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Open Heavy-Flavour Production in Hadron and Photon Collisions

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1 Theory of inclusive single-hadron production

(a) QCD-improved parton model

QCD Lagrangian,

$$\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= -\frac{1}{4}F_{\mu\nu}^{(a)}F^{(a)\mu\nu} + \sum_q \bar{q}(i\not{D} - m_q)q, \\
F_{\mu\nu}^{(a)} &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f_{abc} A_\mu^b A_\nu^c, \\
D_\mu &= \partial_\mu - \frac{i}{2} g_s \lambda^a A_\mu^a,
\end{aligned}$$

contains quarks q and gluons A_μ^a as elementary fields. Asymptotic states (particle beams, targets and produced particles) correspond to hadrons. QCD-improved parton model reconciles QCD with confinement of colour.

- Initial state: Parton density functions (PDF's) $F_a^A(x, M^2)$, where $p_a = x p_A$ and M is factorization scale; nonperturbative input.
- Hard scattering: Partonic cross sections; amenable to perturbative QCD.
- Final state:
  - Inclusive jet production: Partons are clustered according to jet algorithm (parton-hadron duality).
  - Inclusive particle production: FF's $D_c^h(x, M^2)$, where $p_c = p_h/x$ and M is factorization scale; nonperturbative

input if $m_h \lesssim \Lambda_{\mathrm{QCD}}$, calculable in NRQCD if $m_h \gg \Lambda_{\mathrm{QCD}}$.

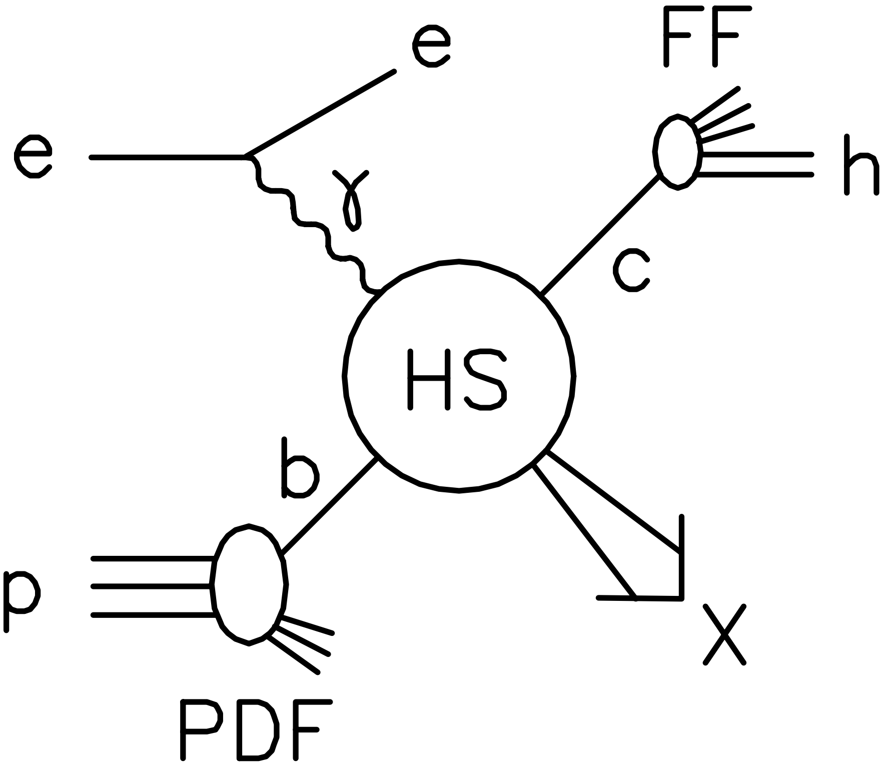


Figure 1: Inclusive photoproduction of single hadrons.

Invariant cross section:

$$\frac{d^3\sigma}{dy\,d^2p_T}(\gamma p \to h + X) \propto \sum_{b,c} \int dx_p \frac{dx_h}{x_h^2} F_b^p(x_p, M_p^2) \times D_c^h(x_h, M_h^2) \frac{d\sigma_{\gamma b \to c}}{dt}(\mu^2, M_p^2, M_h^2),$$

where p_T is transverse momentum and y is rapidity of h.

Photoproduction

Incoming electrons (or positrons) act as source of quasi-real, high-energetic photons. Flux of transverse photons with energy fraction x and virtuality Q^2 is

$$f_\gamma^e(x,Q^2)=\frac{\alpha}{2\pi Q^2}\left[\frac{1+(1-x)^2}{x}-\frac{2m_e^2x}{Q^2}\right].$$

Deep inelastic scattering is greatly suppressed.
Weizsäcker-Williams approximation:

$$F_\gamma^e(x,Q^2_{\mathrm{max}})=\int_{Q^2_{\mathrm{min}}}^{Q^2_{\mathrm{max}}}dQ^2 f_\gamma^e(x,Q^2)$$

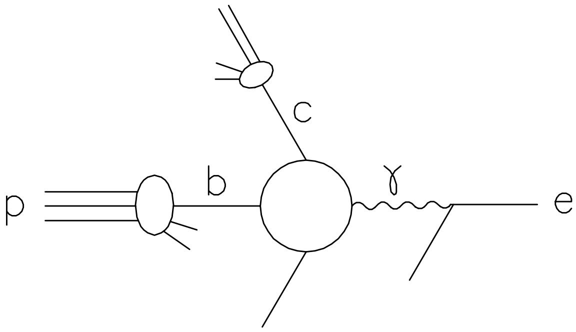
$$=\frac{\alpha}{2\pi}\left[\frac{1+(1-x)^2}{x}\ln\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}}+2m_e^2x\left(\frac{1}{Q^2_{\mathrm{max}}}-\frac{1}{Q^2_{\mathrm{min}}}\right)\right],$$

where $Q^2_{\mathrm{min}}=m_e^2x^2/(1-x)$.

- Tagged events: $Q^2_{\mathrm{max}}\approx 0.01$ GeV^2.
- Untagged events: $Q^2_{\mathrm{max}}\approx 4$ GeV^2.

Longitudinal flux $\lesssim$ 2% of the transverse flux.

Direct photoproduction

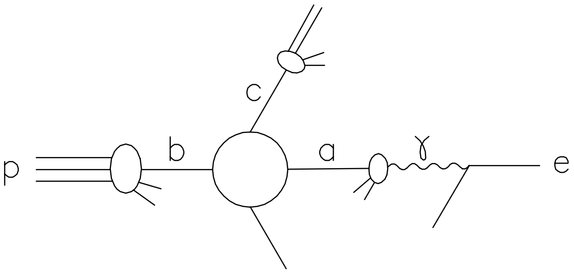


$$\frac{d\sigma}{dt}(\gamma b \rightarrow c + X)$$

$$\mathcal{O}(\alpha\alpha_s)$$

Mostly backward events with large p_T.

Resolved photoproduction



$$F_a^\gamma \times \frac{d\sigma}{dt}(ab \rightarrow c + X)$$

$$F_a^\gamma \propto \frac{\alpha}{2\pi} \ln \frac{M_\gamma^2}{\Lambda_{\mathrm{QCD}}^2}$$

$$\mathcal{O}(\alpha/\alpha_s) \times \mathcal{O}(\alpha_s^2)$$

Mostly forward events with small p_T.

NLO formalism (massless case)

$\gamma p \to h + X$ via resolved photoproduction:

$$\frac{d^3\sigma}{dy\,d^2p_T} = \frac{1}{\pi}\sum_{a,b,c}\int dx_\gamma dx_p \frac{dx_h}{x_h^2}\,F_a^\gamma(x_\gamma, M_\gamma^2) F_b^p(x_p, M_p^2)$$
$$\times\, D_c^h(x_h, M_h^2)\left[\frac{d\sigma^0_{ab\to c}}{dt}(s,t,\mu^2)\,\delta\left(1+\frac{t+u}{s}\right)\right.$$
$$\left.+\,\frac{\alpha_s(\mu^2)}{2\pi}K_{ab\to c}(s,t,u,\mu^2,M_\gamma^2,M_p^2,M_h^2)\,\theta\left(1+\frac{t+u}{s}\right)\right],$$

where $s=(p_a+p_b)^2$, $t=(p_a-p_c)^2$ and $u=(p_b-p_c)^2$. NLO corrections[1] $K_{ab\to c}$ comprise 16 channels and their $t \leftrightarrow u$ crossed counterparts.

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[1] F. Aversa, P. Chiappetta, M. Greco and J.Ph. Guillet, Nucl. Phys. B **327** (1989) 105.

$\gamma p \longrightarrow h + X$ via direct photoproduction:

$$F_a^\gamma(x_\gamma, M_\gamma^2) \longrightarrow \delta(1 - x_\gamma),$$
$$d\sigma^0_{ab\to c} \longrightarrow d\sigma^0_{\gamma b\to c},$$
$$K_{ab\to c} \longrightarrow K_{\gamma b\to c}.$$

At LO, only Bethe-Heitler process $\gamma g \longrightarrow q\bar{q}$ and Compton process $\gamma q \longrightarrow gq$ contribute. NLO corrections[2] $K_{\gamma b\to c}$ comprise 8 channels.

Effect of NLO corrections:

- Reduction of dependence on μ, M_γ, M_p, M_h (theoretical uncertainty).
- Sizeable shift in cross section in certain regions of phase space (K factor).

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[2] P. Aurenche, R. Baier, A. Douiri, M. Fontannaz and D. Schiff, Nucl. Phys. B **286** (1987) 553.

Treatment of divergences

Ultraviolet, infrared and collinear divergences are extracted using dimensional regularization ($n = 4 - 2\epsilon$).

$\Big[\delta\sigma^0_{ab\to c}$ CT

UV div. $1/\epsilon$
renormalization
μ

$+$

$K^0_{ab\to cd}\,\delta\left(1 + \frac{t+u}{s}\right)$ virt. RC

IR div. $1/\epsilon^2$
cancellation

$+$

$K^0_{ab\to cde}\,\theta\left(1 + \frac{t+u}{s}\right)\Big]$ real RC

coll. div. $1/\epsilon$
factorization
M_γ, M_p, M_h

$\times$

$F^{\gamma 0}_a(x_\gamma) F^{p0}_b(x_p) D^{h0}_c(x_h)$ bare PDF, FF

$$
\begin{aligned}
&= K_{ab\to c}(\mu^2, M_\gamma^2, M_p^2, M_h^2) \\
&\times F^\gamma_a(x_\gamma, M_\gamma^2) F^p_b(x_p, M_p^2) D^h_c(x_h, M_h^2).
\end{aligned}
$$

Factorization

After elimination of UV and IR divergences, subtract collinear ones in the following way:

$$\begin{aligned} K^0_{ab\to c}(s,t,u) &- H^{(S)}_{a\to i}(x,M_\gamma^2)\,\frac{d\sigma^0_{ib\to c}}{dt'}(s',t')\bigg|_{p_i=xp_a} \\ &- H^{(S)}_{b\to i}(x,M_p^2)\,\frac{d\sigma^0_{ai\to c}}{dt'}(s',t')\bigg|_{p_i=xp_b} \\ &- H^{(T)}_{i\to c}(x,M_h^2)\,\frac{d\sigma^0_{ab\to i}}{dt'}(s',t')\bigg|_{p_i=p_c/x}, \end{aligned}$$

with universal factorization kernels

$$H^{(S)}_{a\to i}(x,M_\gamma^2) = -\frac{1}{\bar\epsilon}\left(\frac{\mu^2}{M_\gamma^2}\right)^{\bar\epsilon} P^{(S)}_{a\to i}(x) + \frac{\alpha_s(\mu^2)}{2\pi} f_{a\to i}(x),$$

$$H^{(T)}_{i\to c}(x,M_h^2) = -\frac{1}{\bar\epsilon}\left(\frac{\mu^2}{M_h^2}\right)^{\bar\epsilon} P^{(T)}_{i\to c}(x) + \frac{\alpha_s(\mu^2)}{2\pi} d_{i\to c}(x),$$

where $1/\bar\epsilon = 1/\epsilon + \ln(4\pi) - \gamma_E$ and $P^{(S,T)}_{a\to b}(x)$ are spacelike/timelike Altarelli-Parisi splitting functions.

Form of collinear divergence just depends on splitting $a \longrightarrow b$, but not on process $AB \longrightarrow h + X$ (factorization theorem). Thus, PDF's and FF's are universal, and QCD-improved parton model is predictive.

$f_{a\to i}(x)$ and $d_{i\to c}(x)$ define factorization scheme.

- $\overline{\mathrm{MS}}$ scheme: $f_{a\to i}(x) = d_{i\to c}(x) = 0$.
- DIS scheme: NLO structure function $F_2(x,Q^2)$ of νN DIS has same form as in naïve parton model, i.e.

$$f_{g\to q} = \frac{1}{2}\left\{\left[x^2 + (1-x)^2\right]\ln\frac{1-x}{x} + 8x(1-x) - 1\right\},$$

  etc.
- DIS$_\gamma$ scheme:

$$f_{\gamma\to q}(x) = 2N_c e_q^2 \frac{\alpha}{\alpha_s} f_{g\to q}(x).$$

We adopt $\overline{\mathrm{MS}}$ scheme.

DGLAP equations

The μ^2 evolution of $D_a^h(x,\mu^2)$ is ruled by the Altarelli-Parisi equations,

$$\frac{\mu^2 d}{d\mu^2} D_a^h(x,\mu^2) = \sum_b \int_x^1 \frac{dz}{z} P_{a\to b}^{(T)}\left(\frac{x}{z}, \alpha_s(\mu^2)\right) D_b^h(x,\mu^2),$$

with timelike $a \longrightarrow b$ splitting functions

$$\begin{aligned} P_{a\to b}^{(T)}\left(x, \alpha_s(\mu^2)\right) &= \frac{\alpha_s(\mu^2)}{2\pi} P_{a\to b}^{(0,T)}(x) \\ &+ \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 P_{a\to b}^{(1,T)}(x) + \mathcal{O}\left(\alpha_s^3\right). \end{aligned}$$

Numerical solution methods:

- Brute-force evolution in x space.
- Mellin transform technique, i.e.

$$\begin{aligned} F(n) &= \int_0^1 dx\, x^{n-1} f(x) \quad (n = 1, 2, \ldots), \\ f(x) &= \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} dn\, x^{-n} F(n), \end{aligned}$$

where γ is such that integration path is right of all singularities. Renders convolutions to products.

(b) Fragmentation functions

General strategy for determination

- Make ansatz for $D_a^h(x, \mu_0^2)$ at starting scale μ_0.
- Determine input parameters by fitting two sets of e^+e^- data at respective scales $\mu = \sqrt{s}$.
- Determine also Λ_{QCD} from scaling violation.
- Test scaling violation (Altarelli-Parisi equations) through comparison with e^+e^- data at a third scale $\mu = \sqrt{s}$.
- Test universality of FF's (factorization theorem) through comparisons with data of ep, $p\bar{p}$, $\gamma\gamma$ etc. scattering at scale $\mu = p_T$.

FF's for $D^{*\pm}$ mesons

Determine realistic $C \longrightarrow D^{*\pm}$ FF's by fitting high-s data on $e^+e^- \longrightarrow D^{*\pm} + X$.[3] Must also consider $B \longrightarrow D^{*\pm}$. Adopt $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ from charged-hadron analysis.

- Standard set (S), with 6 fit parameters ($Q = C, B$):

$$D_Q^{D^{*\pm}}(x, \mu_0^2) = N x^\alpha (1-x)^\beta .$$

- Mixed set (M), with 5 fit parameters:

$$\begin{aligned} D_C^{D^{*\pm}}(x, \mu_0^2) &= \frac{N}{x[1 - 1/x - \epsilon/(1-x)]^2}, \\ D_B^{D^{*\pm}}(x, \mu_0^2) &= N x^\alpha (1-x)^\beta . \end{aligned}$$

Table 1: χ^2_{DF} of fits to LEP1 data on $e^+e^- \longrightarrow D^{*\pm} + X$.

set	total	ALEPH			OPAL		
		sum	c	b	sum	c	b
LO S	0.81	0.73	1.29	0.65	0.48	1.02	0.82
NLO S	0.79	0.78	1.31	0.63	0.42	1.08	0.63
LO M	1.03	1.05	2.11	0.77	0.47	1.29	0.75
NLO M	0.95	1.10	2.05	0.74	0.40	1.11	0.47

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[3] J. Binnewies, B.A. Kniehl and G. Kramer, Phys. Rev. D **58** (1998) 034016.

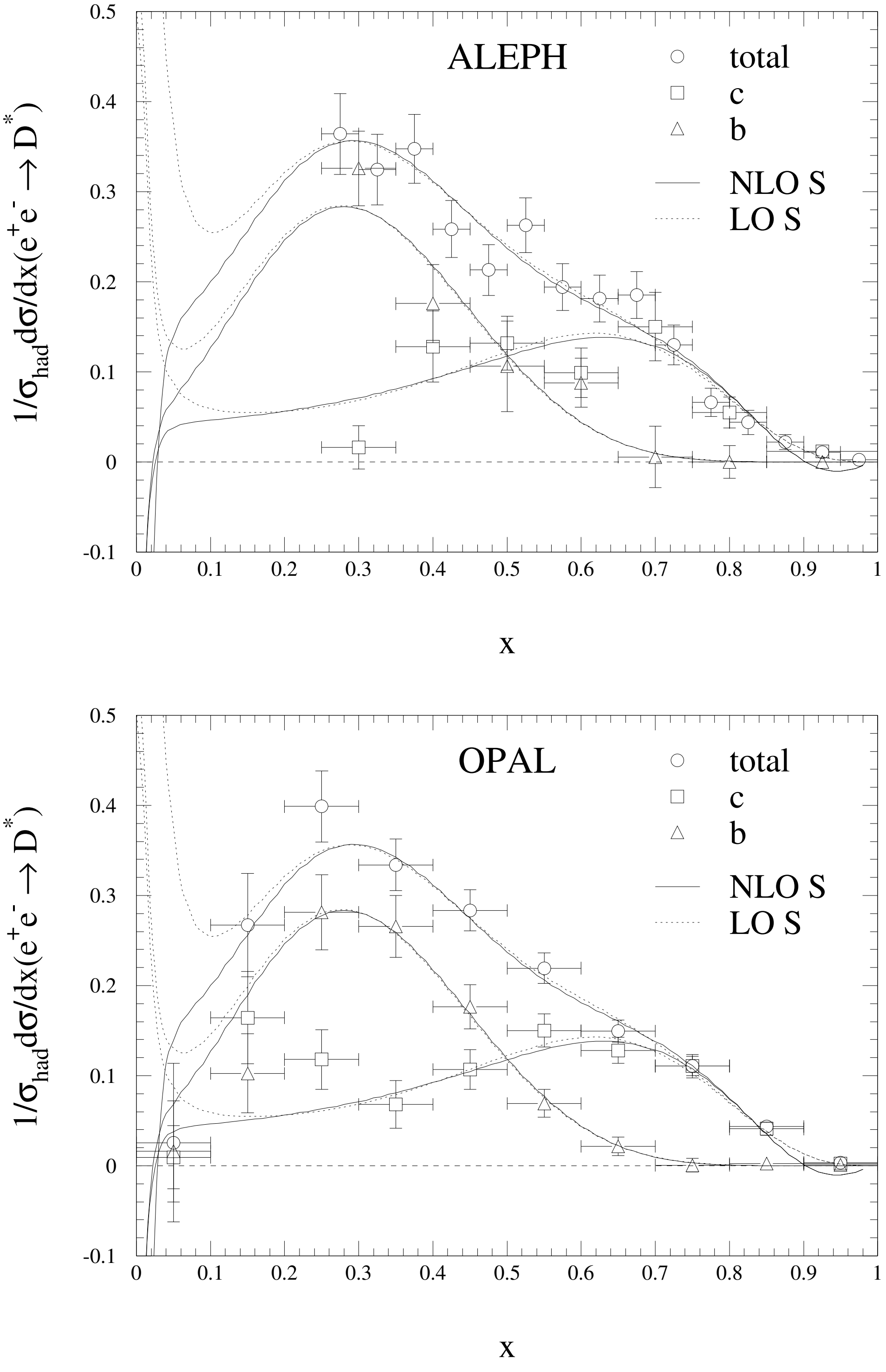

Figure 2: Comparison of the LEP1 data on $e^+e^- \to D^{*\pm} + X$ with the LO S (dashed) and NLO S (solid) calculations.

$$\epsilon = \begin{cases} 0.0926 & \text{LO M,} \\ 0.0674 & \text{NLO M.} \end{cases}$$

Cf. $\epsilon = 0.06^{-0.01}_{+0.02}(\mathrm{stat.})^{-0.01}_{+0.02}(\mathrm{syst.})$ from analysis[4] of PEP and PETRA data with Lund Monte Carlo.

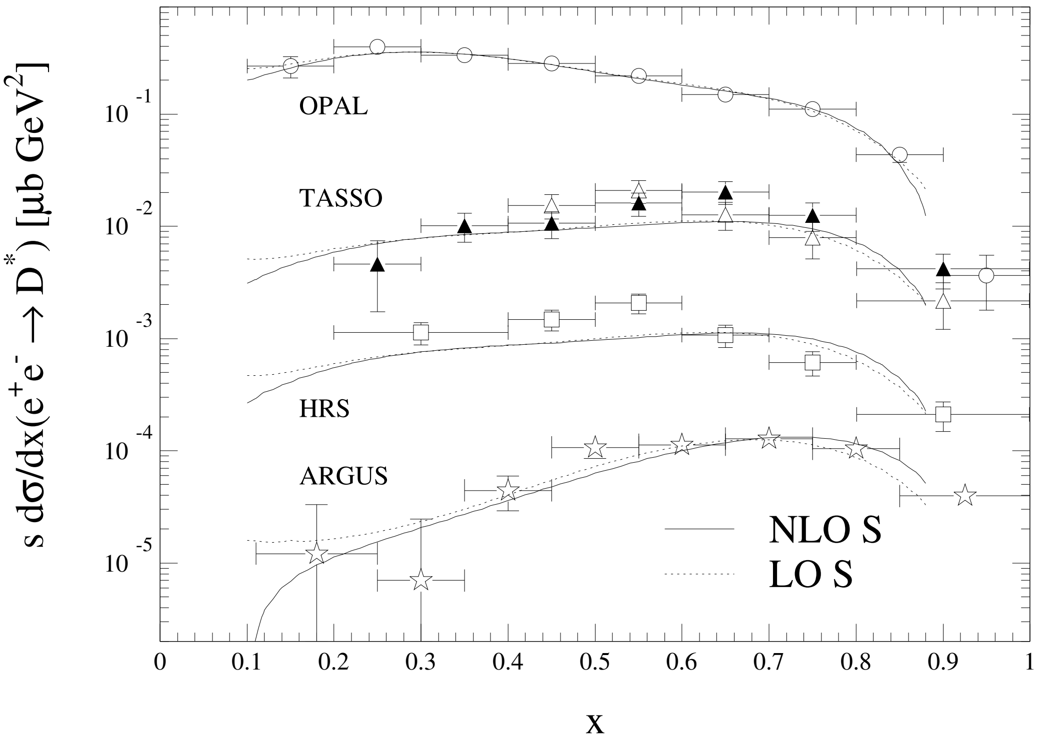

Figure 3: Comparison of the ARGUS, HRS, TASSO and OPAL data on $e^+e^- \to D^{*\pm} + X$ with the LO S (dashed) and NLO S (solid) calculations.

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[4] J. Chrin, Z. Phys. C **36** (1987) 163.

(c) Schemes

- Focus on inclusive heavy-meson production.
- Appropriate scheme depending on p_T/m_Q?
- Heavy-quark fragmentation?

Massive-Q scheme:[5] $ab \to Q\overline{Q} + X$

- $n_l = n_f - 1$ massless flavors a, b treated in $\overline{\mathrm{MS}}$ renormalization and factorization scheme, appear in PDFs.
- Q treated in OS scheme (OS mass and WFR CTs, decoupling in $\alpha_s(\mu)$ for $\mu \ll m_Q$), not *intrinsic*.
- No collinear divergences related to outgoing Q line. ⇝ No factorization. ⇝ No conceptual necessity for FFs.

⊕ Valid for $0 \leq p_T \lesssim \mathrm{few} \times m_Q$. ⇝ σ_{tot} well defined.

⊕ Appropriate for t (no fragmentation).

⊖ Breaks down for $p_T \gg m_Q$ due to would-be collinear divergences $\propto \alpha_s \ln(p_T^2/m_Q^2)$.

⊖ FFs introduced ad hoc to match D and B data. No AP evolution, no universality. ⇝ Different $\epsilon_{\mathrm{Peterson}}$ for different scales, types of experiment. ⇝ Global data analysis unfeasible.

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[5] P. Nason, S. Dawson, R.K. Ellis, Nucl. Phys. **B327** (1989) 49; **B335** (1990) 260 (E); W. Beenakker, H. Kuijf, W.L. van Neerven, J. Smith, Phys. Rev. **D40** (1989) 54; M. Drees, M. Krämer, J. Zunft, P.M. Zerwas, Phys. Lett. **B306** (1993) 371; B.A. Kniehl, M. Krämer, G. Kramer, M. Spira, Phys. Lett. **B356** (1995) 539; M. Cacciari, M. Greco, B.A. Kniehl, M. Krämer, G. Kramer, M. Spira, Nucl. Phys. **B466** (1996) 173.

n_f-flavor $\overline{\mathrm{MS}}$ scheme:[6] $ab \to c + X$ with $c \to Q$ meson

• n_f massless flavors a, b, c treated in $\overline{\mathrm{MS}}$ renormalization and factorization scheme, appear in PDFs.

• Collinear divergences related to outgoing c line factorized into nonperturbative FFs.

$\oplus$ $\alpha_s^{n+1,n} \ln^n(p_T^2/m_Q^2)$ terms resummed by AP evolution. $\rightsquigarrow$ Valid for $p_T \gtrsim \mathrm{few} \times m_Q$.

$\oplus$ Scaling violation and universality of FFs guaranteed by factorization theorem. $\rightsquigarrow$ Unique $\epsilon_{\mathrm{Peterson}}$. $\rightsquigarrow$ Global data analysis possible.

$\ominus$ $(m_Q/p_T)^n$ terms not included. $\rightsquigarrow$ Breaks down for $p_T \lesssim m_Q$. $\rightsquigarrow$ No σ_{tot}.

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[6] J. Binnewies, B.A. Kniehl, G. Kramer, Phys. Rev. **D58** (1998) 014014; 034016.

Perturbative FFs:[7] $ab \longrightarrow c + X$ with $c \longrightarrow Q$

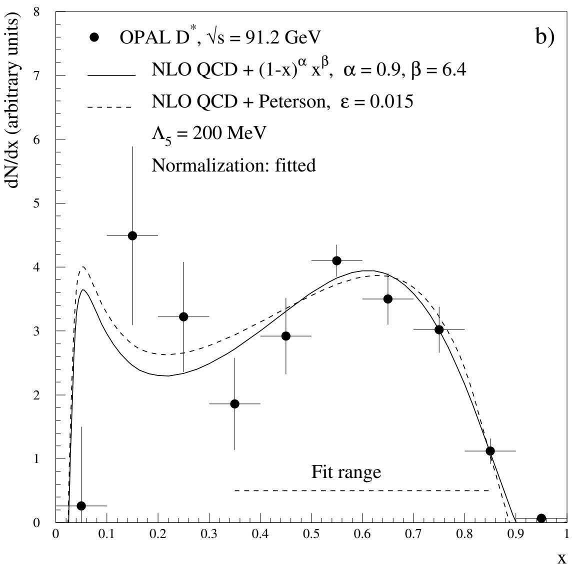
- Match

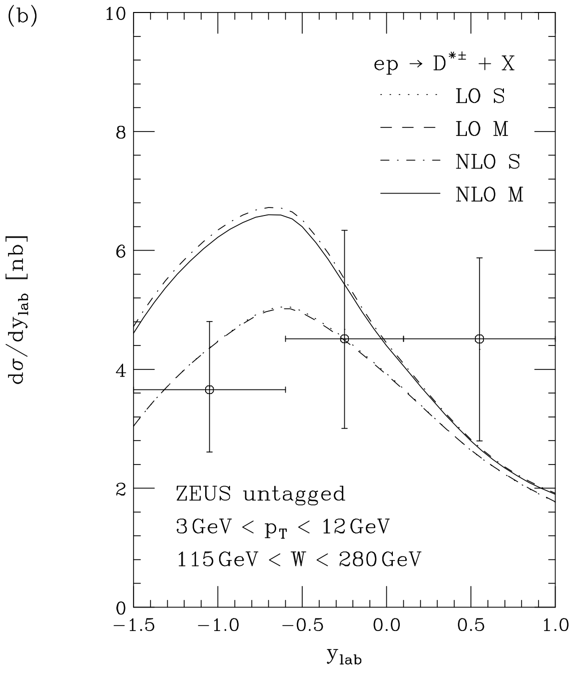
$$\frac{d\sigma_{e^+e^-\to Q\overline{Q}+X}}{dx}(x, s, m_Q^2) = \sum_c \int_x^1 \frac{dz}{z} D_c^Q\left(\frac{x}{z}, \mu^2, m_Q^2\right) \times \frac{d\sigma_{e^+e^-\to c+X}}{dz}(z, s, \mu^2).$$

- Can incorporate PFFs by change of scheme.[8]
- Still need nonperturbative FFs to match data.

⊖ $d\sigma_{e^+e^-\to Q\overline{Q}+X}/dx < 0$ for $x \gtrsim 0.9$! $\rightsquigarrow$ Low-quality fit.

⊖ Unsatisfactory perturbative stability.





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[7] B. Mele, P. Nason, Nucl. Phys. **B361** (1991) 626; J.P. Ma, Nucl. Phys. **B506** (1997) 329; J. Binnewies, B.A. Kniehl, G. Kramer, Z. Phys. **C76** (1997) 677; M. Cacciari, M. Greco, Phys. Rev. **D55** (1997) 7134.

[8] B.A. Kniehl, G. Kramer, M. Spira, Z. Phys. **C76** (1997) 689.

"FONLL":[9]

- $\mathrm{FONLL} = \mathrm{FO} + G(m_Q, p_T) \times (\mathrm{RS} - \mathrm{FOM0})$

FO: massive-Q scheme; α_s and evolution of gluon PDF in n_f-flavor $\overline{\mathrm{MS}}$ scheme; no intrinsic Q.
RS: n_f-flavor $\overline{\mathrm{MS}}$ scheme with PFFs.
FOM0: $m_Q \to 0$ limit of FO.
$G(m_Q, p_T)$: arbitrary function with $G(m_Q, p_T) \longrightarrow 1$ for $m_Q/p_T \longrightarrow 0$, e.g. $G(m_Q, p_T) = p_T^2/(p_T^2 + 25m_Q^2)$.

- RS and FOM0 evaluated at $p_T \to m_T = \sqrt{p_T^2 + m_Q^2}$.

⊖ $(\mathrm{RS} - \mathrm{FOM0})$ abnormally large.

⊖ For $p_T \lesssim \mathrm{few} \times m_Q$ problems of massive-Q scheme (non-universality of FFs).

⊖ For $p_T \gtrsim \mathrm{few} \times m_Q$ problems of PPFs (low-quality fit, unsatisfactory perturbative stability).

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[9] M. Cacciari, M. Greco, P. Nason, JHEP 05 (1998) 007; M. Cacciari, P. Nason, Phys. Rev. Lett. **89** (2002) 122003.

“NLO n_f-flavor scheme”:[10]

• Massive-Q scheme with would-be collinear singularities of form $\alpha_s \ln(p_T^2/m_Q^2)$ $\overline{\mathrm{MS}}$-subtracted.

• Residual $\alpha_s \ln(p_T^2/\mu_F^2)$ terms small for $\mu_F \approx p_T$.

• $\alpha_s \ln(\mu_F^2/m_Q^2)$ terms absorbed into PDFs and FFs, and resummed by AP evolution.

• In e^+e^- and direct $\gamma\gamma$ collisions, subtraction terms coincide with those generated by PPFs. Other processes?

$\oplus$ Naturally interpolates between massive-Q and n_f-flavor $\overline{\mathrm{MS}}$ schemes.

$\oplus$ Factorization theorem[11] in operation also for $p_T \lesssim \mathrm{few} \times m_Q$.
$\rightsquigarrow$ Universality of FFs.

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[10] G. Kramer, H. Spiesberger, Eur. Phys. J. **C22** (2001) 289; **C28** (2003) 495; B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger, hep–ph/0410289.

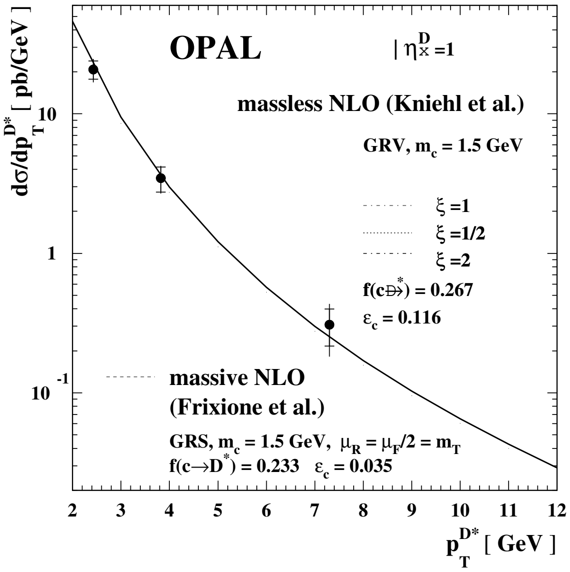
[11] J.C. Collins, Phys. Rev. **D58** (1998) 094002.

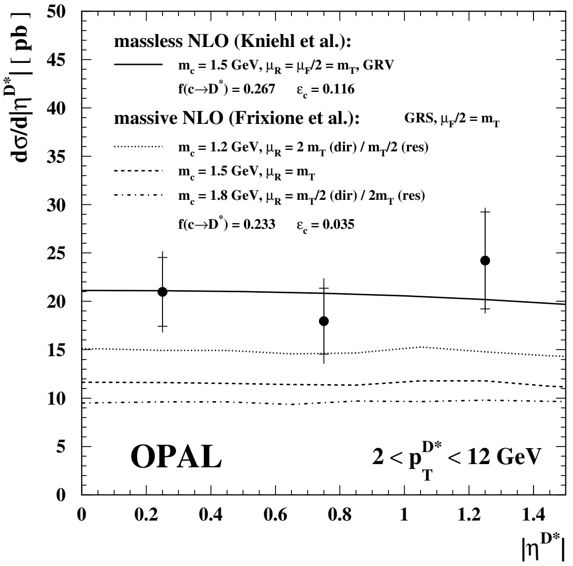
2 Phenomenology

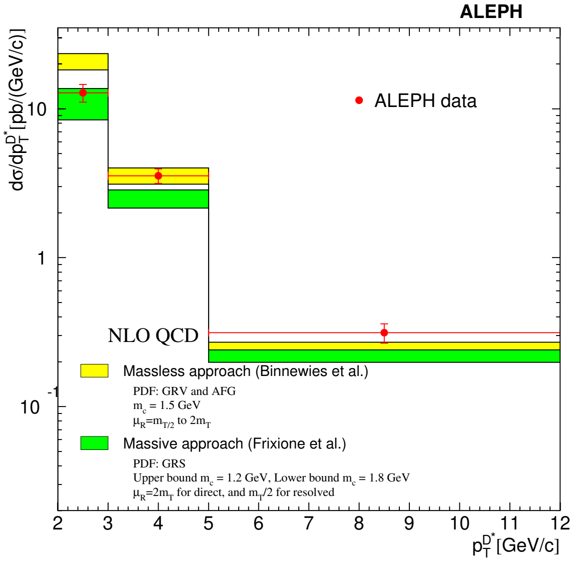
(a) $D^{*\pm}$ mesons

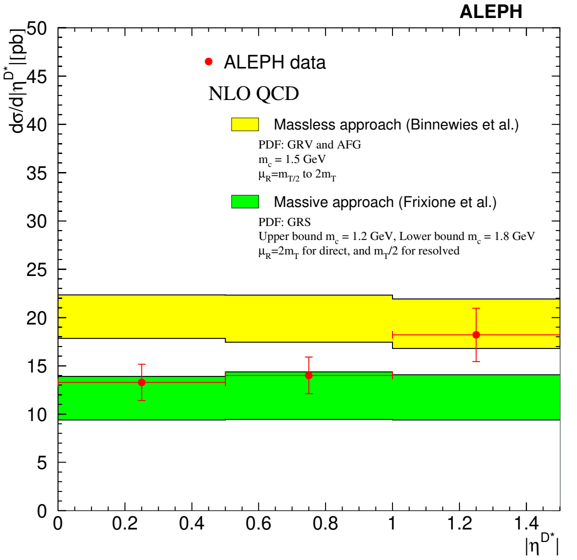
$\gamma\gamma \to D^{*\pm} + X$ at LEP2:[12]

- Massive-Q and n_f-flavor $\overline{\mathrm{MS}}$ schemes:









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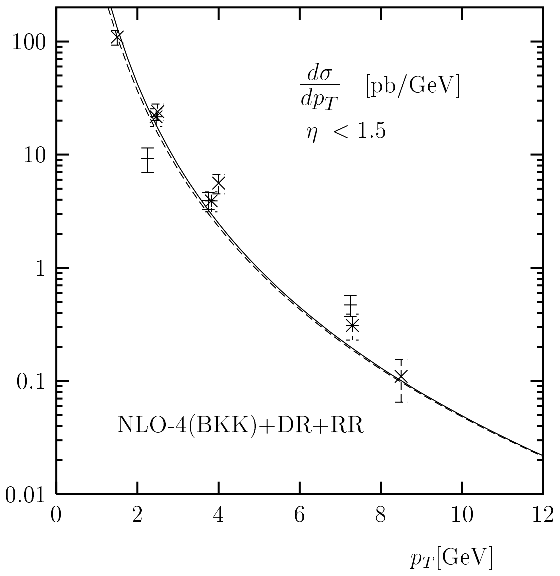
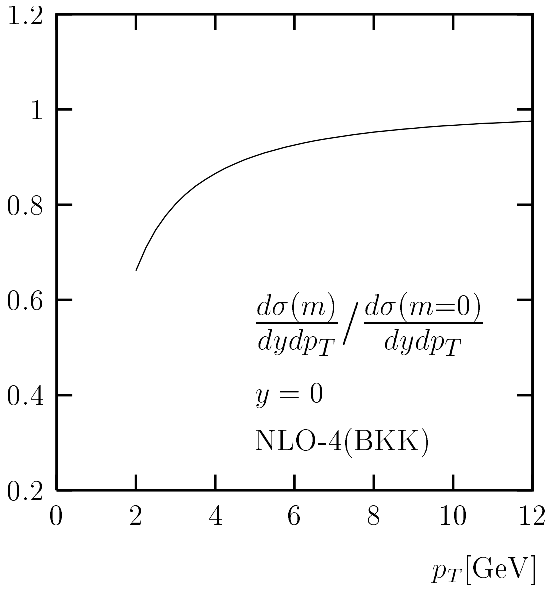
[12] OPAL Collaboration, G. Abbiendi et al., Eur. Phys. J. **C16** (2000) 579; ALEPH Collaboration, A.B. Ngac, private communication.

- NLO 4-flavor scheme:



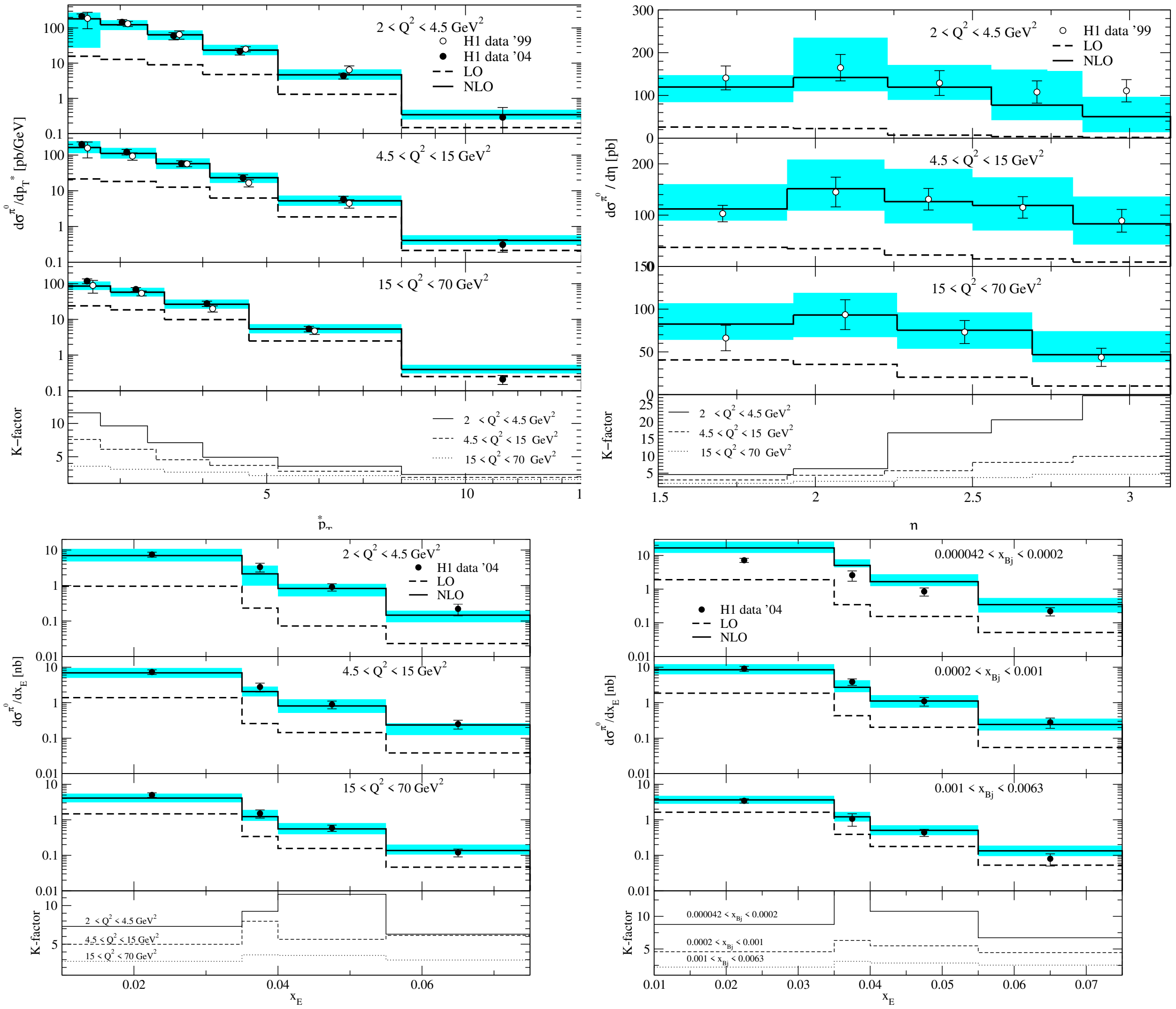
Massive (dash), massless (solid).



Ratio for direct channel.

G. Kramer, H. Spiesberger, Eur. Phys. J. **C22** (2001) 289.

$ep \to eh + X$ in DIS at HERA:[13] 5-flavor $\overline{\mathrm{MS}}$ scheme B.A. Kniehl, G. Kramer, M. Maniatis, hep-ph/0411300.



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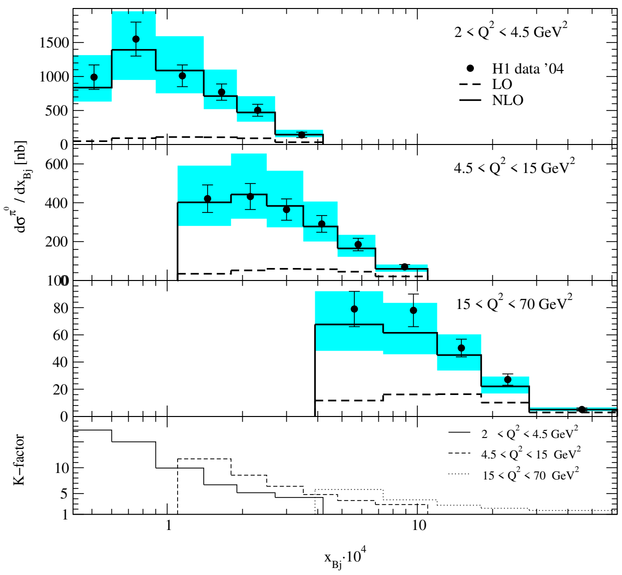
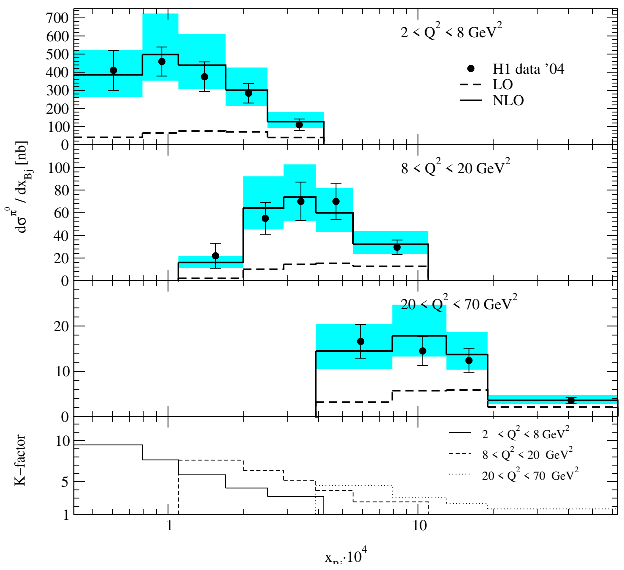
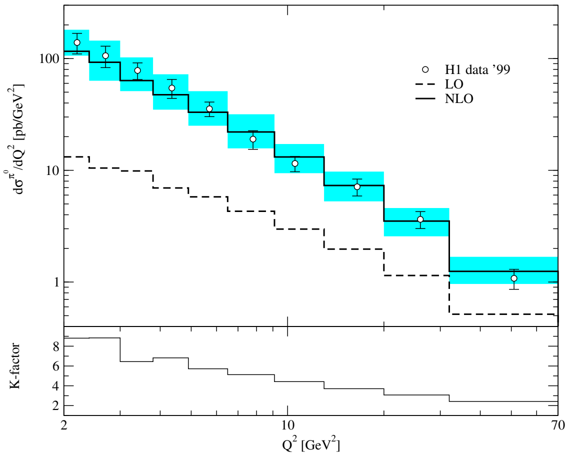
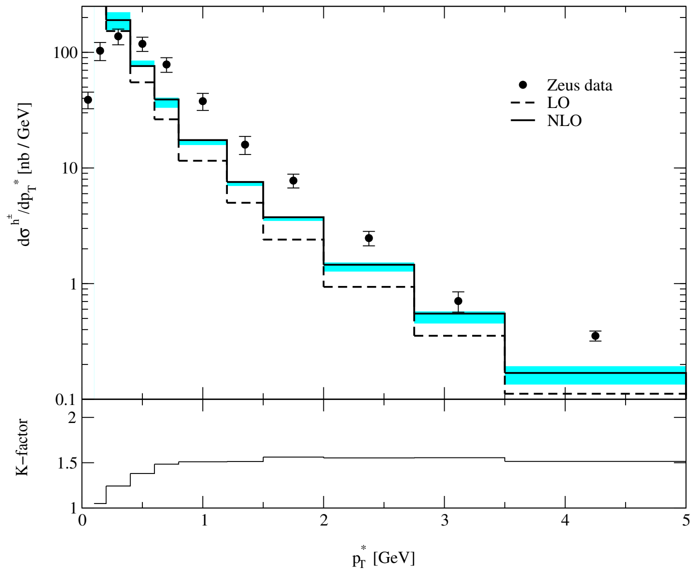
[13] H1 Collaboration, C. Adloff et al., Phys. Lett. B 462 (1999) 440; H1 Collaboration, A. Aktas et al., Eur. Phys. J. C 36 (2004) 441; ZEUS Collaboration, M. Derrick et al., Z. Phys. C 70 (1996) 1.

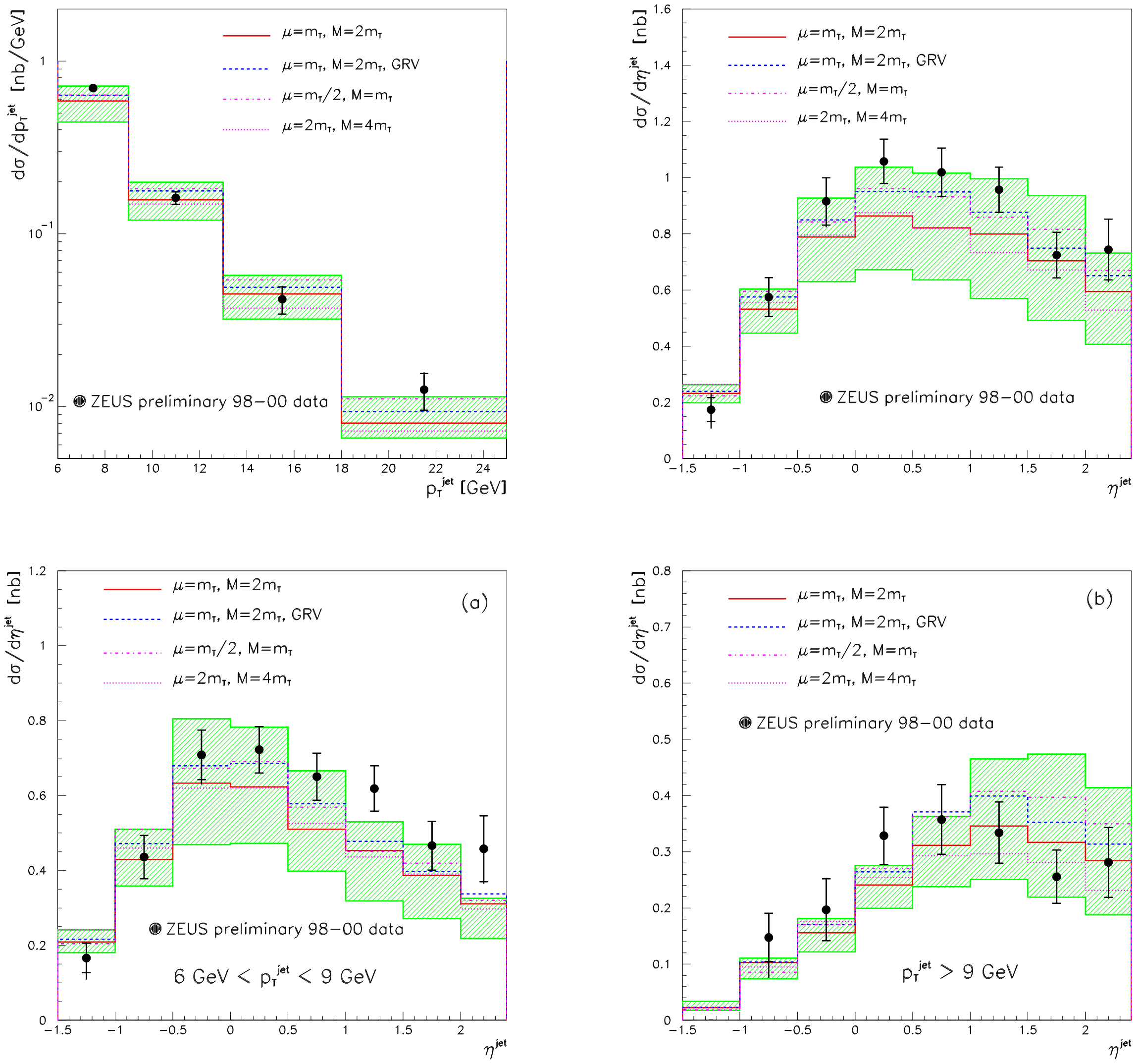

2 < Q² < 4.5 GeV²
H1 data '04
LO
NLO
4.5 < Q² < 15 GeV²
15 < Q² < 70 GeV²
dσ^π0 / dx_Bj [nb]
K–factor
2 < Q² < 4.5 GeV²
4.5 < Q² < 15 GeV²
15 < Q² < 70 GeV²
x_Bj·10⁴
1500
1000
500
0
600
400
200
100
80
60
40
20
10
5
1
10


2 < Q² < 8 GeV²
H1 data '04
LO
NLO
8 < Q² < 20 GeV²
20 < Q² < 70 GeV²
dσ^π0 / dx_Bj [nb]
K–factor
2 < Q² < 8 GeV²
8 < Q² < 20 GeV²
20 < Q² < 70 GeV²
x_Bj·10⁴
700
600
500
400
300
200
100
0
80
60
40
20
10
5
1
10


H1 data '99
LO
NLO
dσ^π0/dQ² [pb/GeV²]
K-factor
Q² [GeV²]
100
10
1
8
6
4
2
2
10
70


Zeus data
LO
NLO
dσ^h±/dp_T* [nb / GeV]
K–factor
p_T* [GeV]
100
10
1
0.1
2
1.5
1
0
1
2
3
4
5

$ep \to eD^{*\pm}j + X$ in photoproduction at HERA:[14] 5-flavor $\overline{\mathrm{MS}}$ scheme G. Heinrich, B.A. Kniehl, Phys. Rev. **D70** (2004) 094035.



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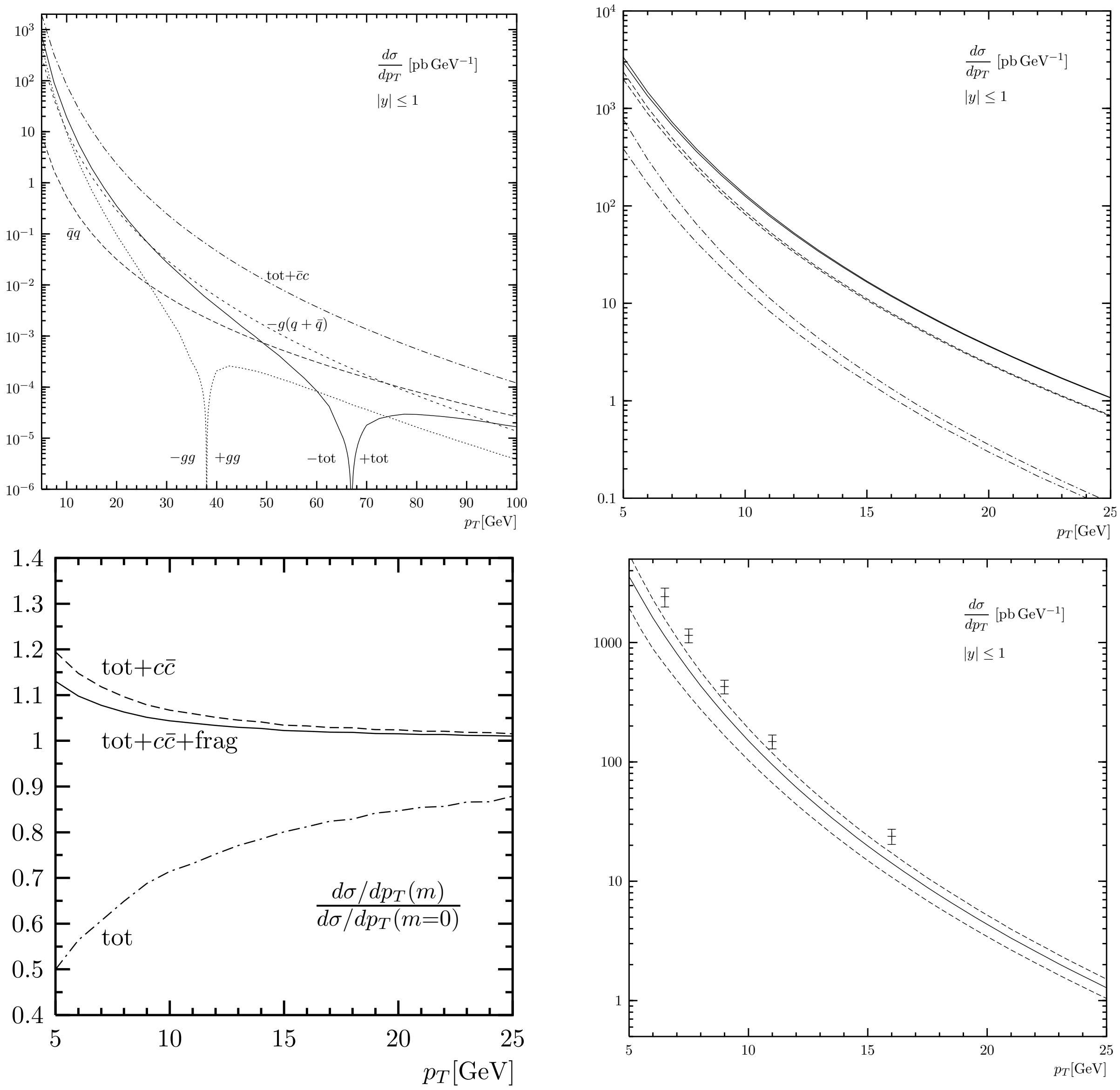
[14] ZEUS, ICHEP'04.

$p\overline{p} \to D^{*\pm} j + X$ in at the Tevatron:[15] NLO 4-flavor scheme

B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger, hep–ph/0410289, PRD (accepted).



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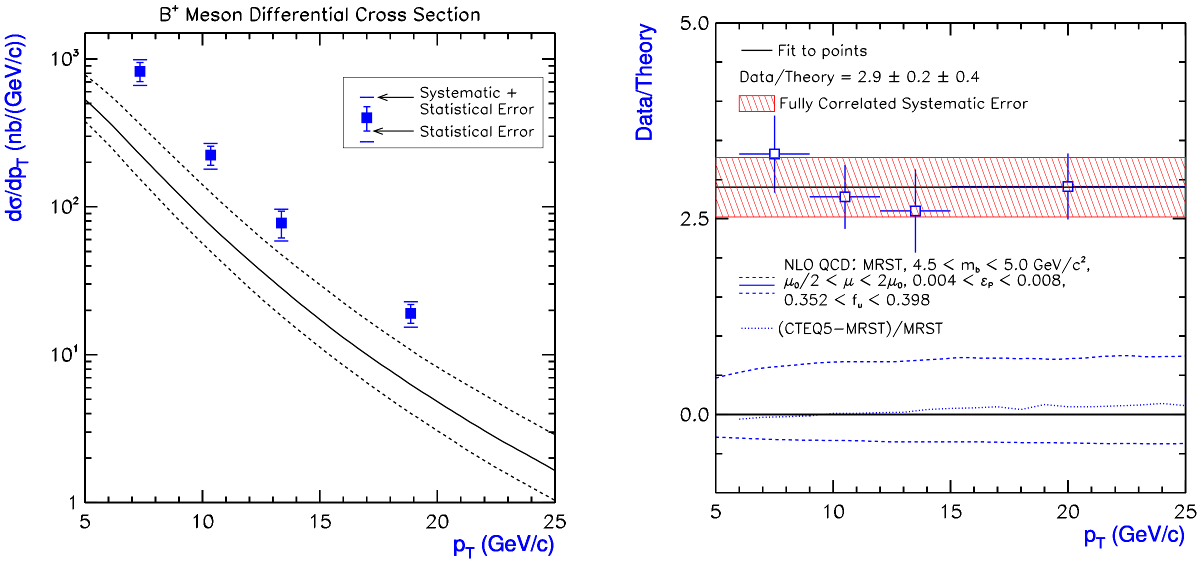
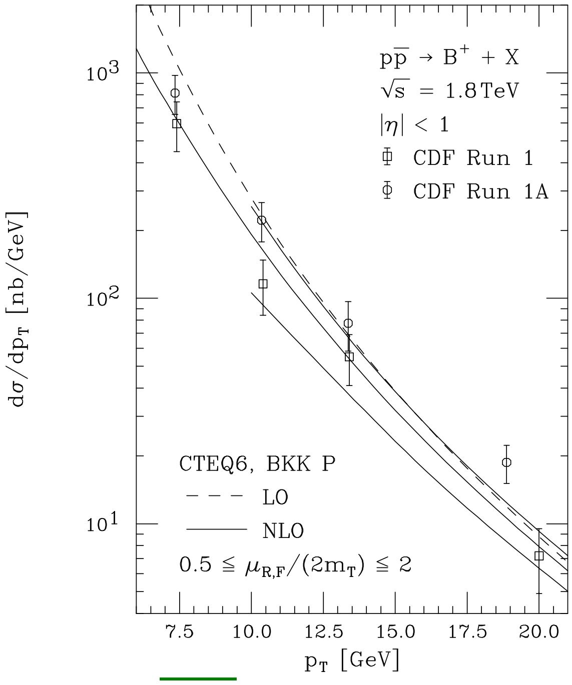
[15] CDF Collaboration, D. Acosta et al., Phys. Rev. Lett. **91** (2003) 241804.

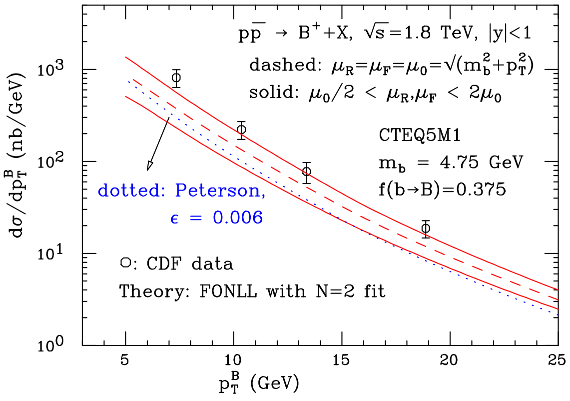
(b) B mesons

$p\bar{p} \longrightarrow B^+ + X$ at the Tevatron:[16]



Massive-Q scheme





5-flavor $\overline{\mathrm{MS}}$ scheme

FONLL, only $\langle x \rangle_{\mathrm{LEP1}}$ used

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[16] CDF Collaboration, D. Acosta et al., Phys. Rev. **D65** (2002) 052005.

3 Summary and outlook

- Massive-Q scheme (with conventional $\epsilon_{\rm Peterson}$) dramatically undershoots data of B hadro-, lepto-, and photoproduction.
- Nonperturbative FFs crucial to describe $D^{*\pm}$ and B data.
- AP evolution and universality of FFs requisite for global data analysis. Both lacking in massive-Q scheme and for $p_T \lesssim \mathrm{few} \times m_Q$ in FONLL!
- NLO n_f-flavor scheme introduces collinear factorization in massive-Q framework.
- Implementation of m_Q effects near threshold, kinematic constraints on threshold behaviour, ACOT(χ),[17]. . .

Expect exciting new data from HERA-II, Tevatron Run II, B factories, LHC, TESLA, . . . !

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[17] W.-K. Tung, S. Kretzer, C. Schmidt, J. Phys. **G28** (2002) 983.