Soft Gluon Resummation Effects in Single Graviton Production at the LHC in the Randall-Sundrum Model

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Outline

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- LO Results
- NLO QCD Corrections
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Introduction

- Now search for extra dimensions has been one of the major objects at the LHC, since its physical effects can appear at the TeV energy scale.

- Two major classes of Extra Dimensions models:
  - **"Flat" (factorizable) ED**
    - Large ED(LED) (Arkani-Hamed, Dvali & Dimopoulos)
    - TeV^{-1} ED (variation of LED)
    - Universal ED(UED) (Appelquist, Cheng & Dobrescu)
    - ...
  - **"Warped" (non-factorizable) ED**
    - Randall-Sundrum(RS) model
    - ...

- For LED and RS model, only graviton can propagate into the extra dimensions. In TeV^{-1} ED, some gauge bosons can also live in the extra dimensions. For UED, all the SM fields can propagate into the extra dimensions.

- Moreover, there are also variants involving SUSY or SUSY breaking, and Higgsless model which utilizes the boundary condition in extra dimensions to break the electroweak symmetry.

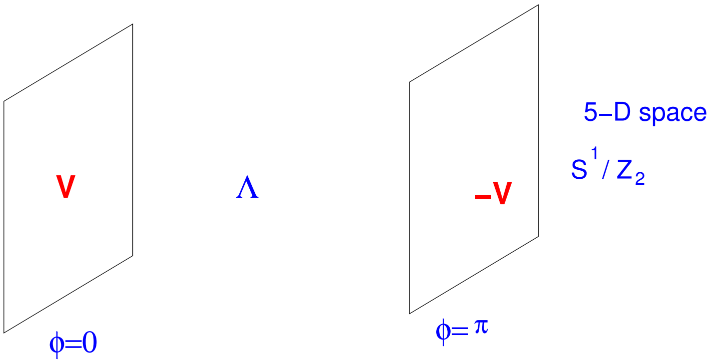
- In our work, we concentrate on the RS model.

- In the RS model, the extra dimension is assumed to be an S_1/Z_2 orbifold, which has two fixed points, $\theta = 0$ and $\theta = \pi$. At each fixed point, there is a 3-brane, and the brane at $\theta = \pi$ corresponds to the brane we live on, while the one at $\theta = 0$ is the high energy brane. Between the two 3-branes is a slice of AdS space, where only the graviton can propagate into.

- the 4-dimensional metric is the function of the coordinate of the 5th dimension, i.e.

$$ds^2 = e^{-2kr_c|\phi|}(\eta_{\mu\nu} + \frac{2}{M_*^{3/2}} h_{\mu\nu}) dx^\mu dx^\nu - r_c^2 d\phi^2, \ 0 \leq |\phi| \leq \pi,$$

  where k is a scale of order of the Planck scale and relates the 5-dimensional Planck scale M_* to the cosmological constant, r_c is the compactification radius, and $h_{\mu\nu}$ is the graviton.


V
Λ
−V
5−D space
S1 / Z2
φ=0
φ= π

- Compared with the LED model, the RS model present a different solution to the gauge Hierarchy problem: the physical mass m of a field on the brane where our world live on, is related to the fundamental mass parameter m_0 as following

$$m = e^{-kr_c\pi} m_0,$$

thus the hierarchy problem can be solved if $kr_c \sim 12$.

- In the RS model, there also exist KK towers of massive spin-2 gravitons which can interact with the SM fields:

$$\mathcal{L} = -\frac{1}{\overline{M}_P} T^{\alpha\beta}(x) h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x)$$

where

$$\Lambda_\pi = e^{-kr_c\pi}\overline{M}_P = \frac{m_1 \overline{M}_P}{x_1 k},$$

and is at the electroweak scale.

- Thus the coupling of the massless graviton $h^{(0)}$ is suppressed by the Planck scale, and the ones of the massive graviton $h^{(n)}$ by Λ_π, but which is only TeV.

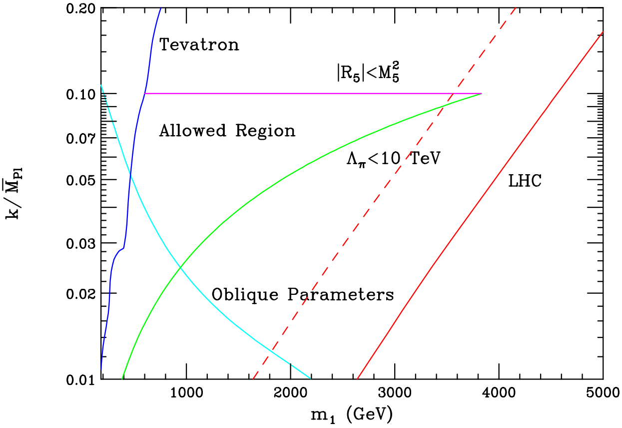
- The masses of the nth graviton KK excitation modes are also at the electroweak scale, which are given by

$$m_n = kx_n e^{-kr_c\pi} = m_1 \frac{x_n}{x_1},$$

where the x_n's are the nth roots of the first order Bessel function.

- The graviton sector of the RS model is completely determined by the two parameters m_1 and $k/\overline{M}_P$.

- Current constraints for the parameters of the RS model are from the theoretical requirement, the low energy precise measurement and also the data from Tevatron, from which we expect $0.01 \leq k/\overline{M}_P < 0.1$ and $\Lambda_\pi \leq 10\,\mathrm{TeV}$.



0.20
0.10
0.07
0.05
0.03
0.02
0.01
1000
2000
3000
4000
5000
$k/\overline{M}_{Pl}$
m_1 (GeV)
Tevatron
$|R_5|<M_5^2$
Allowed Region
$\Lambda_\pi<10$ TeV
LHC
Oblique Parameters

- In the RS model, the lightest massive graviton can have a mass of several hundred GeV, and may be produced copiously at the LHC. More importantly, it has much larger couplings to the SM particles than the ones in the ADD model, thus it may decay into observable particles and hence be detected.
- There have been detailed analysis(B. C. Allanach, et al., JHEP **0212**, 039 (2002)) which demonstrate that using channels $pp \to h^{(n)} \to e^+e^-, \gamma\gamma...$, we can probe the massive graviton in the RS model with masses up to several TeV.

- The analyses are based on the LO results, in order to improve the precision of the theoretical predictions, the higher order QCD effects are necessary.

  In the ADD model and the RS model, the NLO QCD corrections to the virtual graviton production at the LHC have been discussed(e.g. Prakash Mathews et al., Nucl.Phys. **B713**, 333 (2005).), however, the K factors contributed from different parts, the scale dependence and the PDF uncertainty for above processes needs further studies.

- Moreover, we study the transverse momentum distribution of the massive graviton at NLO in QCD, and all order soft gluon resummation effects on the distribution to give reasonable predictions.

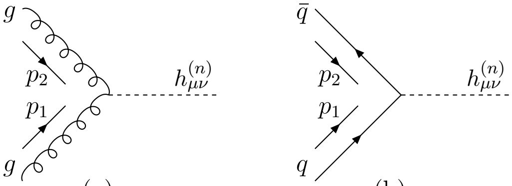
LO results

- The LO amplitude of the partonic process $g^a_\rho(p_1) g^b_\sigma(p_2)$, $q_r(p_1)\bar{q}_s(p_2) \to h^{(n)}_{\mu\nu}$:

$$M^{(0)}_{gg} = -\frac{i\delta_{ab}\mu_r^{4-n}}{\Lambda_\pi} \times \Big[p_1 \cdot p_2 C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma} + E_{\mu\nu,\rho\sigma}(p_1, p_2)\Big] \epsilon^a_\rho(p_1)\epsilon^b_\sigma(p_2)\epsilon^{s*}_{\mu\nu}(p_1+p_2),$$

$$M^{(0)}_{q\bar{q}} = -\frac{i\delta_{rs}\mu_r^{4-n}}{4\Lambda_\pi} \times \bar{v}(p_2)\Big[\gamma_\mu(p_{1\nu} - p_{2\nu}) - \eta_{\mu\nu}(\not{p}_1 - \not{p}_2) + (\mu \leftrightarrow \nu)\Big] u(p_1)\epsilon^{s*}_{\mu\nu}(p_1+p_2),$$

where $C_{\mu\nu,\rho\sigma}$, $D_{\mu\nu,\rho\sigma}$ and $E_{\mu\nu,\rho\sigma}(p_1, p_2)$ are the coefficients in the couplings between the graviton and gluons, which can be found in (T. Han et al., Phys. Rev. **D59**, 05006 (1999))



(a) (b)

- For the polarization sum of the massive graviton,we have

$$\sum_{s=1}^{5} \epsilon^{s}_{\mu\nu}(k)\epsilon^{s*}_{\alpha\beta}(k) = P_{\mu\nu\alpha\beta},$$

where

$$P_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{n-1}\eta_{\mu\nu}\eta_{\alpha\beta}\right) + \ldots,$$

the dots represent terms proportional to the graviton momentum k_μ, and since $k^\mu T_{\mu\nu} = 0$, give a vanishing contribution to the amplitude. For convenience, below we define

$$\chi \equiv \frac{2}{n-1} = \frac{2}{3-2\epsilon}.$$

- In order to avoid introducing external ghost lines while summing over the gluon helicities, we limit ourselves to the sum over the physical polarizations of the gluons, i.e.

$$P_i^{\mu\nu} = \sum_T \epsilon_T^{\mu}(k_i)\epsilon_T^{\nu}(k_i) = -g^{\mu\nu} + \frac{n_i^{\mu}k_i^{\nu} + k_i^{\mu}n_i^{\nu}}{n_i \cdot k_i} - \frac{n_i^2 k_i^{\mu}k_i^{\nu}}{(n_i \cdot k_i)^2},$$

where the index i (=1,2) labels the two external gluons, and $n_i \neq k_i$ is an arbitrary vector. This polarization sum obeys the transversality relations

$$k_{i\mu}P^{\mu\nu} = P^{\mu\nu}k_{i\nu} = n_{i\mu}P^{\mu\nu} = P^{\mu\nu}n_{i\nu} = 0.$$

- Partonic cross sections:

$$\hat{\sigma}^{(LO)}_{gg} = \frac{1}{2s} 2\pi\delta(s - m_n^2)\overline{|M^{(0)}_{gg}|}^2 = \frac{(2-\epsilon)\pi}{32\Lambda_\pi^2}\delta(1-\hat{\tau}),$$

$$\hat{\sigma}^{(LO)}_{q\bar{q}} = \frac{1}{2s} 2\pi\delta(s - m_n^2)\overline{|M^{(0)}_{q\bar{q}}|}^2 = \frac{(1-\epsilon)\pi}{24\Lambda_\pi^2}\delta(1-\hat{\tau}),$$

where $\hat{\tau} \equiv m_n^2/s$.

- LO total cross sections:

$$\sigma^{(LO)} \equiv \sigma^{(LO)}_{gg} + \sigma^{(LO)}_{q\bar{q}},$$

$$\sigma^{(LO)}_{gg} = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \frac{1}{2}\left[G_{g/p}(x_1,\mu_f)G_{g/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\right]\hat{\sigma}^{(LO)}_{gg},$$

$$\sigma^{(LO)}_{q\bar{q}} = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \left[G_{q/p}(x_1,\mu_f)G_{\bar{q}/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\right]\hat{\sigma}^{(0)}_{q\bar{q}},$$

where $\tau_0 \equiv m_n^2/S_0$, $S_0 = (14\mathrm{TeV})^2$.

NLO QCD corrections

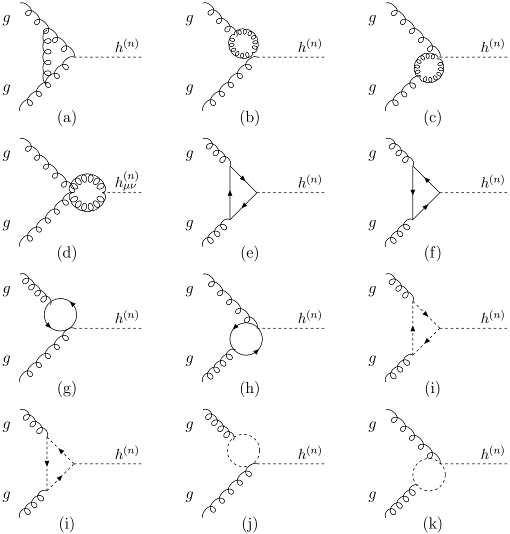
- The NLO QCD corrections consist of the following contributions: the virtual corrections, the real contributions arising from the radiation of a real gluon or a massless (anti)quark, and the contributions of mass factorization.

- We use dimensional regularization (DREG) in $n = 4 - 2\epsilon$ dimensions to regulate the ultraviolet (UV) and infrared (IR) divergences.

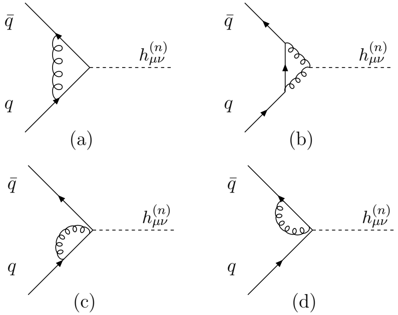
- For the partonic cross section, the total virtual corrections can be written as

$$\hat{\sigma}^{V}_{gg,q\bar{q}} = \hat{\sigma}^{unren}_{gg,q\bar{q}} + \hat{\sigma}^{con}_{gg,q\bar{q}},$$

where the first part in the right hand contains the radiative corrections from the one-loop vertex box diagrams, and the second part is the contributions from the counterterms involving only the the wavefunction renormalization constant for the external fields.



g
g
$h^{(n)}$
(a)
(b)
(c)
$h_{\mu\nu}^{(n)}$
(d)
(e)
(f)
(g)
(h)
(i)
(i)
(j)
(k)



$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(a)
$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(b)
$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(c)
$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(d)

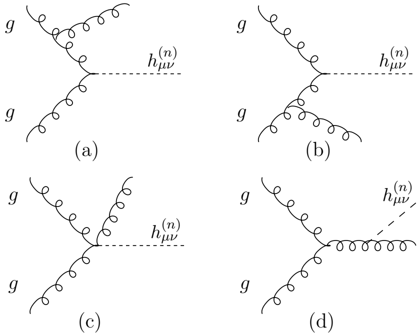
- The $\mathcal{O}(\alpha_s)$ virtual corrections to the partonic cross section:

$$\hat{\sigma}^V_{gg} = 2C_\epsilon \frac{g_s^2(2-\epsilon)}{32\pi\Lambda_\pi^2}\delta(1-\hat{\tau}) \times \left(\frac{-3}{8}\frac{1}{\epsilon_{IR}^2} - \frac{33}{48}\frac{1}{\epsilon_{IR}} + \frac{n_f}{24}\frac{1}{\epsilon_{IR}} + \frac{1}{8}\pi^2 - \frac{203}{96} + \frac{35 n_f}{288}\right),$$

$$\hat{\sigma}^V_{q\bar{q}} = 2C_\epsilon \frac{g_s^2(1-\epsilon)}{24\pi\Lambda_\pi^2}\delta(1-\hat{\tau}) \times \left(\frac{-1}{6}\frac{1}{\epsilon_{IR}^2} - \frac{1}{3}\frac{1}{\epsilon_{IR}} + \frac{1}{18}\pi^2 - \frac{5}{6}\right),$$

with $C_\epsilon \equiv \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{m_n^2}\right)^\epsilon$

- Here, the IR divergences include the soft divergences and the collinear divergences. The soft divergences are canceled after adding the real emission corrections, and the remaining collinear divergences can be absorbed into the redefinition of PDF



g
g
$h_{\mu\nu}^{(n)}$
(a)
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$h_{\mu\nu}^{(n)}$
(b)
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g
$h_{\mu\nu}^{(n)}$
(c)
g
g
$h_{\mu\nu}^{(n)}$
(d)

- For the real gluon emission sub-processes $g(p_1)g(p_2),\ q(p_1)\bar{q}(p_2) \to g(p_3)h^{(n)}_{\mu\nu}$, the partonic cross sections is

$$\hat{\sigma}^{real}_{gg,q\bar{q}} = \frac{1}{2s}\int \overline{|M^{real}_{gg,q\bar{q}}|}^2 d\Gamma_2,$$

with

$$d\Gamma_2 = \frac{1}{8\pi\Gamma(1-\epsilon)}\left(\frac{4\pi\mu_r^2}{m_n^2}\right)^{\epsilon}(\hat{\tau})^{\epsilon}(1-\hat{\tau})^{\epsilon} \times v^{-\epsilon}(1-v)^{-\epsilon}dv,$$

where

$$\begin{aligned} v &\equiv \frac{1}{2}(1+\cos\theta), \\ t &\equiv (p_1-p_3)^2 = -s(1-\hat{\tau})(1-v), \\ u &\equiv (p_2-p_3)^2 = -s(1-\hat{\tau})v, \end{aligned}$$

$$\overline{|M_{gg}^{real}|}^2 = \frac{3\times 8}{8\times 8}\frac{1}{4(1-\epsilon)^2}\frac{4g_s^2}{\Lambda_\pi^2 tu}\times\Big\{\epsilon t^2 u[26-9(4-2\epsilon)-2(-5+2(4-2\epsilon))\epsilon\chi]$$
$$+\epsilon^2\chi tu^2[26-9(4-2\epsilon)-2(-5+2(4-2\epsilon))]-\frac{(1-\epsilon)}{4}s^3[16-6(4-2\epsilon)+4\epsilon^2\chi]$$
$$-\frac{(1-\epsilon)}{2}t^3[16-6(4-2\epsilon)+4\epsilon^2\chi]-\frac{(1-\epsilon)}{2}s^2(t+u)[16-6(4-2\epsilon)+4\epsilon^2\chi]$$
$$-\frac{(1-\epsilon)}{2}u^3[16-6(4-2\epsilon)+4\epsilon^2\chi]-\frac{(1-\epsilon)}{4s}(t^2+tu+u^2)^2[16-6(4-2\epsilon)+4\epsilon^2\chi]$$
$$+\frac{s}{2}[2\epsilon tu(26-9(4-2\epsilon)-6\epsilon\chi)-\frac{3}{2}(1-\epsilon)(t^2+u^2)[16-6(4-2\epsilon)+4\epsilon^2\chi]\Big\},$$

- Combining the contributions of the virtual corrections and the real gluon emission, we still have the collinear divergences, which can be absorbed into the redefinition of the PDF at NLO, in general called mass factorization.
- In the $\overline{\mathrm{MS}}$ scheme, the scale dependent PDF $G_{\alpha/p}(x,\mu_f)$ is given by

$$G_{\alpha/p}(x,\mu_f) = G_{\alpha/p}(x) + \sum_{\beta}(-\frac{1}{\epsilon})\left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{\mu_f^2}\right)^{\epsilon}\right]\int_x^1 \frac{dz}{z}P_{\alpha\beta}(z)G_{\beta/p}(x/z).$$

- Here, for the real gluon emission sub-processes $gg \to h_{\mu\nu}^{(n)}$, we first consider only the contributions from p_{gg}, and we can get the relevant counterterm arising from the PDF redefinition as following:

$$\delta\hat{\sigma}_{gg} = 2 \times \frac{\alpha_s}{2\pi}C'_{\epsilon}\frac{(2-\epsilon)}{32\Lambda_\pi^2\epsilon}zP_{gg}^{(0)}(z),$$

with $C'_{\epsilon} \equiv \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{\mu_f^2}\right)^{\epsilon}$, and $z = \hat{\tau}$.

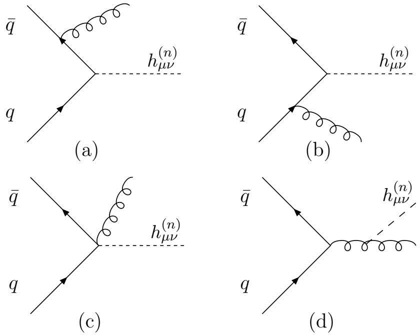
- Summing up the virtual, real emission and PDF redefinition contributions, we have the IR finite results

$$\hat{\sigma}_{gg}^{NLO} = \hat{\sigma}_{gg}^{real} + \hat{\sigma}_{gg}^{V} + \delta\hat{\sigma}_{gg} = \frac{(2-\epsilon)\alpha_s}{32\Lambda_\pi^2} C_\epsilon \frac{m_n^2}{s} \times$$

$$\Bigg\{ 6\ln\left(\frac{m_n^2}{\mu_f^2}\right)\left[\frac{\hat{\tau}}{(1-\hat{\tau})_+} + \frac{1-\hat{\tau}}{\hat{\tau}} + \hat{\tau}(1-\hat{\tau})\right] + \ln\left(\frac{m_n^2}{\mu_f^2}\right)\left(\frac{11}{2} - \frac{n_f}{3}\right)\delta(1-\hat{\tau})$$

$$+\left(\pi^2 - \frac{203}{12} + \frac{35 n_f}{36}\right)\delta(1-\hat{\tau}) + 12\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

$$+6\left[-1 + \frac{1-\hat{\tau}}{\hat{\tau}} + \hat{\tau}(1-\hat{\tau})\right]\ln\frac{(1-\hat{\tau})^2}{\hat{\tau}} - \frac{6\ln\hat{\tau}}{1-\hat{\tau}} - \frac{3}{2} - \frac{11}{2\hat{\tau}} + \frac{3\hat{\tau}}{2} + \frac{11\hat{\tau}^2}{2}\Bigg\}.$$



$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(a)
$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(b)
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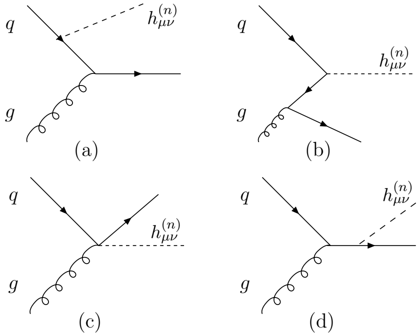
$h_{\mu\nu}^{(n)}$
q
(c)
$\bar{q}$
$h_{\mu\nu}^{(n)}$
q
(d)



q
g
$h_{\mu\nu}^{(n)}$
(a)
(b)
(c)
(d)

- Similarly, we have

$$\hat{\sigma}_{q\bar{q}}^{NLO} = \hat{\sigma}_{q\bar{q}}^{real} + \hat{\sigma}_{q\bar{q}}^{V} + \delta\hat{\sigma}_{q\bar{q}} = \frac{(1-\epsilon)\alpha_s}{24\Lambda_\pi^2} C_\epsilon \frac{m_n^2}{s}$$
$$\times \Big\{ \frac{4}{3} \ln\left(\frac{m_n^2}{\mu_f^2}\right) [\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + \frac{3}{2}\delta(1-\hat{\tau})] + \frac{4}{3}(-5 + \frac{\pi^2}{3})\delta(1-\hat{\tau})$$
$$+ \frac{16}{3}\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - \frac{4}{3}(1+\hat{\tau}) \ln\frac{(1-\hat{\tau})^2}{\hat{\tau}} - \frac{8}{3}\frac{\ln\hat{\tau}}{1-\hat{\tau}} + \frac{16}{9\hat{\tau}} - \frac{16\hat{\tau}^2}{9} \Big\}.$$

- $$\hat{\sigma}_{gq}^{NLO}(=\hat{\sigma}_{g\bar{q}}^{NLO}) = \hat{\sigma}_{gq}^{real} + \delta\hat{\sigma}_{gq} = \frac{\alpha_s}{96\Lambda_\pi^2} C_\epsilon \frac{m_n^2}{s}$$
$$\times\left\{[4\frac{1+(1-\hat{\tau})^2}{\hat{\tau}} + ((1-\hat{\tau})^2+\hat{\tau}^2)]\ln\left(\frac{m_n^2}{\mu_f^2}\right)\right.$$
$$\left.+[4\frac{1+(1-\hat{\tau})^2}{\hat{\tau}} + ((1-\hat{\tau})^2+\hat{\tau}^2)]\ln\left(\frac{(1-\hat{\tau})^2}{\hat{\tau}}\right) + \frac{9}{2} - \frac{6}{\hat{\tau}} + 9\hat{\tau} - \frac{7\hat{\tau}^2}{2}\right\}.$$

- The NLO total cross section for $pp \to h^{(n)}$:

$$
\begin{aligned}
\sigma^{(NLO)} &= \sigma_{gg}^{(LO)} + \sigma_{q\bar{q}}^{(LO)} + \sigma_{gg}^{(NLO)} + \sigma_{q\bar{q}}^{(NLO)} + \sigma_{gq}^{(NLO)} + \sigma_{g\bar{q}}^{(NLO)} \\
&= \sigma_{gg}^{(LO)} + \sigma_{q\bar{q}}^{(LO)} \\
&+ \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \frac{1}{2}\Big[G_{g/p}(x_1,\mu_f)G_{g/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\Big]\hat{\sigma}_{gg}^{(NLO)} \\
&+ \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \Big[G_{q/p}(x_1,\mu_f)G_{\bar{q}/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\Big]\hat{\sigma}_{q\bar{q}}^{(NLO)} \\
&+ \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \Big[G_{q/p}(x_1,\mu_f)G_{g/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\Big]\hat{\sigma}_{gq}^{(NLO)} \\
&+ \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \Big[G_{g/p}(x_1,\mu_f)G_{\bar{q}/p}(x_2,\mu_f) + (x_1 \leftrightarrow x_2)\Big]\hat{\sigma}_{g\bar{q}}^{(NLO)}.
\end{aligned}
$$

Transverse Momentum Distributions

- The corresponding fixed order result of the transverse momentum distribution is only valid when q_T is not too small compared with the mass of the massive graviton m_n. If $q_T \ll m_n$, large logarithms like $\ln(m_n^2/q_T^2)$ will appear and will dominate over the cross section.
- In order to make use of the perturbation theory with the existence of large logarithms at each order, one must reorganize the perturbative expansion to resum the large terms. In this paper, we use the Collins-Soper-Sterman (CSS) resummation formalism to calculate all order soft gluon effects on the transverse momentum distribution.

- $q_T \sim Q$:

$$\frac{d\sigma}{dq_T^2 dy} \sim \alpha_s(u_1 + u_2\alpha_s + u_3\alpha_s^2 + ...)$$

- $q_T << Q$ (Leading Log terms):

$$\frac{d\sigma}{dq_T^2 dy} \sim \frac{\alpha_s}{q_T^2}[v_1 \ln(\frac{Q^2}{q_T^2}) + v_2\alpha_s \ln^3(\frac{Q^2}{q_T^2}) + v_3\alpha_s^2 \ln^5(\frac{Q^2}{q_T^2}) + ...],$$

- More generally:

$$\frac{d\sigma}{dq_T^2 dy} \propto \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \ln^m(Q^2/q_T^2),$$

$$\frac{d\sigma}{dq_T^2 dy} \propto \frac{1}{q_T^2} \quad [\quad \alpha_s(L+1) \\ +\alpha_s^2(L^3+L^2+L+1) \\ +\alpha_s^3(L^5+L^4+L^3+L^2+L+1)+......],$$

where $L \equiv \ln(Q^2/q_T^2)$

- Resummation:

$$\frac{d\sigma}{dq_T^2 dy} \propto \frac{1}{q_T^2}[\alpha_s Z_1 + \alpha_s^2 Z_2 + \alpha_s^3 Z_3 + ...]$$

$$\alpha_s Z_1 = \alpha_s(L+1) + \alpha_s^2(L^3+L^2) + \alpha_s^3(L^5+L^4) + ...,$$

$$\alpha_s^2 Z_2 = \alpha_s^2(L+1) + \alpha_s^3(L^3+L^2) + ...,$$

$$\alpha_s^3 Z_3 = \alpha_s^3(L+1)...$$

$$Z_{n+1}/Z_n = \alpha_s$$

- In the CSS formalism, the differential cross section we are considering can be written as

$$\frac{d\sigma}{dq_T^2 dy}(\mathrm{total}) = \frac{d\sigma}{dq_T^2 dy}(\mathrm{resum}) + Y(q_T, m, x_1^0, x_2^0),$$

where

$$\frac{d\sigma}{dq_T^2 dy}(\mathrm{resum}) = \sum_{\alpha,\beta} \frac{d\sigma_{\alpha\beta}}{dq_T^2 dy}(\mathrm{resum})$$

$$Y(q_T, m, x_1^0, x_2^0) = \sum_{ab} Y_{ab}(q_T, m, x_1^0, x_2^0),$$

with $\alpha\beta = gg, q\bar{q}$, $ab = gg, q\bar{q}, q(\bar{q})g$.

- And the resummed part can be expressed as an inverse Fourier transformation

$$\frac{d\sigma_{\alpha\beta}}{dq_T^2 dy}(\mathrm{resum}) = \frac{1}{2}\sigma^0_{\alpha\beta}\frac{1}{2\pi}\int d^2\mathbf{b} \exp\left(i\mathbf{b}\cdot\mathbf{q}_T\right) W_{\alpha\beta}(b, m, x_1^0, x_2^0)$$

$$= \sum_{\alpha,\beta}\frac{1}{2}\sigma^0_{\alpha\beta}\int_0^\infty bdbJ_0(bq_T)W_{\alpha\beta}(b, m, x_1^0, x_2^0),$$

with

$$W_{\alpha\beta}(b, m, x_1^0, x_2^0) = \tilde{f}_{\alpha/A}(x_1^0, C_3/b)\tilde{f}_{\beta/B}(x_2^0, C_3/b)$$
$$\times \exp\left\{-\int_{C_1^2/b^2}^{C_2^2 m^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{C_2^2 m^2}{\bar{\mu}^2}A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right]\right\},$$

where $\mathbf{b}$ is the impact parameter conjugating to $\mathbf{q}_T$, J_0 is zero order Bessel function of the first kind, and $x_1^0 = e^y m_n/\sqrt{s}$, $x_2^0 = e^{-y} m_n/\sqrt{s}$.

- $C_i (i = 1, 2, 3)$ are constants of order 1 which are by convention chosen to be

$$C_1 = C_3 = 2e^{-\gamma_E} \equiv b_0, \; C_2 = 1.$$

- $\tilde{f}$ is the convolution of the PDFs and the coefficient functions C

$$\tilde{f}_{\alpha/h}(x, \mu) = \sum_{\gamma} \int_x^1 \frac{dz}{z} C_{\alpha\gamma}(z, \alpha_s(\mu)) f_{\gamma/h}(x, \mu).$$

- The coefficients A, B and C can be expanded to series in α_s

$$\begin{aligned} A(\alpha_s) &= \sum_{n=1}^{\infty} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n, \\ B(\alpha_s) &= \sum_{n=1}^{\infty} B^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n, \\ C_{\alpha\beta}(z, \alpha_s) &= \sum_{n=0}^{\infty} C_{\alpha\beta}^{(n)}(z) \left(\frac{\alpha_s}{\pi}\right)^n, \end{aligned}$$

  and they can be calculated order by order in perturbative theory.

- In our case, since the massive graviton is colorless, thus the lowest order coefficients is the same as the ones in the case of $gg \to H^0$ and Drell-Yan. For the gg channel, we have

$$A^{(1)} = 2N_c = 6, \qquad B^{(1)} = -2\beta_0 = (33 - 2n_f)/6,$$
$$C^{(0)}_{\alpha\beta}(z) = \delta_{\alpha\beta}\delta(1-z),$$

and for the $q\bar{q}$ channel, we have

$$A^{(1)} = C_F = \frac{4}{3}, \qquad B^{(1)} = -\frac{3}{2}C_F = -2,$$
$$C^{(0)}_{\alpha\beta}(z) = \delta_{\alpha\beta}\delta(1-z).$$

With these coefficients, we can actually sum up all terms like $\alpha_s^n L^{2n-1}$ and $\alpha_s^n L^{2n-2}$.

- However, the resummed part is still not able to be calculated perturbatively. The reason is that in the resummation part, the integral over the impact parameter b extends to infinity, while the integrand involves the strong coupling constant α_s and the PDFs at scale b_0/b, where they are not well defined if b is large enough so that b_0/b enters non-perturbative region.
- One can use a cut-off b_{max} and regard the effects from $b > b_{\text{max}}$ as non-perturbative input. Practically, one can replace $W(b)$ by

$$\widetilde{W}(b) = W(b_*) F_{\text{NP}}(b),$$

where

$$b_* = \frac{b}{\sqrt{1 + (b/b_{\text{max}})^2}},$$

and $F_{\text{NP}}(b)$ parameterizes the non-perturbative effects. Since b_* never exceeds b_{max}, $W(b_*)$ can be calculated perturbatively, and the theoretical uncertainty mainly relies on the function F_{NP}.

- Landry, Brock, Nadolsky and Yuan (BLNY) proposed the form

$$F_{\mathrm{NP}} = \exp\left\{-b^2\left[g_1 + g_2 \ln\frac{m}{2Q_0} + g_1 g_3 \ln(100 x_1^0 x_2^0)\right]\right\},$$

They take $b_{\max} = 0.5\mathrm{GeV}^{-1}$, $Q_0 = 1.6\mathrm{GeV}$ and the parameters $g_i (i = 1, 2, 3)$ are fitted to the available Drell-Yan data, which are given by

$$g_1 = 0.21, \qquad g_2 = 0.68, \qquad g_3 = -0.60.$$

- The Y term, is the remaining contributions which are not resummed. Since it contains no large logarithms, it can be reliably computed by fixed order truncation of the perturbative series

$$Y_{ab} = \frac{d\sigma_{ab}}{dq_T^2 dy}(\mathrm{pert}) - \frac{d\sigma_{ab}}{dq_T^2 dy}(\mathrm{asym}),$$

where the first term in the right hand is the fixed-order perturbative results, and the second term is the asymptotic part of the differential cross section, defined as the terms which are at least as singular as $1/q_T^2$ when $q_T \to 0$, which can be got by expanding the resummed part.

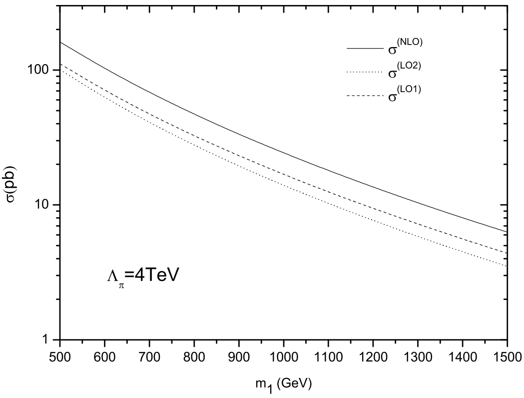
Numerical results

- We choose the input parameters as Λ_π and m_1. For $\Lambda_\pi = 4\,(8)\,TeV$, from current constraints, we have $150\,\mathrm{GeV} < m_1 < 1.5\,\mathrm{TeV}$ $(300\,\mathrm{GeV} < m_1 < 3\,\mathrm{TeV})$.
- For the NLO total cross sections $\sigma^{(NLO)}$ and the contributions from different parts, the NLO ($\overline{\mathrm{MS}}$) PDFs is used throughout this paper.
- For the LO results, we define two cross sections as following:

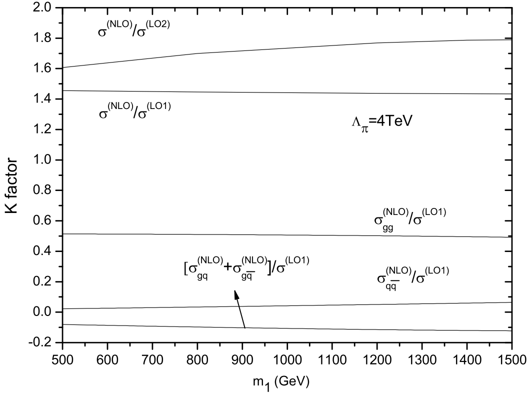
$$\sigma^{(\mathrm{LO1})}: \quad \text{LO partonic cross section convoluted with NLO } (\overline{\mathrm{MS}}) \text{ PDFs;}$$
$$\sigma^{(\mathrm{LO2})}: \quad \text{LO partonic cross section convoluted with LO PDFs,}$$

and correspondingly two K factors:

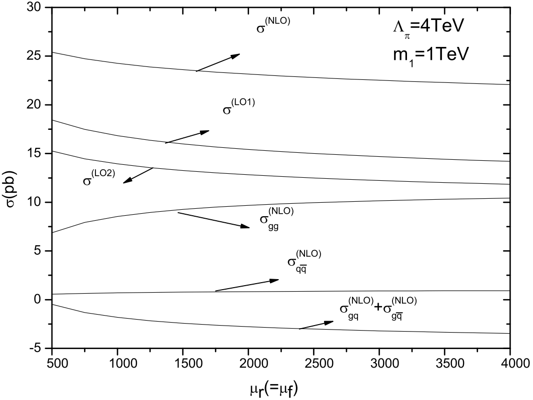
$$K_1 = \frac{\sigma^{(\mathrm{NLO})}}{\sigma^{(\mathrm{LO1})}}, \qquad K_2 = \frac{\sigma^{(\mathrm{NLO})}}{\sigma^{(\mathrm{LO2})}}.$$



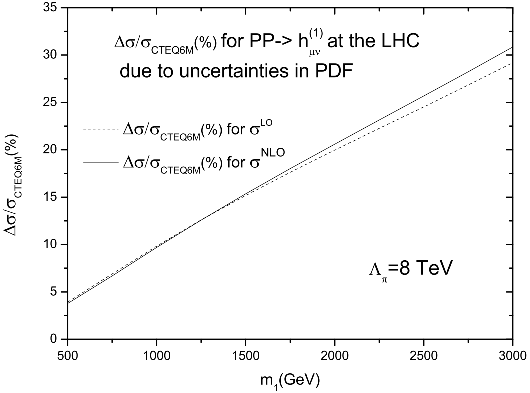
Dependence of the total cross sections for the first KK graviton excitation mode direct production at the LHC on m_1.



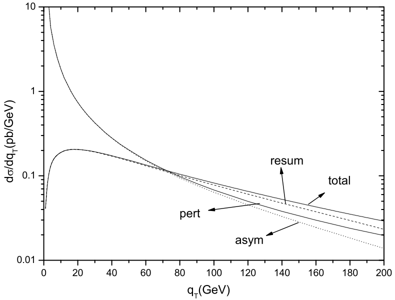
Dependence of the K-factor on m_1.



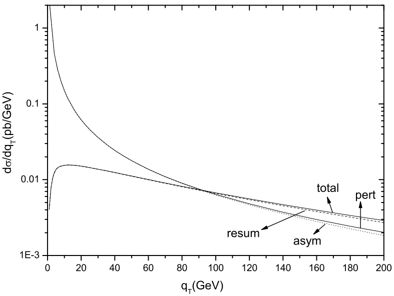
Dependence of the total cross sections for the first KK graviton excitation mode direct production at the LHC on $\mu_r = \mu_f$.



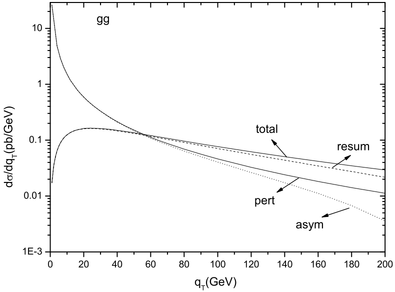
The PDF dependence of the total cross sections for the first KK graviton excitation mode direct production at the LHC, as functions of m_1.



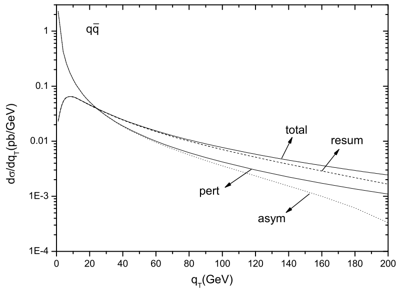
The transverse momentum distribution of the first KK graviton excitation mode from $pp \to h_{\mu\nu}^{(1)}$ process at the LHC, assuming $\Lambda_\pi = 4\,\text{TeV}$ and $m_1 = 1\,\text{TeV}$.



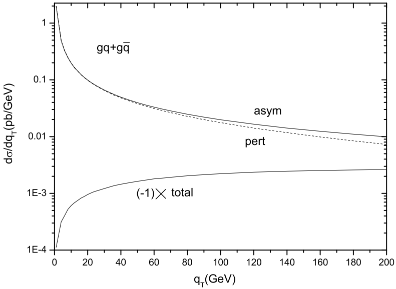
The transverse momentum distribution of the first KK graviton excitation mode from $pp \to h_{\mu\nu}^{(1)}$ process at the LHC, assuming $\Lambda_\pi = 4\,\text{TeV}$ and $m_1 = 2\,\text{TeV}$.



The gg part of the transverse momentum distribution of the first KK graviton excitation mode, assuming $\Lambda_\pi = 4\,\text{TeV}$ and $m_1 = 1\,\text{TeV}$.



The $q\bar{q}$ part of the transverse momentum distribution of the first KK graviton excitation mode, assuming $\Lambda_\pi = 4\,\text{TeV}$ and $m_1 = 1\,\text{TeV}$.



The gq and $g\bar{q}$ part of the transverse momentum distribution of the first KK graviton excitation mode, assuming $\Lambda_\pi = 4\,\text{TeV}$ and $m_1 = 1\,\text{TeV}$.

Conclusion

- we have calculated the next-to-leading order total cross section and transverse momentum distribution of single massive graviton production at the LHC in the RS model, including all-order soft gluon resummation effects.
- The LO total cross sections are in general over several pb in most of the parameter space, and can reach 100 pb when $m_1 = 500\,\mathrm{GeV}$.
- The NLO corrections enhance significantly the total cross sections, which is in general several tens percent, and reduce efficiently the dependence of the total cross sections on the renormalization/factorization scale.

- We have also examined the uncertainty in total cross sections due to the PDF uncertainties, and found that the uncertainty in NLO cross sections is slightly larger than that in LO ones, especially at large m_1.
- For the transverse momentum distribution, within the CSS resummation formalism, we resum the logarithmically-enhanced terms at small q_T to all orders up to NLO logarithmic accuracy. Combined with the fixed order calculations, we give consistent predictions for both small q_T and large q_T.

Thank You!