Physique des neutrinos (théorie)
Cours 1

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- les neutrinos dans le Modèle Standard
- neutrinos massifs - Dirac versus Majorana
- mélange de saveur - matrice PMNS
- oscillations dans le vide et violation de CP
- propagation des neutrinos dans la matière
- échelle de masse et nature des neutrinos
- neutrinos stériles

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Neutrinos in the electroweak Standard Model

Gauge group : $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\rm QED}$

(couplings) (g) (g') $(e = g\sin\theta_W)$

The spontaneous breaking of the electroweak gauge group leads to 2 massive gauge bosons (W±, Z) and 1 massless gauge boson (the photon γ)

Fermions : come in three generations (family replication)

LH fermions → SU(2) doublets $(T^3 = \pm 1/2)$ → couple to the W

RH fermions → SU(2) singlets $(T^3 = 0)$ → do not couple to the W

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L \qquad Y = -1$$

$$e_R \quad \mu_R \quad \tau_R \qquad Y = -2$$

$$Q = T^3 + \frac{Y}{2}$$

Leptons from different generations are distinguished by their flavour (e, μ, τ), which labels the charged lepton mass eigenstates

Neutrinos are special fermions :

1) have only weak interactions $\Rightarrow$ very small interaction rate (this is why detecting them is so difficult)

2) no electric charge $\Rightarrow$ can be their own antiparticles (Majorana fermions)

3) the SM as originally defined contains no RH neutrino, since only ν_L (or more precisely the left-helicity neutrino) has been observed [Goldhaber 1958]
$\Rightarrow$ neutrinos are massless in the SM

A fermion mass term involves both chiralities:

$$\mathcal{L}_{\rm mass} = -m\,\bar\psi\psi = -m\,(\bar\psi_L\psi_R + \bar\psi_R\psi_L) \qquad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

[way out: Majorana mass term, but cannot be generated in the SM]

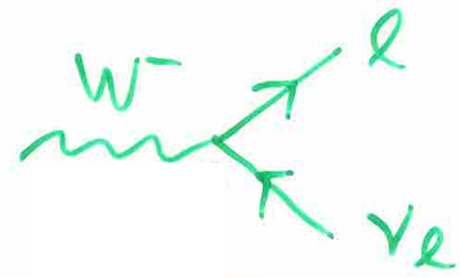
4) neutrinos are actually massive, but their masses are much smaller (< 1 eV) than the ones of charged leptons and quarks; also their mixing angles (PMNS matrix) are large, while those of the quarks (CKM matrix) are small

Neutrinos interactions

Only couple to the W and the Z bosons :

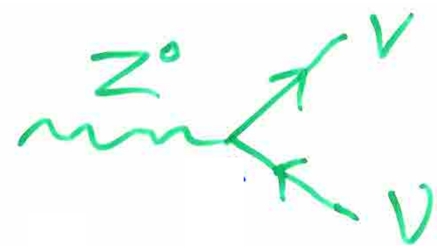
$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha=e,\mu,\tau} \bar{e}_{\alpha L}\gamma^\mu \nu_{\alpha L}$$

$$= \frac{g}{\sqrt{2}} W_\mu^- \left(\bar{e}_L\gamma^\mu\nu_{eL} + \bar{\mu}_L\gamma^\mu\nu_{\mu L} + \bar{\tau}_L\gamma^\mu\nu_{\tau L}\right)$$



$$\mathcal{L}_{\mathrm{NC}} = \frac{g}{2\cos\theta_W} Z_\mu \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L}$$

$$= \frac{g}{2\cos\theta_W} Z_\mu \left(\bar{\nu}_{eL}\gamma^\mu\nu_{eL} + \bar{\nu}_{\mu L}\gamma^\mu\nu_{\mu L} + \bar{\nu}_{\tau L}\gamma^\mu\nu_{\tau L}\right)$$



θw = angle de Weinberg $\quad \sin^2\theta_W = \dfrac{g'^2}{g^2 + g'^2}$

Neutrinos only couple to the W and the Z bosons :

$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}} W_\mu^- \left(\bar{e}_L \gamma^\mu \nu_{eL} + \bar{\mu}_L \gamma^\mu \nu_{\mu L} + \bar{\tau}_L \gamma^\mu \nu_{\tau L}\right)$$

$$\mathcal{L}_{\mathrm{NC}} = \frac{g}{2\cos\theta_W} Z_\mu \left(\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{\nu}_{\mu L} \gamma^\mu \nu_{\mu L} + \bar{\nu}_{\tau L} \gamma^\mu \nu_{\tau L}\right)$$

All SM interactions (including the charged lepton couplings to the photon and the Z, and their Yukawa couplings) preserve lepton number

$$L = \sum_{\alpha=e,\mu,\tau} \left(N_{e_\alpha^-} + N_{\nu_\alpha} - N_{e_\alpha^+} - N_{\bar{\nu}_\alpha}\right) = L_e + L_\mu + L_\tau$$

(accidental global symmetry of the SM; follows from gauge and Lorentz invariance + renormalizability; not a fundamental symmetry)

Thus e.g. $\pi^+ \to \mu^+ \nu_\mu$ is allowed, but $\pi^+ \to \mu^+ \bar{\nu}_\mu$ is forbidden

In the absence of neutrino masses, lepton flavour (i.e. the individual quantum numbers L_e, L_μ, L_τ) is also exactly conserved. Neutrino masses induce lepton flavour violating (LFV) transitions $\nu_\alpha \to \nu_{\beta\neq\alpha}$ (oscillations), but also LFV processes like $\mu^+ \to e^+ \gamma$ and $\mu^+ \to e^+ e^+ e^-$, which however are extremely suppressed in the absence of new physics

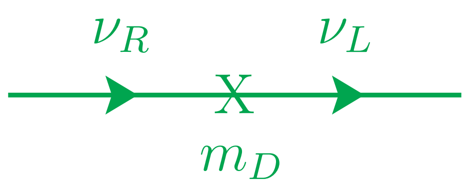
Massive neutrinos – Dirac versus Majorana

Dirac mass term

The simplest way to describe a massive neutrino is to add a ν_R to the SM and to write a Dirac mass term, as for the other fermions:

$$\mathcal{L}^{\text{Dirac}}_{\text{mass}} = -m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \right) \equiv -m_D \, \bar{\nu}_D \nu_D \qquad \nu_D \equiv \nu_L + \nu_R$$

The massive neutrino ν_D is a Dirac fermion (2 independent chiralities)



$$\Delta L = 0 \qquad \Delta T^3 = \frac{1}{2}$$

not invariant under $SU(2)_L \times U(1)_Y$ but can be generated from a Yukawa coupling to the SM Higgs doublet (which has weak isospin 1/2)

$$\mathcal{L}_{\text{Yuk.}} = -y_D \, \bar{L} \, i\sigma^2 H^* \nu_R + \text{h.c.} \quad \longrightarrow \quad m_D = y_D \frac{v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad m_\nu \lesssim 1\,\text{eV} \;\; \Rightarrow \;\; y_D \lesssim 10^{-11}$$

caveat : possible to write a Majorana mass term for ν_R $\Rightarrow$ end up with two Majorana neutrinos rather than one Dirac neutrino (see later)

Majorana mass term

Instead of introducing ν_R, form a RH spinor from ν_L

$$\nu_R^c \equiv C\,\overline{\nu}_L^T \qquad \sim \text{ CP conjugate of } \nu_L$$

C = charge conjugation matrix; defines the charge conjugate of a Dirac spinor

$$\psi(x) \;\rightarrow\; \psi^c(x) \equiv C\bar{\psi}^T(x) \quad \text{describes the corresponding antifermion}$$

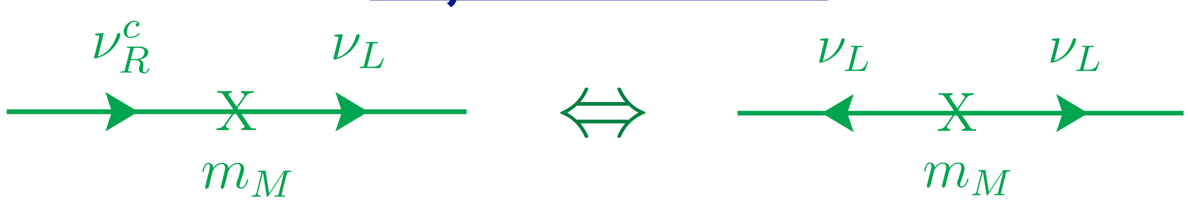
$\Rightarrow$ the existence of a LH neutrino (ν_L) implies the existence of a RH antineutrino ($\nu_R^c \sim \bar{\nu}_R$)

Can write a Majorana mass term :

$$\mathcal{L}_{\rm mass}^{\rm Maj.} = -\frac{1}{2}\, m_M \left(\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L\right) \equiv -\frac{1}{2}\, m_M\, \bar{\nu}_M \nu_M \qquad \nu_M \equiv \nu_L + \nu_R^c$$

The massive neutrino $\nu_M = \nu_L + \nu_R^c$ satisfies the Majorana condition

$\nu_M = \nu_M^c$ $\rightarrow$ Majorana fermion



$\Delta L = 2 \qquad \Delta T^3 = 1$

A Majorana mass term violates lepton number (signature of a Majorana neutrino) and cannot be generated from a coupling to the SM Higgs doublet

$\Rightarrow$ neutrino masses require an extension of the SM

Dirac versus Majorana neutrino

A Dirac neutrino is different from its antiparticle ($\nu \neq \nu^c$)

$\Rightarrow$ describes 4 degrees of freedom: $\nu\uparrow, \ \nu\downarrow, \ \bar{\nu}\uparrow, \ \bar{\nu}\downarrow$

Described by a 4-component spinor $\nu_D = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$ with independent LH and RH components

A Majorana neutrino satisfies the condition $\nu = \nu^c = C\bar{\nu}^T$

$\Rightarrow$ describes only 2 degrees of freedom: $\nu\downarrow, \ \bar{\nu}\uparrow$

Can be described by a 4-component spinor $\nu_M = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$, but the LH and RH components are not independent as $\nu_M = \nu_M^c$: $\nu_R = C\bar{\nu}_L^T$

The Majorana condition is inconsistent with any conserved additive quantum number: if ψ possesses a conserved quantum number q,

$$\psi \to e^{i\theta q} \quad \Rightarrow \quad \psi^c \to e^{-i\theta q}$$

Thus only neutrinos (not quarks, charged leptons) can be Majorana fermions

For the same reason, one cannot rephase a Majorana neutrino

How to distinguish Majorana from Dirac neutrinos?

Dirac and Majorana neutrinos have the same gauge interactions, since weak interactions only involve ν_L and its antiparticle $\nu_R^c \sim \bar{\nu}_R$ (ν_R, if it exists, is a gauge singlet and does not interact at all)

For the same reason, oscillations probabilities are the same for Dirac and Majorana neutrinos (production and detection are weak interaction processes)

The only practical difference between Dirac and Majorana neutrinos lies in their mass term, which violates lepton number by 2 units in the Majorana case

→ the Majorana nature of neutrinos can be established in $\Delta L = 2$ processes such as neutrinoless double beta decay

How to account for neutrino masses?

Simplest possibility: add a RH neutrino to the SM

In addition to the Dirac mass term $-m_D\,\bar{\nu}_L N_R + \mathrm{h.c.}$, must write a Majorana mass term for the RH neutrino, which is allowed by all (non-accidental) symmetries of the SM (or justify its absence):

$$-\frac{1}{2}\,M\,\bar{N}_L^c N_R + \mathrm{h.c.} \;=\; -\frac{1}{2}\,M\,N_R^T C N_R + \mathrm{h.c.} \qquad \Delta L = 2 \qquad \Delta T^3 = 0$$

[only lepton number, if imposed, can forbid this term]

<u>Mass eigenstates :</u> write the mass terms in a matrix form and diagonalize

$$\mathcal{L}_{\mathrm{mass}} \;=\; -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{N}_L^c\end{pmatrix}\begin{pmatrix}0 & m_D\\ m_D & M\end{pmatrix}\begin{pmatrix}\nu_R^c\\ N_R\end{pmatrix} + \mathrm{h.c.}$$

$$= -\frac{1}{2}\begin{pmatrix}\bar{\nu}_{L1} & \bar{\nu}_{L2}\end{pmatrix}\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}\begin{pmatrix}\nu_{R1}^c\\ \nu_{R2}^c\end{pmatrix} + \mathrm{h.c.}$$

where $$\begin{cases}\nu_{L1} \;=\; \cos\theta\,\nu_L - \sin\theta\,\nu_L^c\\ \nu_{L2} \;=\; \sin\theta\,\nu_L + \cos\theta\,\nu_L^c\end{cases}$$

Defining $\nu_{Mi} \equiv \nu_{Li} + \nu_{Ri}^c$ (such that $\nu_{Mi} = \nu_{Mi}^c$), one can see that the mass eigenstates are 2 Majorana neutrinos with masses m1 and m2 :

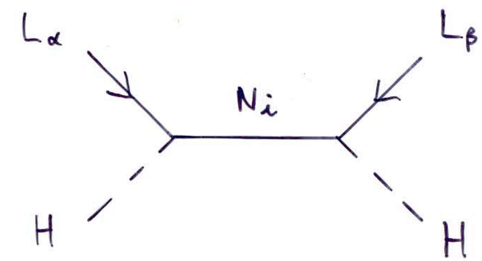
$$\mathcal{L}_{\rm mass} = -\frac{1}{2} \sum_{i=1,2} m_i \, \bar{\nu}_{Li} \nu_{Ri}^c + \text{h.c.} = -\frac{1}{2} \sum_{i=1,2} m_i \, \bar{\nu}_{Mi} \nu_{Mi}$$

<u>"Seesaw" limit :</u> $M \gg M_W \gtrsim m_D$

(N_R = gauge singlet $\Rightarrow$ M unconstrained by electroweak symmetry breaking)

$$m_1 \simeq -m_D^2/M \ll M_W \qquad m_2 \simeq M \gg M_W$$

$$\sin\theta \simeq -\frac{m_D}{M} \ll 1 \quad \Rightarrow \quad \nu_{L1} \simeq \nu_L \,, \quad \nu_{L2} \simeq N_L^c$$



→ the light Majorana neutrino is essentially the SM neutrino

→ natural explanation of the smallness of neutrino masses

<u>New physics interpretation :</u> M = characteristic scale of the new physics responsible for lepton number violation – might be related to Grand Unification: the fermion content of SO(10) includes a RH neutrino in addition to the SM fermions, and B-L is a generator of SO(10)

Alternative mechanisms of neutrino (Majorana) mass generation :

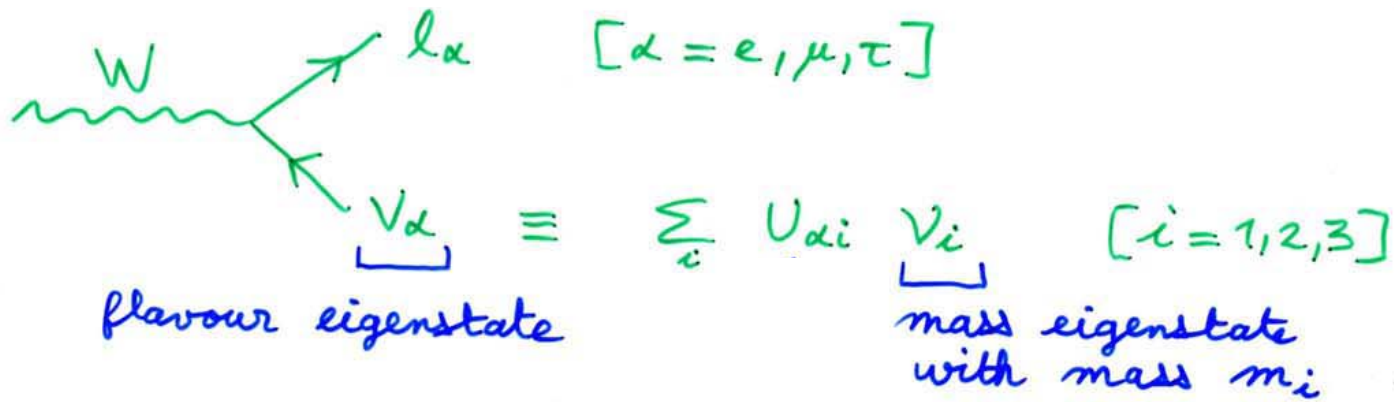
- other versions of the seesaw mechanism with heavy SU(2) triplets (scalar or fermionic)

- radiative models: neutrino masses generated at the one-loop (Zee) or two loop level (Babu-Zee)

- more exotic: supersymmetric models with R-parity violation (in which lepton number is violated), extra spatial dimensions...

Flavour mixing – PMNS matrix

When neutrinos are massive, possibility of flavour mixing : the neutrino to which a given charged lepton (e, μ or τ) couples via the W is not a mass eigenstate, but a coherent superpositions of mass eigenstates



As for quarks, the origin of flavour mixing is the mismatch between the basis of gauge (or flavour) eigenstates and of mass eigenstates. The relative rotation is the lepton mixing matrix, known as PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

Physical parameters in $U_{\rm PMNS}$

U is a 3x3 unitary matrix ⇒ 3 mixing angles and 6 phases (not all physical)

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} W_\mu^- \sum_\alpha \bar{e}_{\alpha L} \gamma^\mu \nu_{\alpha L} = \frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha,i} \bar{e}_{\alpha L} \gamma^\mu U_{\alpha i} \nu_{iL}$$

(i) if neutrinos are Dirac fermions : analogous to quarks and CKM

can rephase the lepton fields $e_{\alpha L} \to e^{i\phi_\alpha} e_{\alpha L}\ ,\quad \nu_{iL} \to e^{i\phi_i} \nu_{iL}$ and absorb the phases in the PMNS matrix, so that CC interactions are unaffected

$$U_{\alpha i} \to e^{i(\phi_\alpha - \phi_i)} U_{\alpha i}$$

⇒ removes 2x3 - 1 = 5 relative phases ⇒ a single physical phase $\delta_{\rm PMNS}$

(i) if neutrinos are Majorana fermions : cannot rephase the neutrino fields, since this would affect the Majorana condition

$$U_{\alpha i} \to e^{i\phi_\alpha} U_{\alpha i}$$

⇒ removes only 3 phases ⇒ 3 physical phases : 1 "Dirac" phase $\delta_{\rm PMNS}$ and 2 "Majorana" phases

Standard parametrization of the PMNS matrix

Analogous to CKM: written as the product of three rotations with angles θ_{23}, θ_{13} and θ_{12}, the second (complex) rotation depending on the phase δ

$$U \equiv U_{23}U_{13}U_{12} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P$$

P is the unit matrix in the Dirac case, and a diagonal matrix of phases containing 2 independent phases ϕ_i in the Majorana case

$$c_{ij} = \cos\theta_{ij}\ , \ \ s_{ij} = \sin\theta_{ij}$$

$$\theta_{ij} \in [0, \pi/2]\,, \quad \delta \in [0, 2\pi[\,, \quad \phi_i \in [0, \pi[$$

$\delta \neq 0, \pi \ \Rightarrow$ CP violation in oscillations: $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

The Majorana phases play a role only in $\Delta L = 2$ processes like neutrinoless double beta decay

Neutrino oscillations in vacuum and CP violation

Oscillations are a quantum-mechanical process due to neutrino mass and mixing. An (ideal) oscillation experiment involves 3 steps:

1) production of a pure flavour state at t = 0 (e.g. a ν_μ from $\pi^+ \to \mu^+ \nu_\mu$)

This flavour state is a coherent superposition of mass eigenstates determined by the PMNS matrix, e.g. in the 2 flavour case

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

2) propagation

Each mass eigenstate, being an eigenstate of the Hamiltonian in vacuum, evolves with its own phase factor $e^{-iE_i t}$ $\Rightarrow$ modifies the coherent superposition, which is no longer a pure flavour eigenstate:

$$|\nu(t)\rangle = -\sin\theta\, e^{-iE_1 t}|\nu_1\rangle + \cos\theta\, e^{-iE_2 t}|\nu_2\rangle$$

3) detection via a CC interaction which identifies a specific flavour

probability amplitude : $\langle\nu_e|\nu(t)\rangle = -\cos\theta\sin\theta e^{-iE_1 t} + \cos\theta\sin\theta e^{-iE_2 t}$

oscillation probability : $P(\nu_\mu \to \nu_e; t) = |\langle\nu_e|\nu(t)\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{E_2 - E_1}{2}\, t\right)$

2-flavour oscillations in vacuum

Assuming ultra-relativistic neutrinos $L \simeq ct$

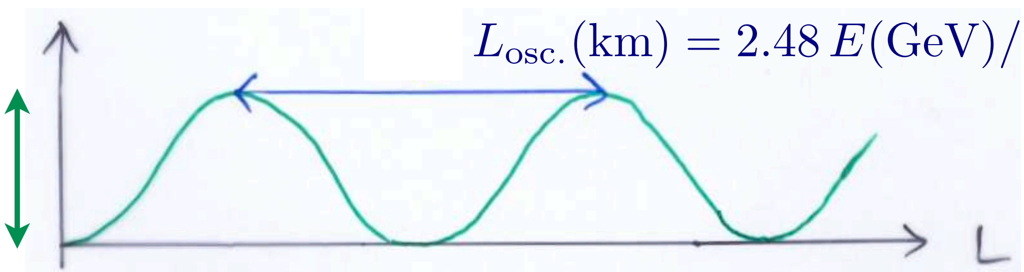
$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \quad \Rightarrow \quad \frac{E_2 - E_1}{2} \simeq \frac{m_2^2 - m_1^2}{4p}$$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

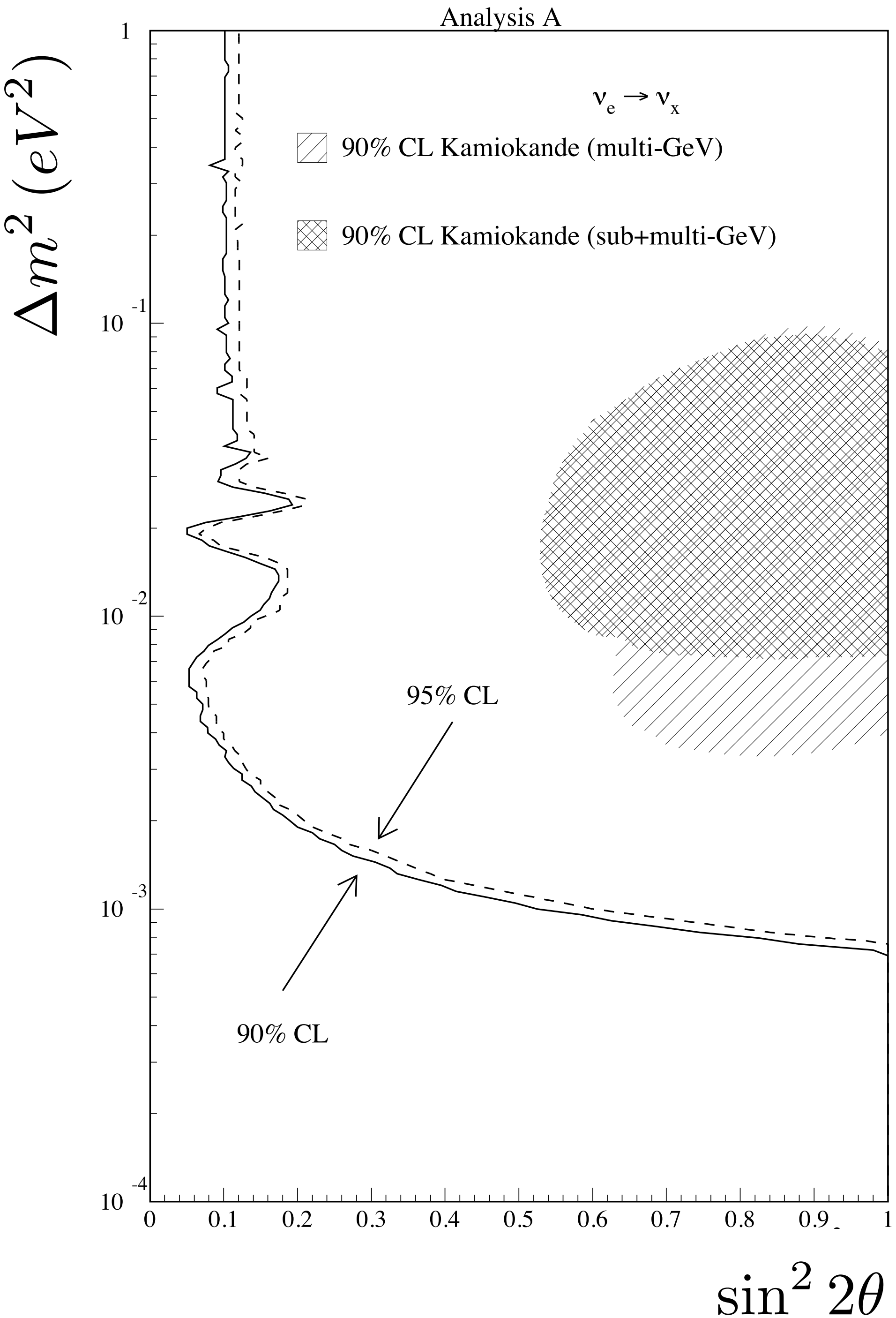
$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$L_{\text{osc.}}(\text{km}) = 2.48\, E(\text{GeV})/\Delta m^2(\text{eV})$$

$\sin^2 2\theta$



A typical exclusion curve (CHOOZ) :



CHOOZ Collaboration, hep-ex/9907037

3-flavour oscillations in vacuum

$$\nu_\alpha(x) = \sum_i U_{\alpha i}\, \nu_i(x)$$ **(fields)** $\Rightarrow$ $$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\, |\nu_i\rangle$$ **(states)**

and for antineutrinos $$|\bar\nu_\alpha\rangle = \sum_i U_{\alpha i}\, |\bar\nu_i\rangle$$

1) production: $|\nu(t=0)\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$

2) propagation: $|\nu(t)\rangle = \sum_i U^*_{\alpha i}\, e^{-iE_i t} |\nu_i\rangle$

3) detection: $\langle \nu_\beta | \nu(t)\rangle = \sum_j U_{\beta j} \langle \nu_j | \nu(t)\rangle = \sum_i U_{\beta i} U^*_{\alpha i}\, e^{-iE_i t}$

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu(t)\rangle|^2 = \left|\sum_i U_{\beta i} U^*_{\alpha i}\, e^{-iE_i t}\right|^2$$

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)$$

$$+ 2 \sum_{i<j} \mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right)$$

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{ji}L}{4E}\right)$$

$$+ 2\sum_{i<j} \mathrm{Im}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{ji}L}{2E}\right)$$

The second term is CP-odd and changes sign for antineutrino oscillations

CP violation in oscillations in vacuum

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$ at leading order in Δm^2_{21} :

$$\Delta P_{\alpha\beta} = \pm 8\, J \left(\frac{\Delta m^2_{21} L}{2E} \right) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right), \qquad J \equiv \mathrm{Im}\left[U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2} \right]$$

Jarlskog invariant: $$J = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$$

→ conditions for CPV:

$$\theta_{ij} \neq 0\,, \quad \delta \neq 0, \pi\,, \qquad m_1 \neq m_2\,,\; m_2 \neq m_3\,,\; m_3 \neq m_1$$

muon to electron neutrino channel

$$P(\nu_\mu \to \nu_e) \; = \; -\,4 \sum_{i<j} \mathrm{Re}\left[U_{ei}U^*_{\mu i}U^*_{ej}U_{\mu j}\right] \sin^2\left(\frac{\Delta m^2_{ji}L}{4E}\right) - \; 4\,J\left(\frac{\Delta m^2_{21}L}{2E}\right)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

At second order in Δm^2_{21} and θ_{13} :

$$P(\nu_\mu \to \nu_e) \; \simeq \; \sin^2\theta_{23}\sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cos^2\theta_{23}\sin^2 2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right)$$

$$+\,\frac{1}{2}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\delta\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{2E}\right)$$

$$-\;\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

The last term is CP-odd and switches sign for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations