Spin Dressing Studies for nEDM@SNS

Raymond Tat

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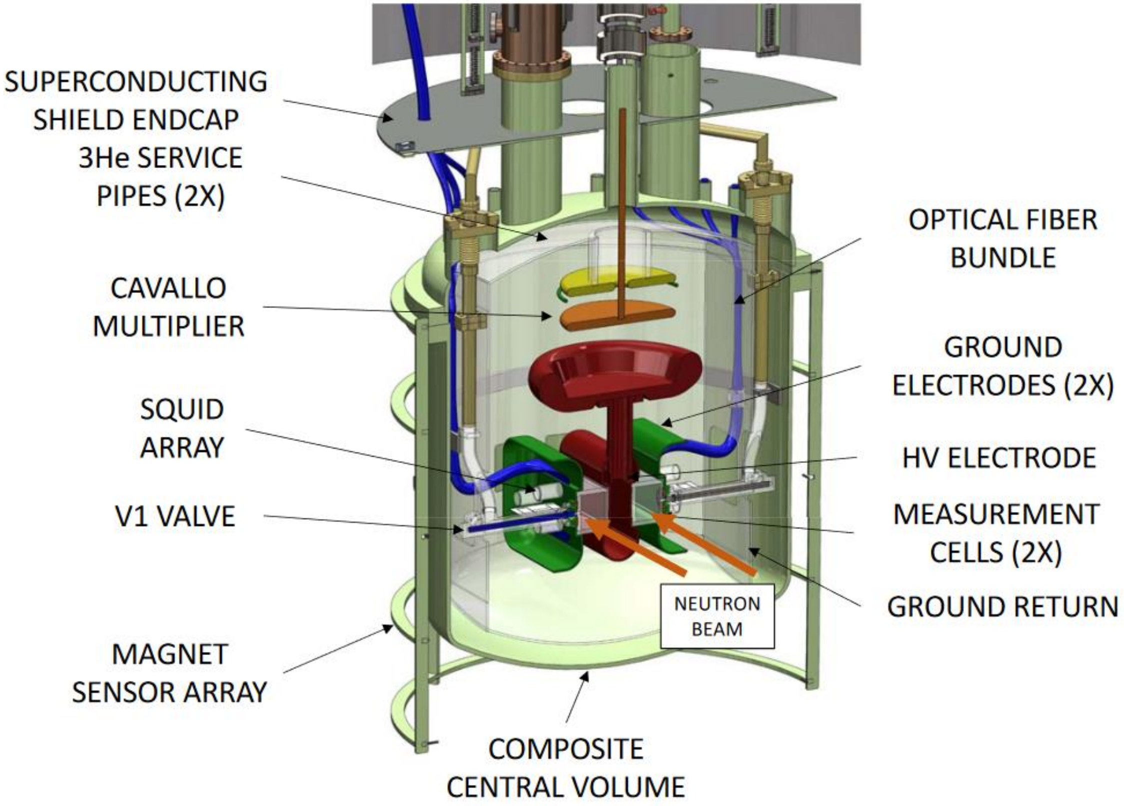
nEDM@SNS Overview

- Polarized ultracold neutrons and helium-3 atoms precess in a magnetic and electric field.
- The spin-dependent capture rate is measured.

$$n + {}^3He \rightarrow {}^3H + p + 764\ \mathrm{keV}$$



M.W. Ahmed *et al* 2019 *JINST* **14** P11017

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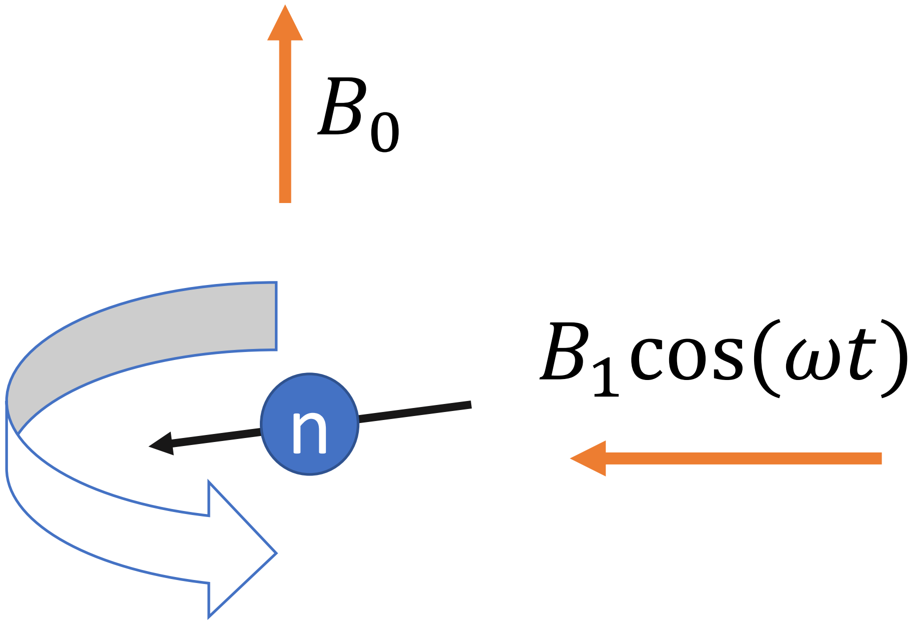
Spin Dressing – why and how

- Applying an off-resonant RF magnetic field perpendicular to the static holding field "dresses" the spins – modifying the gyromagnetic ratio γ



$$\gamma' \approx J_0\left(\frac{\gamma B_1}{\omega}\right)\gamma$$

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Spin Dressing – why and how

- Idea: Apply an RF magnetic field to make the helium-3 atoms and neutrons precess at the same frequency.

- $\vec{B} = B_0\hat{z} + B_1\cos(\omega t)\hat{x}$
- $\frac{\gamma_3}{\gamma_n} \approx 1.1$
- $\gamma' \approx J_0\left(\frac{\gamma B_1}{\omega}\right)\gamma$

Parameter	Approximate Value
ω	$2\pi \times 1000$ Hz
ω_0	$2\pi \times 100$ Hz
B_0	30 mG
B_1	400 mG

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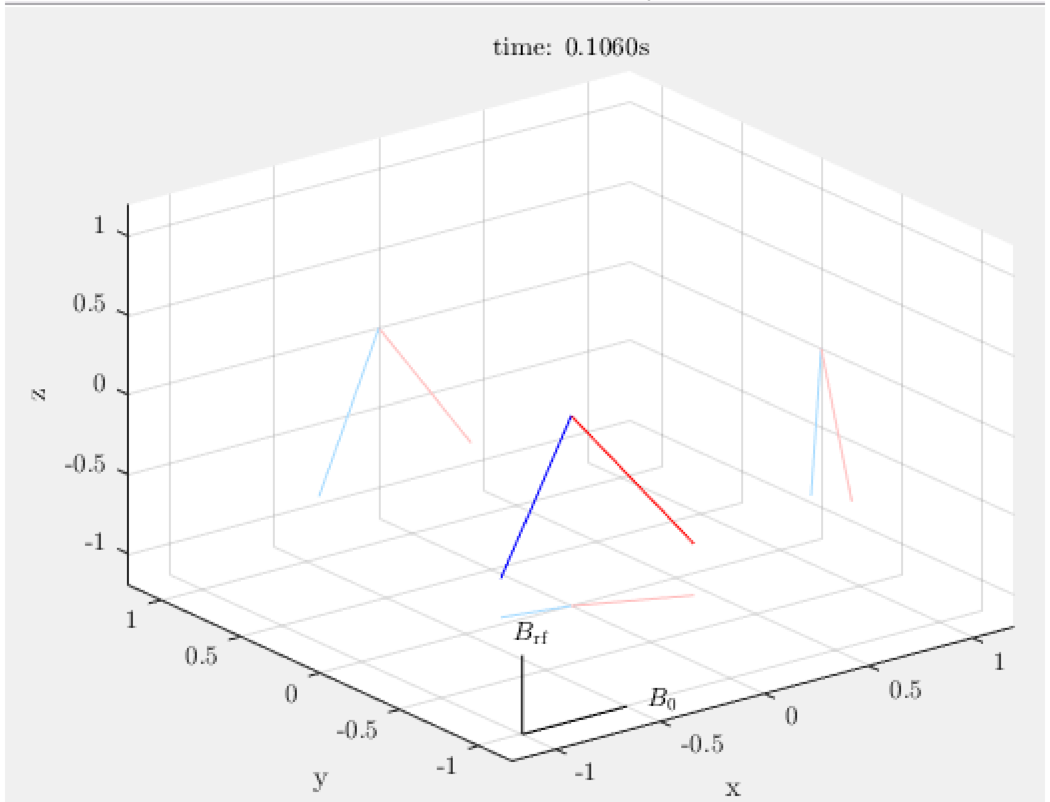
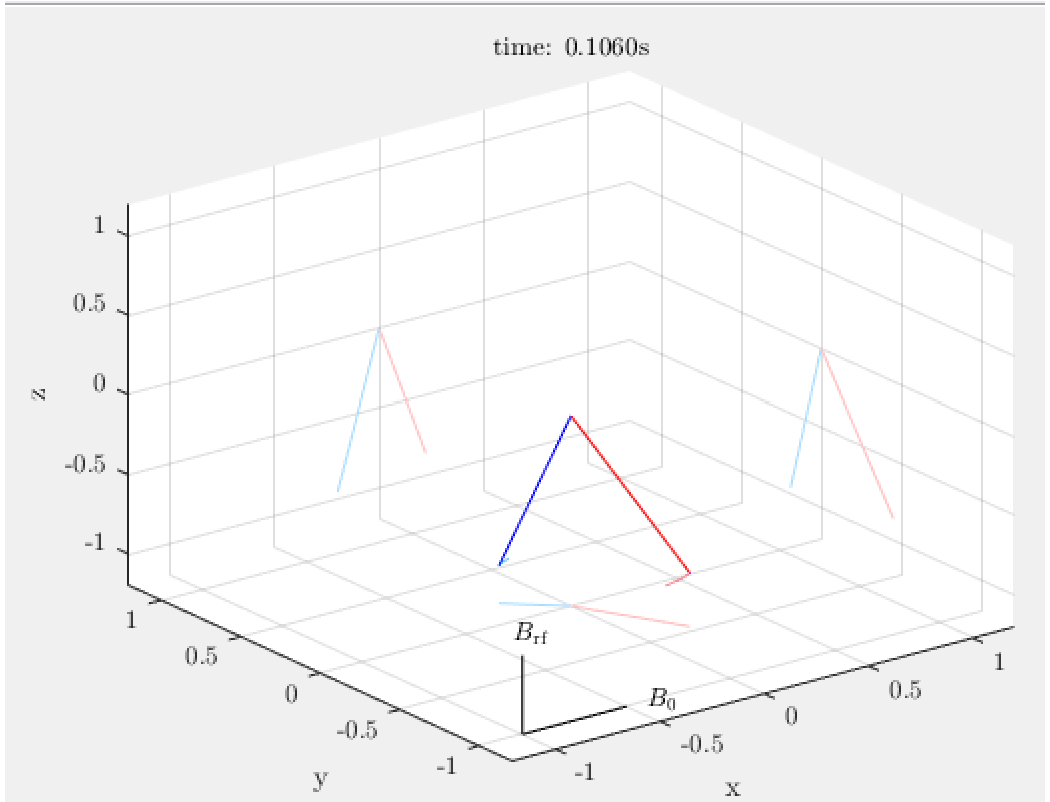
Critical Dressing Animation

ω=1000 rad/s



ω=10,000 rad/s



GIF credit: Ezra Webb

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Noise from the Dressing Field

- What if there is noise in the spin dressing field?
- $\vec{B} = B_0\hat{z} + B_1 \cos(\omega t)\,\hat{x} + \delta B(t)\hat{x}$
- Two cases to consider:
    1. All neutrons in a cell experience the same $\delta B(t)$ (e.g. current fluctuations)
    2. Neutrons in a cell experience different $\delta B(t)$ (e.g. magnetic field gradients)
- We want relaxation times/frequency shifts in terms of power spectrum $S(\omega)$ of $\delta B(t)$.

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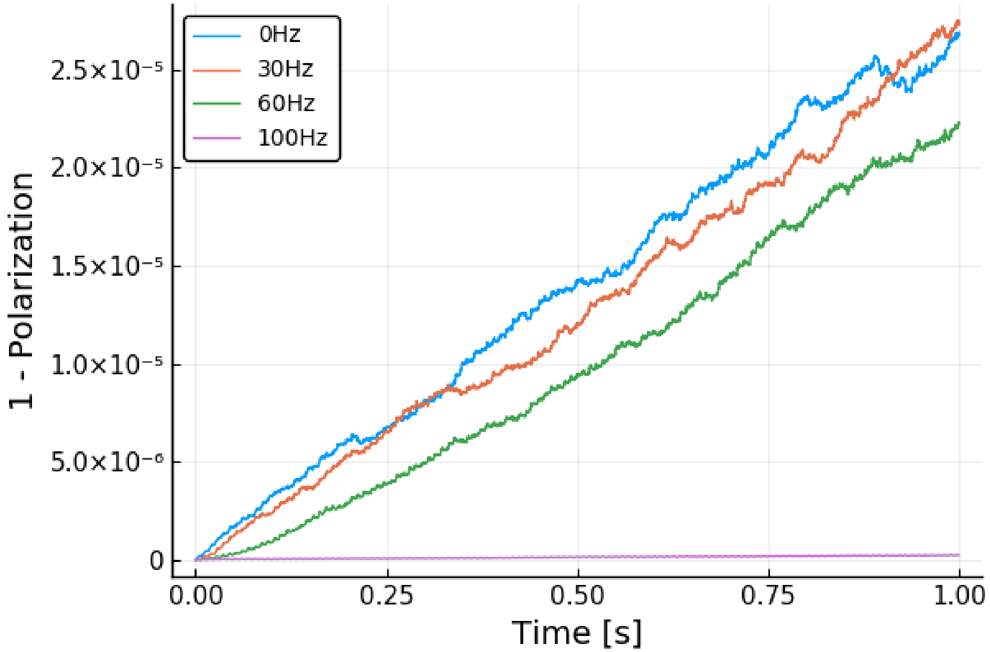
Monte Carlo Simulation

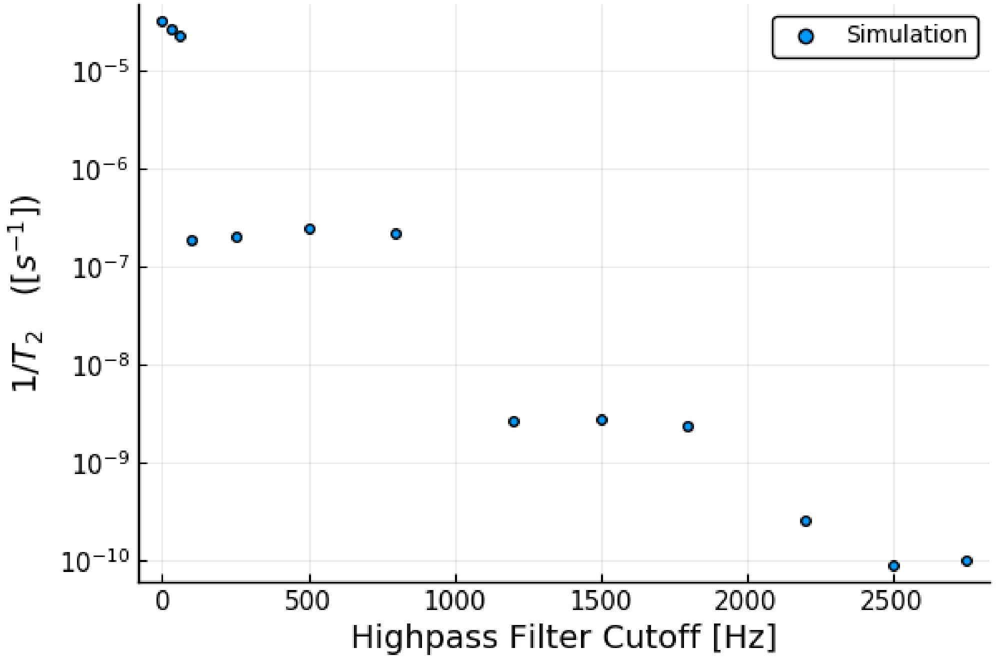
- Simulate Bloch equations numerically with randomly chosen $\delta B(t)$.
  - I use an adaptive Runge-Kutta integrator of order 5.
  - To generate $\delta B(t)$, I start with white noise and apply highpass filters to get the desired spectrum.

$$\frac{d\vec{\sigma}}{dt} = \gamma\vec{\sigma} \times \left(B_0\hat{z} + \left(B_1\cos(\omega t) + \delta B(t)\right)\hat{x}\right)$$

Simulation Results (1/2)

- (1 – Polarization) vs. time (left) and $\frac{1}{T_2}$ vs. cutoff frequency (right)
- Effect of noise is mostly concentrated at a few frequencies.





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*In the case where all neutrons in a cell feel the same $\delta B(t)$, T_2 should be interpreted not as transverse relaxation time but a run-to-run variance of the spin vectors

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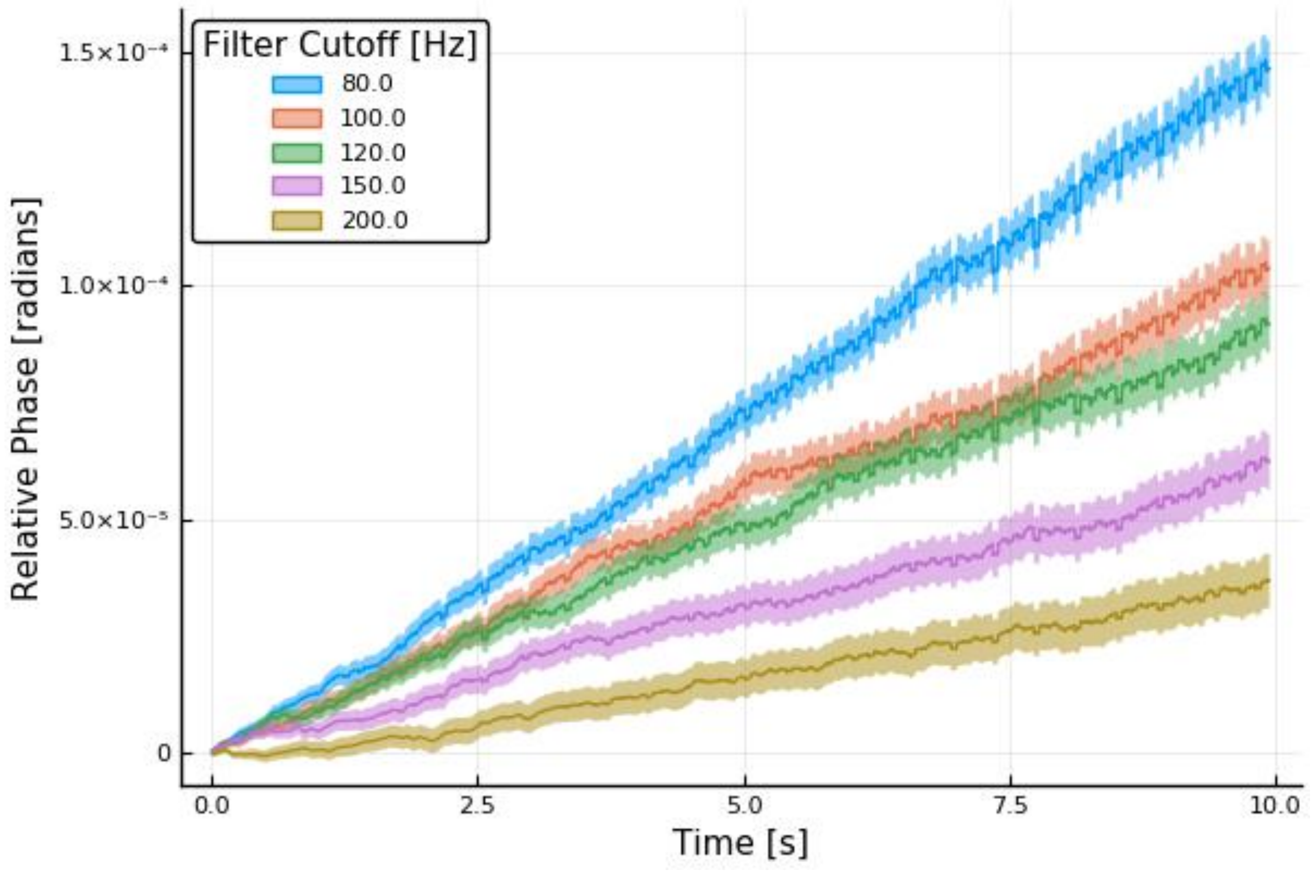
Simulation Results (2/2)

- Colored noise can also induce a phase shift
- Shown: average noise-induced phase shift, as compared to a neutron with no noise applied.
- Effect is much smaller than the noise on the previous slide.



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Previous Results Predicting Noise with Perturbation Theory

- Apply 2nd-order time-dependent perturbation theory, and assume $\omega \gg \omega_0$ (see Swank 2018, "Spin-dressed relaxation and frequency shifts from field imperfections")
- Result: $\frac{1}{T_2} = \frac{\gamma^2}{4} S(\omega_0) + \left(\frac{\gamma J_1(x)\omega_0}{\omega}\right)^2 S(\omega)$

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New Results

- Previous results don't capture the effect of noise at 2000 Hz and 3000 Hz. What's missing?
- Answer: $O\left(\frac{\omega_0}{\omega}\right)$ effects.
    - Time-**independent** perturbation theory gives the first-order correction to the eigenstates of the dressed spin Hamiltonian.

$$H = \underbrace{\omega a^\dagger a + \frac{\Omega}{2}\sigma_x(a + a^\dagger)}_{\text{Diagonalize Exactly}} + \underbrace{\frac{\omega_0}{2}\sigma_x}_{\text{1st order time-independent}} + \underbrace{\frac{\gamma}{2}\delta B(t)\sigma_x}_{\text{2nd order time-dependent}}$$

Diagonalize Exactly

1st order time-independent

2nd order time-dependent

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Perturbation Theory Result (1/2)

$$\frac{1}{T_2} = \frac{\gamma^2}{4} S(\omega_0') + \left(\frac{\gamma J_1(x)\omega_0}{\omega}\right)^2 S(\omega) + \frac{1}{4}\left(\frac{\gamma J_2(x)\omega_0}{2\omega}\right)^2 [S(2\omega - \omega_0') + S(2\omega + \omega_0')]$$

$$x \equiv \gamma B_1/\omega$$

$$\delta\omega = -\frac{\gamma^2}{4\pi}\int_{-\infty}^{\infty} d\omega' \frac{S(\omega')}{\omega' - \omega_0'} - \frac{1}{4\pi}\left(\frac{\gamma J_2(x)\omega_0}{2\omega}\right)^2 \int_{-\infty}^{\infty} d\omega' S(\omega') \left(\frac{1}{\omega' - 2\omega + \omega_0'} + \frac{1}{\omega' - 2\omega - \omega_0'}\right)$$

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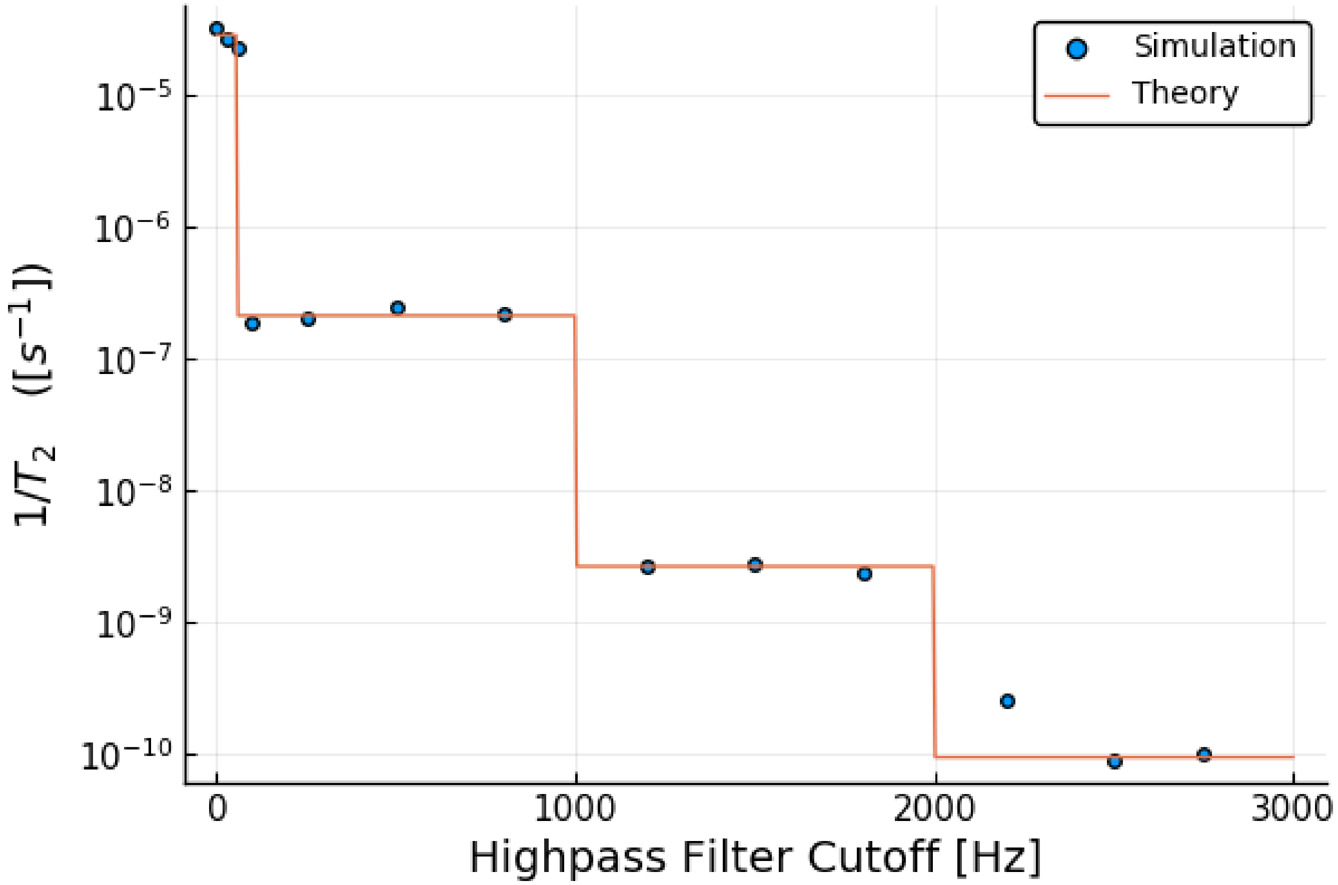
Perturbation Theory Result (2/2)

- In nEDM@SNS, we measure the capture rate which is proportional to $1 - P_n P_3(\sigma_n \cdot \sigma_3)$

$$\mathrm{Var}_{cl}(\sigma_1 \cdot \sigma_2)(t) = \frac{1}{2}(\gamma_1 - \gamma_2)^2 \left|\hat{z} \times (\vec{b_1} \times \vec{b_2})\right|^2 S(\omega_0')t + 2\left(\frac{\gamma_1 J_1(x_1)\omega_1 - \gamma_2 J_1(x_2)\omega_2}{\omega}\right)^2 \left|\hat{z} \cdot (\vec{b_1} \times \vec{b_2})\right|^2 S(\omega)t + \frac{1}{2}\left(\frac{\gamma_1 J_2(x_1)\omega_1 - \gamma_2 J_2(x_2)\omega_2}{2\omega}\right)^2 \left|\hat{z} \times (\vec{b_1} \times \vec{b_2})\right|^2 \left[S(2\omega - \omega_0') + S(2\omega + \omega_0')\right] t$$

Comparing Theory with Simulation (1/2)



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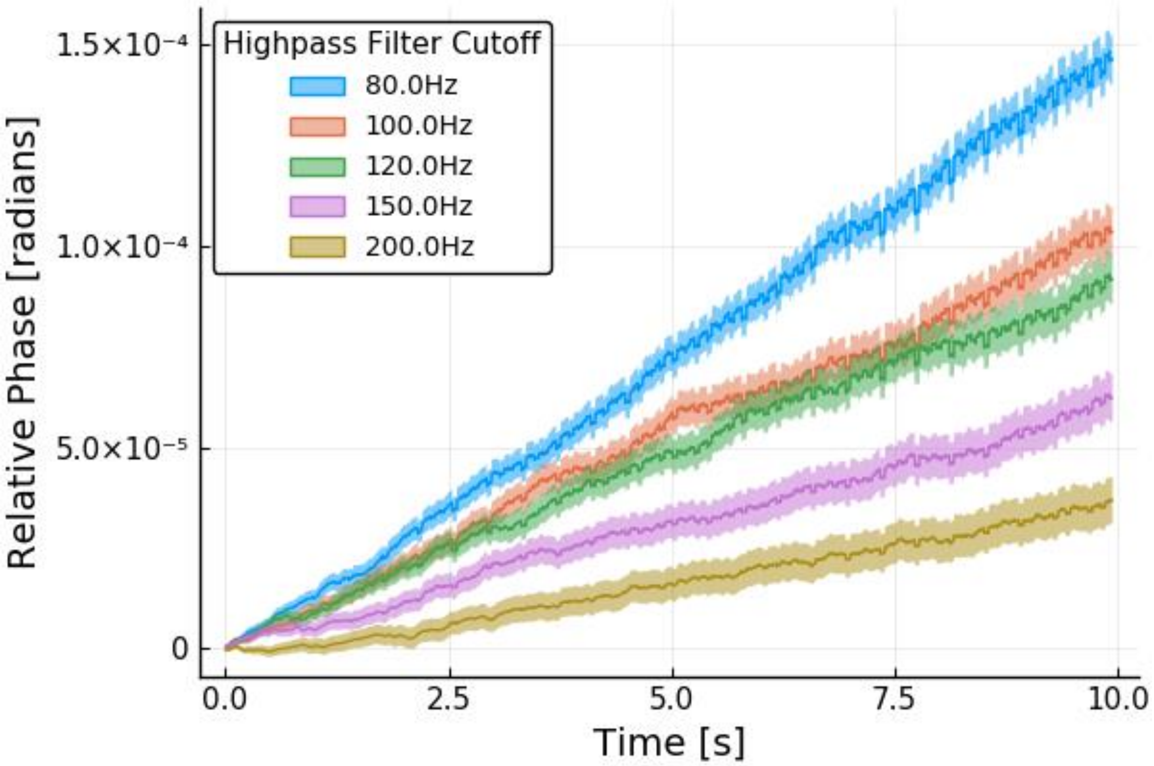
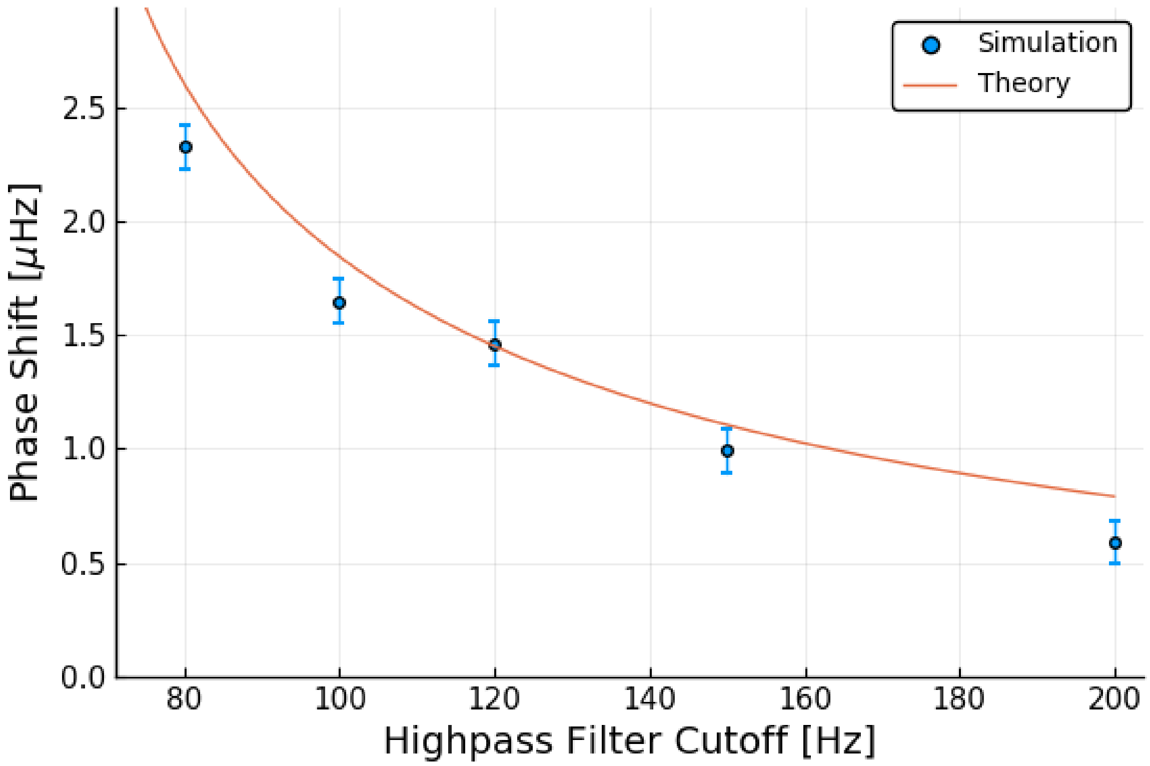
Comparing Theory with Simulation (2/2)





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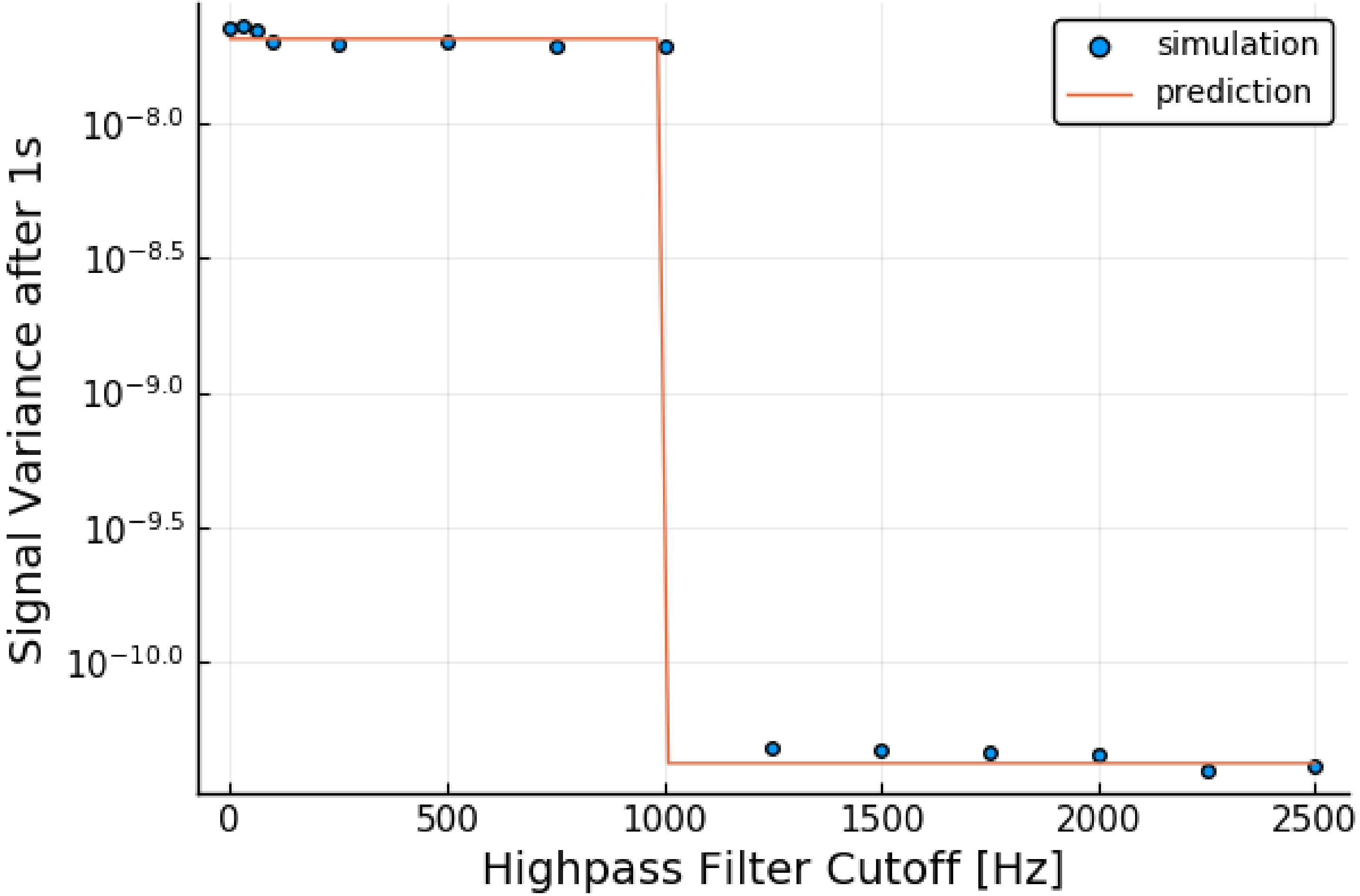
Signal Variance



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Where to go from here?

- Using filters and other electronics to improve the power supply stability
- Precisely measuring the gradients between the two cells
- Robust modulated dressing

Concluding Remarks

- Using perturbation theory, we can calculate relaxation and frequency shifts in the critical dressing mode of nEDM@SNS in terms of the noise power spectrum.
- These results can be applied both to current fluctuations in the dressing coil, or to spatial gradients in the magnetic fields.

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Appendix: Time-Independent Perturbation Theory

$$H_0 = \omega a^\dagger a + \frac{\Omega}{2}\sigma_x(a + a^\dagger)$$

$$H_z = \frac{\omega_0}{2}\sigma_z$$

$$U \equiv D\left(\frac{\Omega}{2\omega}\right)|+_x\rangle\langle +_x| + D\left(\frac{\Omega}{2\omega}\right)^\dagger |-_x\rangle\langle -_x|$$

$$D(\eta) \approx \sum_{n,q} J_q(2\eta\sqrt{\lambda})\,|n+q\rangle\langle n|$$

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