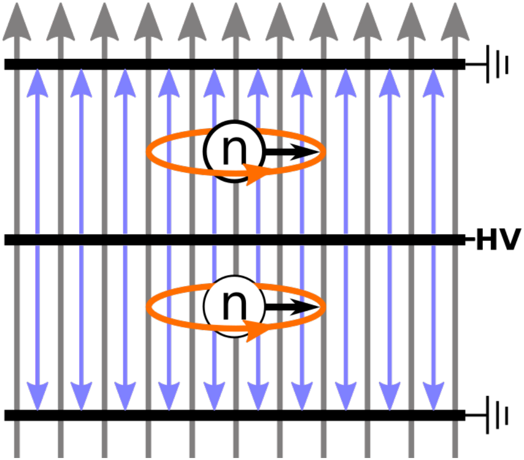
Magnetic Shim Coils for the TUCAN nEDM Experiment

M. McCrea
TUCAN Collaboration

University of Winnipeg

2021/02/19

Motivation for the Shim Coils



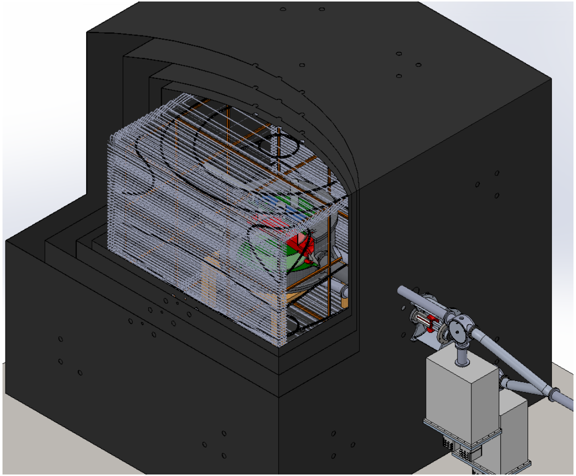
- As the ultra-cold neutrons (UCN) will travel throughout the measurement cells during the measurement process, any inhomogeneities in the magnetic field will change the instantaneous precession rate of neutrons at each new location depolarizing the UCN.
- Some magnetic field shapes will not only depolarize the neutrons in the cells, but also contribute to a false nEDM measurement. For example when using a mercury comagnetometer,

$$d_{Hg}^{false} = -\frac{\hbar\gamma_n\gamma_{Hg199}}{2c^2}\langle xB_x + yB_y\rangle. \quad (1)$$

Setting the Uniformity Limits

- As the nEDM measurement process requires polarized neutrons, the length of time that the neutron polarization must be maintained sets the limits on the required magnetic field homogeneity.
- To reach a neutron $T_2 > 10,000\ \mathrm{s}$ the following requirements can be placed on the field uniformity:
  - $\Delta B_z < 140\ \mathrm{pT}$, where ΔB_z is the difference in the B_z minimum and maximum in a cell.
  - $\sigma(B_z) < 40\ \mathrm{pT}$, where $\sigma(B_z)$ is the standard deviation of the field over a cell.
- For specific systematics such as d_{Hg}^{false}, the systematic can be calculated for a specific field with the goal that it is smaller than the goal accuracy of the measurement.

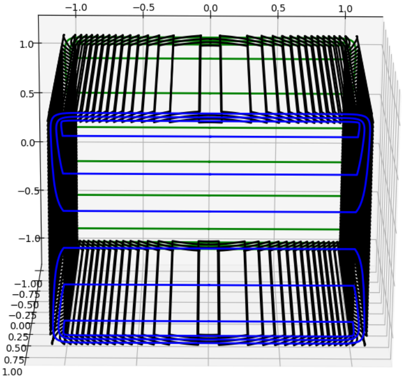
Interior Coils



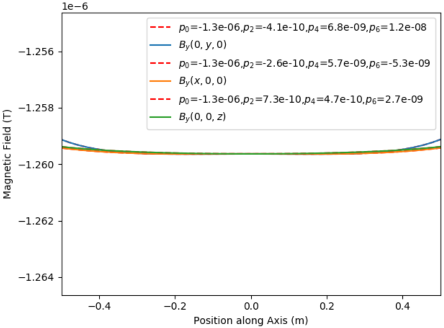
4 Different Classes of coils are used in the interior of the MSR:

- B_0 Holding Field Coil (grey)
- $n \times n$ Coil Array (orange)
- $G_{\ell,0}$ Systematic Coils (black)
- B_1 $\pi/2$ AC Coil (not shown)

Holding Field - B0 Coil



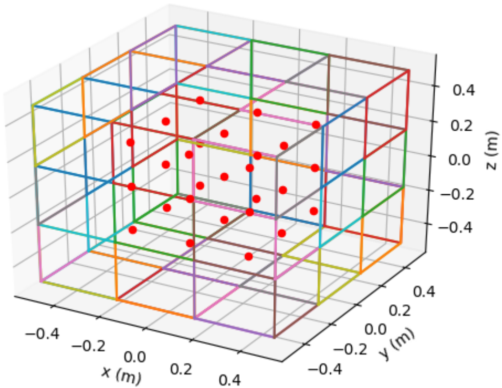
By careful selection of the coil wire placements highly uniform fields can be obtained.



- This coil will provide the 1 μT holding field.
- A self shielded square cosine theta coil is used to reduce coupling to the surrounding shielding.

See Extra Slides at the end for a brief description of method.

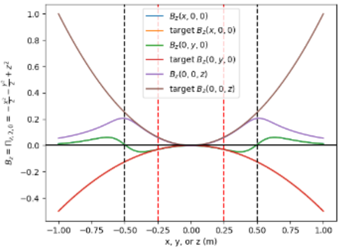
Arbitrary Field Inhomogeneities - $n \times n$ Coil Array



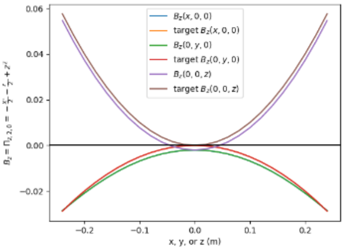
An example coil array shown with an 3×3 array of 9 coils on each cube face. Adjacent coils are drawn with different colors.

- The purpose of this coil array is to cancel arbitrary field inhomogeneities inside the magnetically shielded room(MSR).
- It is constructed as an array of square coils arranged symmetrically on the surface of a cube.
- The cancellation method is performed by using a fitting method to determine the required current in each coil to best cancel the measured fields at the selected measurement points, the 27 red circles in the example.

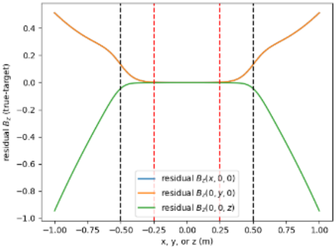
Example Field Cancellation



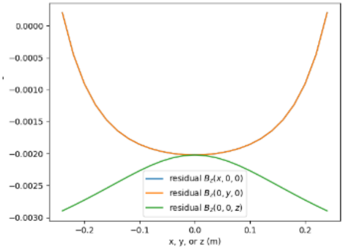
(a) B_z field.



(b) zoomed in B_z field.



(c) residual B_z field.



(d) zoomed in residual B_z field.

- $B_z(x,y,z) = z^2 - \frac{1}{2}(x^2 + y^2)$
- Black dashed lines are the coil array edges.
- Red dashed lines on left hand plots is zoom extents on right hand plots.

Results from 3 × 3 Array on Applied Fields

(ℓ, m) or dipole	ΔB_z (pT)		$\sigma(B_z)$ (pT)		$\langle B_T^2 \rangle$ (nT^2)	
	before correction	after correction	before correction	after correction	before correction	after correction
(1,0)	455	6	140	0.6	0.26	3×10^{-6}
(1,1)	1790	106	458	9	0.36	2×10^{-4}
(2,0)	500	58	106	7	0.04	4×10^{-4}
(2,1)	700	16	120	2	0.02	5×10^{-5}
(2,2)	1065	116	230	12	0.17	4×10^{-4}
dipole 1	1140	232	244	14	0.53	7×10^{-4}
dipole 2 upper	500	20	109	3	0.47	3×10^{-5}
dipole 2 lower	220	17	50	3	0.47	3×10^{-5}
dipole 3 upper	870	37	187	8	0.16	1×10^{-4}
dipole 3 lower	400	41	87	4	0.16	1×10^{-4}
(3,1)	200	61	31	6	0.004	1×10^{-4}
(3,3)	340	65	61	5	0.02	2×10^{-4}
(4,2)	123	59	14	6	0.0005	2×10^{-4}
goal		140		40		0.1

What are the $G_{\ell,m}$'s?

- For magnetic fields in volumes containing no electric currents it can be convenient to describe the field in the terms of a polynomial sum of the form:

$$B(x,y,z) = \sum_{l,m} G_{l,m} \begin{pmatrix} \Pi_{x,l,m}(x,y,z) \\ \Pi_{y,l,m}(x,y,z) \\ \Pi_{z,l,m}(x,y,z) \end{pmatrix} \tag{2}$$

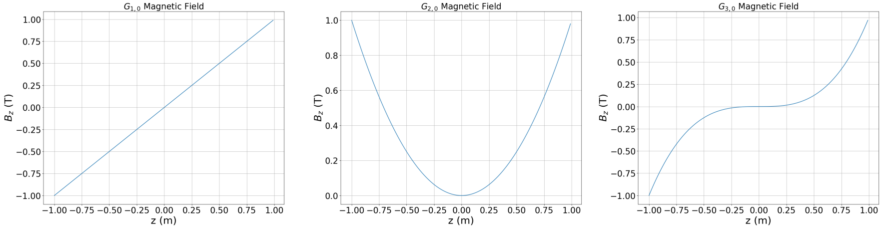
  where each term is a harmonic solution of the Laplace equation.
- See some example terms below:

l	m	Π_x	Π_y	Π_z
0	−1	0	1	0
0	0	0	0	1
0	1	1	0	0
1	−2	y	x	0
1	−1	0	z	y
1	0	$-\frac{1}{2}x$	$-\frac{1}{2}y$	z
1	1	z	0	x
1	2	x	$-y$	0

source: C. Abel et al., Phys. Rev. A 99, 042112 (2019)

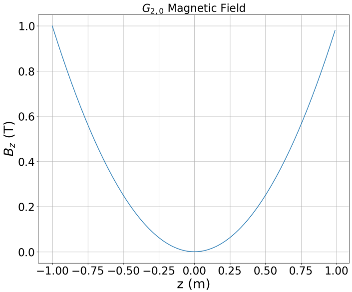
Purpose of G_{lm} Coils

- The goal of the the G_{lm} coils is to cancel or selectively induce specific multipole moments that are related to significant systematics effects in the nEDM measurement.
- Being able to produce these magnetic multipole moments at a high purity will allow dedicated systematic runs to be performed.
- It is believed that 3 such coils are required corresponding to the three lowest order multipole moments, $G_{1,0}$, $G_{2,0}$, and $G_{3,0}$, should be sufficient for the experiment.

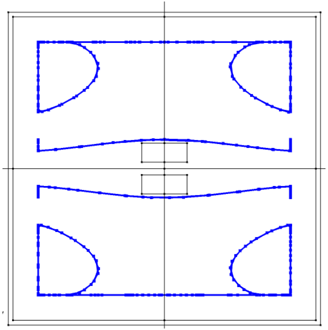


$G_{\ell,m}$ Coil Shapes

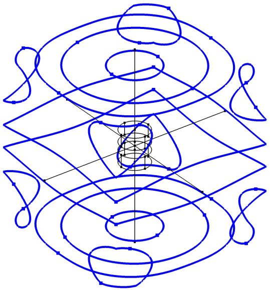
Field Shape



Coil Side View



Coil Isometric View

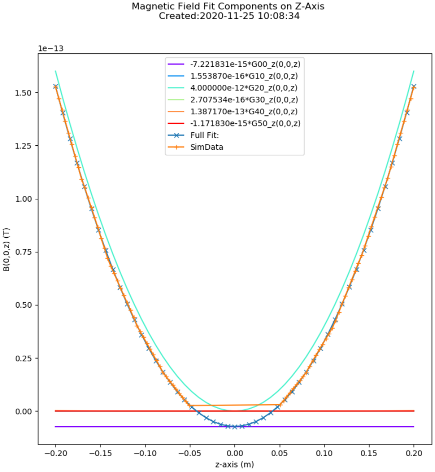


$G_{\ell,m}$ Coil Metrics

The primary metric for these coils is then how well the specific field shape is recreated over the measurement cells.

- Relative Harmonic Contributions
  - By fitting a series of $G_{\ell,m}$ terms to the field produced from a coil configuration the purity of the mode can be examined.
- Calculate the specific systematics that would be produced by a pure multipole to determine the cancellation efficiency.
  - For example, $d_{Hg}^{false} = -\frac{\hbar\gamma_n\gamma_{Hg199}}{2c^2}\langle xB_x + yB_y\rangle$

Example $G_{\ell,m}$ Evaluation



Relative $G_{\ell,0}$	
$G_{\ell,0}$	$G_{\ell,0}/G_{2,0}$
$G_{0,0}$	0.002
$G_{1,0}$	0.00004
$G_{2,0}$	1.0
$G_{3,0}$	0.00007
$G_{4,0}$	0.03
$G_{5,0}$	0.0003

Comparision to Ideal			
$B_z(T)$	Simulated	Ideal	Difference
mean	3.03e-14	3.79e-14	0.76e-14
$\sigma(B_z)$	4.87e-14	4.78e-14	0.09e-14
ΔB_z	21.69e-14	15e-14	6e-14
$\langle xB_x + yB_y \rangle$	-8.30e-15	-8.16e-15	0.14e-15

$$d_{Hg}^{false} \propto \langle xB_x + yB_y \rangle$$

Conclusions

- Based on the required measurement accuracy metrics were set for the magnetic field uniformity and shapes to determine when a coil would function as intended for the experiment.
- A B_0 coil model has been demonstrated that provides a highly uniform central field.
- An $n \times n$ array of square coils has been demonstrated to be able to cancel a significant range of field inhomogeneities.
- A $G_{\ell,m}$ coils design method has been created and is able to generate specific magnetic field modes with a high purity.
- For each type coil configurations have been simulated that meet the field uniformity requirements.
- From these simulations we are confident to move forward with the mechanical design aspects of the coils.

Extra Slides

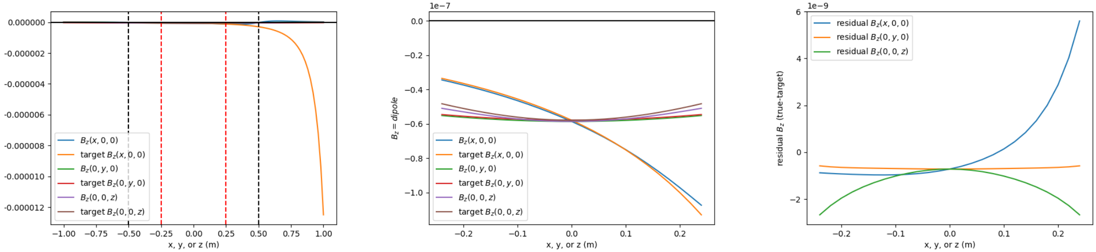
List of Coils

To provide perform the measurement of the precession frequency and to correct for the field inhomogeneities several coils are required:

- B_0 Coil
  - Provides the uniform $1\mu T$ holding field inside the MSR
- $n \times n$ Coil Array
  - An array of square coils to be used to cancel arbitrary magnetic gradients in the MSR interior.
- $G_{\ell,0}$ coils
  - Cubic coils that will be used to cancel and examine specific field multipoles that contribute to the largest systematic effects.
- B_1 Coil
  - For applying the oscillating field for the $\pi/2$ spin flip in the Ramsey method

These coils must be constructed inside the MSR in such as way as to provide the require field shapes, but also to not interfere with the other require sub-systems and to allow access for experimental setup and maintenance.

$n \times n$ Additional Fit and Residuals - Dipole



(a) B_z field. (b) zoomed in B_z field. (c) zoomed in residual B_z field.

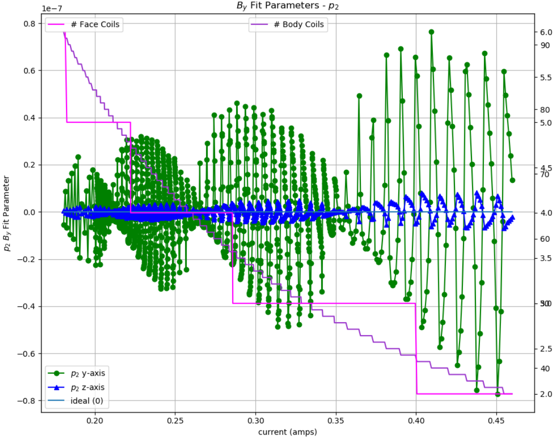
Figure: Magnetic fields and residuals, plotted along three axes, for an applied dipole field. This shows a less symmetric field cancellation than was shown on the previous plot.

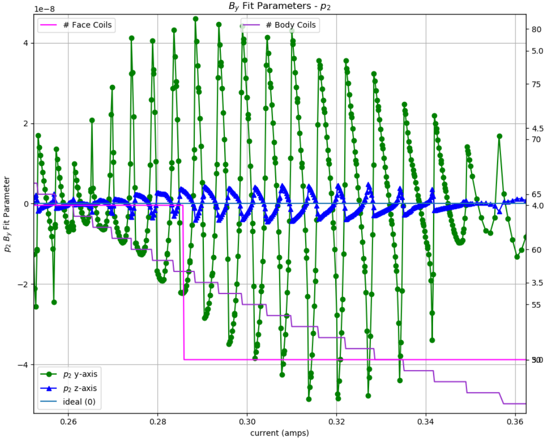
Method for Coil Design Using Scalar Potentials

1. Determine the magnetic boundary conditions on the coil surface for the desired field.
2. Calculate the scalar potential for the boundary conditions on on the coil surface.
    - The Comsol magnetic field no currents (mfnc) module can be used to calculate the scalar potential in the presence of magnetically permeable materials such as the MSR housing to prevent field distortions from using the free space solutions in the presence of the MSR.
3. Take the difference in the scalar potential from the solution in the interior and exterior volumes of the coil surface.
4. Process this through a python code to calculate the wire positions from the equipotential levels.
5. Take the wires back into Comsol to calculate the produced magnetic fields in the presence of the MSR.
6. Compute Metrics to evaluate the usefulness of the produced field.
7. If the required metrics are not met, adjust the wire positions to improve the field and recalculate the fields and metrics.

B_0 Coil Changes in Fit Parameters with Wire Position Changes

By evaluating the coil metric, such as the p_2 fit over a wide range of current spacings in the equipotential levels useful coils closer to the ideal can be found.

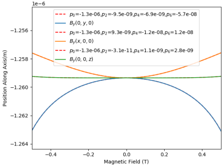




$$B_y(x, y, z) = p_0 + p_2x^2 + p_4x^4 + p_6x^6$$

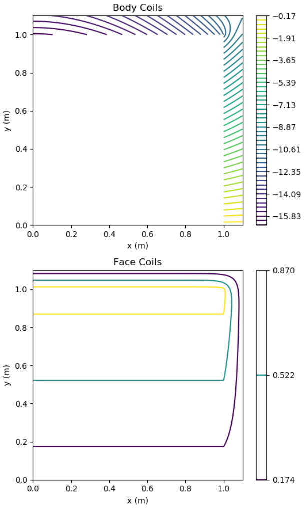
same plot, zoomed in

Example Change in Fields For Small Coil Changes

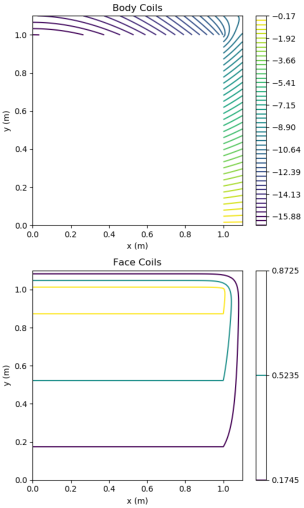


$p_2 = -9.5 \times 10^{-9}$

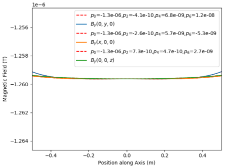
Note that the coil positions changes are small in the coil plots, but the p_2 fit term is 20 times larger on the left hand coil.



current spacing 0.348 A



current spacing 0.349 A



$p_2 = 4 \times 10^{-10}$