Luca Fabbri

University of Genoa

TORSION, SPIN-CONNECTION, SPIN AND SPINOR FIELDS

LPSC, Grenoble, 15th of June 2016

Relativity in general requires a connection; connections in general are not symmetric: so

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Curvature as usual

$$G^{\mu}{}_{\rho\sigma\pi} = \partial_{\sigma}\Gamma^{\mu}_{\rho\pi} - \partial_{\pi}\Gamma^{\mu}_{\rho\sigma} + \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\lambda}_{\rho\pi} - \Gamma^{\mu}_{\lambda\pi}\Gamma^{\lambda}_{\rho\sigma}$$

If torsion is present beside the metric, then metric and connections are independent, and analogously TETRADS and SPIN-CONNECTION

$$\omega^{i}{}_{p\alpha} = e^{i}_{\sigma}(\Gamma^{\sigma}_{\rho\alpha} e^{\rho}_{p} + \partial_{\alpha} e^{\sigma}_{p})$$

are independent variables: the torsion and curvature tensor

$$Q^{i}{}_{\alpha\rho} = -\left(\partial_{\alpha} e^{i}_{\rho} - \partial_{\rho} e^{i}_{\alpha} + e^{p}_{\rho}\omega^{i}{}_{p\alpha} - e^{p}_{\alpha}\omega^{i}{}_{p\rho}\right)$$
$$G^{a}{}_{b\sigma\pi} = \partial_{\sigma}\omega^{a}_{b\pi} - \partial_{\pi}\omega^{a}_{b\sigma} + \omega^{a}_{j\sigma}\omega^{j}_{b\pi} - \omega^{a}_{j\pi}\omega^{j}_{b\sigma}$$

are the strengths (as Hehl said, we believe in Poincaré group and in gauging, so we have to believe in gauging the Poncaré group).

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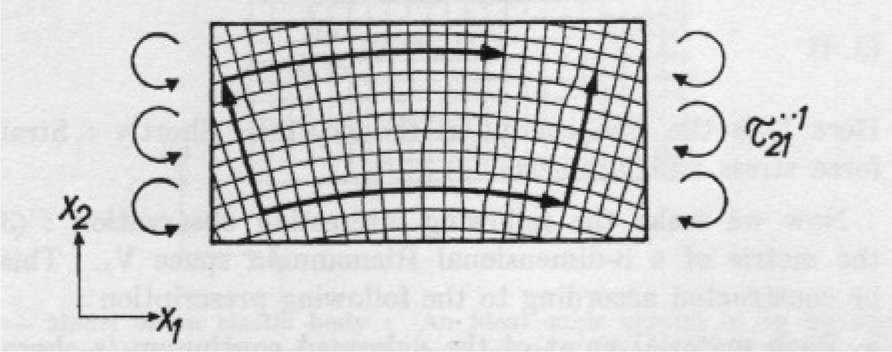
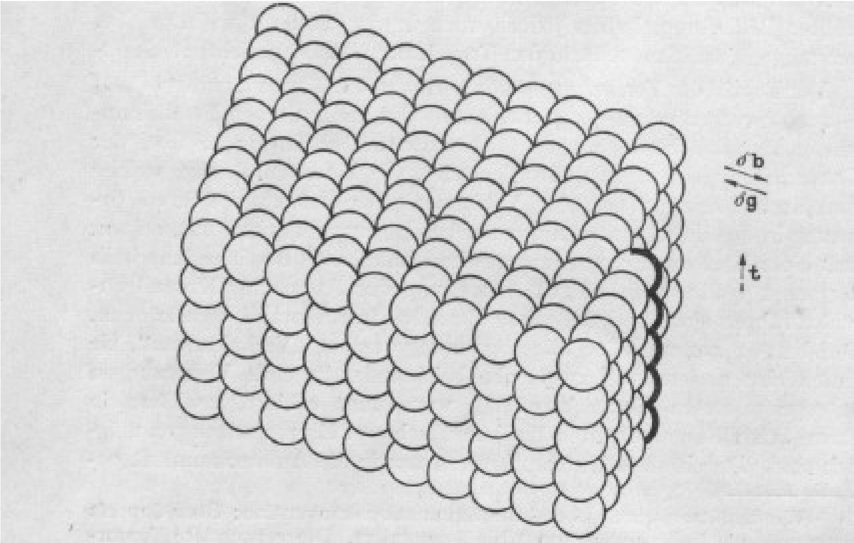
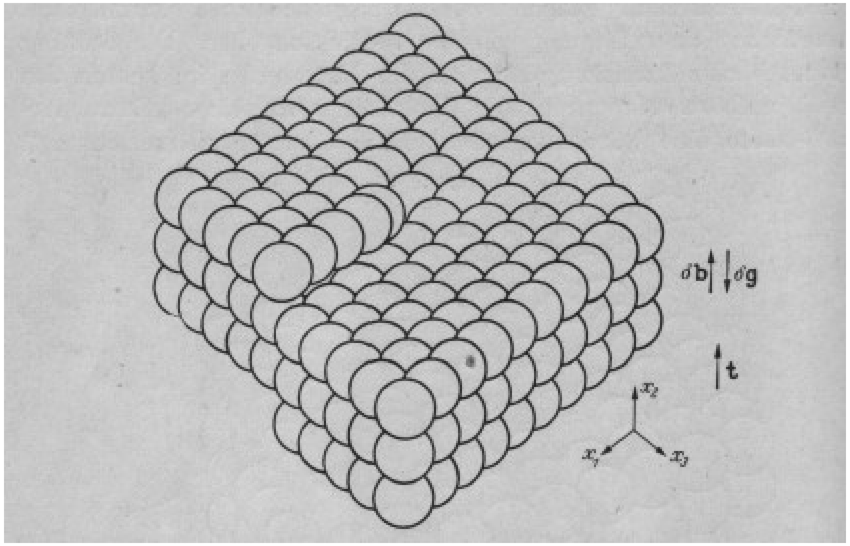
Commutators of covariant derivatives are

$$[D_{\sigma}, D_{\pi}]V^{\mu} = Q^{\theta}{}_{\sigma\pi} D_{\theta} V^{\mu} + G^{\mu}{}_{\rho\sigma\pi} V^{\rho}$$

In the infinitesimal parallelogram torsion produces disclination likewise the curvature produces dislocation (Cosserat media).







So there are many interpretations for torsion and curvature: in particular for the spacetime the Principle of Equivalence provides a natural interpretation for the curvature, but there is no such principle providing a similar interpretation for torsion.

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For the spacetime, torsion is model-dependent, but there is a model that is priviledged with respect to all others:

SPIN

Field equations coupling geometrical quantities to material ones

$$\nabla_\mu F_a^{\mu\nu}$$

$$J_a^\nu$$

Field equations coupling
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$$\nabla_\mu F_a^{\mu\nu} \qquad G_{\alpha\nu}$$

$$J_a^\nu \qquad T_{\alpha\nu}$$

Field equations coupling geometrical quantities to material ones

$$\nabla_\mu F_a^{\mu\nu} \qquad G_{\alpha\nu} \quad Q_{\alpha\nu\sigma}$$

$$J_a^{\nu} \qquad T_{\alpha\nu} \quad S_{\alpha\nu\sigma}$$

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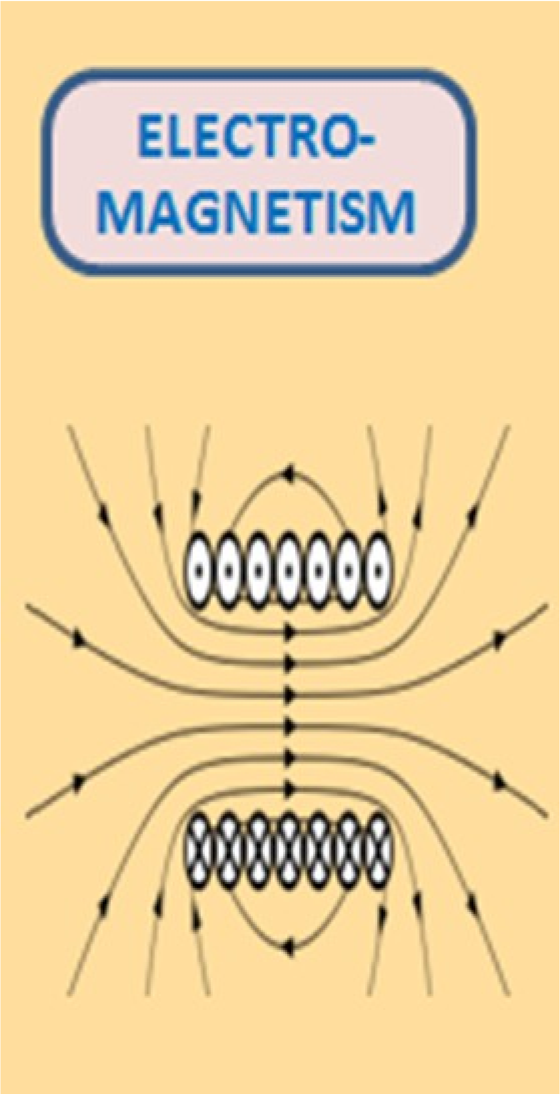

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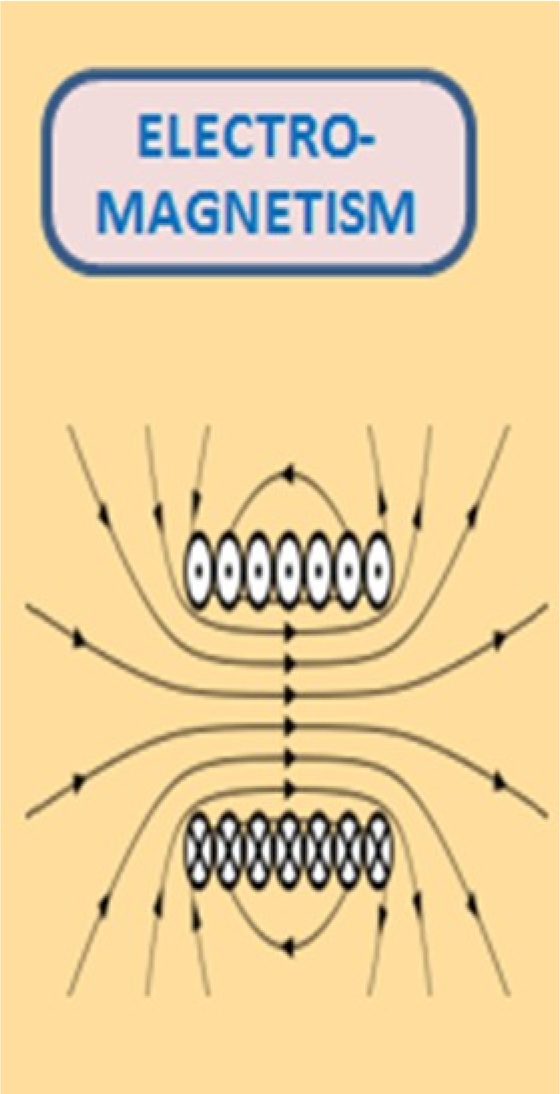

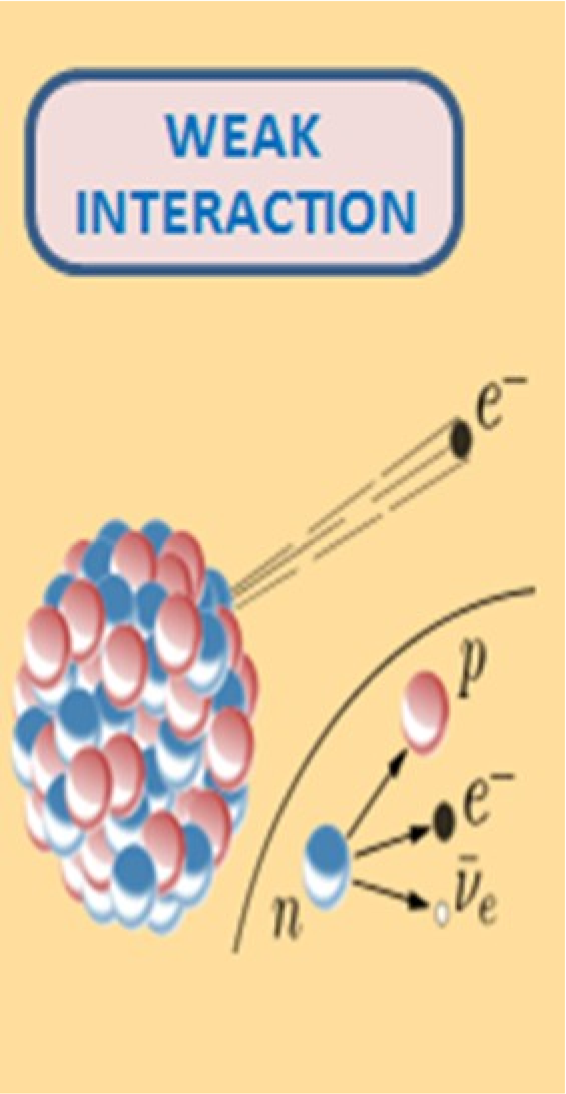

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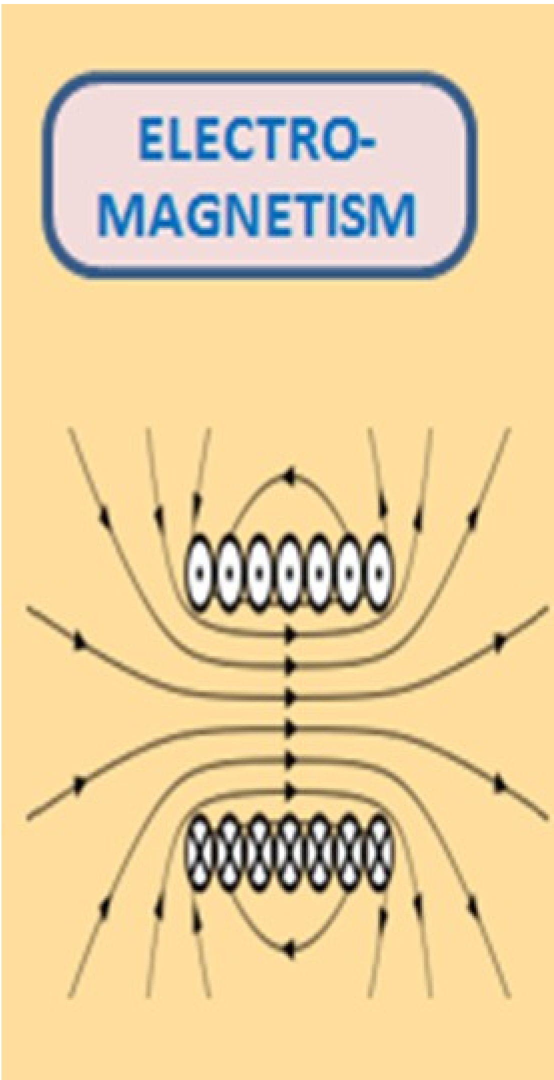
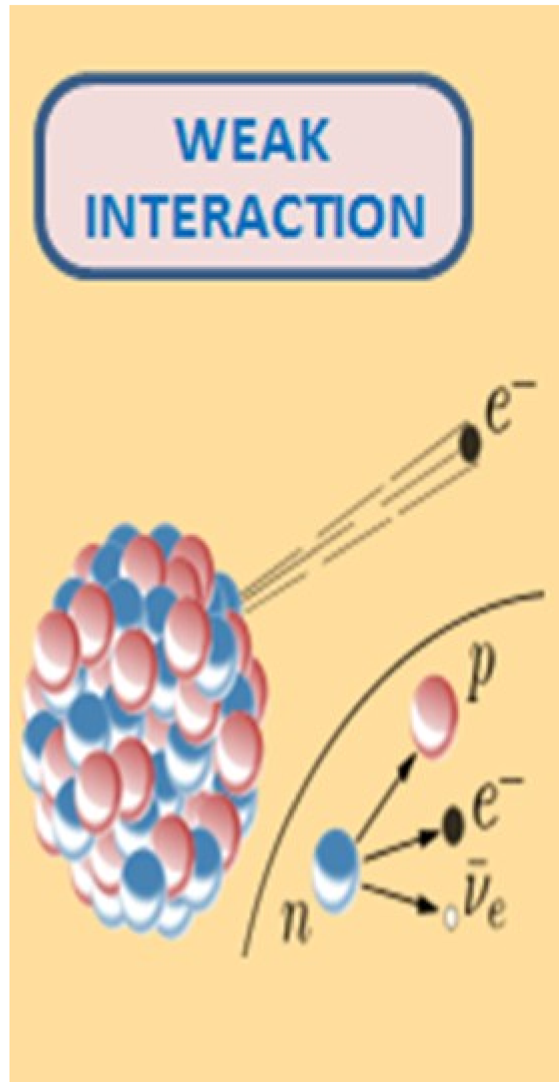
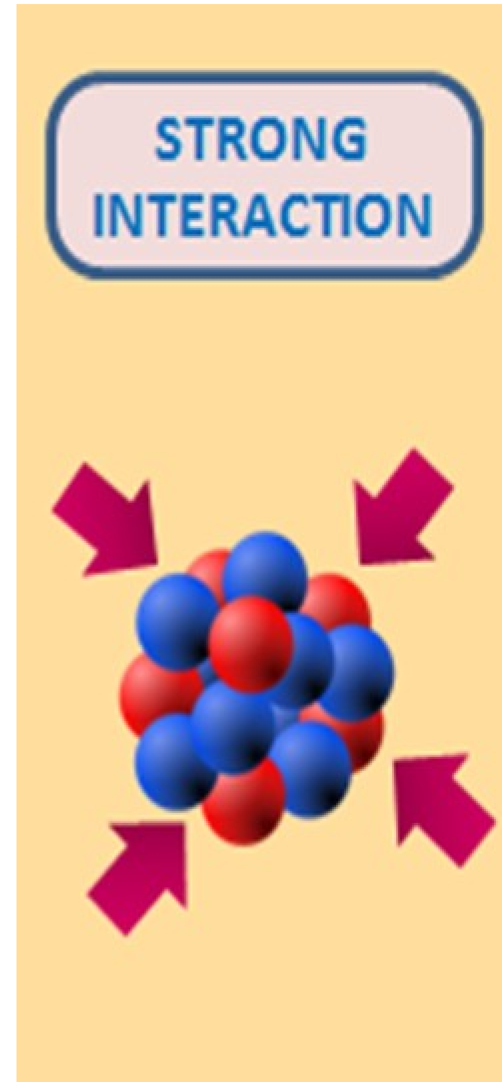






LHCb

LHCb

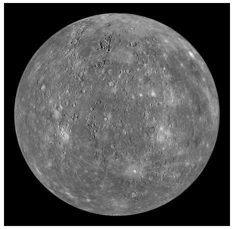
ALICE

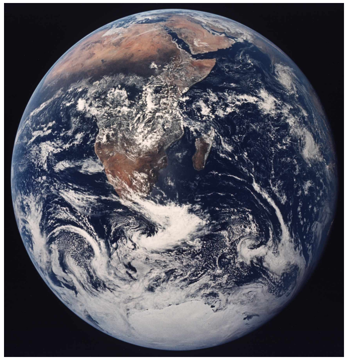
LHCb

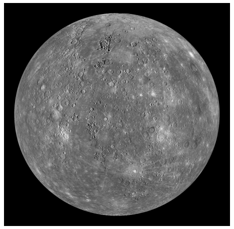
ALICE

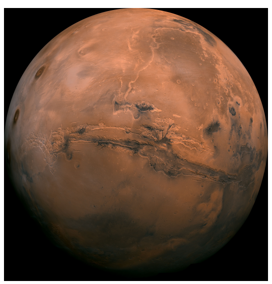
CMS

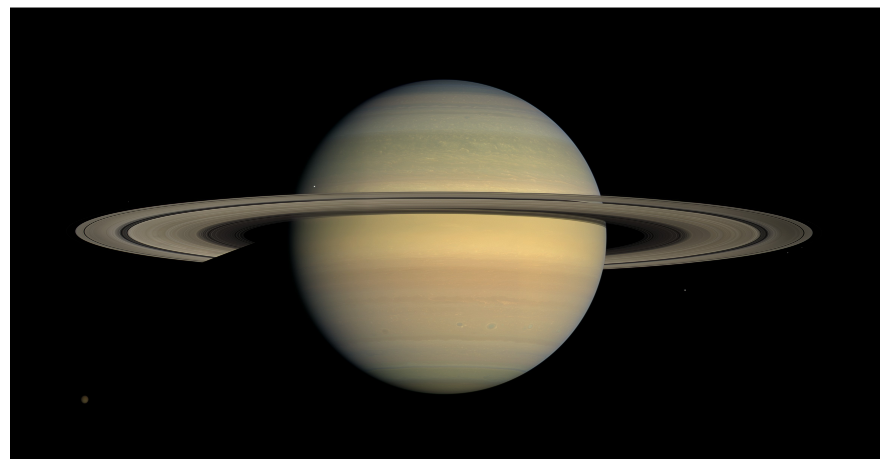
Compact Muon Solenoid

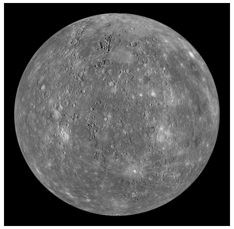


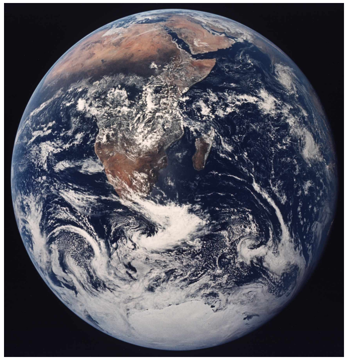
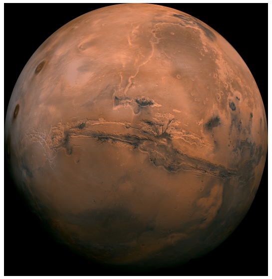
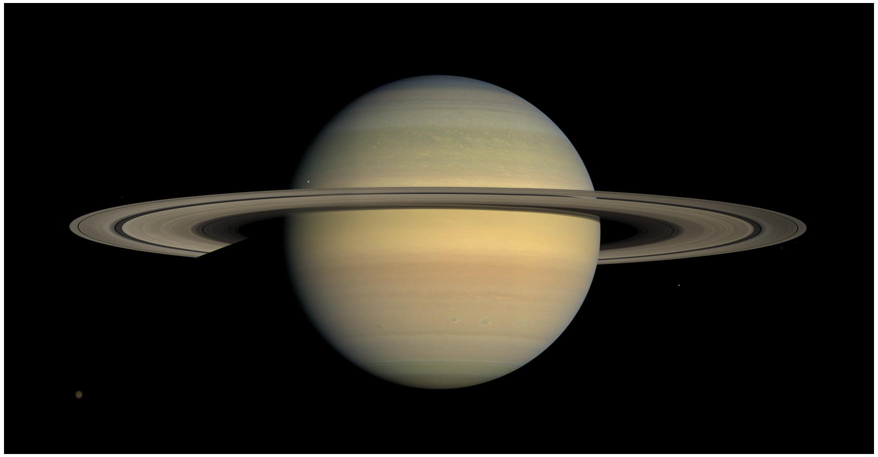
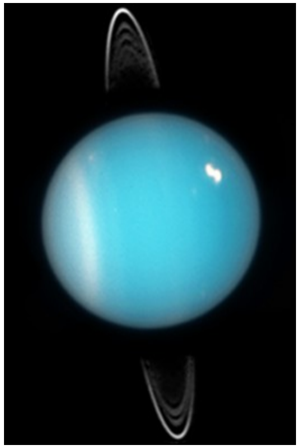
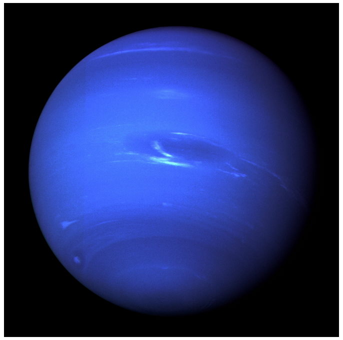




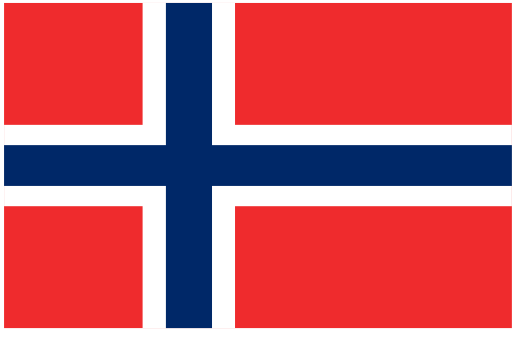
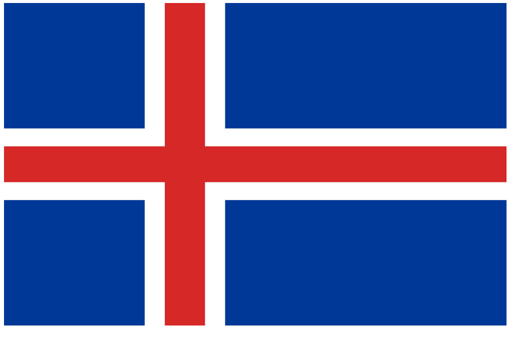




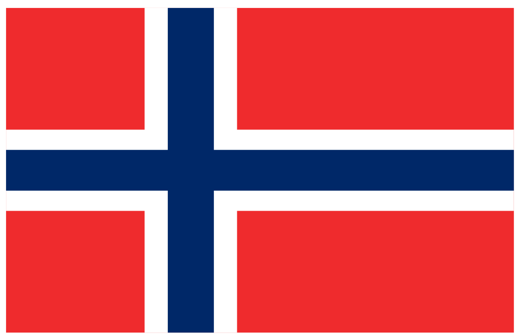
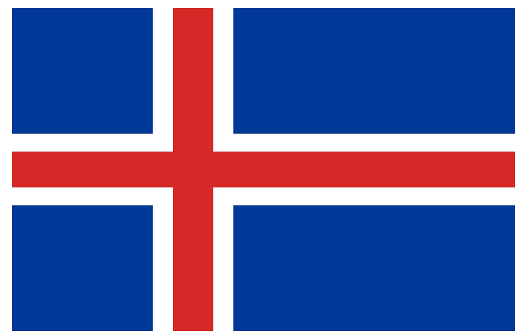








CULINARY MAP OF EUROPE ACCORDING TO
FRANCE 2015
from Yanko Tsvetkov's Atlas of Prejudice
www.atlasofprejudice.com
HEALTH HAZARD
MEATBALLS "INVENTORS"
ICE CUBE SALAD
YOUR SALAD IS FRENCH!
SOGGY PASTRY
THE VAST ATLANTIC OCEAN
ROTTEN CABBAGE
POTATO MUNCHERS
"FRENCH" FRIES
WHOLE-GRAIN URANIUM
PROPER CUISINE
MONEY
MARIE ANTOINETTE
TOO MUCH PEPPER
OVERRATED
FAMINE
RICE
SUGAR-GLAZED CROISSANTS
CHALKY CHEESE
STICKY PASTRY

Einstein gravity is based on the assumption that curvature is coupled to energy

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = 8\pi k T_{\mu\nu}$$

from the Lagrangian L=R(g).

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The Sciama-Kibble completion of Einstein gravity maintains in the most general case Einstein's spirit with spin coupled to torsion as curvature is coupled to energy

$$Q^{\rho\mu\nu} = -16\pi k S^{\rho\mu\nu}$$

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The most general SKE gravity for completely antisymmetric torsion

$$Q^{\rho\mu\nu}=-aS^{\rho\mu\nu}$$

$$\frac{b}{2a}\left(\frac{1}{4}\delta^{\mu}_{\nu}Q^{2}-\frac{1}{2}Q^{\mu\alpha\sigma}Q_{\nu\alpha\sigma}+D_{\rho}Q^{\rho\mu}{}_{\nu}\right)+\left(G^{\mu}{}_{\nu}-\frac{1}{2}\delta^{\mu}_{\nu}G-\lambda\delta^{\mu}_{\nu}\right)=\left(\frac{b+a}{2}\right)T^{\mu}{}_{\nu}$$

from the Lagrangian L=G(g,Q)+Q^2 (with two constants).

The Jacobi-Bianchi identities can be worked out into the conservation laws

$$D_\mu T^{\mu\rho} + Q_\mu T^{\mu\rho} - T_{\mu\sigma} Q^{\sigma\mu\rho} + S_{\beta\mu\sigma} G^{\sigma\mu\beta\rho} = 0$$
$$D_\rho S^{\rho\mu\nu} + Q_\rho S^{\rho\mu\nu} + \frac{1}{2} T^{[\mu\nu]} = 0$$

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These are verified by the spin and energy

$$S^{\rho\mu\nu} = \frac{i\hbar}{4} \overline{\psi} \{\boldsymbol{\gamma}^\rho, \boldsymbol{\sigma}^{\mu\nu}\} \psi$$ → completely antisymmetric

$$T^\mu{}_\nu = \frac{i\hbar}{2} \left(\overline{\psi} \boldsymbol{\gamma}^\mu \boldsymbol{D}_\nu \psi - \boldsymbol{D}_\nu \overline{\psi} \boldsymbol{\gamma}^\mu \psi \right)$$ → non-symmetric

once the Dirac SPINOR Field equation

$$i\hbar \boldsymbol{\gamma}^\mu \boldsymbol{D}_\mu \psi - m\psi = 0$$

assigned in terms of the usual Lagrangian.

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Torsion is as fundamental as the spin is, which is as fundametal as spinors are.

We can vary the Lagrangian to get field equations, then integrate torsion.

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The DESK theory for matter is equivalent to the Nambu--Jona-Lasinio model

$$i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi+\tfrac{3a}{16}\hbar^{2}\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\gamma}\psi\boldsymbol{\gamma}_{\mu}\boldsymbol{\gamma}\psi-m\psi\equiv$$

$$\equiv i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-\tfrac{3a}{16}\hbar^{2}\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi\boldsymbol{\gamma}_{\mu}\psi-m\psi\equiv$$

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\end{aligned}
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For models with propagating torsion, this can be done only for very massive torsion.

In general, torsion is a massive axial-vector Proca field.

Consider spherically symmetric backgrounds: the most general spinor must be

$$\psi_{\pm} = \left(\begin{array}{c} \rho e^{i\frac{\alpha}{2}} \\ \pm\rho e^{i\frac{\alpha}{2}} \\ \eta e^{i\frac{\beta}{2}} \\ \pm\eta e^{i\frac{\beta}{2}} \end{array} \right)$$

It is possible to see that the Dirac equation becomes

$$\left(m + \tfrac{3}{4}\eta\rho\cos\left(\tfrac{\alpha-\beta}{2}\right)\right)(\gamma_3\psi_{\pm}) - \gamma i\left(\tfrac{3}{4}\eta\rho\sin\left(\tfrac{\alpha-\beta}{2}\right)\right)(\gamma_3\psi_{\pm}) -$$
$$-\tfrac{1}{\sqrt{Ar^2}}\left(\gamma\tfrac{i}{A}\left(iq\phi + \tfrac{\partial}{\partial t}\right)\left(\sqrt{Ar^2}\psi_{\mp}\right) \pm \tfrac{i}{B}\tfrac{\partial}{\partial r}\left(\sqrt{Ar^2}\psi_{\mp}\right)\right) \mp$$
$$\mp\tfrac{1}{r}\left[\tfrac{1}{\sqrt{\sin\theta}}\left(\tfrac{\partial}{\partial\theta}\left(\sqrt{\sin\theta}\psi_{\pm}\right) \pm \tfrac{i}{\sin\theta}\tfrac{\partial}{\partial\varphi}\left(\sqrt{\sin\theta}\psi_{\pm}\right)\right)\right] = 0$$

These expressions can eventually be written in the form

$$\frac{\partial}{\partial\theta}\ln\left(\frac{\eta^2}{\rho^2}\right) \pm \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}(\alpha-\beta) = 0$$

$$\frac{\partial}{\partial\theta}(\alpha-\beta) \mp \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}\ln\left(\frac{\eta^2}{\rho^2}\right) = 0$$

$$\frac{\partial}{\partial\theta}(\alpha+\beta) \pm \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}\ln\left(\eta^2\rho^2\right) = 0$$

$$2\cot\theta + \frac{\partial}{\partial\theta}\ln\left(\eta^2\rho^2\right) \mp \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}(\alpha+\beta) = 0$$

in which the above constraints lead to a geometrical contradiction.

Dirac delta distributions are spherically symmetric: the absence of spherically symmetric solutions implies that Dirac delta distributions are not solutions.

The issues about non-renormalizability coming from point-like particles is circumvented. L. Fabbri, *Int. J. Theor. Phys.* **52**, 634 (2013).
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Somewhat obvious, because spin is internal structure.

How reasonable is to have Dirac field condensates at galactic scales? This is not a new idea and it has been used in varius ways

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M. P. Silverman, R. L. Mallett, *Gen. Rel. Grav.* **34**, 633 (2002).

For the symmetries and approximations valid for galactic systems, one has

$$\mathrm{div}\vec{a} \approx -\mathrm{div}\ \mathrm{grad}V \approx -R_{tt} \approx 4\pi k \left[m\overline{\psi}\psi - i\hbar\left(\overline{\psi}\boldsymbol{\gamma}^t\boldsymbol{\nabla}_t\psi - \boldsymbol{\nabla}_t\overline{\psi}\boldsymbol{\gamma}^t\psi\right)\right] \approx$$
$$\approx -4\pi k\left[m\overline{\psi}\psi + \frac{3}{8}a\hbar^2\overline{\psi}\psi\overline{\psi}\psi + i\hbar\left(\vec{\boldsymbol{\nabla}}\overline{\psi}\cdot\vec{\gamma}\psi - \overline{\psi}\vec{\gamma}\cdot\vec{\boldsymbol{\nabla}}\psi)\right)\right]$$

or also the non-relativistic limit

$$\mathrm{div}\vec{a} \approx -4\pi k(m\phi^\dagger\phi + \frac{3}{8}a\hbar^2\phi^\dagger\phi\phi^\dagger\phi)$$

and the non-relativistic Schroedinger field equation

$$\frac{\hbar^2}{2m}\vec{\nabla}\cdot\vec{\nabla}\phi - \frac{9a^2\hbar^4}{512m}|\phi^\dagger\phi|^2\phi - \frac{3a\hbar^2}{16}\phi^\dagger\phi\phi + (E-m)\phi \approx 0$$

For high-density condensates, centripetal acceleration in spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 a\right) \approx \frac{3}{2}\pi k a \hbar^2 u^4$$

while for the condensate we get

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)\right] - \frac{9\hbar^2 a^2}{256}u^5 \approx 0$$

for which a solution is given by

$$u = \sqrt{\frac{8}{3\hbar a r \sin\theta}}$$

which can be substituted to give the tangential velocity as

$$v^2 \approx \frac{32\pi k}{3a}$$

mimicking Dark Matter behaviour. L.Fabbri, *Int.J.Mod.Phys.D* **22**, 1350071 (2013)

Navarro-Frenk-White profile

$$\rho_{\rm iso}(r) = \frac{\rho_0}{1 + (r/r_c)^2}$$

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Effective field theories

Name	Operator	Coefficient
C1	$\chi^\dagger \chi \bar{q} q$	m_q/M_*^2
C2	$\chi^\dagger \chi \bar{q} \gamma^5 q$	$i m_q/M_*^2$
C3	$\chi^\dagger \partial_\mu \chi \bar{q} \gamma^\mu q$	$1/M_*^2$
C4	$\chi^\dagger \partial_\mu \chi \bar{q} \gamma^\mu \gamma^5 q$	$1/M_*^2$
C5	$\chi^\dagger \chi G_{\mu\nu} G^{\mu\nu}$	$\alpha_s/4M_*^2$
C6	$\chi^\dagger \chi G_{\mu\nu} \tilde{G}^{\mu\nu}$	$i\alpha_s/4M_*^2$
R1	$\chi^2 \bar{q} q$	$m_q/2M_*^2$
R2	$\chi^2 \bar{q} \gamma^5 q$	$i m_q/2M_*^2$
R3	$\chi^2 G_{\mu\nu} G^{\mu\nu}$	$\alpha_s/8M_*^2$
R4	$\chi^2 G_{\mu\nu} \tilde{G}^{\mu\nu}$	$i\alpha_s/8M_*^2$

Name	Operator	Coefficient
D1	$\bar{\chi} \chi \bar{q} q$	m_q/M_*^3
D2	$\bar{\chi} \gamma^5 \chi \bar{q} q$	$i m_q/M_*^3$
D3	$\bar{\chi} \chi \bar{q} \gamma^5 q$	$i m_q/M_*^3$
D4	$\bar{\chi} \gamma^5 \chi \bar{q} \gamma^5 q$	m_q/M_*^3
D5	$\bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q$	$1/M_*^2$
D6	$\bar{\chi} \gamma^\mu \gamma^5 \chi \bar{q} \gamma_\mu q$	$1/M_*^2$
D7	$\bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma^5 q$	$1/M_*^2$
D8	$\bar{\chi} \gamma^\mu \gamma^5 \chi \bar{q} \gamma_\mu \gamma^5 q$	$1/M_*^2$
D9	$\bar{\chi} \sigma^{\mu\nu} \chi \bar{q} \sigma_{\mu\nu} q$	$1/M_*^2$
D10	$\bar{\chi} \sigma_{\mu\nu} \gamma^5 \chi \bar{q} \sigma_{\alpha\beta} q$	i/M_*^2
D11	$\bar{\chi} \chi G_{\mu\nu} G^{\mu\nu}$	$\alpha_s/4M_*^3$
D12	$\bar{\chi} \gamma^5 \chi G_{\mu\nu} G^{\mu\nu}$	$i\alpha_s/4M_*^3$
D13	$\bar{\chi} \chi G_{\mu\nu} \tilde{G}^{\mu\nu}$	$i\alpha_s/4M_*^3$
D14	$\bar{\chi} \gamma^5 \chi G_{\mu\nu} \tilde{G}^{\mu\nu}$	$\alpha_s/4M_*^3$

The case of two spinors we simply replicate the Lagrangian

$$L = R + \frac{1}{128k} W^2 - \frac{a_1}{8} \overline{\psi}_1 \boldsymbol{\gamma}^\mu \boldsymbol{\pi} \psi_1 W_\mu - \frac{a_2}{8} \overline{\psi}_2 \boldsymbol{\gamma}^\mu \boldsymbol{\pi} \psi_2 W_\mu$$

$$- i \overline{\psi}_1 \boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi_1 - i \overline{\psi}_2 \boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi_2 + m_1 \overline{\psi}_1 \psi_1 + m_2 \overline{\psi}_2 \psi_2$$

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The torsion can be integrated

$$L = R - i \overline{\psi}_1 \boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi_1 - i \overline{\psi}_2 \boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi_2 -$$
$$- \frac{1}{2} k a_1^2 \overline{\psi}_1 \boldsymbol{\gamma}^\mu \boldsymbol{\pi} \psi_1 \overline{\psi}_1 \boldsymbol{\gamma}_\mu \boldsymbol{\pi} \psi_1 -$$
$$- \frac{1}{2} k a_2^2 \overline{\psi}_2 \boldsymbol{\gamma}^\mu \boldsymbol{\pi} \psi_2 \overline{\psi}_2 \boldsymbol{\gamma}_\mu \boldsymbol{\pi} \psi_2 +$$
$$+ m_1 \overline{\psi}_1 \psi_1 + m_2 \overline{\psi}_2 \psi_2 -$$
$$- k a_1 a_2 \overline{\psi}_2 \boldsymbol{\gamma}^\mu \boldsymbol{\pi} \psi_2 \overline{\psi}_1 \boldsymbol{\gamma}_\mu \boldsymbol{\pi} \psi_1$$

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$$-\frac{1}{2}ka_2^2\overline{\psi}_2\boldsymbol{\gamma}^\mu\boldsymbol{\pi}\psi_2\overline{\psi}_2\boldsymbol{\gamma}_\mu\boldsymbol{\pi}\psi_2 +$$
$$+m_1\overline{\psi}_1\psi_1 + m_2\overline{\psi}_2\psi_2 -$$
$$-ka_1a_2\overline{\psi}_2\boldsymbol{\gamma}^\mu\boldsymbol{\pi}\psi_2\overline{\psi}_1\boldsymbol{\gamma}_\mu\boldsymbol{\pi}\psi_1$$

The Hamiltonian provides the possibility for flavour oscillation, both kinematically and dynamically

$$\Delta\Phi \approx \left(\Delta m^2 + 2kamN\right)\frac{L}{2E}$$

L. Fabbri and S. Vignolo, *Mod. Phys. Lett.* A**31**, 1650014 (2016)

The least-order torsion gravity with gauge fields for Dirac and scalar fields

$$L=\frac{1}{4}(\nabla_{\alpha}W_{\nu}-\nabla_{\nu}W_{\alpha})(\nabla^{\alpha}W^{\nu}-\nabla^{\nu}W^{\alpha})-\frac{1}{2}M^{2}W^{2}$$
$$-i\overline{L}\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}L-i\overline{R}\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}R-\nabla^{\mu}\phi^{\dagger}\nabla_{\mu}\phi+$$
$$+YW_{\mu}(\overline{L}\boldsymbol{\gamma}^{\mu}L-\overline{R}\boldsymbol{\gamma}^{\mu}R)+\Xi\phi^{2}W^{2}+$$
$$+G(\overline{L}\phi R+\overline{R}\phi^{\dagger}L)+\frac{1}{2}\lambda^{2}\phi^{4}$$

The torsion field equations

$$\nabla_{\alpha}(\nabla^{\alpha}W^{\nu}-\nabla^{\nu}W^{\alpha})+M^{2}W^{\nu}=$$
$$=Y(\overline{L}\boldsymbol{\gamma}^{\nu}L-\overline{R}\boldsymbol{\gamma}^{\nu}R)+2\Xi\phi^{2}W^{\nu}$$

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for large torsion mass can be integrated

$$M^2W^\nu\approx Y(\overline{L}\boldsymbol{\gamma}^\nu L-\overline{R}\boldsymbol{\gamma}^\nu R)+2\Xi\phi^2W^\nu$$

$$L=-i\overline{L}\boldsymbol{\gamma}^\mu\boldsymbol{\nabla}_\mu L-i\overline{R}\boldsymbol{\gamma}^\mu\boldsymbol{\nabla}_\mu R-\nabla^\mu\phi^\dagger\nabla_\mu\phi$$
$$-2(M^2-2\Xi\phi^2)^{-1}Y^2\overline{L}R\overline{R}L+$$
$$+G(\overline{L}\phi R+\overline{R}\phi^\dagger L)+\frac{1}{2}\lambda^2\phi^4$$

The potential has a new minimum

$$\lambda^2 v^2 (M^2 - 2\Xi v^2)^2 - 4\Xi Y^2 \overline{L} R \overline{R} L = 0$$

the unitary gauge $\phi^\dagger = (0, v + H)$ gives rise to the

$$L = -i\overline{\nu}\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \nu - i\overline{e}\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu e + Gv\overline{e}e -$$
$$-\nabla^\mu H \nabla_\mu H + 3\lambda^2 v^2 H^2 +$$
$$-[M^2 - 2\Xi(v+H)^2]^{-1} Y^2 (2\overline{\nu} e_R \overline{e}_R \nu + \tfrac{1}{2}\overline{e}\boldsymbol{\gamma}^\nu e \overline{e}\boldsymbol{\gamma}_\nu e) +$$
$$+G\overline{e}eH + \tfrac{1}{2}\lambda^2 (H + 4v) H^3 + 2\lambda^2 v^3 H + \tfrac{1}{2}\lambda^2 v^4$$

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$$\begin{aligned} L = & -i\overline{\nu}\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \nu - i\overline{e}\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu e + Gv\overline{e}e - \\ & -\nabla^\mu H \nabla_\mu H + 3\lambda^2 v^2 H^2 + \\ & -[M^2 - 2\Xi(v+H)^2]^{-1} Y^2 (2\overline{\nu}e_R \overline{e}_R \nu + \tfrac{1}{2}\overline{e}\boldsymbol{\gamma}^\nu e\overline{e}\boldsymbol{\gamma}_\nu e) + \\ & + G\overline{e}eH + \tfrac{1}{2}\lambda^2 (H + 4v) H^3 + 2\lambda^2 v^3 H + \tfrac{1}{2}\lambda^2 v^4 \end{aligned}$$

The induced mass and cosmological constant

$$m_e = Gv \qquad m_{\mathrm{H}}^2 = 2\lambda^2 v^2 \qquad \Lambda = -\frac{\lambda^2 v^2 M^2}{2\Xi}$$

$$\lambda^2 v^2 M^4 = \Xi Y^2 (|2\overline{\nu}e_R|^2 + |\overline{e}e|^2 + |i\overline{e}\boldsymbol{\pi}e|^2)$$

arXiv:1504.04191

Torsion potential:

1. no singularities
2. Dark Matter dynamics
3. Degenerate-mass neutrino oscillation
4. the cosmological constant and coincidence problem

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Anywhere you have a spinor there can be more to it.

There is no critical mass in torsion community:

- connection 1915, spinors 1928
- the Newton constant misconception
- difficulties intrinsic to the non-linearities

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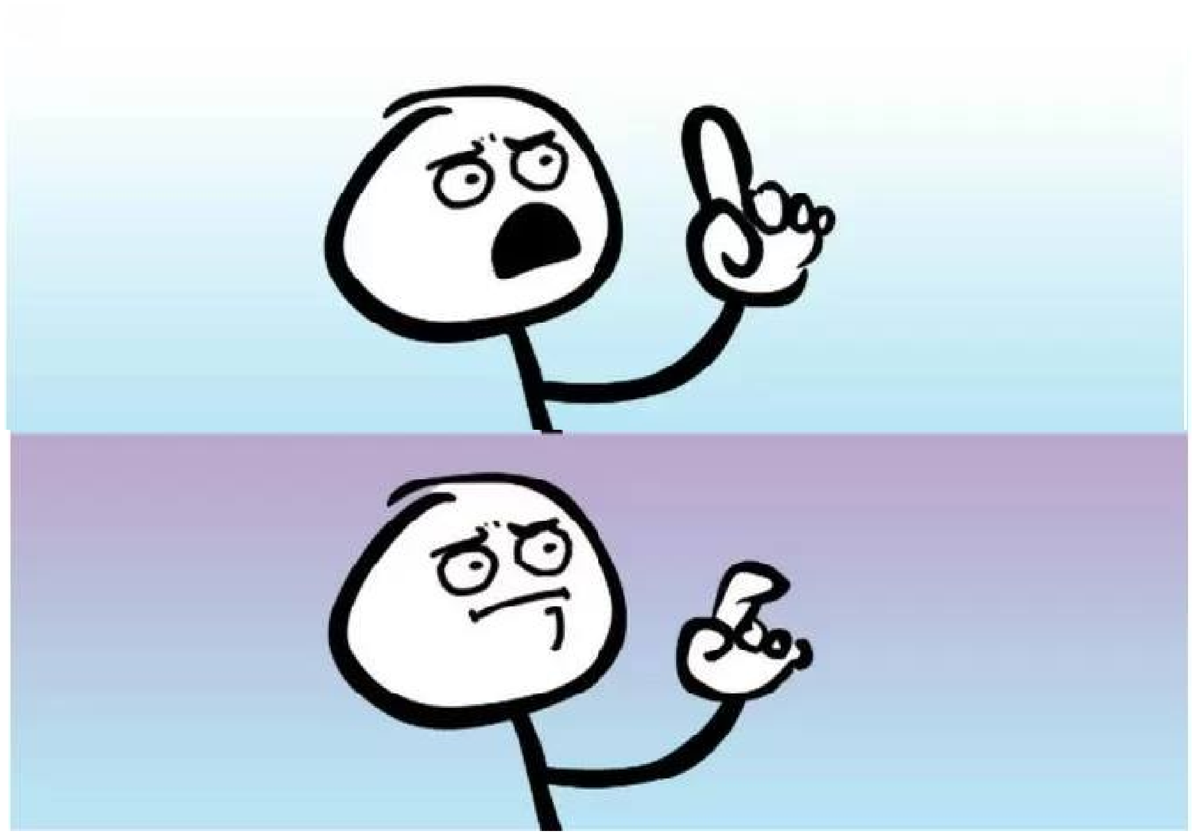
–- Luca: “... so, torsion is natural, despite admittedly it is difficult to treat and we still do not know what it can do.”

–- Colleague: “Nobody intelligent would ever consider anything that is difficut to manage and of which we are not sure about the outcome, regardless its naturality.”

–- Other Colleague (who just had a new-born son): “Well, let me disagree.”



A metaphor

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We are into Naturalness so much that we do not realize when we see it.

Thomas Walter Bannerman Kibble
(23 December 1932 – 2 June 2016)