Magic field option to cancel the false EDM

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Outline

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What is the false EDM?

Spin relaxation theory tells us that a fluctuating transverse magnetic field produces shifts in neutron and Hg precession frequencies

$$\delta\omega = \frac{\gamma^2}{2}\int_0^\infty d\tau \, \mathrm{Im}\left[e^{i\omega\tau}\langle b^*(0)b(\tau)\rangle\right]$$

$$b(\tau) = \left[\mathbf{B}_T(\mathbf{r}(\tau)) + \frac{\mathbf{E}}{c^2}\times\dot{\mathbf{r}}(\tau)\right]\cdot\left[\mathbf{e_x} + i\mathbf{e_y}\right]$$

non-uniform + motional

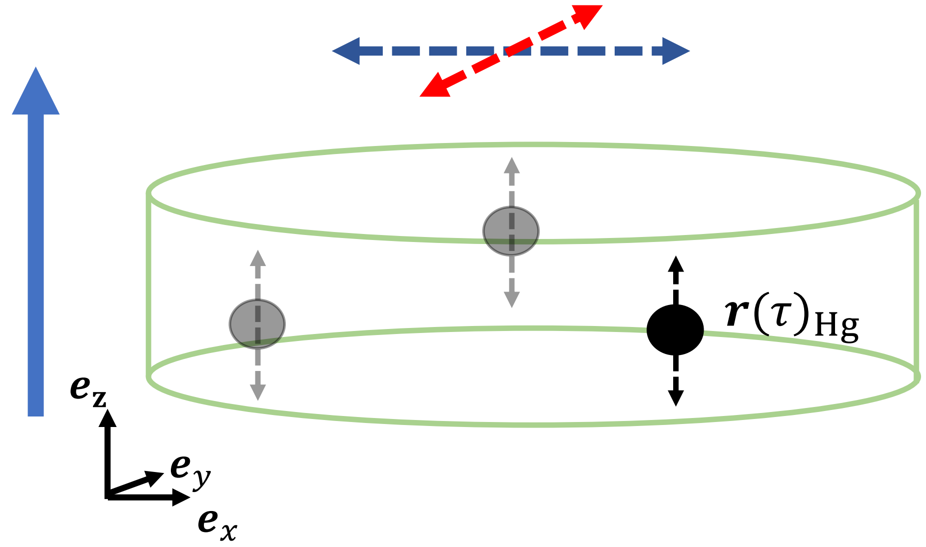
Frequency shifts odd in E (opposite E and B configurations) produce a **false EDM**

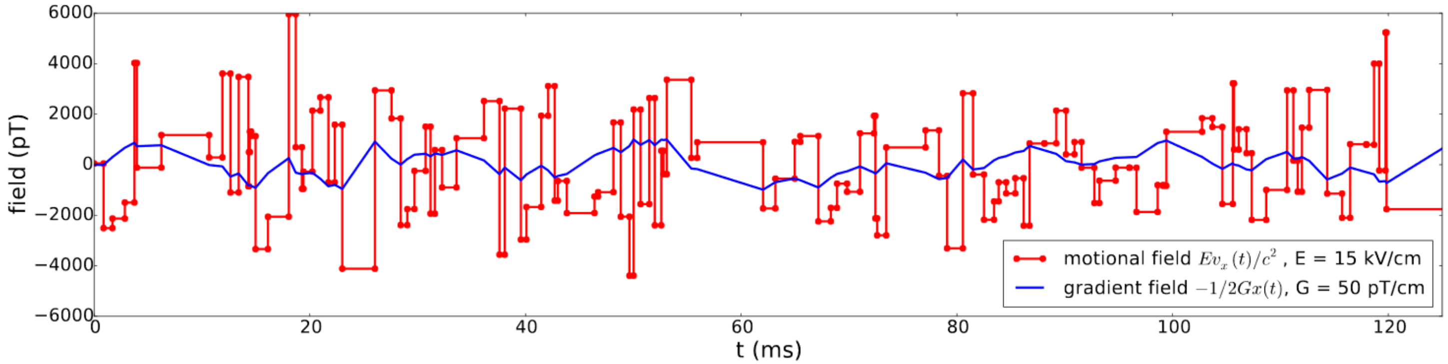
$$d^{\text{false}} = \frac{\hbar}{4|E|}\left(\delta\omega(-E) - \delta\omega(E)\right)$$

$$d_n^{\text{false}}$$

$$d_{n\leftarrow\text{Hg}}^{\text{false}} = \left|\frac{\gamma_n}{\gamma_{\text{Hg}}}\right| d_{\text{Hg}}^{\text{false}}$$

Main contribution to false EDM



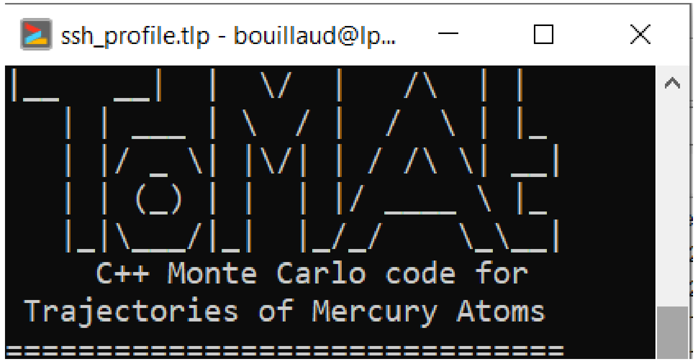


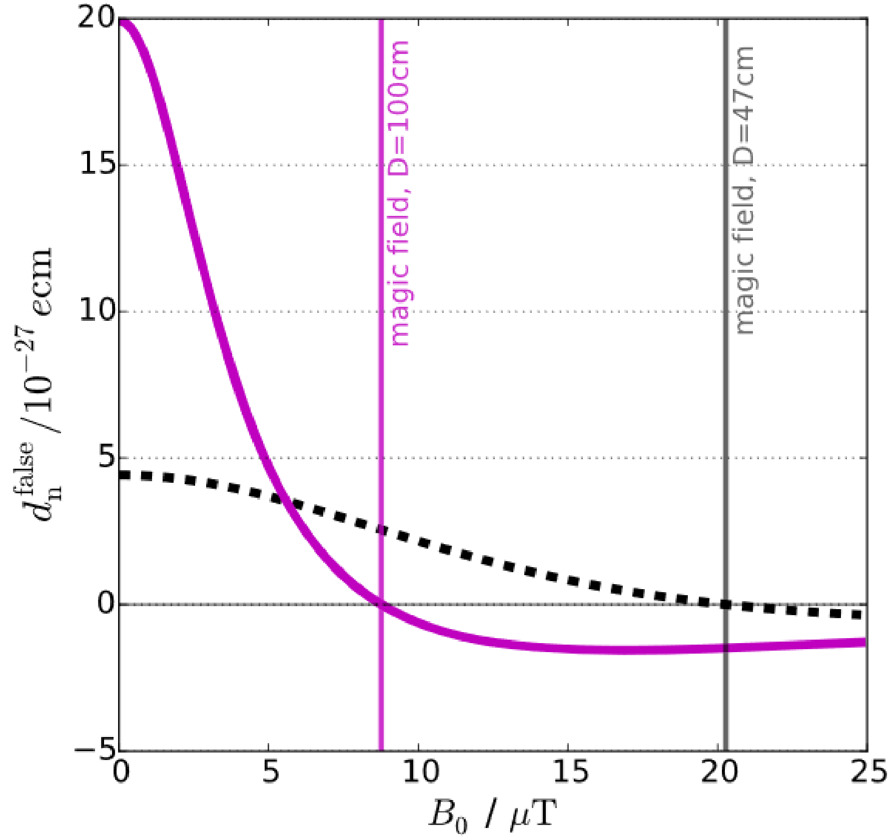
To sum up, the false EDM is produced by the correlation between the motional and the non-uniform transverse fluctuating fields :

$C(\tau)$

$$d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}} = \frac{\hbar\left|\gamma_n\gamma_{\mathrm{Hg}}\right|}{2c^2}\int_0^\infty d\tau\cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

- Can be numerically calculated (TOMAt monte-carlo simulation)





False EDM produced by a linear gradient G_1 as a function of holding field B_0

1. $d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}}(B_0)$ increases with chamber size → concerning for n2EDM
2. $d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}}(B_0)$ has a zero crossing ! We call this the magic value B_{0m}

Motivations for the magic field option

__What we're doing right now:__

$B_0 = 1\mu\mathrm{T}$ → *low frequency regime* approximation
$\omega\tau_c = \gamma_{\mathrm{Hg}} B_0 \tau_c \ll 1$:

$$d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}} = -\frac{\hbar\left|\gamma_n\gamma_{\mathrm{Hg}}\right|}{2c^2}\langle xB_x + yB_y\rangle$$

plug in polynomial expansion of B

$$\mathbf{B}(\mathbf{r}) = B_0\,\mathbf{e_z} + \sum_{l=1}^{+\infty}\sum_{m=-l}^{l} G_{lm}\mathbf{\Pi_{lm}}(\mathbf{r})$$

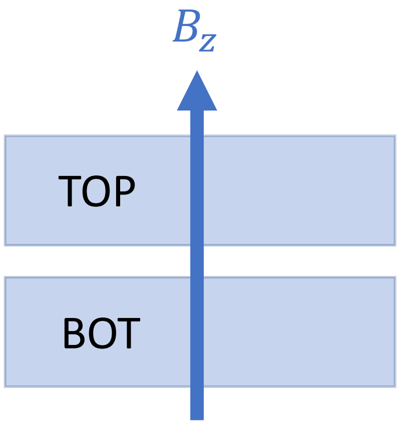
where only $\boldsymbol{l}$**-odd,** $\boldsymbol{m} = \mathbf{0}$ terms generate a false EDM (cylindrical symmetries)

False EDM can be split into 2 contributions:

- *top-bottom gradient*

$$G_{\mathrm{TB}} = \frac{\langle B_z\rangle_{TOP} - \langle B_z\rangle_{BOT}}{H'}$$

- *odd phantom modes* (some linear combinations of $\boldsymbol{l}$**-odd,** $\boldsymbol{m} = \mathbf{0}$ modes that do not produce top-bottom gradient)



__Example:__ With some field configuration

$$\mathbf{B} = B_0\mathbf{e_z} + G_{\mathrm{TB}}\mathbf{\Pi_{10}} + \acute{G}_3\acute{\mathbf{\Pi}}_3 + \acute{G}_5\acute{\mathbf{\Pi}}_5 + \cdots$$

$$d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}} = \frac{\hbar\left|\gamma_n\gamma_{\mathrm{Hg}}\right|}{8c^2}R^2\left(G_{\mathrm{TB}} + \acute{G}_3 + \acute{G}_5 + \cdots\right)$$

1. Introduction

ii. Motivations for the magic field option

	G_{TB}	$\acute{\Pi}_{\mathbf{3}}$	$\acute{\Pi}_{\mathbf{2n+1}}, \boldsymbol{n > 1}$
Expression	$G_{10} - L_3^2 G_{30} + L_5^4 G_{50} - \ldots$	$c_3\left(\mathbf{\Pi}_{10} + \frac{1}{L_3^2}\mathbf{\Pi}_{30}\right)$	$c_{2n+1}\left(\mathbf{\Pi}_{10} - \frac{(-1)^n}{L_{2n+1}^{2n}}\mathbf{\Pi}_{2n+1,0}\right)$
Gradients	G_{TB}	$\acute{G}_{\mathbf{3}}$	$\acute{G}_{\mathbf{2n+1}}, \boldsymbol{n > 1}$
Measurement method	Hg co-magnetometers (online)	Cs magnetometers (online)	Mapper (offline)
Requirement type	Accuracy	Accuracy	Accuracy + reproducibility
Requirement magnitude at $1\mu\mathrm{T}$	$\delta B_{\mathrm{Hg}} < 100\ \mathrm{fT}$	$\delta \acute{G}_3 < 20\ \mathrm{fT/cm}$	$\delta \acute{G}_5 < 20\ \mathrm{fT/cm}$

Currently we operate at $B_0 = 1\mu\mathrm{T}$ → strict requirements on measurement accuracy and reproducibility of $\acute{\Pi}_{\mathbf{2n+1}}$

<u>Another option</u>, the magic field option, is to increase B_0 to a value that cancels or diminishes the false EDM produced by specific phantom modes

We saw earlier that there existed B_{0m} such that $d_{n\leftarrow\mathrm{Hg}}^{\mathrm{false}}(B_{0m}) = 0$ for a linear gradient, let's now look at it in more detail and tackle the general case

Magic field : general approach

B_0 goes to values that **do not allow the *low frequency* approximation**

→ compute **general expression** of $C(\tau)$ numerically (TOMAt) for several $\boldsymbol{B_0}$ **+ $\boldsymbol{\Pi_{2n+1}}$ configurations.**

- $C(\tau)$ written using symmetries of $\boldsymbol{B}$ and properties of correlation functions as a linear combination of correlation functions $C_{ij}(\tau)$ involving trajectories $x(t)$ and $y(t)$:

$$C(\tau) = c_{2n+1}\left[G_{10}\langle x(\tau)x(0)\rangle - G_{2n+1}\frac{(-1)^n}{L_{2n+1}^{2n}}\sum_{i,j,k}\alpha_{ij2k}\frac{H^{2k}}{(2k+1)2^{2k}}\langle x(\tau)x^i(0)y^j(0)\rangle\right]$$

Phantom mode order

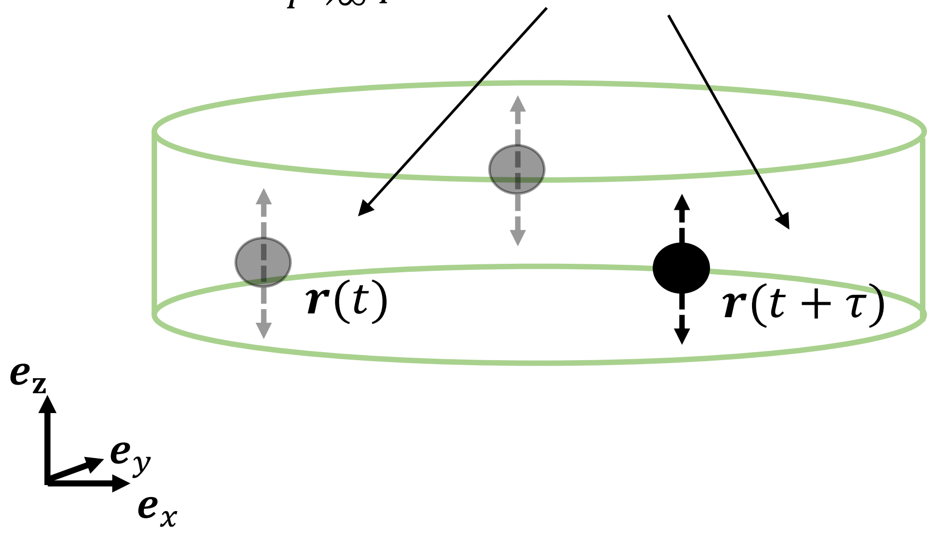
$$i + j + 2k = 2\boldsymbol{n} + 1$$

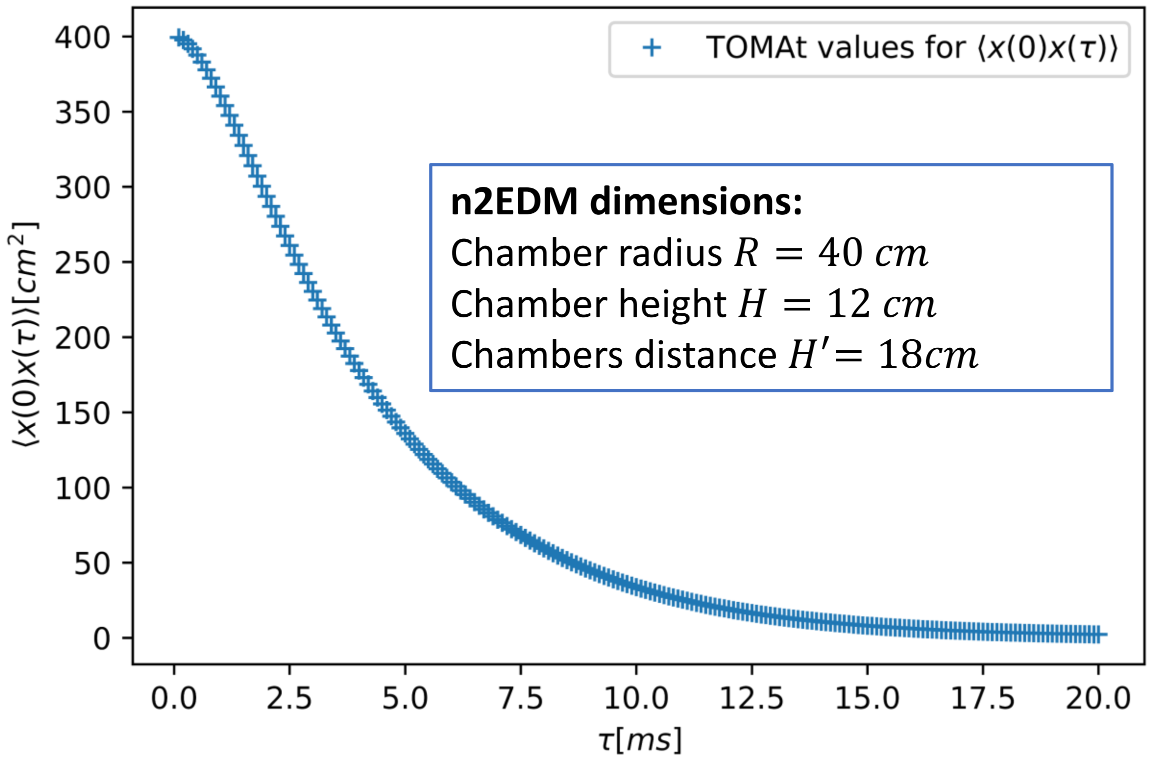
- Correlation terms $C_{ij}(\tau)$ calculated with TOMAt and fitted
- $d_{n\leftarrow \mathrm{Hg}}^{\mathrm{false}}(\omega)$ derived analytically with fitted expressions of $C(\tau)$

→ B_{0m} **for each $\boldsymbol{B_0}$ + phantom mode** configuration

How does TOMAt calculate correlation functions ?

1. Simulates trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$
   - Set of collision points (**assume collisions only with walls**) ⇒ constant velocities between two points
   - Velocities are Maxwell-Boltzman distributed (diffuse reflection)
2. Calculates $\langle x(\tau)x^i(0)y^j(0)\rangle$
   - Uses **ergodicity hypothesis**: average over all particles ⇔ time average of one particle over infinite time: $\lim_{T\to\infty}\frac{1}{T}\int_0^\infty \mathrm{d}t\, x(t)x(t+\tau)$





Correlation between positions $x(0)$ and $x(\tau)$ of one Hg molecule as a function of τ

Correlation function fit

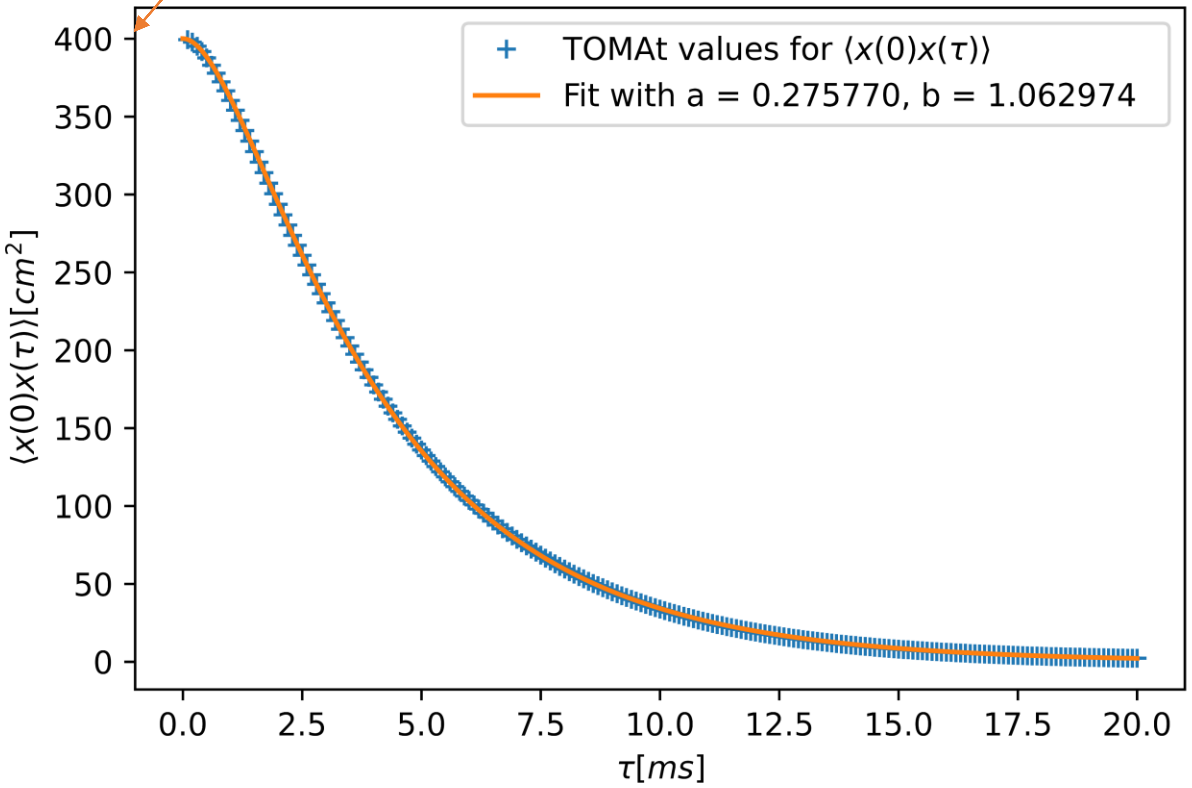
We know we can fit correlation functions with a double exponential model

$$C_{ij}(\tau) = A_{ij}e^{-a_{ij}\tau} - B_{ij}e^{-b_{ij}\tau}$$

Physical intuition :

- Correlation between two positions of one particle at time difference τ goes to 0 as $\tau \to \infty$
- No correlation between velocity and position at identical time $\Rightarrow$ null slope at $\tau = 0$

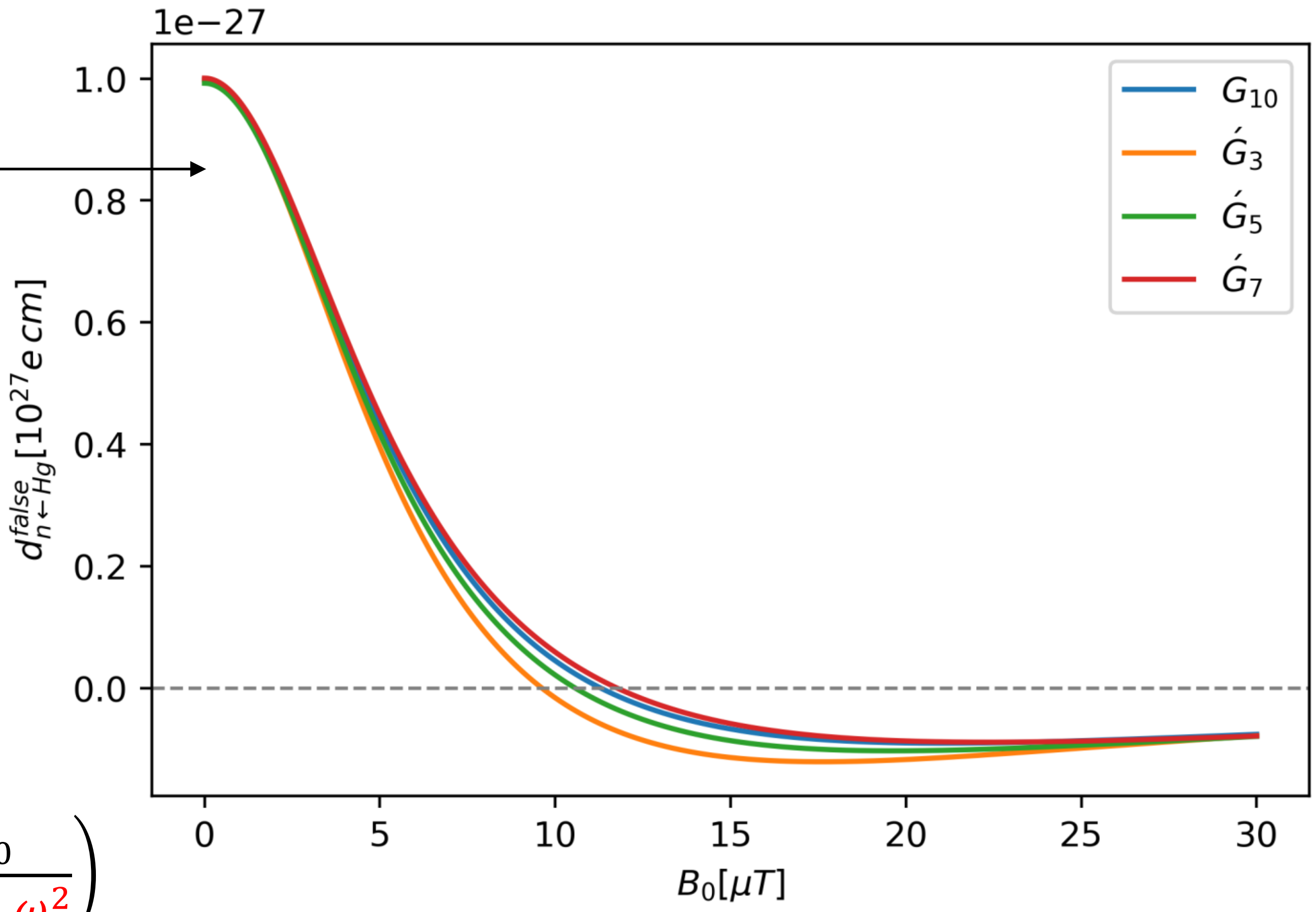
$$C_{10}(0) = \frac{R^2}{4}$$



Example: $C_{10}(\tau) = \langle x(\tau)x(0)\rangle$

Results and magic values

$d_{n\leftarrow \mathrm{Hg}}^{\mathrm{false}(n)}(\omega = \mu_{\mathrm{Hg}} B_0)$ for 4 field configurations: →



$$B_0 \mathbf{e_z} + \acute{G}_{2n+1} \acute{\mathbf{\Pi}}_{2n+1} \qquad n = 0,1,2,3$$

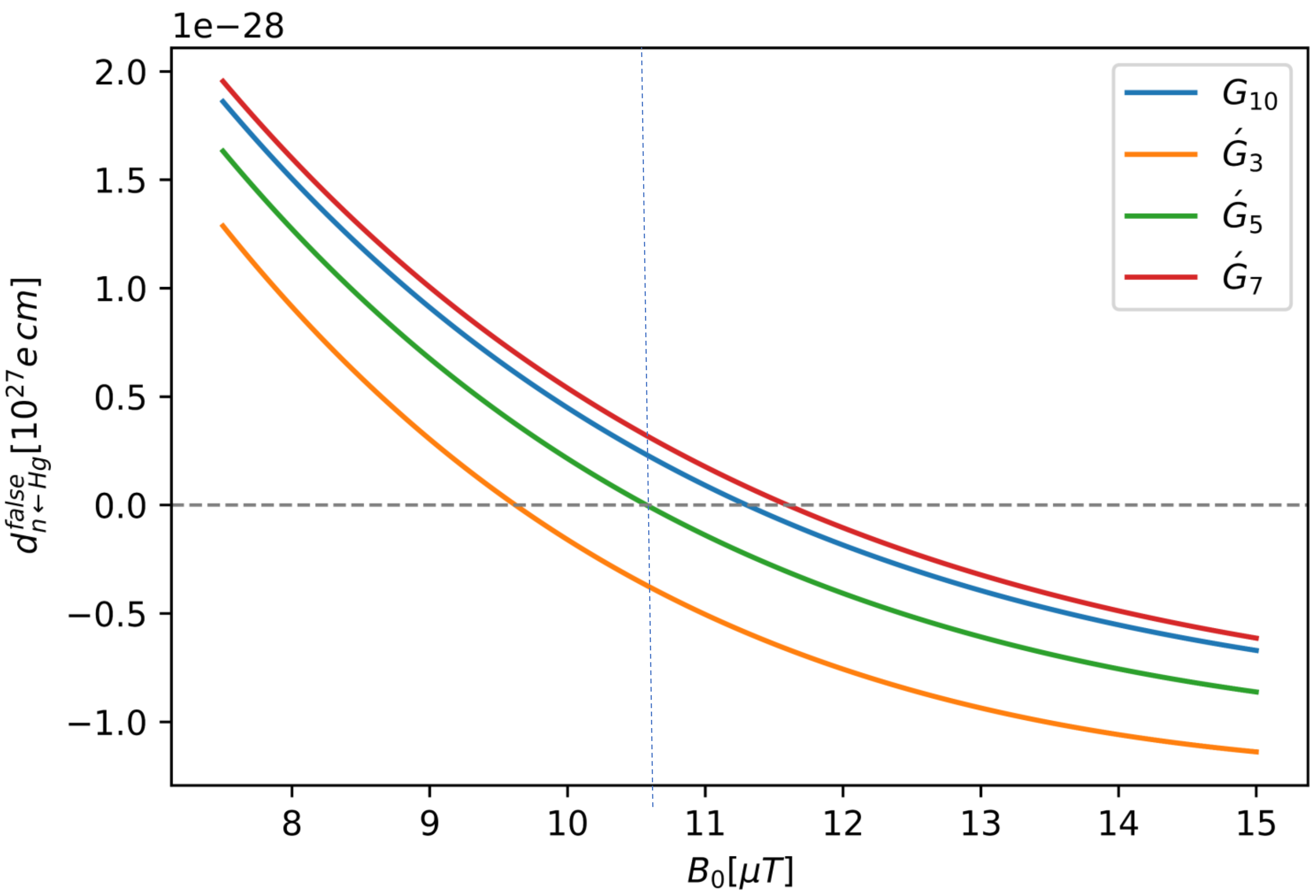
Analytical expression derived with fitted correlation functions:

$$d_{n\leftarrow \mathrm{Hg}}^{\mathrm{false}(n)}(\omega) = \frac{\hbar \left|\gamma_n \gamma_{\mathrm{Hg}}\right|}{2c^2} c_{2n+1} \left[G_{10} \left(A_{10} \frac{a_{10}^2}{a_{10}^2 + \omega^2} - B_{10} \frac{b_{10}^2}{b_{10}^2 + \omega^2} \right) - G_{2n+1} \frac{(-1)^n}{L_{2n+1}^{2n}} \sum_{i,j,2l} \alpha_{ij2l} \frac{H^{2l}}{(2l+1)2^{2l+1}} \left(A_{ij} \frac{a_{ij}^2}{a_{ij}^2 + \omega^2} - B_{ij} \frac{b_{ij}^2}{b_{ij}^2 + \omega^2} \right) \right]$$

Best magic value for a combination of those phantom modes ?

One possibility is to set it to $B_{0m,5}$

- Cancels 5th order mode
- Suppresses 3rd order mode by a factor 30
- Suppresses the 7th order mode by a factor 40



$B_{0m,3} = 9.6\mu\mathrm{T}$ $B_{0m,5} = 10.5\mu\mathrm{T}$ $B_{0m,1} = 11.3\mu\mathrm{T}$ $B_{0m,7} = 11.6\mu\mathrm{T}$

Conclusion

The increased sensitivity of the n2EDM apparatus comes at the price of an intensified false EDM. By increasing the **holding field value** to the **magic value** of one of the less controlled **phantom modes**, the magic field option will allow us to do a measurement with very limited systematics.

On the other hand a higher B_0 is not as easily kept uniform and stable. With $B_{0m} \sim 10\mu\mathrm{T}$, this difficulty increases by an order of magnitude.

Details on TOMAt

Two modules:

1. Simulates trajectories $(x(t), y(t), z(t))$
2. Calculates correlation functions $\langle F(0)G(\tau)\rangle$

- Sequence of collision points $(x(t), y(t), z(t))$
- Assumes constant velocity between two points
- Reflection is either *specular* (normal velocity changes direction) or *diffuse* (new velocities are Maxwell-Boltzman distributed)

- Assume $F(t) = x^a(t)y^b(t)z^c(t)$ and $G(t) = x^i(t)y^j(t)z^k(t)$
- Use ergodicity: $\langle F(0)G(\tau)\rangle = lim_{T\to+\infty}\frac{1}{T}\int_0^T dt\ F(t)G(t+\tau)$
- Split $[0, T]$ into intervals $\Delta T_n = T_{n+1} - T_n$ such that $x(t), y(t), z(t)$ and $x(t+\tau), y(t+\tau), z(t+\tau)$ are linear functions of t for $T_n < t < T_{n+1}$:
$$\langle F(0)G(\tau)\rangle = \frac{1}{T}\sum_n \Delta T_n I_n$$
- Calculate the integral I_n on ΔT_n recursively

Details on fit

Correlation functions satisfy:

- $C_{ij}(0) = \langle x^{i+1} y^j \rangle$
- $\lim_{\tau \to +\infty} C(\tau) = 0$
- $\frac{dC(\tau)}{d\tau}|_{\tau=0} = 0$

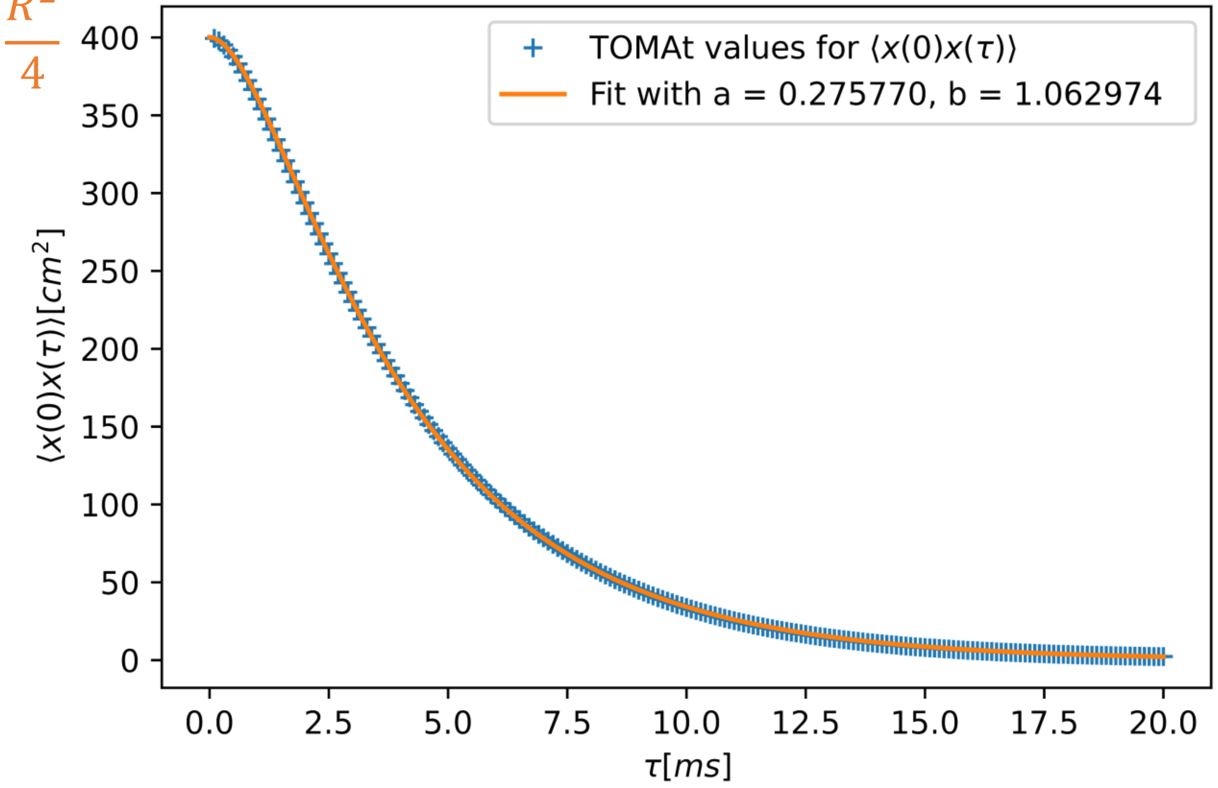
$\Rightarrow$ double exponential fit of correlation terms

$$C_{ij}(\tau) = A_{ij} e^{-a_{ij}\tau} - B_{ij} e^{-b_{ij}\tau}$$

with constrained parameters $a, b, A, B > 0$

- $A_{ij} = \frac{b_{ij} C_{ij}(0)}{b_{ij} - a_{ij}}$
- $B_{ij} = \frac{a_{ij} C_{ij}(0)}{b_{ij} - a_{ij}}$

$C_{10}(0) = \frac{R^2}{4}$



Example

$C_{10}(\tau) = \langle x(\tau)x(0) \rangle$