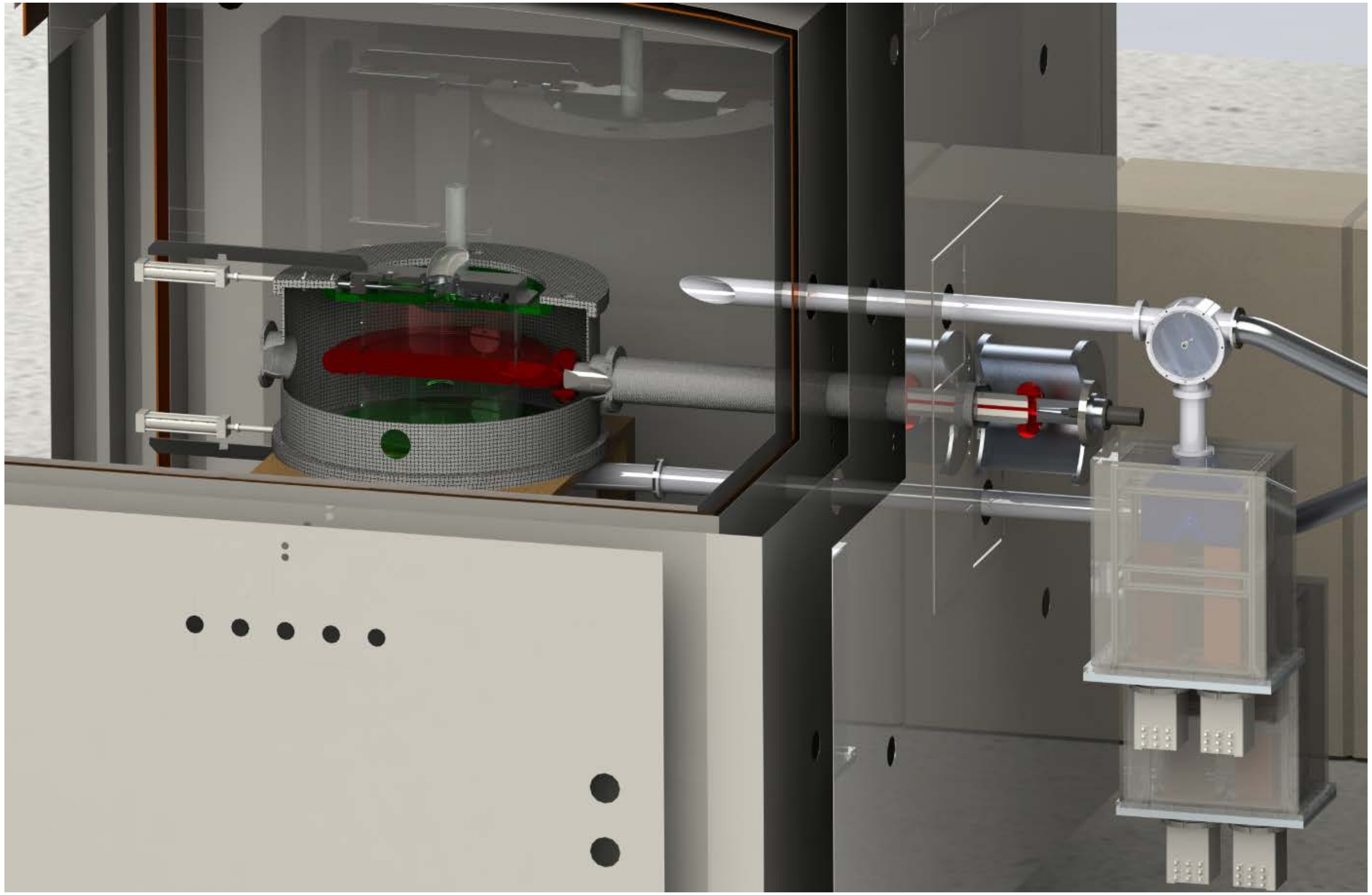
Studies of nEDM Systematics from Collective Rotational Motion of Neutrons

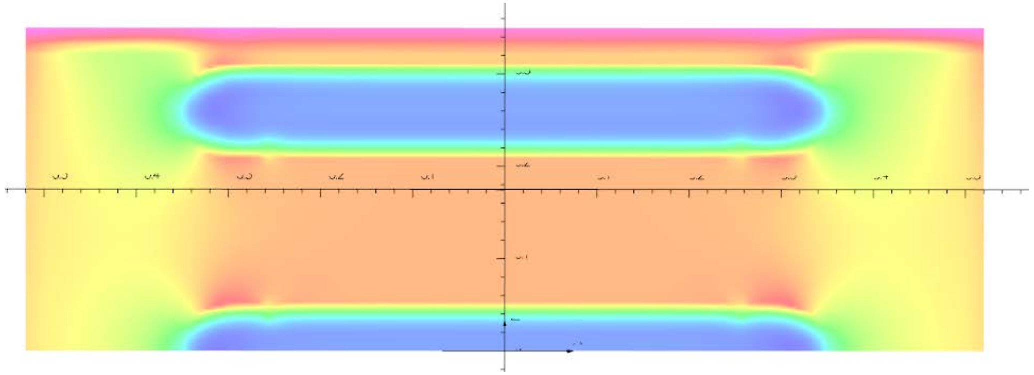
Sean Vanbergen – TUCAN Collaboration – nEDM2021

TUCAN EDM Measurement

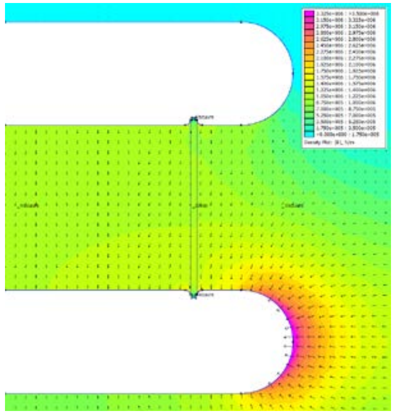
- The TUCAN EDM apparatus is currently in the design and prototyping phase.
- A large part of our work on the design is trying to characterize various systematic and statistical effects.
- We want to develop our understanding within the group and establish our requirements for various elements of the apparatus.



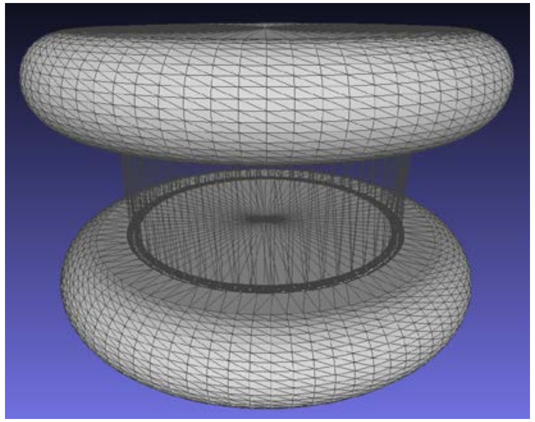
Approach to Systematic Effects



3D Magnetic Field Map - OPERA



2D Electric Field Map - FEMM



STL Meshed Geometry - Solidworks

- Many sources of systematic uncertainty which shift the measured frequency.
- Main cause is imperfections in the electromagnetic fields.
- Our approach to studying systematic effects relies heavily on simulations using PENTrack.
- Producing predictions of systematic magnitudes through Monte Carlo simulations using spin tracking integration of BMT equation

$$\dot{\vec{S}} = \left(-\frac{2\mu}{\gamma\hbar}\overrightarrow{B'} + \overrightarrow{\omega_T}\right) \times \vec{S}$$

Collective Motion vxE

- Effective magnetic field:

$$\vec{B}' = \gamma\vec{B} + (1-\gamma)(\vec{B}\cdot\dot{\vec{x}})\frac{\dot{\vec{x}}}{\dot{x}^2} - \frac{\gamma}{c^2}\dot{\vec{x}}\times\vec{E}$$

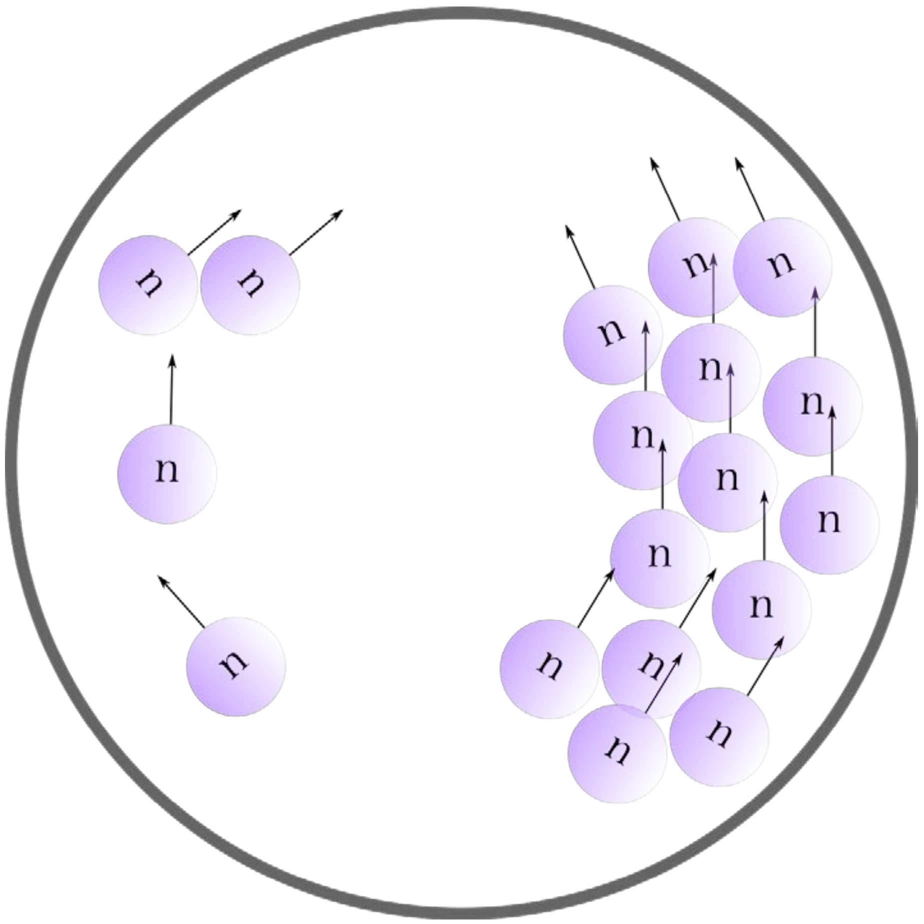
- Radial inhomogeneity E_r in electric field, in combination with velocity v_r about the cell axis of rotation results in a non-zero shift in B' along the B0-axis.

- If there is net rotational motion of the neutrons in the cell, there is an effect which mimics the nEDM:

$$d_{rot}^{f} = \frac{\hbar}{4E}\frac{\gamma_n v_r}{c^2}\left(E_r^{\uparrow\uparrow} - E_r^{\uparrow\downarrow}\right)$$

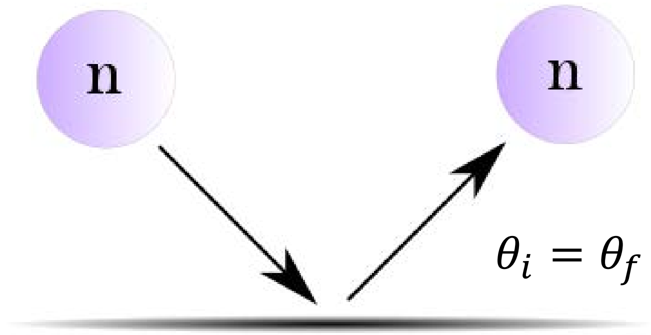
- Depends on initial conditions of neutrons in the cell, so difficult to characterize by measurement.

- Sets a requirement of electric field homogeneity.

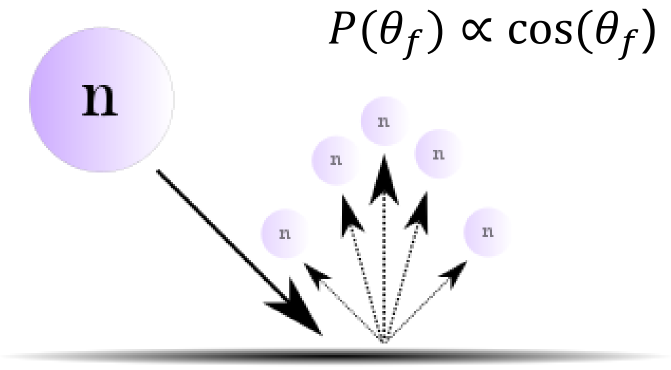


Decay of Net Motion of Stored Neutrons

Specular Reflection



Diffuse (Lambert) Reflection



- Diffuse reflections occur with probability P_L and randomize the motion of the neutrons, causing any net rotational motion to decay with time constant τ

Decay between end of filling and starting precession

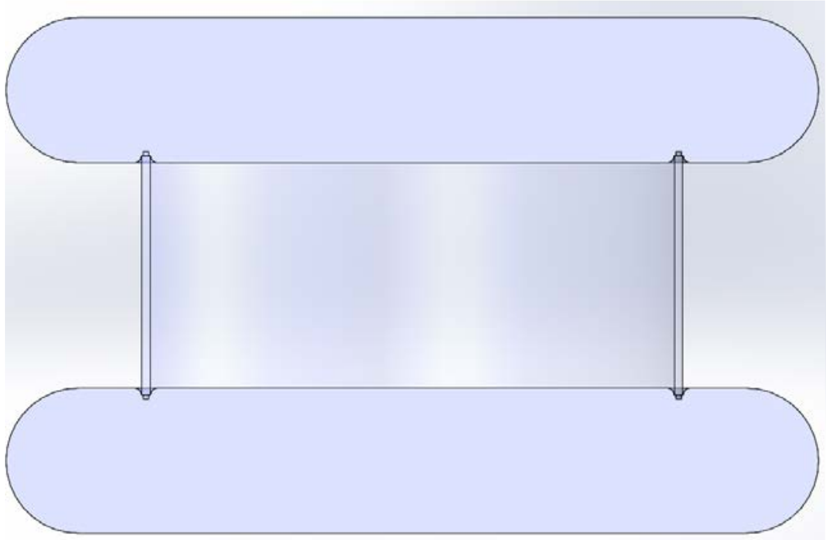
Time-averaged decay during the measurement

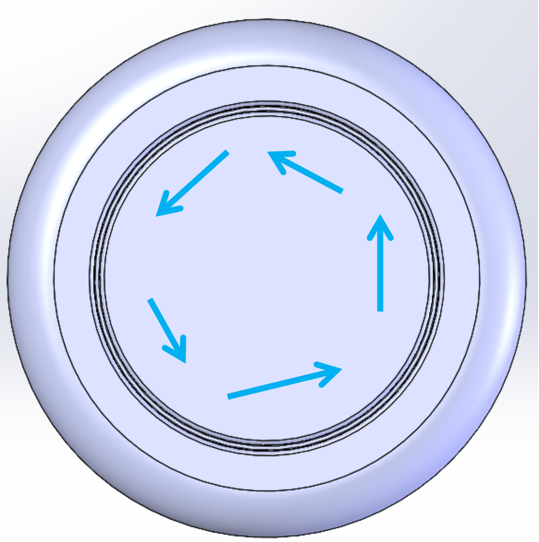
$$d_{rot}^{f} = K e^{-t_{delay}/\tau} \frac{\tau}{T} \frac{\hbar}{4E} \frac{\gamma_n v_{r,init}}{c^2} \left(E_t^{\uparrow\uparrow} - E_t^{\uparrow\downarrow}\right)$$

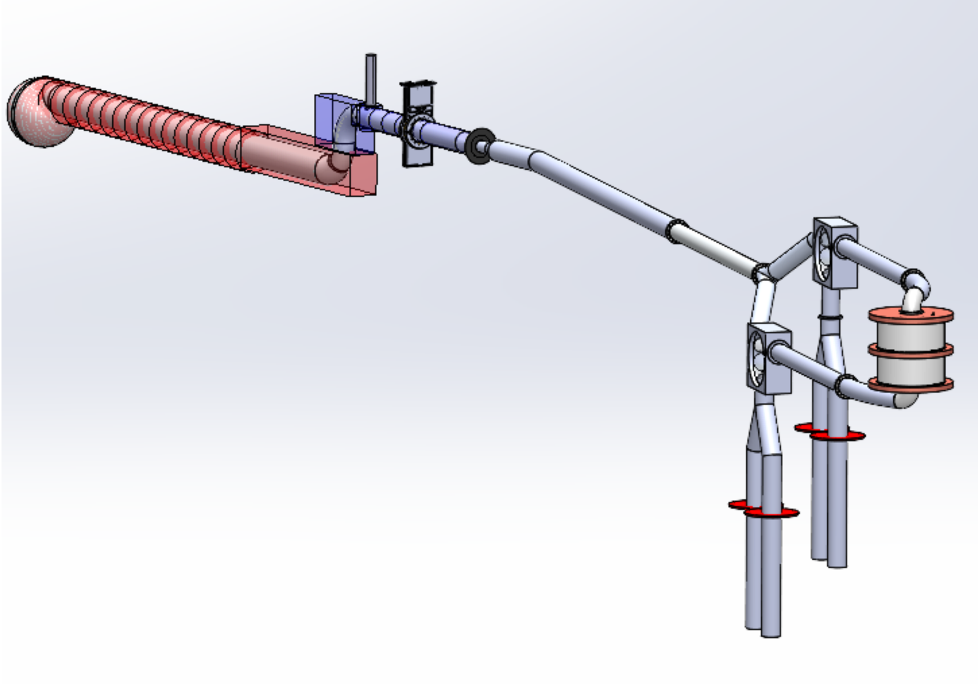
Neutron-averaged decay of motion during filling

Simulation Models

- Simplified geometry for simulating spin precession.
- 2D rotationally symmetric FEA electric field map.
- Neutrons start with all motion counter-clockwise.
- Effect is cell-dependent so we consider only a single cell.







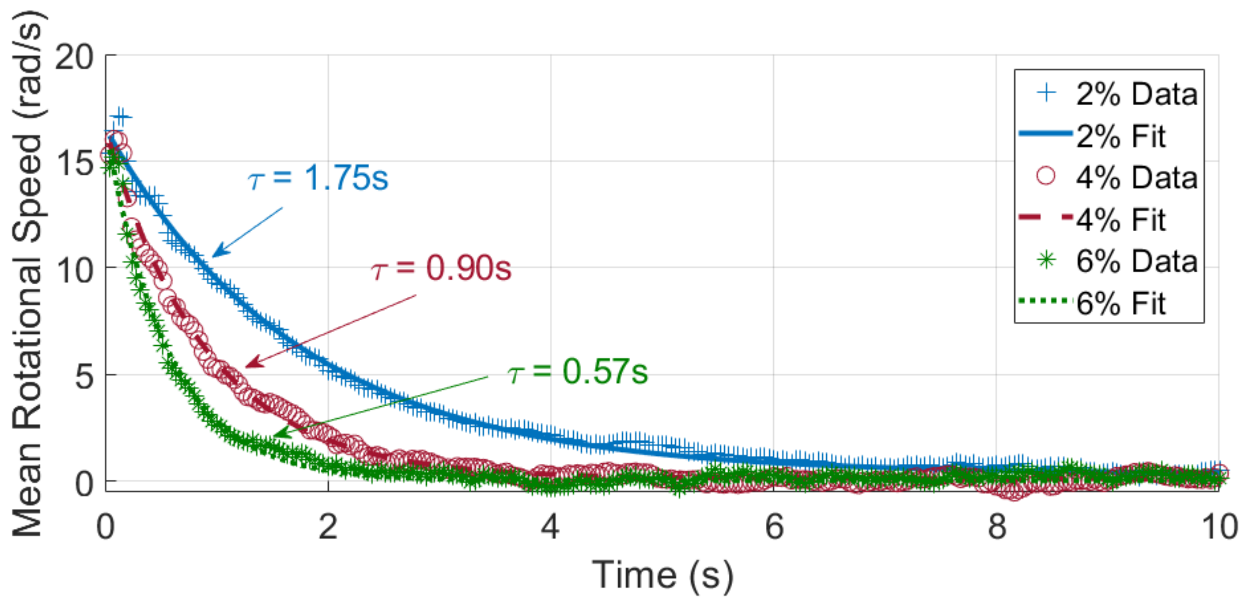
- Complete source to EDM model is used for simulating other necessary information:
  - Expected spectra in the cell
  - Filling curves

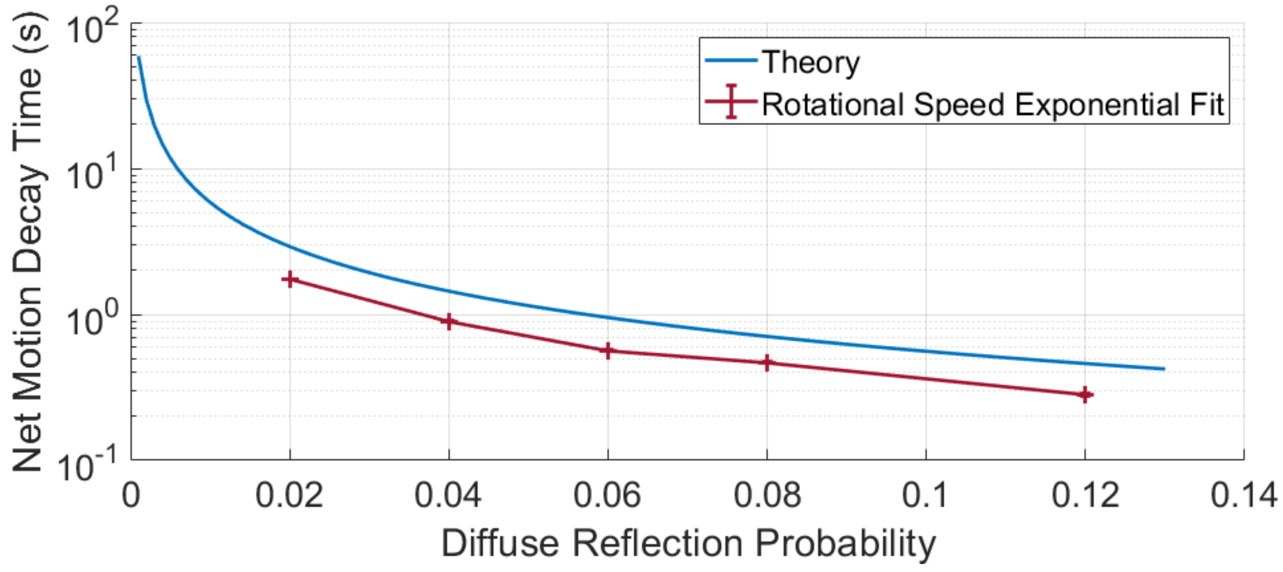
Study of UCN Motion

- Can derive a theoretical estimate of the motion decay time:
  - Assume neutrons follow straight paths between wall bounces (path length depends only on angle)
  - Upon diffuse reflection the motion of a neutron is randomized
  - For a given diffuse reflection probability, how long does it take until only 1/e of the neutrons are still part of the ordered motion?
  - Average over path angles:

$$\bar{\tau} = -\frac{\pi}{2}\frac{R}{v_r \ln(1-P_L)}$$

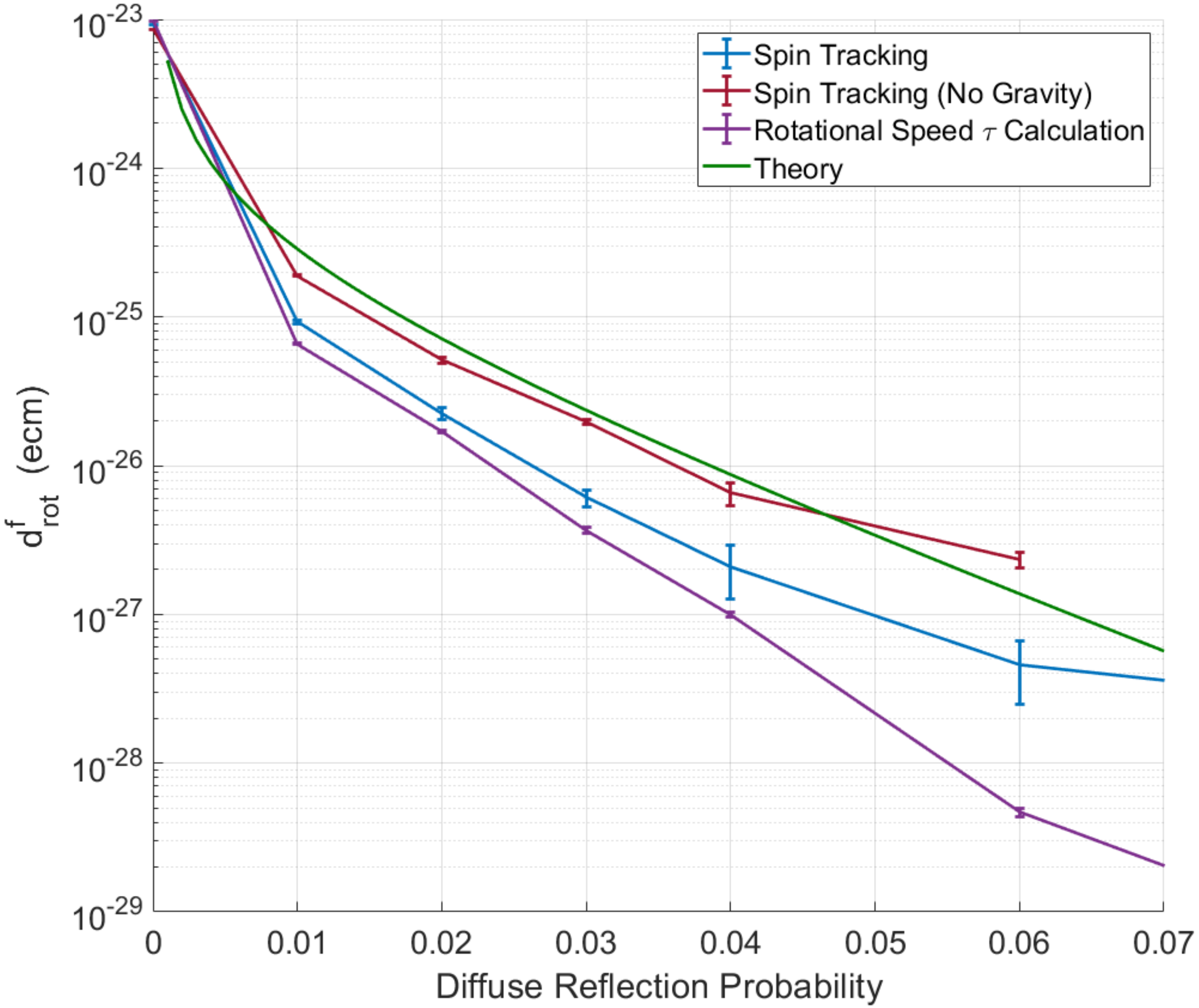
- In simulation, the mean rotational speed is a measure of the "amount of net motion". Fitting an exponential decay gives an estimate of the motion decay time.





S. Vanbergen

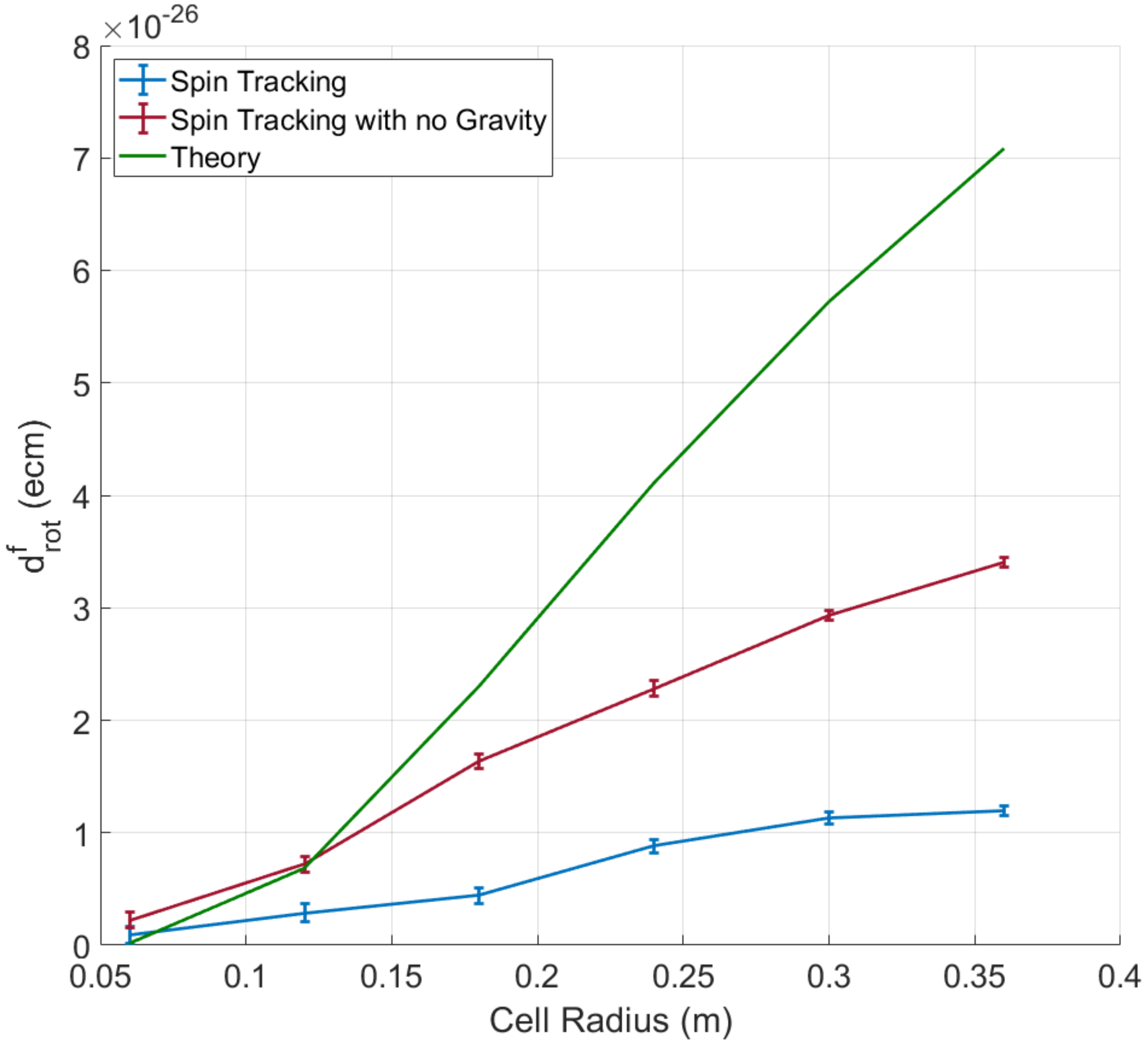
Study of False nEDM Signal



S. Vanbergen

- Results from an R = 18cm cell, 3% average inhomogeneity (E_r/E)
- Complete spin tracking simulation has reasonable agreement with the simulated time constants at low P_L
- Deviates from a pure analytical model, which appears to be mostly due to gravity: reduces time between wall bounces
- Results appear to diverge at larger P_L, but this could just be due to the larger uncertainty, which is difficult to overcome due to the required computation time

Dependence on Cell Radius



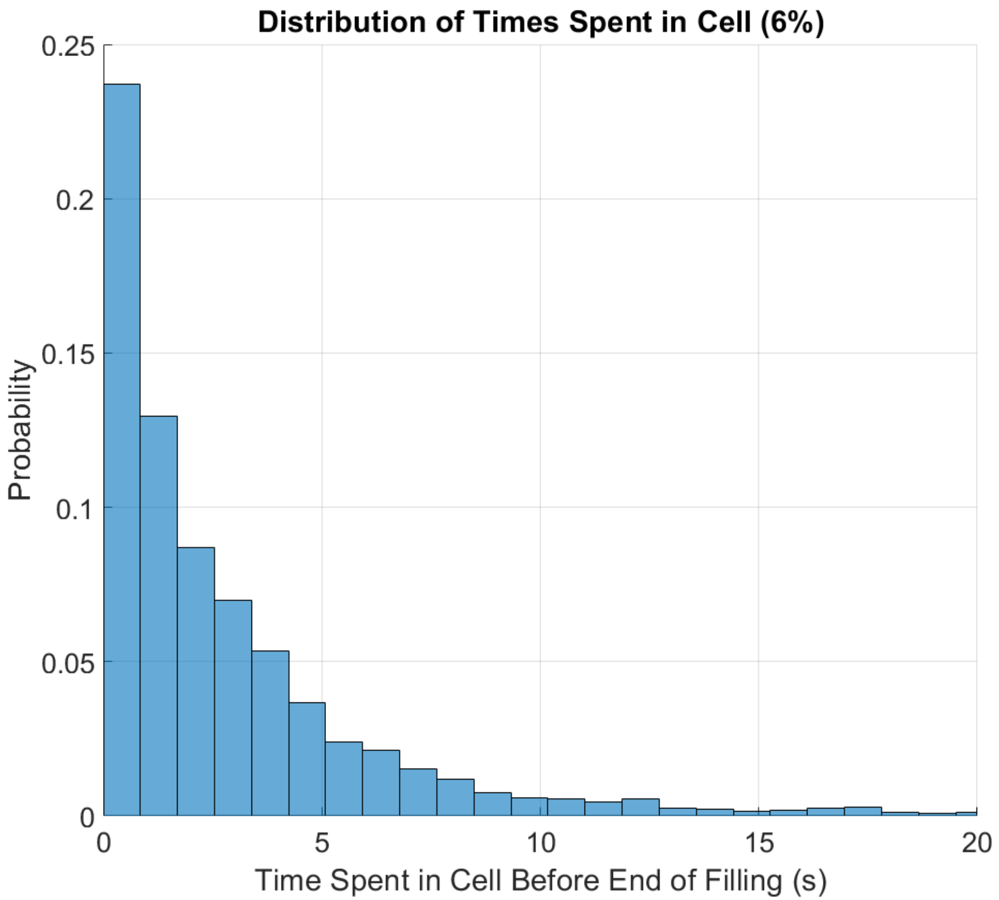
S. Vanbergen

- Effect increases with increasing cell radius
  - Neutrons take longer to decay because of increased time between wall hits
- Plot for 3% diffuse reflection
- Simulation shows that the effect of radius is not as strong as predicted
  - Neutrons do not actually travel from wall to wall, since they contact the top and bottom surfaces of the cell
  - Larger radius does not control the wall bounce frequency

Studies of Cell Filling

- How much net rotation can there actually be?
- Neutrons fill into the cell over time, so some of them spend a significantly longer time in the cell before the start of the measurement
- Simulated filling of the cell using full source model. Found the time that each neutron spends in the cell before the end of filling, *t**. Call the distribution of these times *N(t*).*
- Each bin of neutrons will, by the start of the spin flip, have decayed by $e^{-t^*/\tau}$. Averaging over the time distribution gives an expected average suppression of this systematic:

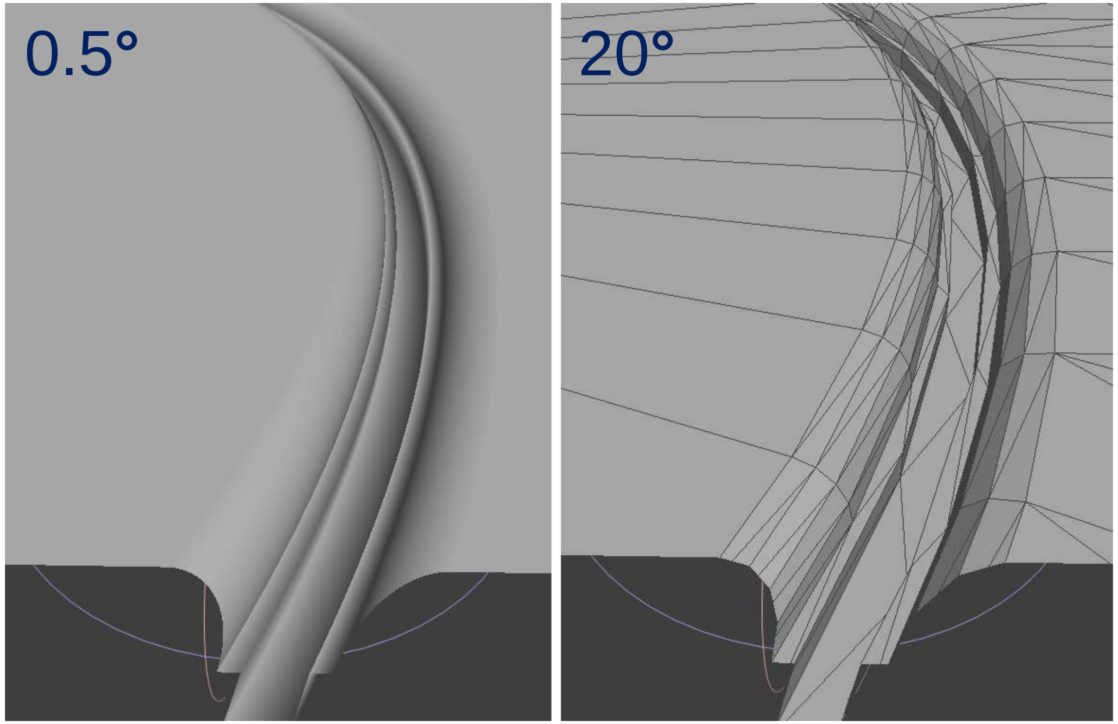
$$K = \int N(t^*)\, e^{-t^*/\tau} dt^*$$



For example: in a cell with 6% diffuse reflection, the false signal without the fill suppression is 5×10^{-28} *e*cm. The fill time suppression reduces this by a factor of 5 to 1×10^{-28} *e*cm.

Limitations of Geometry Mesh

- At 0% diffuse reflection, there is some decay of net motion – worse for coarser meshes
- This should be impossible, as the normals in the cell should all be aligned in the radial direction, and the mechanics of specular reflection thus prevent a change in the azimuthal (rotational) direction
- Caused by deviations of the normal angles in the mesh representation
- At 20º allowed angular deviation of the normals, this corresponded to ~0.4% equivalent diffuse reflection



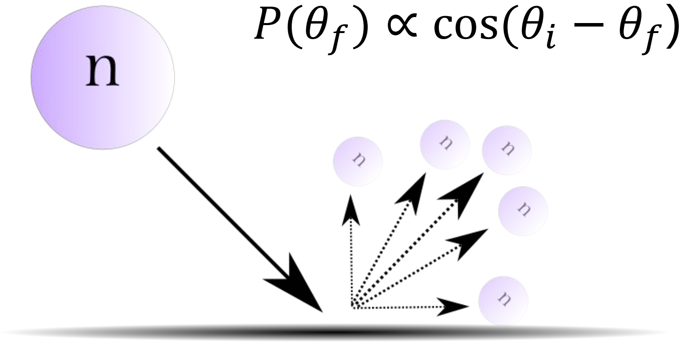
Mesh smoothness for 0.5º and 20º allowed angular deviation in meshing

Summary

- TUCAN is using Monte Carlo simulations with PENTrack to characterize expected systematics in our nEDM design
- Allows us to examine the impact of realistic geometry, fields, and neutron behaviour
- Such studies can be impacted by computational effects
- Expect that this particular systematic effect may contribute on the level of 1.0×10^{-28} *e*cm

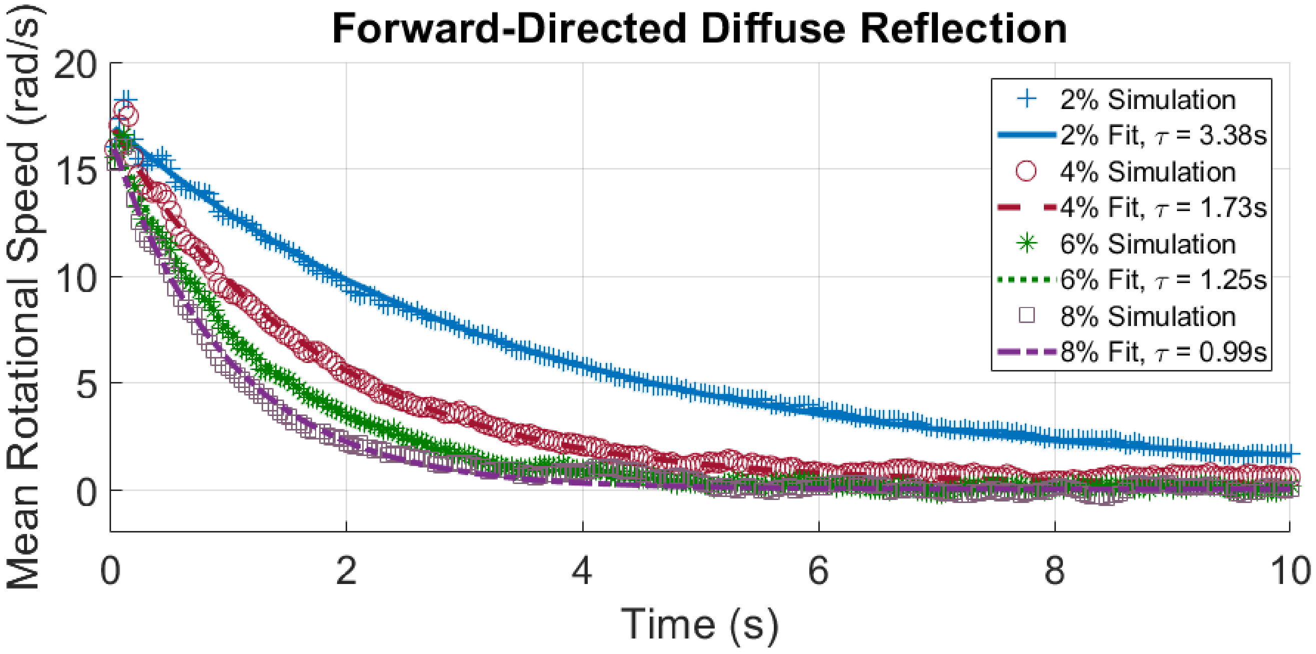
Thank you!

Alternate Reflection Model: Modified Lambert Diffuse Reflection



$$P(\theta_f) \propto \cos(\theta_i - \theta_f)$$

- A more forward-directed model
- Diffuse reflection direction is directed along the specular reflection direction
- Slower decay time



S. Vanbergen