



What is the problem with the cosmological constant?

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What is the problem with the vacuum energy?

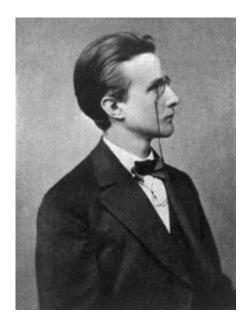
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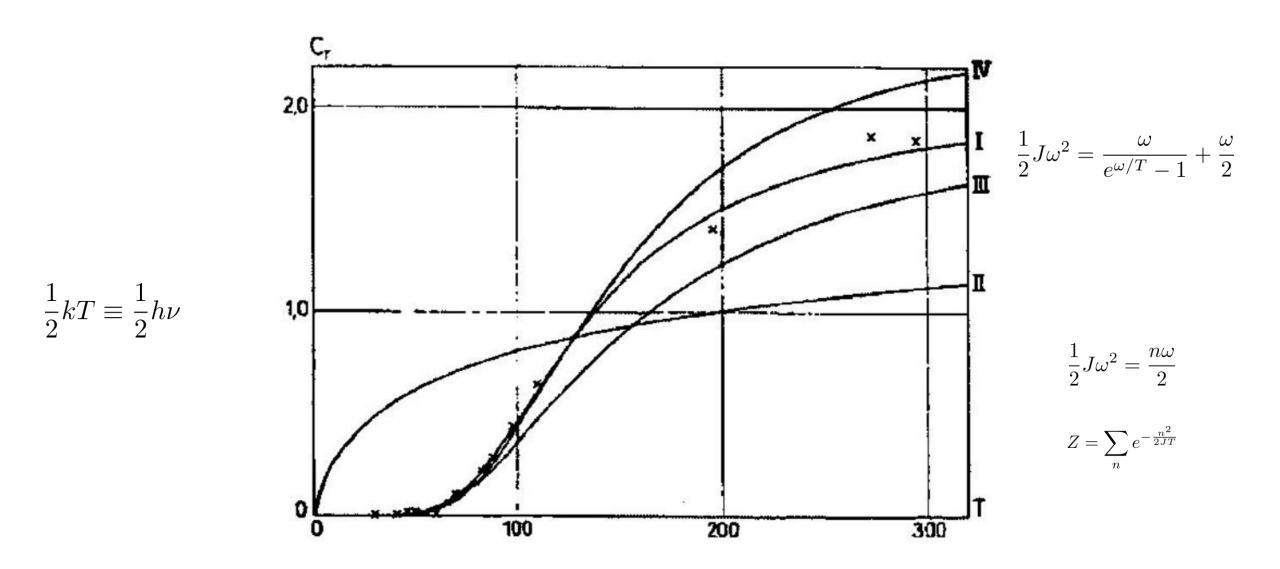


The concept of zero point energy emerges first in the « second theory » of radiation given by Planck as early as 1911, whereby oscillators absorb energy continuously and reemit in a quantised way. This was taken up by Nernst and collaborators in the calculation of the melting point of hydrogen. In modern parlance, zero point energy arises in the spectrum of an harmonic oscillator for n=0:

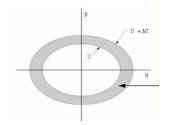


$$E_n = (n + \frac{1}{2})\hbar\omega$$





First experimental evidence for zero point energy: the specific heat of molecular hydrogen (Eucken) fitted by a spectrum with (I) rather than without (II) zero point energy (Einstein and Stern 1913), using Planck's second theory. Recalculated later in 1913 by Ehrenfest using statistical physics and the first Planck theory with no zero point energy.



$$(n+1)^2 - n^2 = 2n+1$$
 $Z = \sum_n (2n+1)e^{-n^2/2JT}$ Reich (1919) 2d rotator Planck 1915

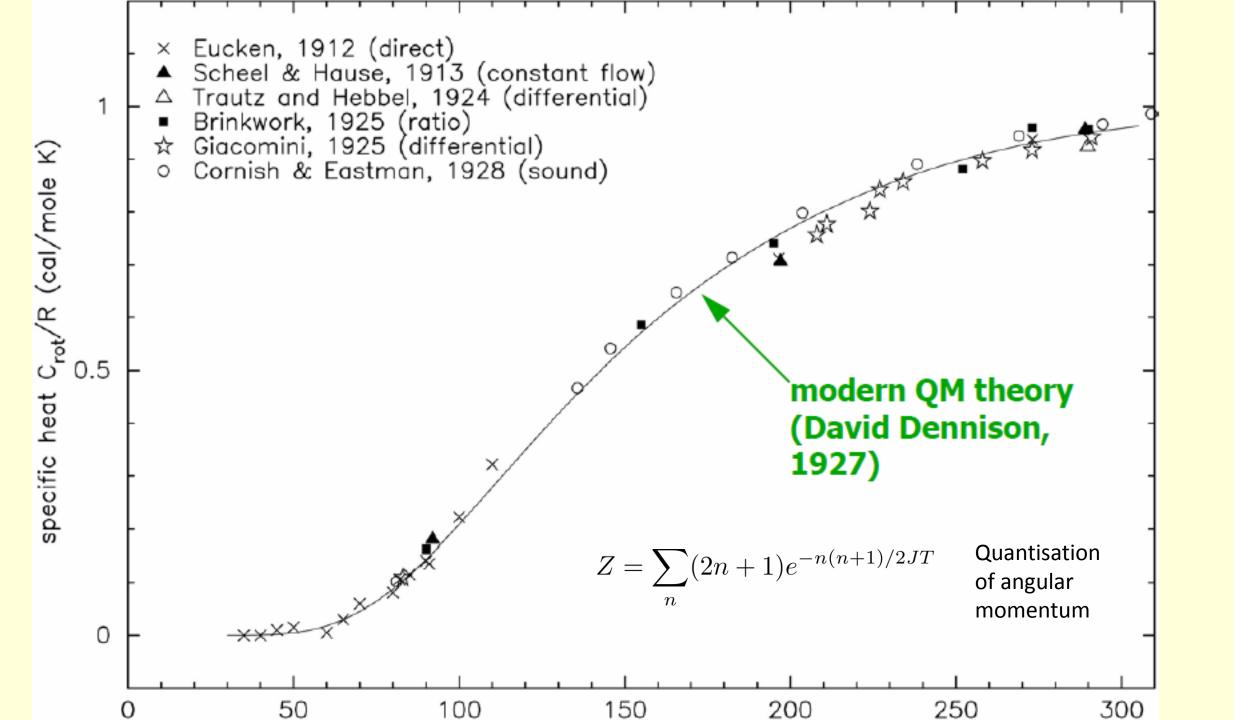
It is probable that the minimum vibrational energy of BO (and doubtless of other) molecules is ½ quantum. In the case of molecular rotational energy, the necessity of using half quanta is already well established. Analogous relations appear in line spectra; e.g. Heisenberg has successfully used half-integral radial and azimuthal quantum numbers in explaining the structure and Zeeman effect of doublets and triplets.⁴⁷

Half quanta

Mulliken, Nature 1925

 $(n+\frac{1}{2})^2 - \frac{1}{4} = n(n+1)$

No zero point energy



A more direct and correct manifestation of the vacuum energy: the Lamb shift

$$< V(r + \delta r) >= V(r) + \frac{1}{6}\Delta V < (\delta r)^{2} >$$

$$fluctors the equation of t$$

Fluctuation of the interaction potential due to the jittering of the electrons in the fluctuating electric field of the nucleus.

Taking the average in the s states (the only ones with a non-vanishing wave function at the origin)

$$\delta E_n = \frac{2\alpha^4}{3} \frac{1}{n^3} < (\delta r)^2 > \qquad \qquad \delta \vec{r} = -\frac{e}{m\omega^2} \vec{E}$$

The fluctuation is given by an integral over all frequencies from IR to UV:

$$<(\delta r)^2>=rac{e^2}{m}\int_0^\infty rac{d\omega}{\omega^4}E^2$$

This integral encapsulates all the worse of Quantum Field Theory: it diverges at 0 and infinity. It also involves the vacuum energy of the electromagnetic field:

$$\langle E^2 \rangle = \int \frac{d^3k}{(2\pi)^3} \omega = \int d\omega E^2$$

This implies that the quantum fluctuations are:

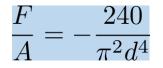
$$<(\delta r)^2>=\frac{2\alpha}{\pi m^2}\int_0^\infty \frac{d\omega}{\omega}$$
 Logarithmic divergences.

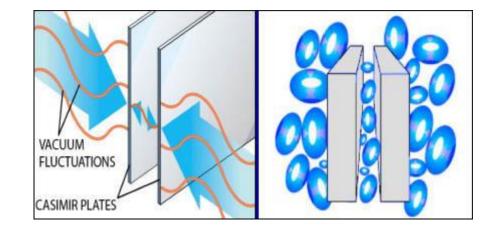
Physically, the theory is valid up to the cut off m (non-relativistic theory) and distances are larger than Bohr's radius:

$$\int_0^\infty \frac{d\omega}{\omega} = \int_{1/a_0}^m \frac{d\omega}{\omega} = \ln \frac{1}{\alpha}$$

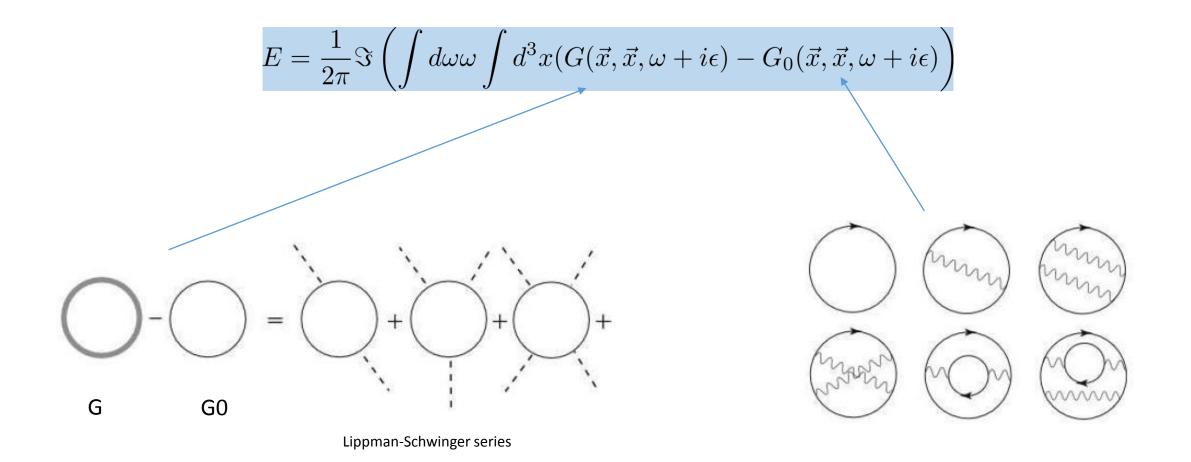
This looks like dirty old tricks... It took all the machinery of renormalisation to really understand what is going on... Clearly the Lamb shift is a consequence of the coupling of the electric field to matter (and the coupling between the quantum fluctuations and matter).

Another clear example is the Casimir effect:



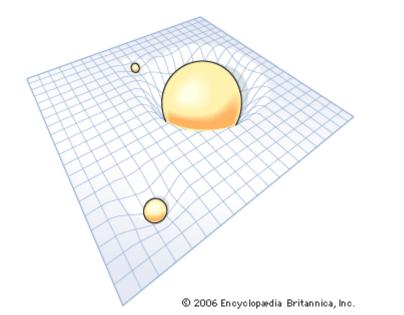


The Casimir effect measures the effect of the two plates (seen mathematically as boundary conditions for the fields) on the quantum fluctuations. It depends on the coupling of the field to matter intrinsically (the boundary conditions).



The Casimir energy is not a measure of the vacuum fluctuations but of the coupling of matter to the quantum fluctuations.

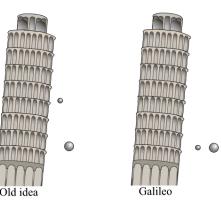
The vacuum energy plays a fundamental role in the physics of the Universe: cosmology.



Equivalence principle: all forms of energy influence the geometry of space-time in the same manner.

 $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T = 8\pi G_N T_{\mu\nu}$

curvature



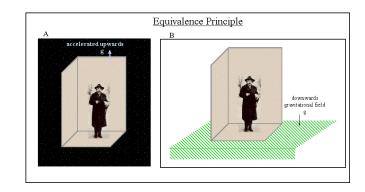
matter

$$T_{ab}^{\rm vac} = -V\eta_{ab}$$

Locally vacuum energy is homogeneous and isotropic

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$$

Locally gravity can be effaced

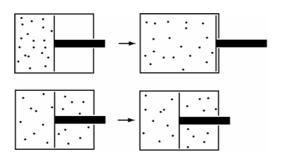


$$T_{\mu\nu}^{\rm vac} = -V g_{\mu\nu}$$

For a fluid:

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$$





Everything happens as though the energy in vacuo would be different from zero. In order that absolute motion, i.e., motion relative to vacuum, may not be detected, we must associate a pressure $p = -\rho c^2$ to the density of energy ρc^2 of vacuum. This is essentially the meaning of the cosmical constant λ which corresponds to a negative density of vacuum according

to $\rho_0 = \lambda c^2 / 4\pi G \approx 10^{-27} \text{ g/cm}^{-3.114}$

Lemaitre (1934)

Not clear whether the convention is different of if Lemaitre thought of negative energies.

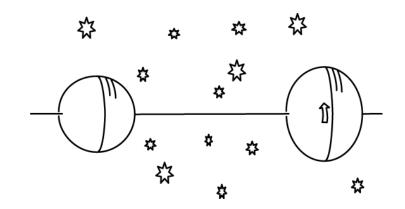
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{\rm vac})$$

The vacuum energy contribution (or cosmological constant) was postulated as early as 1917 by Einstein. It immediately led to great confusion as it admits a solution with no matter and an accelerating expansion rate (de Sitter space 1919):

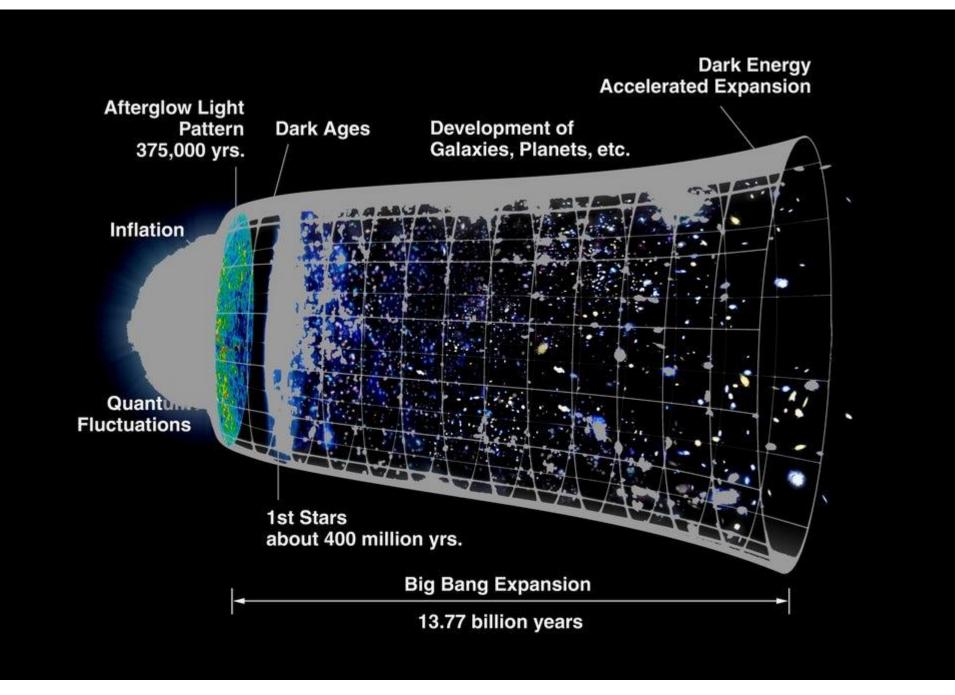
$$ds^{2} = -dt^{2} + a^{2}(t)d\vec{x}^{2}, \quad a(t) = e^{Ht}, \quad H^{2} = 8\pi G_{N}V$$

Does vacuum energy gravitate?

Einstein did not like this solution because it violates « Mach's principle »: inertia (geometry) here is only due to matter there.



 $\frac{-4e4t}{(-3)\left(\frac{2}{R}+\frac{1}{2}\right)} + k(=2)\left(\frac{1+10}{24}\right)$ $\frac{1}{R} - \frac{1}{2}\left(\frac{2}{R}+\frac{1}{2}\right) + \frac{1}{R}$ $\frac{1}{R} - \frac{1}{2}\left(\frac{1+10}{R}\right) - \frac{1}{2} + \frac{1}{R}$ $\frac{1}{R} + \frac{1}{R} - \frac{1}{2}\left(\frac{1+10}{R}\right) - \frac{1}{2} + \frac{1}{R}$ $T = \frac{1}{2}$



The acceleration of the Universe makes the role of vacuum energy fundamental in cosmology:

why?

The dynamics of the expansion of the Universe obeys an analogue of Newton's law:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

Raychaudhuri's equation

which is a consequence of the known physics of the Universe reluing simply on :

- General Relativity
- The cosmological principle
- Four types of energy in the Universe

For a matter or radiation dominated Universe with a simple equation of state :

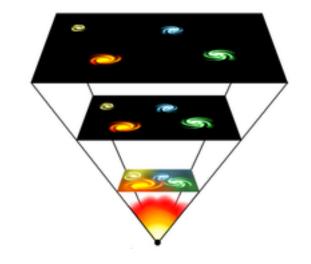
$$p = \omega \rho$$
 $\omega = 0 \text{ or } \frac{1}{3}$

The Universe cannot have an accelerated expansion:

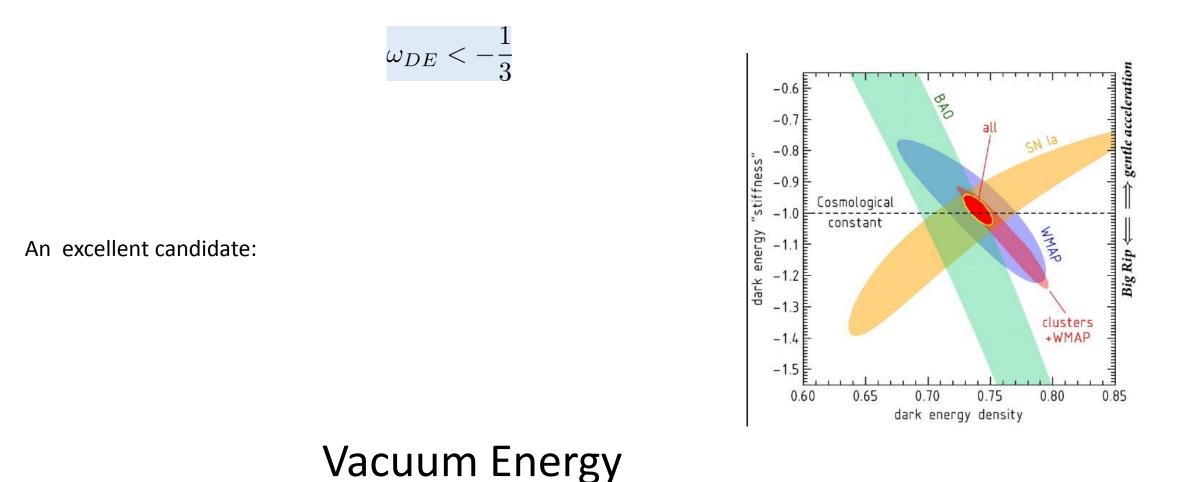
 $\ddot{a} < 0$

Contradicting the observations of the recent Universe (small redshift).

ANY IDEA?



Fortunately for observers, it is enough to postulate that the recent epoch of the Universe is dominated by a new type of matter (quintessence or dark energy) with a negative equation of state:



As a confirmation let us write the first integral of motion: the Friedmann equation (1922)

$$H^{2} = (\frac{\dot{a}}{a})^{2} = \frac{8\pi G_{N}}{3}(\rho + \rho_{\Lambda}) - \frac{k}{a^{2}}$$

Involving matter and the cosmological constant as postulated by Einstein and corresponding to a fluid of negative pressure:

$$p_{\Lambda} = -\rho_{\Lambda}$$

Einstein introduced it in 1917 to describe his "Aristotelean" Universe:

Static and spherical Universe:

$$\rho_m = 2\rho_\Lambda$$
$$R = \sqrt{\frac{1}{8\pi G_N \rho_\Lambda}}$$



As early as 1916 with Nernst's hypothesis that the vacuum is filled with the zero point energy of radiation (always in interaction with matter) and the 1920's with Lenz, Pauli (and Jordan) linking the mysterious cosmological constant to the vacuum energy, we see that the role of the vacuum energy in cosmology begins to shape:

$$\rho_{\Lambda} = \frac{1}{2} \sum_{i} \omega_{i} = \frac{1}{8\pi^{2}} \int_{0}^{m_{e}} dk \ k^{2} \sqrt{k^{2} + m_{e}^{2}}$$

This is the zero point energy, i.e. the vacuum energy, of all the oscillators associated to the known particles: e.g. the electron in the 1920's. As the integral is divergent, Lenz (1926) and later Pauli (1928) cut off the integral at the highest energy envisageable then: the electron mass or the largest energy of known gammarays:

$$\rho_{\Lambda} \sim \frac{m_e^4}{32\pi^2}$$

This immediately leads to a Universe with a radius smaller than the distance to the moon (Lenz). This was long forgotten until Zeldovich (1969) and also because by 1927 the Universe is observed to expand:

Einstein's Universe is not dynamical (Lemaitre-Hubble)



Very modern point of view: only matter particles contribute to the vacuum energy in flat space-time.

Contrary to the eigen-oscillations in a crystal lattice (where theoretical as well as empirical reasons speak to the presence of a zero-point energy), for the eigen-oscillations of the radiation no physical reality is associated to this "zero-point energy" of $\frac{1}{2}hv$ per degree of freedom. We are here doing with strictly harmonic oscillators, and since this "zero-point energy" can neither be absorbed nor reflected – and that includes its energy or mass – it seems to escape any possibility for detection. For this reason it is probably simpler and more satisfying to assume that for electromagnetic fields this zero-point radiation does not exist at all.⁹⁸

Jordan and Pauli (1928)

Despite all this, as early as the 1920's, the problem of the vacuum energy should have been brought to the fore of cosmological research. Indeed one can learn a great deal from its examination:

- $\checkmark\,$ The result seems to depend on an arbitrary UV cut off.
- The result should take into account not only the electron but all the known particles.
- The sole contribution from the proton is larger than the energy at the formation of the elements (Big Bang Nucleosynthesis) preventing one from understanding the Universe's dynamics since then.

It is thus a sheer catastrophe. The first two points have been gradually understood since the 1950's with the advent of modern Quantum Field Theory:

$$\rho_{\Lambda} = \Lambda m_{\rm Pl}^2 + \rho_{\rm transition} + \sum (2j+1)(-1)^{2j} \frac{m_j^4}{64\pi^2} \ln \frac{\mu^2}{m_j^2}$$
Cosmological constant
Phase transitions
Quantum fluctuations

BEWARE OF CUT OFFs

$$\rho_{\Lambda} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \omega_k = \frac{1}{4\pi^2} \int_0^{\Lambda} dk \ k^2 \ \sqrt{k^2 + m^2} \sim \frac{\Lambda^4}{16\pi^2}$$
$$p_{\Lambda} = \frac{1}{6} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{\omega_k} = \frac{1}{12\pi^2} \int_0^{\Lambda} dk \ \frac{k^4}{\sqrt{k^2 + m^2}} \sim \frac{\Lambda^4}{48\pi^2}$$

TOTALLY WRONG !!!

 $p_{\Lambda} = \frac{\rho_{\Lambda}}{3}$

Why: because Lorentz invariance is violated! Need to use a method like dimensional regularisation which preserves Lorentz invariance.

There are three contribution to the vacuum energy: the "latent heat" from phase transitions, e.g. electroweak, the cosmological constant and the vacuum fluctuations. The latter plays the role of a counter term and the measured value of the vacuum energy is:

$\rho_{\Lambda 0}^{1/4} = 2.4 \mathrm{meV}$

This scale is far lower than any of the scales in particle physics and the early Universe (apart from neutrinos) !

Hence the cosmological constant (counter term) must almost cancel all the disparate contributions from all the particles and phase transitions:

WHO ORDERED THAT ?

I.I. Rabi about the muon in 1936

And how could one justify such a fine-tuning?

Weinberg proved a "no-go" theorem which explains why such a tuning is contrived. Let us consider the simplest of all field theories and assume that its ground state represents a valid description of the field content of vacuum:

$$\mathcal{L} = \sum_{i} \frac{(\partial \phi_i)^2}{2} + V(\phi_i)$$



Let us also assume that no fundamental scale appears in this Lagrangian, to avoid any hidden fine-tuning. The ground state minimises the potential :

$$\phi_{i} = \lambda z_{i}, \quad V(\phi_{i}) = \lambda^{4} \tilde{V}(z_{i})$$

$$\tilde{V}(z_{i}) = 0$$

$$\tilde{V}(z_{i}) = y^{ijkl} z_{i} z_{j} z_{k} z_{l}$$

$$\partial_{z_{i}} \tilde{V}(z_{i}) = 0$$

$$for und state , N \text{ equations for}$$

$$g(y^{ijkl}) = 0$$

(N-1) fields!

There must exist a relationship between the couplings for a ground state to be. **BUT** the couplings vary under renormalisation:

$g(y^{ijkl}(\mu)) \neq 0$

The flat direction of vanishing energy parameterised by the dilaton is "lifted" by quantum corrections.

The only ground state is then at the origin:

$$\phi_i = 0$$

Now the scalar gives a mass to the fermions by the Higgs mechanism:

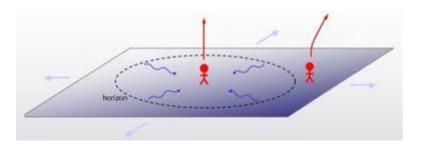
$$\mathcal{L} = x^{ijk} \phi_i \bar{\psi}_j \psi_j \quad \to m_{ij} = x^{kij} \phi_k$$

There would be no massive fermions in the Universe!

One can easily violate the hypotheses of the "no-go" theorem:

- The field configurations are dynamical with a long time attractor.
- A symmetry relates the couplings and a non-trivial vacuum exists.
- Extra dimensions.
- The theory which would describe the vacuum energy could be a non-conventional field theory: global violation of causality.

The latter is not so preposterous as ascertaining the existence of a purely constant vacuum energy requires to know the physics of vacuum inside and outside one's horizon.



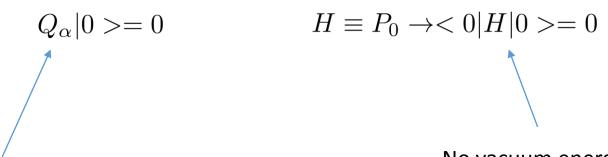
The Old World

Supersymmetry?

Global supersymmetry is a fermionic symmetry whose square is a global translation

$$\{Q_{\alpha}, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}}P_{\mu}$$

As the Hamiltonian is the generator of time translations:



No vacuum energy!

Supersymmetric vacuum

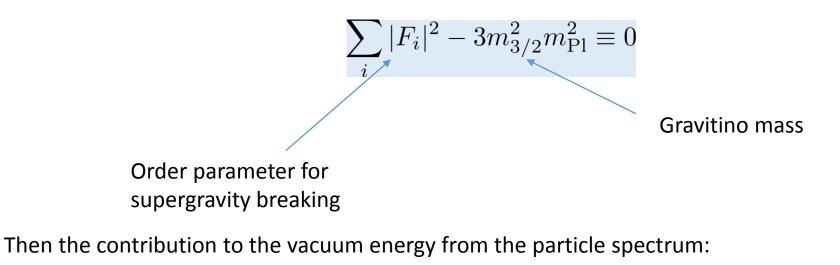
Miraculous!

LHC physics tells us no superpartners seem to exist so far and hence the vacuum is not supersymmetric!

$$M_{\rm SUSY} \ge M_Z$$

Moreover supersymmetry must be local and not global as gravity requires the possibility of local changes of coordinates, and in supergravity the supersymmetric vacuum has a negative energy.....

For decades supergravity models were postulated to have a vanishing vacuum energy :



$$\sum_{i} (2j+1)(-1)^{2j} m_j^4 \sim M_{\text{SUSY}}^4 \qquad \qquad \text{Enormous}$$

EXTRA DIMENSIONS

If the Universe is not 4d, then Weinberg's theorem does not apply. A class of model mixing supersymmetry and extra dimensions was formulated about 15 years ago (SLED).



Let us consider a 6d space-time where our Universe would be as "brane", i.e. a 4d space-time localised at one of the poles of the two extra dimensions (of spherical topology).

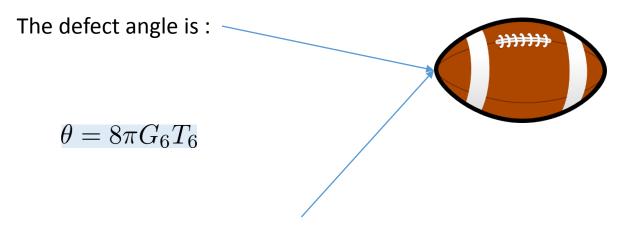
Close to our "brane", the gravitational action is simply:

$$S_{6} = \frac{1}{16\pi G_{6}} \int d^{6}x \sqrt{-g_{6}}R_{6} - T_{6} \int d^{4}x \sqrt{-g_{4}}$$
"tension" = vac

6d Newton constant

"tension" = vacuum energy on the brane

Whatever the brane tension, the 6d Einstein equations tell us that the extra dimension has the shape of a rugby ball with topological defects at the "branes":



Our brane-Universe is at a conical defect.

Moreover the vacuum energy on our brane vanishes:

 $\rho_{\Lambda} = 0$

because the brane tension curves the extra dimensions.

How about Dark Energy?

It is more complex although 6d supergravity is of great help.

Let us first focus on gravity at low energy: as the extra dimensions are "small" of size L, gravity on larger scales is blind to these details and behaves like 4d gravity with:

$$G_6 = 4\pi L^2 G_N$$



6d supersymmetry is broken by the presence of our brane: the vacuum fluctuations in 6d induce a vacuum energy

 $\rho_{\Lambda} \sim L^{-4}$

4d gravity has been tested with no failure down to 1 mm, if

The value of L is not determined and pertains to the thorny issues called "moduli stabilisation", if not stabilised then this extra field would generate a detectable fifth force.... $L \sim 1 \text{ mm}$

The dark energy problem is solved!

The New World

Infrared Fixed Point

It is possible that a pure field theory answer exists, for instance the existence of an infrared fixed point with vanishing vacuum energy:

$$\frac{d\rho_{\Lambda}}{d\ln\mu} = \frac{1}{32\pi^2} \mathrm{Str}m^4$$

In a de Sitter space and perturbatively, the graviton behaves like a massive particle with a mass:

$$m_g^2 = -\frac{\rho_\Lambda}{m_{\rm Pl}^2}$$

Is this allowed? Instabilities? How to embedd this in a massive gravity model?

The resulting renormalisation group equation admits a fixed point in the infrared:

$$\frac{d\rho_{\Lambda}}{d\ln\mu} = \frac{1}{16\pi^2} \frac{\rho_{\Lambda}^2}{m_{\rm Pl}^4}$$

$$\mu \to 0$$
 implies $\rho_{\Lambda} \to 0$

SEQUESTERING

The motivation for this approach is Weinberg's "no-go" theorem and its loopholes. Let us start with the familiar action:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \lambda^4 \mathcal{L}_m(\lambda^2 g_{\mu\nu}, \psi^i) - 2\Lambda \right)$$

where the dilaton λ and the vacuum energy Λ are constant Matter action in space-time.

In the absence of any new ingredient, then the dilaton vanishes and:

$$m_{\psi} = \lambda m_{\psi}^{(0)} \quad \to m_{\psi} = 0$$

To violate Weinberg's no-go theorem, add a *non-local* action:

$$S_{\sigma} = \sigma(\frac{\Lambda}{\lambda\mu})$$

where σ vanishes at the origin and μ is a given scale. This contradicts locality in Quantum Field Theory but in a soft way, i.e. in a sector not involving matter or gravity. The field equations yield:

$$\int d^4x \sqrt{-g} = \frac{1}{2\lambda^4 \mu^4} \sigma'(\frac{\Lambda}{\lambda^4 \mu^4})$$

Finite volume of space-time: space must be finite and time must start with a bang and end in a crunch... This determines a non-vanishing value of the dilaton. The Einstein equations become:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N(T_{\mu\nu} - \frac{\langle T \rangle}{4}g_{\mu\nu})$$

Einstein tensor describing curvature

Energy momentum tensor of matter and vacuum

Cosmological constant term

$$\Lambda = \frac{\langle T \rangle}{4}$$

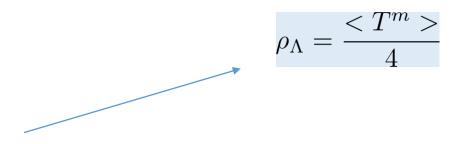
$$< T > = \frac{\int d^4x \sqrt{-gT}}{\int d^4x \sqrt{-g}}$$

Decomposing the energy-momentum tensor:

$$T_{\mu\nu} = T^m_{\mu\nu} - V_\Lambda g_{\mu\nu} - \sum_{\mu\nu} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N (T^m_{\mu\nu} - \frac{\langle T^m \rangle}{4} g_{\mu\nu})$$

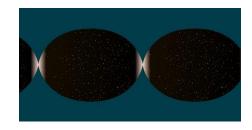
One finds that :

- The "old" problem of the cosmological constant has a solution coming from the stabilisation of the dilaton at a non-vanishing value.
- ✓ The dark energy problem is intimately linked to the violation of causality:



An explicit model with a scalar field:

Necessitates to know all the physics til the crunch...



$$V(\phi) = m^3 \phi$$

The field rolls down slowly until it becomes negative and then: *CRUNCH*

EXTRA SLIDES



Et si la gravité etait modifiée?

Il est possible qu'une partie de l'explication de l'accélération cosmique vienne d'une modification de la relativité générale à grande échelle.

La façon la plus naturelle d'introduire cette modification pourrait être de donner une masse au graviton:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Equation de propagation:

Champ associé au graviton, particule de spin 2 et sans masse.

$$\nabla^{\rho}\nabla_{\rho}(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T) = -16\pi G_N T_{\mu\nu}$$

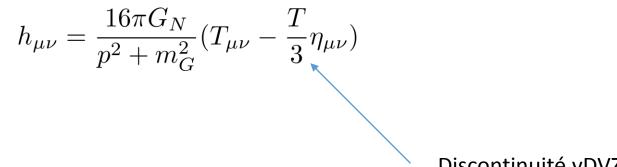
$$a_{\mu\nu} = \frac{16\pi G_N}{p^2} (T_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}T)$$

Fierz-Pauli en 1939 ont ajouté un terme de masse pour le graviton:

$$\mathcal{L}_{FP} = -\frac{m_G^2 m_{\rm Pl}^2}{8} (h_{\mu\nu} h^{\mu\nu} - (h_{\mu}^{\mu})^2)$$

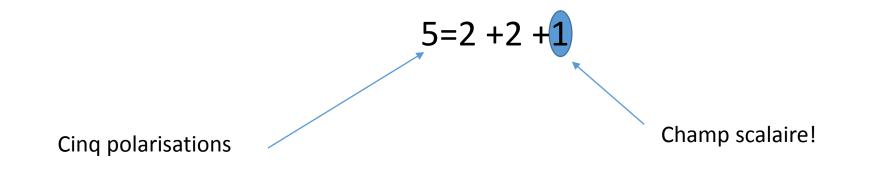


qui modifie la propagation:

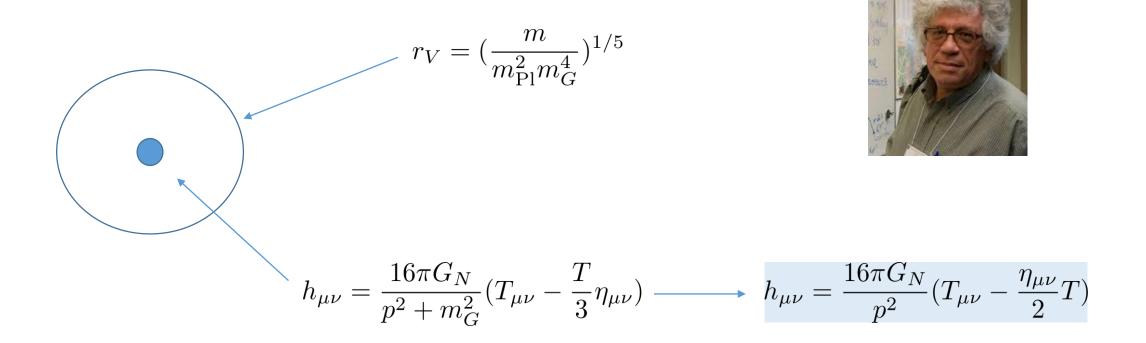


Cette discontinuité est due à l'existence:

Discontinuité vDVZ!



L'existence d'un champ scalaire couplé à la matière implique que le modèle devient non-linéaire proche d'un objet massif. Dans ce cas Vainshtein a suggéré que l'interaction engendrée par le mode scalaire devient négligeable à l'interieur du rayon de Vainshtein:



Mais ces mêmes non-linéarites qui sauvent la gravité massive à courte portée condamnent le modèle a être instable: il possède un





Champ d'énergie négative

Il a fallu attendre les années 2010 pour résoudre ce problème! (cf exposé de Cédric Deffayet)

De nombreuses recherches actuelles ont pour but la construction de modàles violant le théorèm d'Ostrograski: génériquement un système physique dont le Lagrangien dépent de l'accélération et de ses dérivées possède un fantôme.

Une grande classe de modèles avec champ scalaire sans fantôme a été découverte en 1974 par Horndeski puis redécouverte indépendamment récemment. Le Lagrangien est extrèmement complexe et inclut les modèles simples d'énergie noire comme celui des bosons de Goldstone.

Heureusement le Lagrangien se simplifie en utilisant le principe cosmologique:

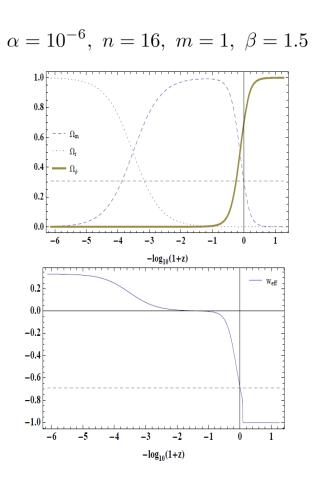
$$\mathcal{L} = a^3 \sum_{i=0}^2 X_i(\dot{\phi}) H^i$$

Ces modèles possèdent un attracteur de de Sitter à grand temps avec taux de Hubble constant.

$$f_0(\dot{\phi}) = -\beta \dot{\phi}^{-m}, \ f_1(\dot{\phi}) = -\alpha \dot{\phi}^n + \beta \dot{\phi}^{-m}, \ f_2(\dot{\phi}) = \alpha \dot{\phi}^n$$

$$3m_{\rm Pl}^2 X_i(\dot{\phi}) = f_i(\dot{\phi}) H_0^{i-2}$$

WHO ORDERED THAT (AGAIN) ?



Un problème pour les physiciens?

Un MERVEILLEUX problème pour les physiciens (théoriciens)!

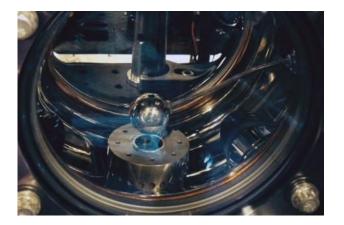
De nombreux obstacles _____ Le théorème "sans issue" de Weinberg

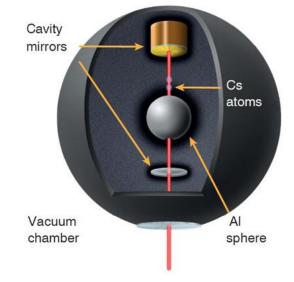
Mais de nombreuses pistes:

- Les dimensions supplémentaires
- La violation de la causalité
- L'auto-ajustement dans les modèles d'Horndeski

Et de la nouvelle physique: la gravité modifiée et ses tests en cosmologie, astrophysique, dans le système solaire ou au laboratoire.

Il est possible de contraindre expérimentalement un champ d'enérgie noire couplé à la matière, et qui donc doit être "écranté" dans le système solaire, au laboratoire.





H. Muller (Berkeley 2016)

Un atome de césium dans le champ d'une sphère d'aluminium aurait une influence scalaire qui modifierait les figures d'interférence atomique. Et donc une accélération anormale, de modification de la gravité, pourrait être détectée.