The Standard Model of the Universe Confronted to Observations: from the Theory of Inflation to Dark Matter and Dark Energy.

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The History of the Universe

It is a history of EXPANSION and cooling down.

EXPANSION: the space itself expands with the time.

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 \quad , \quad a(t) =$ scale factor.

FRW: Homogeneous, isotropic and spatially flat geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succesion of phase transitions towards the less symmetric phases.

Wavelenghts redshift as $a(t) \quad : \quad \lambda(t) = a(t)\, \frac{\lambda(t_0)}{a(t_0)}$

Redshift z : $\quad z + 1 = \frac{a(\text{today})}{a(t)} \quad , \quad a(\text{today}) \equiv 1$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant

Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO
- Acceleration of the Universe expansion: Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.
During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-36}$ sec and ends by $z \sim 10^{29}$ followed by a radiation dominated era.
Energy scale when inflation starts $\sim 10^{16}$ GeV ($\Longleftarrow$ CMB anisotropies) which **coincides** with the GUT scale..

Matter can be effectively described during inflation by a Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi) \right]$.

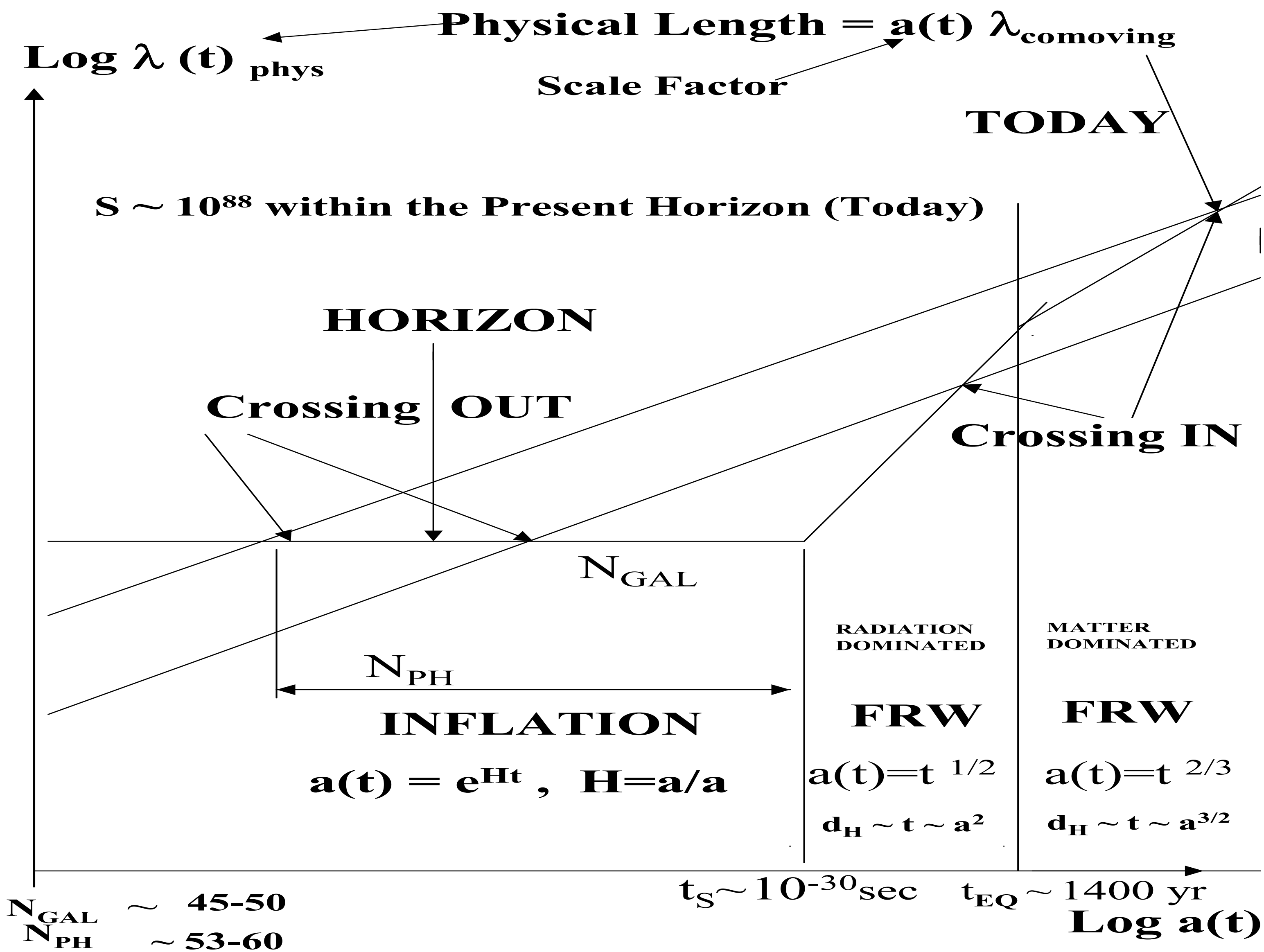
Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Physics during Inflation

- Out of equilibrium evolution in a fastly expanding geometry. Vacuum energy DOMINATES. $a(t) \simeq e^{H\,t}$.
- Extremely high energy density at the scale of $\lesssim 10^{16}$ GeV.
- Explosive particle production due to spinodal or parametric instabilities.
- Quantum non-linear phenomena eventually shut-off the instabilities and stop inflation. Radiation dominated era follows: $a(t) = \sqrt{t}$.
- Huge redshift classicalizes the dynamics: an assembly of (superhorizon) quantum modes behave as a classical and homogeneous inflaton field. Inflaton slow-roll.

D. Boyanovsky, H. J. de Vega, in Astrofundamental Physics, NATO ASI series vol. 562, 2000, Lectures at the Chalonge School, astro-ph/0006446.

Fluctuations Out and In the Horizon.



*Scales CROSS OUT the Horizon and Later COME BACK : UNIQUE for INFLATION

**LARGER SCALES CROSS OUT FIRST and CROSS BACK LATER

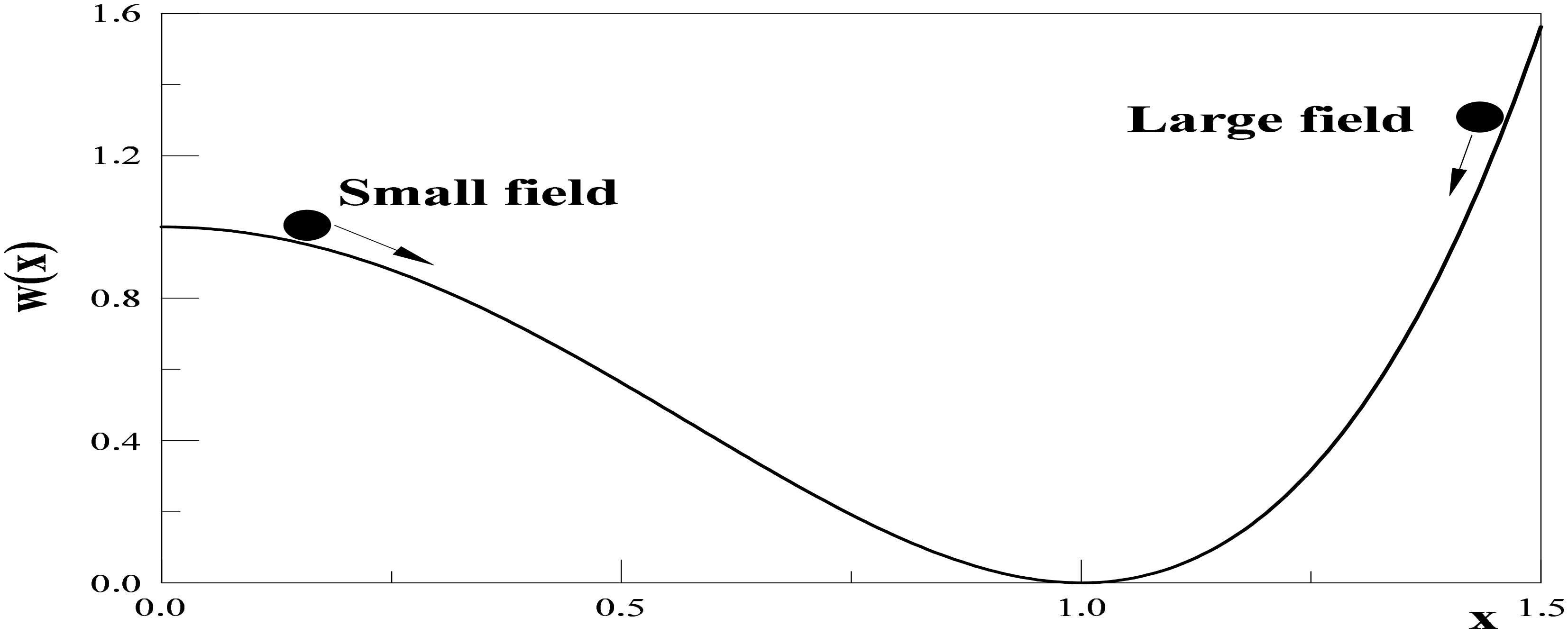
The Theory of Inflation

The inflaton is an effective field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q \; , \; \sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
- ....

Slow Roll Inflaton Models



$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$$V(\phi) = N\ M^4\ w\left(\frac{\phi}{\sqrt{N}\ M_{Pl}}\right)$$

$N \sim 60$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

Slow-roll is needed to produce enough efolds of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\ M_{Pl}} \quad , \quad \tau = \frac{m\ t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\ \sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}}\right)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order one.

Evolution Equations:

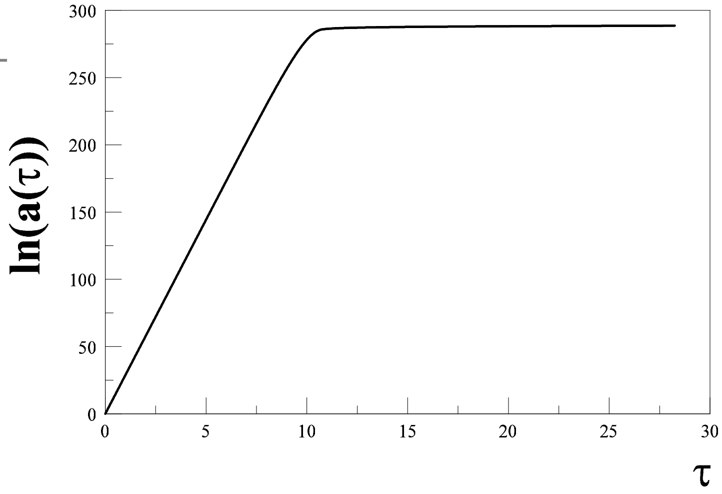
$$\mathcal{H}^2(\tau) = \frac{1}{3}\left[\frac{1}{2\ N}\left(\frac{d\chi}{d\tau}\right)^2 + w(\chi)\right] \quad ,$$

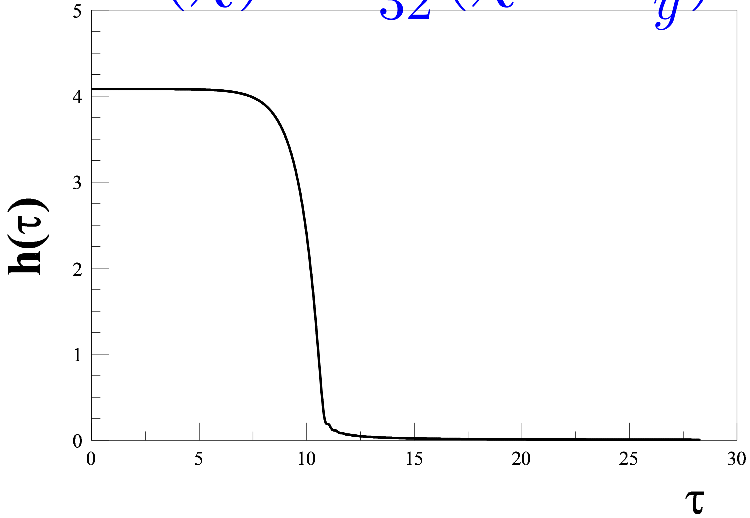
$$\frac{1}{N}\ \frac{d^2\chi}{d\tau^2} + 3\,\mathcal{H}\ \frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \tag{1}$$

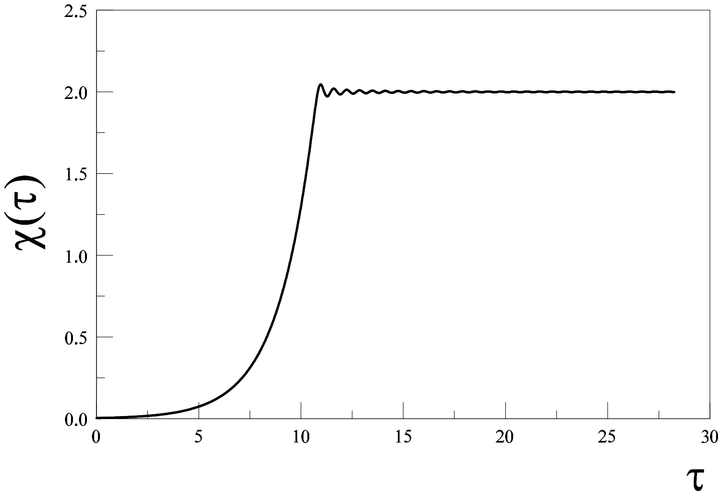
$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$

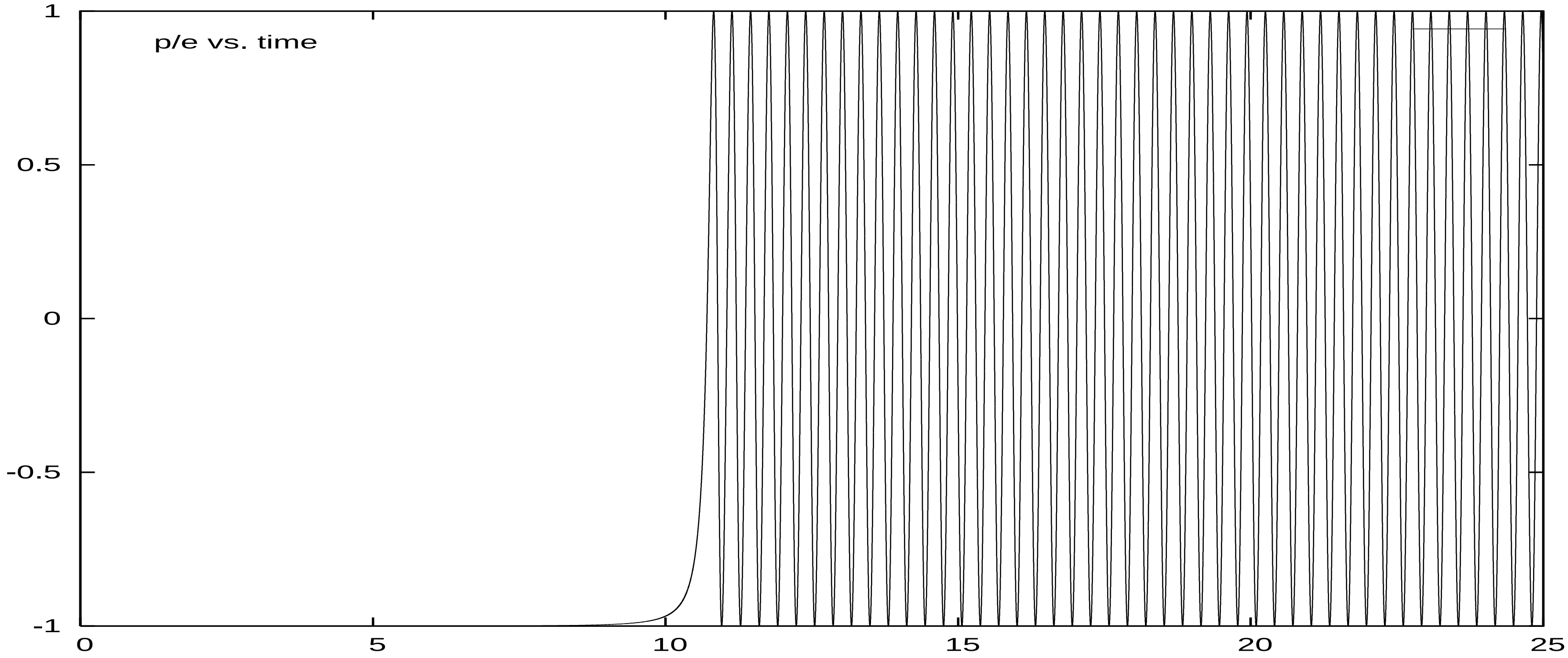






The vacuum energy transforms into particles and inflation is followed in this simplified approach by a matter dominated stage.

Equation of State: pressure/energy density



The equation of state is $p/e = -1$ during inflation.

p/e strongly oscillates between $+1$ and -1 during the matter dominated stage. We have in average $< p/e >= 0$.
We have here neglected spatial gradient terms

$$\frac{(\nabla\phi)^2}{2\; a^2(t)}$$

since $a(t)$ grows exponentially during inflation.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2\ k^{n_s - 1}$.

To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2} \left(\frac{M}{M_{Pl}}\right)^4 \frac{w^3(\chi)}{w'^2(\chi)} .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.470 \pm 0.09) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!

We find the scale of inflation without knowing r !!

The scale M is independent of the shape of $w(\chi)$.

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

$$n_s - 1 = -\frac{3}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N}\,\frac{w''(\chi)}{w(\chi)} \quad , \quad r = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2}\,\frac{w'(\chi)\,w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2}\,\frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2}\,\frac{[w'(\chi)]^2\,w''(\chi)}{w^3(\chi)} \,,$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation it is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\, \chi^3 + G_4\, \chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\; M^4\; w\left(\frac{\phi}{\sqrt{N}\; M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\; \phi^2 + g\; \phi^3 + \lambda\; \phi^4 \; .$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}} \left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60 \; .$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:

$$m = \frac{M^2}{M_{Pl}} \simeq 0.003\; M \quad , \quad m = 2.5 \times 10^{13} \text{GeV}$$

Trinomial Inflationary Models

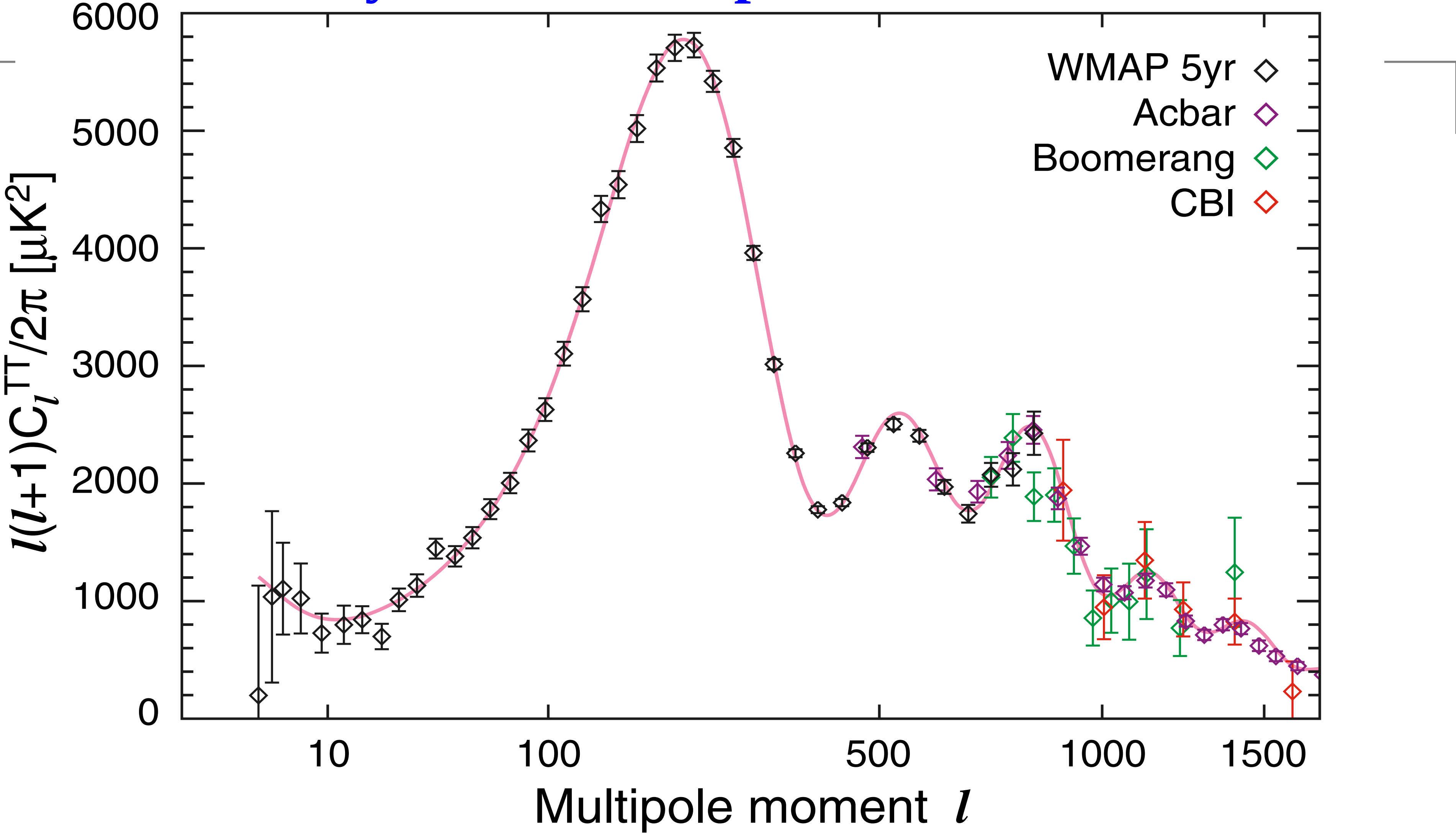
- Trinomial Chaotic inflation:
  $$w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \; .$$
- Trinomial New inflation:
  $$w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\,F(h) \; .$$

h = asymmetry parameter. $w(\text{min}) = w'(\text{min}) = 0$,

$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\,(h^2+1)^{\frac{3}{2}}$.

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

WMAP 5 years data set plus other CMB data



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

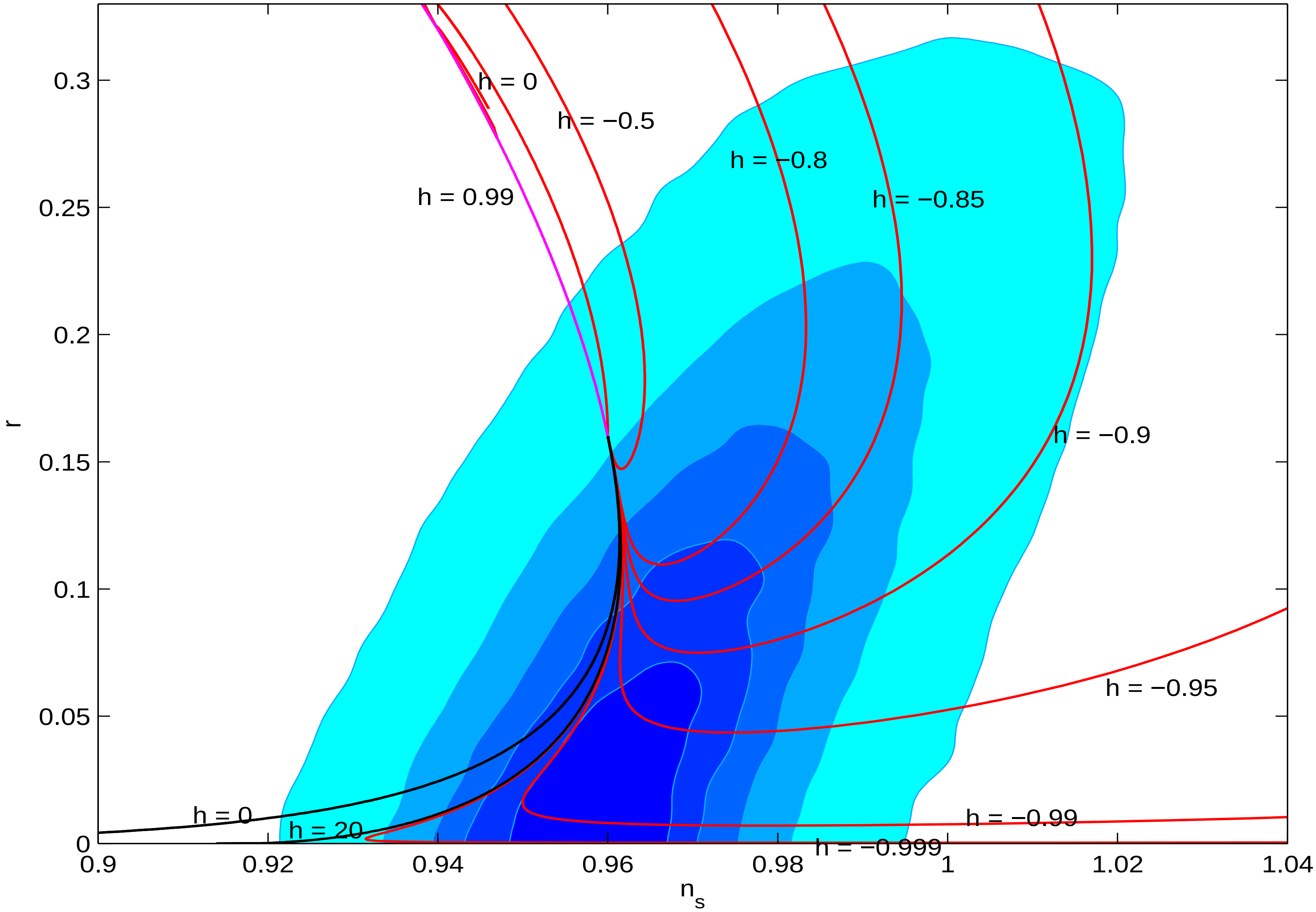
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

The color–filled areas correspond to $12\%, \ 27\%, \ 45\%, \ 68\%$ and 95% confidence levels according to the WMAP3 and Sloan data.
The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for Trinomial New Inflation.



MCMC Results for Trinomial New Inflation.

Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL)
Most probable values: $n_s \simeq 0.956$, $r \simeq 0.055 \Leftarrow$measurable!!
The most probable trinomial potential for new inflation has a moderate nonlinearity with the quartic coupling $y \simeq 1.5 \ldots$ and $h < 0.3$.
We can choose $h = 0$ and we then we find $y \simeq 1.322 \ldots$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$$

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.
Similar results from WMAP5 data. Acbar08 data slightly increases $n_s < 1$ and r.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \; v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.

First observation of SUSY in nature??

De Sitter Geometry and Scale Invariance

The De Sitter metric is scale invariant:

$$ds^2 = \frac{1}{(H\ \eta)^2}\left[(d\eta)^2 - (d\vec{x})^2\right] \ .$$

$\eta =$ conformal time.

But inflation only lasts for N efolds !

Corrections to scale invariance:

$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$

$n_s = 1$ and $r = 0$ correspond to a critical point.

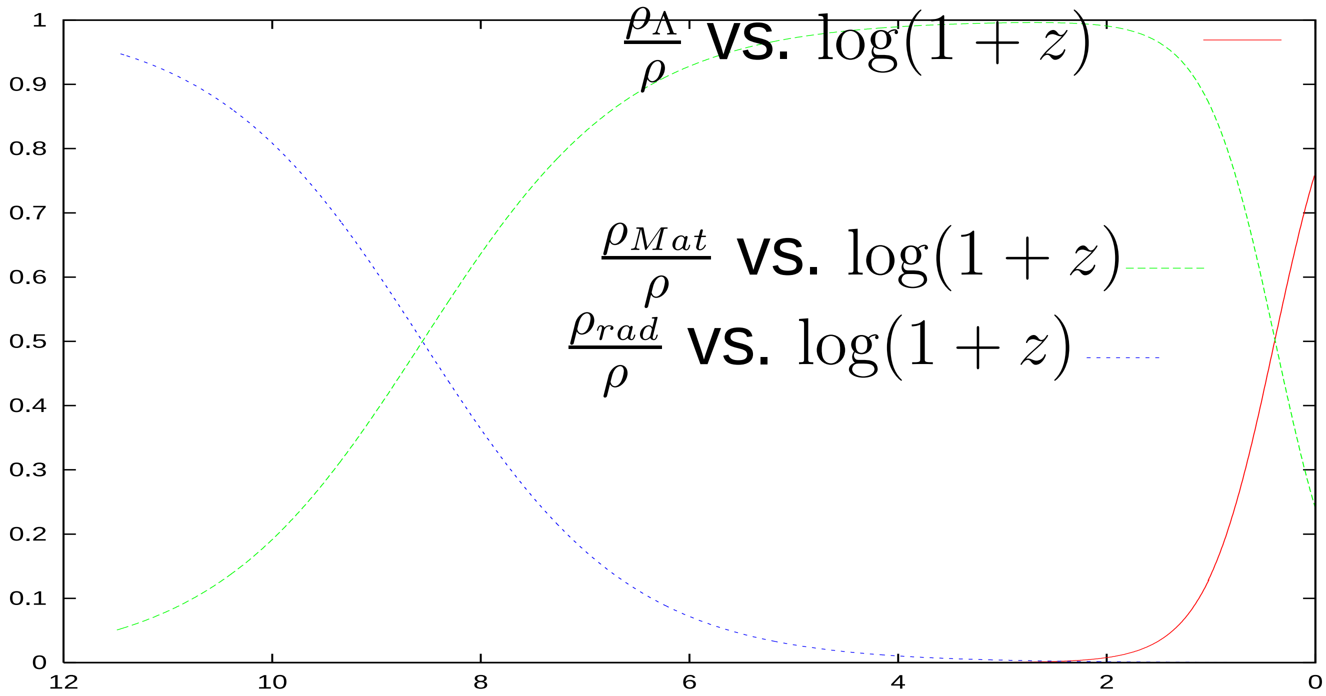
It is a gaussian fixed point around which the inflation model hovers in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.

The quartic coupling:

$$\lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4 \quad , \quad N = \log\frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}, \; T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.

E-W phase transition: $z \sim 10^{15}, T_{\rm EW} \sim 100$ GeV, $t \sim 10^{-11}$ s.

QCD conf. transition: $z \sim 10^{12}, T_{\rm QCD} \sim 170$ MeV, $t \sim 10^{-5}$ s.

BBN: $z \sim 10^9$, $\quad T \simeq 0.1$ MeV, $t \sim 20$ sec.

Rad-Mat equality: $z \simeq 3050, \; T \simeq 0.7$ eV, $t \sim 57000$ yr.

CMB last scattering: $z \simeq 1100, \; T \simeq 0.25$ eV , $t \sim 370000$ yr.

Mat-DE equality: $z \simeq 0.47, \; T \simeq 0.345$ meV , $t \sim 8.9$ Gyr.

Today: $z = 0, \; T = 2.725$K $= 0.2348$ meV $t = 13.72$ Gyr.

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structure at $\sim 2-3$ kpc. DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.
Consider particles that decouple UR at or out of LTE (LTE = local thermal equilibrium).
Distribution function: $f_d[a(t)\, P_f(t)] = f_d[p_c]$.
$P_f(t) = p_c/a(t) =$ Physical momentum.
$p_c =$ comoving momentum.
DM decoupling at LTE: $f_d(p_c) = 1/[\exp[\sqrt{m^2+p_c^2}/T_d] \pm 1]$
In general (out of equilibrium): $f_d(p_c) = f_d\left(\frac{p_c}{T_d}; \frac{m}{T_d}; \ldots\right)$
Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t)\rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} f_d[a(t)\, P_f]}{\int \frac{d^3 P_f}{(2\pi)^3} f_d[a(t)\, P_f]} = \left[\frac{T_d}{m\; a(t)}\right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy} .$$

Velocity Dispersion of Dark Matter particles

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_\gamma$,

$g_d =$ effective # of UR degrees of freedom at decoupling,

$$\sqrt{\langle \vec{V}^2 \rangle}(z) = 0.08875 \, \frac{1+z}{g_d^{\frac{1}{3}}} \, \frac{\text{keV}}{m} \left[\frac{\int_0^\infty y^4 \, f_d(y) \, dy}{\int_0^\infty y^2 \, f_d(y) \, dy} \right]^{\frac{1}{2}} \frac{\text{km}}{\text{s}}$$

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \, f_d[a(t) \, P_f]$

$\rho_{DM}(t) = m \, g \, [T_d^3 / a^3(t)] \int_0^\infty y^2 \, f_d(y) \, \frac{dy}{2\pi^2}$ for $m \gg T_d/a(t)$.

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and therefore:

$$m = 6.46 \text{ eV } g_d / [g \int_0^\infty y^2 \, f_d(y) \, dy]$$

For Fermions decoupling at LTE:

$f_d(y) = 1/[e^y + 1]$ and $m = 3.593 \text{ eV } g_d/g$.

The formula for m

m increases:

a) if the DM particle decouples earlier because g_d increases.
b) if it decouples out of LTE, $f_d(y)$ can favour small momenta and increase $1/[\int_0^\infty y^2 \; f_d(y) \; dy]$.

Special Cases of the formula for m :

Particles decoupling non-relativistically $\Longrightarrow$ Lee-Weinberg (1977) lower bound.

Particles decoupling ultrarelativistically $\Longrightarrow$ Cowsik-McClelland (1972) upper bound.

Phase-space density invariant under universe expansion

$$\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}^2_{phys}(t)\rangle^{\frac{3}{2}}} \stackrel{\text{non}-\text{rel}}{=} \frac{\rho_{DM}}{m^4\, \sigma^3_{DM}} \quad , \quad \sigma_{DM} \equiv \sqrt{\langle \vec{V}^{\,2}\rangle} =$$

computed theoretically from equilibrium distributions.

$\rho_{DM} = 1.107 \times \text{keV}/\text{cm}^3 =$ observed value today.

$$\frac{\rho_{DM}}{\sigma^3_{DM}} \sim 10^3 \; \frac{\text{keV}/\text{cm}^3}{(\text{km/s})^3} \; \left(\frac{m}{\text{keV}}\right)^3 \; g_d \begin{cases} 0.177 & \text{Fermions} \\ 0.247 & \text{Bosons} \end{cases} .$$

$g_d = \#$ of UR degrees of freedom at decoupling.

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields: $\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \; \frac{\text{keV}/\text{cm}^3}{(\text{km/s})^3}$ Gilmore et al. 07.

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

N-body simulations results: $\frac{\rho_s}{\sigma_s^3} \sim 10^{-2} \; \frac{\rho_{DM}}{\sigma^3_{DM}}$.

Mass Estimates of DM particles

Collecting all formulas yields for relics decoupling at LTE:

$$m \sim \frac{2}{g^{\frac{1}{4}}} \text{ keV} \quad , \qquad g_d \geq 500 \, g^{\frac{3}{4}} \quad ,$$

Hence, $T_d > 100$ GeV. $[g = 1 - 4]$.

g_d can be smaller for relics decoupling out of LTE

Let us consider now WIMPS (weakly interactive massive particles): $m \sim 100$ GeV, $T_d \sim 10$ MeV. We find:

$$\frac{\rho_{wimp}}{\sigma^3_{wimp}} \sim 10^{21} \, \frac{\text{keV/cm}^3}{(\text{km/s})^3} \left(\frac{\sqrt{m \, T_d}}{1 \text{ GeV}} \right)^3 g_d \; .$$

Eighteen orders of magnitude larger than the observations in dShps.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark !
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \;\; , \;\; 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Little Bang vs. Big Bang

Similarities: baryon free, entropy is dominated by radiation, longitudinal expansion in RHIC and LHC similar to the Hubble expansion.

Differences: Cosmology:
Local Thermal Equilibrium: $1/H \sim 10^{-5}\,\mathrm{s} \gg t_{QCD} \sim 10^{-23}$ s,
Starting energy density $\gg$ QCD phase transition energy density ~ 1 GeV/fm^3 $\Longrightarrow$
weakly interacting QGP **was initially** present due to asymptotic freedom.

URHIC: expansion time scale $\sim 10^{-22}\,\mathrm{sec} \sim 10\; t_{QCD} \Longrightarrow$
non-equilibrium effects can be relevant.
Strongly interacting *liquid* initially present (‘color glass condensate’).

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$.
  Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$.
  Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 60$.
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016 \; (95\% \text{ CL}) \; , \; r > 0.049 \; (68\% \text{ CL})$.
  The most probable values are $r \simeq 0.055 (\Leftarrow$ measurable !!) $n_s \simeq 0.956$ with a quartic coupling $y \simeq 1.3$.

Summary and Conclusions 2

- The quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited 0.1 efolds before the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy formation. Gigantic black-holes ($M \sim 10^9 \ M_\odot$) as galaxy nuclei, early star formation...

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim 2$ keV.

Sterile neutrinos? Some unknown light particle ??

Need to learn about the physics of light particles (< 1 MeV).

THANK YOU VERY MUCH

FOR YOUR ATTENTION!!

Out of equilibrium Decoupling

Thermalization mechanism: k-modes cascade towards the UV till the thermal distribution is attained.
[D. Boyanovsky, C. Destri, H. J. de Vega, PRD69, 045003 (2004). C. Destri, H. J. de Vega, PRD73, 025014 (2006)]

Hence, before LTE is reached: lower momenta are more populated than at LTE.
An approximate description:

$$f_d(y) = f_{equil}(y/\xi)\ \theta(y_0 - y), \quad \xi < 1 \text{ out of equilibrium}$$

Modes with $p_c > y_0\, T_d$ are empty. [$y = p_c/T_d$].

For fermions: $m = 6.46 \text{ eV } (g_d/g)\ F(\infty)/[\xi^3\ F(y_0/\xi)]$

$F(s) \equiv \int_0^s f_{equil}(w)\ w^2\ dw \quad , \quad F(\infty)/[\xi^3\ F(y_0/\xi)] > 1.$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y} \, \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is small field inflation.

From the above integral: $\quad y = z - 1 - \log z$

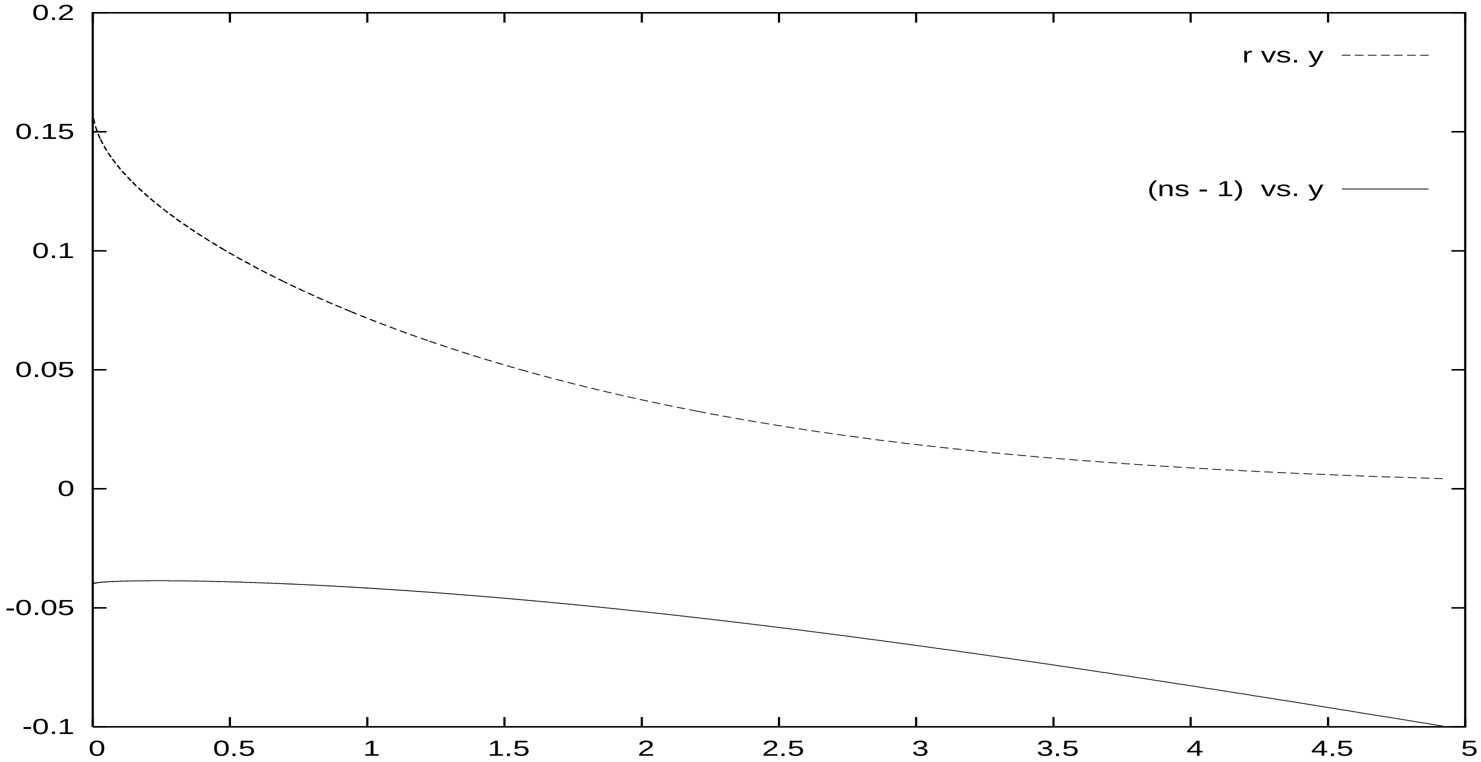
where $z \equiv y \, \chi^2/8$ and we have $0 < y < \infty$ for $1 > z > 0$.

Spectral index n_s and the ratio r as functions of y:

$$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\,y}{N} \frac{z}{(z-1)^2}$$

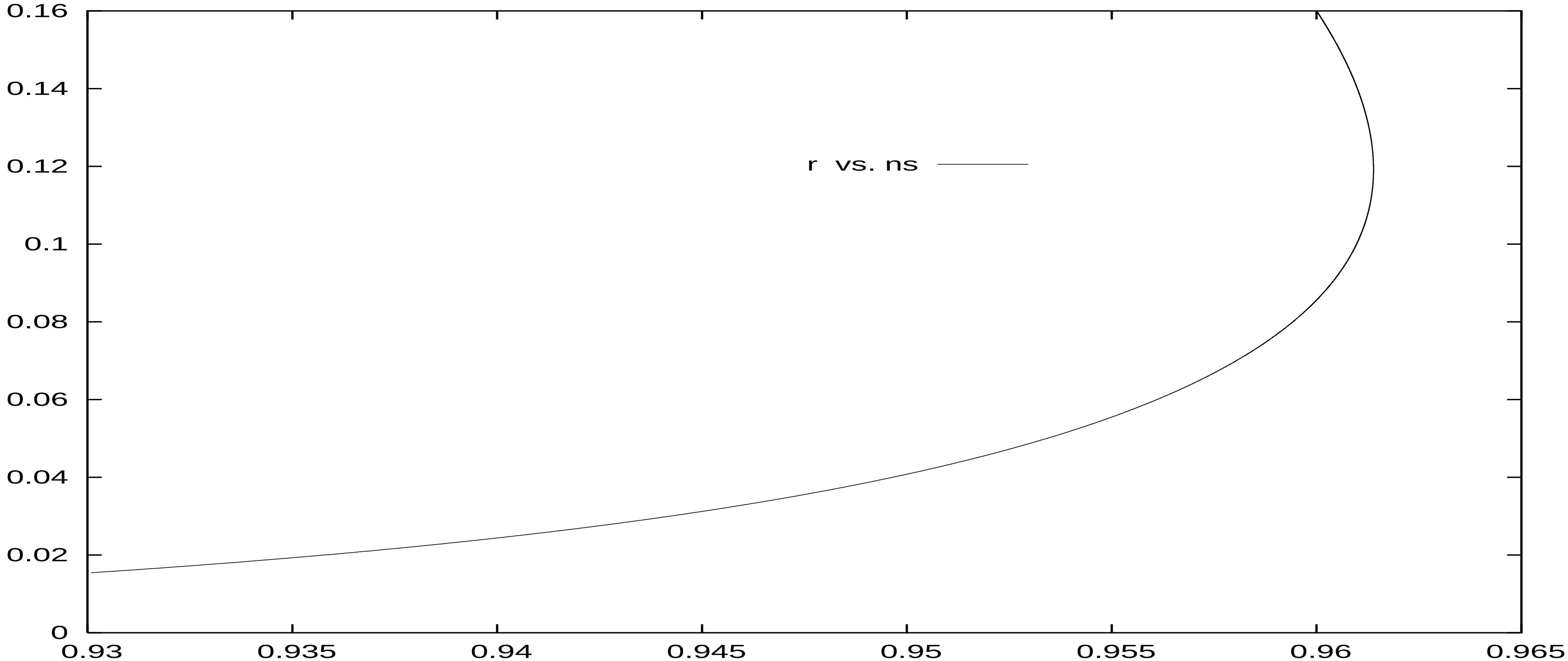
Binomial New Inflation: (y = coupling).

r decreases monotonically with y :
(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y.

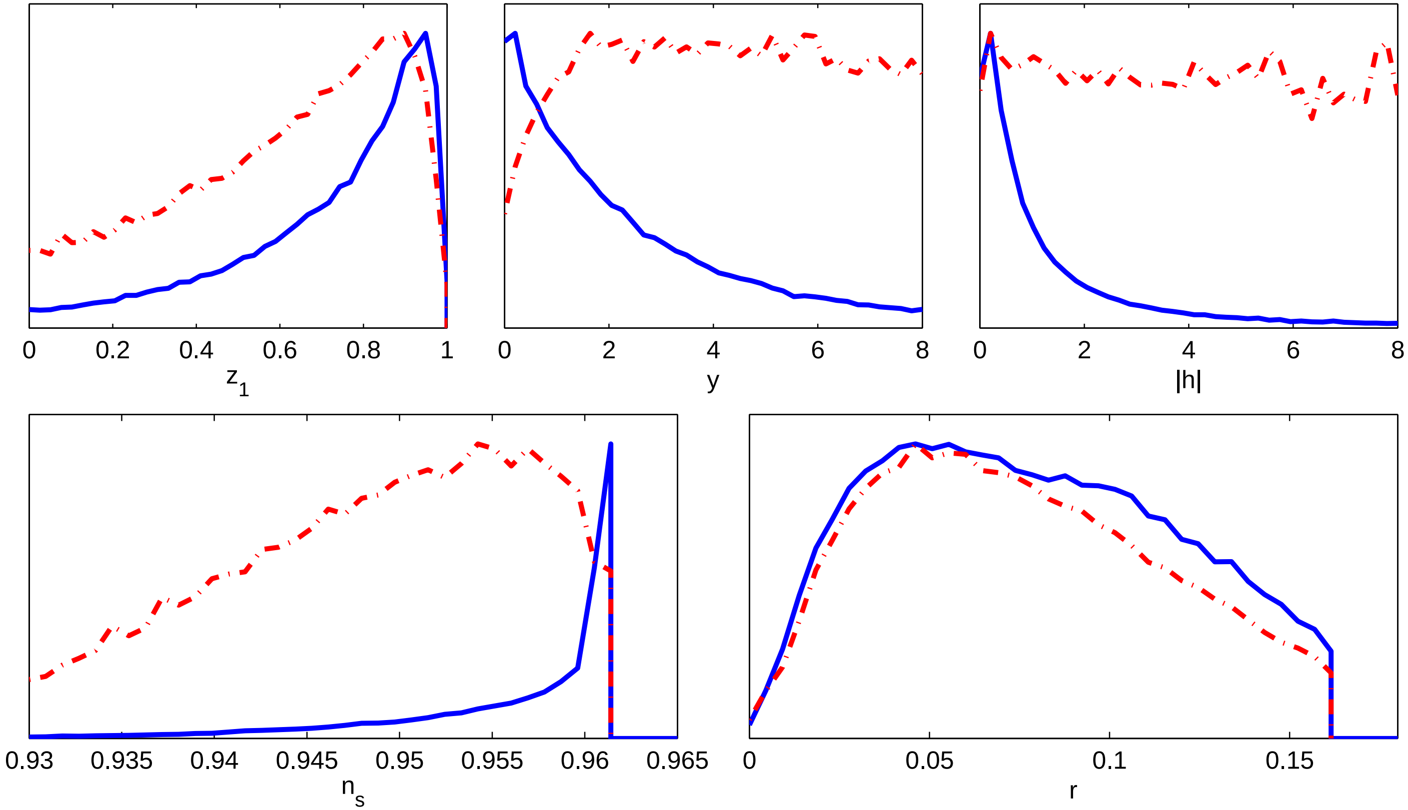
Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Probability Distributions. Trinomial New Inflation.

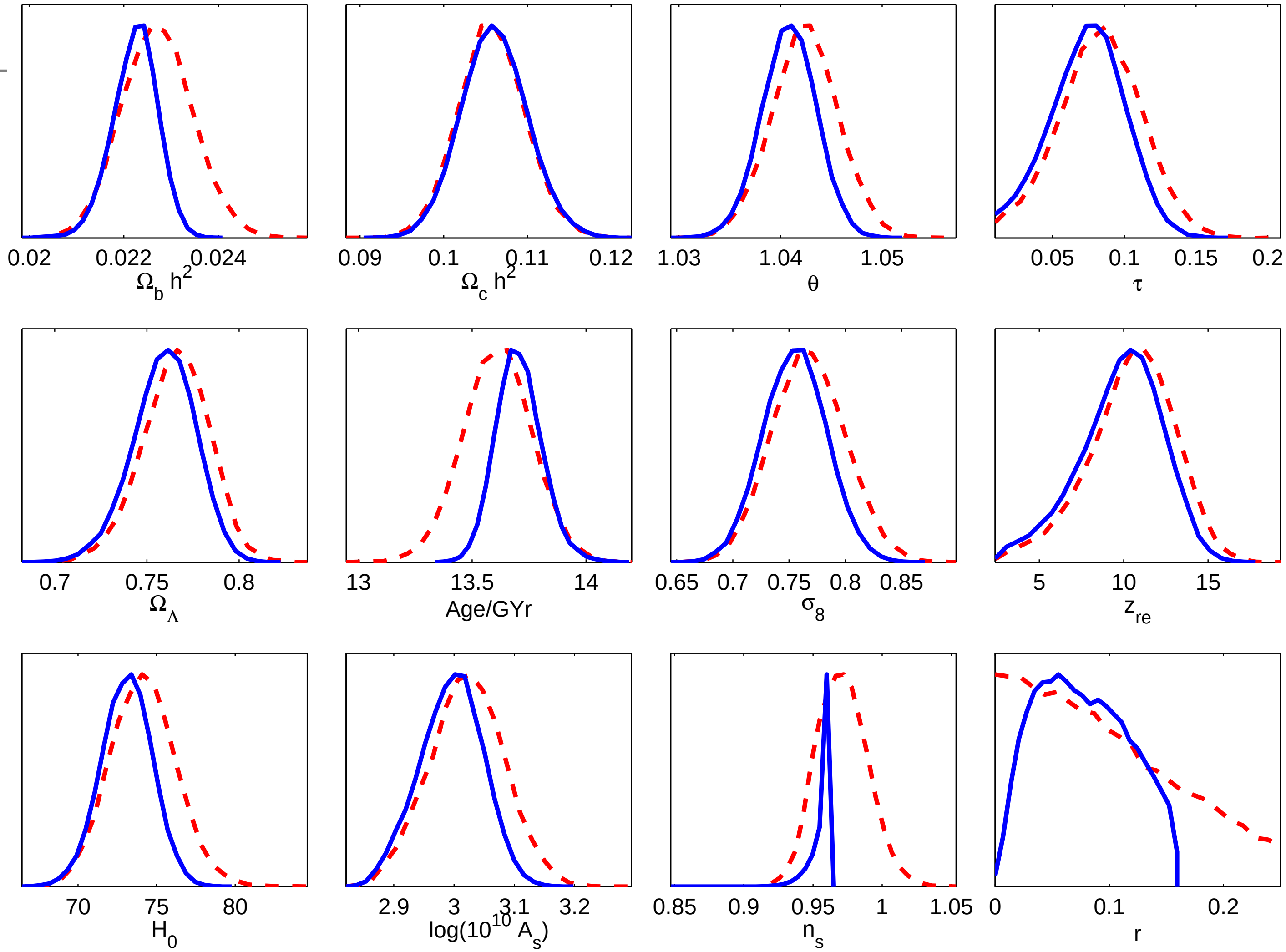


Probability distributions: solid blue curves

Mean likelihoods: dot-dashed red curves.

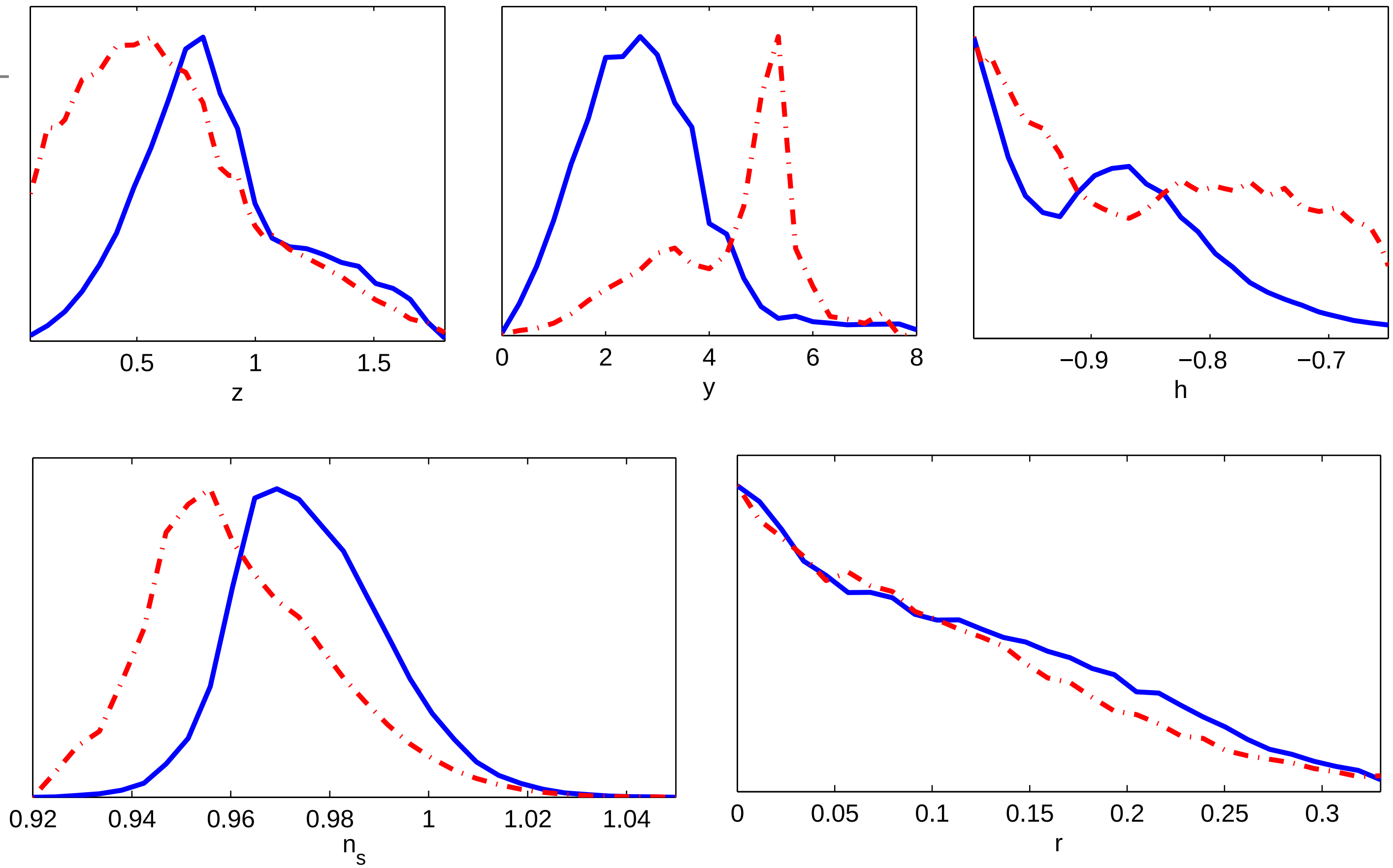
$$z_1 = 1 - \frac{y}{8\left(|h|+\sqrt{h^2+1}\right)^2}\ \chi^2 \ .$$

Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

Probability Distributions. Trinomial Chaotic Inflation.



Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).
The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

The QCD Phase Transition

Hubble scale then: $1/H \sim 10^{-5}$ sec
$\gg t_{QCD} \sim 1/[\alpha_s^2 \, T_{\rm QCD}] \sim 10^{-23}$ sec $\Longrightarrow$ Very fast!.
Hence, the transition happens in thermal equilibrium.

Hubble radius then $c/H \sim 10$ km. $\Longrightarrow$ 1 pc today.

Probably a first order transition: Bubbles of the confined phase appear in the quark-gluon plasma $\Longrightarrow$ hadronization.

Supercooling very short $\sim 10^{-3} \; 1/H$.
Bubble separation $1 \, {\rm cm} \sim 10^{-6} \; c/H$.
Bubbles grow slowly and eventually fill the whole space.
Latent heat of the transition reheats the universe.

BBN happens at $200 \, {\rm secs} \gg 10^{-5}$ sec: signatures ERASED.

D. Boyanovsky, H. J. de Vega, D. J. Schwarz, Ann. Rev. Nucl. Part. Sci. 56, 441(2006), hep-ph/0602002.

Primordial Magnetic Fields

Astrophysical observations show the presence of large scale magnetic fields $\sim \mu G$ correlated on scales up to ~ 1 Mpc (cluster of galaxies).

Origin?: Dynamo mechanisms amplify seed magnetic fields. Typical growth rates $\Gamma \sim$ Gyr^{-1} over time scales $\sim 10 - 12$ Gyr

Origin of Seeds?: Inflation and/or phase transitions.

If the electroweak and/or the chiral phase transitions occurred out of equilibrium they can be a significant source of primordial magnetic fields

Cosmic magnetic fields may be one of the few observational relics of primordial phase transitions.

D. Boyanovsky, H. J. de Vega, M. Simionato ‘Large scale magnetogenesis from a non-equilibrium phase transition in the radiation dominated era’, Phys.Rev.D67,123505(2003).

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x},t) = \Phi_0(t)+\varphi(\vec{x},t), \quad \Phi_0(t) \equiv < \phi(\vec{x},t) >, \quad < \varphi(\vec{x},t) > = 0$$

$$\varphi(\vec{x},t) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\,\pi)^3} \left[a_{\vec{k}}\; \chi_k(\eta)\; e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}\right],$$

$a^\dagger_{\vec{k}},\ a_{\vec{k}}$ are creation/annihilation operators,
$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.
To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3\,H\;\dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0)\;\langle[\varphi(\boldsymbol{x},t)]^2\rangle = 0$$

where $g(\Phi_0) = \frac{1}{2}\,V^{'''}(\Phi_0)$.
The mode functions obey:

$$\chi^{''}_k(\eta) + \left[k^2 + M^2(\Phi_0)\;a^2(\eta) - \frac{a^{''}(\eta)}{a(\eta)}\right]\chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3\;H^2_0\;\eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0\,\eta\,(1-\epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2}\sqrt{-\pi\eta}\; i^{\nu+\frac{1}{2}}\; H_\nu^{(1)}(-k\eta), \quad H_\nu^{(1)}(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle[\varphi(\boldsymbol{x},t)]^2\rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3}\, |\chi_k(\eta)|^2$

$$\frac{1}{2}\langle[\varphi(\boldsymbol{x},t)]^2\rangle = \left(\frac{H_0}{4\,\pi}\right)^2 \left[{\Lambda_p}^2 + \ln\Lambda_p^2 + \frac{1}{\Delta} + 2\,\gamma - 4 + \mathcal{O}(\Delta)\right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\quad \frac{1}{\Delta} =$ infrared pole.

$\langle\dot{\varphi}^2\rangle \quad , \quad \left\langle(\nabla\varphi)^2\right\rangle$ are infrared finite

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi}_0^{\;2} + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R^{''}(\Phi_0)]^2}{64\,\pi^2}\ln\frac{V_R^{''}(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4} \, g_{\mu\nu} \; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle.
Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3\,(4\pi)^2 \, M_{Pl}^2} \left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \; \mathcal{T}_t = -\frac{717}{5}, \; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_{k}|^2 \left\{1+ \right.$$

$$\left. +\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\right\}$$

$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_{k}|^2 \left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_{k}|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_{k}|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.