From general relativity to quantum gravity

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A super-simple, naïve, non-rigorous, "old fashioned", and purely intuitive reminder on GR

Objects with different weights fall at the same speed.

mg=ma , indeed ! ☺
Gravity is « democratic ». No need to think about it as a force. A motion of the frame is enough.

But the motion is accelerated to reproduce (locally) gravity – this is the equivalence principle. What does that change ?

Special relativity teaches us that $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ **is invariant under changes or inertial frames. It is based on *symmetries.***
But its functionnal dependance upon coordinated is *not* invariant upon a change to an accelerated frame !

→ $$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

This defines the metric function encoding the geometry.
The world in not euclidean ! There is curvature ! Space curvature ?

What shall we do ? Just

1) **Determine how to write down physics in a curved world**
2) **Determine how the content of the world fixes the curvature**

Let's adress first point 1).

The generalized covariance principle states that laws of physcis should be those of special relativity in the free-falling frame, that is in the tangent space.

All physics eq. use tensors. So it seems we have nothing to do ! A rank-2, e.g. tensor is just a guy transfomring as :

$$S^{\mu'\nu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} S^{\mu\nu}$$

Maxwell eq. are tensor eq. ! Fine !
(I remind that Poisson eq. + SR → Maxwell eq.)

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad\qquad \partial^{[\sigma} F^{\mu\nu]} = 0$$

Unfortunately … Let's take the derivative of a vector:

$$\partial_\mu V^\nu \to \partial_{\mu'} V^{\nu'} = \left(\frac{\partial x^\mu}{\partial x^{\mu'}}\partial_\mu\right)\left(\frac{\partial x^{\nu'}}{\partial x^\nu} V^\nu\right) = \frac{\partial x^\mu}{\partial x^{\mu'}} \cdot \frac{\partial x^{\nu'}}{\partial x^\nu}\partial_\mu V^\nu + \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial^2 x^{\nu'}}{\partial x^\mu \partial x^\nu} V^\nu .$$

There is a problem … Geometrically, this is related to the path-dependence of the parallel transport.
Let's build a new derivative:

$$\frac{\mathrm{D}q^\mu}{\mathrm{D}s} = \frac{\mathrm{d}q^\mu}{\mathrm{d}s} + \Gamma^\mu{}_{\sigma\rho}\, q^\sigma \left(\frac{\mathrm{d}x^\rho}{\mathrm{d}s}\right).$$

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By covariant-deriving the metric tensor, it is straightforward to express the Christofell as a function of the usual derivatives of the metric field.

$$g_{\mu\nu,\rho} - g_{\rho\mu,\nu} + g_{\nu\rho,\mu} = 2\Gamma_{\nu\mu\rho}.$$

And the Maxwell eq. now read (with ; the covariant derivative)

$$D_\mu F^{\mu\nu} = j^\nu \qquad D^{[\sigma}F^{\mu\nu]} = 0.$$

<u>We can now address point</u> 2). How to define the content. Let's consider a non-relativistic clould of gaz. It's comoving density is :

$$\rho = n \times mc^2$$

But for an observer at speed v, he sees $\rho' = \rho \times \gamma^2$

$\gamma = (1 - v^2/c^2)^{-1}$ **→ we need a rand-2 tensor ! It is called stress-energy,** $T_{\mu\nu}$

Something to generalize the gaussian curvature ?

$$\mathrm{d}v^\alpha = \mathrm{d}i^\delta\,\mathrm{d}j^\gamma\,v^\beta\,R^\alpha{}_{\beta\gamma\delta}$$

Riemann tensor. Can be expressed with second derivatives of the metric.

So let's wonder if there is geometric object that can be put in front of $T_{\mu\nu}$

$$D_\nu T_{\mu\nu} = 0$$

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Let's require that this tensor

- **Is rank 2**
- **Is divergence less**
- **Is made of the metric and Riemann tensor**
- **Is Riemann-linear**
- **Vanishes in flat space**

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \qquad R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu} \qquad R \equiv R^{\alpha}_{\ \alpha}$$

Then let's assume G is proportionnal to T and fix the constant by requiring that the weak field limit agrees with Newtonian mechanics:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **Space become a dynamical entity**
- **No background structure**
- **A machinery to calculate metrics**
- **G/c^4 is small**

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Many other formulations of GR

- **Lagrangian :**

$$S = \frac{1}{2}\int R\sqrt{-g}d^4x$$

- **Tetrads :**

$$S = \frac{1}{2}\int e^I \wedge e^J \wedge F^*$$

- **Holst action :**

$$S = \frac{1}{2}\int (*e \wedge e + \frac{1}{\gamma} e \wedge e)$$

- **Plebansky action :**

$$S = \int_{\Sigma \times R} \epsilon_{ijkl} B^{ij} \wedge F^{kl}(A_a^i) + \phi_{ijkl} B^{ij} \wedge B^{kl}$$

- **Hamiltonian constraint :**

$$H_G = \frac{\epsilon^{abc}\epsilon_{ijk}E_i^a E_j^b F_{ab}^k}{\sqrt{det(q)}} + \frac{\gamma^2+1}{\gamma}\frac{(E_i^a E_j^b - E_j^a E_i^b)(A_a^i - \Gamma_a^i)(A_b^j - \Gamma_b^j)}{\sqrt{det(q)}}$$

Some simple analytical solutions.

1. COSMOLOGY.

Symmetries + immersion in a euclidean 4-space. $\sigma \equiv r/a(t)$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}\sigma^2}{1-k\sigma^2} + \sigma^2\mathrm{d}\Omega^2\right)$$

Then solving Einstein's equations leads to :

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

This determines perfectly the history of the evolution of the Universe.

Singularity unavoidable.

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Some simple analytical solutions.

2. BLACK HOLES. (No hair theorem)

Symmetries + immersion in a euclidean 4-space.

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{r}\right)\mathrm{d}t^2 - \left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$$

A black hole is such that the region r<2GM is accessible. (for a star 2GM is inside the star !)
Space ←→ Time

Coordinates :
- **Free falling**
- **Shell observer**

$$\mathrm{d}s(r=cte) = \mathrm{d}t_{\mathrm{coq}} = \left(1-\frac{2GM}{r}\right)^{1/2}\mathrm{d}t$$

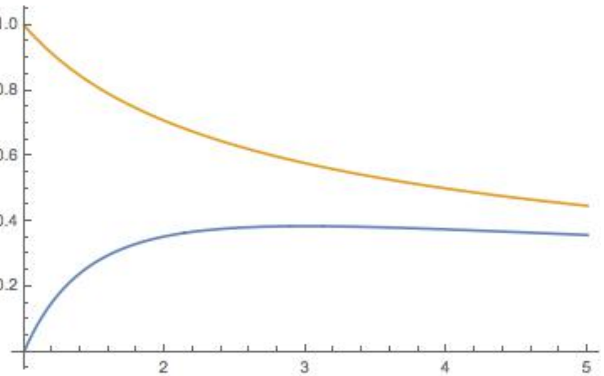
$$\mathrm{d}s(t=cte) = \mathrm{d}r_{\mathrm{coq}} = \left(1-\frac{2GM}{r}\right)^{-1/2}\mathrm{d}r$$

$$\mathrm{d}s^2 = \mathrm{d}t^2_{\mathrm{coq}} - \mathrm{d}r^2_{\mathrm{coq}}$$

Energy : $E = m\left(1-\frac{2GM}{r}\right)\frac{\mathrm{d}t}{\mathrm{d}\tau}$ **So:** $\left(1-\frac{2GM}{r}\right)^2\mathrm{d}t^2 = \mathrm{d}\tau^2$

→ $$\frac{\mathrm{d}r}{\mathrm{d}t} = -\left(1-\frac{2GM}{r}\right)\left(\frac{2GM}{r}\right)^{\frac{1}{2}}$$

$$\frac{\mathrm{d}r_{\mathrm{coq}}}{\mathrm{d}t_{\mathrm{coq}}} = -\left(\frac{2GM}{r}\right)^{\frac{1}{2}}$$



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Inside de black hole …

$$\mathrm{d}t_{\mathrm{chute}} = -\gamma V_{\mathrm{rel}} \times \mathrm{d}r_{\mathrm{coq}} + \gamma \times \mathrm{d}t_{\mathrm{coq}}$$

this is just SR

→

$$\mathrm{d}t_{\mathrm{chute}} = -\frac{\gamma V_{\mathrm{rel}}\mathrm{d}r}{\left(\frac{2GM}{r}\right)^{\frac{1}{2}}} + \gamma\left(1 - \frac{2GM}{r}\right)^{\frac{1}{2}}\mathrm{d}t$$

By replacing the coordinates

And

$$V_{\mathrm{rel}} = \frac{\mathrm{d}r_{\mathrm{coq}}}{\mathrm{d}t_{\mathrm{coq}}} = -\left(\frac{2GM}{r}\right)^{\frac{1}{2}}$$

so:

$$\mathrm{d}t = \mathrm{d}t_{\mathrm{chute}} - \frac{\left(\frac{2GM}{r}\right)^{\frac{1}{2}}\mathrm{d}r}{\left(1 - \frac{2GM}{r}\right)}$$

In Schwarzschild metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t_{\mathrm{chute}}^2 - 2\left(\frac{2GM}{r}\right)^{\frac{1}{2}}\mathrm{d}t_{\mathrm{chute}}\mathrm{d}r - \mathrm{d}r^2$$

For light $ds^2 = 0$ →

$$\frac{\mathrm{d}r}{\mathrm{d}t_{\mathrm{chute}}} = -\left(\frac{2GM}{r}\right)^{\frac{1}{2}} \pm 1$$

Both solutions are negative: even the light emitted outward goes inward !

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BEYOND GR ?

WHY ?

From a theoretical point of view, GR is a purely classical theory.
Power-counting arguments indicate that GR is **not renormalizable** in the standard QFT sense.
Strong-field modifications may provide a solution to this problem: the theory becomes renormalizable if we add **quadratic curvature terms**.
High-energy corrections can **avoid singularities** that are inevitable in GR. Candidate theories of quantum gravity (such as string theory and loop quantum gravity) make potentially testable predictions of how GR might be modified at high energies.

From an observational point of view, cosmological measurements are usually interpreted as providing evidence for **dark matter** and a **nonzero cosmological dark energy**. This poses conceptual issues.
No dynamical solution of the cosmological constant problem is possible within GR. It seems reasonable that ultraviolet corrections to GR would inevitably “leak” down to cosmological scales, showing up as low-energy (infrared) corrections.
Dark matter could be **modified gravity**.

This suggest that GR might have to be modified at both low and high energies.

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HOW ?

Lovelock theorem :

In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term.

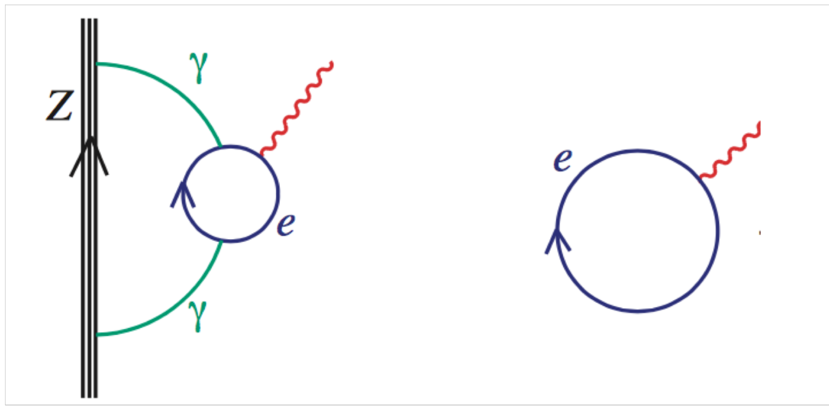
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

This leads to an important point about the acceleration of the Universe.

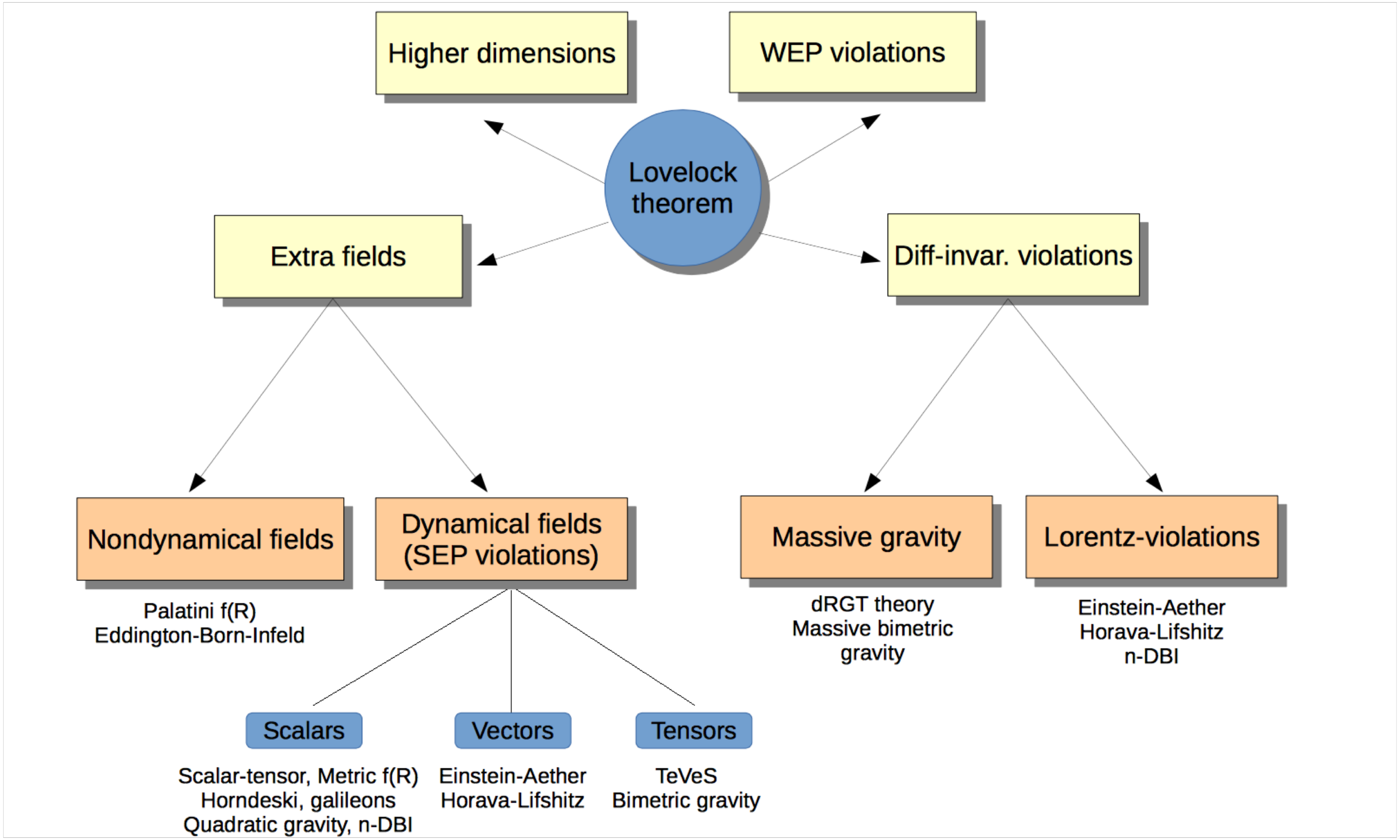
In a universe dominated by the cosmological constant : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}$ $\quad a = a_0 e^{\sqrt{\frac{\lambda}{3}}t}$

It works perfectly ! What was Einstein's greatest blunder ?

This cosmological constant *is not* the quantum fluctuations of vacuum. It is perfectly ok.
The real problem is indeed with vacuum fluctuations !



The theorem contains a number of nontrivial assumptions. Giving up each of these assumptions gives rise to different classes of modified theories of gravity.



(i) Additional fields.

The simplest and most beaten path to circumvent Lovelock's theorem consists of adding extra degrees of freedom. This leaves more options to construct the left-hand side of Einstein's equations, including more than just the metric and connection. Lifting this assumption paves the way for countless possibilities, where the metric tensor $g_{\mu\nu}$ is coupled to extra fundamental (scalar, vector, tensor) fields.

(ii) **Violations of diffeomorphism invariance**.

If we assume that Lorentz invariance is just an emergent symmetry that is broken at high energies in the gravitational sector, a new class of gravity theories can be built. Some of these theories were found to possess a better ultraviolet behavior than GR.

(iii) **Higher dimensions**.

Even retaining all other assumptions of Lovelock's theorem, the Einstein-Hilbert action is not unique in higher dimensions.

(iv) **WEP violations**.

The requirement that the left-hand side of Einstein's equation be divergence-free is dictated by the desire of having a divergence-free $T_{\mu\nu}$ and, in turn, by the weak equivalence principle. Various classes of theories that circumvent Lovelock's theorem only by postulating a nonminimal coupling to the matter sector (and thus violating the weak equivalence principle) have been proposed.

An interesting example : scalar-tensor gravity.

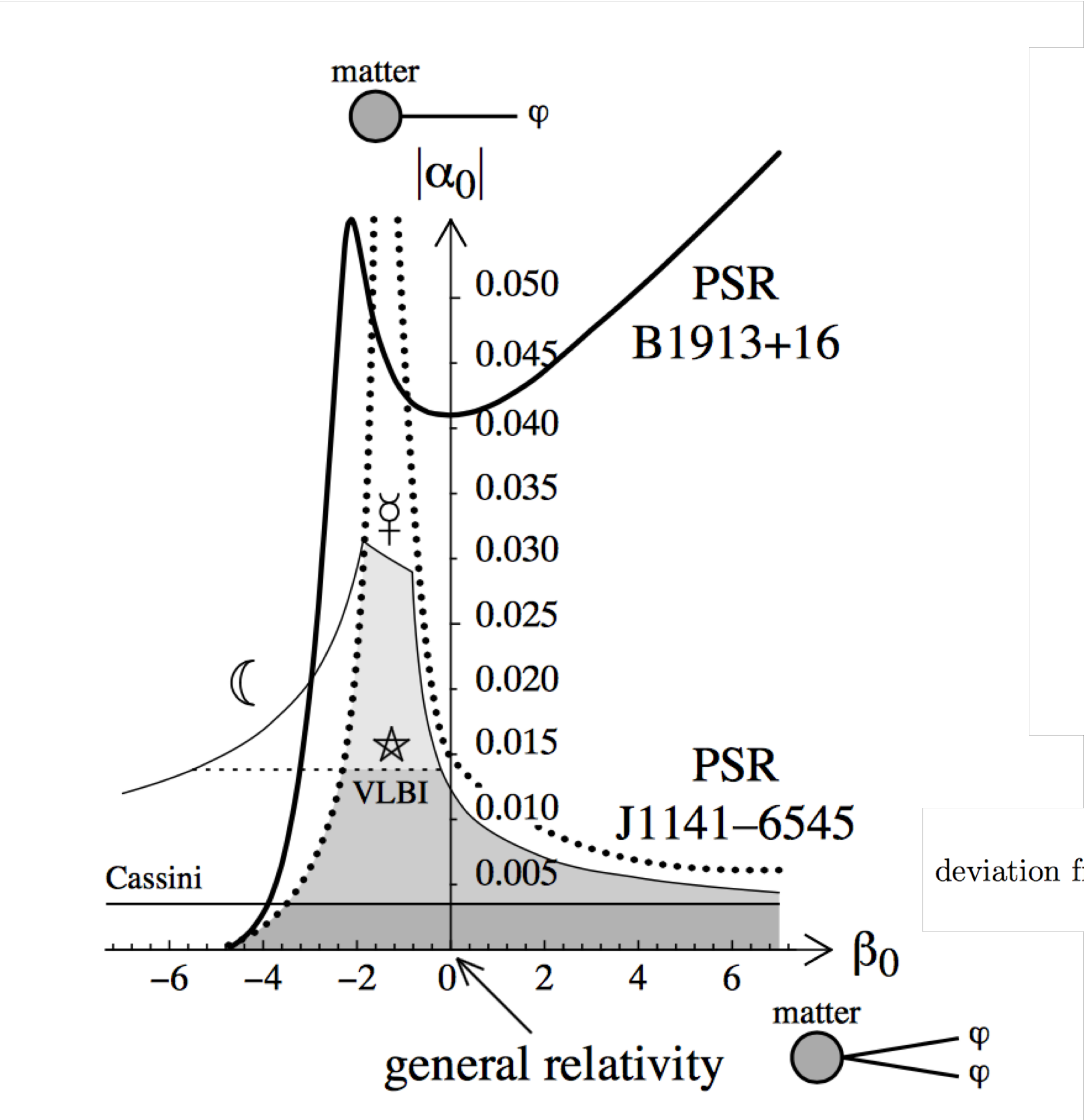
One of the best motivated alternatives to general relativity are scalar-tensor theories, in which the gravitational interaction is mediated by one or several scalar fields together with the usual graviton.

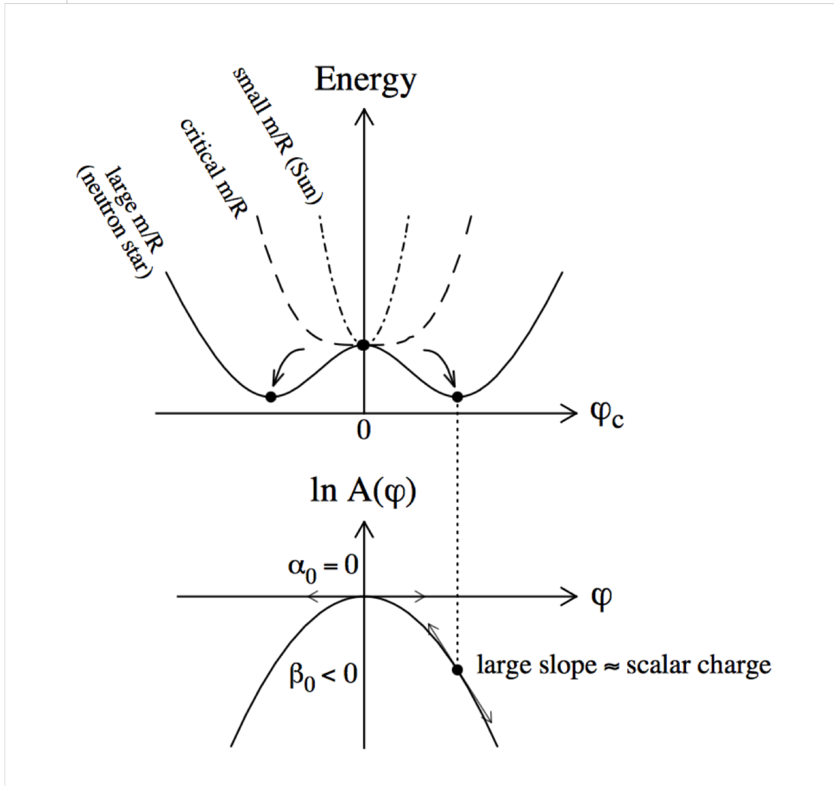
Quite natural:

- Extra dimensions
- respect most of GR's symmetries: conservation laws, constancy of non-gravitational constants, and local Lorentz invariance together with the WEP.
- Scalar fields appear in the inflation theory.

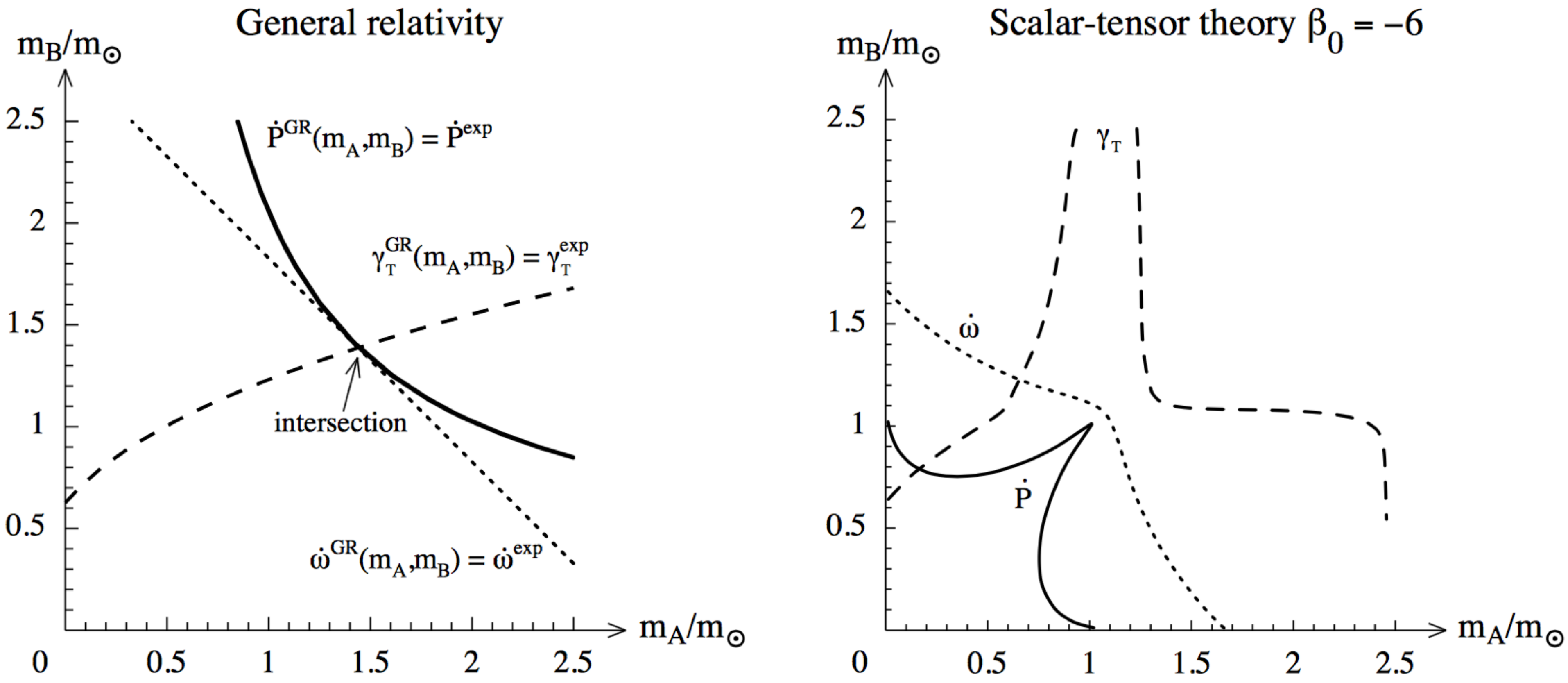
Besides these theoretical and experimental reasons for studying scalar-tensor theories of gravity, one of their greatest interests is to embed GR within a class of mathematically consistent alternatives

$$S = \frac{c^3}{4\pi G} \int \sqrt{-g} \left\{ \frac{R}{4} - \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) \right\} + S_{\text{matter}} \left[\text{matter}; \widetilde{g}_{\mu\nu} \equiv A^2(\varphi) g_{\mu\nu} \right]$$





$$\text{deviation from GR} = \alpha_0^2 \times \left[\lambda_0 + \lambda_1 \frac{Gm}{Rc^2} + \lambda_2 \left(\frac{Gm}{Rc^2}\right)^2 + \cdots\right]$$



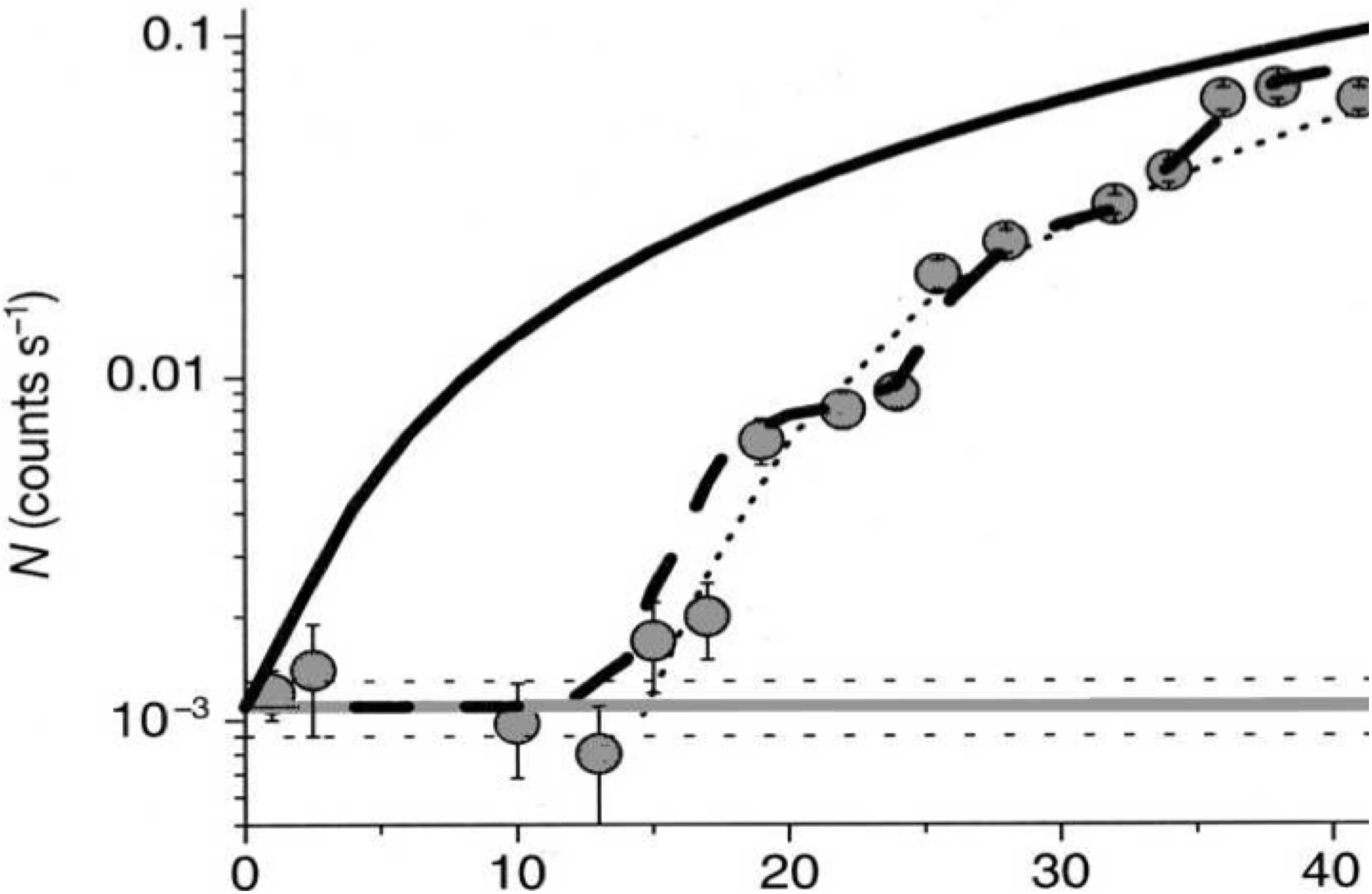
Measured parameters : (i) the Einstein time delay parameter, which combines the second-order Doppler effect together with the redshift due to the companion; (ii) the periastron advance $\dot{\omega}$ (relativistic effect of order v2/c2); and (iii) the rate of change of the orbital period, $\dot{P}$, caused by gravitational radiation damping

Lagrangian reconstruction ? Yes ! The knowledge of the luminosity distance DL(z) and of the density fluctuations δm(z) = δρ/ρ as functions of z suffices to reconstruct both the potential V (ϕ) and the coupling function A(ϕ) !

More general ?

It is possible to couple the scalar fields to the Gauss-Bonnet term. This is the only combination involving powers of the Riemann and/or the Ricci tensors which preserves the spectrum of the theory. f(R) eq. to a new scalar. A function f(R,dR,...,d^nR) of the scalar curvature and its iterated derivatives is equivalent to n + 1 scalar fields in the theory. Functions f(Rμν) or f(Rμνρσ) of the Ricci or the Riemann tensor involve generically an extra massive spin-2 ghost, i.e., an extra negative-energy graviton which spoils the stability of the theory.

$$
\begin{aligned}
S = \frac{c^3}{4\pi G} \int \sqrt{-g} \left\{ \frac{R}{4} - \frac{1}{2} F\Big(g^{\mu\nu} \gamma_{ab}(\varphi^c)\, \partial_\mu \varphi^a \partial_\nu \varphi^b\Big) - V(\varphi^a) \right\} \\
-\hbar \int \sqrt{-g}\, W(\varphi^a) \left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right) \\
+ \sum_i S_{\mathrm{matter}_i} \left[\mathrm{matter}_i \,;\, \widetilde{g}^{(i)}_{\mu\nu} \equiv A_i^2(\varphi^a) g_{\mu\nu}\right] .
\end{aligned}
$$



0.1
0.01
10^{-3}
N (counts s^{-1})
0
10
20
30
40

STRANGE BLACK HOLES

No scale, no hair

Defined by :
- Mass
- Spin
- Charge

(determinable from the outer space)

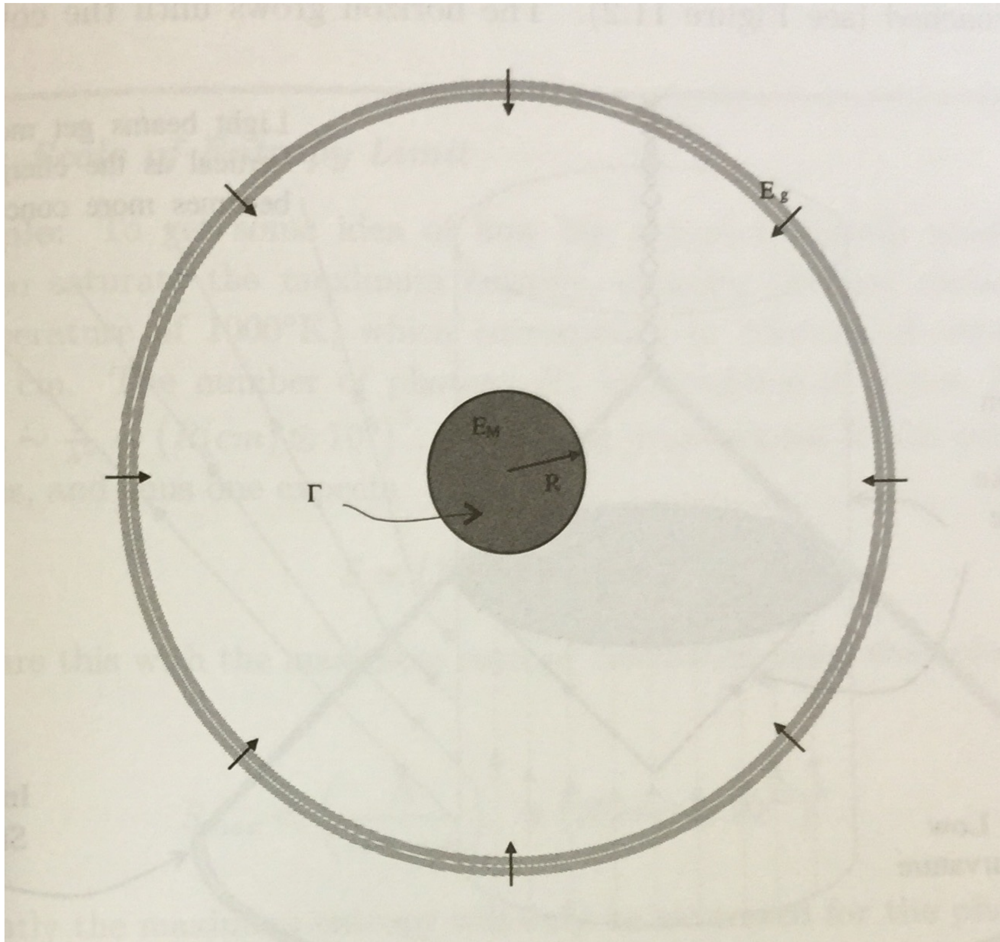
→ **All that is needed for thermodynamics.**

Missing entropy.
Hawking theorem : DA>0

Thought experiment : gaz bottle in a BH. What happens to the entropy ?
$S = q * A$

Generealizd second principle : BH entropy + external entropy cannot decrease

Black holes have a maximum entropy.



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Law of BH Thermodynamics

0) *The temperature of a body at equilibrium is constant*

→ The surface gravity of a BH horizon in constant

1) *Energy conservation. dE = TdS + W (heat <-> work)*

→ $$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ$$

A temperature is missing.

2) *Entropy increases*

→ $$\frac{dA}{dt} \geq 0$$

3) *The absolute 0 cannot be atteinted*

→ It is impossible to form a BH with 0 surface gravity

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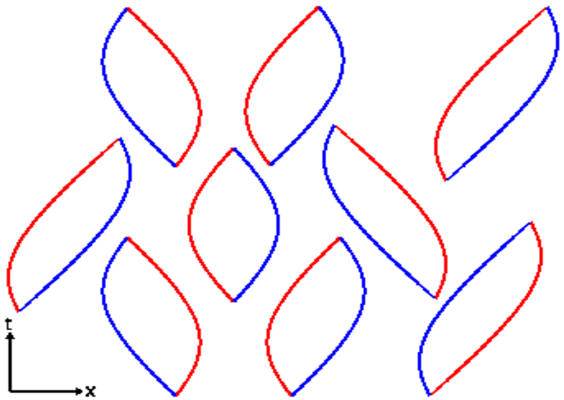
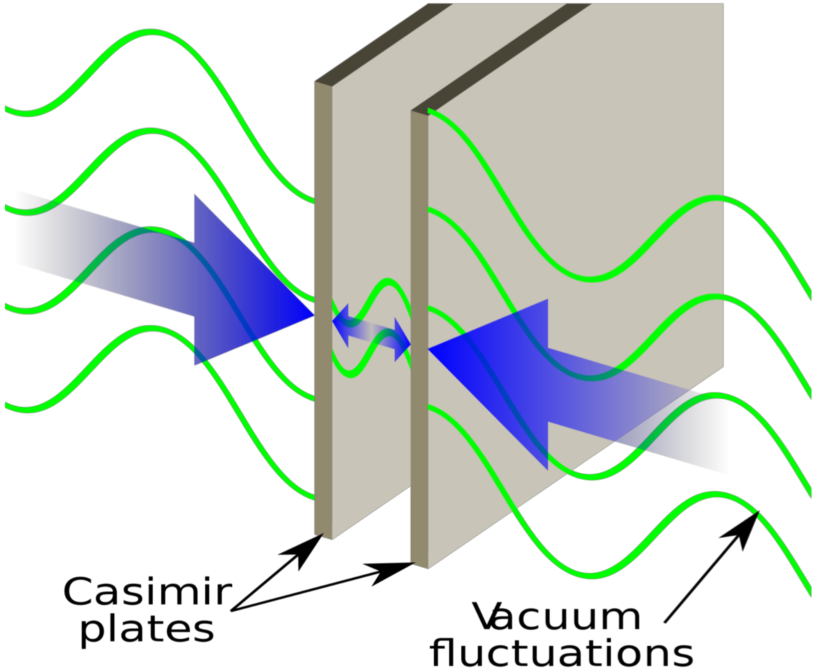
Toward Hawking discovery

Vacuum is not empty

DE*Dt > h.

- Lambshift
- Casimir
- (Scwhinger)





Vacuum is subtle in QFT.
Why doesn'it gravitate ?

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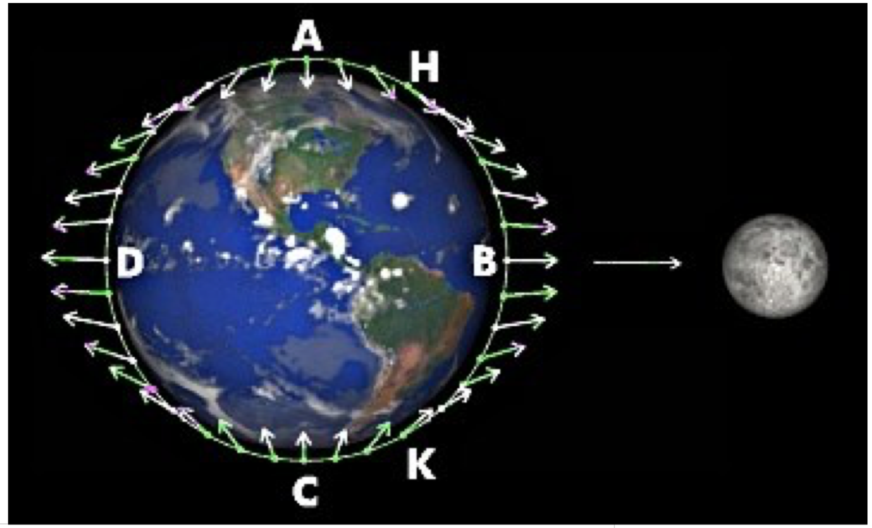
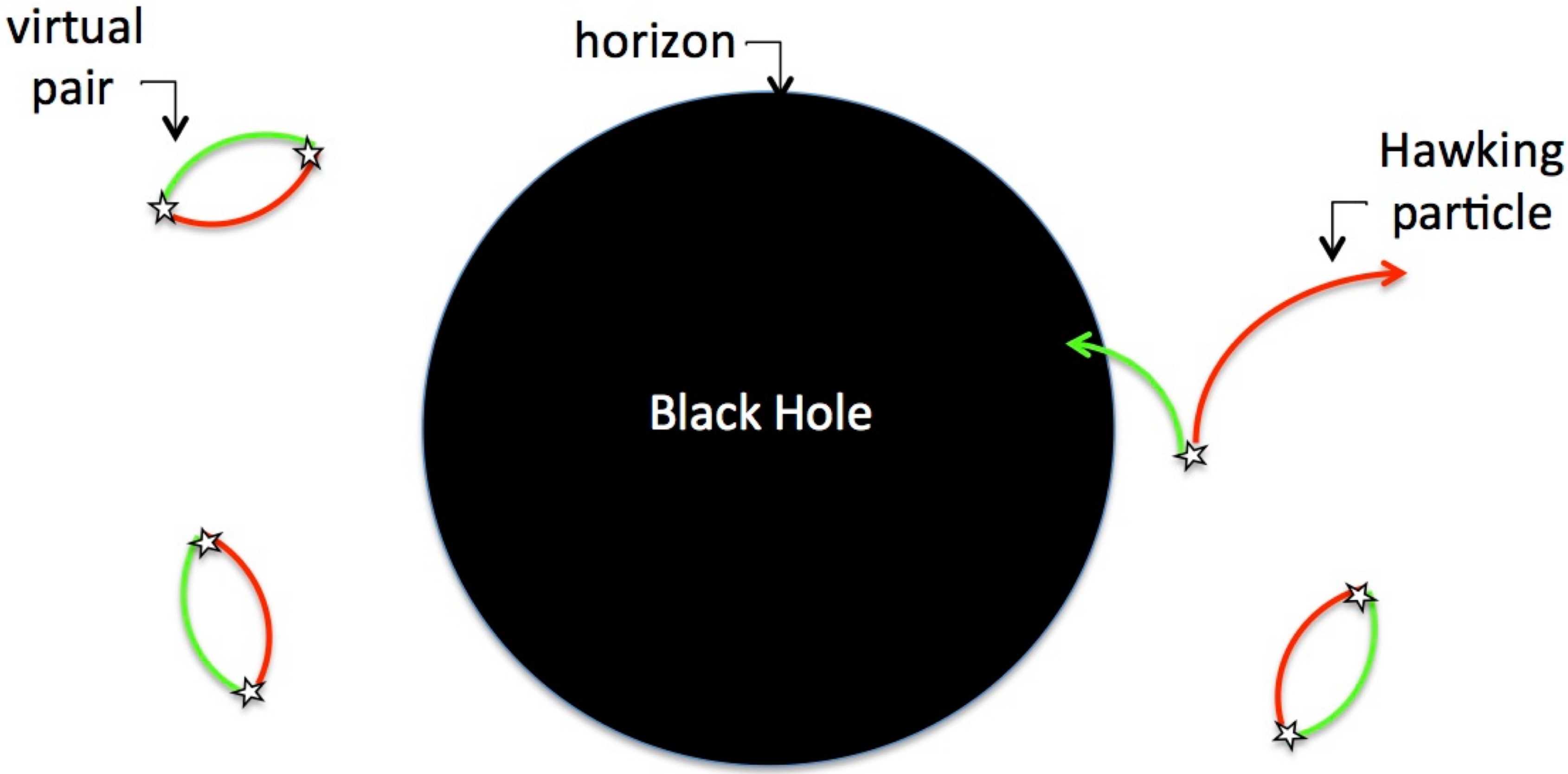
Evaporation of black holes



Coupling between the quantum vacuum and QF

Tidle forces.



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The spectrum is nearly thermal. One of the greatest formula of physics.

$$T_{\rm BH} = \frac{\hbar c^3}{8\pi k_{\rm B} G M}$$

The smaller the mass, the higher the temperature !

$$champ = \frac{GM}{r^2} \qquad r = 2GM \qquad champ \propto \frac{1}{M}$$

(not true for a planet : $M=kr^3$)

Explosive …

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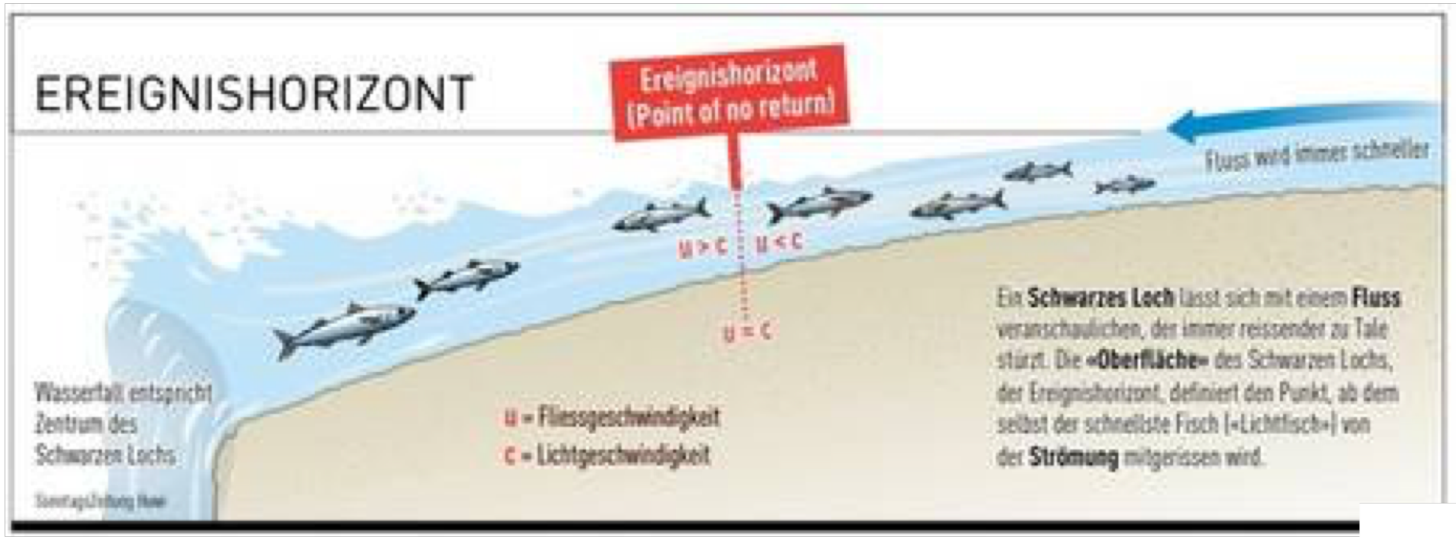
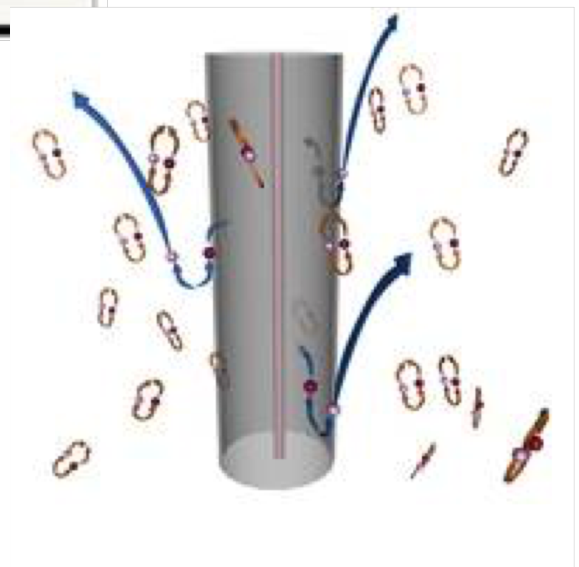
Acoustic black holes and Hawking effect

Observed with accoustic BHs;





Photons → Phonons
General relartivity → hydrodynamics
Horizon (actually 2 for resonance) with a supersonic fluid.
Pb : TH very small → cold fluids (CBE). + LASER.

Not a proof but a strong evidence !

$$g_i = \sum_j \left(\alpha_{ij} f_j + \beta_{ij} f_j^*\right)$$

$$f_i = \sum_j \left(\alpha_{ji}^* g_j - \beta_{ji} g_j^*\right).$$

$$\hat{a}_i = \sum_j \left(\alpha_{ji} \hat{b}_j + \beta_{ji}^* \hat{b}_j^\dagger\right)$$

$$\hat{b}_i = \sum_j \left(\alpha_{ij}^* \hat{a}_j - \beta_{ij}^* \hat{a}_j^\dagger\right).$$

$$\begin{aligned}
\langle 0_f | \hat{n}_{gi} | 0_f \rangle &= \langle 0_f | b_i^\dagger b_i | 0_f \rangle \\
&= \left\langle 0_f \middle| \sum_{jk} \left(\alpha_{ij} \hat{a}_j^\dagger - \beta_{ij} \hat{a}_j\right) \left(\alpha_{ik}^* \hat{a}_k - \beta_{ik}^* \hat{a}_k^\dagger\right) \middle| 0_f \right\rangle \\
&= \sum_{jk} \left(-\beta_{ij}\right) \left(-\beta_{ik}^*\right) \langle 0_f | \hat{a}_j \hat{a}_k^\dagger | 0_f \rangle \\
&= \sum_{jk} \beta_{ij} \beta_{ik}^* \langle 0_f | \left(\hat{a}_k^\dagger \hat{a}_j + \delta_{jk}\right) | 0_f \rangle \\
&= \sum_{jk} \beta_{ij} \beta_{ik}^* \delta_{jk} \langle 0_f | 0_f \rangle \\
&= \sum_j \beta_{ij} \beta_{ij}^*.
\end{aligned}$$

Particles are relative !

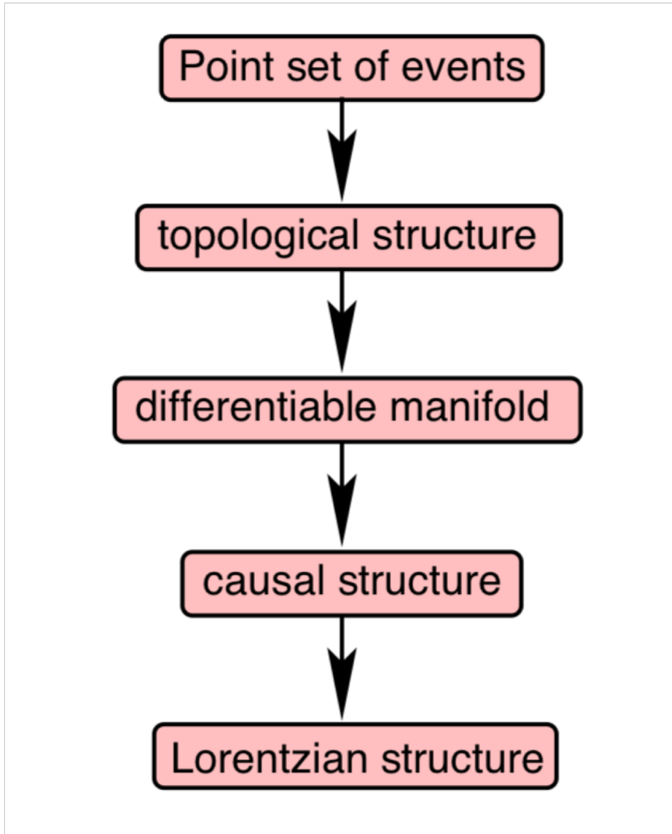
$$\langle 0_f | \hat{n}_{gi} | 0_f \rangle = \sum_j |\beta_{ij}|^2.$$

Real origin of the Hawking effect

QUANTUM GRAVITY

3) Quantum gravity ?

- G=8pi*T
- Singularity theorems
- Initial conditions in cosmology
- Fate of black holes
- Unification of interactions
- Inconsistency of the semi-classical theory
- Avoidance of divergences
- Problem of time



String theory → THE main approach ! (talk by C. Bachas)

Perturbative quantum gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{32\pi G} f_{\mu\nu}$$

In contrast to Yang-Mills theory, gravity is non-renormalizable

$$\mathcal{L}^{(\mathrm{div})}_{2-\mathrm{loop}} = \frac{209\hbar^2}{2880} \frac{32\pi G}{(16\pi^2)^2\epsilon} \bar{R}^{\alpha\beta}{}_{\gamma\delta} \bar{R}^{\gamma\delta}{}_{\mu\nu} \bar{R}^{\mu\nu}{}_{\alpha\beta}$$

GRAVITY AND THE QUANTUM

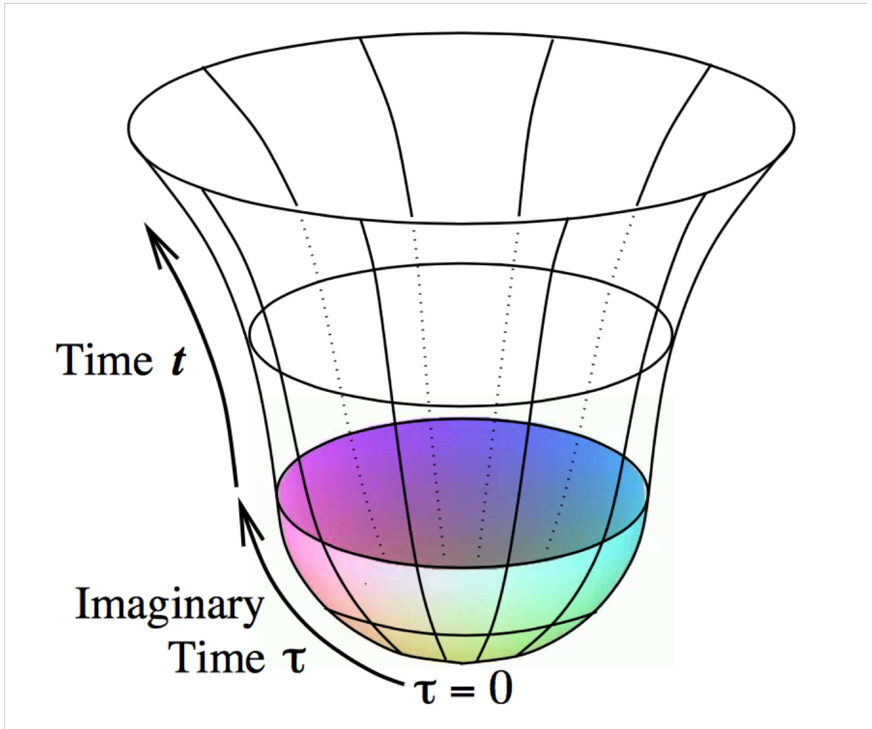
Effective Field theories and renormalisation group

$$V(r) = -\frac{Gm_1 m_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{2r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right)$$

Hope for non-perturbative renormalization.
(Asymptotic freedom)

Path integrals

$$Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\, \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}$$



Dynamical triangulation

GRAVITY AND THE QUANTUM

Canonical quantization

Quantum geometrodynamics

WdW equation

$$H\psi \equiv \left(G\hbar^2 \frac{\partial^2}{\partial\alpha^2} - \hbar^2 \frac{\partial^2}{\partial\phi^2} + m^2\phi^2 \mathrm{e}^{6\alpha} - \frac{\mathrm{e}^{4\alpha}}{G} \right) \psi(\alpha, \phi) = 0$$

$$\hat{H}|\psi >= 0$$

<u>*Loop quantum gravity*</u>

Simple idea.

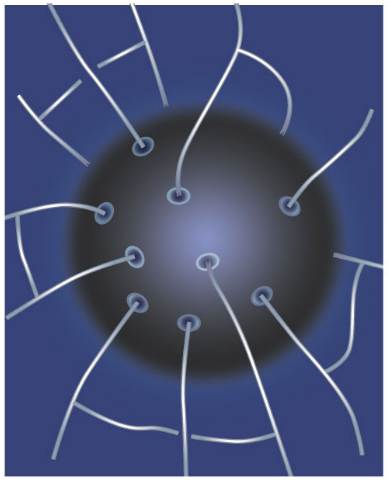
« Can we construct a quantum theory of spacetime based only on the experimentally well confirmed principles of general relativity and quantum mechanics ? » L. Smolin, hep-th/0408048

How to build Loop Quantum Gravity ?

- **Foliation → space metric and conjugate momentum**
- **Constraints (difféomorphism, hamiltonian + SO(3))**
- **Quantification « à la Dirac » → WDW → Ashtekar variables**
- **« smearing » → holonomies and fluxes**

LQC :

- **Aires et volumes quantifiés**
- **Absence de divergences**
- **Limite classique ?**



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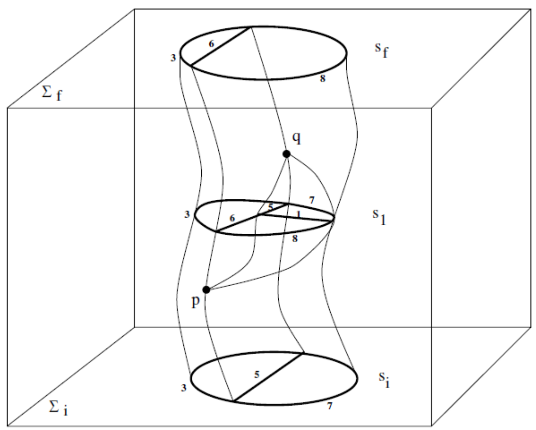
How to build Loop Quantum Gravity 2 ?
If you are a particle physicist…

- **Think of lattice QCD**
- **Define a graph and the Hilbert space : L2(G^L/G^N). The Fock space is obtain by taking the appropriate limit.**
- **In gravity you do the same : HΓ = L2[SU(2)^L/SU(2)^N]. Then H˜Γ = HΓ/ ~ (automorphism group)**
- **Define « natural » operators on L2[SU2]**
- **Gauge invariance + Penrose theorem lead to a simple geometrical interpretation in the classical limit.**
- **Define the spin-network basis (diagonolizes the area and volume operators)**



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Toward Loop Quantum Cosmology

Following Ashtekar

Within the Wheeler, Misner and DeWitt QGD, the BB singularity is not resolved
→ could it be different in the specific quantum theory of Riemannian geometry called LQG?

KEY questions:
- **How close to the BB does smooth space-time make sense ? Is inflation natural ?**
- **Is the BB singularity solved as the hydrogen atom in electrodynamics (Heinsenberg)?**
- **Is a new principle/boundary condition at the BB essential ?**
- **Do quantum dynamical evolution remain deterministic through classical singularities ?**
- **Is there an « other side » ?**

The Hamiltonian formulation generally serves as the royal road to quantum theory. But absence of background metric → constraints, scalar field for evolution.

- **Emergence of time ?**
- **Can we cure small scales and remain compatible with large scale ? 14 Myr is a lot of time ! How to produce a huge repulsive force @ 10^94 g/cm^3 and turn it off quickly.**

LQC

von Neumann theorem ? OK in non-relativistic QM. Here, the holonomy operators fail to be weakly continuous → no operators corresponding to the connections! → new QM

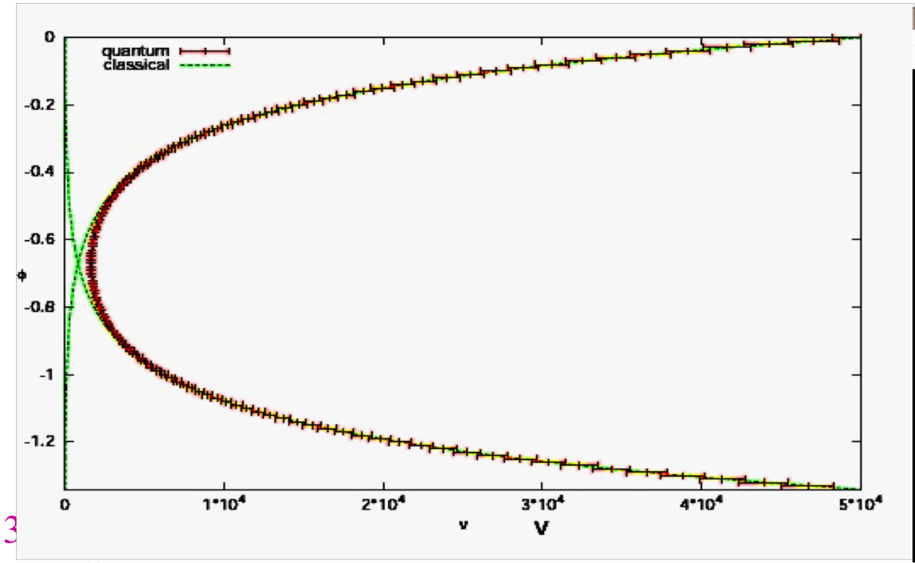
$$\Theta_o \Psi(v,\phi) = -F(v)\ \left(C^+(v)\,\Psi(v+4,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\,\Psi(v-4,\phi)\right)$$

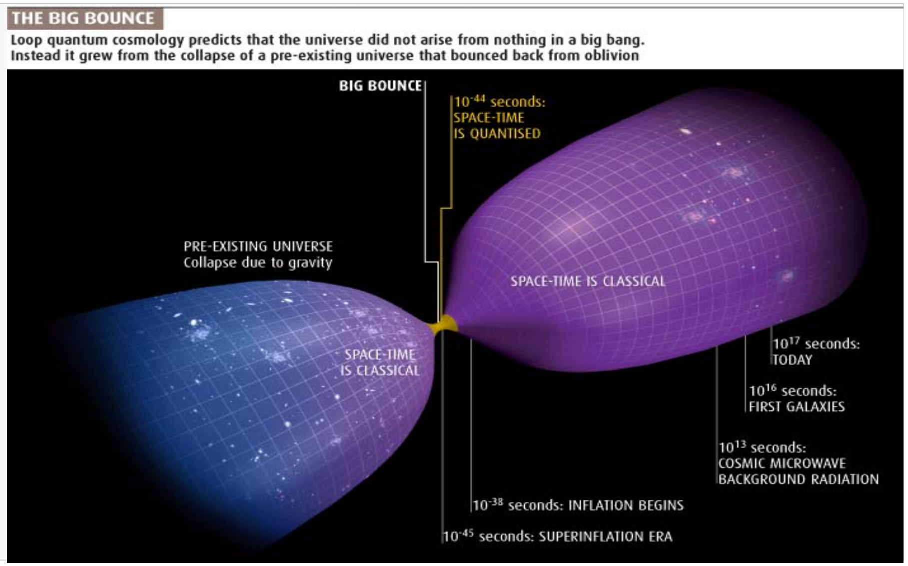
$$\left(\frac{\dot{a}}{a}\right)^2 = (8\pi G\,\rho/3)\,\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

Dynamics studied:

- **Numerically**
- **With effective equations**
- **With exact analytical results**

- **Trajectory defined by expectation values of the observable V is in good agreement with the classical Friedmann dynamics for $\rho<\rho_{Pl}/100$**
- **When $\rho \to \rho_{Pl}$ quantum geometry effects become dominant. Bounce at $0.41\rho_{Pl}$**





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Singularity resolution in LQC: robustness

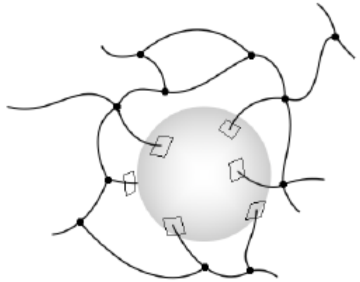
Following P. singh

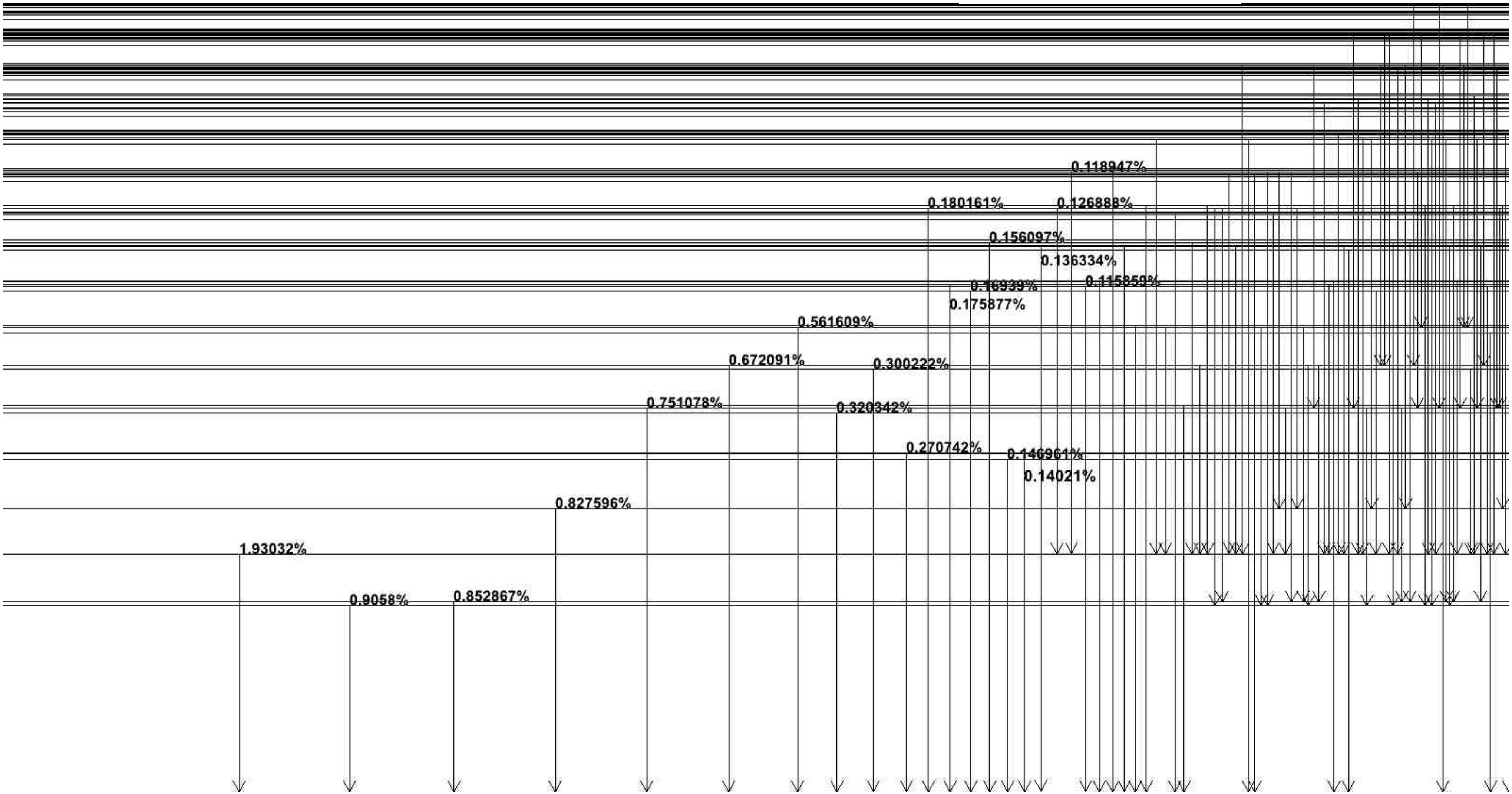
- **Exactly solvable model (flat, isotropic with a massless scalar)**
- **In presence of spatial curvature k = ±1**
- **Bianchi models**
- **Negative cosmological constant**
- **Positive cosmological constant**
- **φ2 inflationary potential**
- **Extremely wide states not corresponding to a classical universe at late times**
- **Non-gaussian and highly squeezed states corresponding to highly quantum universes**

Under quite general conditions conditions, the quantum evolution can be approximated by a continuum effective spacetime description

→ PHENOMENOLOGY, SPETRA

LQC: Black holes





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Quantum gravity might become soon experimental physics

It also obliges us to face philosophical questions.

Falsifiable ? (Is Popper right ? If yes, should he define science forever ?)
Multiverse ?
Extra-dimension are magics ?
Non-empirical corroboration ?
New science war ?
Evolution of the rules ?

Sociological questions too.

Let's not be afraid of deconstruction ☺

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