Quantum states of neutrons

In the gravitational field of Earth

A.Yu. Voronin

P.N. Lebedev Physical Institute Moscow

Plan of the talk

- Quantum motion in the gravitational field of Earth
- Principles of observation of neutron states- how to do it better.
- Neutrons in the wave-guide with ABSORPTION

  a) Flat absorber b) Rough absorber

- Beyond gravitational states – what can we know?

Neutron quantum motion in the Earth gravitational field

- Nesvizhevsky V.V. et. al. Nature 415, 297 (2002)
- Nesvizhevsky V.V. et. al. Phys. Rev. D67 102002 (2003)

Quantum states in the gravitational field of Earth

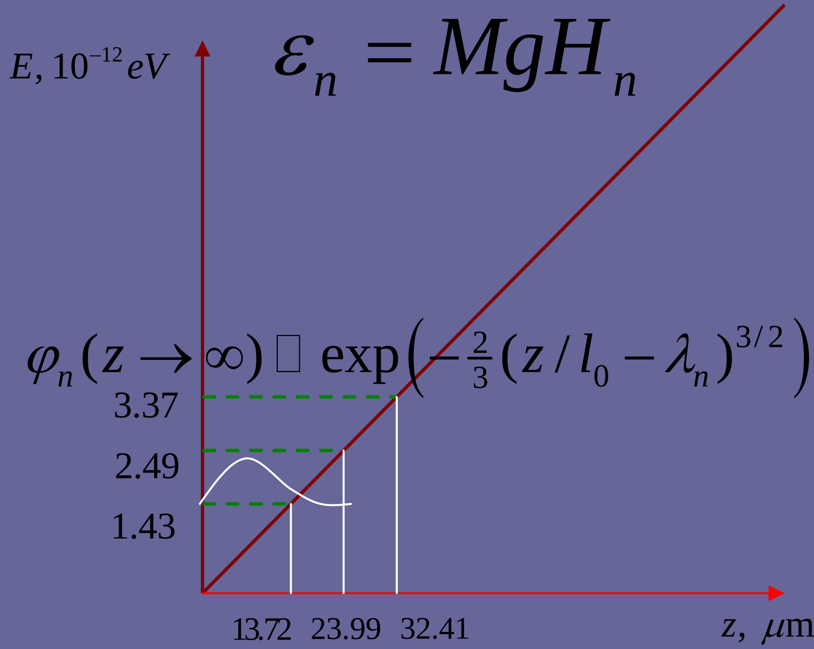
$$\begin{cases}\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2}+Mgz-\varepsilon_n\right]\varphi_n(z)=0\\ \varphi_n(0)=0\quad \varphi_n(\infty)=0\end{cases}$$

$$\varepsilon_n = MgH_n$$

$$\varphi_n(z\to\infty) \sim \exp\left(-\tfrac{2}{3}(z/l_0-\lambda_n)^{3/2}\right)$$

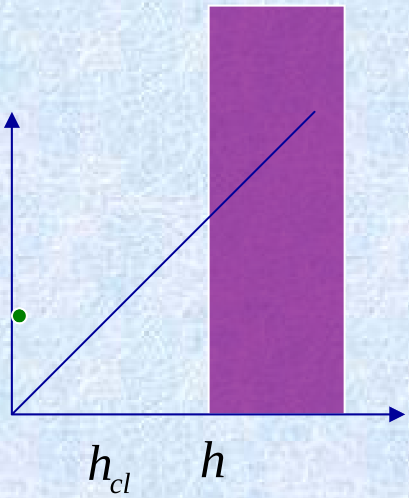
$$\boxed{\varphi_n(z)=C_n Ai(z/l_0-\lambda_n)}$$

$$Ai(-\lambda_n)=0$$



$$\varepsilon_n=\lambda_n\varepsilon \qquad \varepsilon=\sqrt[3]{\frac{\hbar^2 Mg^2}{2}}=0.61\ 10^{-12}\text{eV} \qquad l_0=\sqrt[3]{\frac{\hbar^2}{2M^2 g}}=5.87\ 10^{-6}m$$

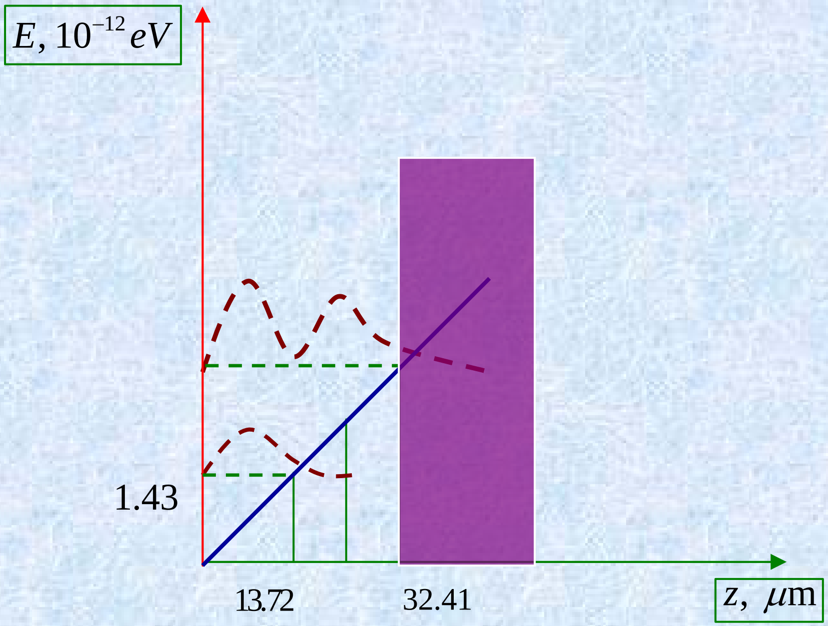
Idea of experiment



$$R_{abs} \approx \omega P_{tun}$$

$$P_{tun} \sim \exp\left(-\frac{4}{3}\frac{(h-h_{cl})^{3/2}}{l_0^{3/2}}\right)$$

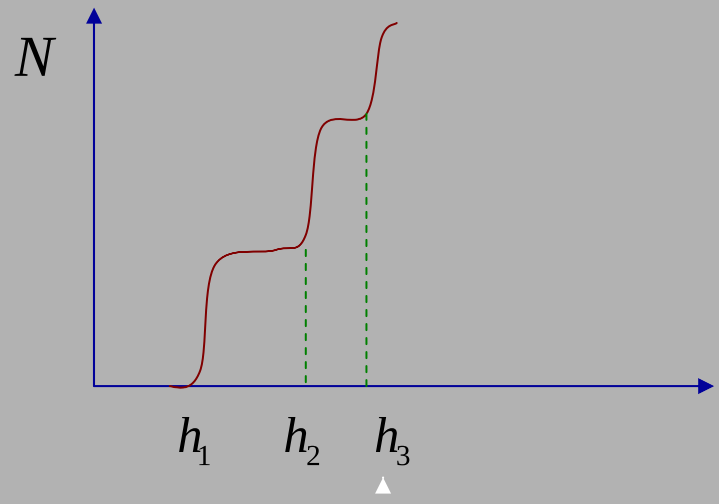
$$\tau = \begin{cases} 0 & h < h_{cl} \\ \omega^{-1}\exp\left(\frac{4}{3}\frac{(h-h_{cl})^{3/2}}{l_0^{3/2}}\right) & h > h_{cl} \end{cases}$$

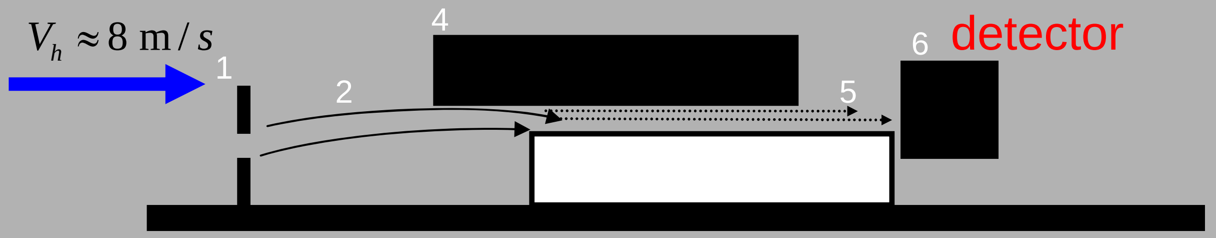


Idea of experiment

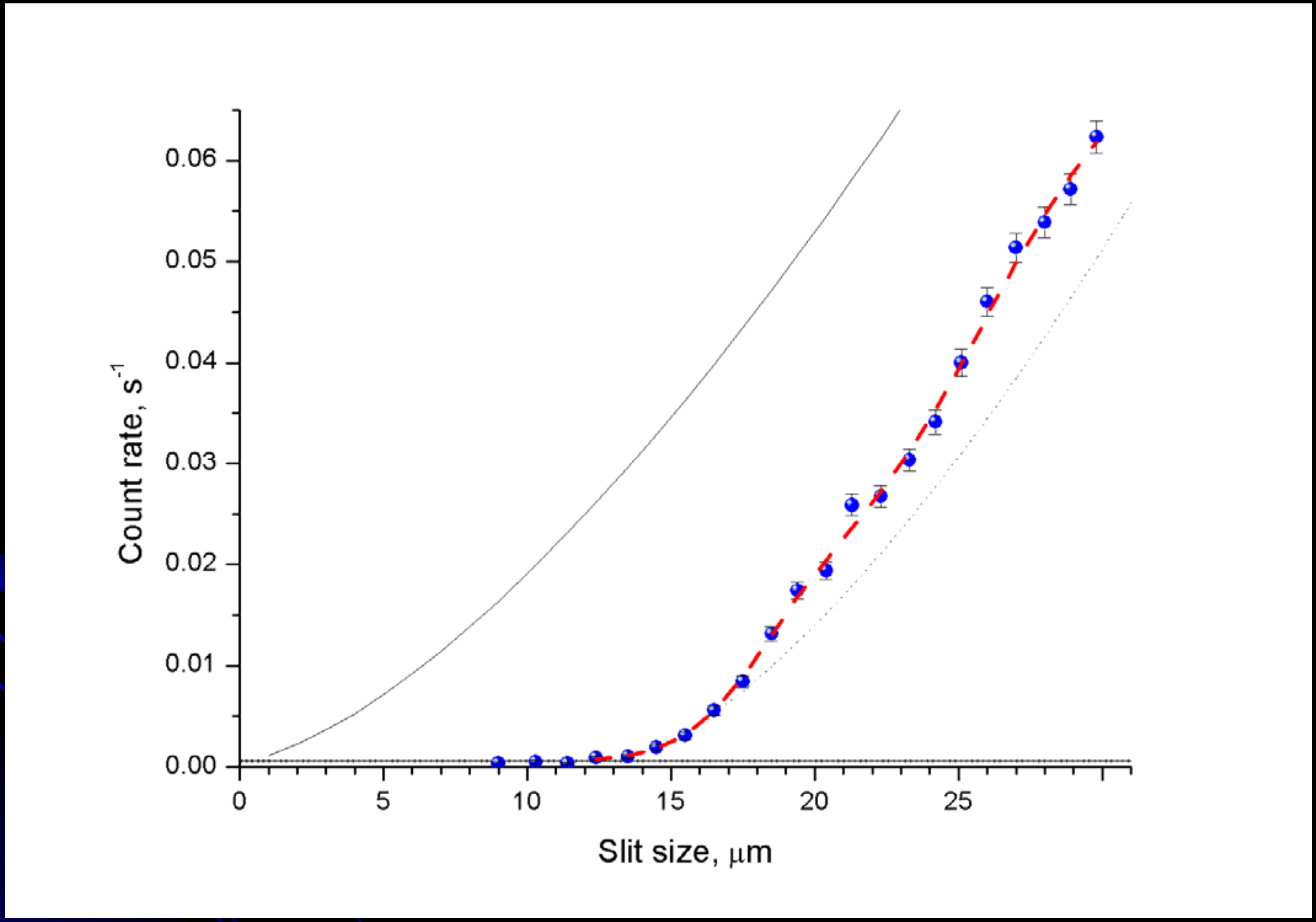
$$\tau = \begin{cases} 0 & h < h_{cl} \\ \omega^{-1} \exp\left(\dfrac{4}{3} \dfrac{(h - h_{cl})^{3/2}}{l_0^{3/2}} \right) & h > h_{cl} \end{cases}$$

Dream





Results



Count rate, s⁻¹
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
5
10
15
20
25
Slit size, μm

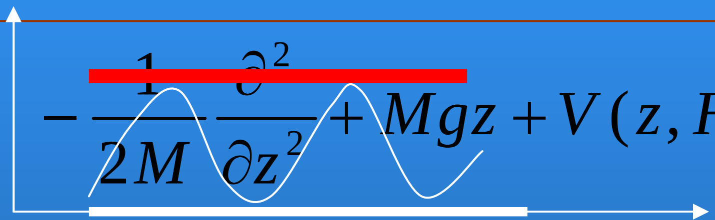
Theoretical problems:

- The neutron absorption mechanism
- How to achieve the best resolution of quantum states
- Roughness – its role
- What can we extract from the experiment

Flat Absorber

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial x^2}-\frac{1}{2M}\frac{\partial^2}{\partial z^2}+Mgz+V(z,H)-iW(z,H)-E\right]\Psi(x,z)=0$$

$$-\frac{1}{2M}\frac{\partial^2}{\partial x^2}\equiv \mathrm{H}_x \qquad -\frac{1}{2M}\frac{\partial^2}{\partial z^2}+Mgz+V(z,H)-iW(z,H)\equiv \mathrm{H}_z$$



$$\mathrm{H}=\mathrm{H}_z+\mathrm{H}_x$$

Absorption=Non-self-adjoint Hamiltonian

$$\mathrm{H}_z\varphi_n(z)=(\varepsilon_n-i\Gamma_n/2)\varphi_n(z)$$

Decaying quasistationary states basis:

$$\Psi(x,z)=\sum_n C_n e^{ik_n x}\varphi_n(z)$$

$$\int\varphi_n^*\varphi_k dz\neq\delta_{nk}\ \text{ but }\int\varphi_n\varphi_k dz=\delta_{nk}$$

Transition through the wave-guide

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial x^2}+\left(\varepsilon_n - i\Gamma_n/2\right)-E\right]\exp(ik_n x)=0$$

$$k_n=\sqrt{2M\left(E-\varepsilon_n+i\Gamma_n/2\right)}\approx P-\frac{\left(\varepsilon_n-i\Gamma_n/2\right)}{V}$$

$$\left|E\right|\gg\left|\varepsilon_n-i\Gamma_n/2\right|\Rightarrow P=\sqrt{2ME}\ ;\ V=\sqrt{2E/M}$$

$$\Psi(x,z)=\sum_n C_n e^{ik_n x}\varphi_n\left(z\right)=e^{iPx}\sum_n C_n\varphi_n\left(z\right)e^{-i\varepsilon_n x/V}e^{-\Gamma_n x/2V}$$

$$F(L)=\int\left|\Psi(L,z)\right|^2 dz=\sum_{n,k}C_n^* C_k\left\langle\varphi_n\middle|\varphi_k\right\rangle e^{i\omega_{nk}\tau}e^{-(\Gamma_n+\Gamma_k)\tau/2}$$

$$\boxed{F(L)\approx\sum_n\left|C_n\right|^2 e^{-\Gamma_n\tau}}$$

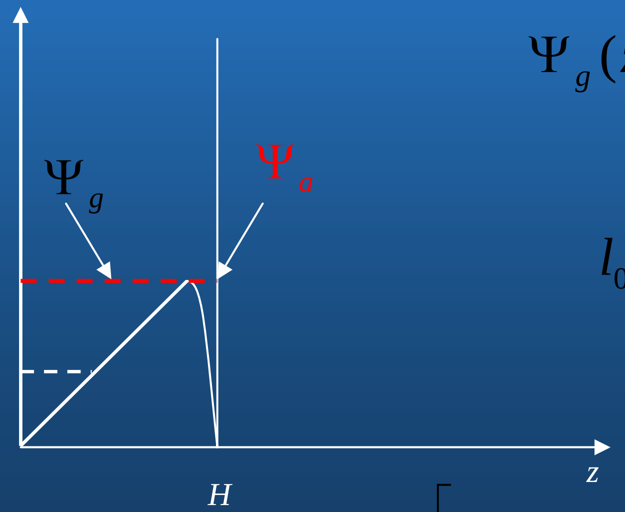
Short-range absorber

$$U(z) = -iW(H-z)\ ;\ W(H) \gg \varepsilon_0\ ;\ U(H-z) \to 0 \text{ when } H-z \gg \rho$$

$$\rho \ll l_0$$

Scattering length approximation works precisely.

Only one parameter of absorber is needed, namely complex scattering length a



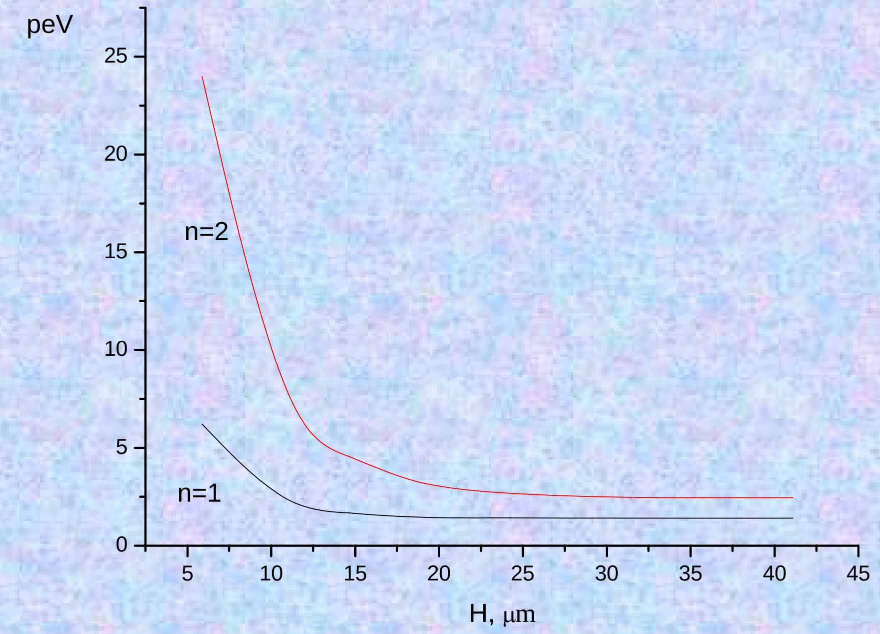
$$\Psi_g(z) = \mathrm{Ai}(z/l_0 - \lambda_n) - S\,\mathrm{Bi}(z/l_0 - \lambda_n)\ ;\ \Psi_g(0) = 0$$

To find λ_n and S:

$$l_0 \gg H - z \gg \rho \qquad \Psi_a \approx 1 + \frac{H-z}{a} \qquad \Psi_g \leftrightarrow \Psi_a$$

$$\mathrm{Ai}(-\lambda_n)\left[\mathrm{Bi}(H/l_0 - \lambda_n) - \frac{a}{l_0}\mathrm{Bi}'(H/l_0 - \lambda_n)\right] = \mathrm{Bi}(-\lambda_n)\left[\mathrm{Ai}(H/l_0 - \lambda_n) - \frac{a}{l_0}\mathrm{Ai}'(H/l_0 - \lambda_n)\right]$$

Neutrons levels



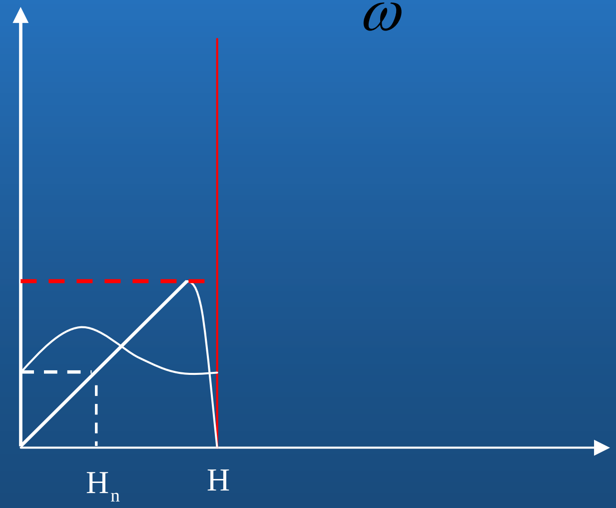
$$\mathrm{Ai}(-\lambda_n)\left[\mathrm{Bi}(H/l_0-\lambda_n)-\frac{a}{l_0}\mathrm{Bi}'(H/l_0-\lambda_n)\right]=\mathrm{Bi}(-\lambda_n)\left[\mathrm{Ai}(H/l_0-\lambda_n)-\frac{a}{l_0}\mathrm{Ai}'(H/l_0-\lambda_n)\right]$$

$$\varepsilon=\lambda(H)\sqrt[3]{\frac{\hbar^2 M g^2}{2}}$$

Through gravitational barrier

$$H > H_n \equiv l_0 \lambda_n$$

$$\Gamma \approx \underbrace{\varepsilon_0 \sqrt{\frac{l_0}{H_n}}}_{\omega} \underbrace{\sqrt{\frac{H - H_n}{l_0}} \exp\left[-\frac{4}{3}\left(\frac{H - H_n}{l_0}\right)^{3/2}\right]}_{D} \underbrace{\frac{4\left|\operatorname{Im} a\right|}{l_0}}_{P}$$



$$H \le H_n$$

$$\Gamma \approx \underbrace{\varepsilon_0 \sqrt{\frac{l_0}{H_n}} \frac{4\left|\operatorname{Im} a\right|}{l_0}}_{\omega P}$$

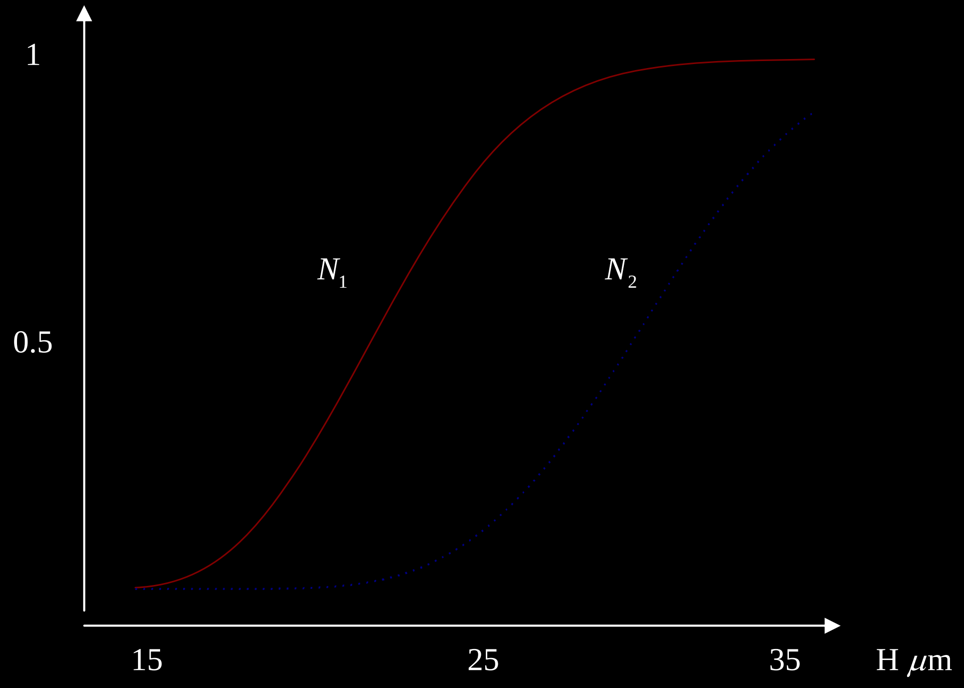
Complex potential

$$V(z,H) - iW(z,H) = (V_0 - iW_0)\frac{1}{1+e^{(H-z)/\rho}}$$

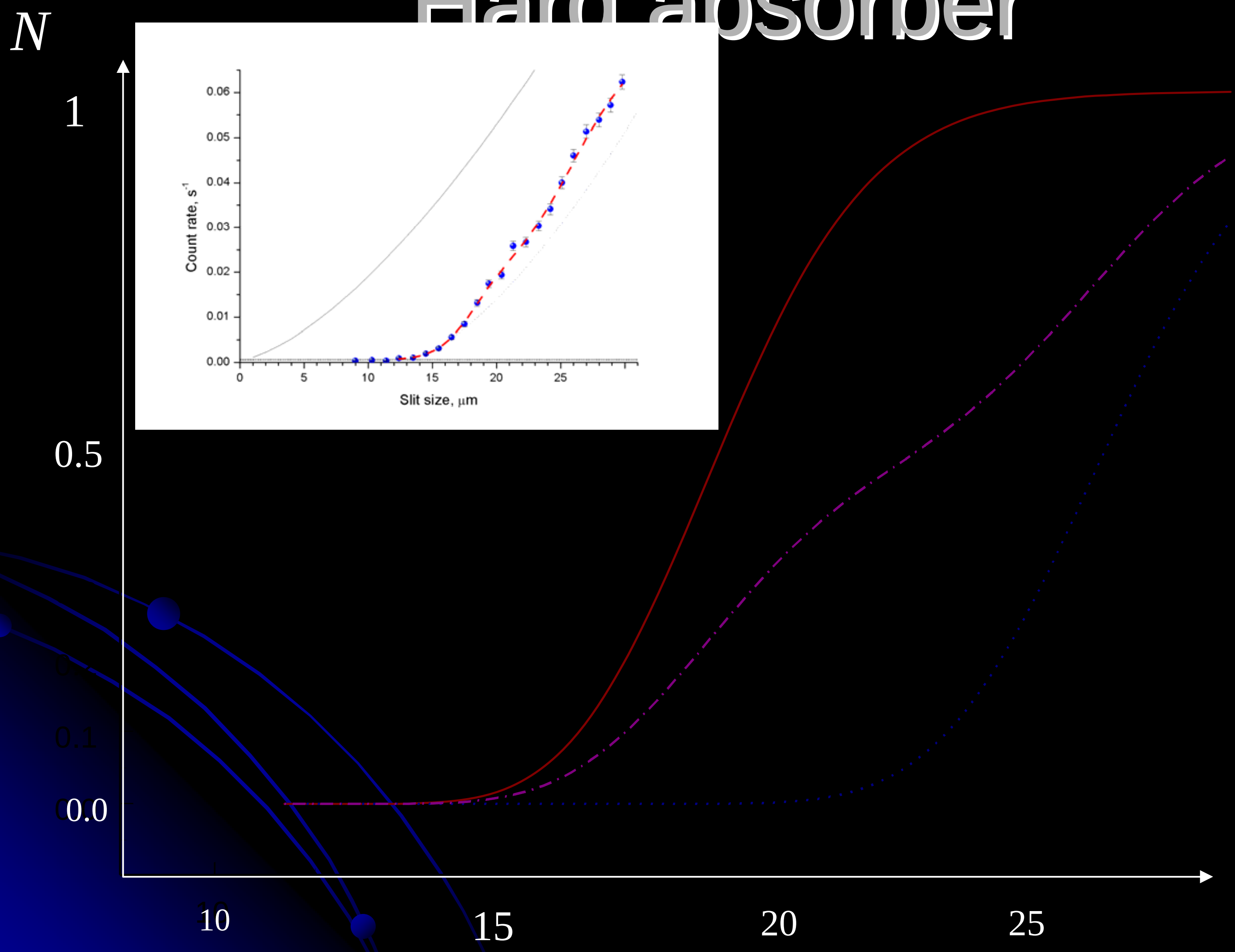
Independent on the depth!

$$10^{-3}\,\mu m < \rho < 1\mu m$$

$$W_0 \sim 10^{-7}\,eV >> 10^{-12}\,eV \qquad \left|\mathrm{Im}\,a\right| = \pi\rho$$



Hard absorber



How to see them better?

$$\Gamma \approx \varepsilon_0 \sqrt{\frac{l_0}{H_n}} \frac{4\left|\mathrm{Im}\, a\right|}{l_0} \qquad \Rightarrow \text{ increase } \frac{\left|\mathrm{Im}\, a\right|}{l_0}!$$

$$V = -iW \exp\left[\frac{z-H}{\rho}\right];\ \rho\sqrt{2MW} \gg 1 \Rightarrow \mathrm{Im}\, a = -\pi\rho$$

Decrease $W \to \varepsilon_0$

Increase ρ

Rough absorber

Good absorber = rough absorber

- Averaged potential
- Non-specula reflections
- Better resolution?

Time-dependent model of neutron absorption

- Horizontal motion is classical=time dependence
- Rough edges scattering = time dependent variation of wall position (boundary condition)
- Fast transversal neutrons are promptly absorbed
- The problem of neutron passage through the slit

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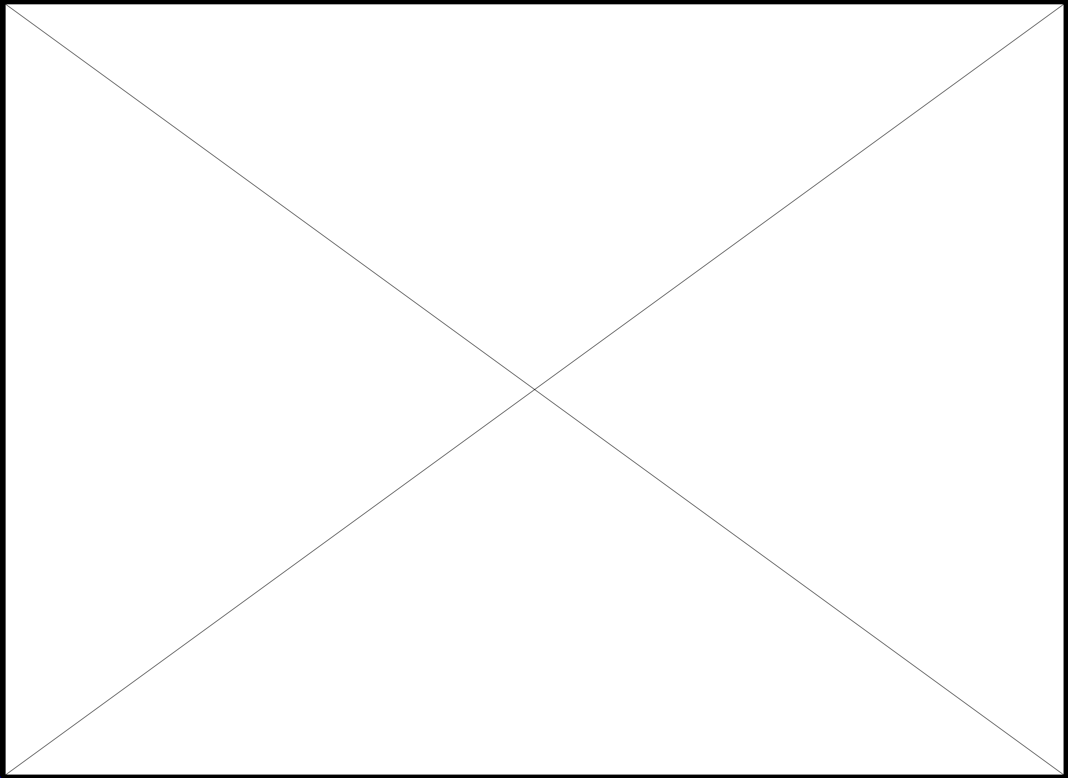
The problem of ionization of particle in the well with vibrating wall.

Oscillating wall



Equations

$$\Psi(t,z)=\sum C_n(t)\varphi_n(H(t),z)\exp(-i\int_0^t \varepsilon_n(\tau)d\tau)$$

$$\begin{cases}\left[-\frac{1}{2M}\frac{\partial^2}{\partial z^2}^n+Mgz-W\theta(-z)-\varepsilon_n(H)\right]\varphi_n(H,z)=0\\ \varphi_n(H(t))=0\end{cases}$$

$$\dot{C}_n(t)=-\frac{dH}{dt}\sum_{k\neq n}\left\langle\varphi_n\frac{\partial\varphi_k}{\partial H}\right\rangle C_k(t)\exp(-i\int_0^t \omega_{kn}(\tau)d\tau);$$

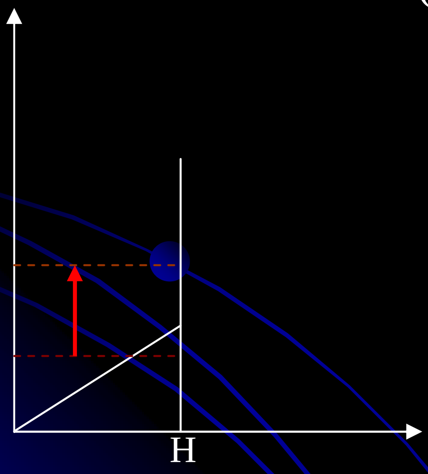
$$\begin{cases}\dot{C}_1(t)=-a\omega\cos(\omega t)\sum_k\left\langle\varphi_1\frac{\partial\varphi_k}{\partial H}\right\rangle C_k(t)\exp(-i\int_0^t \omega_{1k}(\tau)d\tau)\\ \dot{C}_k(t)=a\omega\cos(\omega t)C_1(t)\left\langle\varphi_1\frac{\partial\varphi_k}{\partial H}\right\rangle\exp(i\int_0^t \omega_{1k}(\tau)d\tau)\end{cases}$$

Coupling between states (non-specula reflections)

$$\begin{cases} \varphi(z,H) = C(H)\left[Ai(z/l_0 - \lambda(H)) - SBi(z/l_0 - \lambda(H))\right] \\ \varphi(z=0,H) = 0 \quad \varphi(z=H,H) = 0 \end{cases}$$

Equation for the eigenvalues:

$$Ai(H/l_0 - \lambda(H))Bi(-\lambda(H)) = Ai(-\lambda(H))Bi(H/l_0 - \lambda(H))$$

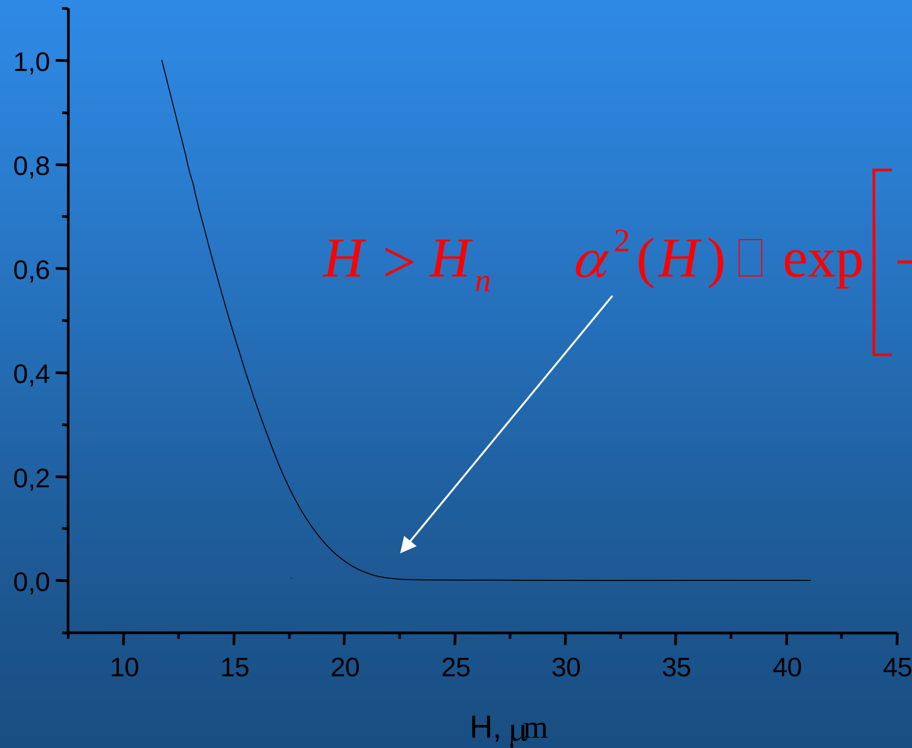


$\left\langle \varphi_k \frac{\partial \varphi_n}{\partial H} \right\rangle$=analytical expression in terms of *Ai* and *Bi*

Coupling as a function of H

$$\alpha(H) = \left\langle \varphi_k \middle| \frac{\partial}{\partial H} \varphi_1 \right\rangle = -\left\langle \varphi_1 \middle| \frac{\partial}{\partial H} \varphi_k \right\rangle = \frac{\sqrt{\partial E_k / \partial H \, \partial E_1 / \partial H}}{E_k - E_1}$$

$$\left| \left\langle \varphi_k \frac{\partial \varphi_1}{\partial H} \right\rangle \right|^2$$



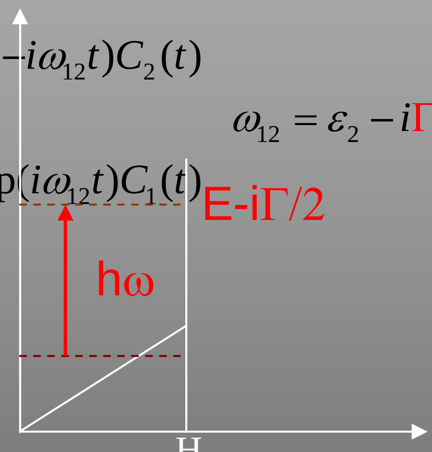
$$H > H_n \qquad \alpha^2(H) \propto \exp\left[-\frac{4}{3} \left(\frac{H - H_n}{l_0} \right)^{3/2} \right]$$

2-state model

Roughness mixes the gravitational state with excited quasistationary state

$$\begin{cases} \dot{C}_1(t) = a\omega\cos(\omega t)\left\langle \varphi_2 \dfrac{\partial \varphi_1}{\partial H} \right\rangle \exp(-i\omega_{12} t) C_2(t) \\ \dot{C}_2(t) = -a\omega\cos(\omega t)\left\langle \varphi_2 \dfrac{\partial \varphi_1}{\partial H} \right\rangle \exp(i\omega_{12} t) C_1(t) \end{cases}$$

$$\omega_{12} = \varepsilon_2 - i\Gamma/2 - \varepsilon_1$$



$$C_1(t) = \exp(-\Gamma t/4)\left[\cos(Rt/2) + \frac{\Gamma}{2R}\sin(Rt/2)\right]$$

$$R = \sqrt{\Omega^2 - \Gamma^2/4} \quad \Omega^2 = \rho^2\omega^2 \left| \left\langle \varphi_2 \frac{\partial \varphi_1}{\partial H} \right\rangle \right|^2$$

$$\begin{cases} |C_1(t)|^2 \to \exp(-\tfrac{\Omega^2}{\Gamma} t) \text{ if } \Omega/\Gamma \to 0 \\ |C_1(t)|^2 \to \exp(-\Gamma t/2)\sin(\delta)\sin(\Omega t/2 + \delta) \text{ if } \Omega/\Gamma \to \infty \end{cases}$$

$$\delta = \text{arccot}(\tfrac{\Gamma}{2\Omega})$$

Rough surface absorption

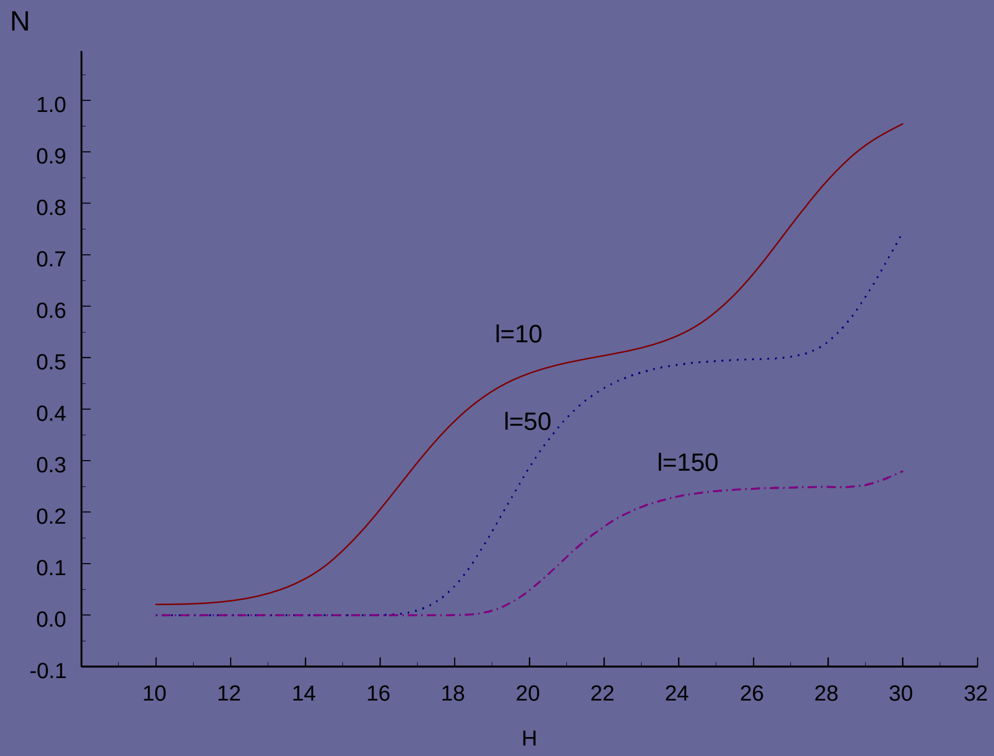
$$H > H_n$$

$$\Gamma \approx \underbrace{\varepsilon_0 \sqrt{\frac{l_0}{H_n}}}_{\omega} \underbrace{\sqrt{\frac{H - H_n}{l_0}} \exp\left[-\frac{4}{3}\left(\frac{H - H_n}{l_0}\right)^{3/2} \right]}_{D} \underbrace{\frac{\rho^2}{l_0 \operatorname{Im} a}}_{P}$$

$$H \le H_n$$

$$\Gamma = \Gamma_{\text{large}} / 2 \;\square\; 1/\tau^{pass}$$

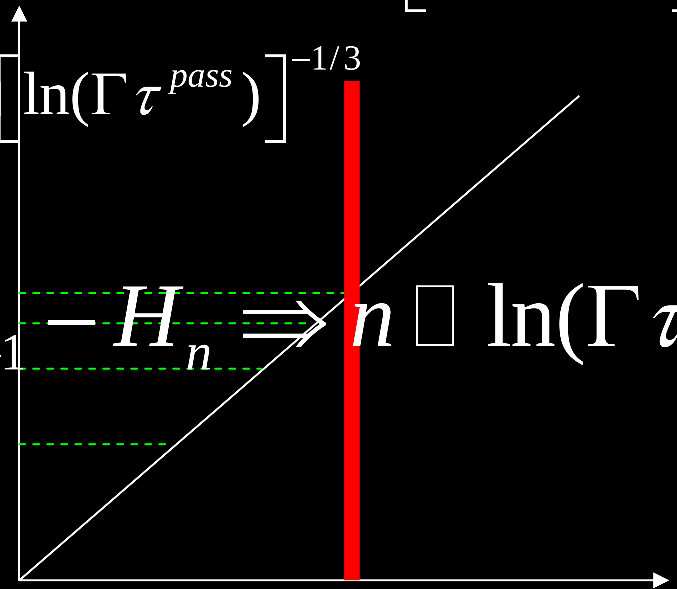
Flux 2 state model



Resolution limits

$$\tilde{H}_n = H_n + \Delta_n \quad \Delta_n \sim l_0 \left[\ln(\Gamma \tau^{pass})\right]^{2/3}$$

$$\delta_n \sim l_0 \left[\ln(\Gamma \tau^{pass})\right]^{-1/3}$$

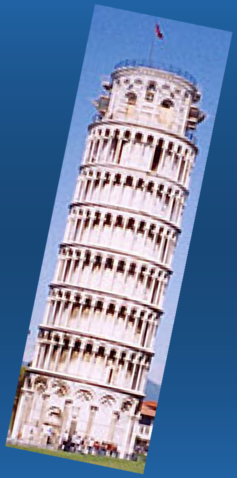


$$\delta_n \sim H_{n+1} - H_n \Rightarrow n \sim \ln(\Gamma \tau^{pass})$$

Quantum Weak Equivalence Principle

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+Mgz-E\right]\varphi(z)=0$$

m – Inertial mass   M-Gravitational mass

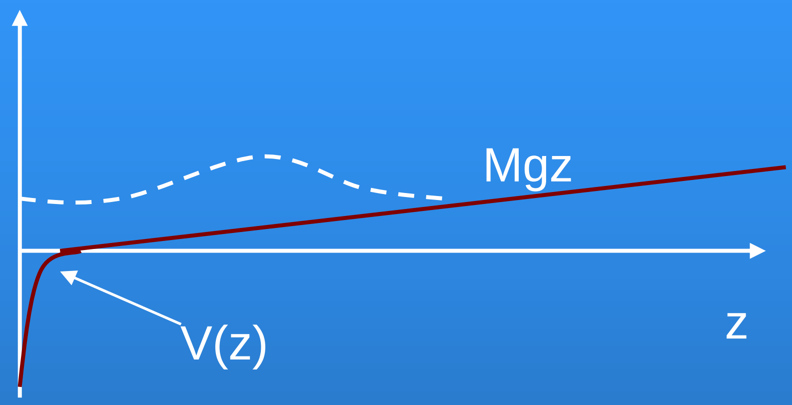


$$\varepsilon_0=\sqrt[3]{\frac{h^2M^2g^2}{2m}} \qquad l_0=\sqrt[3]{\frac{h^2}{2mMg}}$$

$$m=M\Rightarrow\frac{\varepsilon_0}{\hbar}=\sqrt{\frac{g}{2l_0}}$$

$$2\frac{\varepsilon_0^2 l_0}{\hbar^2}=g$$

Additional forces



$$\tilde{\lambda}_n = \lambda_n + a / l_0$$

$$\varepsilon_n = \varepsilon(\lambda_n + \mathrm{Re}\, a / l_0) \quad \Gamma = 2\varepsilon \left|\mathrm{Im}\, a\right| / l_0$$

Criteria: $a \leq l_0$

5-th force limits from neutron gravitational experiment

Nesvizhevsky V.V., Protasov K.V.

H. Abele, A. Westphal, Lect. Notes in Physics 631, 355 (2003)

Some conclusions

- Beautiful and transparent physics
- Interesting tool for studying rich physics of neutron surface interactions
- “Easy” and elegant way to measure the gravitational force acting on neutron and check Quantum Equivalence Principle

THANKS FOR YOUR ATTENTION!