Physique des neutrinos (théorie)
Cours 2

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- les neutrinos dans le Modèle Standard
- neutrinos massifs - Dirac versus Majorana
- mélange de saveur - matrice PMNS
- oscillations dans le vide et violation de CP
- propagation des neutrinos dans la matière
- échelle de masse et nature des neutrinos
- neutrinos stériles

Ecole de Gif 2016: La physique souterraine
Aussois, 19-23 septembre 2016

Neutrino propagation in matter

The interaction of neutrinos with matter (e-, p, n) affect their propagation $\Longrightarrow$ modified oscillation parameters + a new phenomenon: matter-induced flavour conversion in a medium with varying density

Appropriate description: Schrödinger-like equation

$$i \frac{d}{dt} |\nu(t)\rangle = H |\nu(t)\rangle$$

The Hamiltonian H contains a potential term describing the interactions of the neutrinos with the medium and can depend on t

It is convenient to write the Schrödinger equation in the flavour eigenstate basis $\{ |\nu_\alpha\rangle , |\nu_\beta\rangle , \cdots \}$, in which $|\nu(t)\rangle = \sum_\beta \nu_\beta(t) |\nu_\beta\rangle$:

$$i \frac{d}{dt} \nu_\beta(t) = \sum_\gamma H_{\beta\gamma} \nu_\gamma(t)$$

$$\nu_\beta(t) = \langle \nu_\beta | \nu(t) \rangle$$

$$H_{\beta\gamma} = \langle \nu_\beta | H | \nu_\gamma \rangle$$

$\nu_\beta(t)$ is the probability amplitude to find the neutrino in the state $|\nu_\beta\rangle$ at t

if $|\nu(t=0)\rangle = |\nu_\alpha\rangle$, then $P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2$

Vacuum oscillations in the Schrödinger formalism (2-flavour case)

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_0\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad H_0 = U\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}U^\dagger$$

The Hamiltonian in vacuum H_0 is diagonalized by the PMNS matrix

One can check that this reproduces the standard oscillation formula (*)

It is customary to subtract a piece proportional to the unit matrix from H_0 (which only affects the overall phase of the neutrino state vector $|\nu(t)\rangle$, leaving oscillations unchanged) to bring it to the form:

$$H_0 = \begin{pmatrix}-\frac{\Delta m^2}{2E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & 0\end{pmatrix}$$

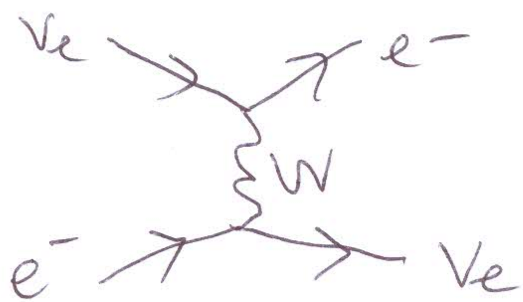
(*) proof: use the fact that $|\nu_\gamma\rangle = \sum_i U^*_{\gamma i}\,|\nu_i\rangle \;\Rightarrow\; \nu_\gamma(t) = \sum_i U_{\gamma i}\,\nu_i(t)$, where $\nu_i(t) \equiv \langle\nu_i|\nu(t)\rangle$ to show that the solution of the Schrödinger equation is $\nu_\gamma(t) = \sum_i U_{\gamma i}\,\nu_i(t)$ with $\nu_i(t) = e^{-iE_i t}\,\nu_i(0) = e^{-iE_i t}\,U^*_{\delta i}\,\nu_\delta(0)$, which reproduces the known oscillation formula

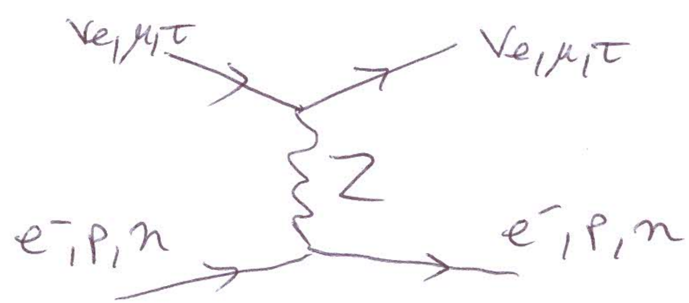
Neutrino propagation in matter

$$H_0 \;\rightarrow\; H_m = H_0 + V$$

V induced by interactions (anti-)neutrinos / e-, p, n of the medium

Relevant interactions: forward elastic scatterings ($\vec{p}_\nu$ unchanged)





CC – only for ν_e

NC – same for $\nu_{e,\mu,\tau}$ $\Rightarrow$ can be subtracted from Hm

$$V_{\rm CC} = \sqrt{2}\, G_F n_e(x)$$

$$V_{\rm NC} = -\frac{G_F}{\sqrt{2}}\, n_n(x)$$

G_F = Fermi constant

In the flavour eigenstate basis:

$$(H_m)_{\beta\gamma} \;=\; (H_0)_{\beta\gamma} + V_\beta\, \delta_{\beta\gamma} \qquad V_\beta = V^\beta_{\rm CC} + V^\beta_{\rm NC}$$

For anti-neutrinos, V has the opposite sign: $V \;\rightarrow\; -V$

For a sterile (= insensitive to weak interactions) neutrino: $V_\beta \;=\; 0$

Example with electron neutrinos and another flavour:

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_m \begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad \begin{cases}\alpha = e\\ \beta = \mu, \tau, s\end{cases}$$

$$H_m = \begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta + \sqrt{2}\,G_F n & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}$$

$$n = \begin{cases} n_e(x) & \text{if } \beta = \mu, \tau \\ n_e(x) - \frac{1}{2}\,n_n(x) & \text{if } \beta = s \end{cases}$$

For anti-neutrinos, $+\sqrt{2}\,G_F n \;\rightarrow\; -\sqrt{2}\,G_F n$

Energy levels in matter and matter eigenstates

In vacuum, the PMNS matrix U relates the flavour eigenstates to the mass eigenstates (= eigenstates of H_0):

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \qquad \begin{matrix} \leftarrow & E_1 = \sqrt{\vec{p}^2 + m_1^2} \\ \leftarrow & E_2 = \sqrt{\vec{p}^2 + m_2^2} \end{matrix}$$

In matter, one defines matter eigenstates = eigenstates of Hm

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U_m^* \begin{pmatrix} |\nu_1^m\rangle \\ |\nu_2^m\rangle \end{pmatrix} \qquad \left.\begin{matrix} \leftarrow & E_1^m \\ \leftarrow & E_2^m \end{matrix}\right\} \begin{matrix} \text{eigenvalues of } H_m = \\ \text{energy levels in matter} \end{matrix}$$

Um contains the mixing angle in matter that diagonalize Hm :

$$H_m = U_m \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$$

By analogy with $\nu_\beta(t) = \langle\nu_\beta|\nu(t)\rangle$, one defines $\nu_i^m(t) = \langle\nu_i^m|\nu(t)\rangle$

(amplitude of probability to find the neutrino in the ith matter eigenstate at t)

then

$$\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

Medium with constant matter density

n(x) = n ⇒ Hm, hence the matter eigenstates $|\nu_i^m\rangle$, energy levels E_i^m and mixing matrix Um, do not depend on t

Using $\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$, one can rewrite the Schrödinger equation for the probability amplitudes $\nu_i^m(t)$

$$i\frac{d}{dt}\begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

which is solved by $\nu_i^m(t) = e^{-iE_i^m t}\,\nu_i^m(0)$, giving

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2 = |\textstyle\sum_i (U_m)_{\beta i}\,\nu_i^m(t)|^2 = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

→ oscillations in matter with constant density are governed by the same formula as in vacuum, with the replacements

$$\theta \to \theta_m\,, \qquad \frac{\Delta m^2}{4E} \to \frac{(E_m^2 - E_m^1)}{2}$$

Oscillation parameters in matter

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E}\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$\cos 2\theta_m = \frac{\left(1-\frac{n}{n_{\rm res}}\right)\cos 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$n_{\rm res} \equiv \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E}$$

for antineutrinos,

$$n \to -n$$

($n = n_e$ if only active neutrinos)

MSW resonance (Mikheev-Smirnov-Wolfenstein):

$$\sin 2\theta_m = 1 \quad \text{for } n = n_{\rm res}$$

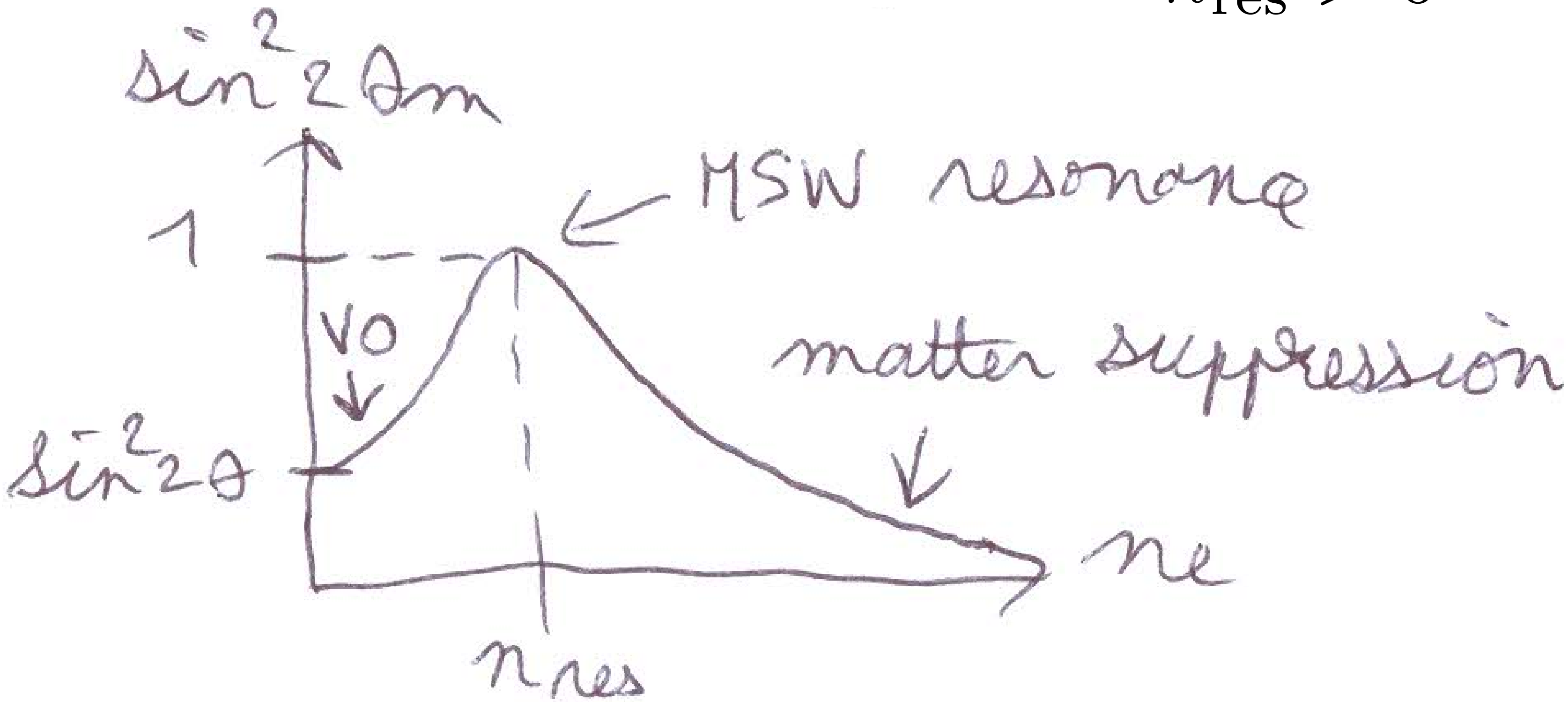
(irrespective of the value of the mixing angle in vacuum θ)

Resonance condition: $$\begin{cases} \Delta m^2 \cos 2\theta > 0 & \text{for neutrinos} \\ \Delta m^2 \cos 2\theta < 0 & \text{for antineutrinos} \end{cases}$$

When neutrino oscillations are enhanced, antineutrino oscillations are suppressed, and vice versa

Different regimes for oscillations in matter :

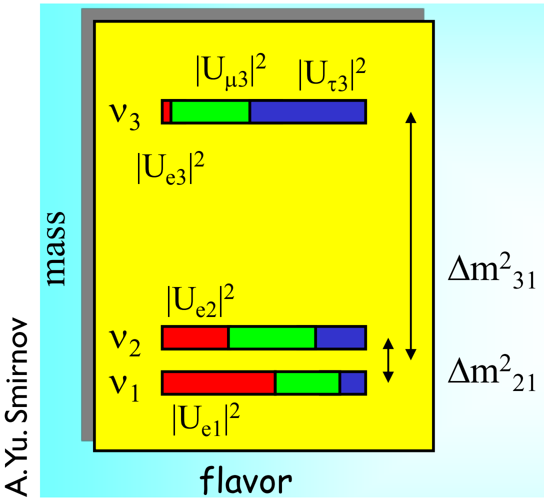
$n_{\rm res} > 0$



- low density ($n \ll n_{\rm res}$) : $\sin 2\theta_m \simeq \sin 2\theta$ $\Rightarrow$ vacuum oscillations
- resonance ($n = n_{\rm res}$) : $\sin 2\theta_m = 1$
- high density ($n \gg n_{\rm res}$) : $\sin 2\theta_m < (\ll) \sin 2\theta$ $\Rightarrow$ oscillations are suppressed by matter effects

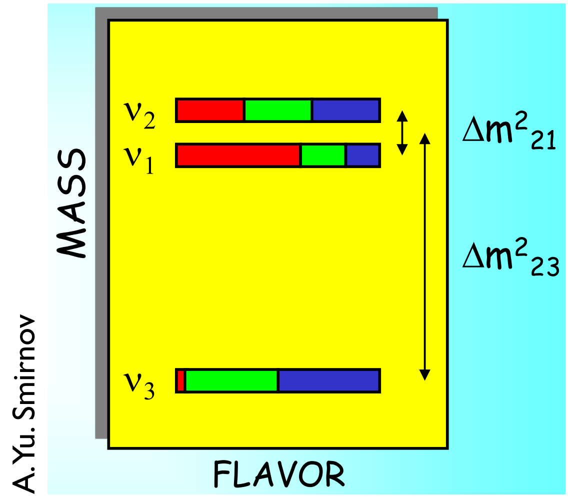
Application: determination of the mass hierarchy in long-baseline experiments

Two mass orderings allowed by experiments:



Normal hierarchy

$$\Delta m^2_{31} > 0$$



Inverted hierarchy

$$\Delta m^2_{31} < 0$$

In vacuum: $P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$

For long baselines (> several 100 km), matter effects cannot be neglected

$$n_{\rm res} = \frac{\Delta m^2_{31}\cos 2\theta_{13}}{2\sqrt{2}\,G_F E} \qquad \begin{cases} n_{\rm res} > 0 & \text{for normal hierarchy} \\ n_{\rm res} < 0 & \text{for inverted hierarchy} \end{cases}$$

If $n_{\rm res}$ is close to the Earth crust density, neutrino (antineutrino) oscillations are enhanced for NH (IH), while antineutrino (neutrino) oscillations are suppressed

[may have to disentangle CP violation from matter effect]

and outdated but informative plot

$$R = \frac{\overline{\nu}_e \to \overline{\nu}_\mu}{\nu_e \to \nu_\mu}$$

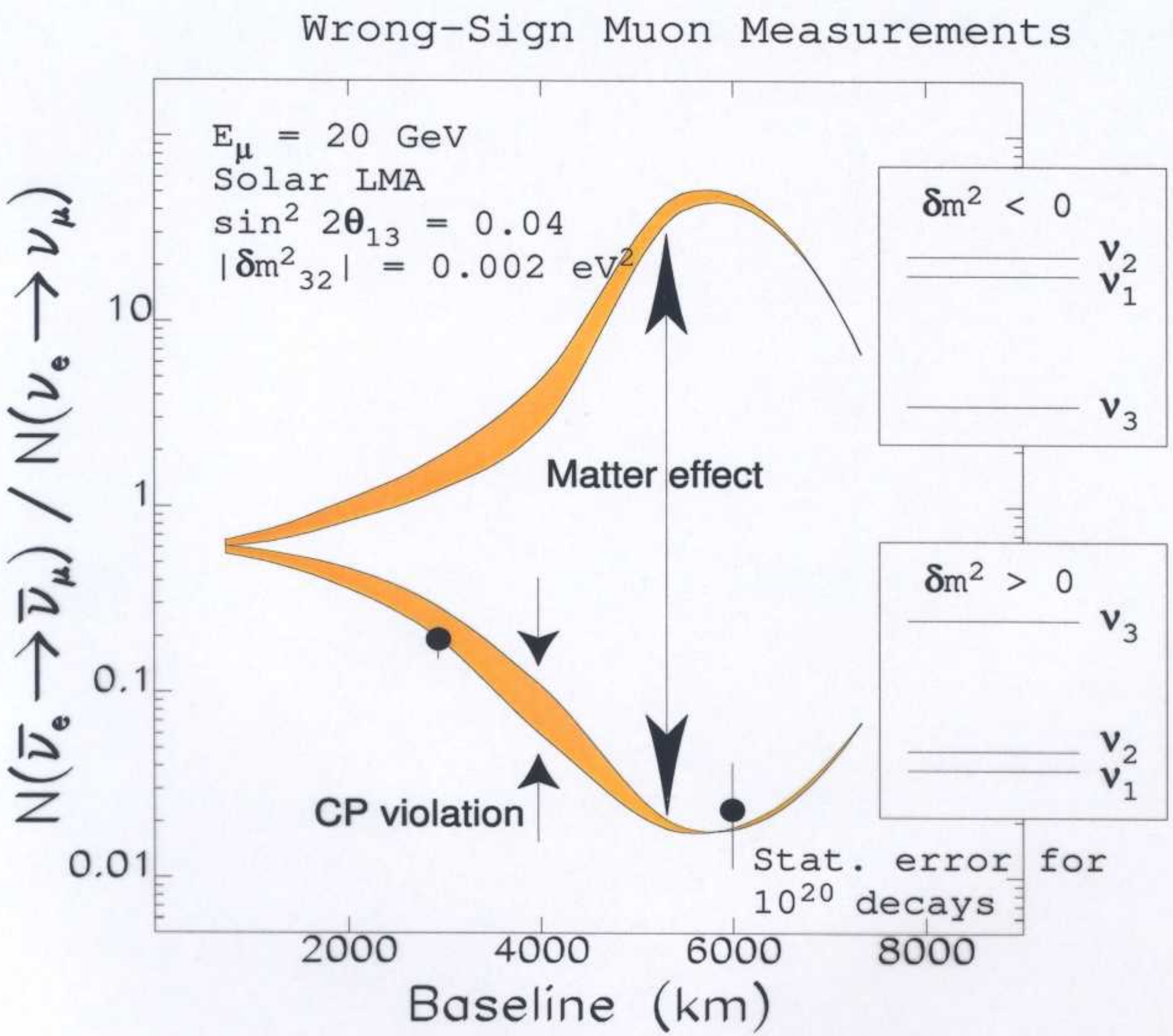


Figure 2: Predicted ratios of $\bar{\nu}_e \to \bar{\nu}_\mu$ to $\nu_e \to \nu_\mu$ rates at a 20 GeV neutrino factory. The statistical error shown corresponds to 10^{20} muon decays of each sign and a 50 kt detector.

- Un baseline de $L = \mathcal{O}$ (3000 km) est nécessaire/optimale

[Barger, Geer, Raja, Whisnant]

Medium of varying density (e.g. Sun)

Now the matter eigenstates, energy levels and mixing angle depend on t

→ "instantaneous" matter eigenstates: $|\nu_i^m(t)\rangle \leftarrow E_i^m(t)$

$$H_m = U_m \begin{pmatrix} E_1^m(t) & 0 \\ 0 & E_2^m(t) \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{pmatrix}$$

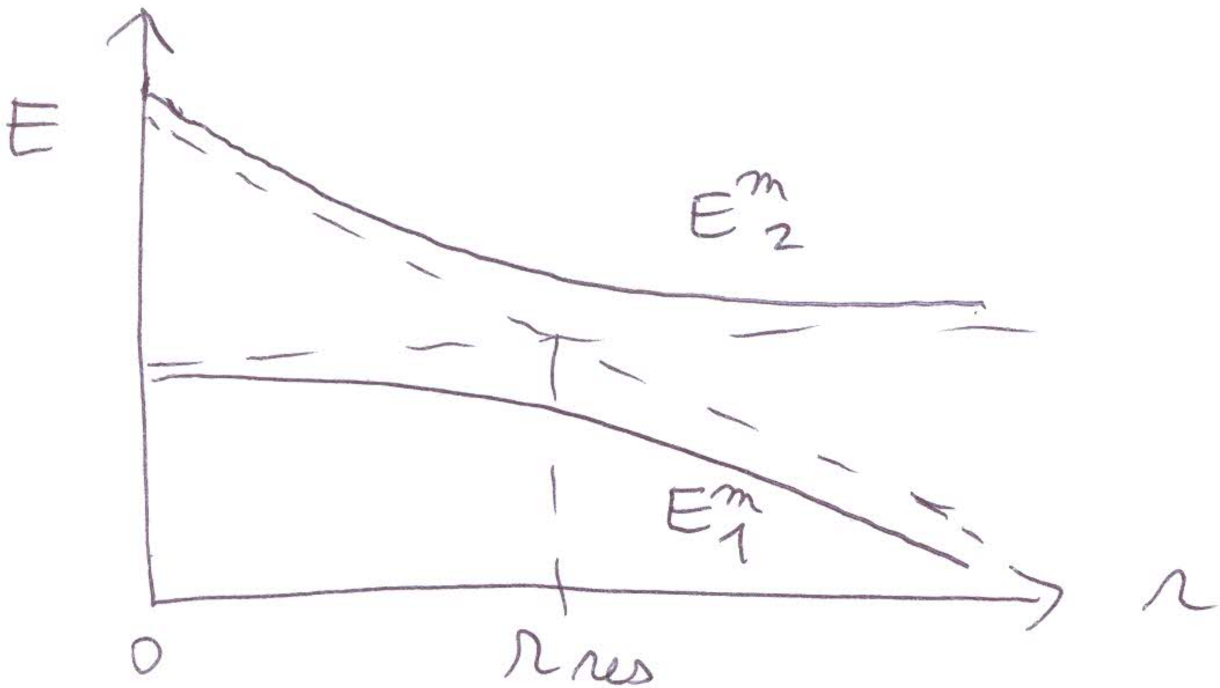
The Schrödinger equation now depends on the time variation of θm:

$$i\frac{d}{dt}\begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m(t) & -i\dot{\theta}_m \\ i\dot{\theta}_m & E_2^m(t) \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

In most physical environments (including the Sun), the evolution is adiabatic (the neutrino state has the time to adjust to the variation of density) and one can neglect $\dot{\theta}_m$ in the Schrödinger equation. A neutrino produced in a given matter eigenstate will stay in the same matter eigenstate during its propagation, but its flavour composition will change

→ adiabatic flavour conversion

"Level crossing" in the Sun (case $n_e\,(r=0) >> n_{res}$)



This is the case for high-energy solar neutrinos (E > 1 MeV)

$$n_e(r=0) \gg n_{\rm res} \;\Rightarrow\; \sin 2\theta_m^0 \simeq 0 \text{ and } \cos 2\theta_m^0 \simeq -1$$

$$\Rightarrow\; \theta_m^0 \simeq \pi/2 \;\Rightarrow\; |\nu_e\rangle \simeq |\nu_2^m(r=0)\rangle$$

⇒ a neutrino produced at the center of the Sun is a quasi pure matter eigenstate and exits the Sun in the eigenstate

$$|\nu_2^m(r=R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\beta\rangle \quad (\beta=\mu,\tau)$$

⇒ a high-energy neutrino produced at the center of the Sun is a quasi pure matter eigenstate and exits the Sun in the eigenstate

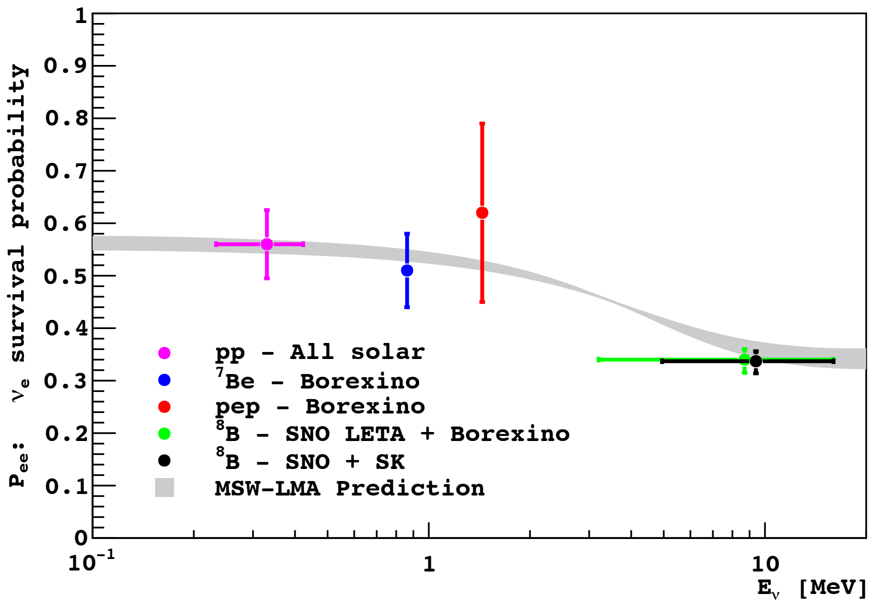
$$|\nu_2^m(r = R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\beta\rangle \qquad (\beta = \mu, \tau)$$

and reaches the Earth as a $|\nu_2\rangle$, giving

$$P_{ee} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta \simeq 0.3$$

For low-energy solar neutrinos, the level-crossing condition is not satisfied ($n_e(r=0) \ll n_{\rm res}$) and matter effects are small

⇒ averaged vaccum oscillations: $P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta \simeq 0.6$



[Borexino Collaboration, arXiv:1308.0443]

The absolute neutrino mass scale

Oscillation experiments measure only mass squared differences
→ information on the neutrino mass scale from beta decay or cosmology

<u>Cosmology</u>

Current upper bound $\sum m_\nu < 0.23\,\mathrm{eV}$ (95%; *Planck*+WP+highL+BAO)

<u>Kinematic measurements (beta decay)</u>

The non-vanishing neutrino mass leads to a distortion of the Ee spectrum close to the endpoint

Tritium beta decay:

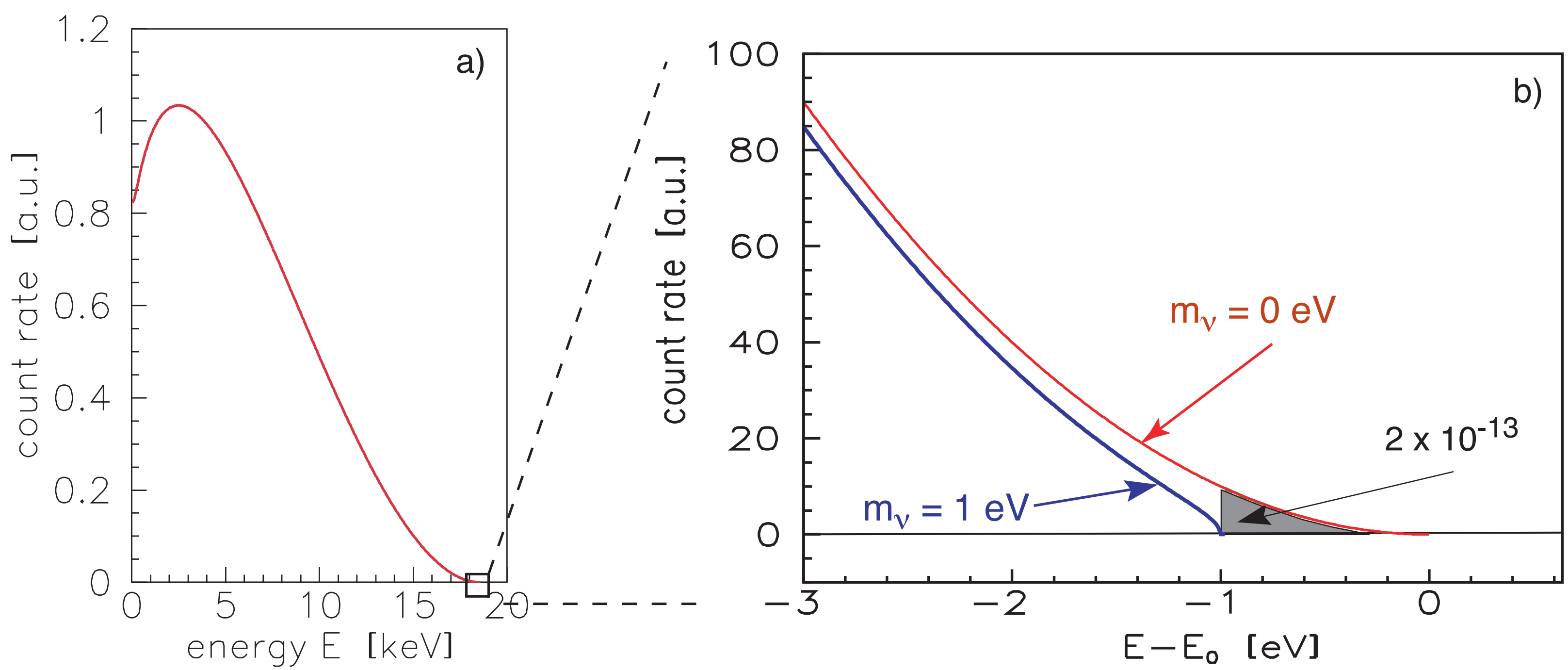
$$^3H \rightarrow \, ^3H_e + e^- + \bar{\nu}_e \qquad\qquad E_0 = m_{^3H} - m_{^3H_e}$$

The electron energy spectrum is given by:

$$\frac{dN}{dE_e} = R(E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2} \qquad\qquad E_e = E_0 - E_\nu$$

Effect of the non-vanishing neutrino mass: $E_e^{max} = E_0 \rightarrow E_0 - m_\nu$

⇒ distorsion of the Ee spectrum close to the endpoint



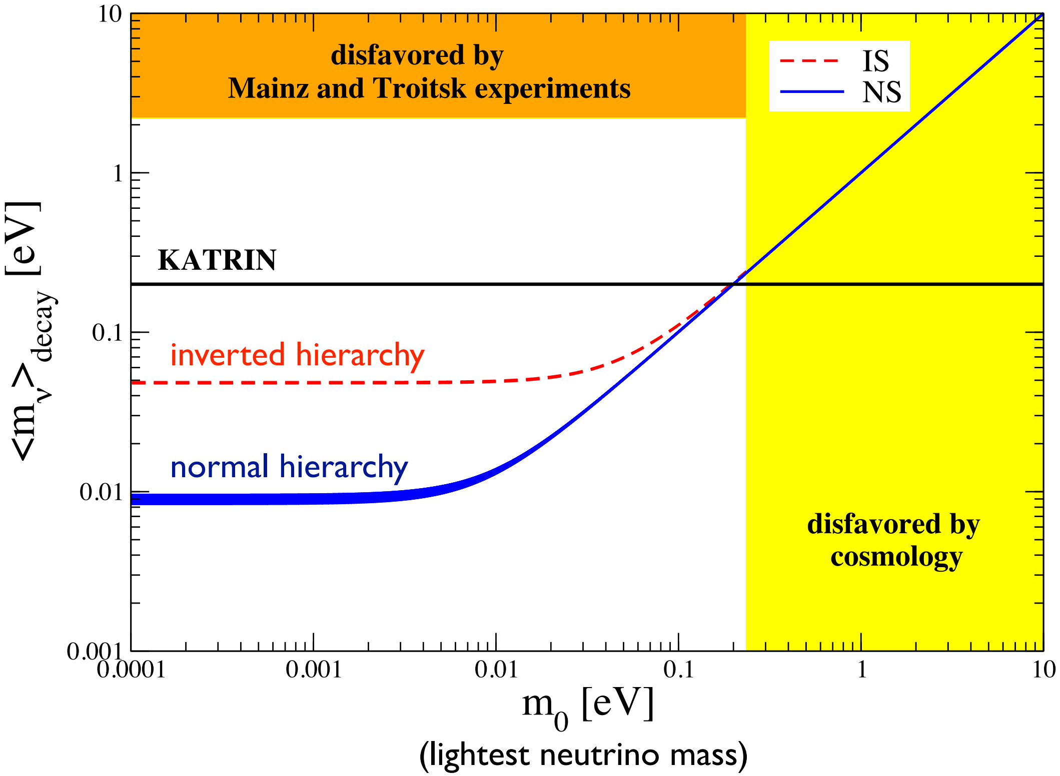
[Katrin Collaboration, hep-ex/0109033]

Present bound (Troitsk/Mainz): $m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})$

KATRIN will reach a sensitivity of about 0.2 eV (5σ discovery potential 0.35 eV)

In pratice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates. However the energy resolution does not allow to resolve them, and what is measured is the effective mass

$$m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$



KATRIN will test only the degenerate case

data taking will start in 2017

Future experiments like Project 8 aim at the 50 meV level

In pratice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates, and

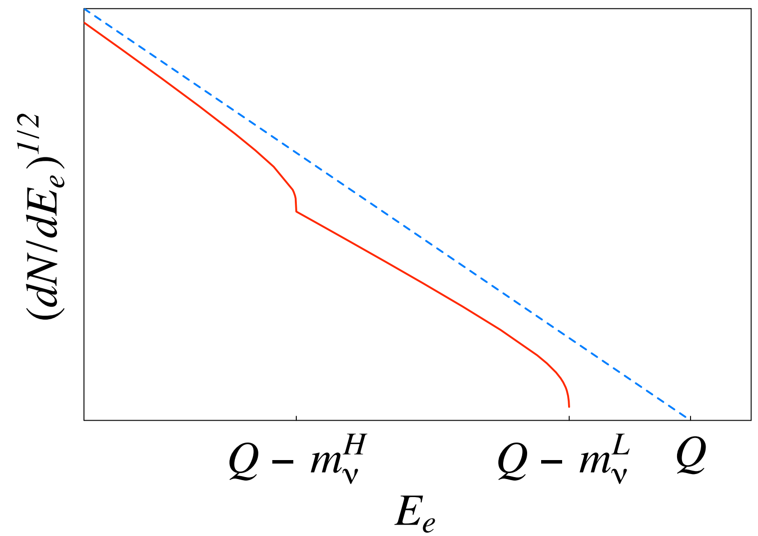
$$\frac{dN}{dE_e} = R(E_e) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E_e)^2 - m_i^2}\, \Theta(E_0 - E_e - m_i)$$

If all mi are smaller than the energy resolution, this can be rewritten as:

$$\frac{dN}{dE_e} = R(E_e) \sqrt{(E_0 - E_e)^2 - m_\beta^2} \qquad m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$

If there is an eV-scale sterile neutrino (comparable to the energy resolution of KATRIN), its mass may be resolved (but difficult measurement):

$$\frac{1}{R(E_e)} \frac{dN}{dE_e} = (1 - |U_{e4}|^2) \sqrt{(E_0 - E_e)^2 - m_\beta^2} + |U_{e4}|^2 \sqrt{(E_0 - E_e)^2 - m_4^2}\, \Theta(E_0 - E_e - m_4)$$

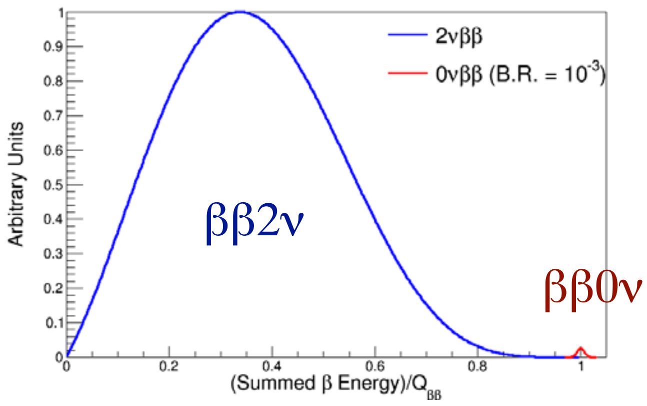


The neutrino nature: neutrinoless double beta decay

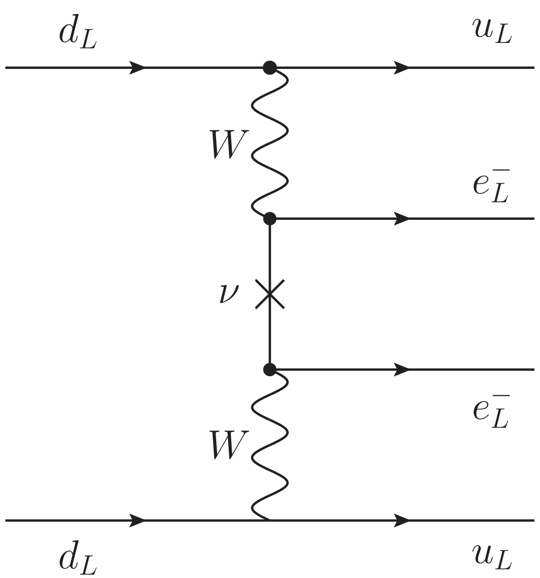
$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$

violates lepton number by 2 units

$\Rightarrow$ possible only for Majorana neutrinos



$$Q_{\beta\beta} = E_1 + E_2$$



Half-life: $$\left[T^{0\nu}_{1/2}\right]^{-1} = \Gamma^{0\nu} = \left(|\langle m_\nu \rangle| / m_e\right)^2 \; G^{0\nu} \; \left|M^{0\nu}\right|^2$$

integrated phase-space factor ↗ ↖ nuclear matrix element (NME)

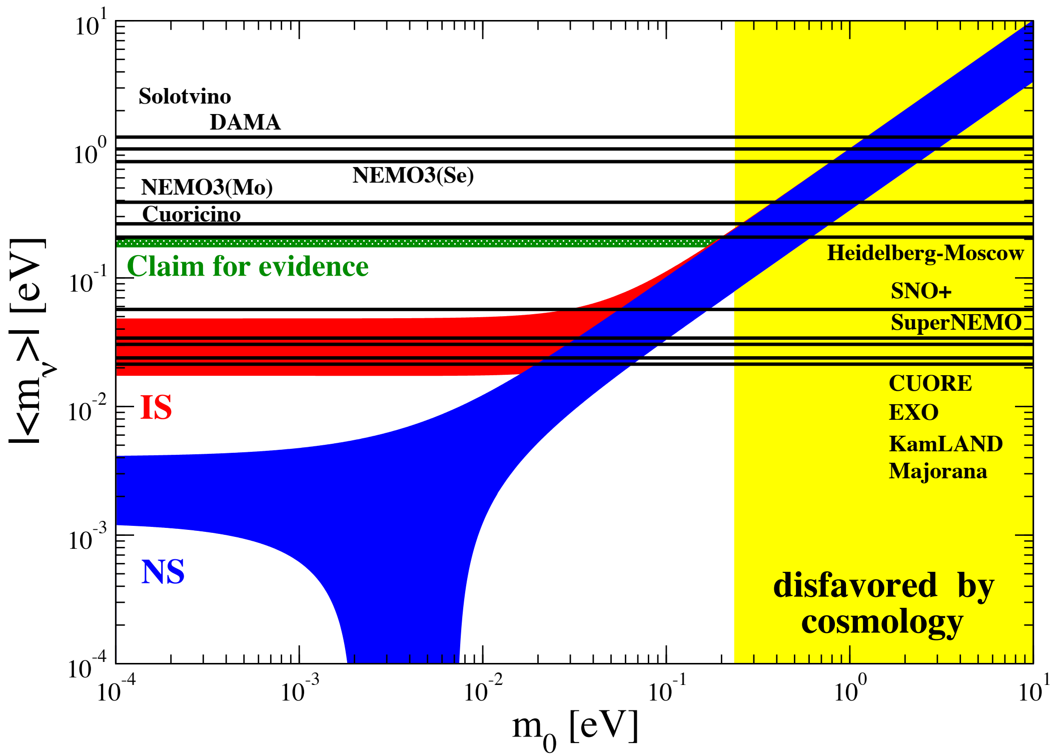
Sensitive to the effective mass parameter:

$$\langle m_\nu \rangle \equiv \sum_i m_i U_{ei}^2 = m_1 c_{13}^2 c_{12}^2 e^{2i\alpha_1} + m_2 c_{13}^2 s_{12}^2 e^{2i\alpha_2} + m_3 s_{13}^2$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

IH

NH



[Vergados et al., arXiv:1205.0649]

(experimental constraints as of May 2012 / optimistic future sensitivities)

Figure 6: (Color online) We show the allowed range of values for $|\langle m_\nu \rangle|$ as a function of the lowest mass eigenstate m_0 using the three standard neutrinos for the cases of normal (NS, $m_0 = m_1$) and inverted (IS, $m_0 = m_3$) spectrum of neutrino masses. Also shown are the current experimental limits and the expected future results [183] (QRPA NMEs with CD-Bonn short-range correlations and $g_A^{eff} = 1.25$ are assumed [174, 184]).

- need to reach 10 meV to exclude IH (lower bound on $\langle m_\nu \rangle$)
- need to reach few meV to test NH (if no mass degeneracy)
- if unlucky (m1 ~ 1-10 meV), may not observe ββ0ν even if neutrinos are Majorana (cancellation in $\langle m_\nu \rangle$)

Sterile neutrinos

Several experimental anomalies suggest the existence of sterile neutrinos

<u>LSND (1993-1998)</u>: $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations

Excess of $\bar{\nu}_e$ events over background at 3.8 σ

Not observed by KARMEN

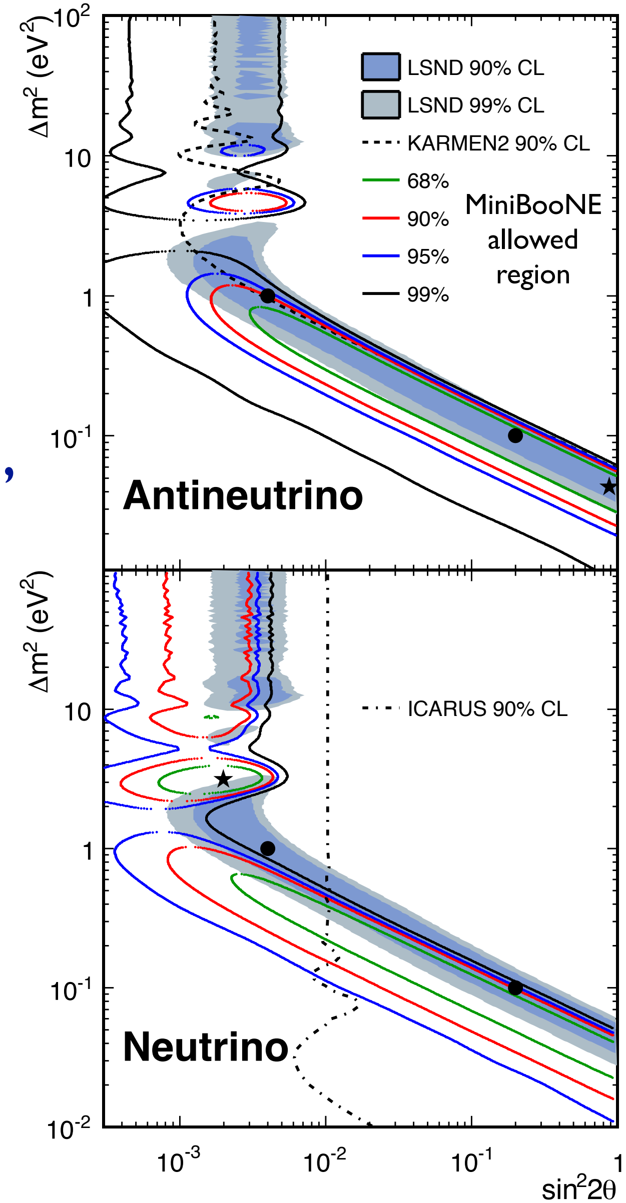
<u>MiniBooNE (2002-2012):</u>

$\nu_\mu \to \nu_e$ data: no excess in the 475-1250 MeV range, but unexplained 3σ excess at low energy

$\bar{\nu}_\mu \to \bar{\nu}_e$ data: $\bar{\nu}_e$ excess in the E > 475 MeV region consistent with LSND-like oscillations, but not very significant (2.8 σ)

A low-energy excess is also seen

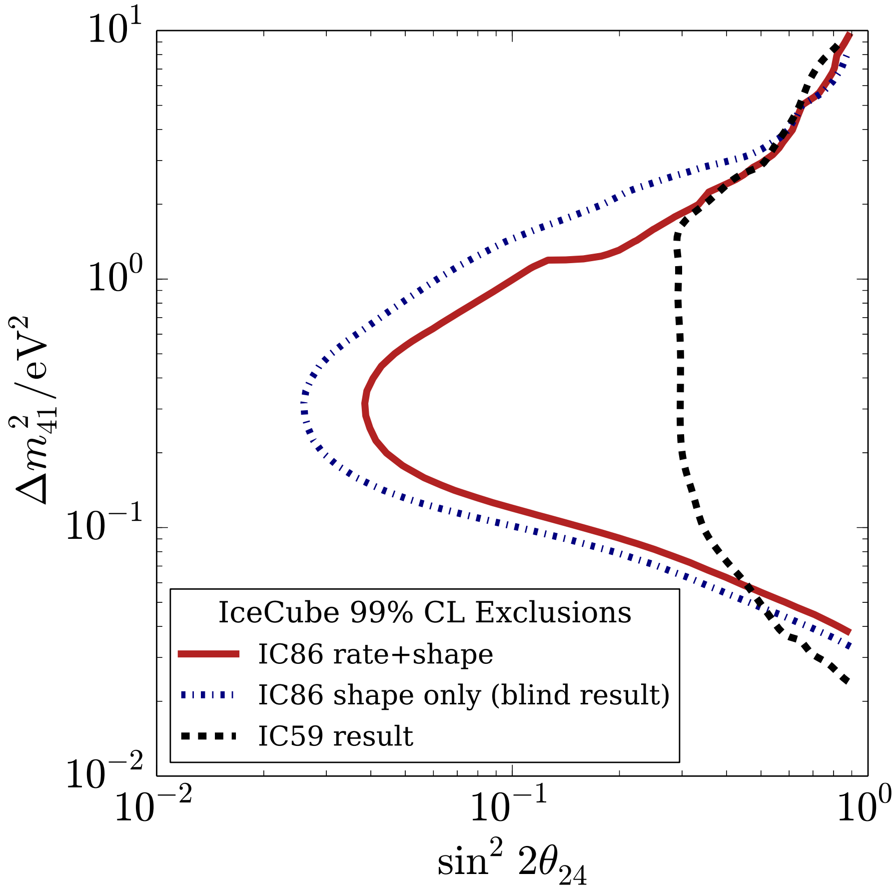
→ unconclusive



[MiniBooNE 2-neutrino fit using the full 200-1250 MeV data, arXiv:1303.2588]

IceCube (2016):

excludes most of the LSND + MiniBooNE allowed region through ν_μ disappearance



[IceCube Collaboration, arXiv:1605.01990]

FIG. 4. Results from IceCube sterile neutrino searches (regions to the right of the contours are excluded). The dot-dashed blue line shows the result of the original analysis based on shape alone, while the solid red line shows the final result with a normalization prior included to prevent degeneracies between the no-steriles hypothesis and sterile neutrinos with masses outside the range of sensitivity. The dashed black line is the exclusion range derived from an independent analysis of data from the 59-string IceCube configuration.

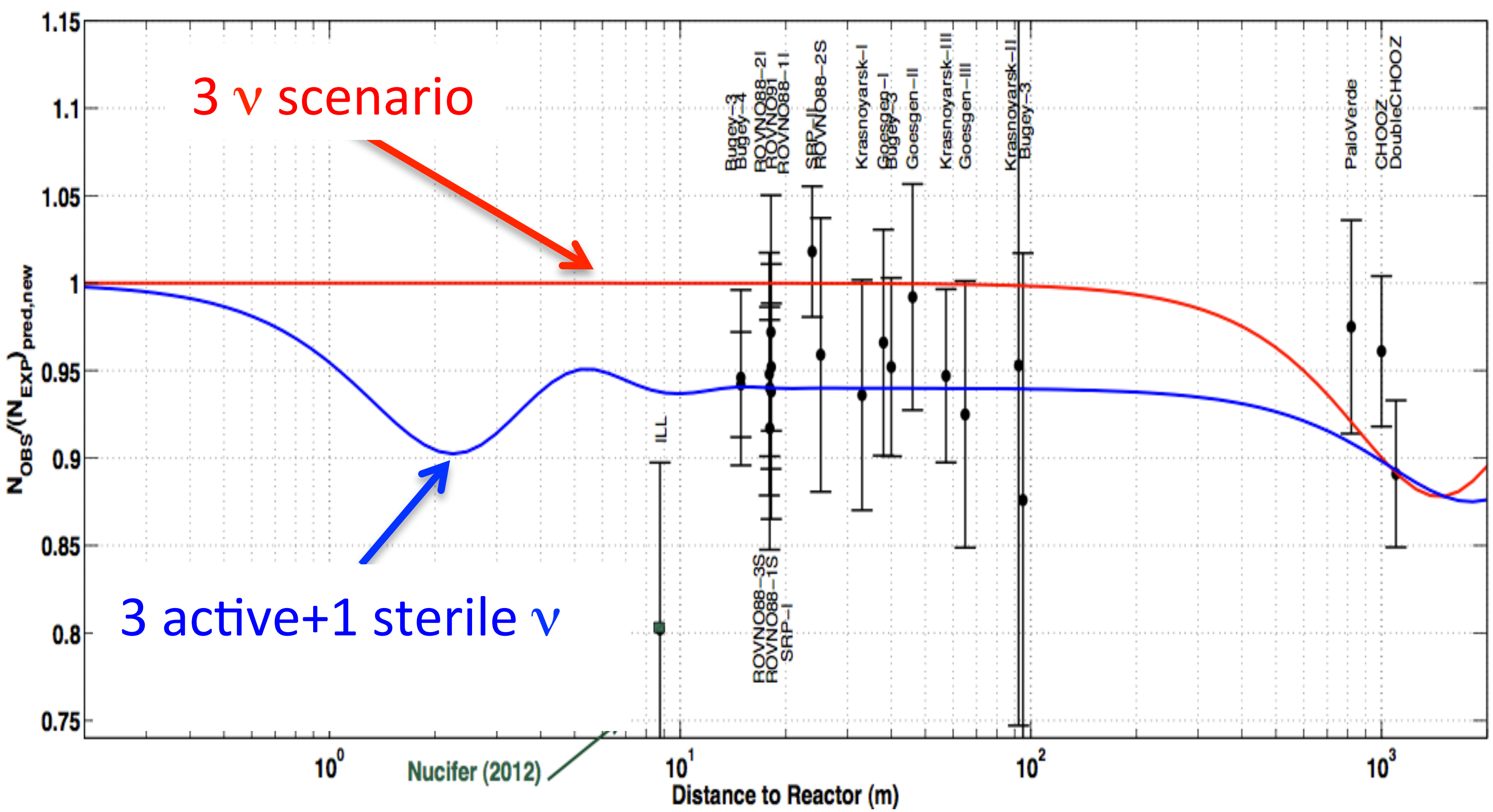
Reactor antineutrino anomaly (2011):

New computation of the reactor antineutrino spectra
[Th. Mueller et al., 2011 - P. Huber, 2011]

⇒ increase of the flux by about 3%

⇒ deficit of antineutrinos in SBL reactor experiments

Mean observed to predicted rate 0.943 ± 0.023 [G. Mention et al., arXiv:1101.2755]



[D. Lhuillier, talk at IPA 2016]

Gallex-SAGE calibration experiments:

Calibration of the Gallex and SAGE experiments with radioactive sources
$\Rightarrow$ observed deficit of ν_e with respect to predictions
R = 0.86 ± 0.05

All these anomalies suggest oscillations with a new $\Delta m^2 \gtrsim 1\,\mathrm{eV}^2$

However, no coherent picture of all data with an additional (or even 2) sterile neutrinos: tension between appearance (LSND/MiniBooNE antineutrino data) and disappearance experiments (reactors + ν_μ disappearence experiments, including IceCube) + tension between LSND and MiniBooNE neutrino data

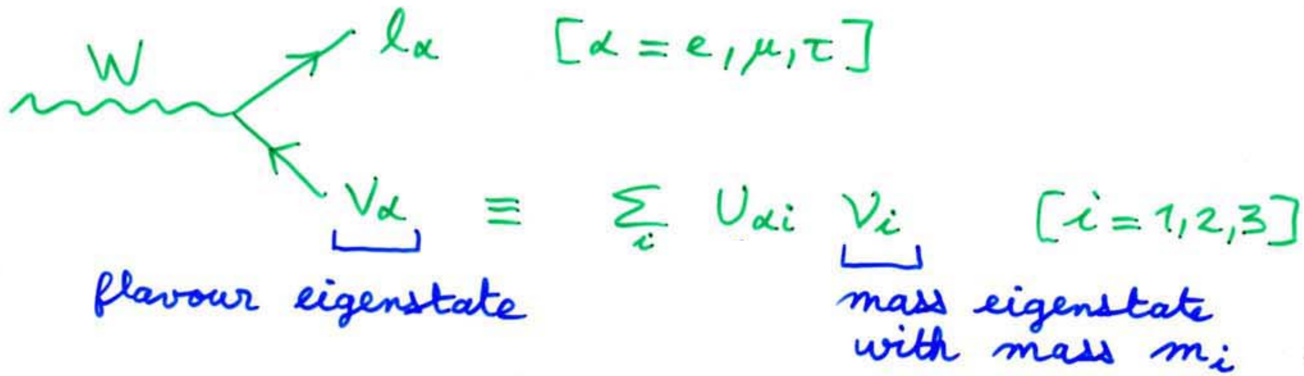
Also, cosmology leaves little room for a sterile neutrino:

$N_{\mathrm{eff}} = 3.15 \pm 0.23 \quad (68\%\,\mathrm{C.L.})$ [Planck 2015 + BAO]

→ need experimental clarification

Active-sterile neutrino mixing

Standard case (3 flavours):



Add a sterile neutrino:

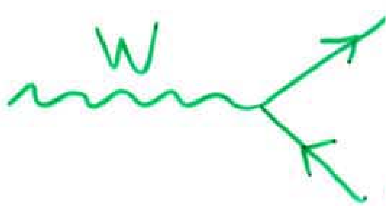
$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i}\, \nu_i \qquad [\alpha = e, \mu, \tau]$$

ν_s flavour eigenstate

ν_4 mass eigenstate (m4)

U = 4x4 unitary matrix

Only ν_e, ν_μ, ν_τ couple to electroweak gauge boson, but all four mass eigenstate are produced in a beta decay:



e^-

$$\nu_e = \sum_{i=1}^{4} U_{ei}\, \nu_i$$

We are interested in short baseline oscillations with

$$\frac{\Delta m^2_{41} L}{4E} \lesssim 1 \quad \Longrightarrow \quad \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \gg \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right), \; \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

→ approximate $\Delta m^2_{31} = \Delta m^2_{21} = 0 \,, \quad \Delta m^2_{43} = \Delta m^2_{42} = \Delta m^2_{41} \equiv \Delta m^2_{\rm SBL}$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\alpha} &\simeq 1 - 4\left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2\right)|U_{\alpha 4}|^2 \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\ &\equiv 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \end{aligned}$$

where $\sin^2 2\theta_{\alpha\alpha} \equiv 4\,(1 - |U_{\alpha 4}|^2)|U_{\alpha 4}|^2$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &\simeq -4\,\mathrm{Re}\left[\left(U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3}\right) U^*_{\alpha 4} U_{\beta 4}\right] \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\ &\equiv \sin^2 2\theta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \end{aligned}$$

where $\sin^2 2\theta_{\alpha\beta} \equiv 4\,|U_{\alpha 4} U_{\beta 4}|^2$

Tension between appearance (LSND + MiniBooNE antineutrino data) and disappearance experiments (reactors, ν_μ disappearence experiments)

Reactors: $$P_{\bar{\nu}_e \to \bar{\nu}_e} \simeq 1 - \sin^2 2\theta_{ee} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

require relatively small $\sin^2 2\theta_{ee} \equiv 4\,(1 - |U_{e4}|^2)|U_{e4}|^2 \simeq 4\,|U_{e4}|^2$

(using info from solar neutrino data)

CDHS, IceCube: $$P_{\nu_\mu \to \nu_\mu} \simeq 1 - \sin^2 2\theta_{\mu\mu} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

require relatively small $\sin^2 2\theta_{\mu\mu} \equiv 4\,(1 - |U_{\mu 4}|^2)|U_{\mu 4}|^2 \simeq 4\,|U_{\mu 4}|^2$

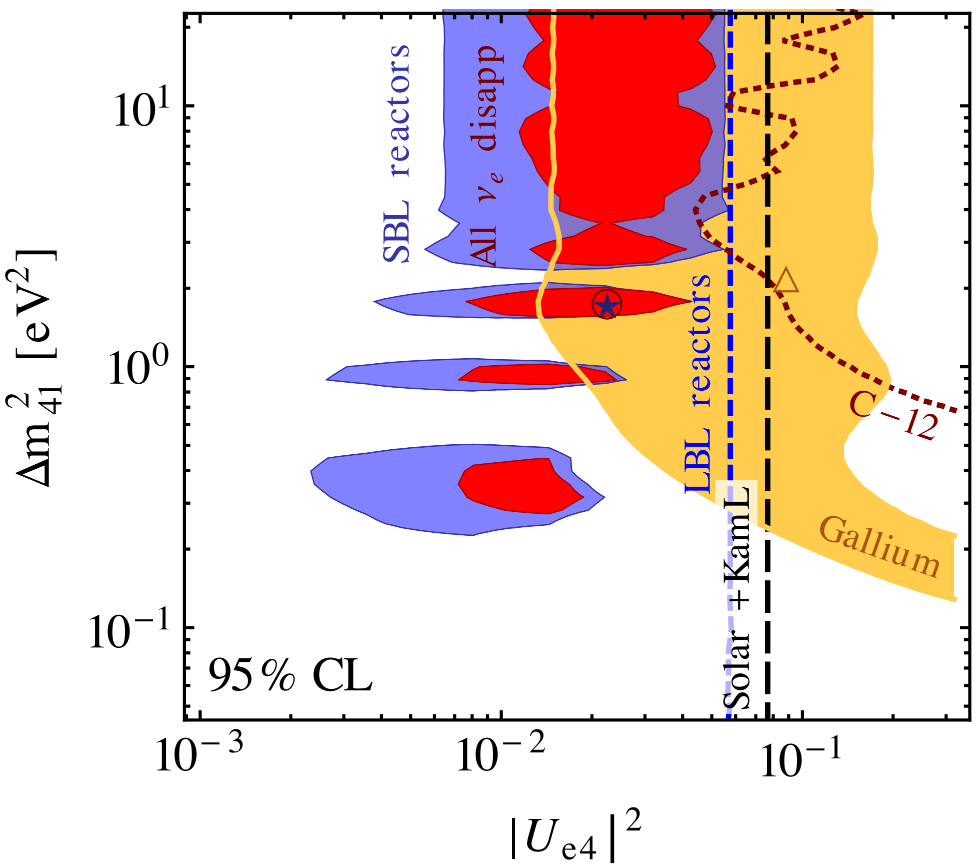
(using info from atm. neutrino data)

Appearance experiments (LSND + MiniBooNE antineutrino data):

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq \sin^2 2\theta_{e\mu} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

require relatively large $\sin^2 2\theta_{e\mu} \equiv 4\,|U_{e4} U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}$

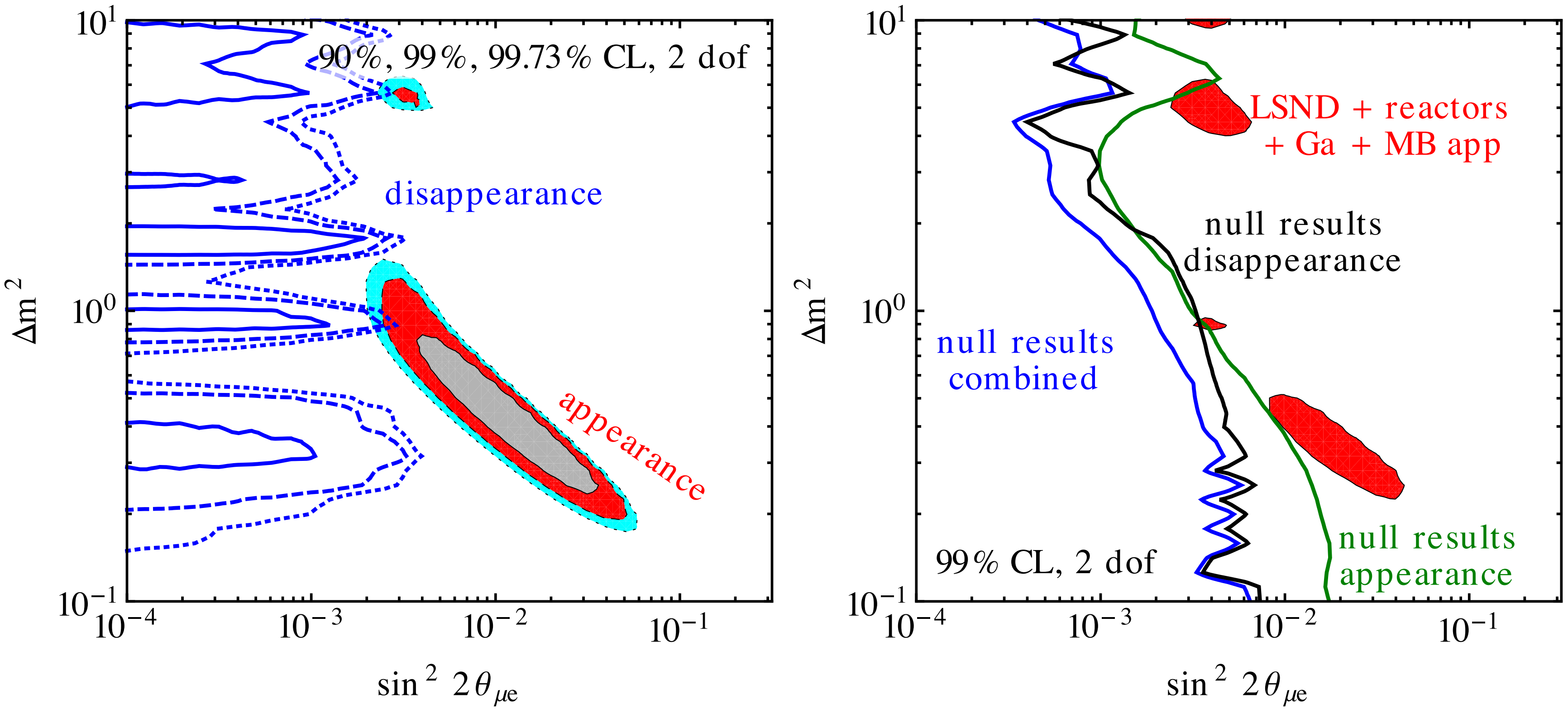
Parameter region allowed by $\nu_e/\bar{\nu}_e$ disappearance data



[Kopp et al., arXiv:1303.3011]

Figure 2. Allowed regions at 95% CL (2 dof) for 3+1 oscillations. We show SBL reactor data (blue shaded), Gallium radioactive source data (orange shaded), ν_e disappearance constraints from ν_e–^{12}C scattering data from LSND and KARMEN (dark red dotted), long-baseline reactor data from CHOOZ, Palo Verde, DoubleChooz, Daya Bay and RENO (blue short-dashed) and solar+KamLAND data (black long-dashed). The red shaded region is the combined region from all these ν_e and $\bar{\nu}_e$ disappearance data sets.

Pre-IceCube tension between appearance and disappearance data



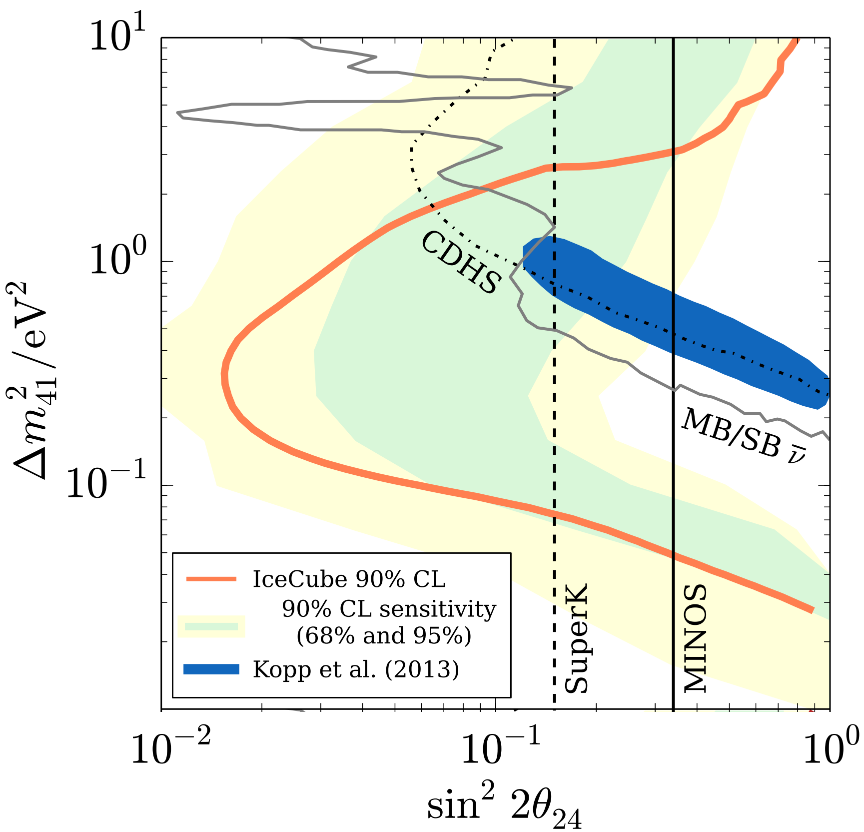
[Kopp et al., arXiv:1303.3011]

Figure 8. Results of the global fit in the 3+1 scenario, shown as exclusion limits and allowed regions for the effective mixing angle $\sin^2 2\theta_{\mu e} = 4|U_{e4}|^2|U_{\mu 4}|^2$ and the mass squared difference Δm^2_{41}. Left: Comparison of the parameter region preferred by appearance data (LSND, MiniBooNE appearance analysis, NOMAD, KARMEN, ICARUS, E776) to the exclusion limit from disappearance data (atmospheric, solar, reactors, Gallium, CDHS, MINOS, MiniBooNE disappearance, KARMEN and LSND ν_e–^{12}C scattering). Right: Regions preferred by experiments reporting a signal for sterile neutrinos (LSND, MiniBooNE, SBL reactors, Gallium) versus the constraints from all other data, shown separately for disappearance and appearance experiments, as well as their combination.

Tension between IceCube and appearance data

IceCube constraints vs appearance

[IceCube Collaboration, arXiv:1605.01990]



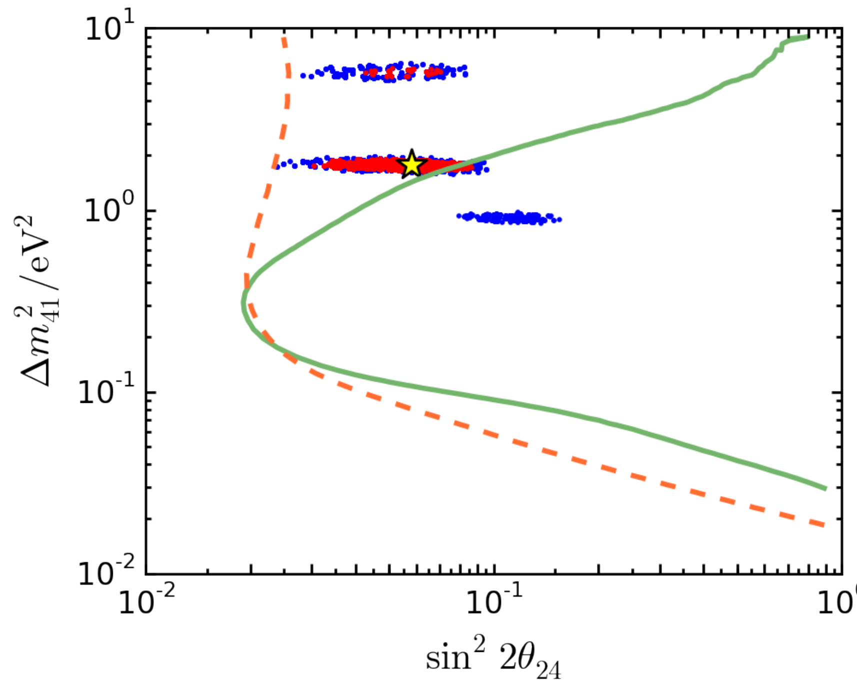
the appearance region (blue) assumes
$|U_{e4}|^2 = 0.023$ (best fit from Kopp. et al.)

the IceCube contour (orange) assumes
$U_{e4} = U_{\tau 4} = 0 \quad (\theta_{14} = \theta_{34} = 0)$

Global fit of SBL data + IceCube

(includes both appearance and disappearance data)

[G.H. Collin et al., arXiv:1607.00011]



SUPPL. FIG. 1: The solid (dashed) line represents the 90% C.L. IceCube limit when calculated with $\theta_{34} = 0°$ ($\theta_{34} = 15°$). The result of the SBL+IC global fit is overlaid, Red – 90% CL; blue–99% CL.