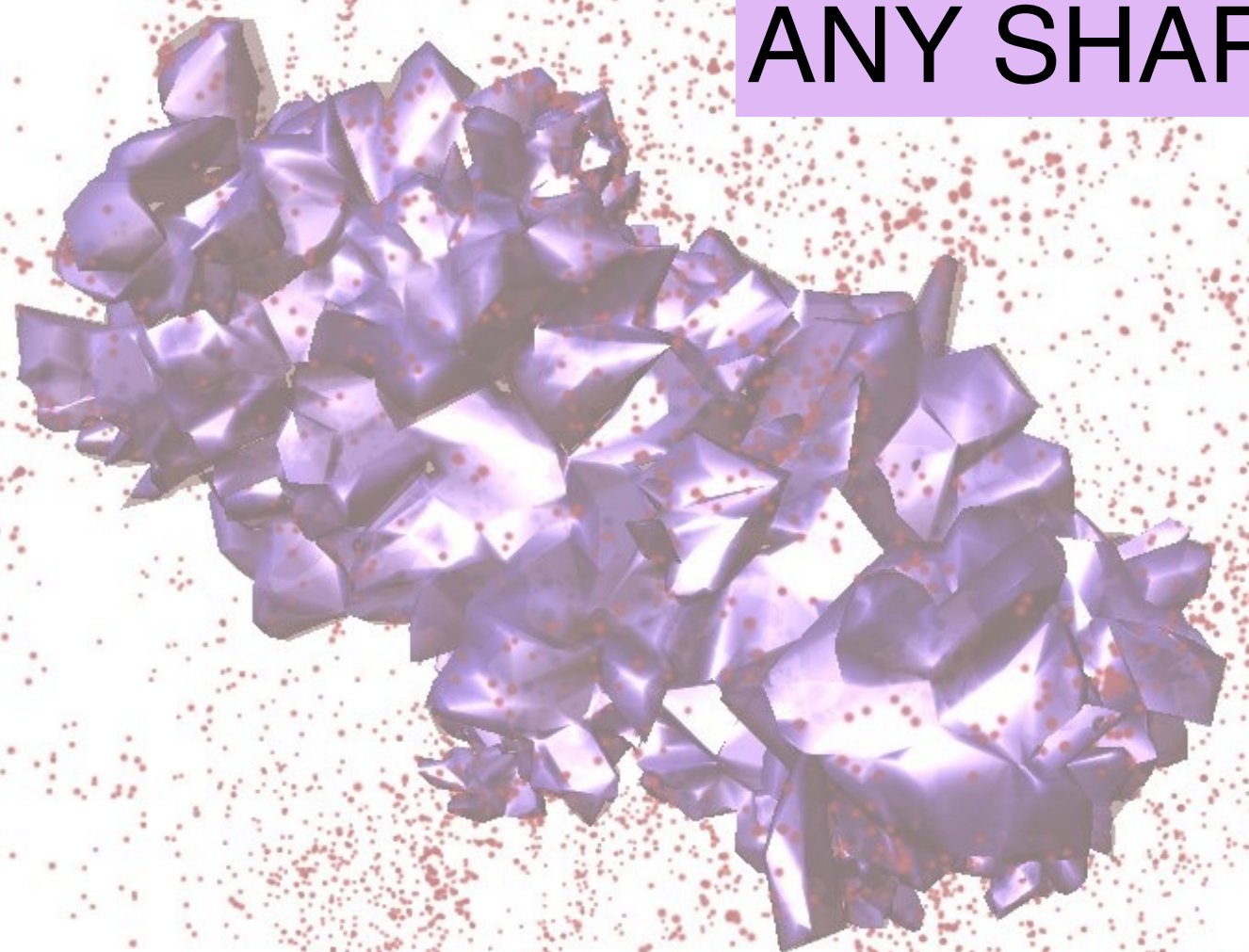


Density cuts:

- 1) merge if  
 $\rho_{\text{link}} < 0.2$   
 $\rho_{\text{mean}}$
- 2) density in  
 $R_{\text{eff}}/4 < -0.8$

+ exclude voids  
below mps

ANY SHAPE



*Credit: Sutter et al. 2012*

+ it takes into account survey  
boundaries and masks

We have voids,  
how can we measure  
expansion with them?



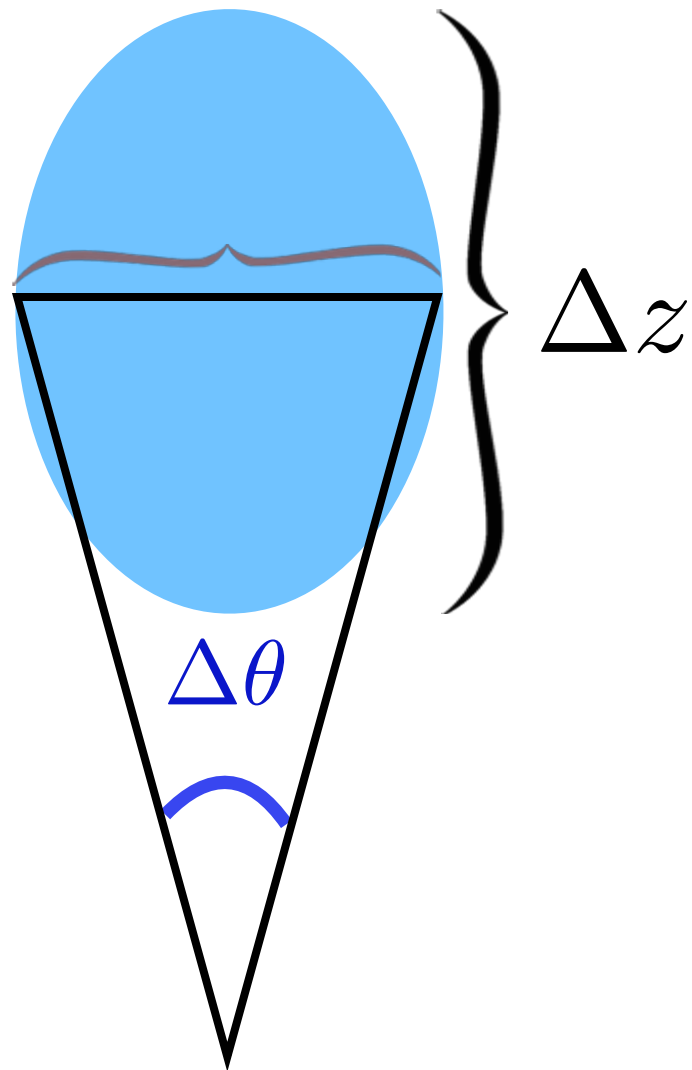
# Standard objects



Known  
luminosity

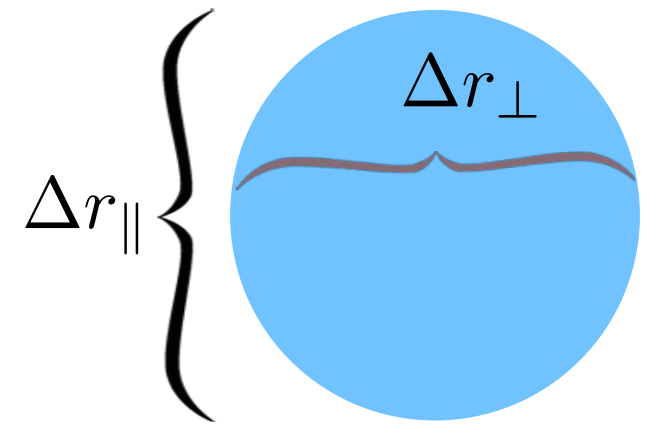
$$\frac{\text{Known ratio:}}{\text{radial size}} = \text{angular size}$$

What we measure:



Relation?

Real length scales:



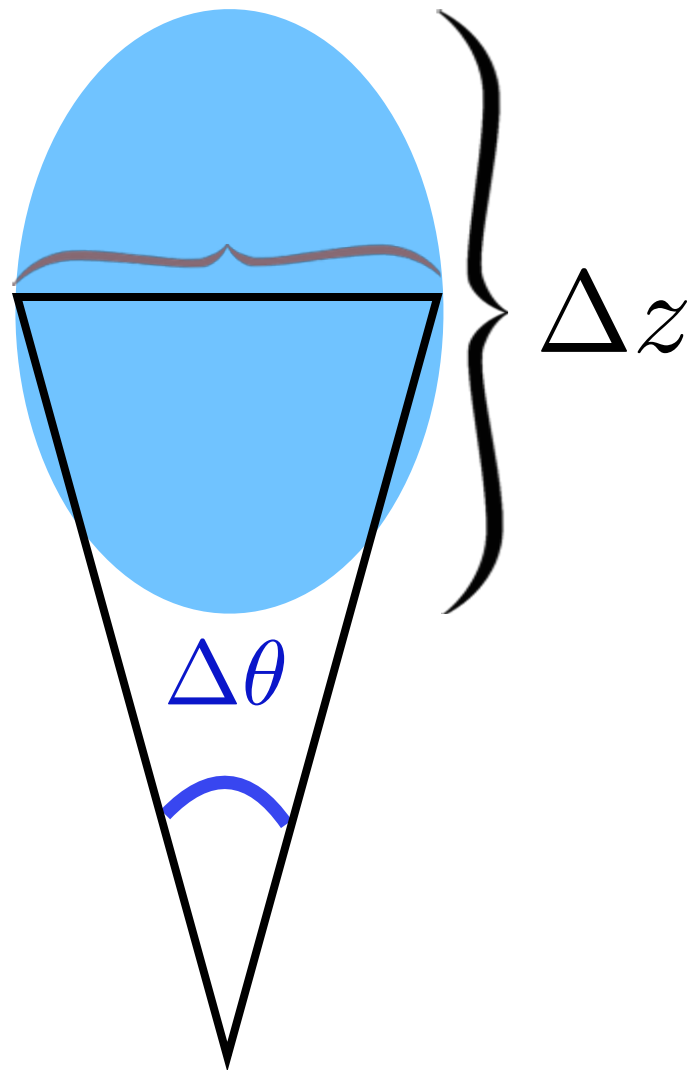
*physical sizes of the object*

$\Delta r_{\perp}$  *in the transverse direction*

$\Delta r_{\parallel}$  *in the longitudinal direction*

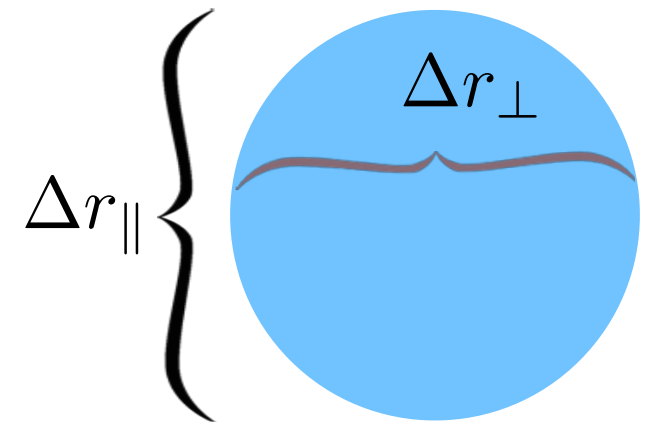


What we measure:



Relation?

Real length scales:



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$\Delta r_{\perp}$  *in the transverse direction*

$\Delta r_{\parallel}$  *in the longitudinal direction*

Cosmology, of course...

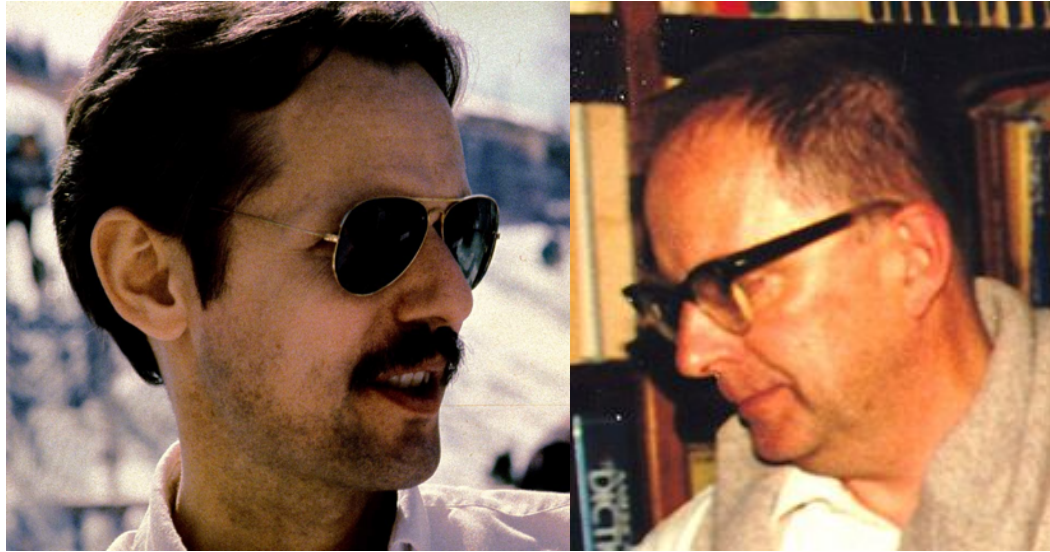
$$\Delta r_{\perp} = D_A(z) \Delta\theta$$

*angular diameter  
distance*

$$c\Delta z = H(z) \Delta r_{\parallel}$$

*Hubble parameter*

# Alcock-Paczynski test (1979)

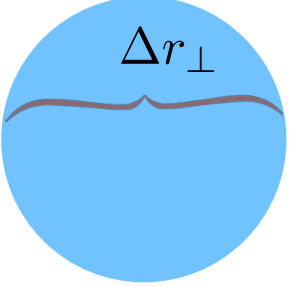




# Alcock-Paczynski test (1979)



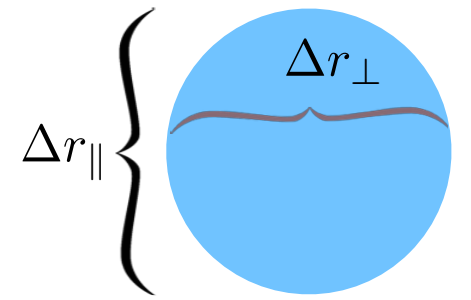
$$\Delta r_{\perp} = \Delta r_{\parallel}$$

$\Delta r_{\parallel}$  { 

# Alcock-Paczyński test (1979)



$$\Delta r_{\perp} = \Delta r_{\parallel}$$



*what we  
know*

$$\frac{c\Delta z}{\Delta\theta} = D_A(z)H(z)$$

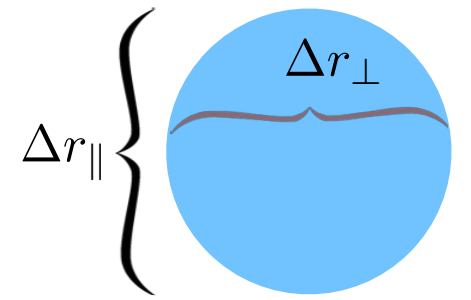
*what we  
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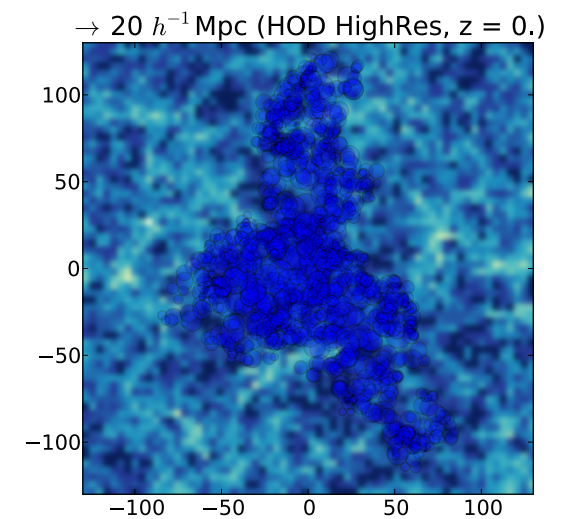
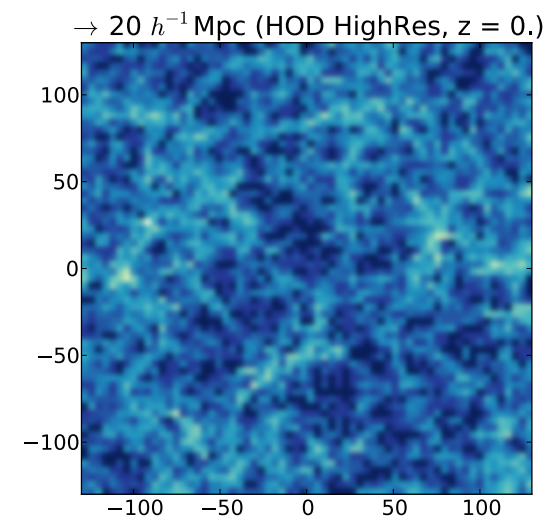
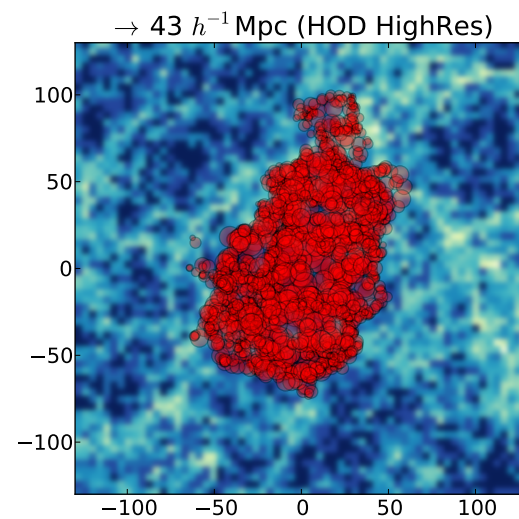
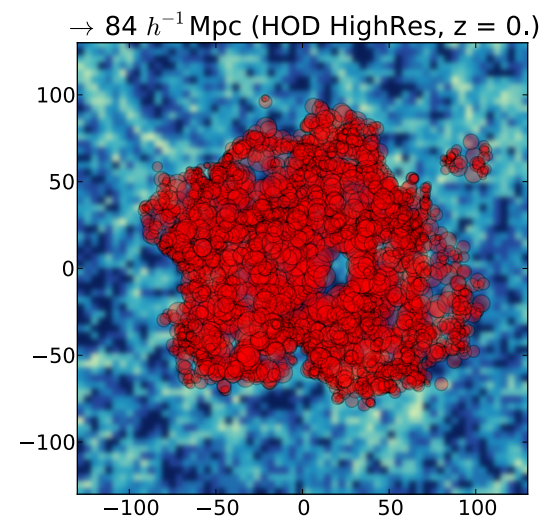
To perform the test we measure stretch  $e_V(z)$

The deviations from fiducial cosmology  
cause geometrical distortions.



1995

# Barbara Ryden intuition: apply the Alcock-Paczyński test on voids

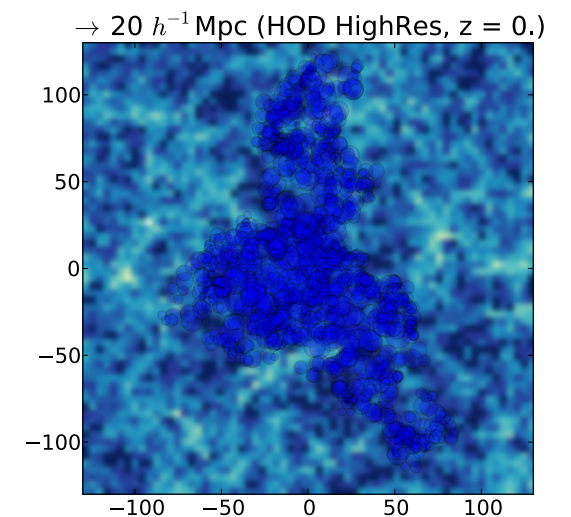
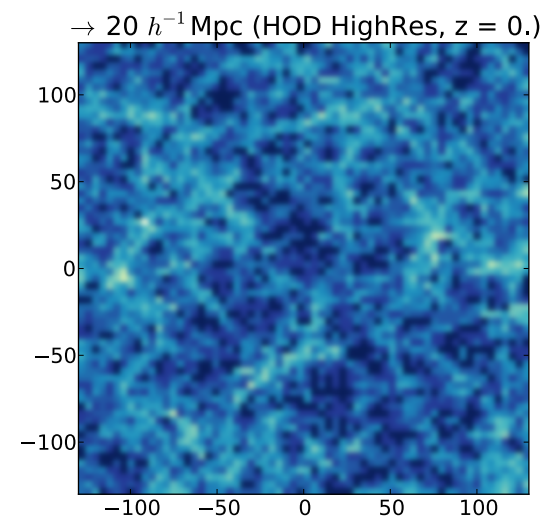
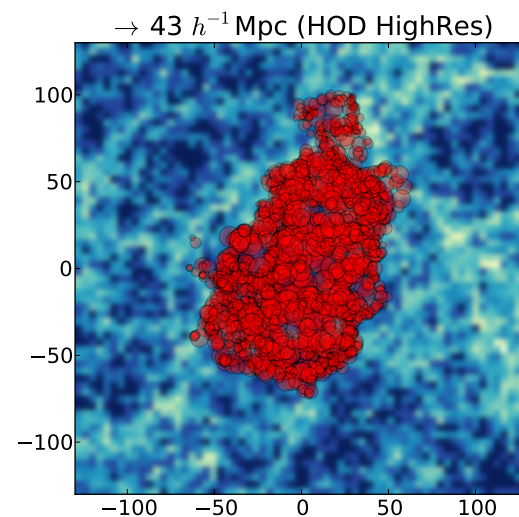
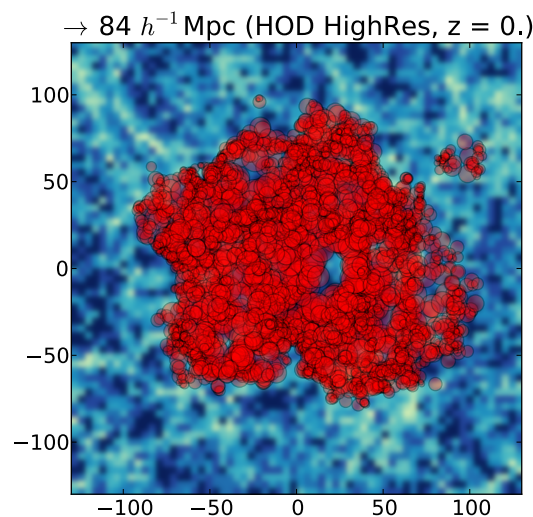






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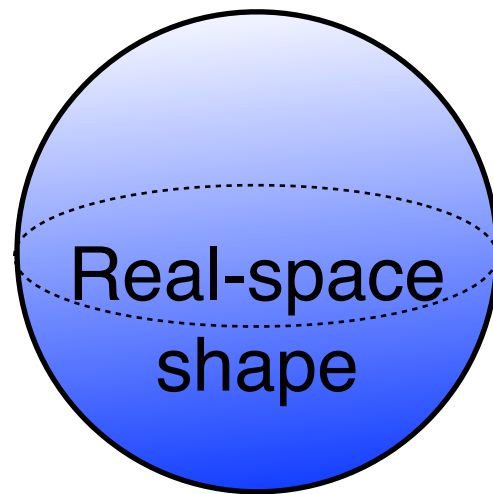
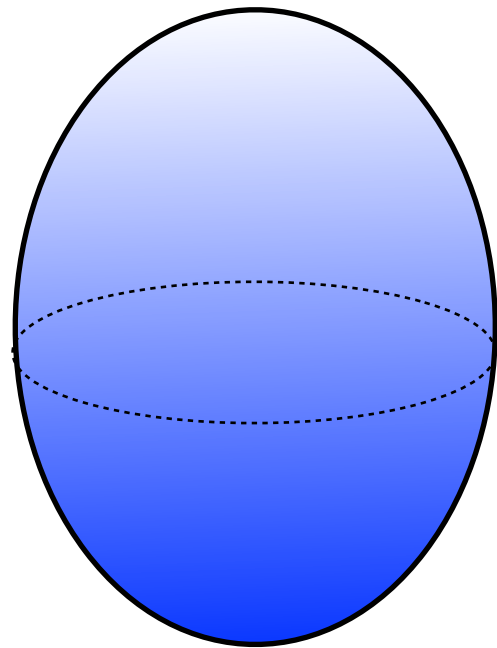


Voids have different shapes but **spherical** average shape in an isotropic and homogeneous universe!

We can use **stacked voids** for the test  
=> promising with new surveys.



# The void shape tells us the cosmology



$$cz = H_0 d + v \cos \theta$$



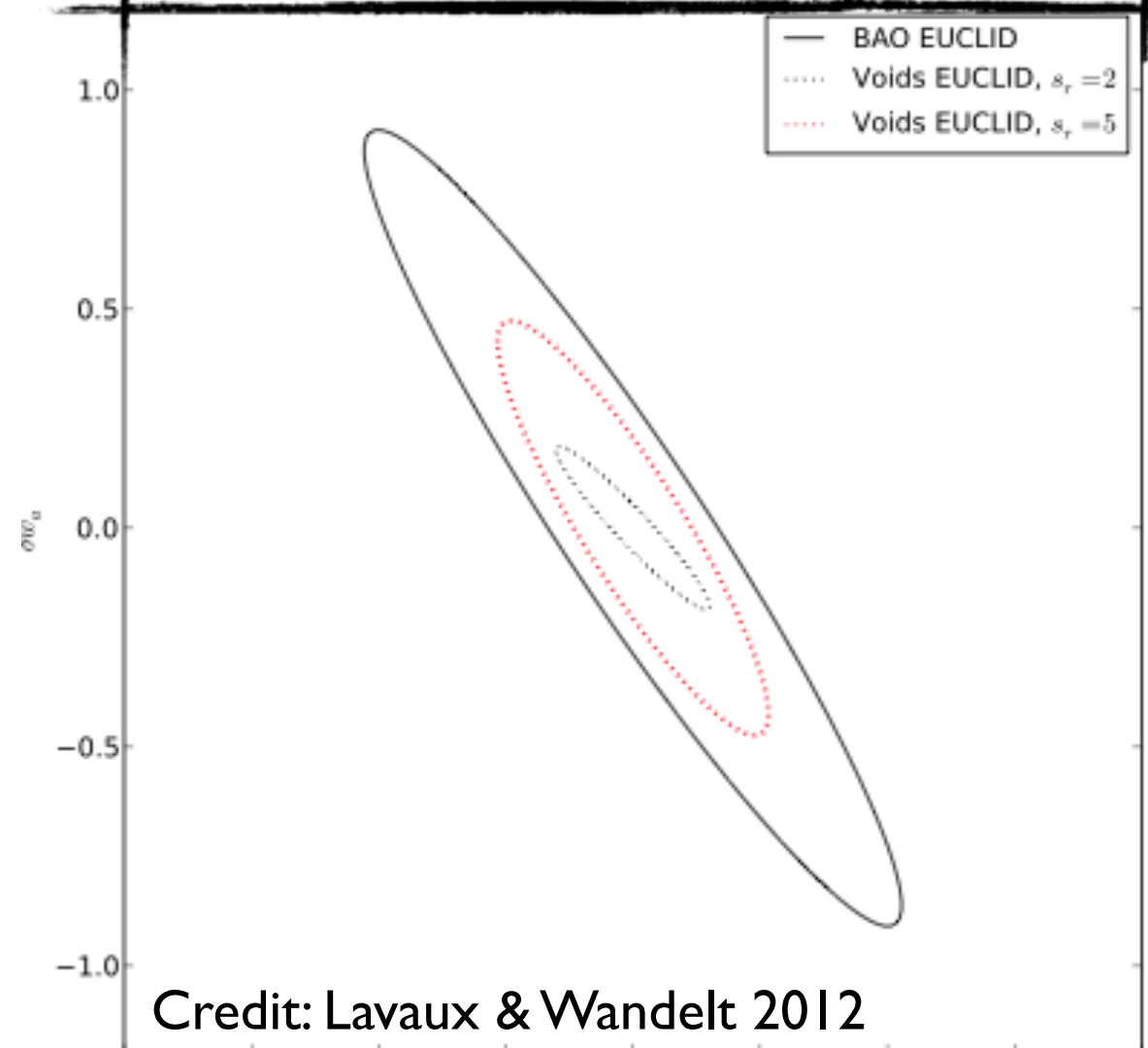
$z_{\text{measured}}$

$z_{\text{cosmo}}$

*But... the velocities of galaxies also affect that shape.*

**Velocities will be our main source of systematics!**

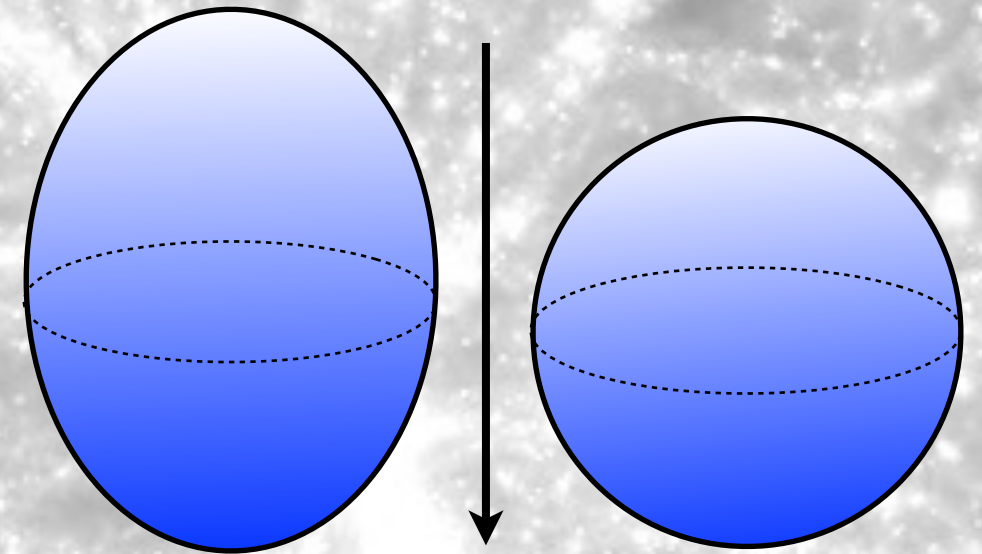
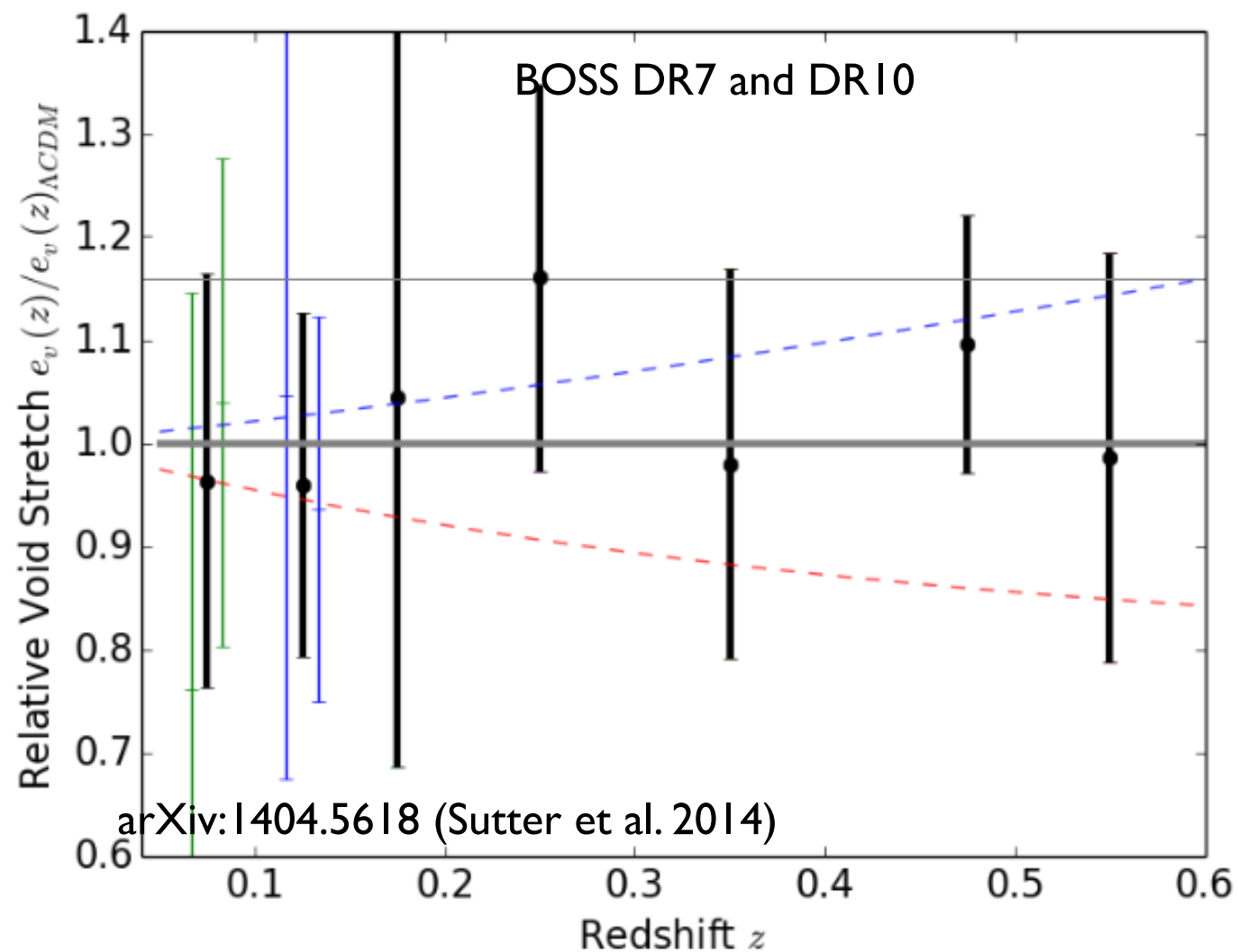
AP test with void stacks has the potential to beat BAO in constraining DE





Current situation?

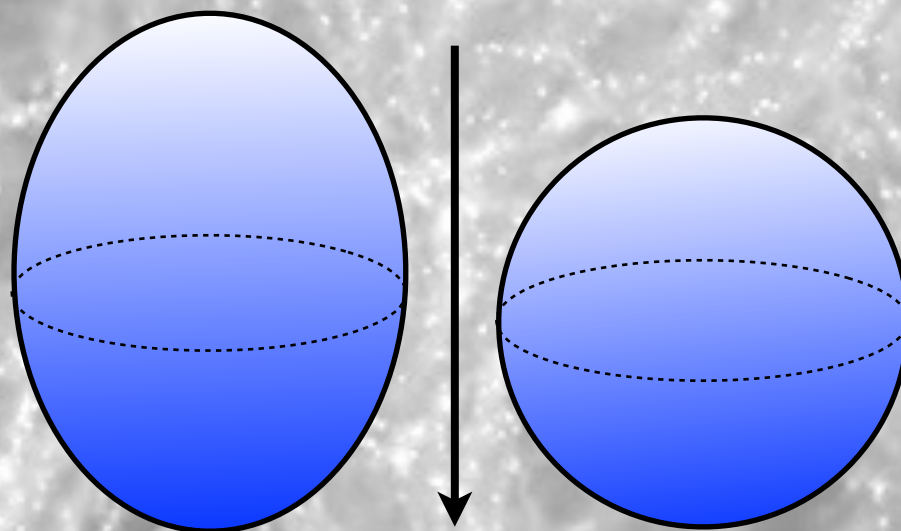
We have a measurement!



How to turn  
this into a  
precise  
constraint?

We can reduce systematics by:

- 1) better modeling of the real space shape
- 2) studying the effect of peculiar velocities

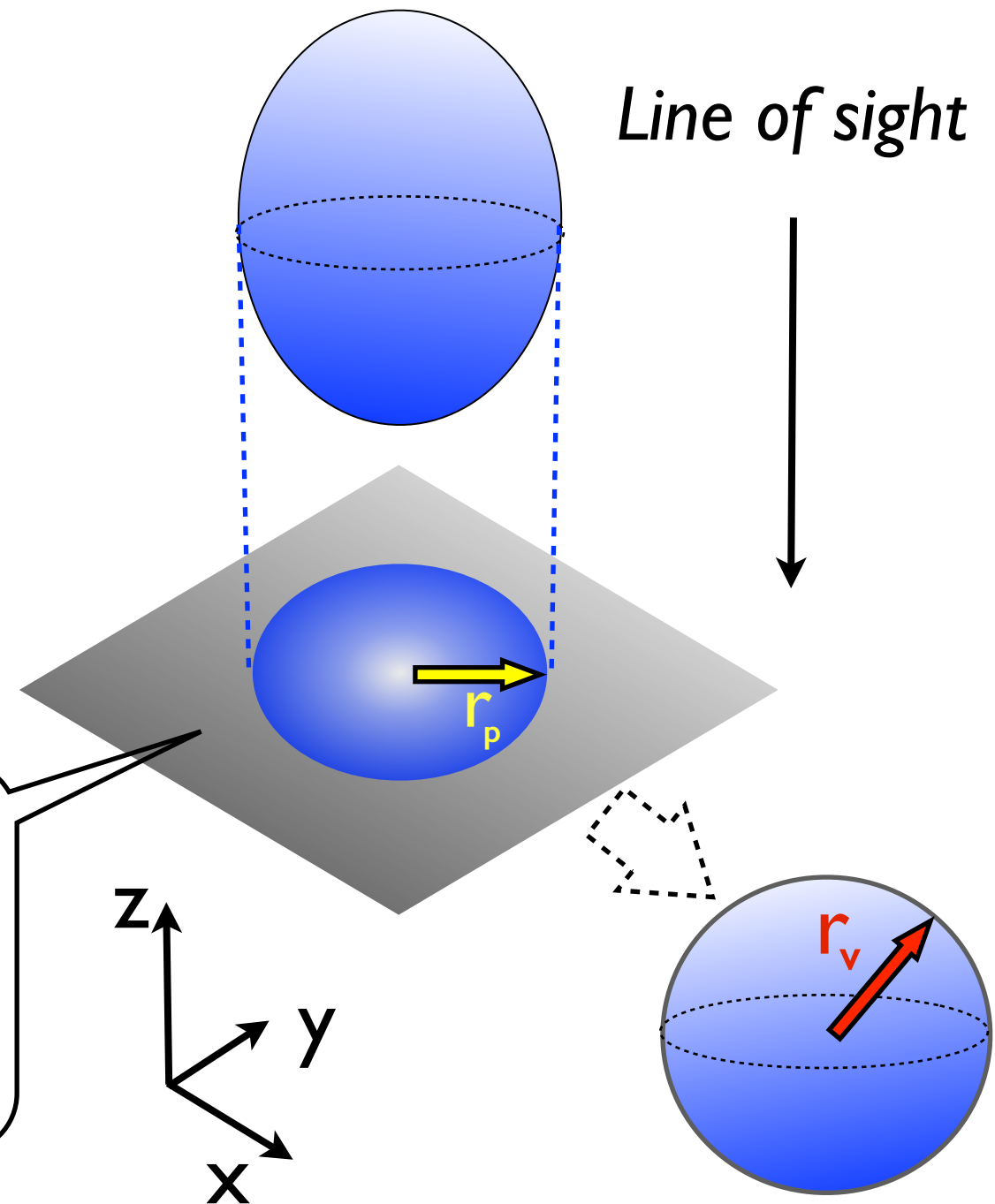


# The method to get the spherical profile

## Key idea

Projecting the 3D distribution along the line of sight, the contribution of peculiar velocities disappears.

From this projection we reconstruct a 3D profile without the contribution of peculiar velocities.

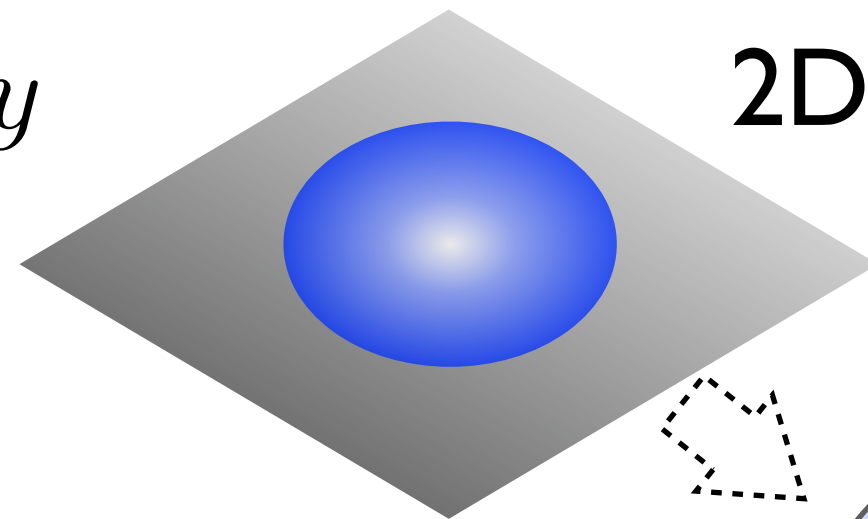


We can obtain the SPHERICAL density profile of stacked voids in real space.

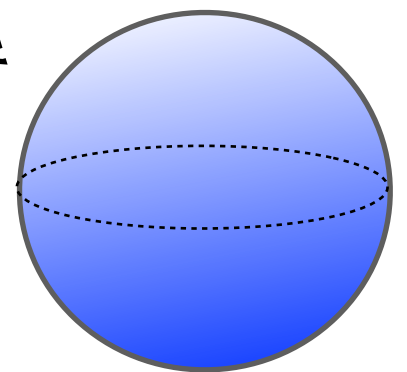


# The Abel inverse transform

$$g(r) = -\frac{1}{\pi} \int_r^1 \frac{I'(y)}{\sqrt{y^2 - r^2}} dy$$



3D



To test the reconstruction we need a class of functions for which the inverse is known: Abel Pairs

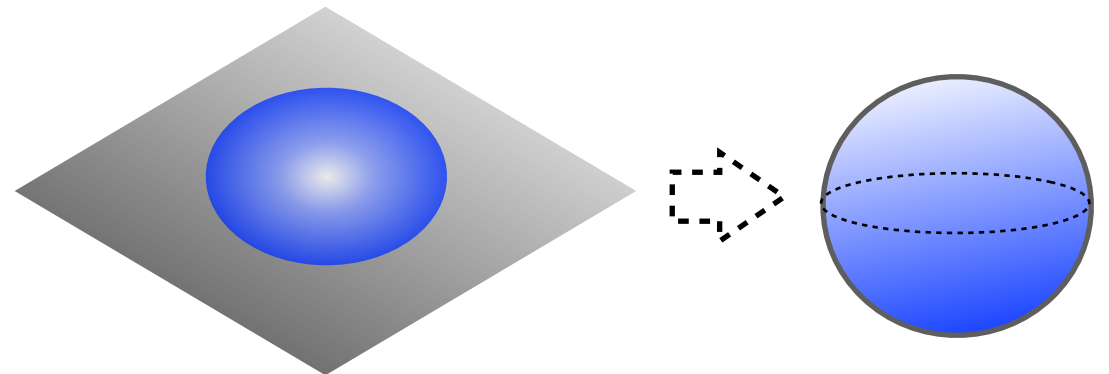
But...

# Result I

## Fighting ill-conditioning

$$g(r) = -\frac{1}{\pi} \int_r^1 \frac{I'(y)}{\sqrt{y^2 - r^2}} dy$$

Abel inverse transform:  
mathematically well-defined  
but **ill-conditioned**!

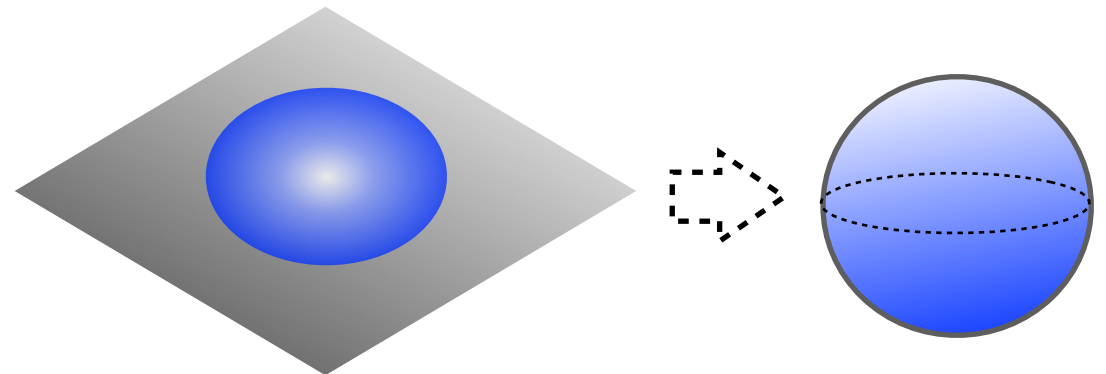
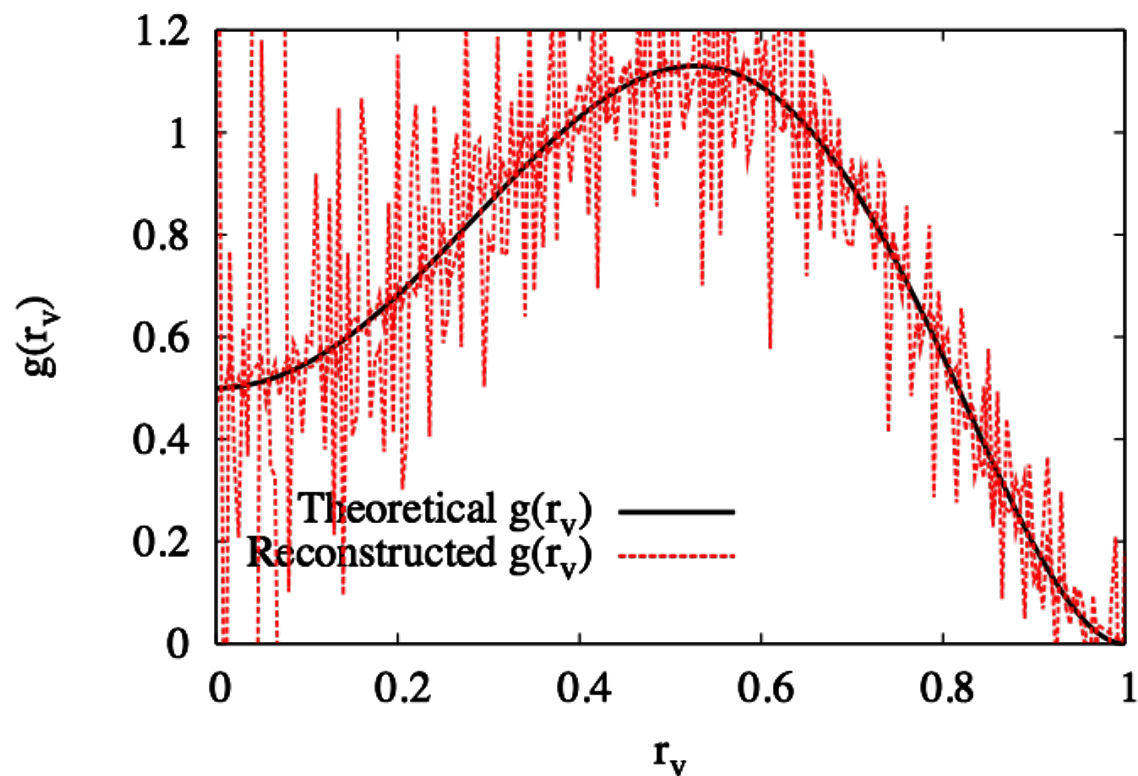


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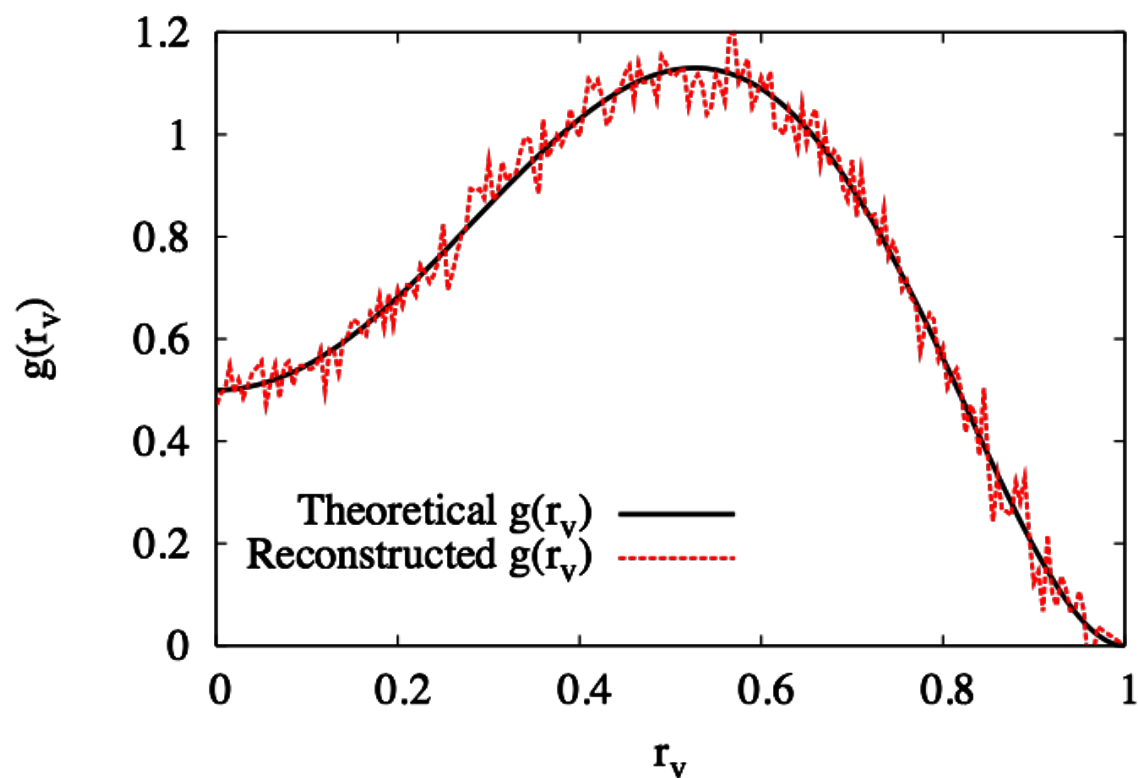


# Result I

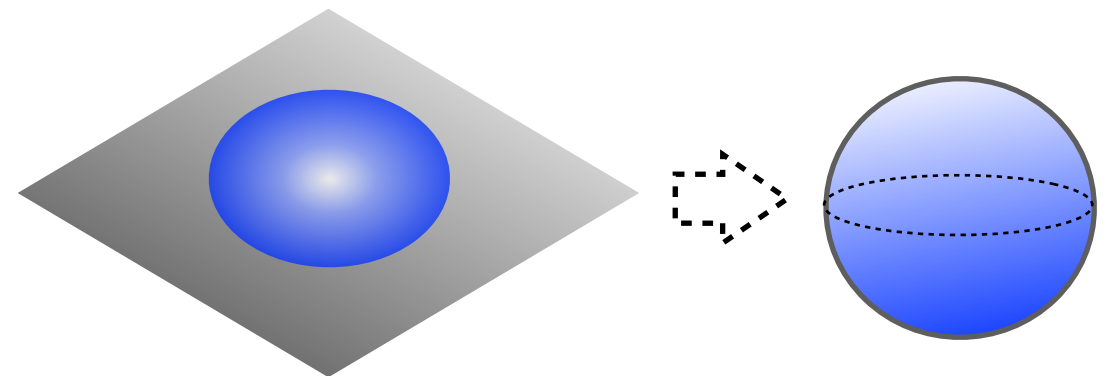
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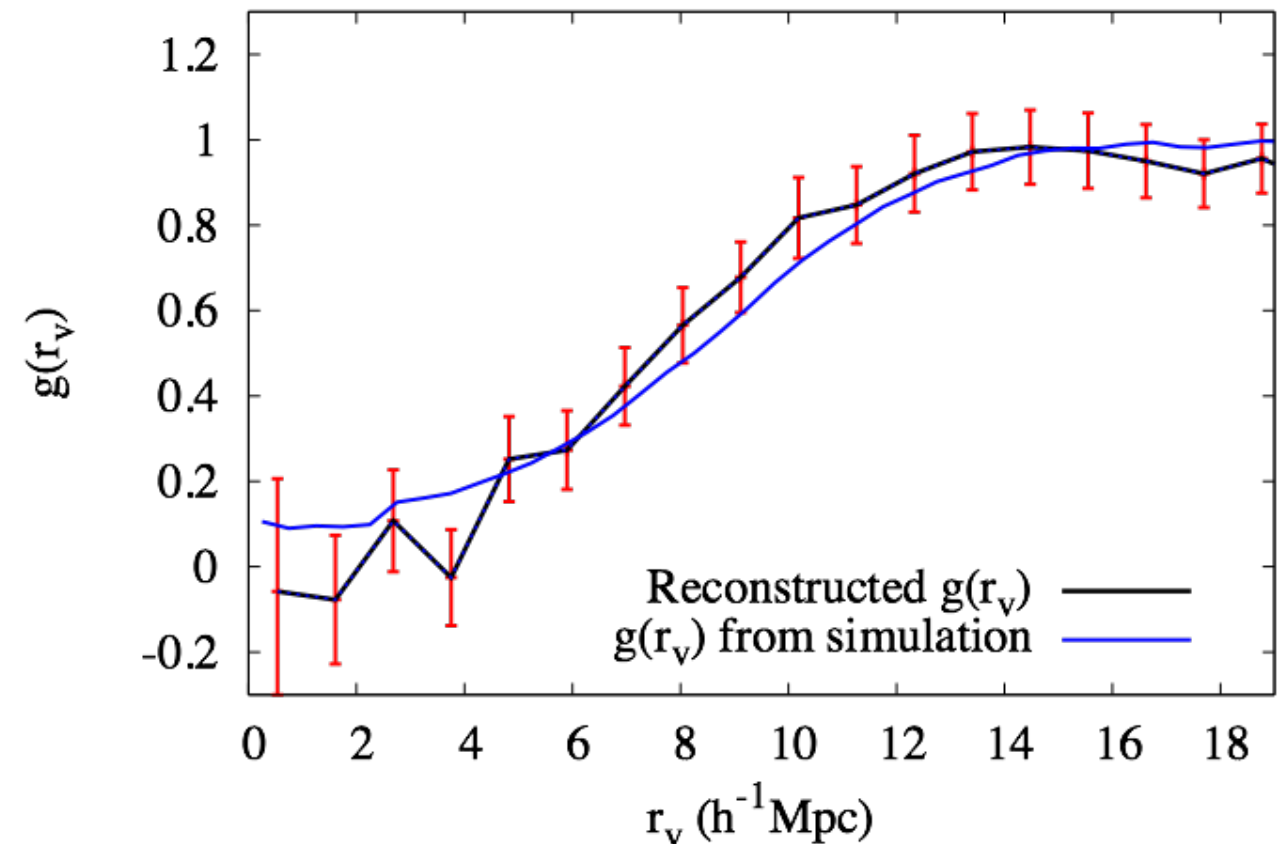
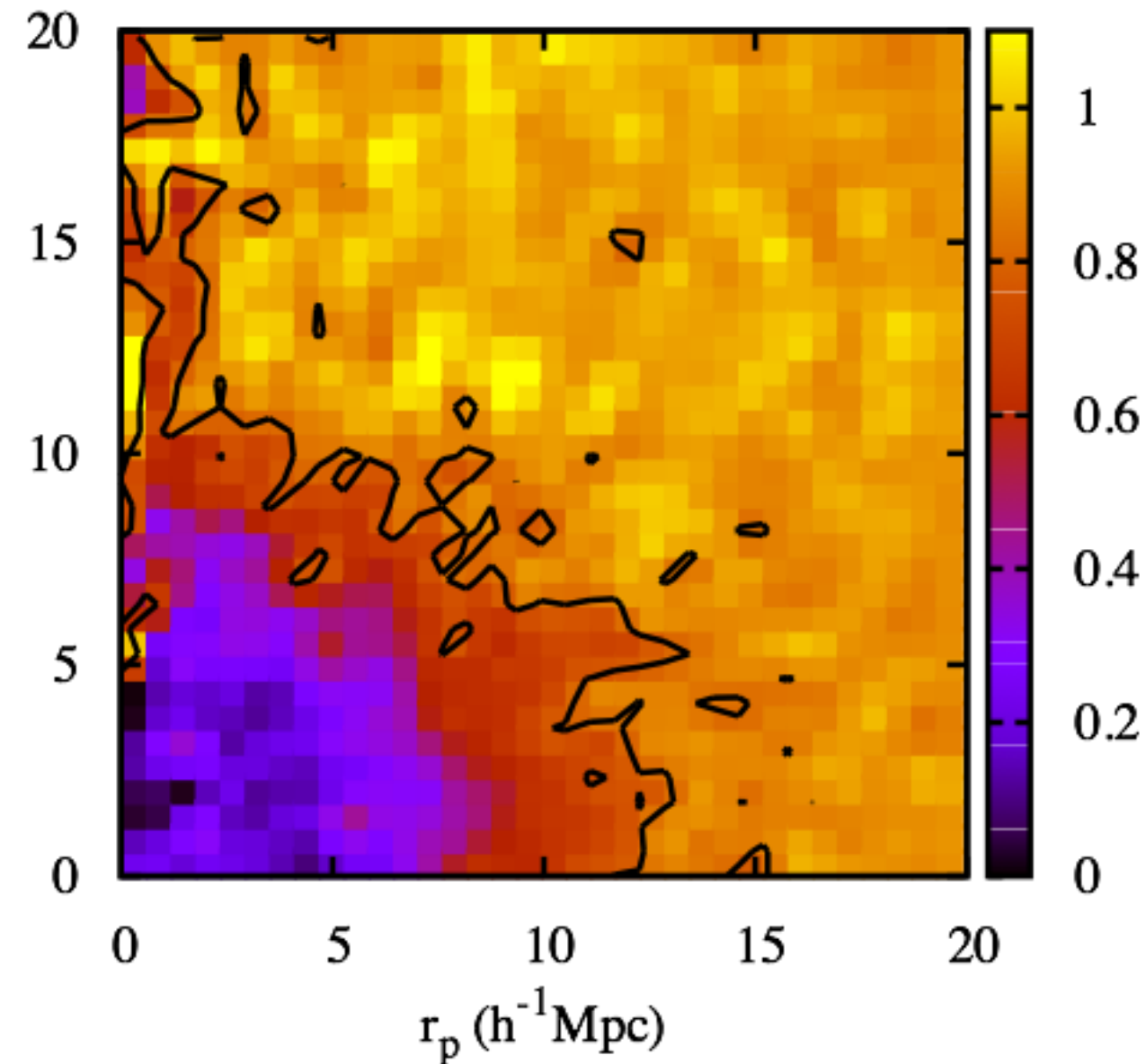
Reconstruction was with an Abel pair,  
so it is a particular case



**RESULT:**  
**Very good**  
**reconstruction!**

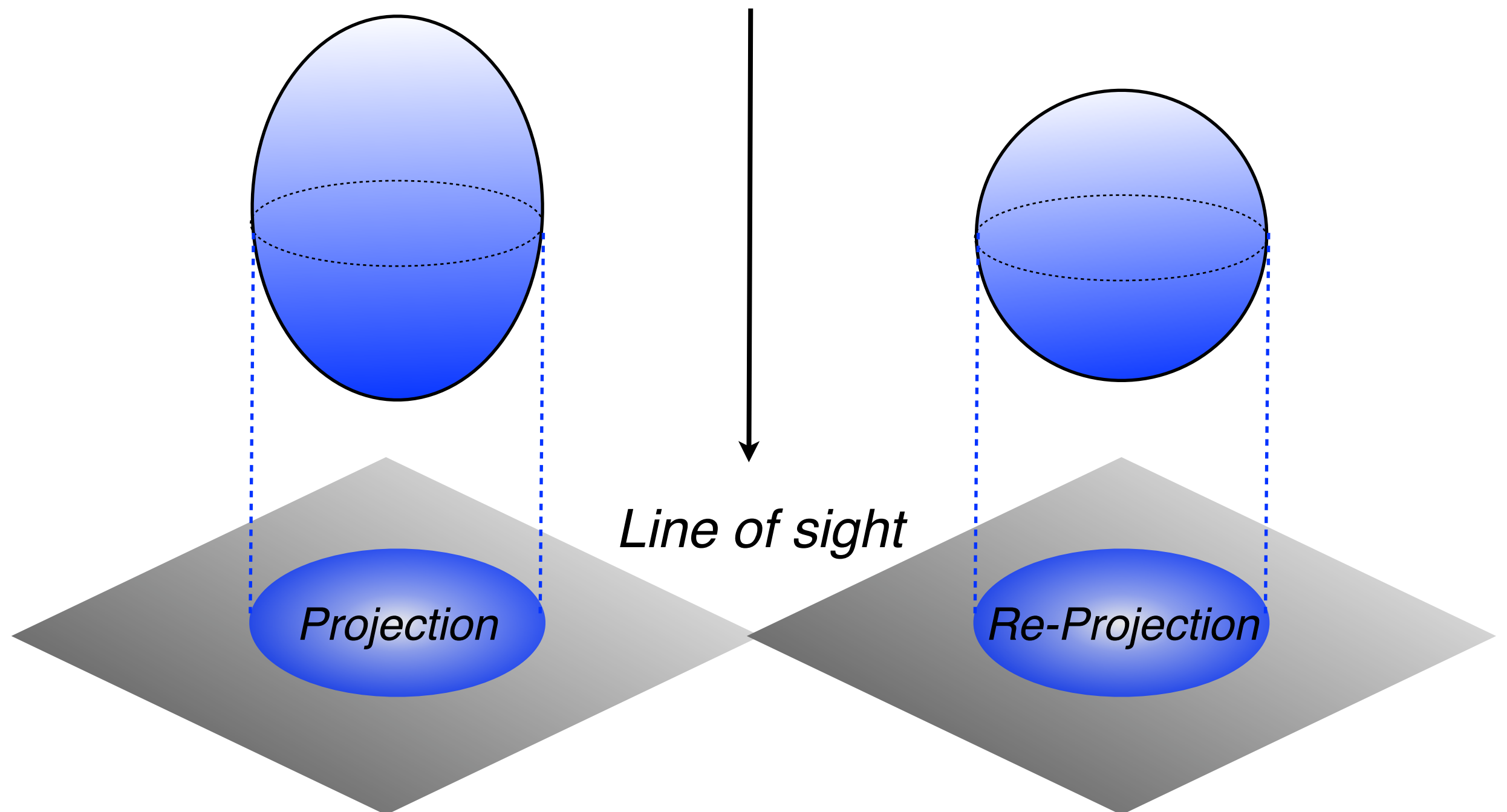
## Result II

## The full simulated stacked void



*Stacking from 10 to 12 Mpc/h*

# The sanity check for the reconstruction



Check the reconstruction



# Reconstruction from stacked void with HOD model

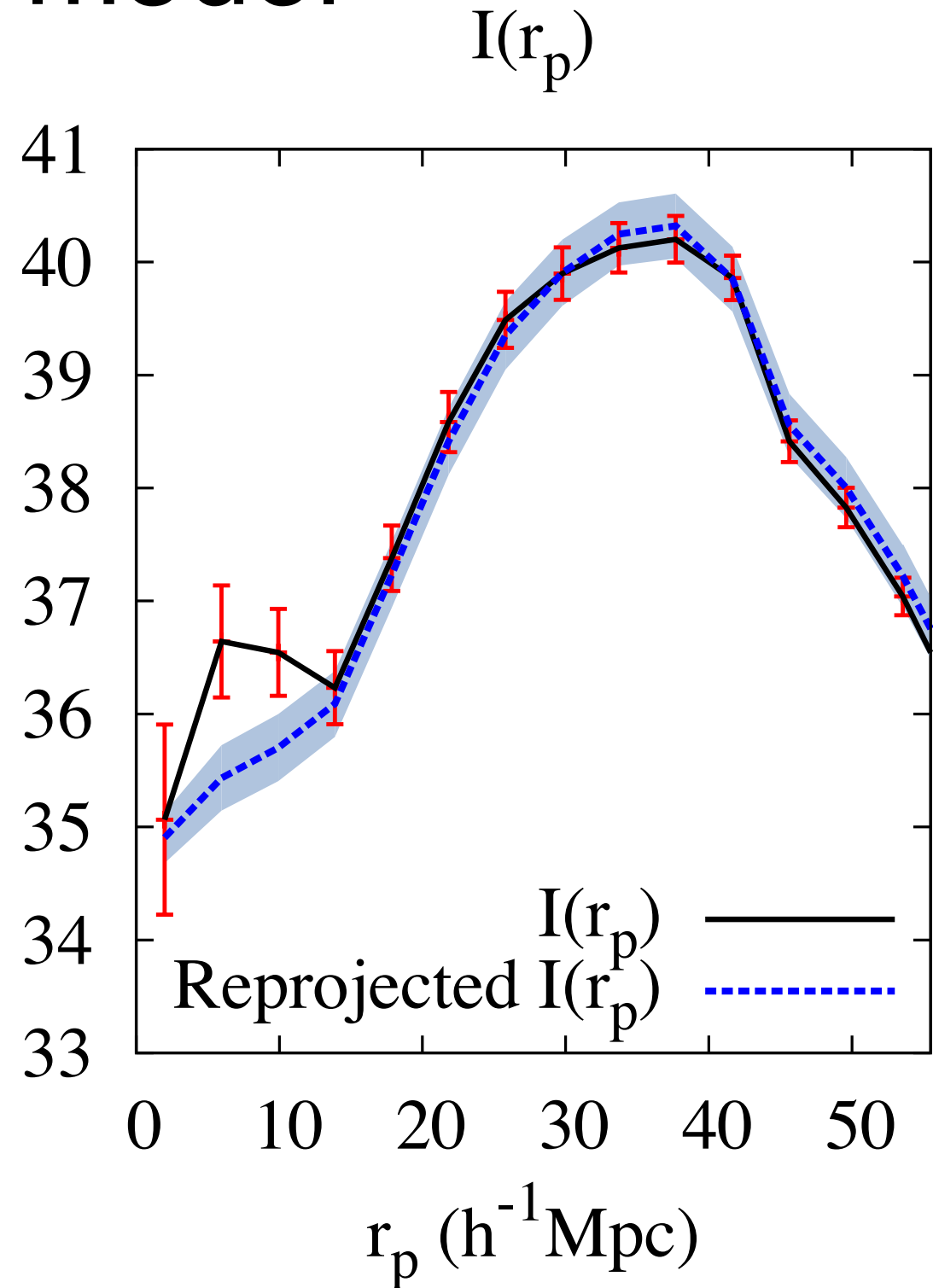
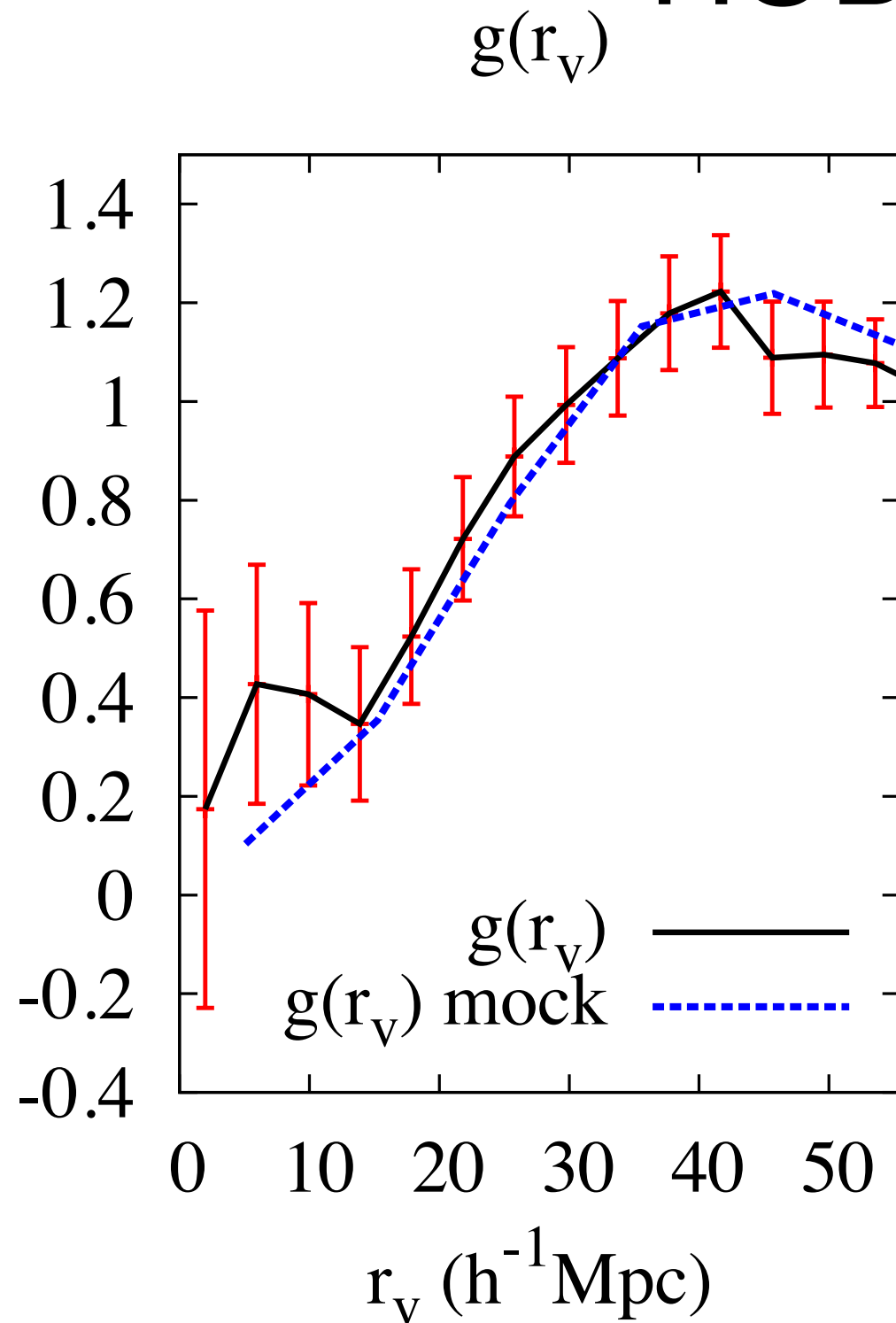
$$\langle N_{\text{cen}}(M) \rangle = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right]$$
$$\langle N_{\text{sat}}(M) \rangle = \langle N_{\text{cen}}(M) \rangle \left( \frac{M - M_0}{M'_1} \right)^\alpha$$

Rockstar halo finder  
(Behroozi et al. 2013)

+ HOD model assigns central and satellite  
galaxies to a dark matter halo (Zheng et al. 2007)

## Matching the features of SDSS DR7

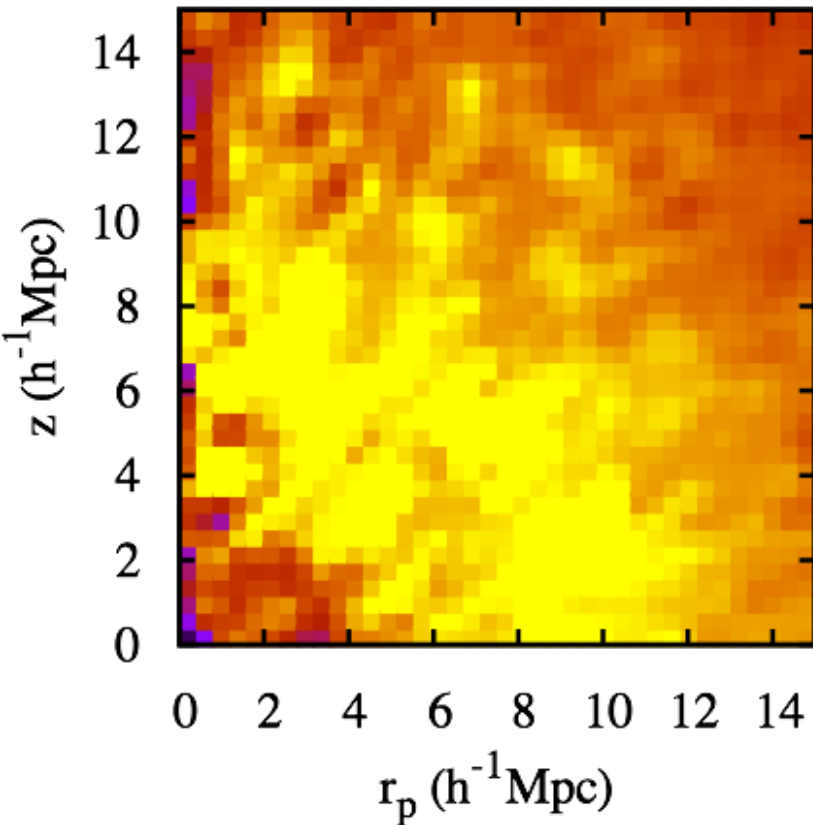
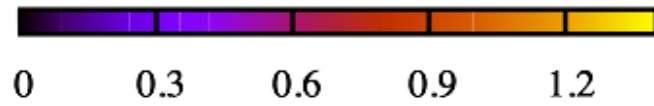
# Reconstruction from stacked void of HOD model



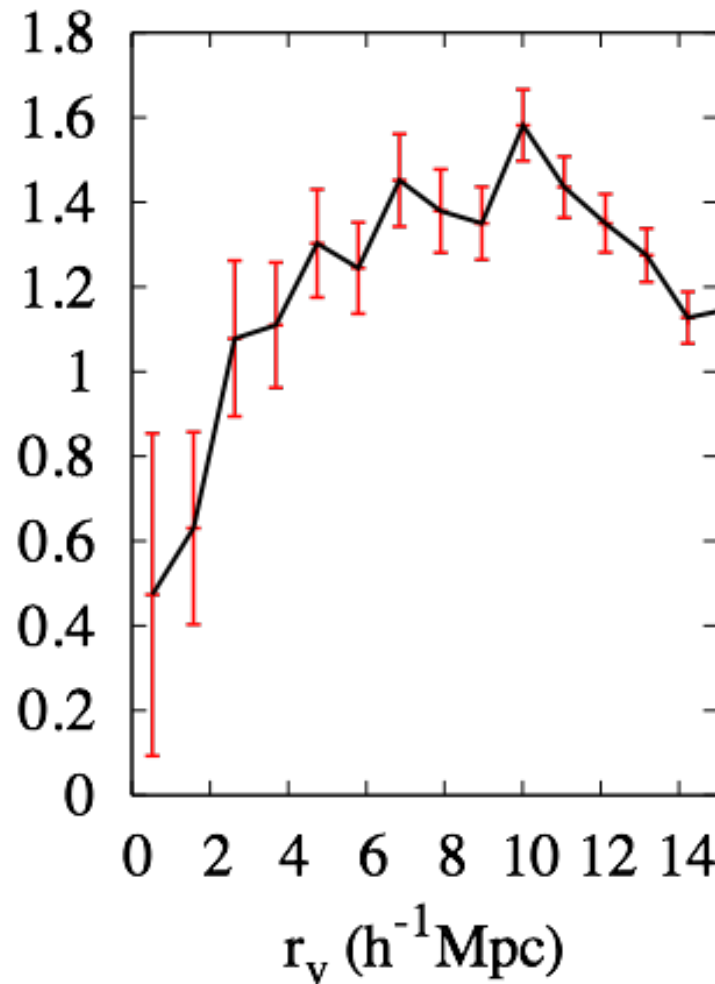
# Result III

# REAL DATA from SDSS!!!

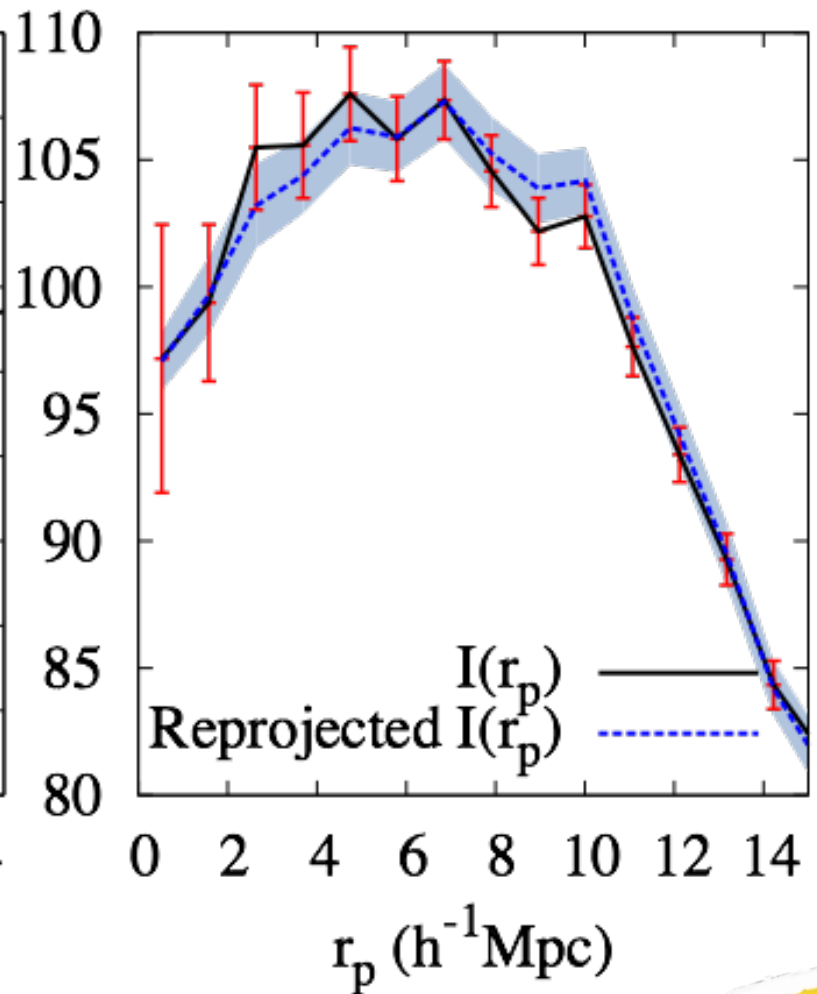
Dim (5-15 Mpc/h)



$g(r_v)$



$I(r_p)$

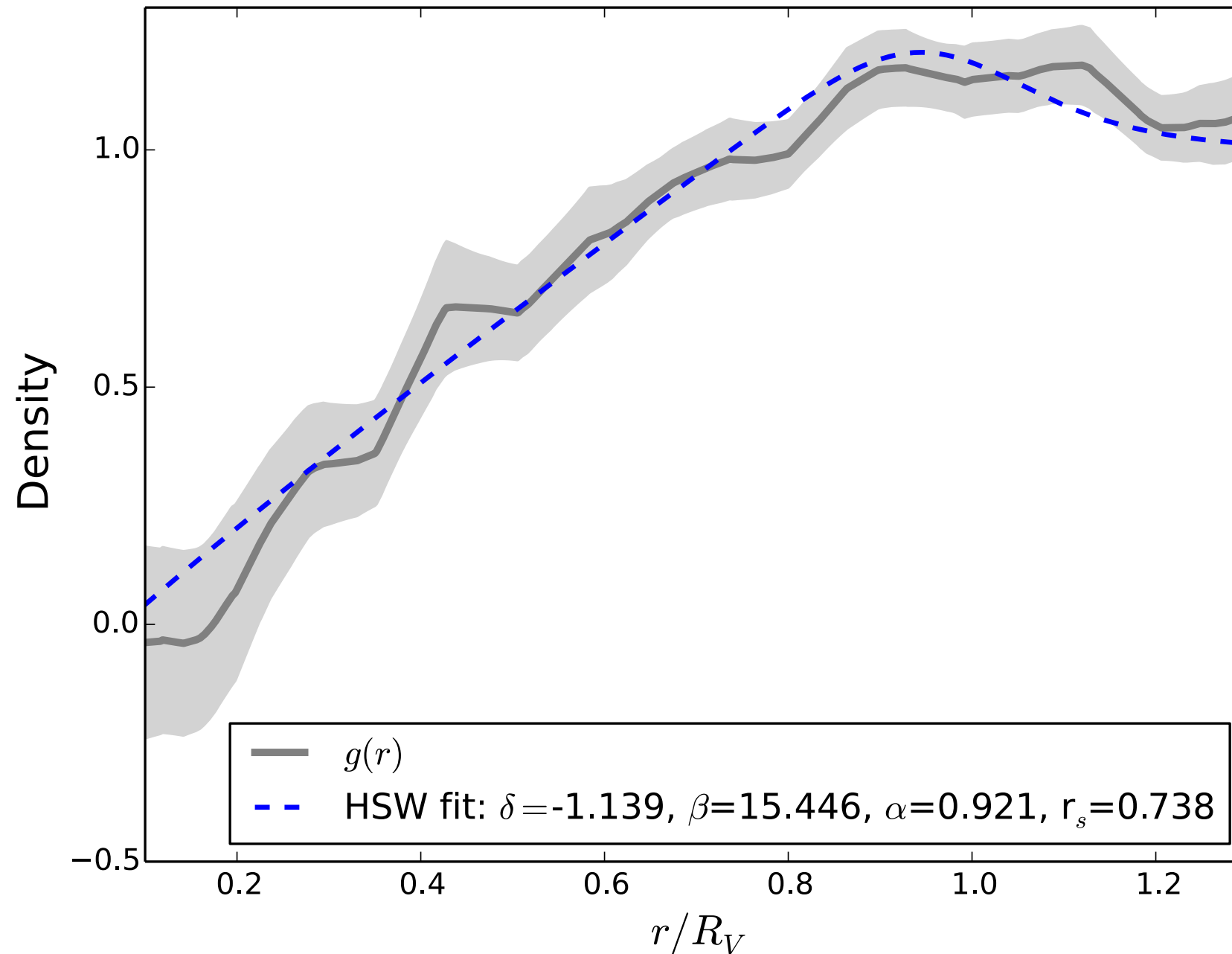


Stack radius	Redshift	Dataset	Galaxies	Voids
5-15	0.05-0.10	dim2	173929	173
10-15	0.05-0.10	dim2	43527	41
20-25	0.10-0.15	bright1	21241	17
25-45	0.15-0.20	bright2	51913	37

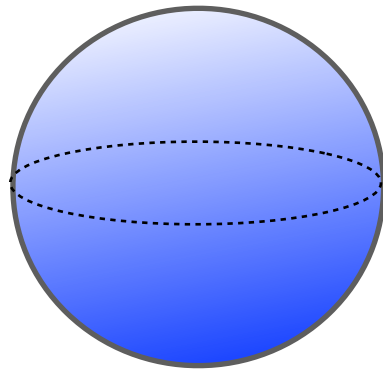
Model independent!  
No assumption about RD



# Average real space void from SDSS DR7 matches simulations

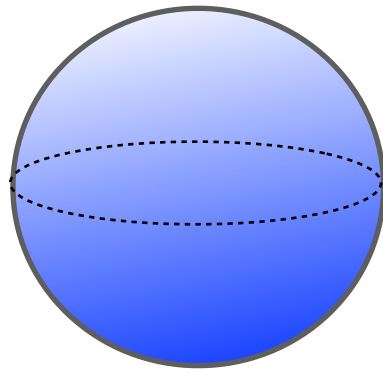


# What do we know about voids?

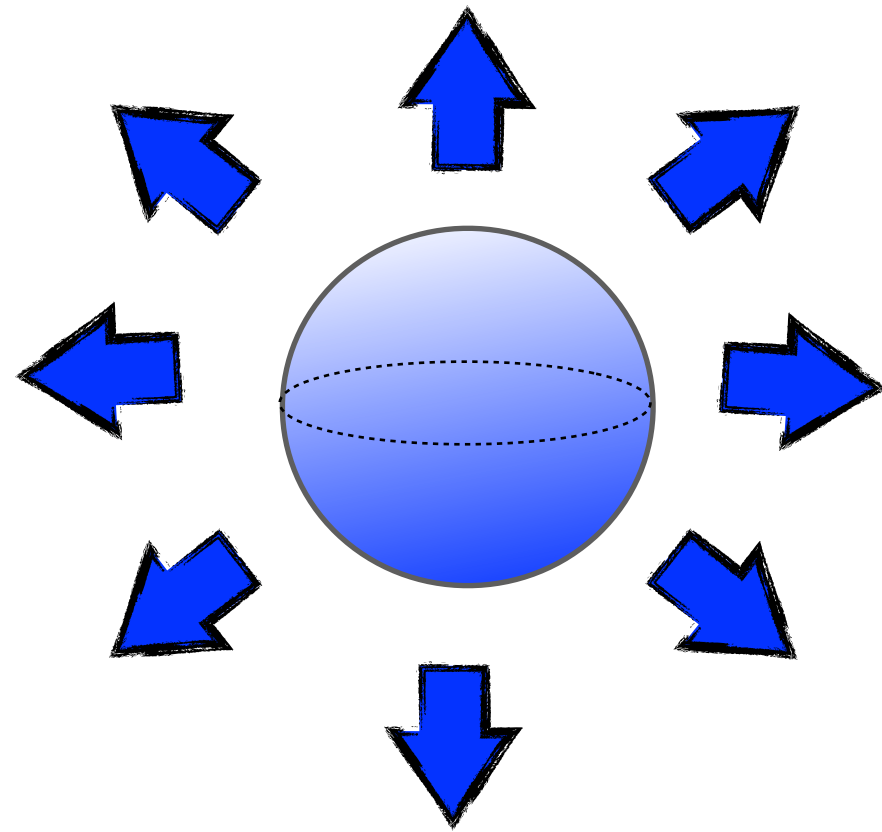


STATIC!!!!

# What do we know about voids?



STATIC!!!!



DYNAMICS ?????



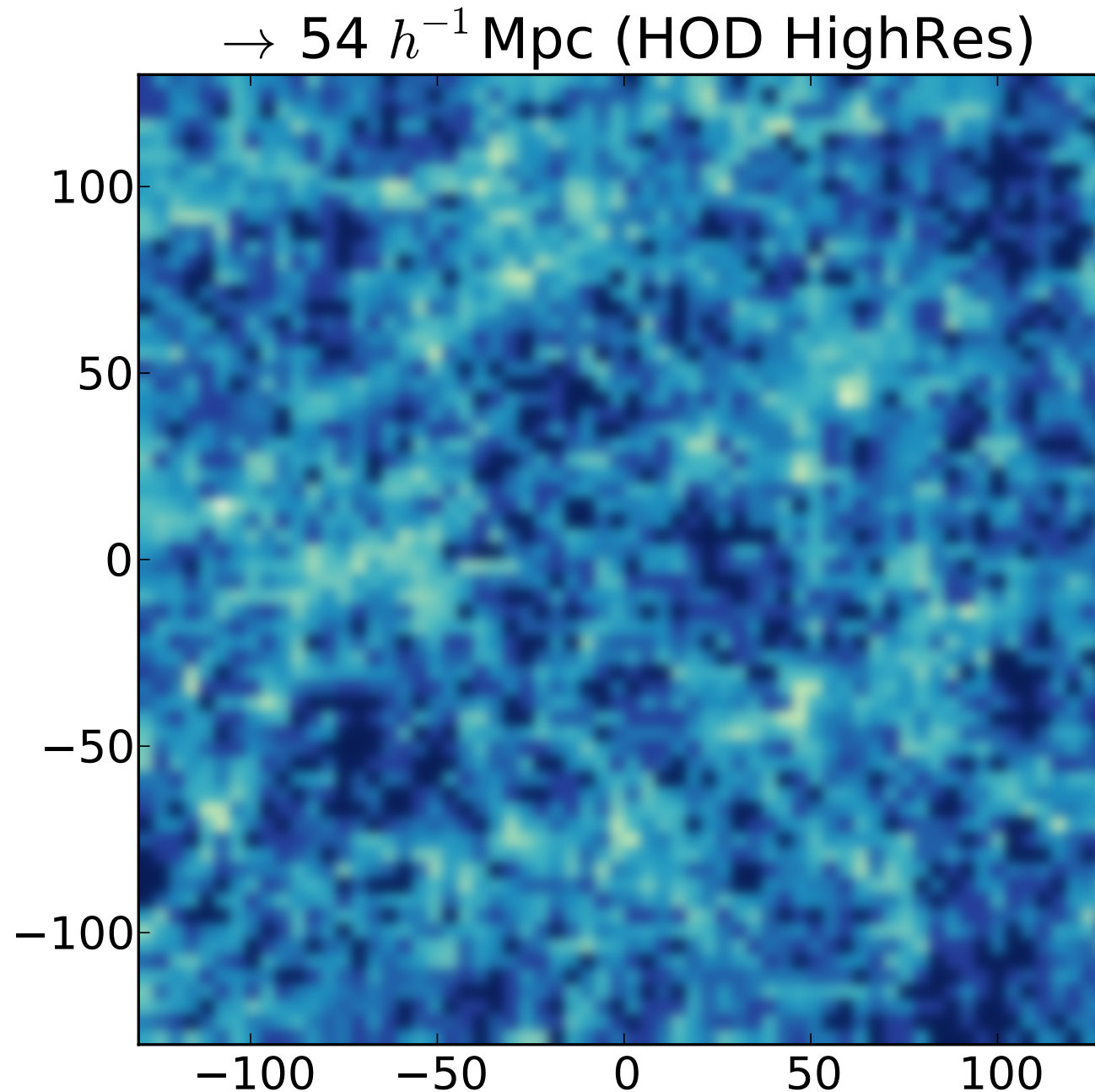
“Really” looking at voids...



How do velocities impact the way the void finder selects voids?

# Let's give a look at voids...

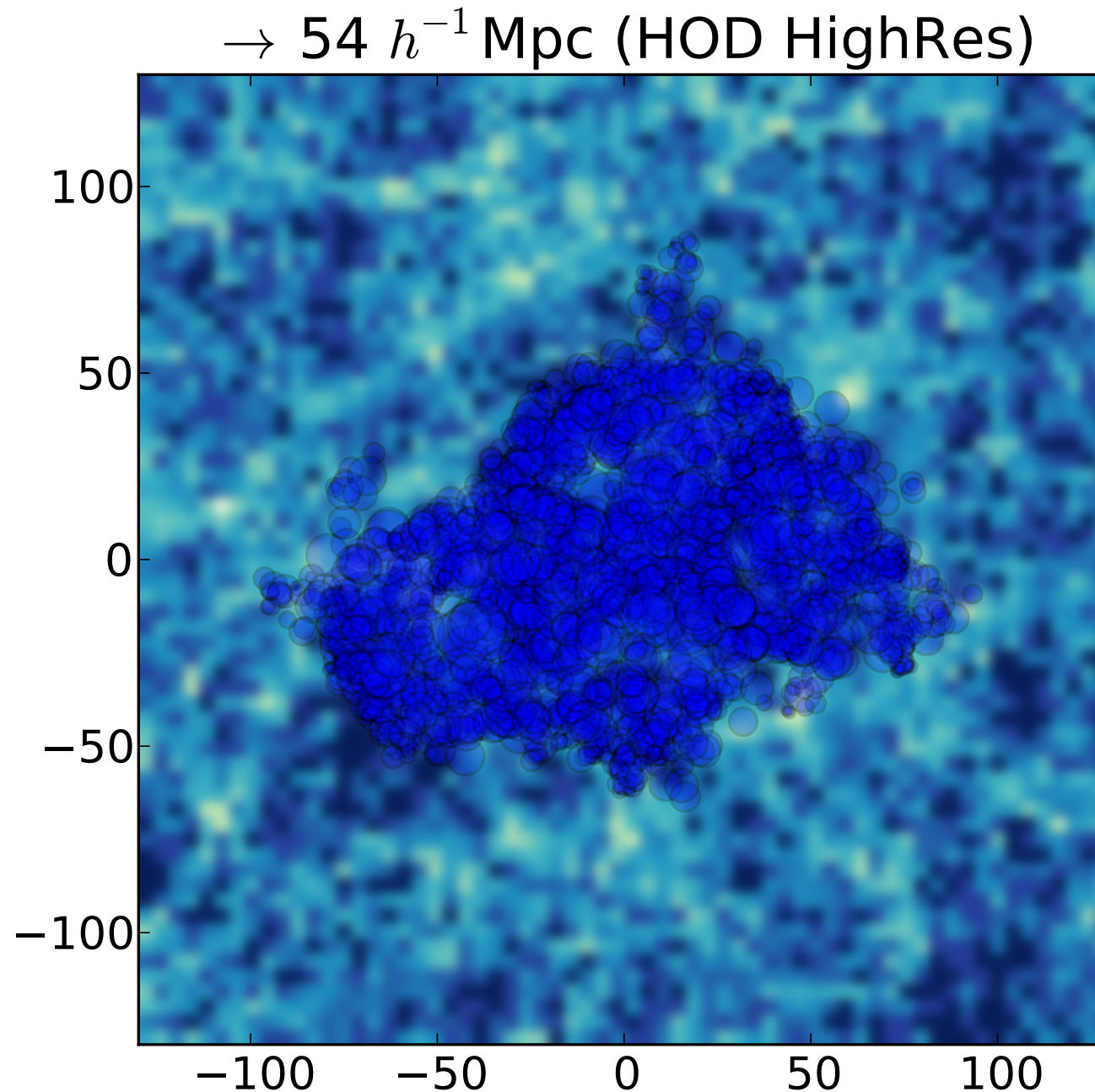
HOD nopv  
versus  
HOD + pv



Is the cosmological signal washed out by velocities in a certain kind of voids? Can we identify them and boost the cosmological signal?

# Let's give a look at voids...

HOD nopv  
versus  
HOD + pv

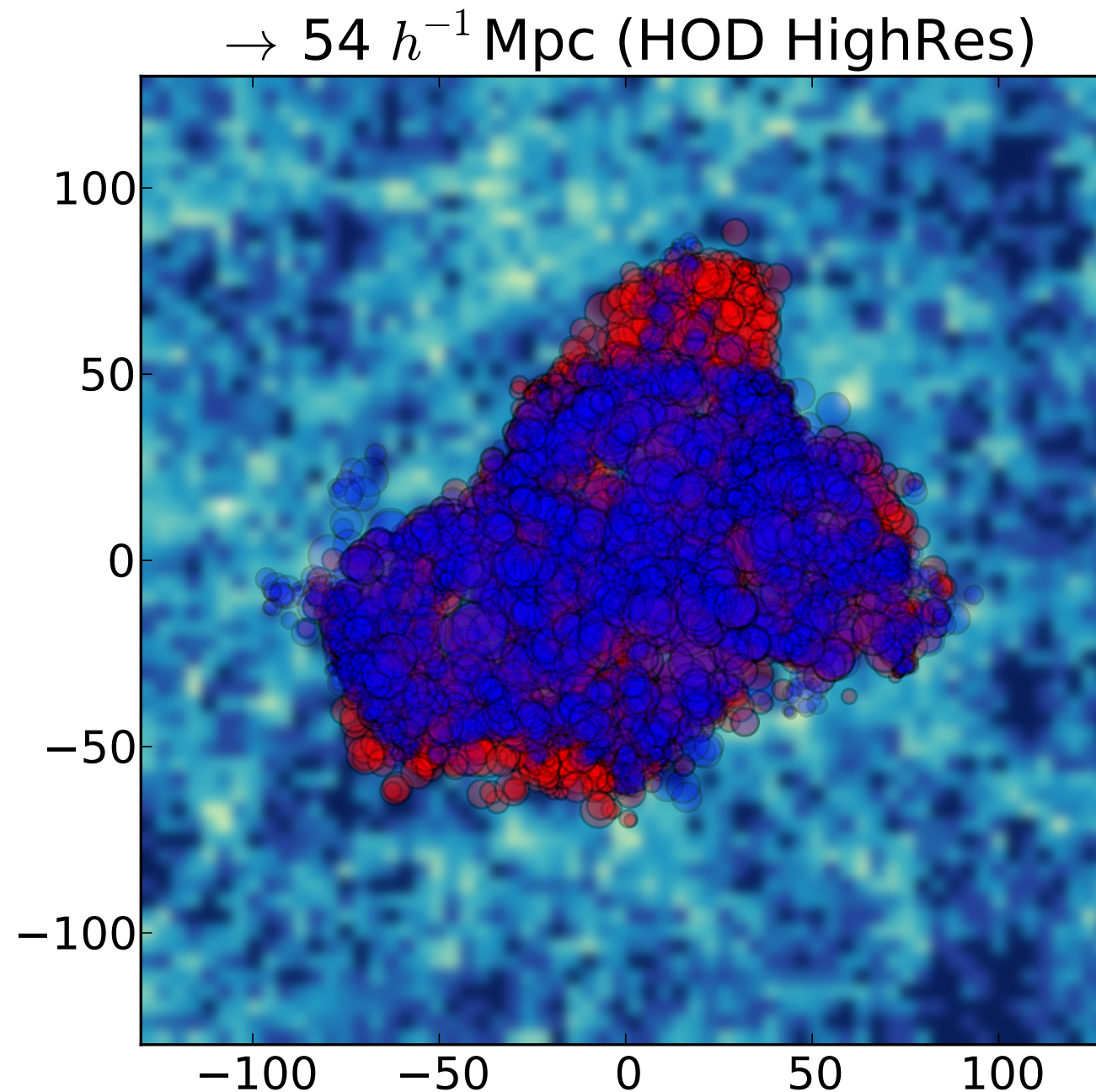


Is the cosmological signal washed out by velocities in a certain kind of voids? Can we identify them and boost the cosmological signal?



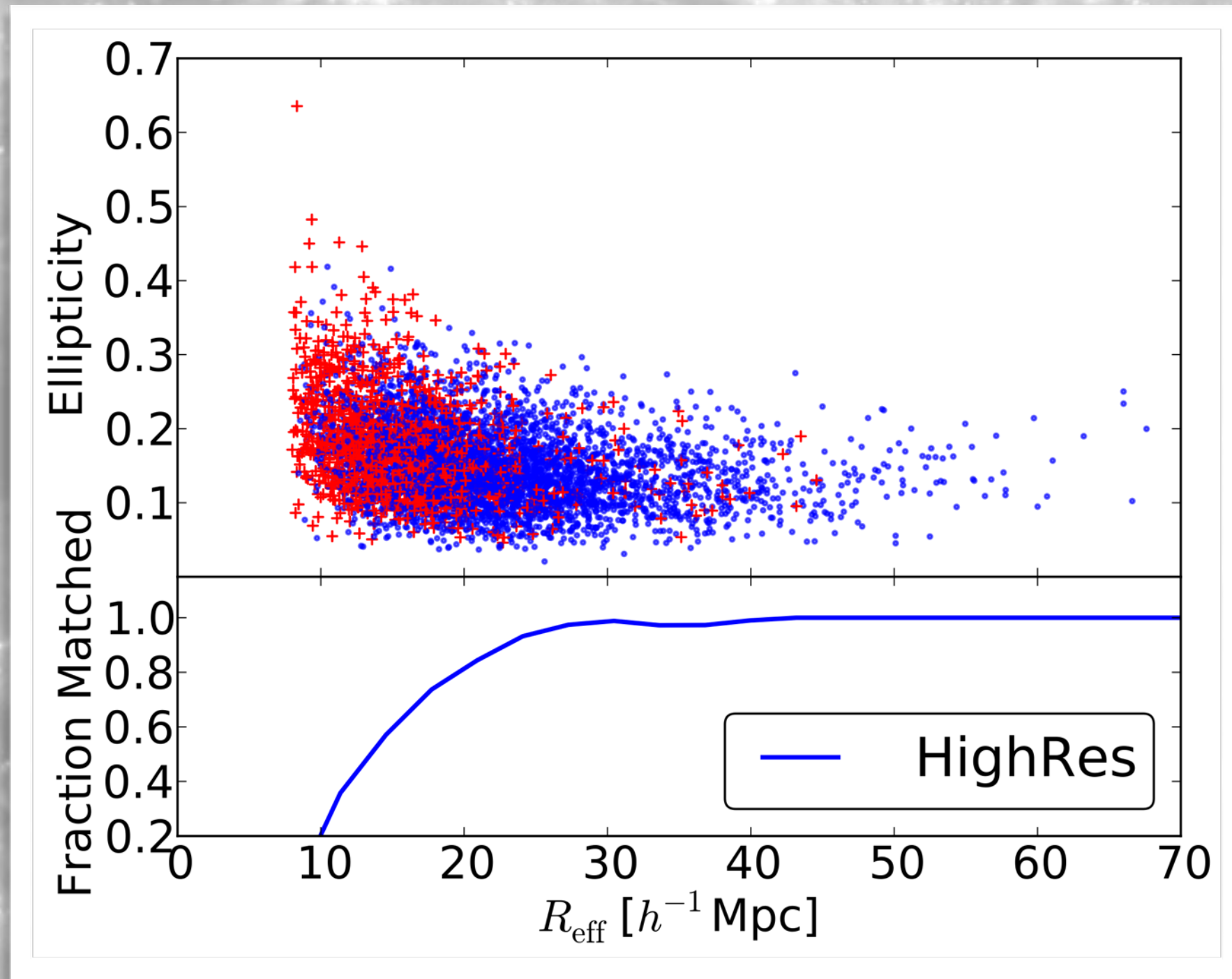
# Let's give a look at voids...

HOD nopv  
versus  
HOD + pv



Is the cosmological signal washed out by velocities in a certain kind of voids? Can we identify them and boost the cosmological signal?

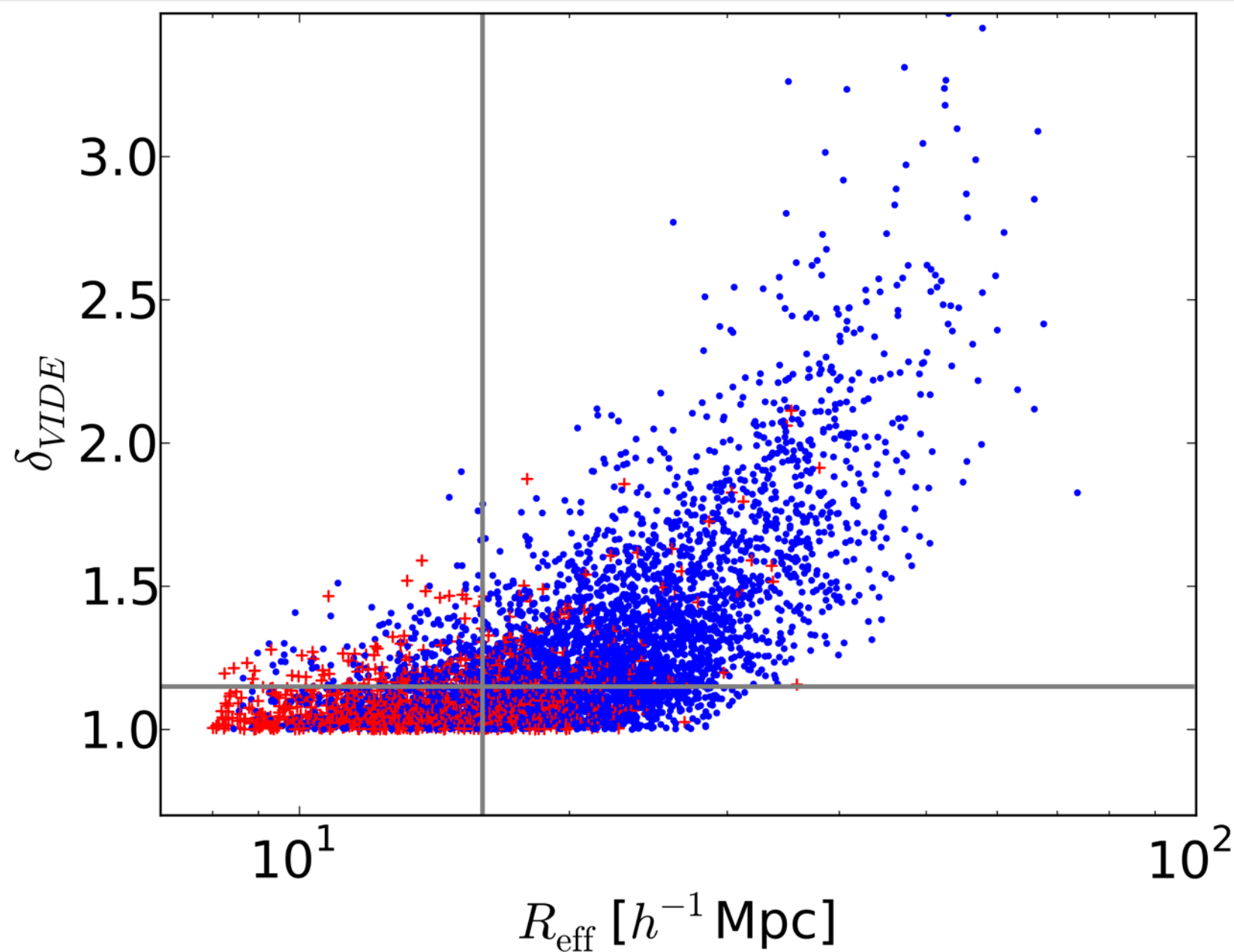
# Which voids are affected most?



The number of voids without match  
is high for small voids.



# Identify them by properties

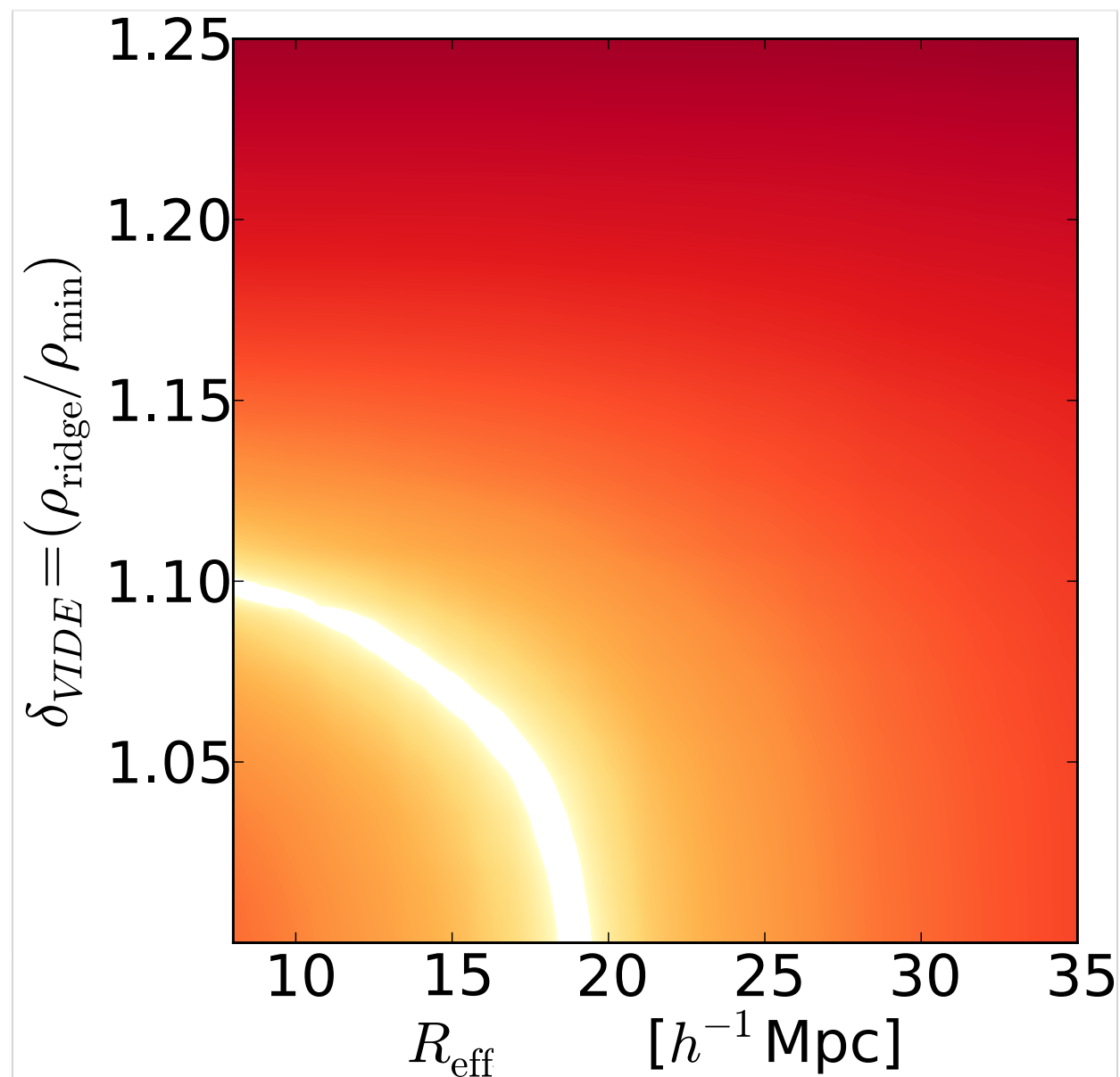


Applying cuts  
on these  
quantities we  
can boost the  
signal to noise  
for cosmological  
signal.

NB: depends on goal



# Optimal cuts for real surveys!



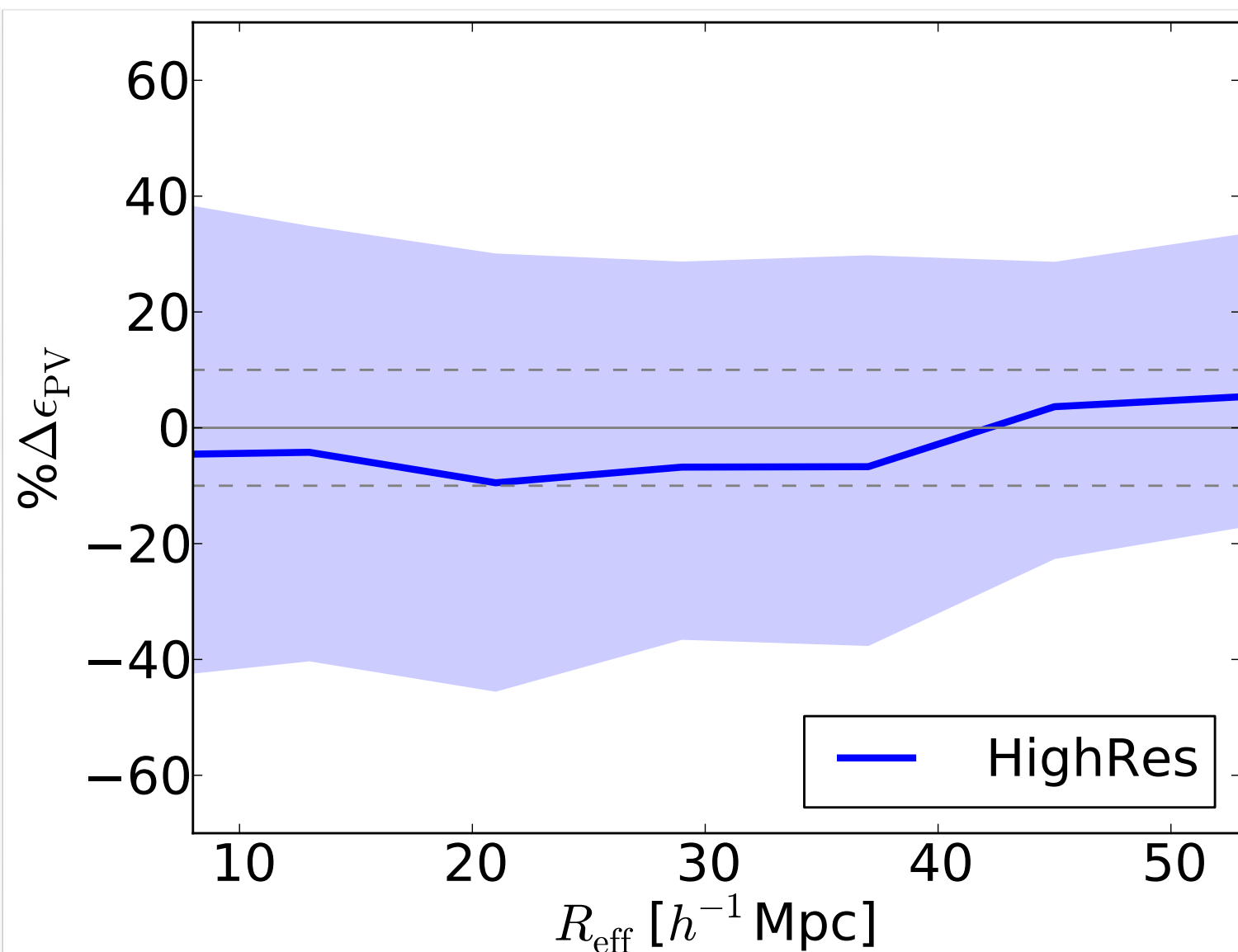
*Maximize the number of  
unmatched removed  
and the number of  
matched kept*

$$\eta = \frac{N_{\text{unmatch removed}}}{N_{\text{match kept}}}$$

We only keep voids very mildly affected  
by velocities! This correction does not  
need any prior knowledge!

# Mastering the effects of peculiar velocities

Exclude the affected voids, but what about the others? Can we correct the properties of other voids for the effects of velocities?



Direct ellipticity  
correction  
for the AP test  
application



# Guidelines to boost the cosmological information



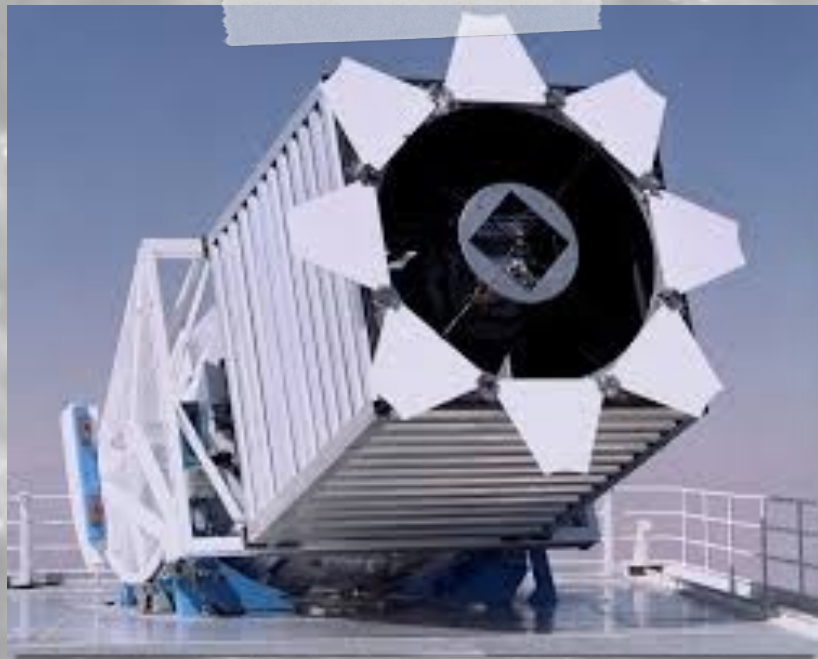
Apply **cuts** on **radius** and  $\delta_{VIDE}$  that match our physical sense => boost the signal to noise for cosmological information.



Direct ellipticity correction for the AP test application



Cuts optimized for current surveys!



*eBOSS: extended Baryon Oscillation Spectroscopic Survey*  
*French Participation Group (~20)*  
*IN2P3: APC, CPPM, LPNHE*  
*INSU: IAP, LAM*  
*CEA: IRFU/SPP*

BOSS provided us with an amazing number of galaxies, to be increased by eBOSS



Set of tools to **beat systematics** and ... what about increasing statistics?



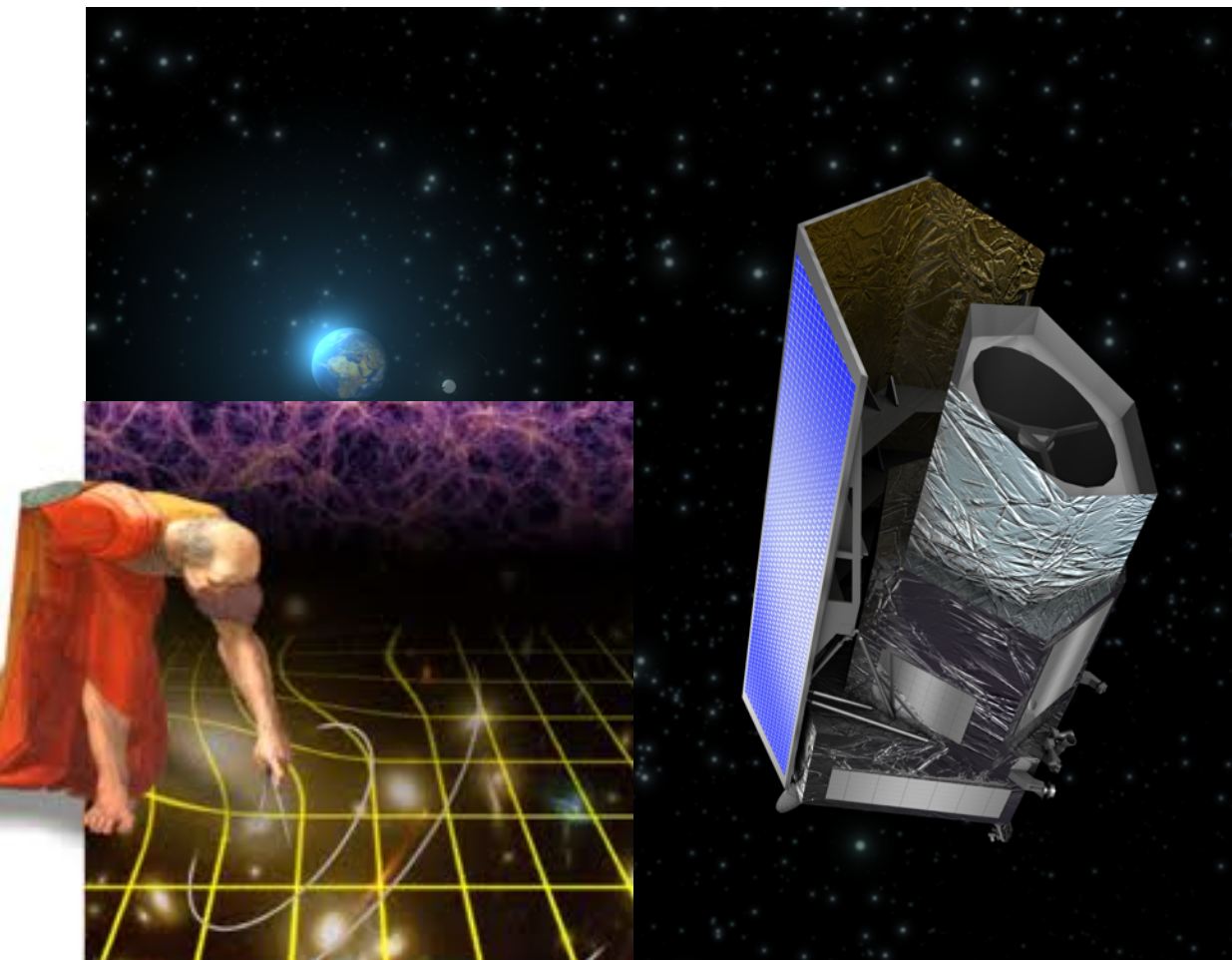
For example let's go.... Back to the future  
to bet on LSS with upcoming surveys

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*SDSS DR7*       $6.7 \cdot 10^5$       galaxies

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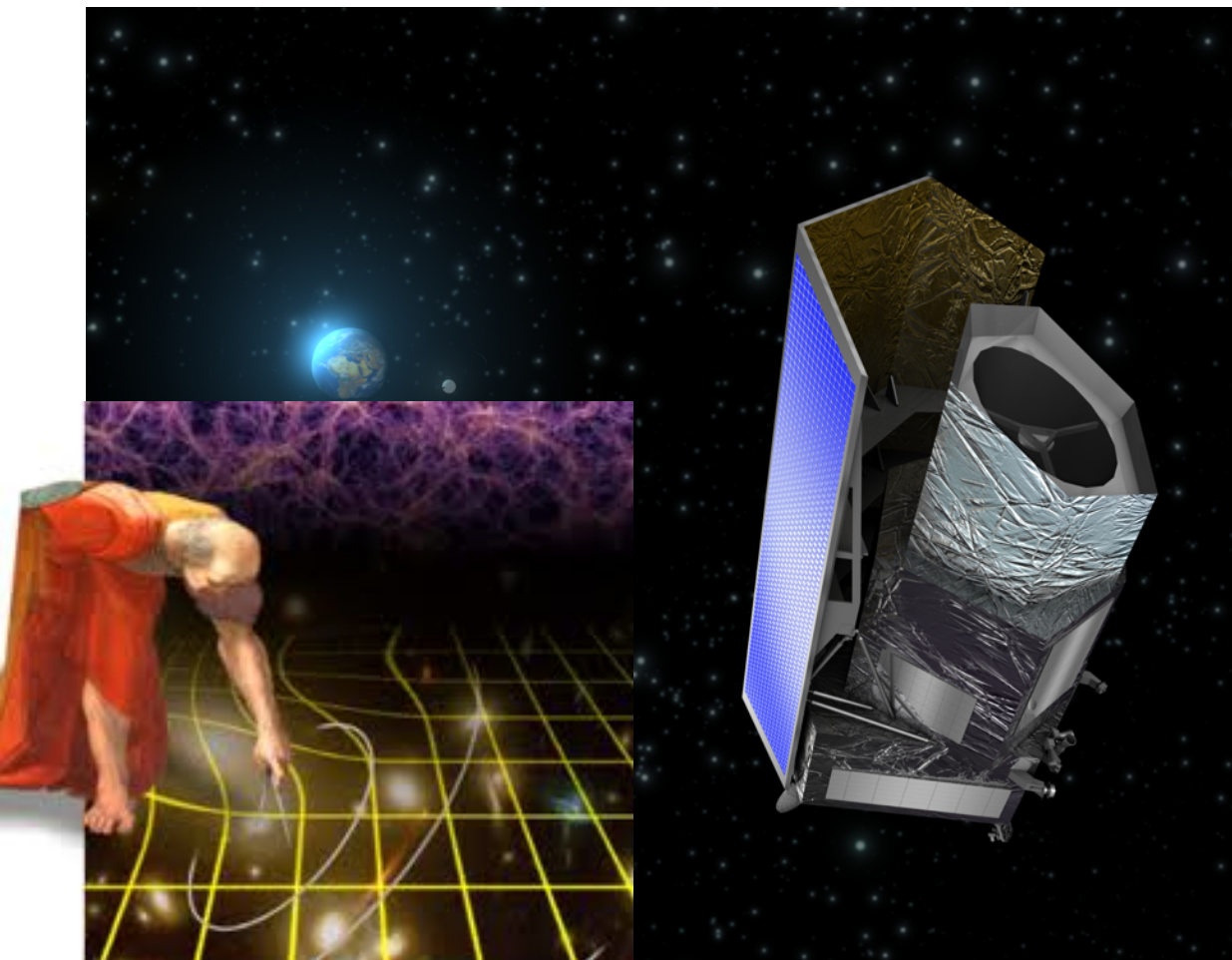


**EUCLID**       $5.0 \cdot 10^7$



For example let's go.... Back to the future  
to bet on LSS with upcoming surveys

*SDSS DR7*       $6.7 \cdot 10^5$       galaxies



**EUCLID**       $5.0 \cdot 10^7$

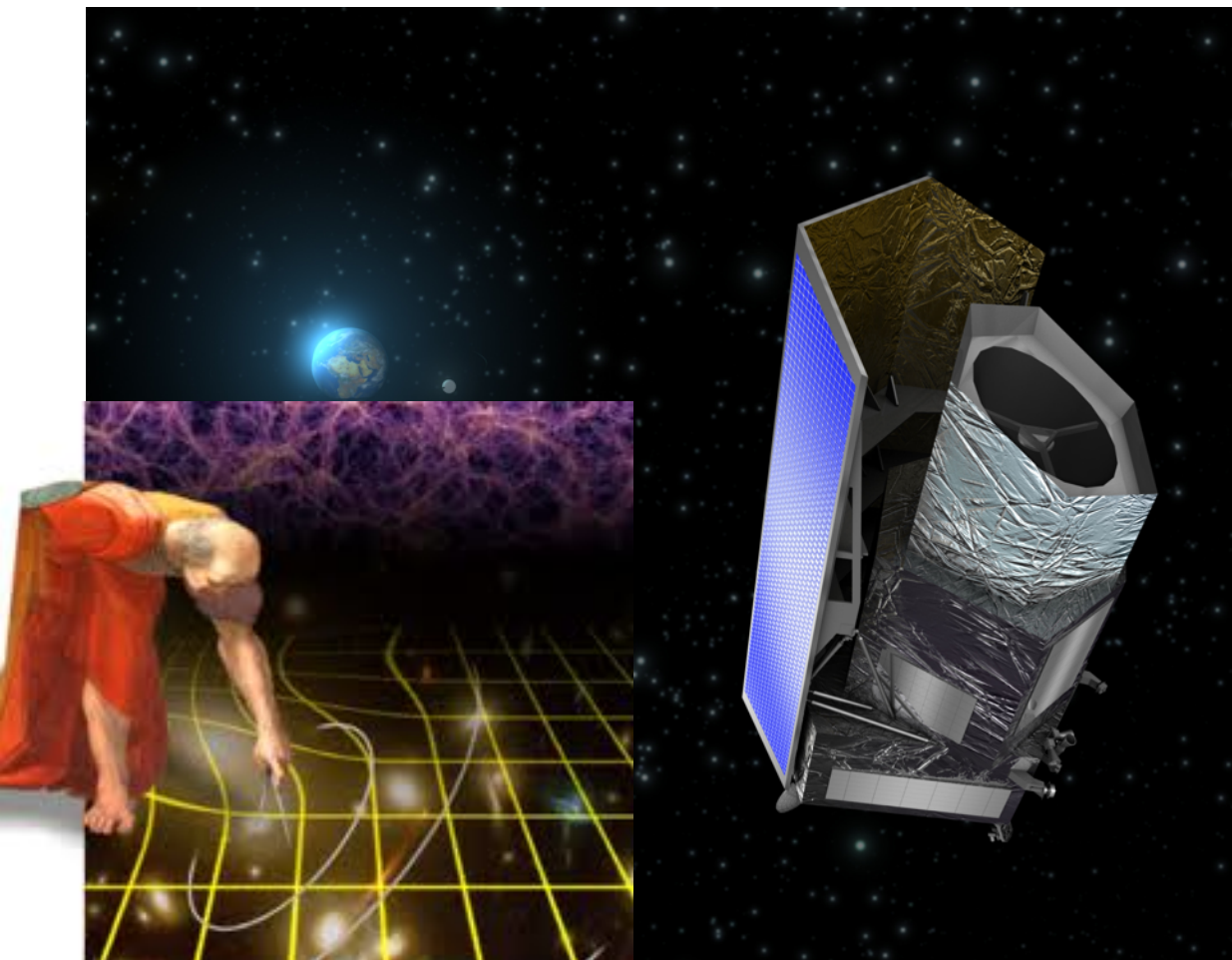
**WFIRST**

$2.0 \cdot 10^7$



# For example let's go.... Back to the future to bet on LSS with upcoming surveys

*SDSS DR7*       $6.7 \cdot 10^5$       galaxies



**EUCLID**       $5.0 \cdot 10^7$

**WFIRST**

$2.0 \cdot 10^7$



Real-space density profiles of increased precision  
+ a huge statistic for AP test and abundances

# Theory

Sheth Van de Weygaert  
excursion set model for  
void abundance (2004)

+

# Simulation

Tuned on Euclid to obtain the  
parameter of the model and  
marginalise on parameter

+

# Survey

Take into account  
features such as  
galaxy number density,  
survey area, redshift  
covering



# Theory

Sheth Van de Weygaert  
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+

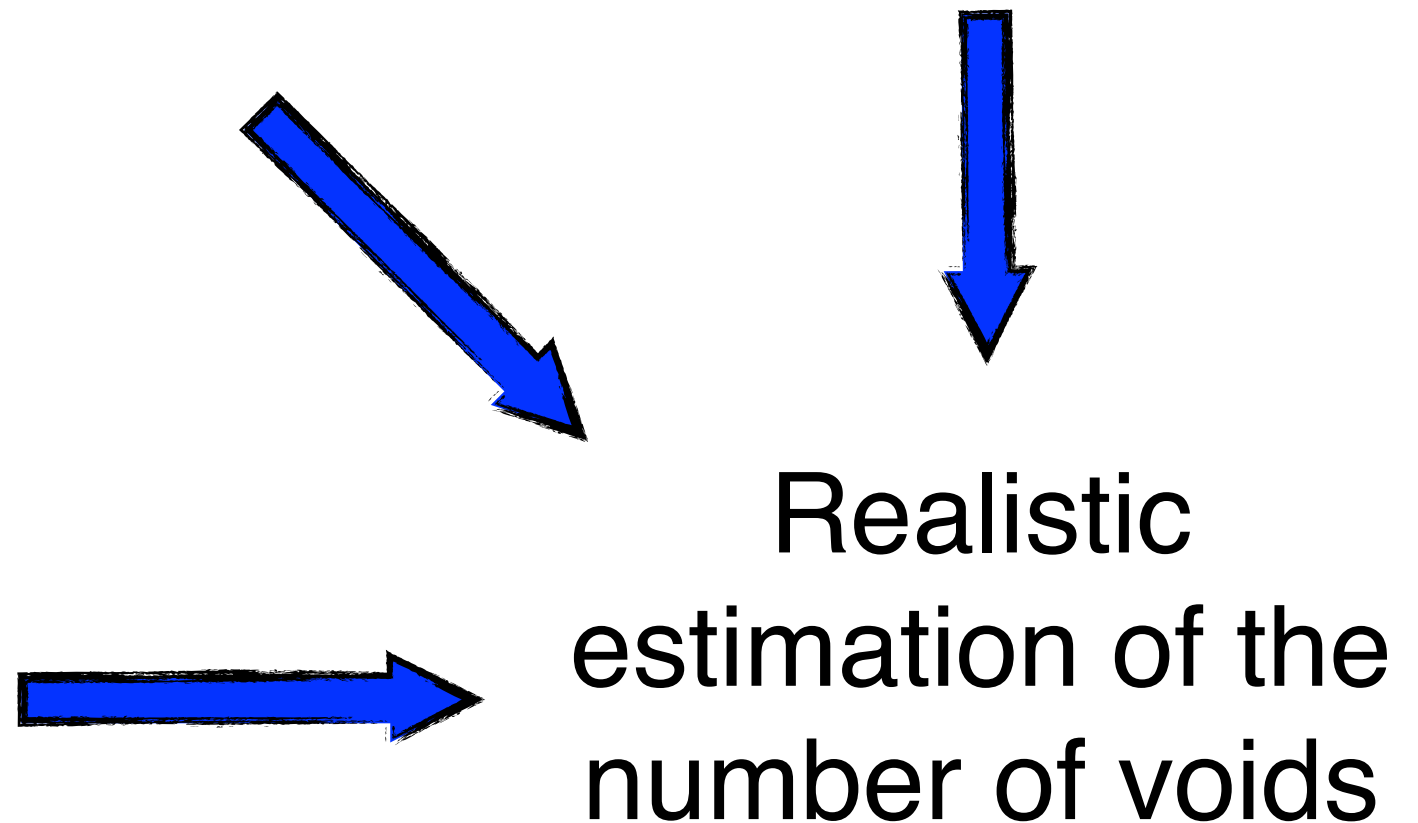
# Simulation

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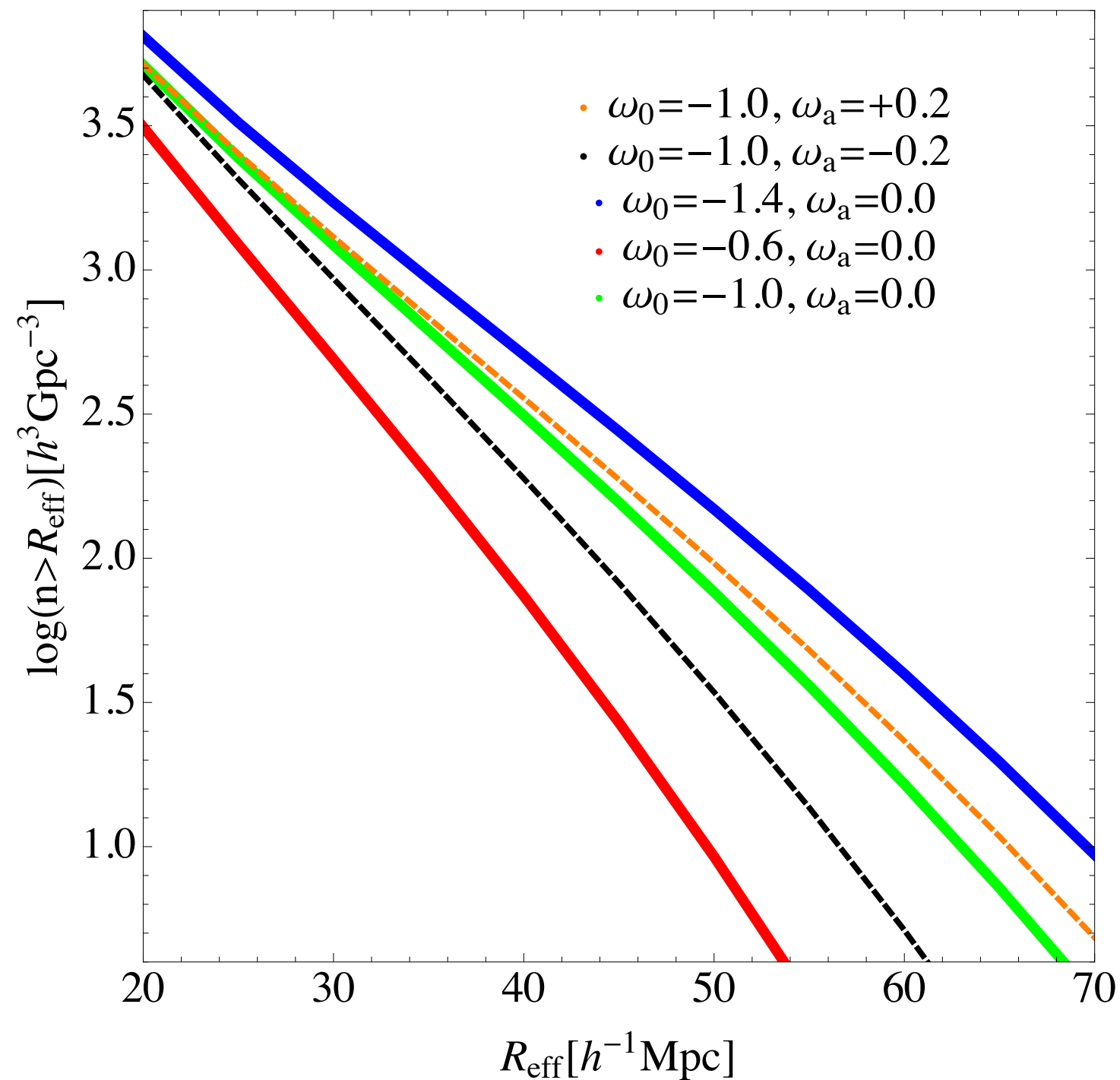
+

# Survey

Take into account  
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galaxy number density,  
survey area, redshift  
covering

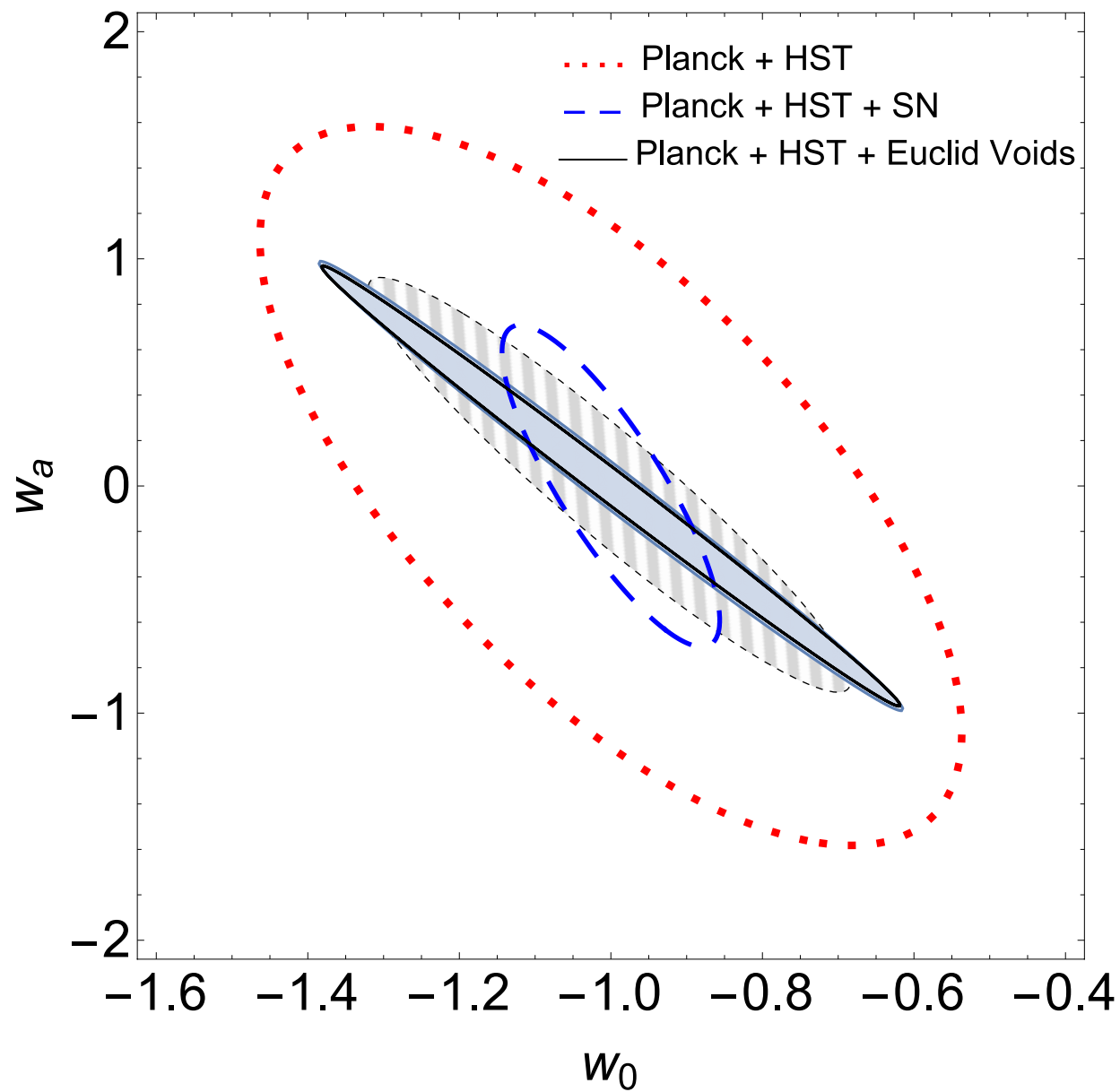


# Abundances to constrain Cosmology



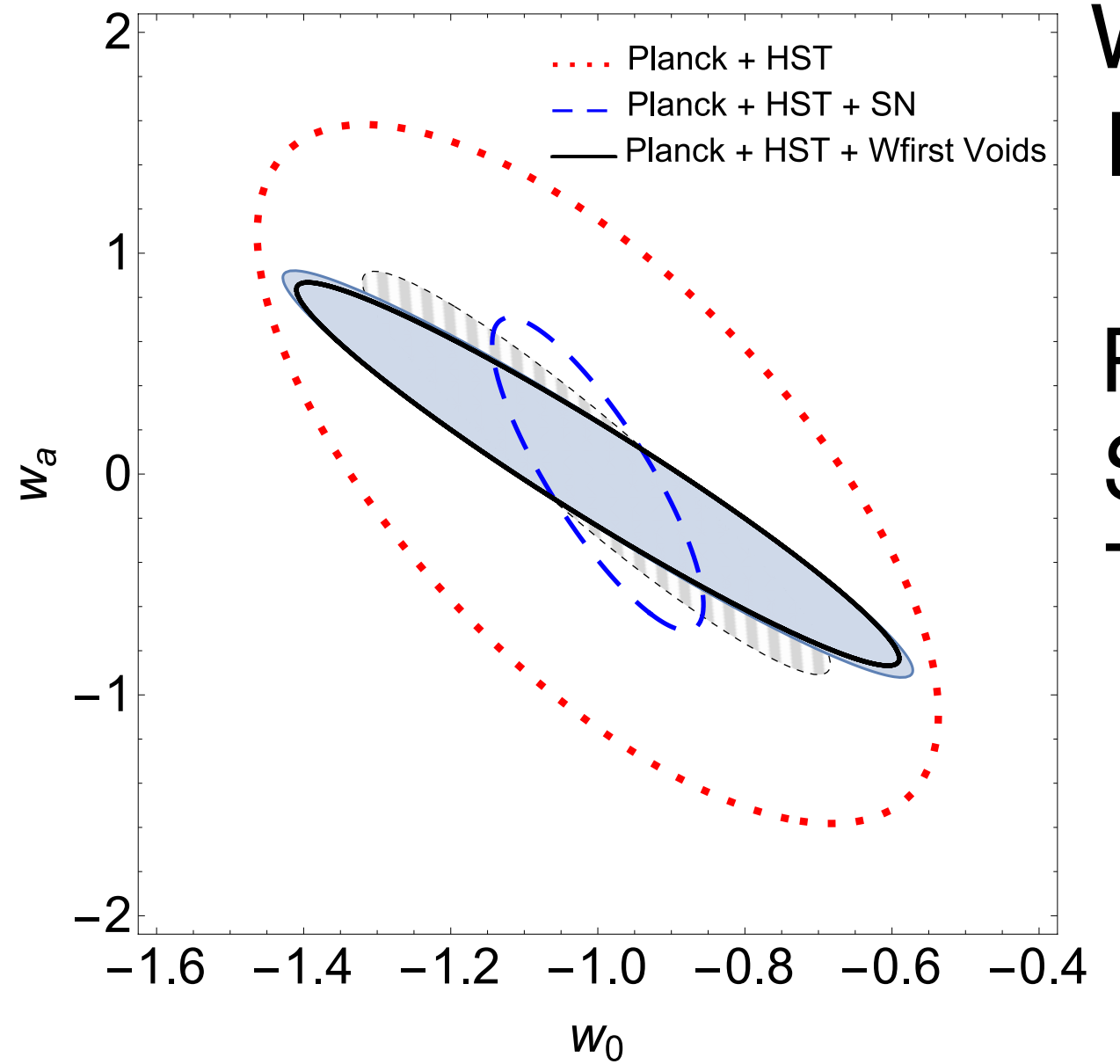
# Comparing future surveys

E  
U  
C  
L  
I  
D



$$7.8 \times 10^5$$

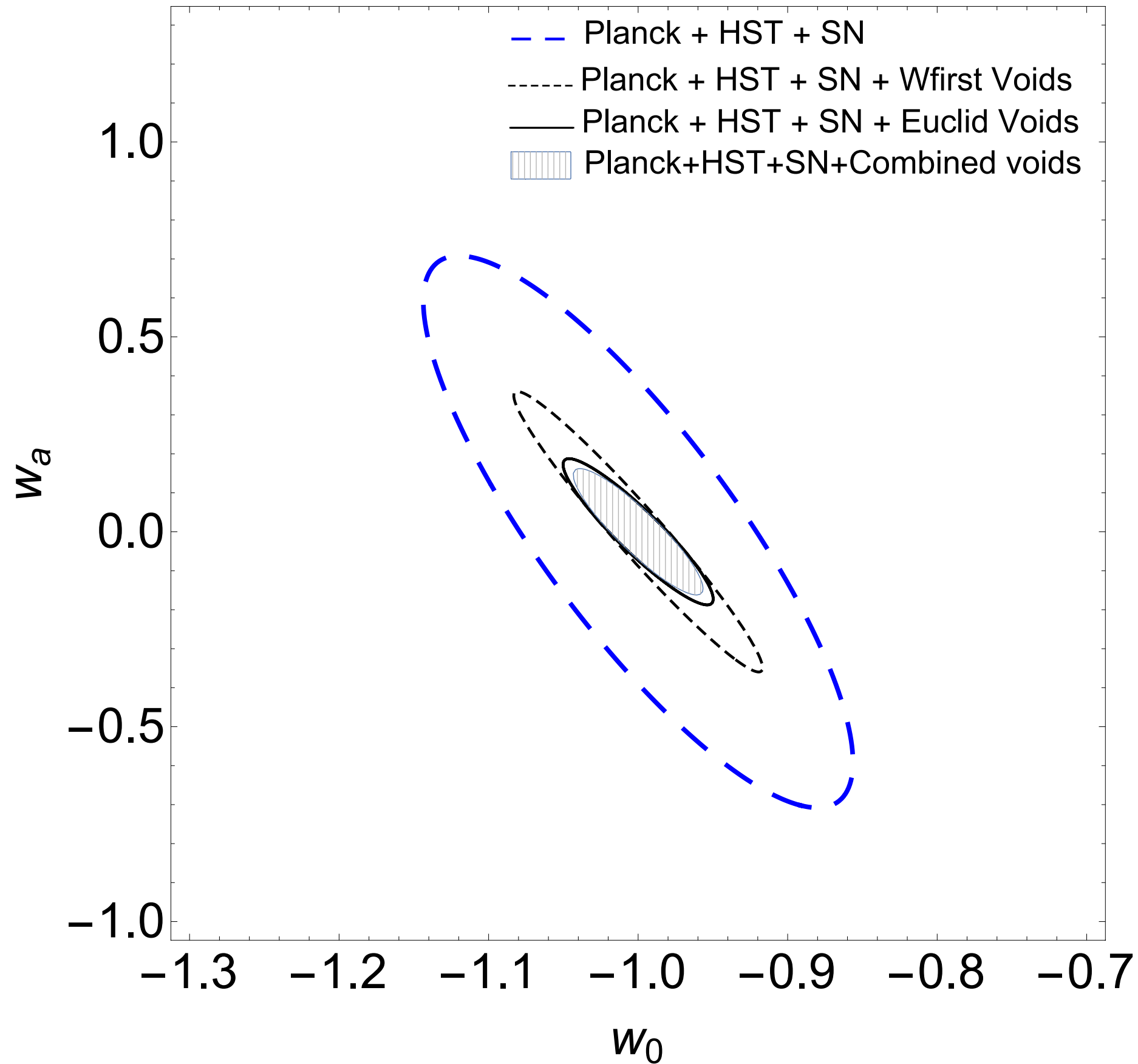
W  
F  
I  
R  
S  
T



$$2.5 \times 10^5$$



# Combining future surveys



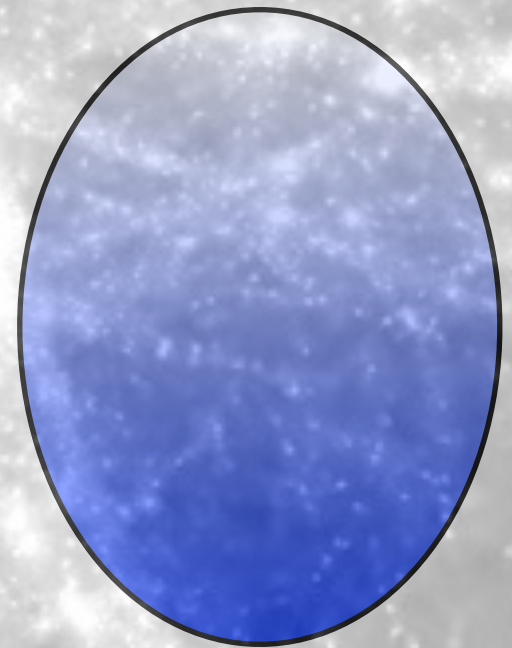
# Conclusion

AP test

Voids as a **new tool** to constrain cosmology in the era of large surveys.

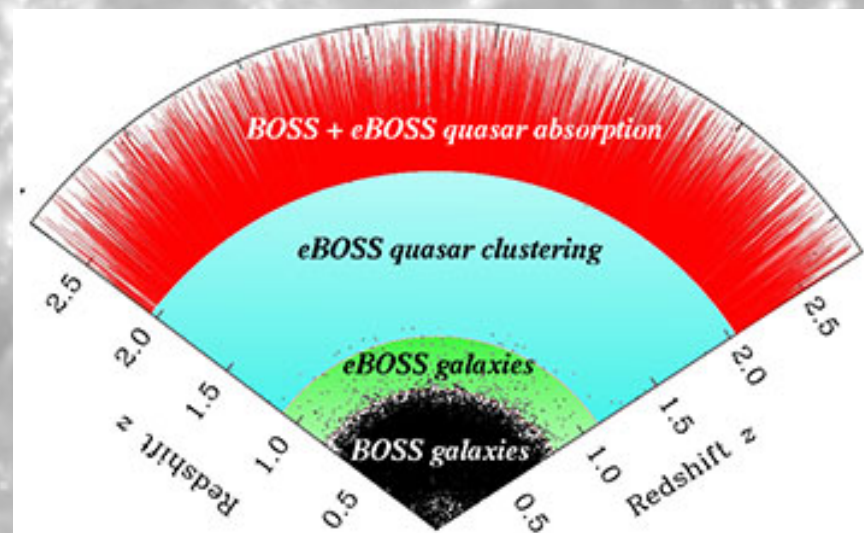
First ever **real space density profile** of voids from real data and guidelines for treatment of systematics (velocity)

Forecast for void **abundance** with Euclid



Combine!! LSST

BOSS and eBOSS (high  $z$ ) apply these innovative techniques





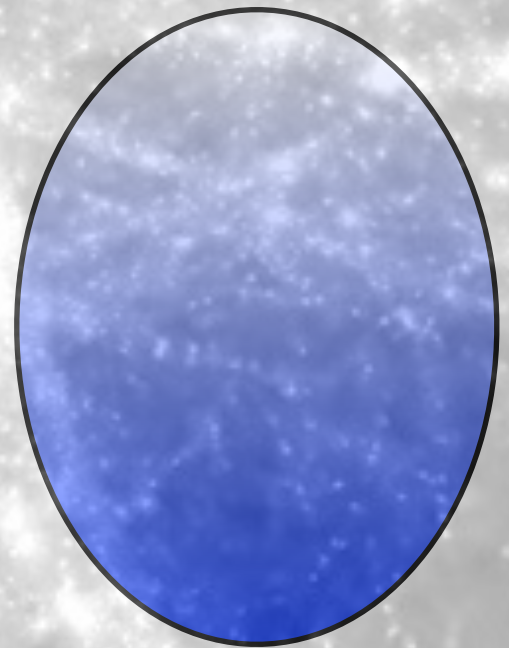
# Conclusion

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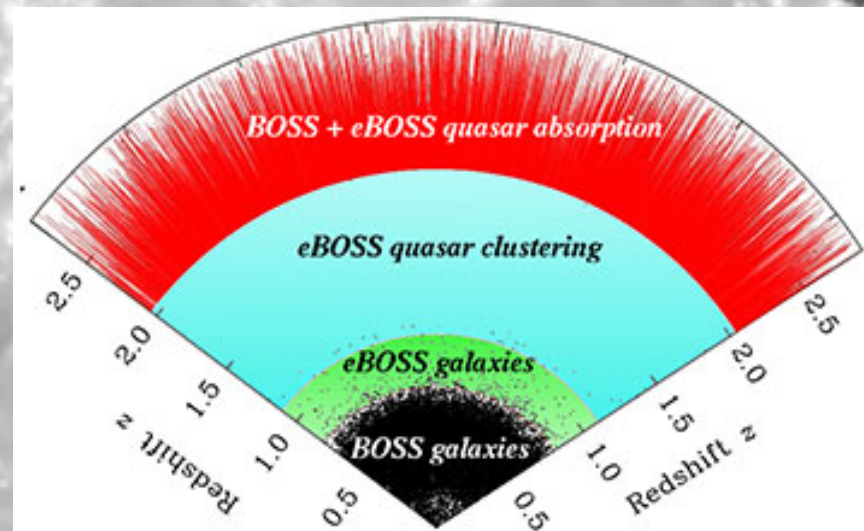
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Thank you!



# Supplementary slides



“Just” understanding the evolution of the universe.

(My goal as a cosmologist.)





“Just” understanding the evolution of the universe.

(My goal as a cosmologist.)



What is he  
looking at??



# How do we calculate abundances?

Number density of voids  
within a given mass range

$$\frac{M^2 n(M, z) dM}{\rho_{\text{back}}(0) M} = \nu f(\nu) \frac{d\nu}{\nu},$$

$$\nu = \frac{\delta_v^2}{\sigma^2(M)}$$

dynamics of void  
formation  
(void in void)

$$\sigma^2(M, a) = \int_0^\infty \frac{k^3 P_\delta(k, a)}{2\pi^2} \left| \tilde{W}(k R_{\text{Lag}}(M)) \right|^2 \frac{dk}{k}.$$

$$R_L = 1.7 R_E$$

$$R_L = (1 + \delta_{\text{void}})^{1/3} R_E$$

$$R_{\text{min}}^{\text{Eul}} = \text{Max}[2R_{\text{mps}}; R_{\text{Eul}, \sigma \simeq 1}]$$

$$\sigma(R_{\text{Lag}}, z) \simeq 1$$

# How do we calculate abundances?

Number density of voids  
within a given mass range

$$\frac{M^2 n(M, z)}{\rho_{\text{back}}(0)} \frac{dM}{M} = \nu f(\nu) \frac{d\nu}{\nu},$$

Linearly  
extrapolated  
underdensity

$$\nu = \frac{\delta_v^2}{\sigma^2(M)}$$

dynamics of void  
formation  
(void in void)

$$\sigma^2(M, a) = \int_0^\infty \frac{k^3 P_\delta(k, a)}{2\pi^2} \left| \tilde{W}(k R_{\text{Lag}}(M)) \right|^2 \frac{dk}{k}.$$

Density variance  
inside a sphere  
with given mass

$$R_L = 1.7 R_E$$

$$R_L = (1 + \delta_{\text{void}})^{1/3} R_E$$

$$R_{\text{min}}^{\text{Eul}} = \text{Max}[2R_{\text{mps}}; R_{\text{Eul}, \sigma \simeq 1}]$$

$$\sigma(R_{\text{Lag}}, z) \simeq 1$$

Press-Schechter formalism & excursion set  
It gives the number density of objects for which  
the linearly extrapolated density exceed a  
threshold

It considers a mass element; this element will  
belong to a halo of given mass ( $>M$ ) if the  
density fluctuation centred in the element and  
filtered over a sphere of radius prop to  
 $M^{1/3}$  has  $\delta > \delta_c$

It gives the fraction of collapsed objects with  
mass  $>M$  corresponding to volume samples  
where the initial density fluctuation  $> \delta_c$

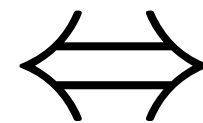


# Alcock-Paczyński test



$$\Delta r_{\perp} = \Delta r_{\parallel}$$

$\Delta r_{\parallel} \left\{ \begin{array}{c} \Delta r_{\perp} \end{array} \right.$



$$D_A(z) \Delta \theta = \frac{c \Delta z}{H(z)}$$

*what we  
know*

*what we  
don't know*

$$\frac{c \Delta z}{\Delta \theta} = D_A(z) H(z)$$

The cosmological model is telling us what we don't know:

Assuming a flat universe,  $D_A(z)$  and  $H(z)$  are then related to the dark energy density through

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_X \exp \left[ 3 \int_0^z \frac{1+w(z)}{1+z} dz \right]}$$

$$= \sqrt{\frac{\Omega_m H_0^2}{1 - \Omega_X}} \sqrt{\Omega_m (1+z)^3 + \Omega_X \exp \left[ 3 \int_0^z \frac{1+w(z)}{1+z} dz \right]}, \quad (3)$$

$$D_A(z) = \frac{c}{1+z} \int_0^z \frac{dz}{H(z)}, \quad (4)$$

To perform the  
Alcock-Paczyński  
test we measure  
the stretch

$$e_V(z) = \frac{c}{H_0} \frac{\Delta z}{\Delta d}$$

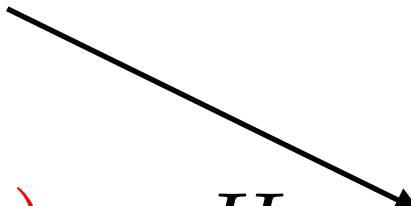
$$e_V(z) = \frac{\Delta z}{z \Delta \theta} \quad \Delta d = \frac{cz \Delta \theta}{H_0}$$

The deviations from fiducial cosmology  
cause geometrical distortions.

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Alcock-Paczyński test we measure the stretch

$$E(z) = \frac{H(z)}{H_0}$$

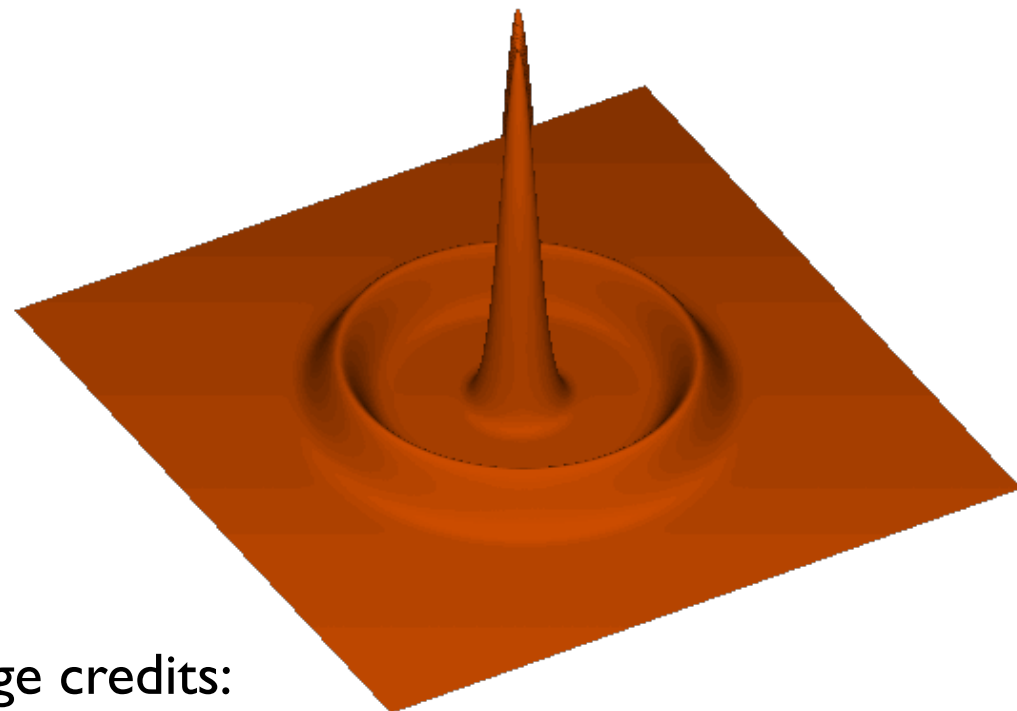
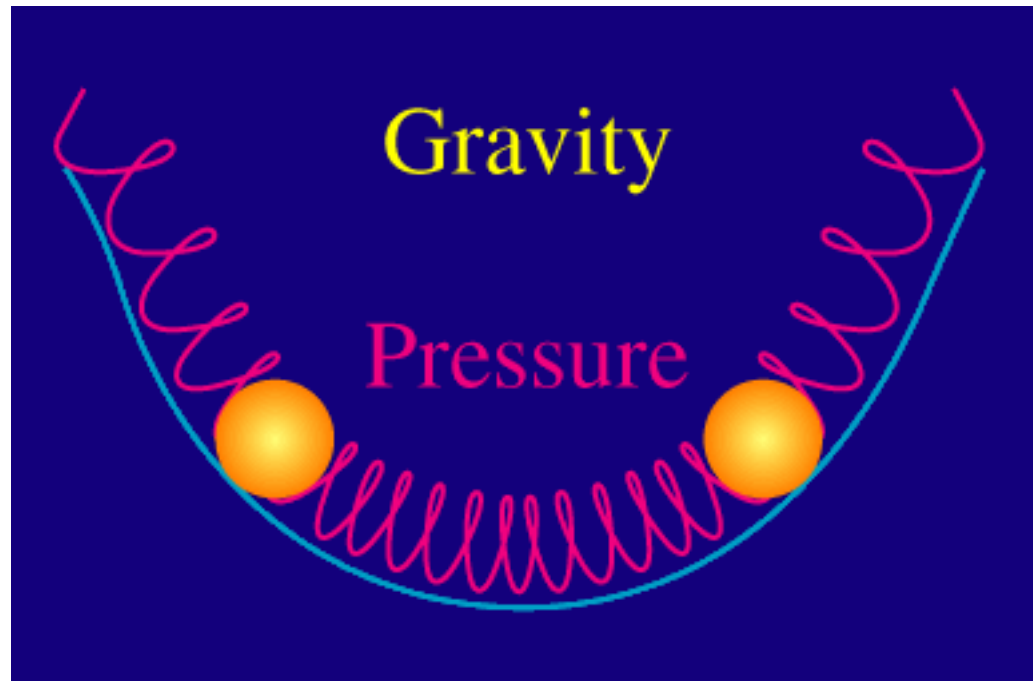
$$\frac{\delta z}{\delta d} = \frac{H_0}{c} \frac{H_0}{c} \frac{D_A(z) E(z)}{z} = \frac{H_0}{c} e_v(z)$$


$$e_v(z) = \frac{c}{H_0} \frac{\delta z}{\delta d}$$



# Baryonic Acoustic Oscillations

*Primordial plasma*



Overdense region  
Dark matter, baryons, photons



Oscillations of baryons  
and photons



Decoupling: Photons diffuse  
away, pressure ends

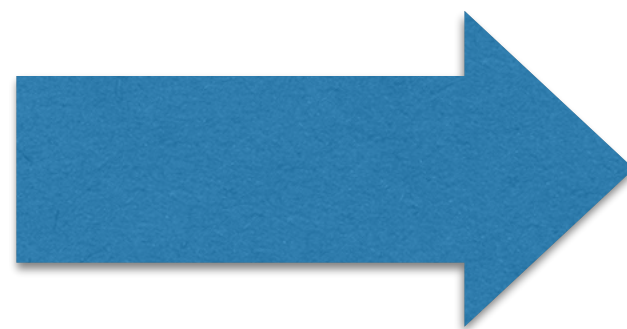


Shell of baryonic matter at  
fixed radius

*BAO can be considered an absolute AP test (i.e. with standard ruler)*

A standard ruler  $\left\{ \begin{array}{l} \Delta r_{\perp} = 150 Mpc \text{ (theory)} \\ \Delta r_{\perp} = D_A(z) \Delta \theta \end{array} \right.$

$$D_A(z) H(z) \quad (\text{our friend AP test})$$


$$H(z)$$

# Chevallier-Polarski-Linder parametrisation

$$w(z) = w_0 + w_a \frac{z}{z+1}$$



*Methods for the AP test: Method for AP test fits real space shape to redshift shape.*

### Method 1: better but needs more voids

It starts from a radial profile, fits it to an ellipse. In this ellipse equation there are  $\delta z$  and  $\delta d$ , so we get stretch (by comparing with the void)  $e_v(z)$ ; its real space shape to redshift shape.

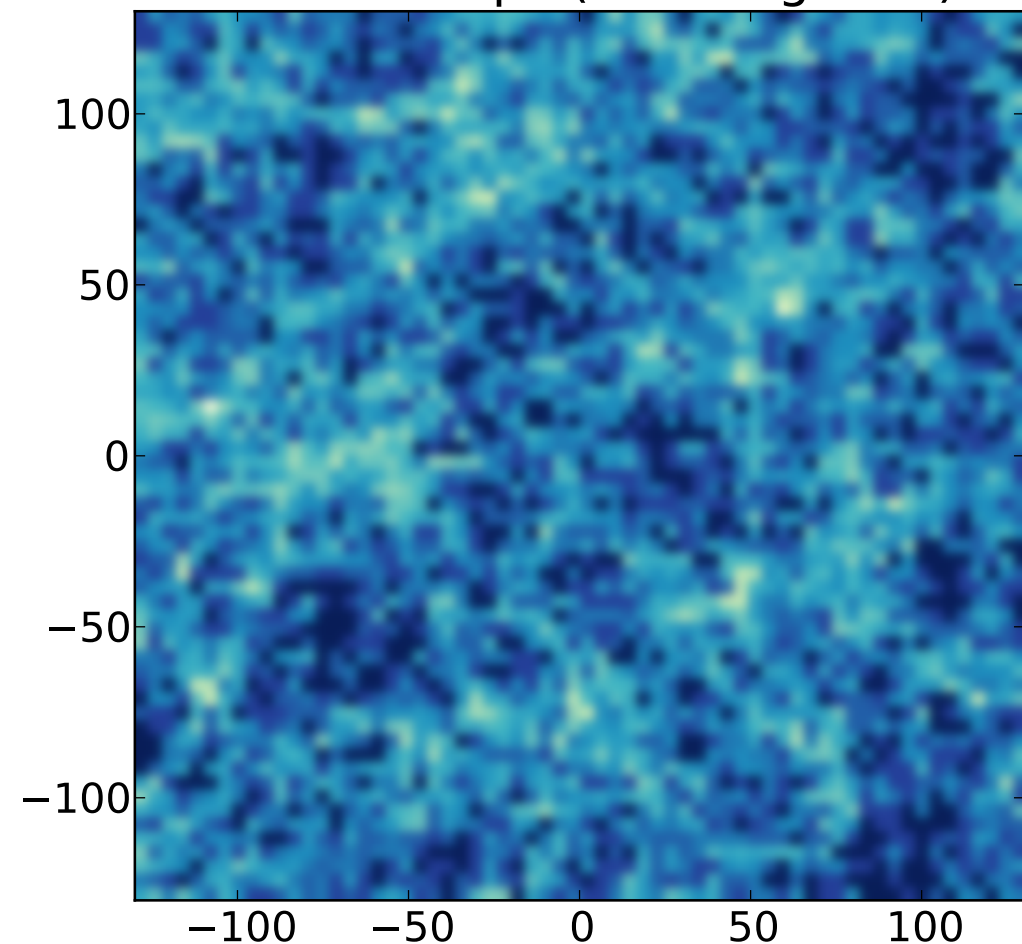
### Method 2: worse but works with less voids

We transform the line of sight coordinate by a factor proportional to  $e_v(z)$ , until it matches the void, so we get stretch...



From stretch we constrain  $D_A(z)H(z)$

→  $54 h^{-1} \text{Mpc}$  (HOD HighRes)



BOX:  $1 h^{-1} \text{Gpc}$  side

$1024^3$  particles

Mass resolution:

$7.36 \times 10^{11} h^{-1} M_{\odot}$

Rockstar halo finder  
(Behroozi et al. 2013)

Dense: halos above  $1.47 \times 10^{12} h^{-1} M_{\odot}$

Sparse:  $1.2 \times 10^{13} h^{-1} M_{\odot}$

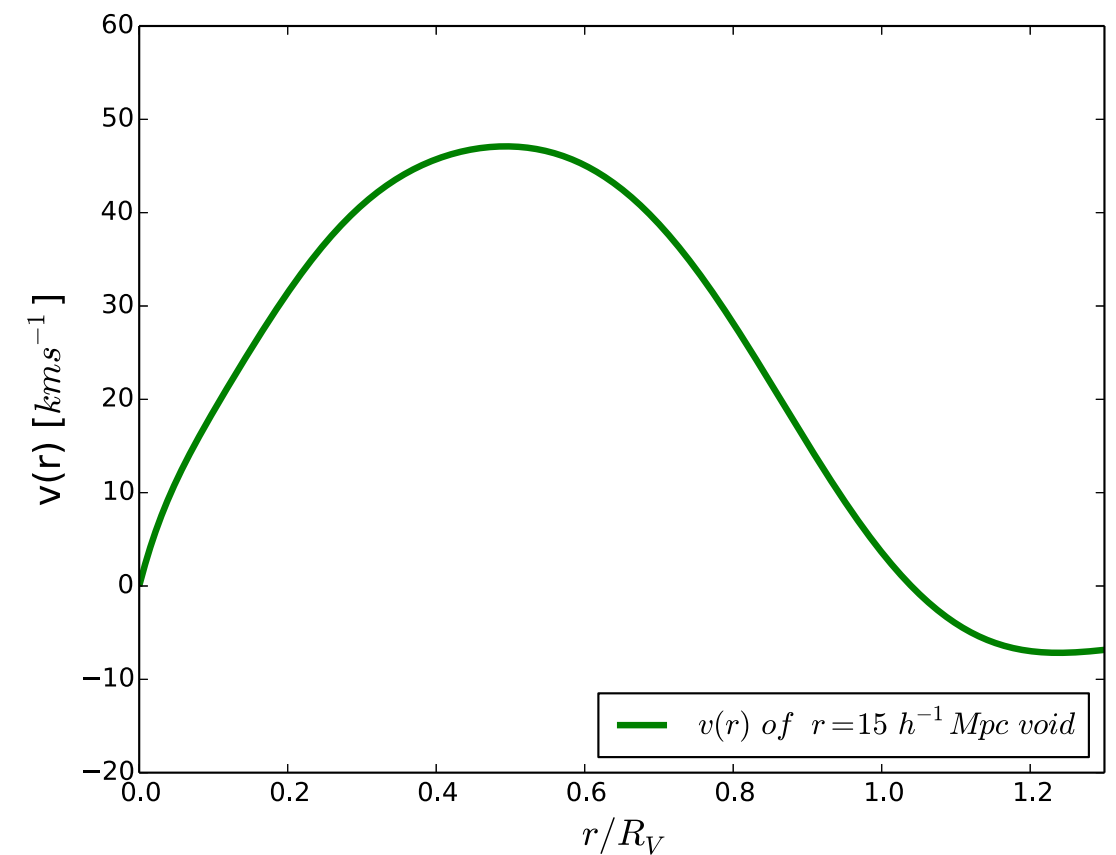
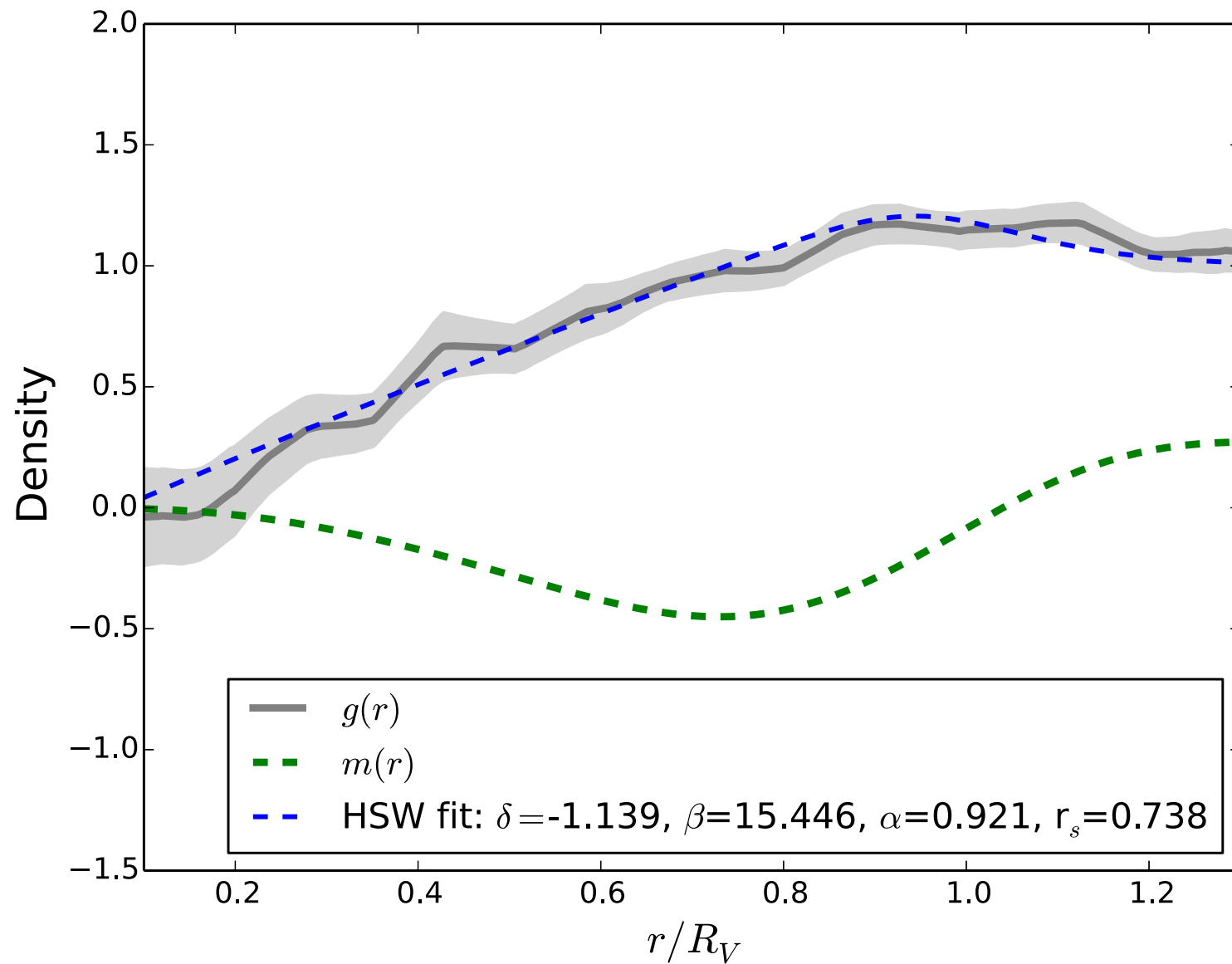
HOD modeling  $\Rightarrow$  mock catalog

WMAP 7-year cosmological parameters

Simulation: 2HOT code, adaptive treecode

N-body method, standard symplectic integrator (Quinn et al. 1997)

# A prediction for mass and velocity



mass

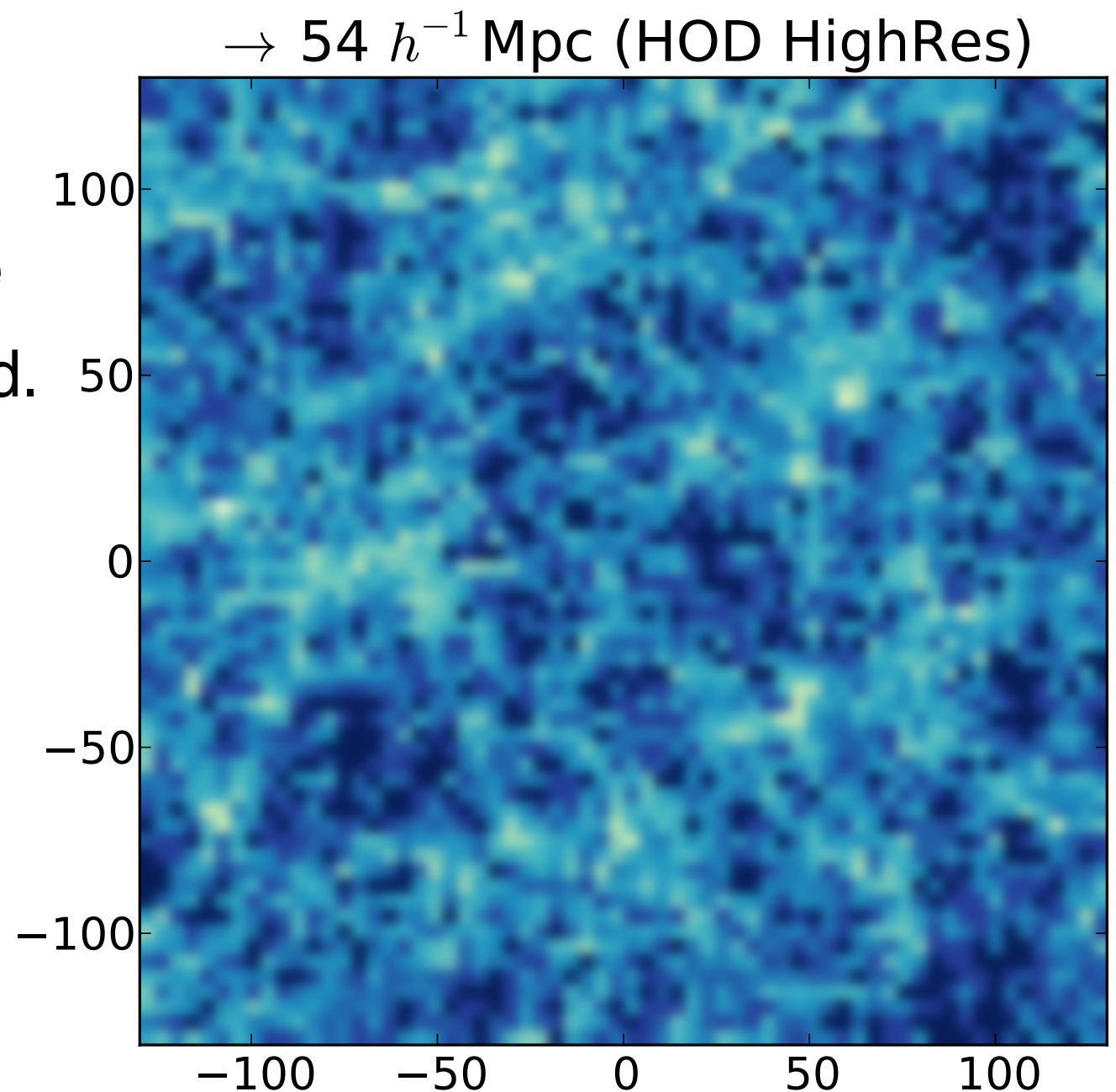
velocity



# Matching algorithm

A "potential match" is any void whose macrocenter lies within the Voronoi volume of the original void.

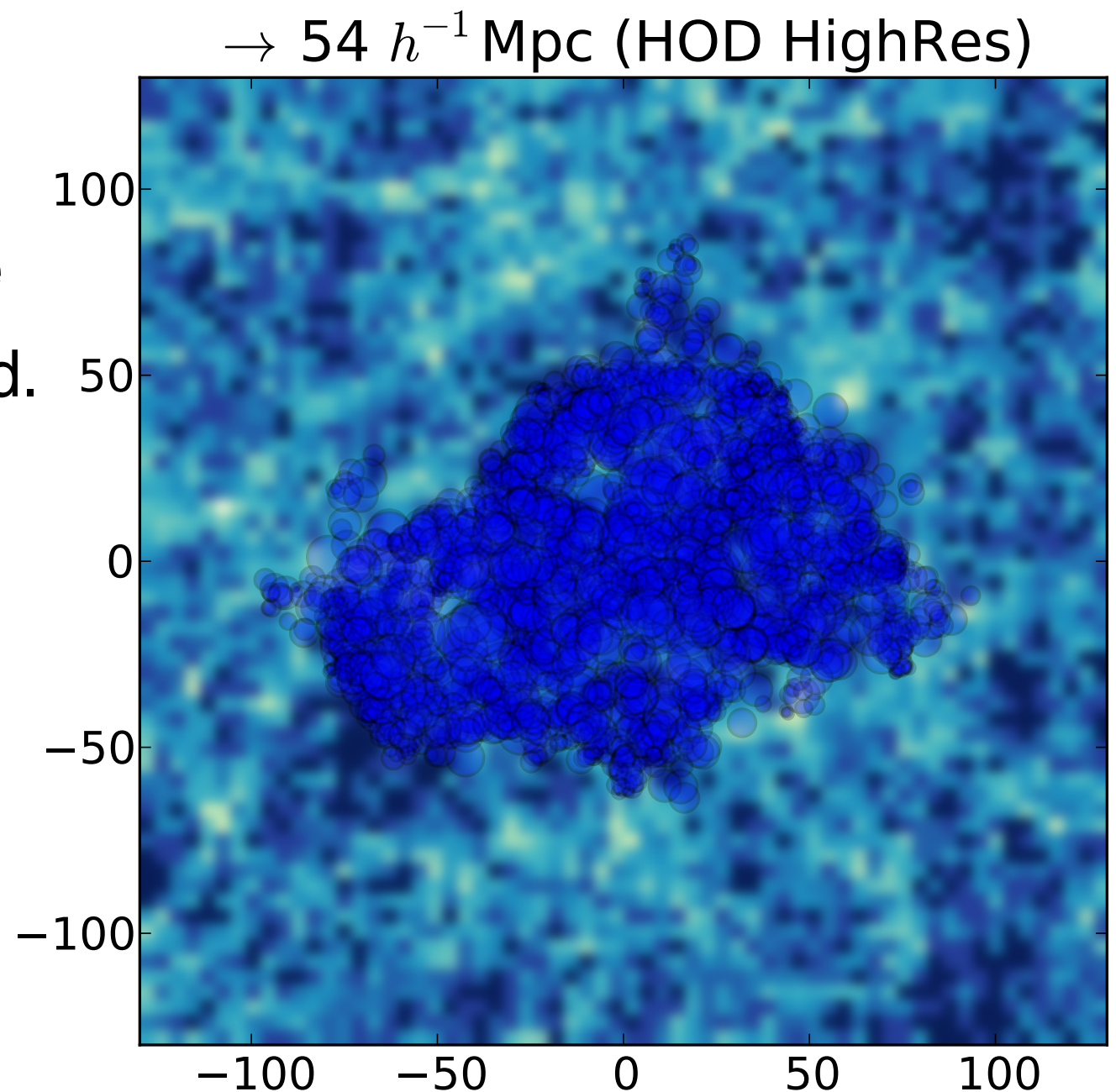
The best match is the one with major cells overlap



# Matching algorithm

A "potential match" is any void whose macrocenter lies within the Voronoi volume of the original void.

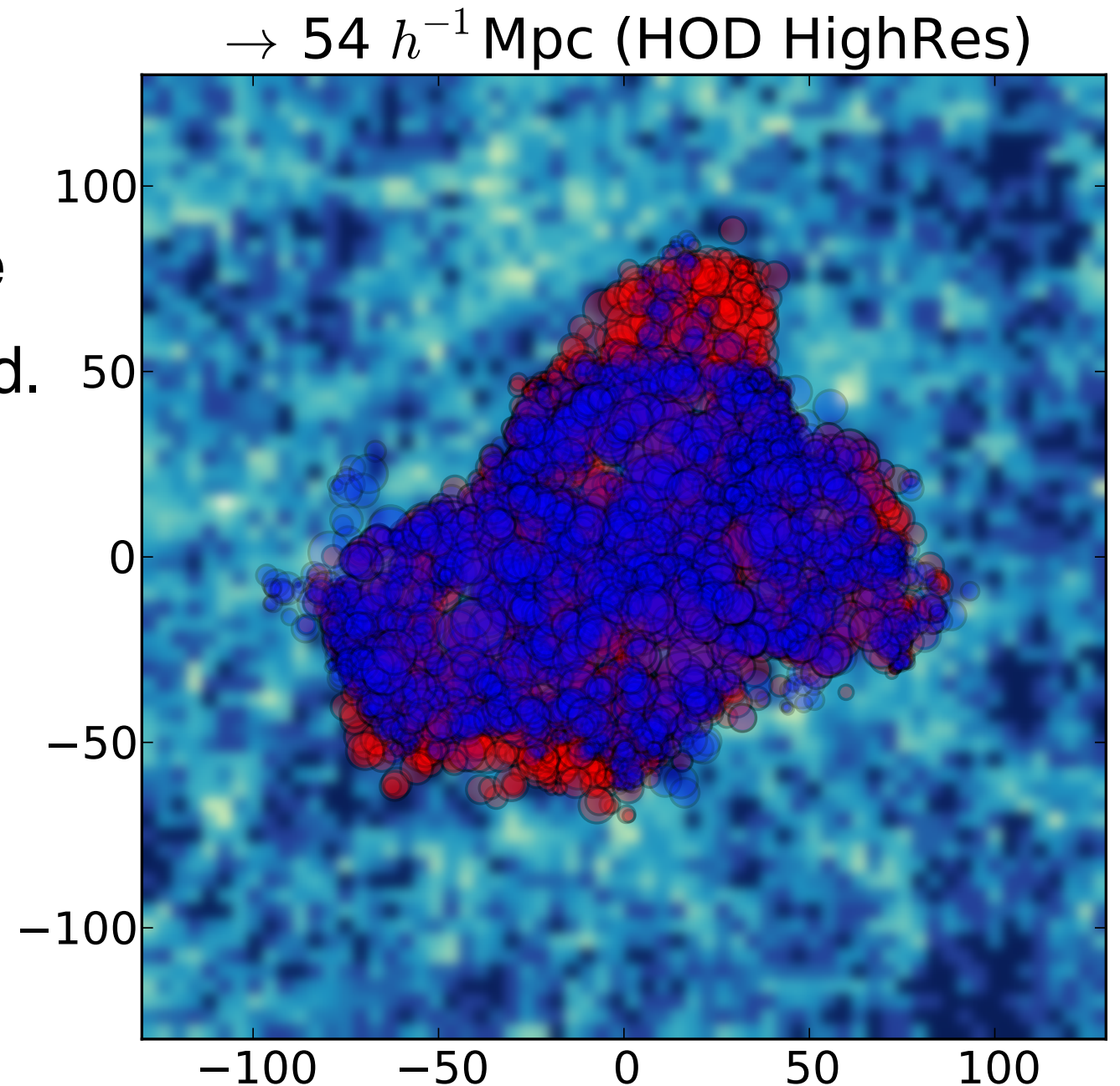
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# Matching algorithm

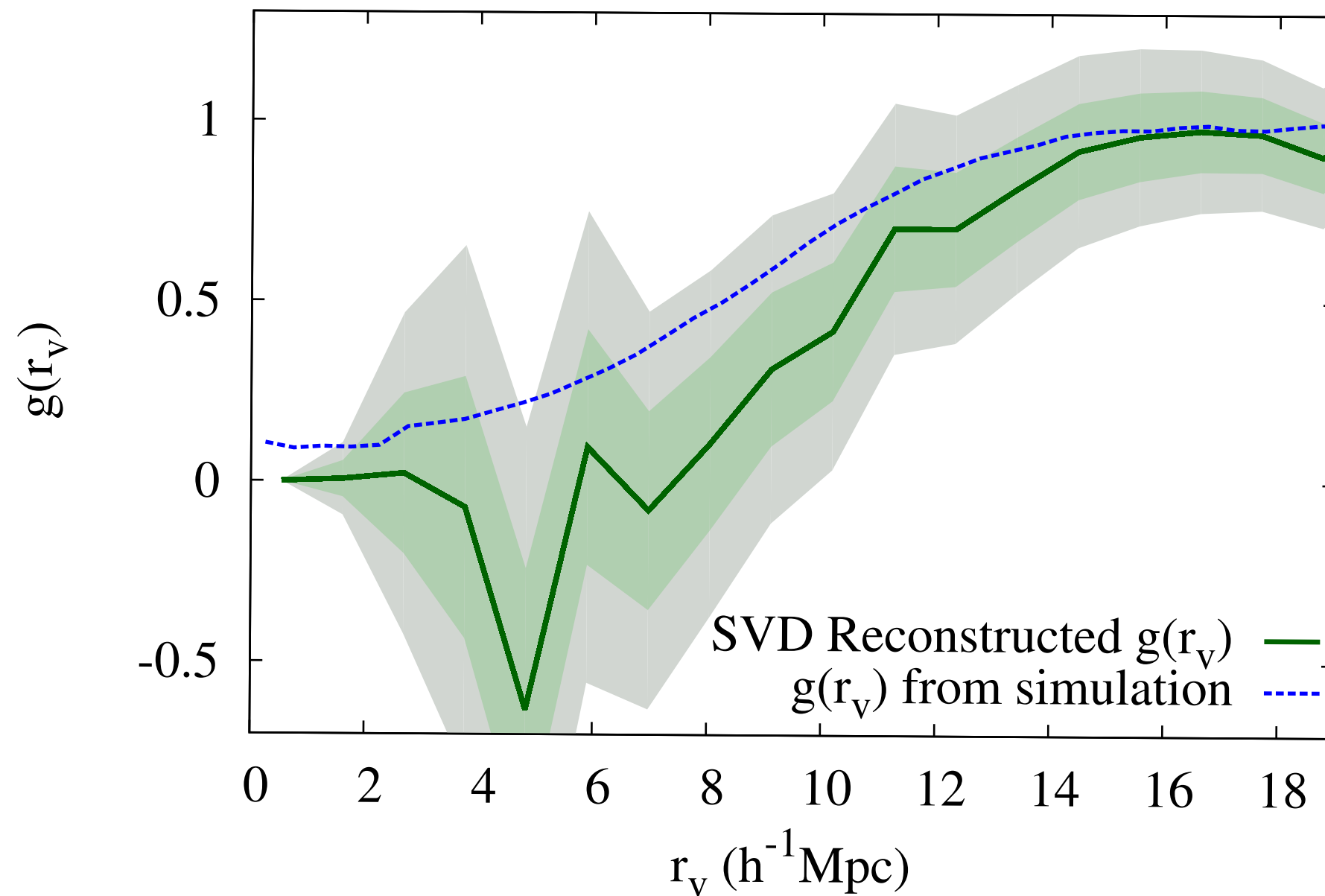
A "potential match" is any void whose macrocenter lies within the Voronoi volume of the original void.

The best match is the one with major cells overlap





# SVD reconstruction



$$M = \frac{4\pi}{3} R_{\text{Lag}}^3 \rho_{\text{back}}(0) = (1 + \Delta_V) \frac{4\pi}{3} R_{\text{Eul}}^3 \rho_{\text{back}}(0)$$

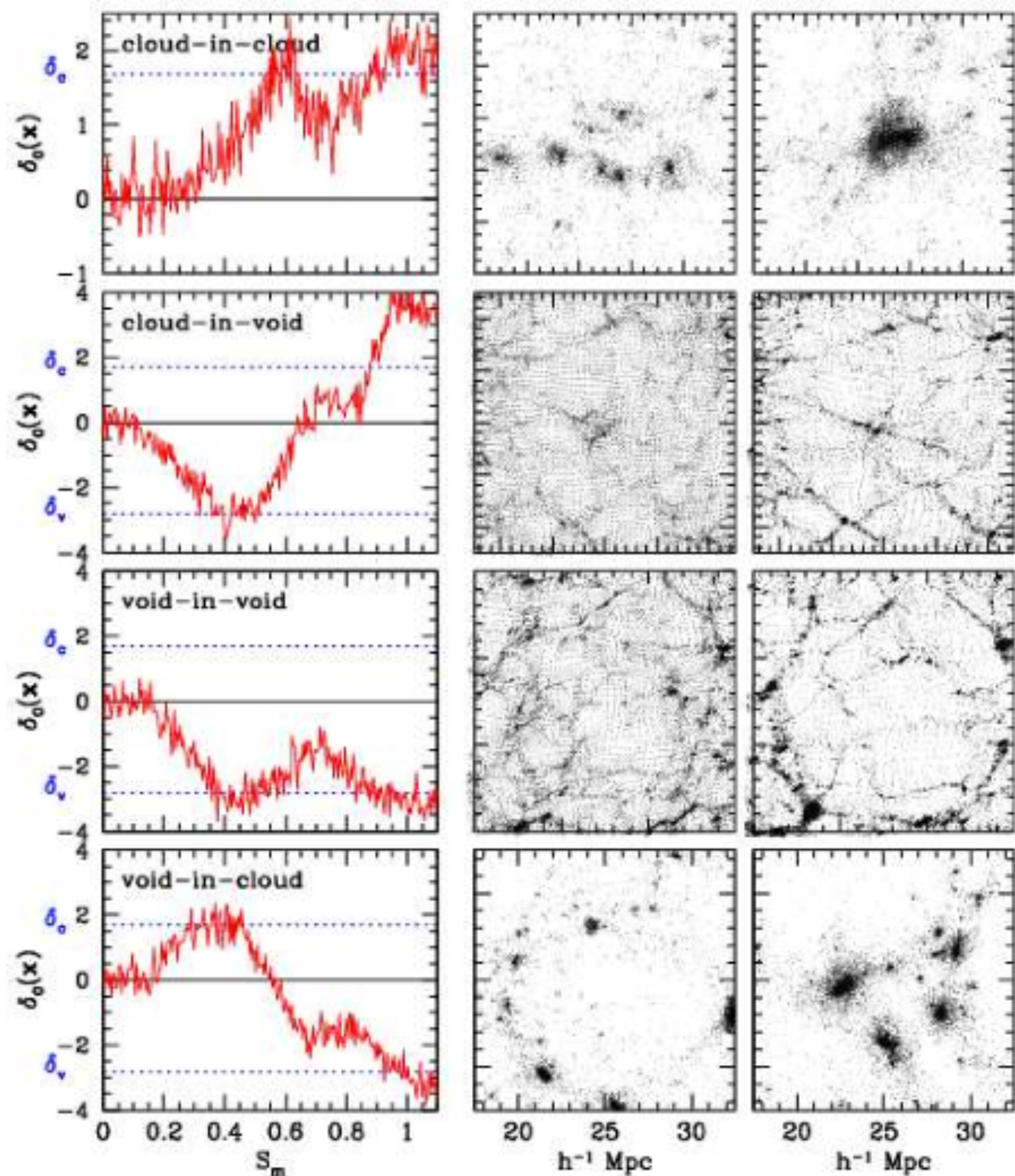
We use the extension of Press-Schechter for voids, proposed by Sheth and Van de Weygaert.

There is a turn over at small radii, because small voids collapse. We put ourselves above that turn over in radius (otherwise we should have assumed the two barrier excursion set model also with  $d_c$ )+ we consider  $2M_{\text{ps}}$  as limit.



# Sheth & Van de Weygaert 2004

Another natural choice is  $\delta_{ta} = 1.06$ ; this ignores all voids that are within regions which are beginning to turnaround, even though they may still have non-negligible sizes, and so underestimates the abundance of large voids. Accounting more carefully for the effect of the void-in-cloud problem is the subject of ongoing work.



**Figure 6.** Four mode (extended) excursion set formalism. Each row illustrates one of the four basic modes of hierarchical clustering: the *cloud-in-cloud* process, *cloud-in-void* process, *void-in-void* process and *void-in-cloud* process (from top to bottom). Each mode is illustrated using three frames. Leftmost panels show ‘random walks’: the local density perturbation  $\delta_0(\mathbf{x})$  as a function of (mass) resolution scale  $S_m$  (cf. Fig. 5) at an early time in an N-body simulation of cosmic structure formation. In each graph, the dashed horizontal lines indicate the collapse barrier  $\delta_c$  and the shell-crossing void barrier  $\delta_v$ . The two frames on the right show how the associated particle distribution evolves. Whereas halos within voids may be observable (second row depicts a halo within a larger void), voids within collapsed halos are not (last row depicts a small void which will be squeezed to small size as the surrounding halo collapses). It is this fact which makes the calculation of void sizes qualitatively different from that usually used to estimate the mass function of collapsed halos.

# Sheth & Van de Weygaert

apply this on voids, considering that a void is formed when  $\delta_v$ , the linearly extrapolated UNDERDENSITY is reached (when a shell forms around a sphere)