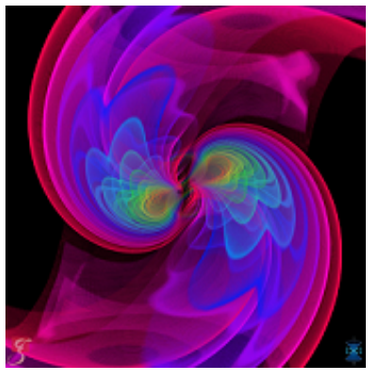
String Theory, Holography and Quantum Gravity

Constantin **BACHAS**

(ENS, Paris)



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Grenoble 3-7 décembre 2018

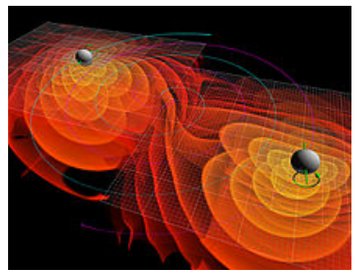
Contents

1. Black Holes & Quantum

Black holes: fascinating at **all scales**

- Sagittarius A region of Galactic center: supermassive $\sim 4 \times 10^6 \ M_\odot$

- LIGO/Virgo: merger of 29+36= 62 $M_\odot$



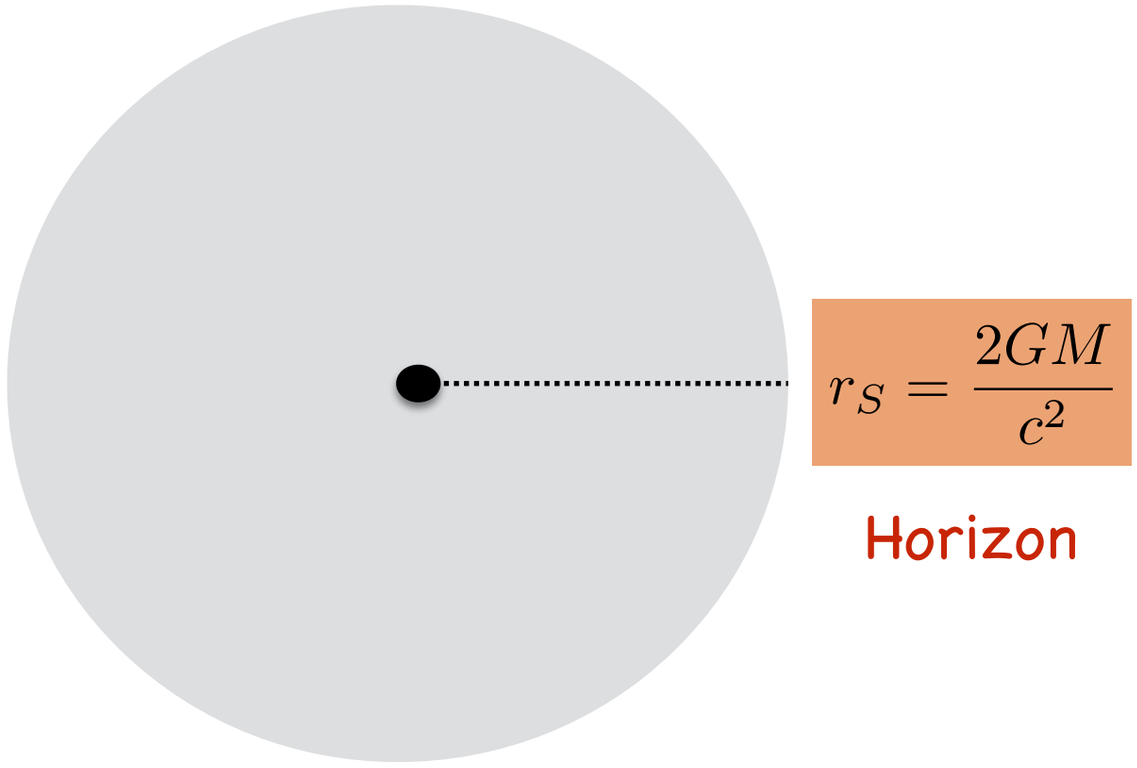
GW150914

- Mount-Everest-size BHs: dark matter ? $\sim 10^{-15} M_\odot$

None of them needs Quantum Theory, **or does it ?**

Schwarzschild solution of the Einstein equations:

Distant observer



$$r_S = \frac{2GM}{c^2}$$

Horizon

$$ds^2 = -(1 - \frac{r_s}{r})c^2dt^2 + (1 - \frac{r_s}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

slow down

In-falling observer:

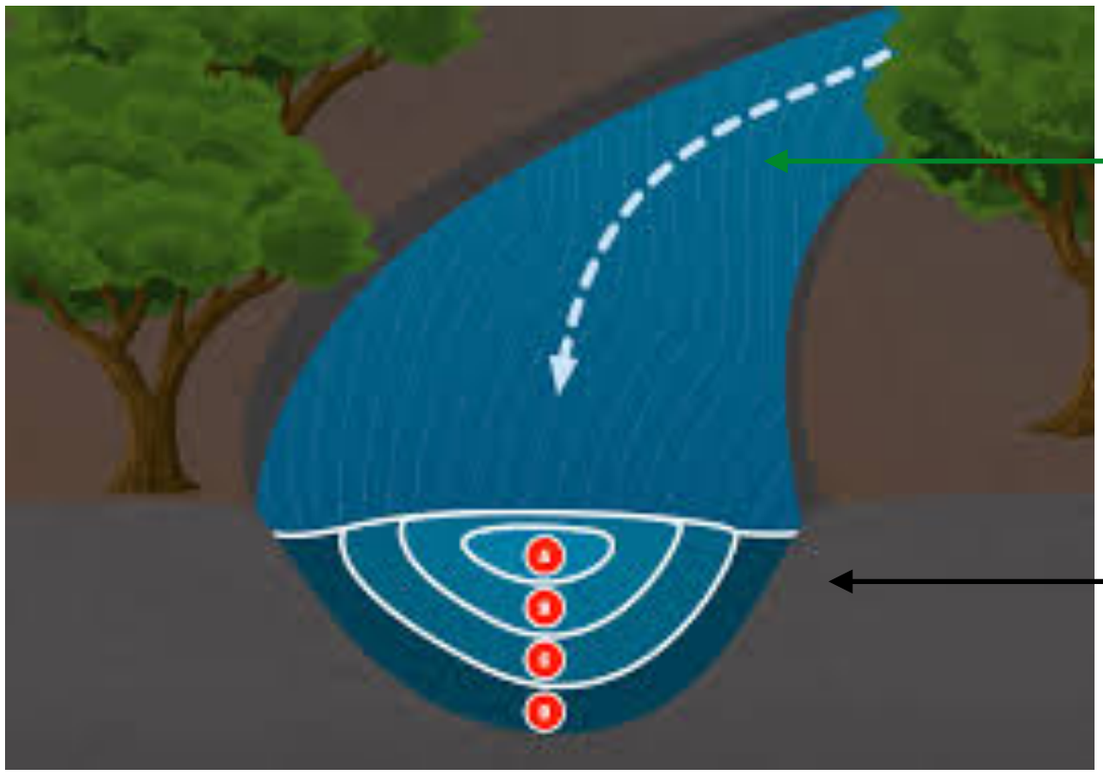
accelerates towards BH, but
feels nothing in local inertial frame

Einstein's principle of equivalence

On horizon of 10 million $M_\odot$ BH : same tidal forces as on earth's surface

> Passing the horizon seems very innocent while it is happening. It's like being in a rowboat above Niagara Falls. If you accidentally pass the point where the current is moving faster than you can row, you are doomed. But there is no sign—DANGER! POINT OF NO RETURN—to warn you. Maybe on the river there are signs but not at the horizon of a black hole.
>
> (Lenny Susskind, CA Literary Review)

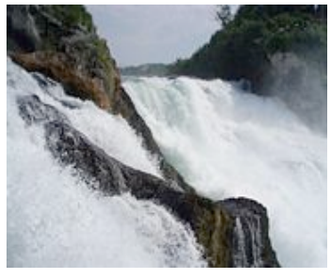
Acoustic «**black hole**»



Horizon

$V_{\text{flow}} > V_{\text{sound}}$

Singularity



Enters Hawking 1974:

BHs emit radiation like **hot bodies** at

$$k_B T_H = \frac{\hbar c^3}{8\pi G M}$$

and carry a (Bekenstein-Hawking) **entropy**

$$S_{BH} = \frac{c^3}{G\hbar}\,\frac{1}{4}(Area_H)$$

so that $c^2 dM = T_H dS_{BH}$ 1rst law of thermodynamics

Quantum origin, negligible for solar BH $T_H \sim 6 \times 10^{-8} K \times \frac{M_\odot}{M}$

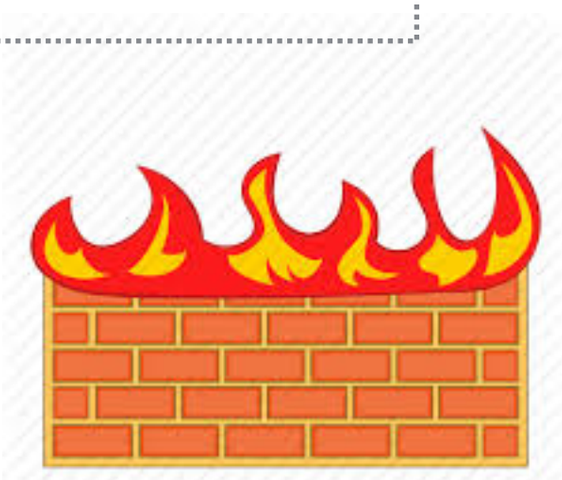
But what does the infalling observer see ?

For comparison: Universe **opaque** to cosmic rays travelling with $E > 6 \times 10^{19} eV$ through the $T \simeq 2.7K$ bath of primordial photons

Greisen, Kuz'min, Zatsepin Cutoff

$$\gamma_{CMB} + p \rightarrow p + \pi^0$$

Could the observer cruising through the hot horizon encounter likewise a **Firewall ?**



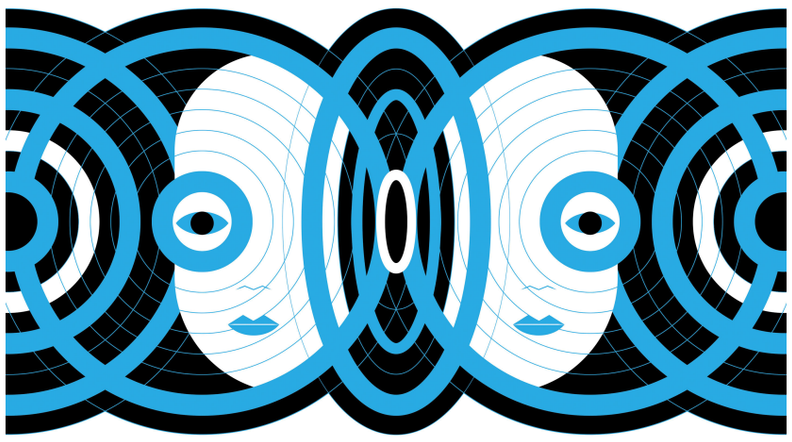
Almheiri, Marolf, Polchinski, Sully '12

Answering this question could give key for unlocking

the theory of **quantum gravity**

& may be of more than intellectual interest

[we could be cruising through a horizon at the moment]



https://www.quantamagazine.org/black-hole-echoes-would-reveal-break-with-einsteins-theory-20180322/

Black Hole Echoes Would Reveal Break With Einstein's Theory

2. Holography

The Bekenstein-Hawking entropy formula has a

universal geometric form:

$$S_{BH} = \frac{c^3}{G\hbar}\,\frac{1}{4} \times Area_H$$

All theories, spin, charge, & any dimension

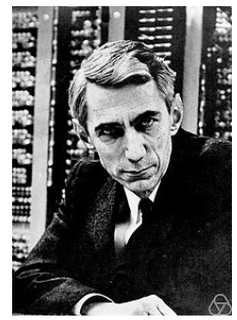
This is counter-intuitive: for normal matter

$$S \propto Volume$$

Recall:

One of the revolutions of 20th century physics:

Thermodynamic entropy = (quantum) **information entropy**



Boltzmann **Shannon**

For a binary digit (q-bit, spin-1/2, coin flip)

$$S = -p_0 \log p_0 - p_1 \log p_1$$

$$S = \begin{cases} 0 \\ \log 2 \end{cases}$$

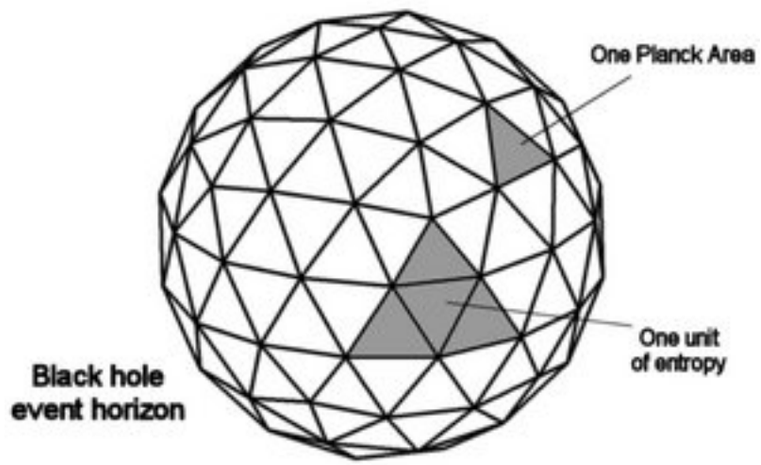
frozen (certainty) $T = 0$

maximal capacity (uncertainty) $T = \infty$

Key property of entropy: it is **extensive**

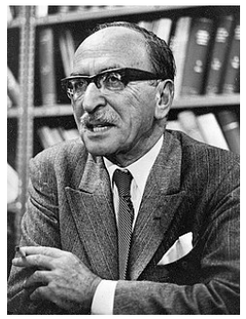
$\underbrace{110}_{N}$ q-bits have information capacity $N \log 2$

« Normally » $N \propto Volume$



but BH formula hints that information is **stored on the Horizon**

Gabor
’48 ’49



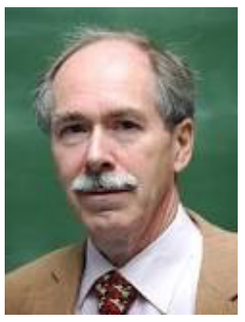
Holographic screen:

(by recording both phases & amplitudes)
the screen stores
3d information on a 2d surface

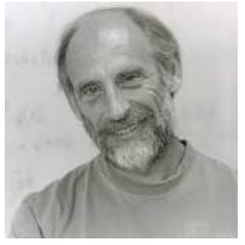
by analogy:

BH Horizon is a Holographic screen:

possible to describe the interior
from its surface



’t Hooft ‘93



Susskind ‘94

But what are these degrees of freedom on the BH horizon ?

String theory provides an answer, at least for charged **near-extremal** BHs

Reproduce the Bekenstein-Hawking formula

Resolve the paradox of the infalling observer

3. BHs in String Theory and AdS/CFT

Holography promoted to a precise mathematical statement

in the framework of **String Theory**

I will here try to convey the line of argument

without entering in the detailed mathematics

https://simonsingh.net

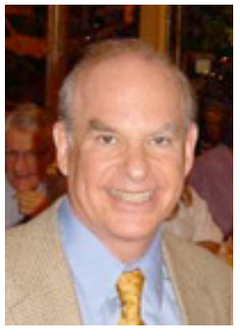
By early 90's : accumulated evidence that string theory is a

perturbatively consistent theory of QG (like Quantum Electrodynamics)

Veneziano

Yoneya

Scherk



Schwarz

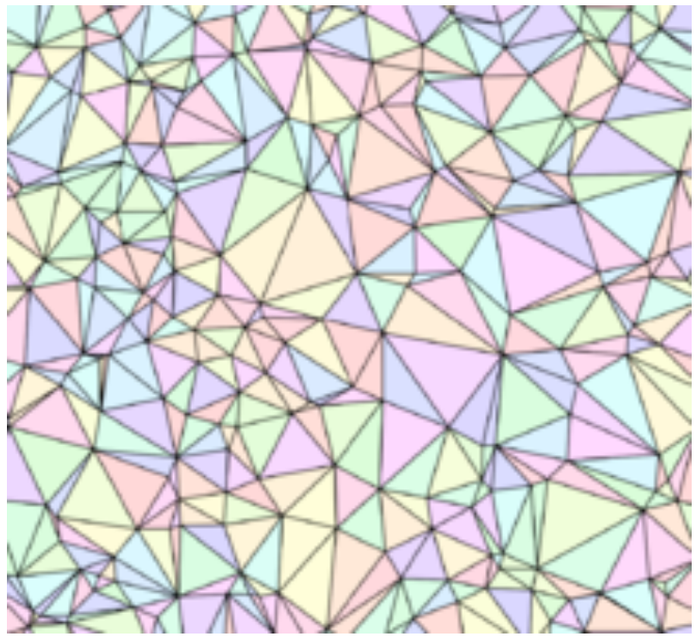
Green

Scattering of gravitons respects **Unitarity** & **Causality**

no loss of probability

nothing faster than light

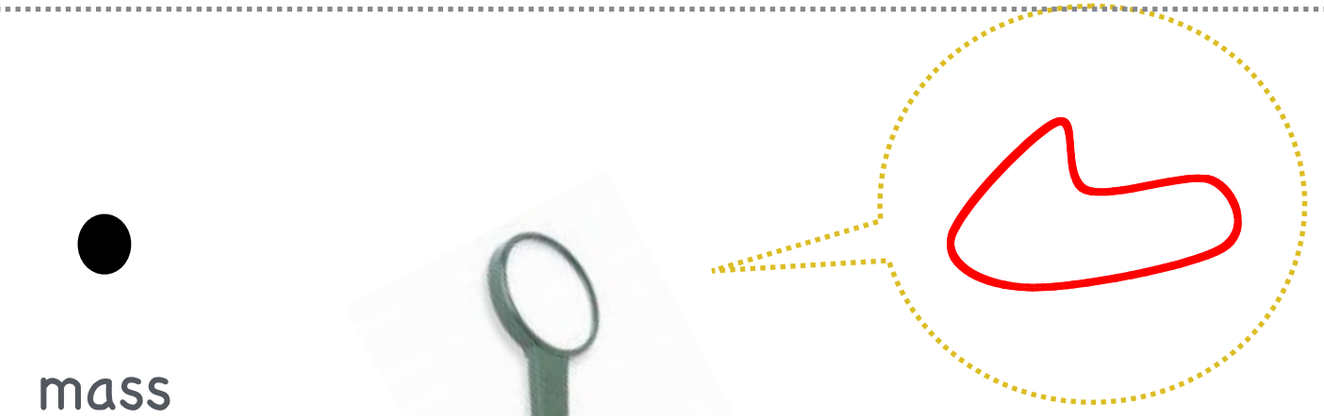
- May sound easy, but it is **NOT** !



$\ell_{\rm Planck}$

Granular spacetimes will violate these principles at astronomical scales if they do so at $\ell_{\rm Planck}$

- The same logic led to the prediction of Brout-Englert-Higgs boson discovered at the LHC of CERN.



mass
spin
charge

CM energy
angular momentum
KK momentum

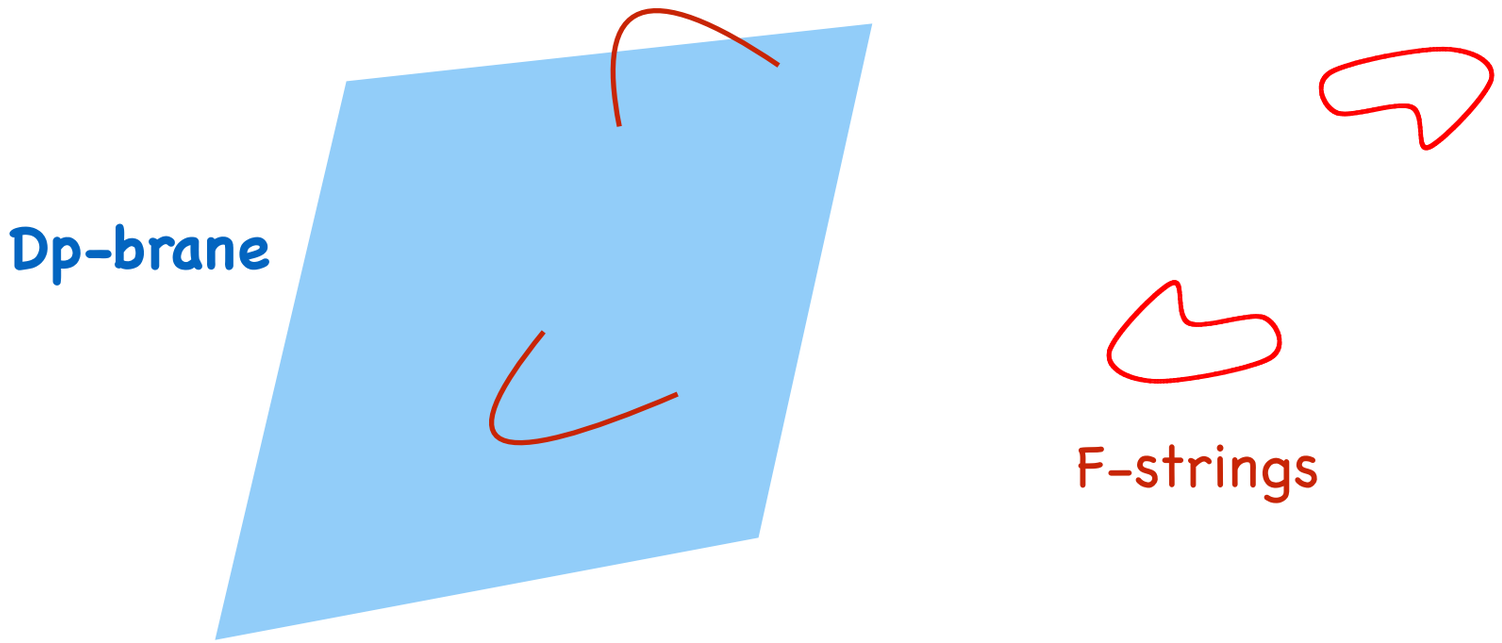
In string theory: **point** particles are replaced by vibrating **strings**

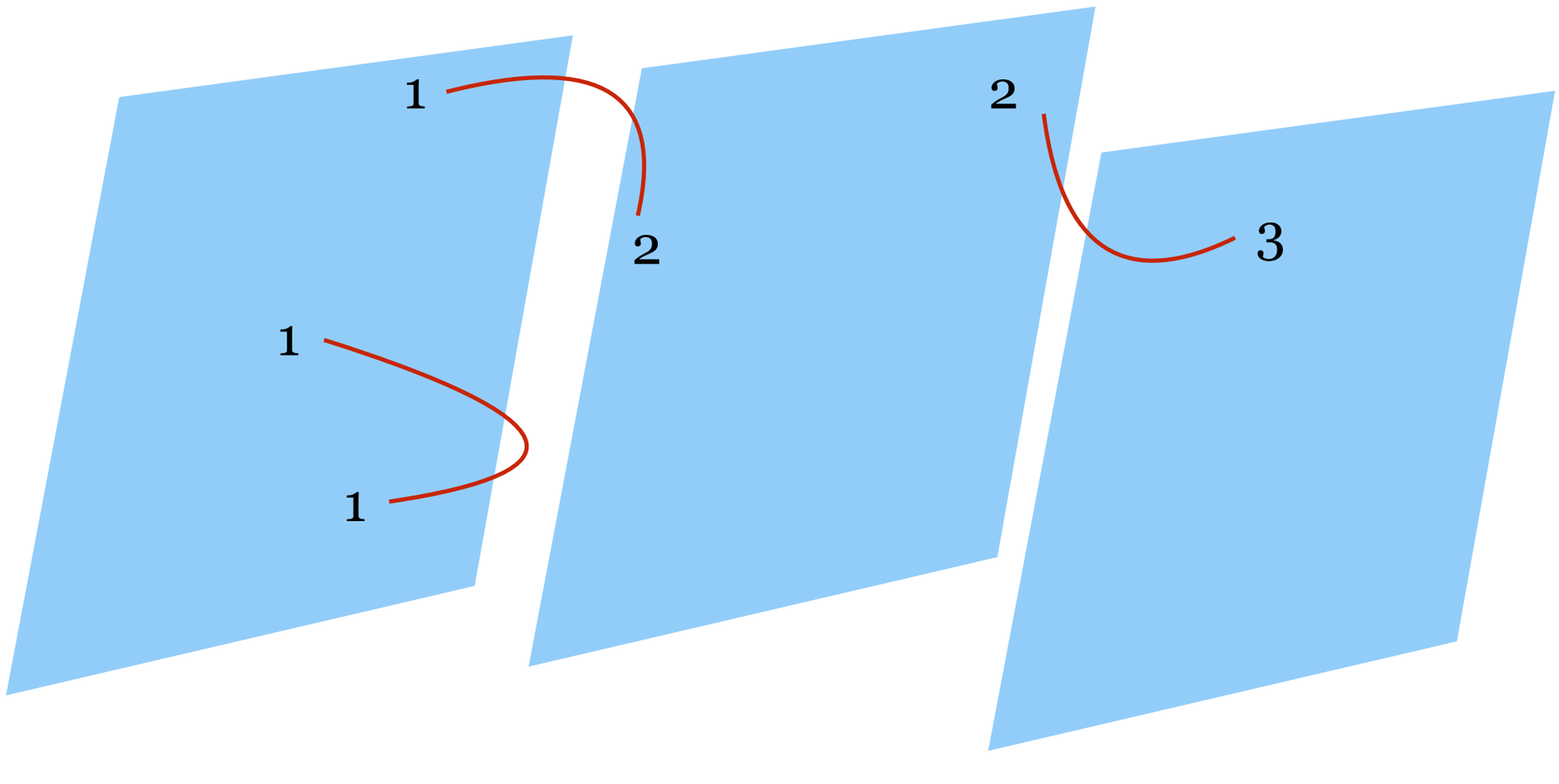
A striking universal feature: the lowest-lying string states include a **massless spin-2 particle** that is

Einstein's **graviton**

A crucial step was taken in '95 by Polchinski's discovery of **D-branes**

He postulated that a theory of **closed** strings has spacetime defects (**solitonic** excitations) on which **open** strings can end





For N D-branes: the open strings are $N \times N$ matrices

The low-E open-string theory is **Yang-Mills** theory

a.k.a. Standard Model

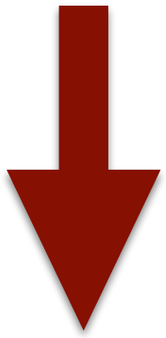
Neveu

Scherk

D-branes are **solitons** of string theory with a microscopic description

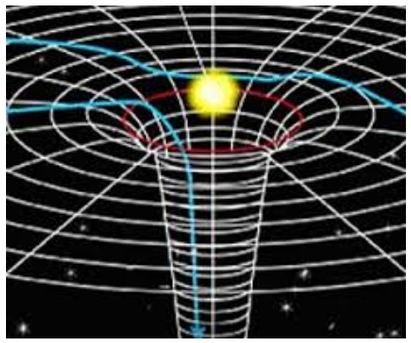
non-dispersive lumps of energy/mass

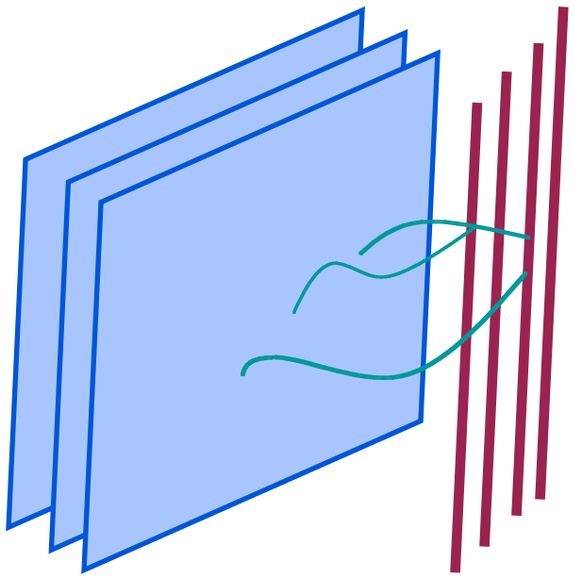
Black Holes are the solitons of gravity



A microscopic model of **Black Holes** ?

BH





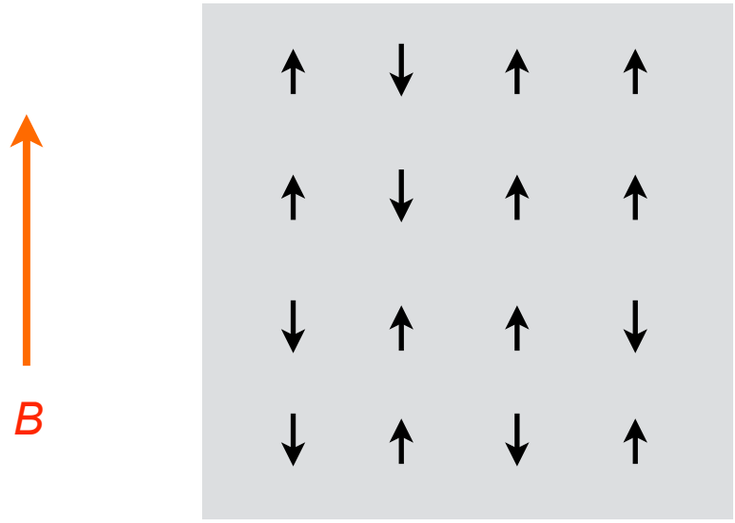
Strominger + Vafa '96

exhibited the first microscopic model of (near extremal, 3-charge 5d) **BH** that reproduced the BH formula

$$S \simeq 2\pi\sqrt{N N_1 N_5} = \frac{Area_H}{4G}$$

cf normal ferromagnet:

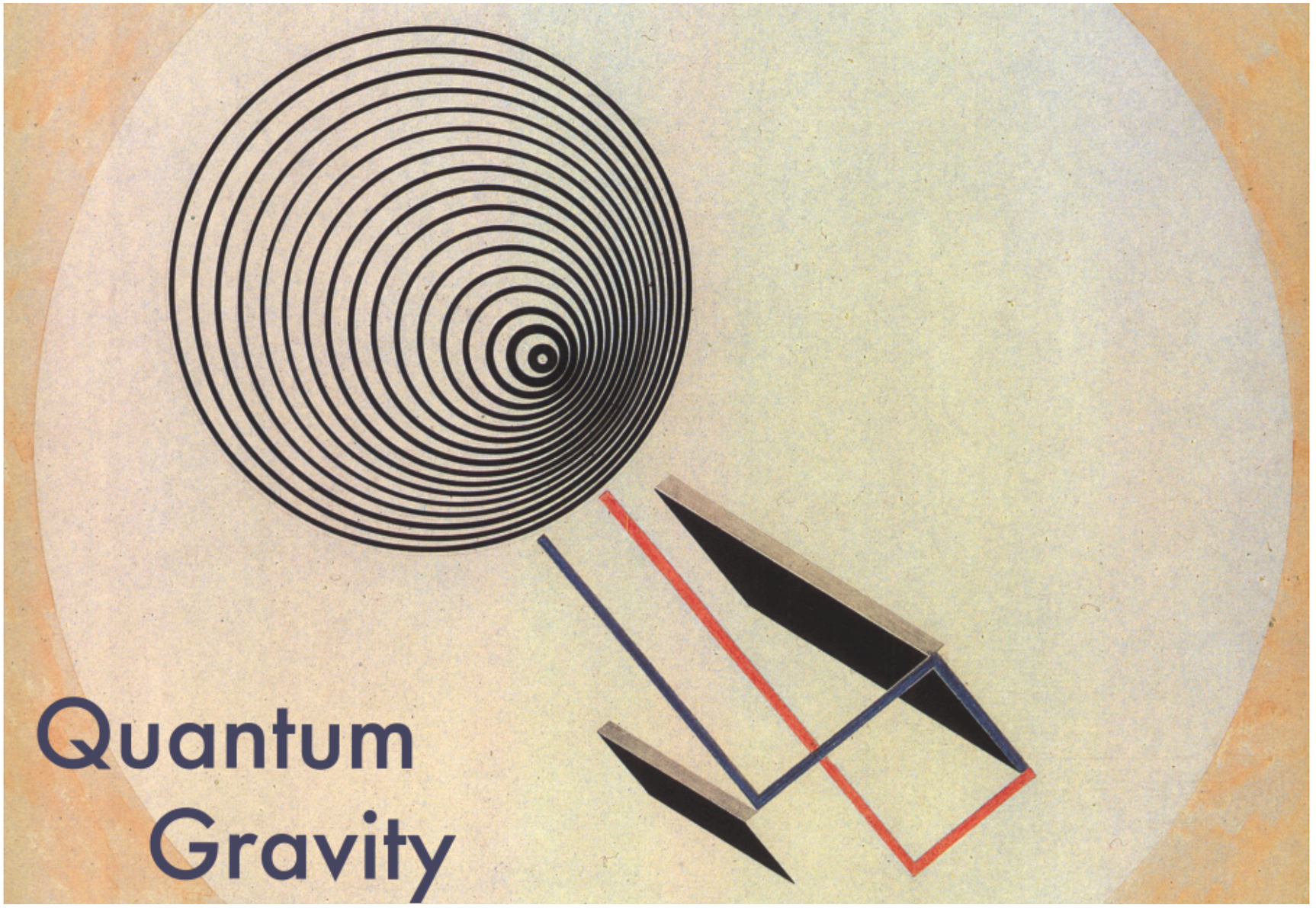
Thermodynamics versus Statistical Mechanics



equation of state

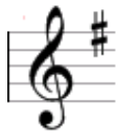
$S(B,T)$

derived from
microscopic Hamiltonian



an artist's view

IHES october 2017



The last step taken in the famous paper of Maldacena

« The Large N limit of superconformal field theories and supergravity » ’97

+ two companion papers

Gubser, Klebanov, Polyakov « Gauge theory correlators from non-critical string theory »

Witten « Anti-de Sitter space and holography »

Why did the matching of entropies work ?

A more general description of quantum horizons ?

A key feature of a **Black Hole** is the infinite Horizon **redshift:**

$$ds^2 = -(1 - \frac{r_s}{r})c^2dt^2 + (1 - \frac{r_s}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

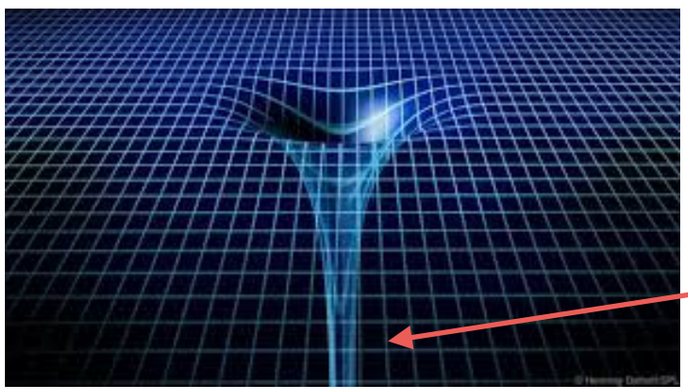
$r \to \infty$

flat Minkowski

$$\frac{r}{r_S} - 1 = \delta \to 0$$

$$ds^2 = r_S^2\left(-\delta dt^2 + \frac{d\delta^2}{\delta} + ds^2(sphere)\right)$$

`throat' geometry



$$\tau = t\sqrt{\delta}$$

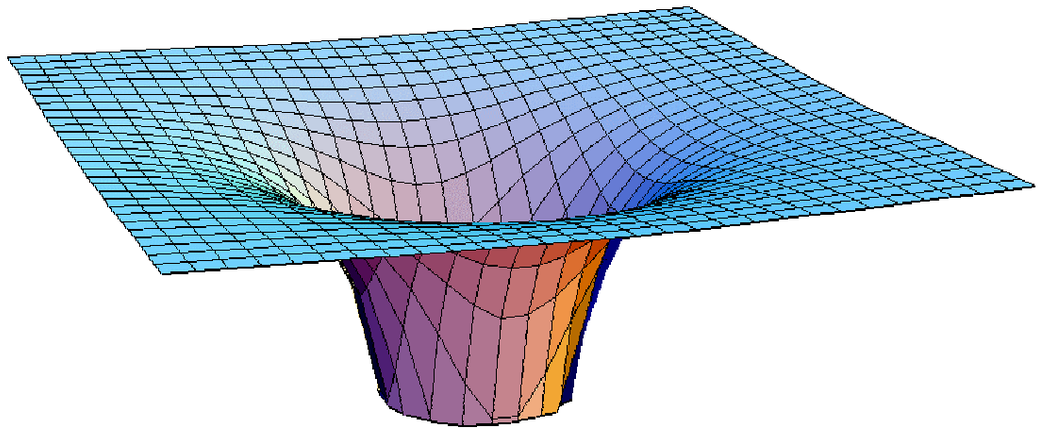
invariant time

$$e^{i\omega\tau} = e^{i\omega\sqrt{\delta}\,t}$$

A mathematically simpler model of a BH is a **Black 3-brane**

whose near-horizon geometry is $\mathrm{AdS}_5 \times \mathrm{S}^5$

Maldacena considered all low-energy excitations of this system

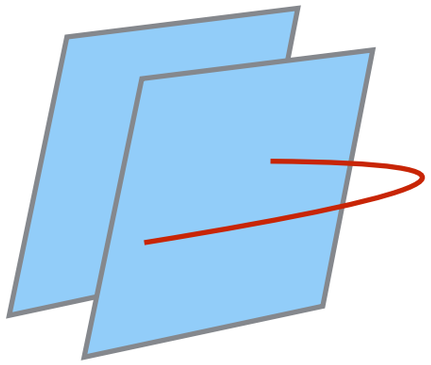


free gravity waves

⊕

light because of
huge redshift

Closed-string theory in AdS5xS5



free gravity waves

⊕

SU(N) Yang-Mills

This led to a mathematically-sharp conjecture

of **holographic duality**:

supersymmetric Yang-Mills theory in d=4 dimensions

type IIB string theory in $\mathrm{AdS}_5 \times \mathrm{S}^5$

This has been generalized to many other systems

Also called **AdS/CFT correspondence**

or **gauge/gravity duality**

4. What next ?

Holographic duality led to development of new tools for calculations at **strong coupling**

In the previous example

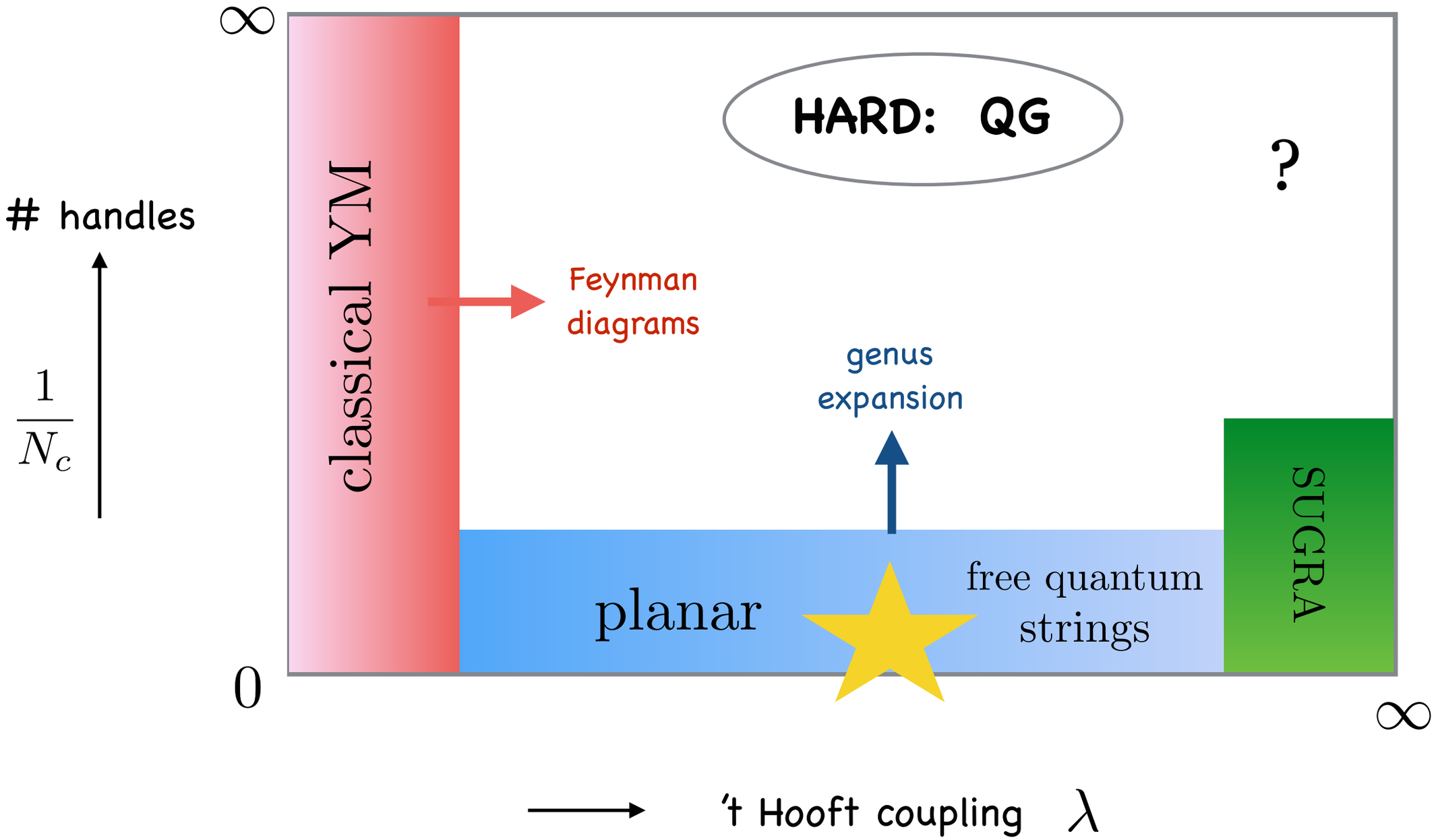
String-theory parameters: $\left(\frac{L_{\rm AdS}}{\ell_{\rm Planck}}\right)^4$, $\left(\frac{L_{\rm AdS}}{\ell_s}\right)^4$

$\Big\|$ by holographic duality

YM-theory parameters: $\frac{1}{N_c}$, $\lambda = g^2_{\rm YM} N_c$

't Hooft coupling

Different regions of computability, they agree where they can be compared



spectra matched by integrability

Minahan,Zarembo,
Beisert, Gromov, Kazakov, Vieira . . .

Computing and resumming Feynman diagrams in 4d YM theory is extremely tedious, and of practical importance

for QCD backgrounds at LHC

String theory (& integrability) led to very efficient resummations

One example: scaling dimension of the (spin=twist=2) **Konishi** operator in $\mathcal{N}=4$ sYM

Konishi operator from quantum spectral curve:

Marbeu,Volin arXiv:1411.4758

$g = \sqrt{\lambda}/4\pi$

130 000 Feynman graphs !

$$
\begin{aligned}
\Delta \;=\;& 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\big) \\
&+g^{10}\big(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 - 8640\,\zeta_5 + 30240\,\zeta_7\big) \\
&+g^{12}\big(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7 - 489888\,\zeta_9\big) \\
&+g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5 \\
&\quad - 993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big) \\
&+g^{16}\Big(54408192 - 83496960\,\zeta_3 + 7934976\,\zeta_3^2 + 1990656\,\zeta_3^3 - 19678464\,\zeta_5 - 4354560\,\zeta_3\,\zeta_5 \\
&\quad - 3255552\,\zeta_3^2\,\zeta_5 + 2384640\,\zeta_5^2 + 21868704\,\zeta_7 - 6229440\,\zeta_3\,\zeta_7 + 22256640\,\zeta_5\,\zeta_7 \\
&\quad + 9327744\,\zeta_9 + 23224320\,\zeta_3\,\zeta_9 + \frac{65929248}{5}\,\zeta_{11} - 106007616\,\zeta_{13} - \frac{684288}{5}\,Z_{11}^{(2)}\Big) \\
&+g^{18}\Big(-1014549504 + 1140922368\,\zeta_3 - 51259392\,\zeta_3^2 - 20155392\,\zeta_3^3 + 575354880\,\zeta_5 \\
&\quad - 14294016\,\zeta_3\,\zeta_5 - 26044416\,\zeta_3^2\,\zeta_5 + 55296000\,\zeta_5^2 + 15759360\,\zeta_3\,\zeta_5^2 - 223122816\,\zeta_7 \\
&\quad + 34020864\,\zeta_3\,\zeta_7 + 22063104\,\zeta_3^2\,\zeta_7 - 92539584\,\zeta_5\,\zeta_7 - 113690304\,\zeta_7^2 - 247093632\,\zeta_9 \\
&\quad + 119470464\,\zeta_3\,\zeta_9 - 245099520\,\zeta_5\,\zeta_9 - \frac{186204096}{5}\,\zeta_{11} - 278505216\,\zeta_3\,\zeta_{11} - 253865664\,\zeta_{13} \\
&\quad + 1517836320\,\zeta_{15} + \frac{15676416}{5}\,Z_{11}^{(2)} - 1306368\,Z_{13}^{(2)} + 1306368\,Z_{13}^{(3)}\Big) \\
&+g^{20}\Big(16445313024 - 13069615104\,\zeta_3 - 1509027840\,\zeta_3^2 + 578949120\,\zeta_3^3 \\
&\quad - 14929920\,\zeta_3^4 - 11247547392\,\zeta_5 + 1213581312\,\zeta_3\,\zeta_5 + 1234206720\,\zeta_3^2\,\zeta_5 \\
&\quad - 70170624\,\zeta_3^3\,\zeta_5 - 1390279680\,\zeta_5^2 - 654842880\,\zeta_3\,\zeta_5^2 + \frac{6966252288}{175}\,\zeta_5^3 \\
&\quad + 377212032\,\zeta_7 - 1610841600\,\zeta_3\,\zeta_7 + 154680192\,\zeta_3^2\,\zeta_7 + 222341760\,\zeta_5\,\zeta_7 \\
&\quad + 133788672\,\zeta_3\,\zeta_5\,\zeta_7 + 868662144\,\zeta_7^2 + 4915257984\,\zeta_9 - 332646912\,\zeta_3\,\zeta_9 \\
&\quad - 91072512\,\zeta_3^2\,\zeta_9 + 1099699200\,\zeta_5\,\zeta_9 + 2275620480\,\zeta_7\,\zeta_9 + \frac{9793211904}{5}\,\zeta_{11} \\
&\quad - 2334572928\,\zeta_3\,\zeta_{11} + 2713772160\,\zeta_5\,\zeta_{11} - \frac{787483944}{175}\,\zeta_{13} + 3372969600\,\zeta_3\,\zeta_{13} \\
&\quad - \frac{4308536566944}{875}\,\zeta_{15} - 21661960320\,\zeta_{17} + \frac{752219136}{5}\,Z_{11}^{(2)} - \frac{5070791808}{175}\,Z_{13}^{(2)} \\
&\quad - \frac{7159104}{7}\,Z_{13}^{(3)} + \frac{2716063488}{175}\,Z_{15}^{(2)} - \frac{17895168}{25}\,Z_{15}^{(3)} + 11943936\,\zeta_3\,Z_{11}^{(2)}\Big) + \mathcal{O}(g^{22})\,, \qquad (85)
\end{aligned}
$$

where $Z_a^{(n)}$ denote single-valued MZV's written in the basis [63]

Closer to experiment for relativistic fluids:

- **viscocity/entropy** of 'holographic' fluids

$$\frac{\eta}{s} \leq \frac{\hbar}{4\pi k_B}$$

Kovtun, Policastro, Son, Starinets

better than perturbative QCD for quark-gluon plasma

- quantum anomalies & transport in Weyl semi-metals

Banerjee etal; Erdmenger et al; Landsteiner . . .

Behavior of strongly-coupled **ordinary matter** controlled by near-horizon **gravitational physics** !

But how about the opposite ?

What does AdS/CFT say about the paradoxes of quantum gravity?

- Quantum coherence/Unitarity
- Causality/Locality
- Principle of Equivalence

Which (if any) should we abandon near the horizon ?

(No proof but) serious argument that coherence/unitarity continue to hold:

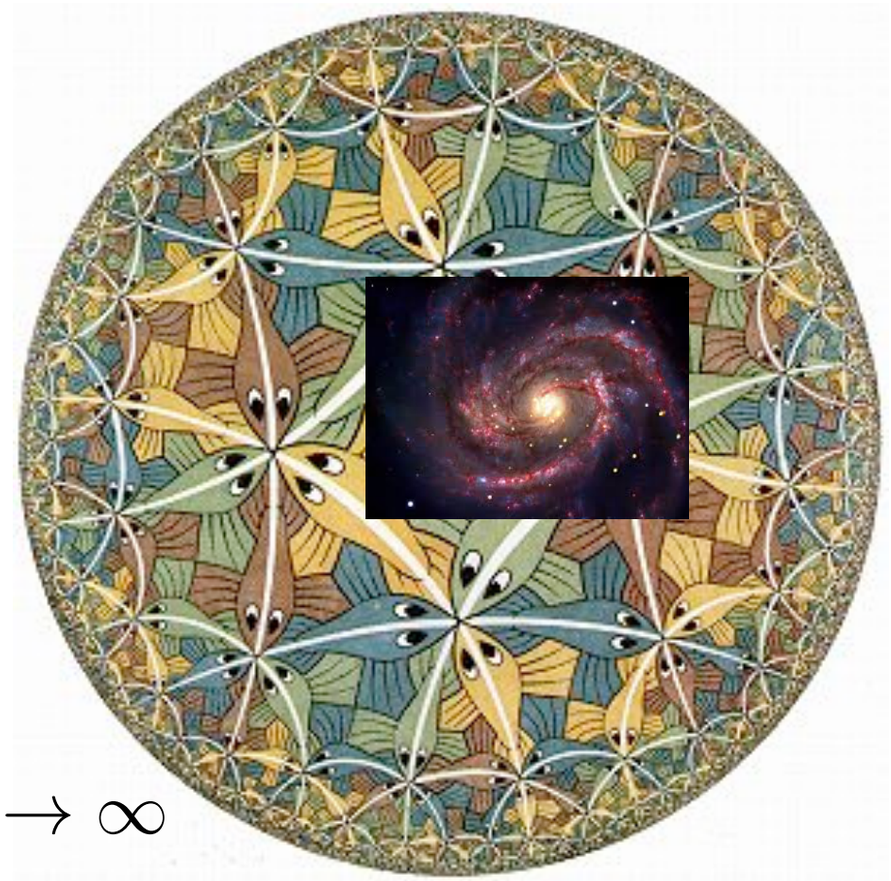
AdS/CFT gives a (partly) **background independent** formulation of QG

Boundary asymptotically AdS

as ordinary Quantum Field Theory with a unitary S-matrix.

Although hard to trace it in detail, information cannot be lost !

M.C. Escher, *Circle Limit III*, 1959.
strictly-speaking this is EAdS2



$\rho \to \infty$

AdS is a **gravitational trap**

in global coordinates

$$ds^2 = d\rho^2 + e^{2\rho}(-dt^2 + d\vec{x}d\vec{x})$$

infinite **blueshift** repels
from boundary

But in interior anything goes :

(Small) **black holes** form and evaporate, **singularities** appear etc
All this is described by YM theory !

How to describe the fate of an in-falling observer in Yang-Mills theory ?

Between Causality/Locality and Equivalence principle should one give ?

Actively researched and debated, many ideas - no consensus

Fuzzballs: horizon and singularity absent in higher dimensions

(smooth `fingered' geometries where space ends)

Mathur, . . .

Firewalls: Equivalence Principle breaks down at horizon Almheiri et al

State dependence: experimental equipment of infalling observer depends on the quantum state of the BH

Papadodimas + Raju

Entanglement & Geometry: geometry result of quantum entanglement

ER=EPR ?

Ryu + Takayanagi

Quantum chaos: BHs scramble information at a maximal rate;

Rigorous bound on growth of chaos: $\lambda_L \leq 2\pi k_B T/\hbar$

Saturating the bound: guide to model Schwarzchild horizons

Kitaev;
Maldacena+ Shenker + Stanford
. . .

. . . story still unfolding

Closing Remarks

- Of various QG proposals, **string theory** is the most conservative
  (gives up no basic principles of QM; has Einstein theory as limit)

- **Holographic** duality relates QG in AdS box to normal QFT

- We now have a model in which to analyze **BH `paradoxes'**

  Red herring ? Experimental signature ?