Do virtual particles really exist?

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Outline

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1. Introduction
2. Metaphysical(?) considerations
3. The problem
4. An alternative point of view
5. Back to QFT
6. (No–)Conclusions

A simple answer

When Luca kindly proposed me to give a talk titled *Do virtual particles really exist?* I was strongly tempted to prepare the shortest talk ever:

NO

Any question?

Sources of confusion

Soon after I realized that things are not so simple.

- The concept of *virtual particles* is largely used, both in didactical practice and in divulgative science stuff;
- It's used in reasearch too!
- In the end, what really means *do virtual particles exist?*
- That brings immediately to maybe deeper questions: *do elementary particles exist at all? What an elementary particle really is?*
- It's clear that we'll quickly slide into metaphysics; in fact a question about the *existence* of something is an *ontological* question.
- Luckily enough, I strongly believe that metaphysics, understood in the proper way, is fundamental to physics.

Outline

Semi–phylosophical introduction

WARNING! Just my definitions. I'am not a philosopher.

- *Realism*: an objective world, independent of human mind[1], perceptions and/or desires, exists out there;
- *Naive Realism*: our everyday experiences and perceptions can fournish, even if only through a *limiting procedure*, striking analogies able to capture and describe elements of reality whose phenomena can be attained only by using *extended senses* (microscopes, telescopes, LHC and so forth);
- *Scientific Realism*: our best scientific theories give true or *approximately*[2] true descriptions of observable and unobservable[3] aspects of a mind–indepentent world.

[1] Which, by the way, is part of the world.

[2] *Hic sunt leones.*

[3] And here tigers too.

We are...

- *Realists*: I hope so. . .
- *Naive Realists*: We cannot.[4]
- *Scientific Realists*: That's my own position, even if I arrived here after a contamination with (neo–)kantian ideas.

[4] See later.

A bit of history

Leaving aside ancient phylosophical ideas, the first elementary particle recognized as such was the *electron*. After Crookes, Schuster, Thomson and other physicists work (1880-1900) it became clear that small bunches of matter with a well defined mass m and an electric charge $-e$ do in fact exist inside atoms.

Looking for a mental representation of an electron, I think it was really easy to consider the idea of a material point.

Why we cannot be Naive Realists (no more)

We all known the concept of *material point*. For example we know that, under specific circumstances and approximations, even a planet can be considered as a *material point*.

From a certain point of view, we can (at least, a physicist at the very beginning of 20^{th} century could) consider an *electron* like a very small tennis–ball with some intrinsic properties: for example a well defined mass m and an electric charge $-e$.

I know (to a certain extent, I'm not Federer) the behaviour of a tennis–ball; *limiting procedure* $\rightarrow$ the electron behaves as a very very small tennis–ball. My everyday experiences give me a striking and perfect analogy to understand electrons.

But then...

Problems with "classical" definition of an electron

Take the electron to be a sphere of radius a and electric charge $-e$ uniformly distributed over the surface. We can compute the energy of the E.M. field generated by an electron *at rest*[5]

$$E_{\mathrm{EM}} = \frac{e^2}{2a} \to \infty \quad \text{when } a \to 0$$

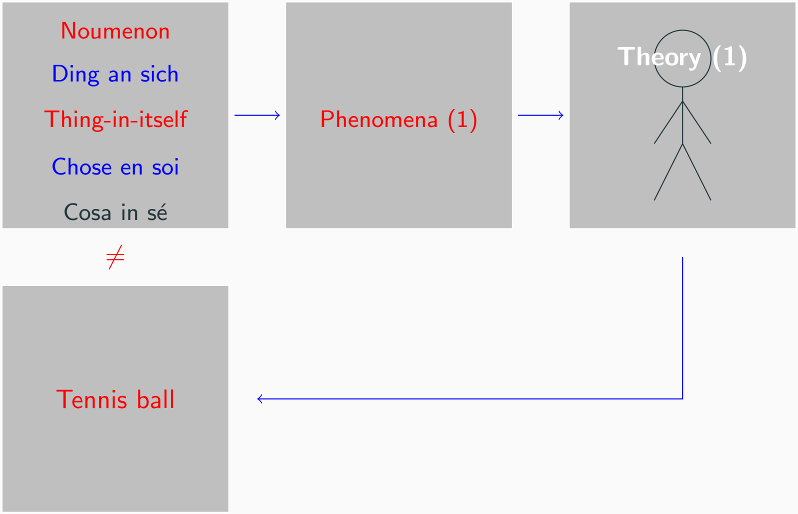
Well, energy is classically defined up to an arbitrary additive constant. But you get also, assuming that all of the electron mass is given by EM effects,

$$m = \frac{2e^2}{3ac^2}$$
$$a = \frac{2e^2}{3mc^2}$$

But then relativistic inconsistencies come out. We cannot conceive an electron as a small charged tennis–ball.

[5] See *Feynman Lectures, vol II, chap 28.*

(My) scientific realism point of view (1/3)



Quantum days

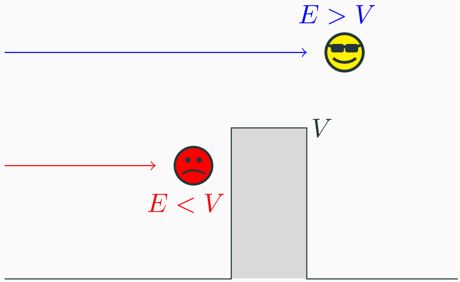
There is of course much more.

It soon became clear that electrons, under some circumstances, show a wave–like behaviour.

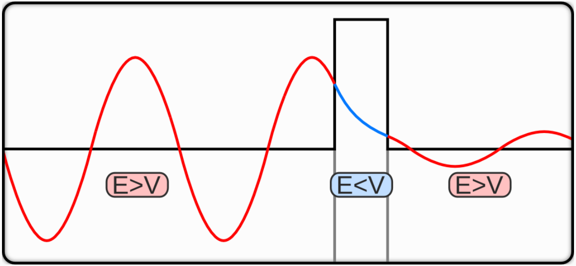
From an historical point of view all this started with a theoretical step (de Broglie, Schrödinger), not with an experimental one.

Atomic spectra $\rightarrow$ *Bohr atomic model* $\rightarrow$ *de Broglie, Schrödinger*.

Tunnel effect: classical particles point of view



Tunnel effect: quantum waves point of view



First quantum days, 2

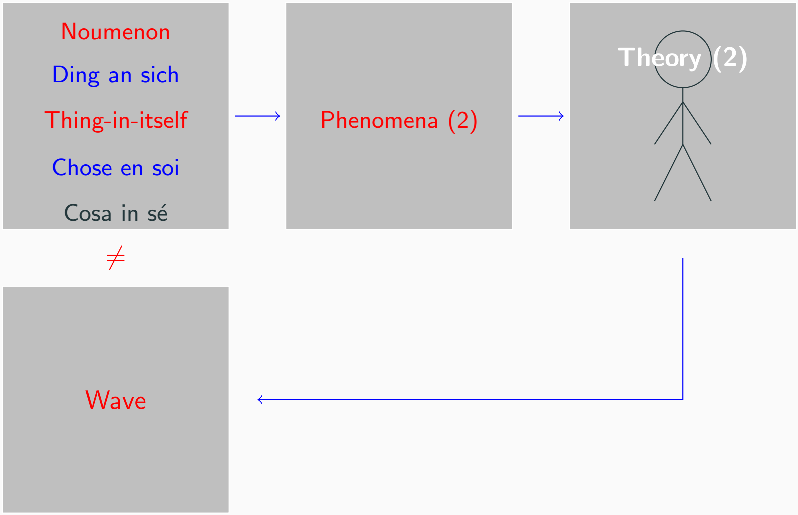
But we know waves! What I mean is that we have a clear mental representation of what a wave is. So, can we build a wave model of the electron?

Unfortunately we cannot: we never detect an electron spread out a bit here and a bit there. When we detect it, we always observe a single well localized particle.

Moreover, Schrödinger equation is wrong from a relativistic point of view[6].

[6] More on this later.

(My) scientific realism point of view (2/3)



So what an elementary particle is?

Since the first days of quantum mechanics (if not before) physicist had to abandon what I called *Naive Realism*.

We were obliged to forget the picture of an elementary particle as a very small and indivisible bunch of matter, or as a wave spread here and there.

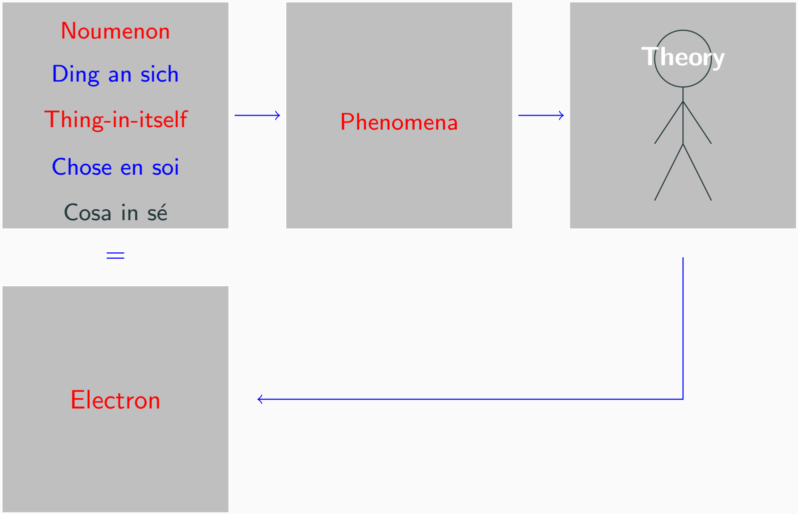
As Wigner teach us, an elementary particle of mass m and spin s is *an irreducible representation* (m, s) *of the Poincaré group*[7]. That's what the theory tells us, and that is what an elementary particle is: to the best of our knowledge, up to now.

I think that this is the best example I can find of Scientific Realism.

[7] See for example P. Ramond, *Field Theory. A modern primer*, Second edition.

(My) scientific realism point of view (3/3)

An electron is what it is.



Outline

Naive questions

The first naive questions we have to answer are: *what on the earth a virtual particle is? Where this concept come from?*

To answer, we have to return to the glorious days of second quantization.

The first attempt to obtain a relativistic version of the Schrödinger equation was the Klein–Gordon equation:

$$(\Box + m^2)\phi(x) = 0$$

It comes from the correspondence principle:

$$E^2 = m^2 + p^2 \qquad (c = \hbar = 1)$$

$$p \to -i\nabla \qquad E \to i\,\partial/\partial t$$

Second quantization 1/3

Unfortunatly we cannot interpret $\phi(x)$ as the wave–function of a single particle:

- negative probabilities;
- negative energy states.

We can expect some trouble in the search for a relativistic and quantum description of a point particle. Indeed, relativity associates a momentum scale $p = mc$ to a particle of mass m. But the uncertainty relations $\Delta x \cdot \Delta p \sim \hbar$ tell us that for length scales smaller than the Compton wavelength $\lambda = \hbar/mc$, the concept of a point particle may suffer difficulties. Analyzing the position of the particle with a greater accuracy requires an energy momentum of the same order of the rest mass, thus allowing the creation of new particles[8].

[8] Itzykson–Zuber, *Quantum Field Theory*, page 46 of the first edition.

Second quantization 2/3

The difficulties eventually led Dirac to his equation: but still problems with negative energy states.

To write down a relativistic and quantum theory of elementary particle physics we had to discard particles in favour of fields.

$\phi(x) \to$ scalar field, not wavefunction. We put the system in a box of linear size L remembering that ϕ is not only a field, but an operator–valued one.

Introduce commutation relations and Fourier expansion:

$$[\phi(\mathbf{x}, t), \dot{\phi}(\mathbf{x}', t)] = i\delta(\mathbf{x} - \mathbf{x}')$$

$$\phi(x) = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_{\mathbf{k}}} \right)^{1/2} \left[a(\mathbf{k}) e^{-ik\cdot x} + a^{\dagger}(\mathbf{k}) e^{ik\cdot x} \right]$$

$$\omega_{\mathbf{k}} = k_0 = \sqrt{\mathbf{k}^2 + m^2}$$

Second quantization 3/3

We find harmonic oscillator commutation relations

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta_{\mathbf{k},\mathbf{k}'}$$

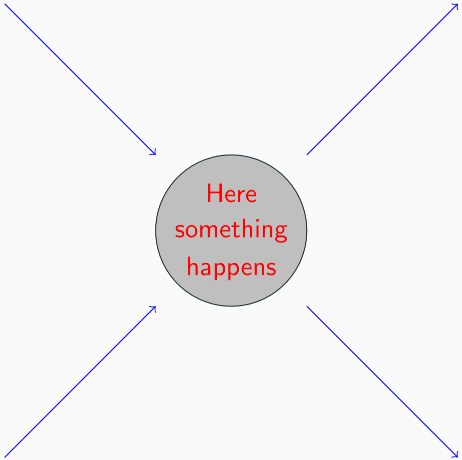
so that $a(\mathbf{k})$ and $a^\dagger(\mathbf{k}')$ are destruction and creation operators:

$$a^\dagger(\mathbf{k})|0\rangle = |\mathbf{k}\rangle$$
$$a(\mathbf{k})|\mathbf{k}\rangle = |0\rangle$$

Following the stardard path given by textbooks we arrive to the concept of Feynman propagator.

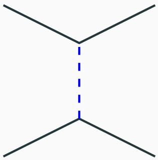
Moreover, a particle becomes a quantum of the *field*, i.e. something coming out from the vacuum when we act with a creation operator of the given field.

The tipical situation



Warning!

To illustrate how these propagators arise, we shall consider qualitatively nucleon–nucleon scattering. In this process there will be two nucleons but no mesons present in the initial and final states (i.e. before and after the scattering). The scattering, i.e. the interaction, corresponds to the exchange of virtual mesons between the nucleons. The simplest such process is the one–meson exchange[9].



[9] Mandl–Shaw, *Quantum Field Theory*, pag. 55 of the first edition.

The perturbative point of view

It seems that the best description we can give of this situation is a perturbative one.

From this point of view: why shouldn't a virtual particle exist? After all it is created from the vacuum, in a vertex interaction, by a creation operator, just like a real particle is.

It doesn't respect energy–momentum dispersion relations, but who cares?

If we follow this path, it seems that in a perturbative solution of interacting QFT the concept of virtual particle is unavoidable.

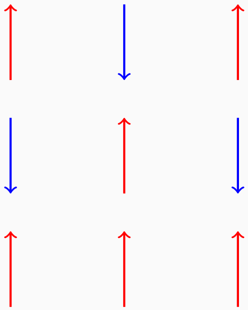
The problem is that we can end up with a virtual photon at rest. Can we consider this strange animal a virtual photon, even if it is not detectable, not even in principle?

Outline

Ising model – 1/5

Let me (apparently) change subject: *the Ising Model*.

$$\mathcal{H} = -\sum_{\langle x,y\rangle} \sigma_x \sigma_y - h \sum_x \sigma_x \qquad \sigma = \pm 1$$



Ising model – 2/5

$$Q = \sum_{\{\sigma\}} e^{-\beta\mathcal{H}}$$

$$\langle M \rangle = \frac{1}{Q} \sum_{\{\sigma\}} \left(\sum_x \sigma_x \right) e^{-\beta\mathcal{H}} = \frac{1}{\beta} \left[\frac{\partial \ln Q}{\partial h} \right]_{h=0}$$

- Solved in $d = 1$: no phase transition;
- Solved in $d = 2$ at $h = 0$: second order phase transition;
- Not yet solved in $d > 2$;
- Most important quantities are *critical exponents*, because they are *universal*.

Ising model – 3/5

Critical exponents dictate the behaviour of the system near criticality:

$$\langle \sigma_x \sigma_y \rangle \propto e^{-|x-y|/\xi} \qquad \xi \sim |T - T_c|^{-\nu}$$
$$\langle \sigma_x \sigma_y \rangle \propto |x-y|^{\eta} \qquad \text{at } T = T_c$$

And so on.

Scaling relations do exist: only two critical exponents are independent. Let's take ξ and η.

Ising model – 4/5

Renormalization Group transformation:

$$\phi(x) = \frac{1}{|B(x)|} \sum_{y \in B(x)} \sigma_y$$

Imagine you iterate the transformation (we start from an infinite lattice, you can iterate as many times as you want). $\phi(x)$ becomes a *real* variable, it's no more restricted to ± 1 values. Let's pretend the nearest neighbour coupling survive (with some renormalization, maybe): but the transformation induces self–coupling for the ϕ *field*:

$$\mathcal{H} = \sum_x \left[-\sum_{\nu=1}^{d} \phi(x)\phi(x+\nu) + \mu\phi^2(x) + \lambda\phi^4(x) + \cdots \right]$$

We can then use:

- translational invariance;
- redefinitions of the couplings $\mu,\ \lambda$;
- continuum limit

and we obtain

$$\mathcal{H} = \int \mathrm{d}^d x \left\{ \frac{1}{2} \left[\partial_\mu \phi(x)\right]^2 + \frac{1}{2}\mu_0^2 \phi^2(x) + \frac{\lambda}{4!}\phi^4(x) \right\}$$

This is the Landau-Ginzburg model for ferromagnetism. We know that it is in the *same universality class of the Ising model*, so we can think to use a perturbative expansion around the free case $(\lambda = 0)$ in order to obtain the critical exponents in the Ising universality class. It's a very well known procedure in *Statistical* Field Theory[10].

[10]See for example *G. Parisi, Statistical Field Theory*.

Statistical Field Theory 1/3

Let's say we want to compute the two–point function

$$\tilde{G}(x) = \langle \phi(x)\phi(0) \rangle$$

in order to evaluate ν and η.

First of all, we can easily go to Fourier space:

$$\tilde{G}(x) = \int \frac{\mathrm{d}^d p}{(2\pi)^d}\, e^{-ip\cdot x}\, G(p)$$

Next we expand in powers of the coupling constant λ:

$$G(p) = G_0(p) + \lambda G_1(p) + \lambda^2 G_2(p) + \cdots$$

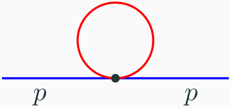
where

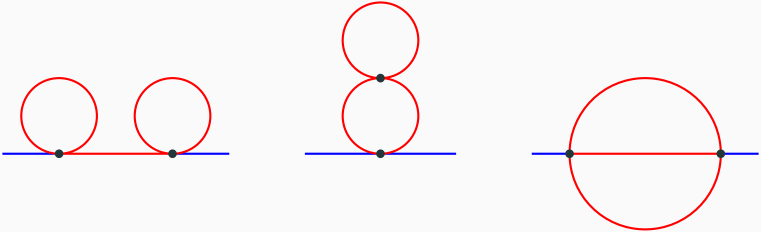
$$G_0(p) = \frac{1}{p^2 + \mu_0^2}$$

Does it resemble something already known? Sure, it is the *Euclidean propagator* of a free scalar particle of mass μ_0.

Statistical Field Theory 2/N

It comes out (not so surprisingly at this point) that you can organize your perturbative computations with the aid of diagrammatic tools:

$G_1(p) \propto$ 



In case you are wondering why they resemble so much Feynman diagrams: it's because *they are* Feynman diagrams.

You can still reorganize your computation considering only the so called One–Particle–Irreducible diagrams. In this way you obtain

$$G(p) = \frac{1}{G_0^{-1}(p) - \Sigma(p)}$$

where $\Sigma(p)$, the self–energy, is of course a power series in λ. Then in principle (I'm leaving aside details about renormalization) you can get ν and η by looking at the behaviour of $G(p)$ for $p \to 0$ and $\mu^2 \to 0$, where μ is some kind of *renormalized* mass.

But remember: we started from a spin system; we are studing *ferromagnetism*. No particles at all from the very beginning. So, then: why a perturbative diagrammatic tool should imply the existence of *virtual particles*?

Outline

Quantum Field Theory on a Lattice 1/N

Generating functional for a QFT:

$$\mathcal{Z} = \int [\mathcal{D}\phi]\, e^{-\frac{i}{\hbar} S[\phi, \partial_\nu \phi]}$$

$[\mathcal{D}\phi]$ is the (generally not so well defined) functional measure, ϕ stands for the collection of all fields entering in the theory, S is the classical action.

It is called *generating functional* because by functional differentiating respect to external sources we can recover all the n–point functions of the theory.

The expectation value of an observable $\mathcal{A}[\phi, \partial_\nu \phi]$ is

$$\langle \mathcal{A} \rangle = \frac{1}{\mathcal{Z}} \int [\mathcal{D}\phi]\, \mathcal{A}[\phi, \partial_\nu \phi]\, e^{-\frac{i}{\hbar} S[\phi, \partial_\nu \phi]}$$

Quantum Field Theory on a Lattice 1/3

Take as an example ϕ^4 scalar theory.

$$\mathcal{L}[\phi,\, \partial_\mu \phi] = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m_0^2\phi^2(x) - \frac{\lambda_0}{4!}\phi^4(x)$$

$$S[\phi,\, \partial_\mu \phi] = \int \mathrm{d}t\, \mathrm{d}^3x\, \mathcal{L}[\phi,\, \partial_\mu \phi]$$

Now set $t = -ix_0$ (Wick rotation). The metric becomes euclidean. The euclidean Action is in fact an Hamiltonian:

$$S_{\mathrm{E}} = \int \mathrm{d}^4x \left\{ \frac{1}{2}\left[\partial_\mu \phi(x)\right]^2 + \frac{1}{2}m_0^2\phi^2(x) + \frac{\lambda_0}{4!}\phi^4(x) \right\}$$

But this is exactly the same expression we saw before.

Quantum Field Theory on a Lattice 2/3

In natural units ($c = \hbar = 1$) we get

$$\mathcal{Z} = \int [\mathcal{D}\phi] \, e^{-S_{\mathrm{E}}}$$

This is the *canonical partition function* of a statistical system.

A general result: a *quantum field theory* in d space–time dimensions can be rewritten as a *statistical field theory* in $d+1$ space dimensions.

The theory has to be renormalized.

The first step is to regularize the theory, for example by using a *lattice*. The lattice spacing a acts as an ultraviolet cut–off $\Lambda \sim 1/a$.

$$[\mathcal{D}\phi] \to \prod_x \mathrm{d}\phi(x) \qquad \partial_\nu \phi(x) \to \frac{\phi(x+\nu) - \phi(x)}{a}$$

Quantum Field Theory on a Lattice 3/3

After some steps you obtain

$$S = \sum_{x} \left\{ -\beta \sum_{\nu} [\varphi(x)\varphi(x+\nu)] + \varphi^2(x) + \lambda[\varphi^2(x) - 1]^2 \right\}$$

The lattice spacing a has been absorbed in the definition of the lattice field φ and of the lattice couplings $(\beta,\, \lambda)$.

If now you send $\lambda \to \infty$ you obtain that $\varphi^2 = 1$, and so

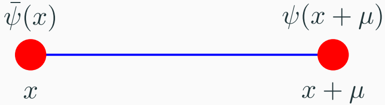
$$S = -\beta \sum_{\langle x,y \rangle} \varphi(x)\varphi(y)$$

which is the Ising Hamiltonian, including $\beta = 1/T$ (I set $k = 1$).

If you want to write a lattice gauge theory, with local invariance, you have to implement a discrete *parallel transport*.

Let's take QED as an example. Put free fermions on a lattice and sobstitute (carefully) derivatives with finite differences. You'll end up with terms of this kind:

$$\bar{\psi}(x)\psi(x+\mu)$$



Under a *local* gauge transformation this term will go into

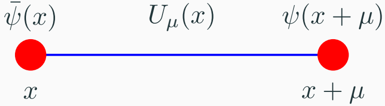
$$\bar{\psi}(x)e^{-i\alpha(x)}e^{i\alpha(x+\nu)}\psi(x+\nu)$$

and the Lagrangian is no more invariant.

Lattice Gauge Theory 2/7

Solution: define a new field, $U_\mu(x)$, which lives on the link (x, μ). This field has values in the *local symmetry group* (in this case U(1)) so that under a local gauge transformation it transforms in this way:

$$U_\mu(x) \to e^{i\alpha(x)} U_\mu(x) e^{-i\alpha(x+\mu)}$$



$$\bar{\psi}(x) U_\mu(x) \psi(x+\mu) \to \bar{\psi}(x) U_\mu(x) \psi(x+\mu)$$

The new term is invariant under local gauge transformation.

Lattice Gauge Theory 3/7

Since I know that $U_\mu(x)$ is an element of the U(1) abelian group, let me write it in this way:

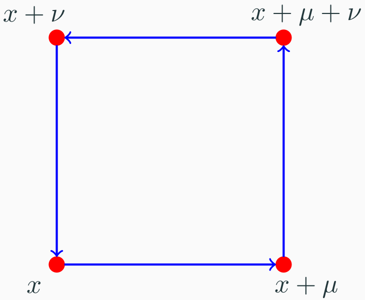
$$U_\mu(x) = e^{iag_0 A_\mu(x)}$$

where a is the lattice spacing and g_0 is some not–yet specified constant.

Spoiler alert: since I know the story, I'll start from the end (it's easyer).

$$S = \frac{a^4}{2g_0^2} \sum_x \sum_{\mu>\nu} \mathrm{Re}\left[U_{\mu\nu}(x)\right]$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x)$$



A plaquette.

If you now expand the exponentials and send $a \to 0$ you obtain

$$S = \frac{1}{4g_0^2}\int \mathrm{d}^4x\, F_{\mu\nu}(x)F_{\mu\nu}(x) + \mathcal{O}(a^2)$$

with

$$F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

In this way you can define a covariant derivative on the lattice.

Lattice Gauge Theory 5/7

Recipe to build a Lattice Gauge Theory from a QFT in the continuum:

- Wick rotate to Euclidean space;
- replace continuum with a mesh of points (a lattice);
- put matter fields on sites;
- put gauge fields on links as elements of the gauge group (not of the algebra);
- replace gauge covariant derivative with gauge covariant finite differences.

You are ready to put your theory on a computer and to do a Monte Carlo simulation in order to extract non–perturbative observables.

$$\mathcal{Z} = \int \left\{\mathcal{D}\left[U, \bar{\psi}, \psi\right]\right\} \exp\left(-S_W[U] - \bar{\psi}M[U]\psi\right)$$

Monte Carlo simulation: obtain equilibrium configurations through a Markovian process, compute observables over these configurations.

A problem: it's not so easy to represent anticommuting variables on a computer.

Solution: Matthews–Salam formula:

$$\int \left\{\mathcal{D}\left[\bar{\psi}, \psi\right]\right\} \exp\left(-\bar{\psi}M[U]\psi\right) = \det M[U]$$

$$\mathcal{Z} = \int \left\{\mathcal{D}\left[U\right]\right\} \exp\left(-S_W[U] + \operatorname{Tr}\ln M[U]\right)$$

Lattice QCD 7/7

When you do a simulation the effect of the quark field and of the gluon field is taken into account *automagically*, in a non–perturbative way (or, if you prefer, to all orders of perturbation theory).

So the question is: where virtual particles are in this game?

Outline

(No–)Conclusions

From a perturbative point of view, it seems impossible to avoid the use of virtual particle concept.

On the other hand, from a non–perturbative perspective you'll never see something like a virtual particle.

Thanks