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Les Houches lectures on EFT

Lectures given at the 4th International Workshop on Searches for a Neutron Electric Dipole Moment

14-19 February 2021


CENSORED

Timetable

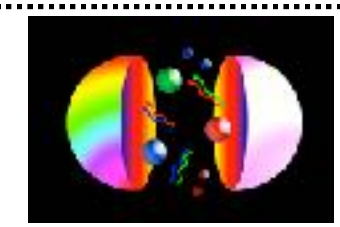
2 lectures on EFT, at a fairly elementary level (I hope)

- Lecture 1 (Tuesday 10:30-12:00): Philosophy and Landscape of EFTs
- Lecture 2 (Thursday 10:30-12:00): CP-violation in EFT

100 TeV ???

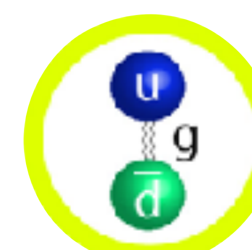
100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

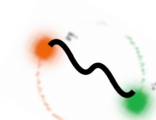


2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c

1 GeV γ, ν_i, e, μ + hadrons



100 MeV γ, ν_i, e, μ + pions and kaons



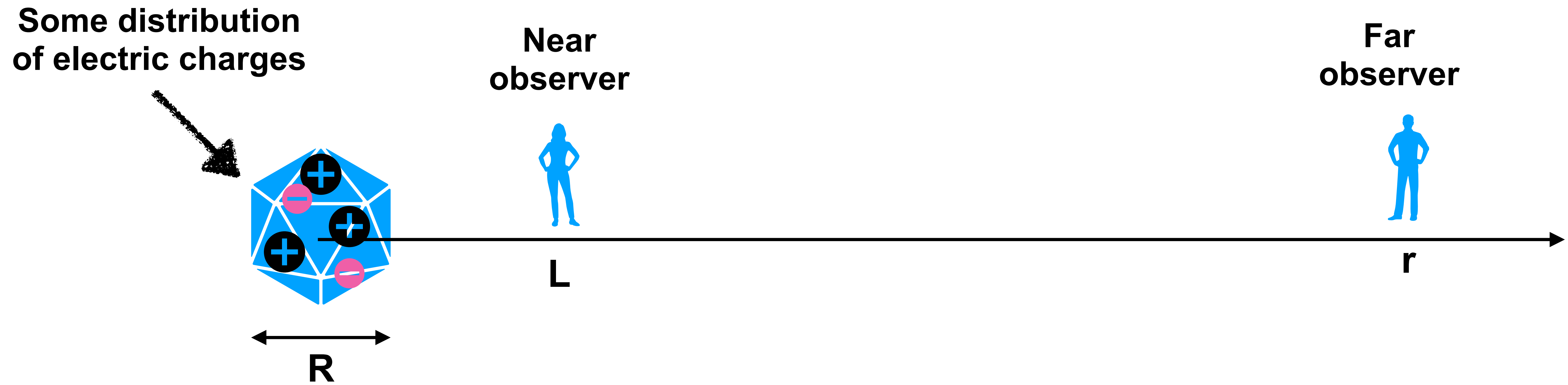
1 MeV γ, ν_i, e

γ, ν_i

Lecture 1

Philosophy of EFT

Role of scale in physical problems



Near observer, L~R, needs to know the position of every charge to describe electric field in her proximity

Far observer, r >> R, can instead use multipole expansion: $V(\vec{r}) = \frac{q}{r} + \frac{\vec{d}\cdot\vec{r}}{r^3} + \frac{Q_{ij}r_i r_j}{r^5} + \dots$

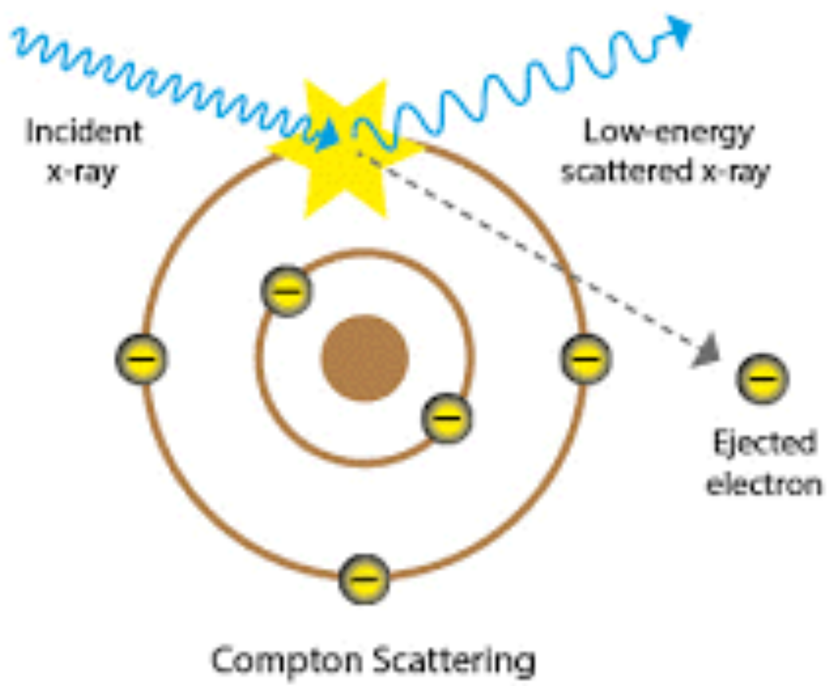
~1/r ~R/r^2 ~R^2/r^3

Far observer is able to describe electric field in his vicinity using just a few parameters: the total electric charge, eventually the dipole moment, ….

Higher order terms in the multipole expansion are suppressed by powers of the small parameter (R/r). One can truncate the expansion at some order depending on the value of (R/r) and experimental precision

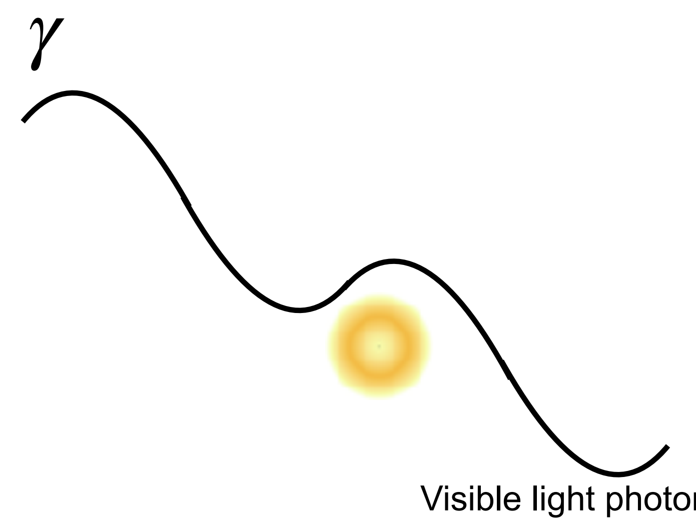
Far observer, like Molière's Mr. Jourdain, discovers that he has been using EFT all his life

Scale in microscopic problems



$$= \frac{1}{m_e\alpha}$$

X-ray photons see the atomic structure and scatter on the orbiting electrons



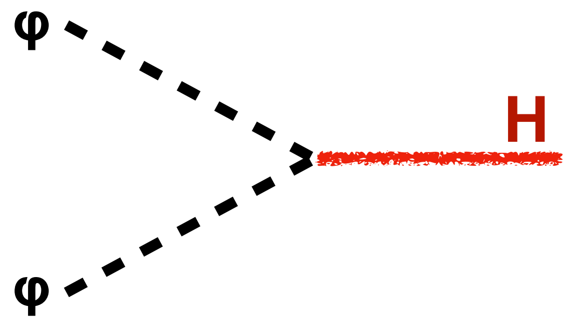
$$= \frac{10}{m_e\alpha}$$

Lower-energy photons see atoms as neutral objects which are basically transparent

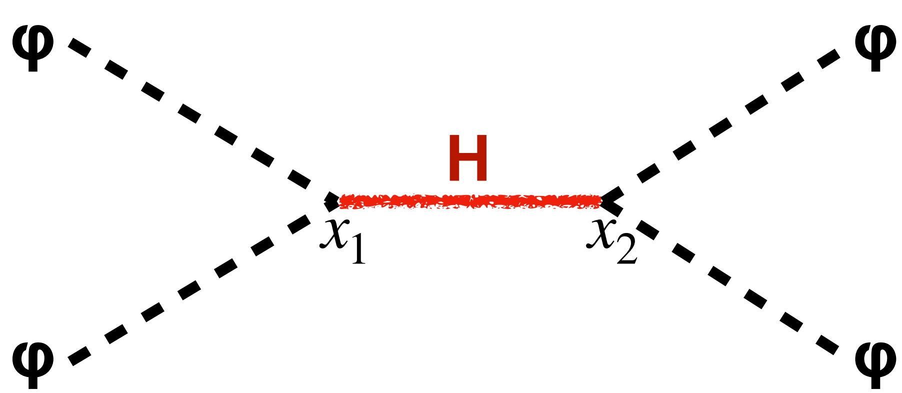
(that's how the universe becomes transparent to photons right after recombination)

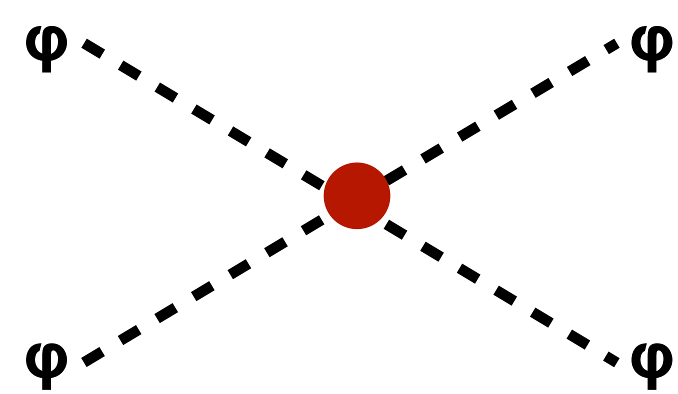
Scale in quantum field theory

Consider a theory of a light particle φ interacting with a heavy particle H



Heavy particle H propagator in coordinate space: $P(x_1, x_2) \sim \exp(-m_H |x_1 - x_2|)$





At small distance scales, $|x_1-x_2| << 1/m_H$, the heavy particle H propagates. Force acting between light particles φ

At large distance scales, $|x_1-x_2| >> 1/m_H$, propagation of the heavy particle H suppressed. Interaction looks like a delta function potential

$$m_H \sim \Delta E \ll \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \ll 1$$

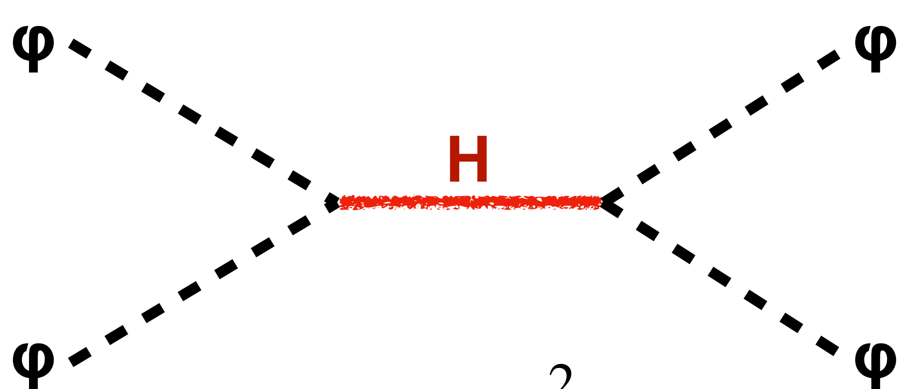
$$m_H \sim \Delta E \gg \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \gg 1$$

Scale in quantum field theory

Consider a theory of a light particle φ interacting with a heavy particle H

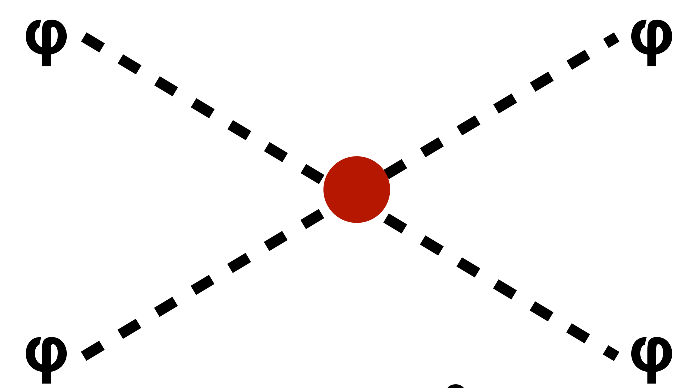
Heavy particle H propagator in momentum space:

$$P(p^2) \sim \frac{1}{p^2 - m_H^2} = \begin{cases} \dfrac{1}{p^2} & p^2 \gg m_H^2 \\ -\dfrac{1}{m_H^2} & p^2 \ll m_H^2 \end{cases}$$



$$\mathscr{M} \sim \frac{g^2}{p^2}$$

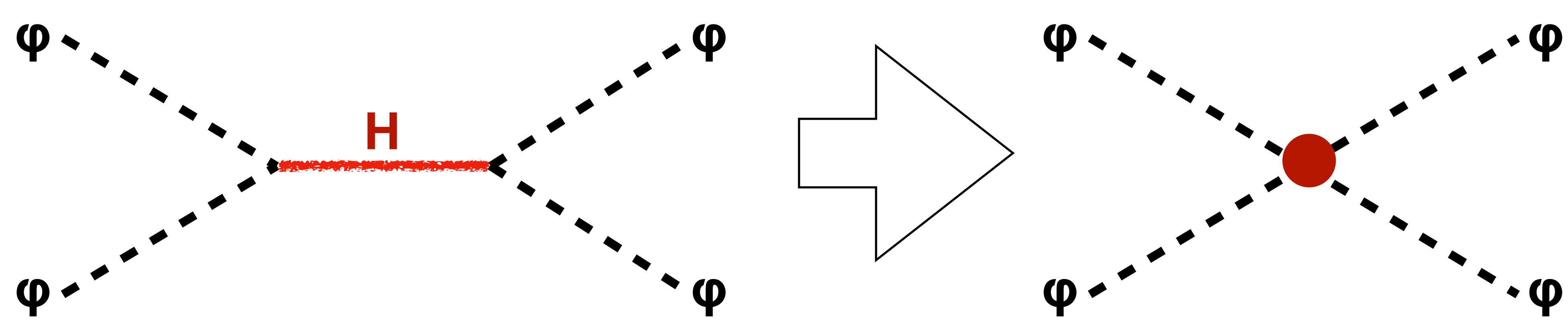
At large momentum scales, $p^2 >> m_H^2$, we see propagation of the heavy particle H. Force acting between light particles φ



$$\mathscr{M} \sim \frac{g^2}{m_H^2}$$

At small momentum scales, $p^2 << m_H^2$, propagation of the heavy particle H effectively leads to a contact interaction between light particles φ

Scale in particle theory



- Processes probing distance scales >> $1/m_H$, equivalently energies scales << m_H, cannot resolve the propagation of H

- Then, intuitively, exchange of heavy particle H between light particles φ should be indistinguishable from a contact interaction of φ

- In other words, the <u>effective theory</u> describing φ interactions should be well approximated by a local Lagrangian, that is, by a polynomial in φ and its derivatives

This is the generic way how the effective theory description arise in particle physics, which will be repeated in all the examples that follow

Effective field theory

How to build an EFT

Bottom up

**Starting with a set of particles
we build the Lagrangian
describing all their possible interactions
obeying a prescribed set of symmetries
and organised in a consistent expansion**

Top down

**Starting with a given theory
(effective or fundamental)
we integrate out degrees of freedom
heavier than some prescribed mass scale**

Intermezzo: Dimensional analysis

- Effective Lagrangians by construction must contain infinite number of terms. Therefore any useful EFT comes with a set of power counting rules which allow one to organize the Lagrangian in a consistent expansion and single out the most relevant terms

- Relativistic effective theories are obtained by integrating out heavy fields H with mass of order Λ, and the inverse of the latter provides a natural expansion parameter to organize the effective Lagrangian.

- The effective Lagrangian is then organized according to canonical dimensions of its interactions terms, where the powers of the mass scale multiplying each term are identified with Λ. The observables computed fare then expanded in E/Λ where E is the typical energy scale of the experiment

- Warning: different power counting rules may apply to non-relativistic theories, or relativistic systems with one heavy component (such as e.g. B-mesons), or to theories with non-linearly realized symmetry. These cases will be discussed later.

Dimensional analysis

To isolate UV and IR limits, consider rescaling of spacetime coordinates

$$x \to \xi x'$$

$\xi \to 0$ is zooming in on small distances (UV limit)
$\xi \to \infty$ is zooming in on large distances (IR limit)

$$S = \int d^4x \left((\partial_\mu \phi)^2 - m^2\phi^2 - \lambda\phi^4 - \frac{c}{\Lambda^{n+d-4}}\phi^n \partial^d \right)$$
$$\to \int d^4x' \left(\xi^2 (\partial'_\mu \phi)^2 - m^2 \xi^4 \phi^2 - \lambda \xi^4 \phi^4 - \frac{c\xi^{4-d}}{\Lambda^{n+d-4}}\phi^n \partial'^{\,d} \right)$$

Since path integral is dominated by kinetic terms
to easily compare the original and rescaled actions
it is convenient normalize the kinetic terms canonically

$$\phi(x) \to \xi^{-1}\phi'(\xi x)$$

$$S \to \int d^4x' \left((\partial'_\mu \phi')^2 - m^2 \xi^2 \phi'^2 - \lambda \phi'^4 - \frac{c_{n,d}\xi^{4-d-n}}{\Lambda^{n+d-4}} \phi'^n \partial'^{\,d} \right)$$

Dimensional analysis

$$\phi(x) \to \xi^{-1}\phi'(\xi x)$$

$$x \to \xi x'$$

$$S = \int d^4x \left((\partial_\mu \phi)^2 - m^2\phi^2 - \lambda\phi^4 - \frac{c_{n,d}}{\Lambda^{n+d-4}}\phi^n \partial^d \right)$$

$$\to \int d^4x' \left((\partial'_\mu \phi')^2 - m'^2 \phi'^2 - \lambda' \phi'^4 - \frac{c'_{n,d}}{\Lambda^{n+d-4}} \phi'^n \partial'^d \right)$$

Mass term is relevant operator: it gets more important in IR

$$m'^2 = m^2 \xi^2$$

$$\lambda' = \lambda$$

$$c'_{n,d} = c_{n,d}\xi^{4-d-n}$$

Quartic coupling is marginal operator: it is (approximately) the same in UV and in IR

Higher dimensional interactions (for d+n>4) are irrelevant operators: they get less important in IR

Power counting in relativistic EFT, determining the importance of various interactions, can be organized based on canonical dimension of interactions

Dimensional analysis cheat sheet

Relativistic field theory

$$S = \int d^4x \left[\partial_\mu \phi^\dagger \partial_\mu \phi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + i\bar{\psi}\gamma_\mu \partial_\mu \psi + \ldots \right]$$

$$[\partial] = \text{mass}^1$$

$$[\phi] = \text{mass}^1$$

$$[A_\mu] = \text{mass}^1$$

$$[\psi] = \text{mass}^{3/2}$$

Euler-Heisenberg EFT

Rules of the game

- **Quantum Mechanics + Poincaré invariance = relativistic Quantum Field Theory**
- **Degrees of freedom: a massless spin-1 photon with 2 polarizations (neutrinos are ignored in this discussion)**
- **U(1) gauge invariance**
- **Validity regime up for photon energies smaller than the cutoff scale Λ**
- **No other mass scale in the EFT except for Λ**

Starting from these principles, we will build an EFT for the photon in systematic expansion in $1/\Lambda$

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{D=4} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

E.g. 2-to-2 photon scattering amplitude calculated from this Lagrangian must have form

$$\mathscr{M}(\gamma\gamma \to \gamma\gamma) = a_0 + a_1\frac{E_\gamma}{\Lambda} + a_2\frac{E_\gamma^2}{\Lambda^2} + a_3\frac{E_\gamma^3}{\Lambda^3} + a_4\frac{E_\gamma^4}{\Lambda^4} + \dots$$

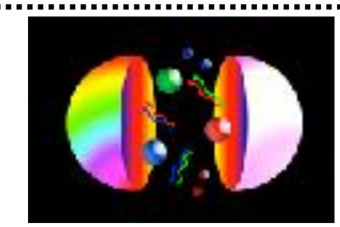
For $E_\gamma \ll \Lambda$ each consecutive term is more suppressed, therefore the expansion makes sense

100 TeV ???

100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

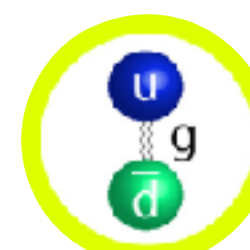
5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c

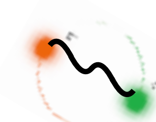


1 GeV γ, ν_i, e, μ + hadrons

100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e



γ, ν_i

Illustration #1

Euler-Heisenberg EFT

We will start with a bottom up approach, and then connect it to the top down approach

Euler-Heisenberg EFT

Consider effective theory for photons propagating in vacuum with $E_\gamma << 2m_e \approx 1$ MeV

- **At these energies all charged particles are integrated out, hence the effective Lagrangian must be a function of only the photon field A_μ**
- **Photons are massless, so the only explicit mass scale in this construction is the EFT cutoff scale Λ**
- **Gauge and Lorentz invariance requires the effective Lagrangian to be a function of the field strength $F_{\mu\nu}$ and its derivatives**

$$\mathscr{L}_{\text{eff}} = \mathscr{L}\left(F_{\mu\nu}, \tilde{F}_{\mu\nu}, \partial_\mu, \Lambda\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$$

We will build the effective Lagrangian as an expansion in $1/\Lambda$

$$\mathscr{L}_{\text{eff}} = \Lambda^2 \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

Here D denotes the canonical dimension of each term
(no odd dimensions because $[F_{\mu\nu}]=2$, and derivatives must always come in pairs)

Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = \Lambda^2 \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

D=2: $F_{\mu\mu} = \tilde{F}_{\mu\mu} = 0$ **No possible invariants thus** $\mathscr{L}_{D=2} = 0$

D=4: **One invariant** $F_{\mu\nu}F_{\mu\nu}$

$\tilde{F}_{\mu\nu}\tilde{F}_{\mu\nu} = F_{\mu\nu}F_{\mu\nu}$

$F_{\mu\nu}\tilde{F}_{\mu\nu}$ **is a total derivative**

$\partial_\mu \partial_\nu F_{\mu\nu} = 0$

$$\mathscr{L}_{D=4} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

the numerical coefficient is pure convention, except for the sign, which is required to avoid ghost instability

D=6: **Again, no non-trivial invariants! Hence** $\mathscr{L}_{D=6} = 0$

$$F_{\mu\nu}F_{\nu\rho}F_{\rho\mu} = 0 = F_{\mu\nu}F_{\nu\rho}\tilde{F}_{\rho\mu} = \dots$$

$$F_{\mu\nu}\partial_\alpha F_{\mu\alpha}\partial_\beta F_{\nu\beta} = 0$$

$\mathscr{L}_{D=6} = cF_{\mu\nu}\Box F_{\mu\nu}$ **can be eliminated by the change of variables** $A_\mu \to A_\mu + \frac{2c}{\Lambda^2}\Box A_\mu$

Non-trivial interactions between photons can arise only at order $1/\Lambda^4$ in the EFT!

Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

D=8: **The most general non-redundant Lagrangian at D=8 is**

Lagrangian's interactions are called operators in this context

$$\mathscr{L}_{D=8} = c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})$$

The Lagrangian's free parameters are called the <u>Wilson coefficient</u> in this context

Other possible structures can be shown to be <u>redundant</u>, that is they can be eliminated or expressed by the three above

e.g. $F_{\mu\alpha}F_{\alpha\nu}F_{\mu\beta}F_{\beta\nu} = \frac{1}{4}(F_{\mu\nu}F_{\mu\nu})^2 + \frac{1}{2}(F_{\mu\nu}\tilde{F}_{\mu\nu})^2$

The high-school version of the same Lagrangian:

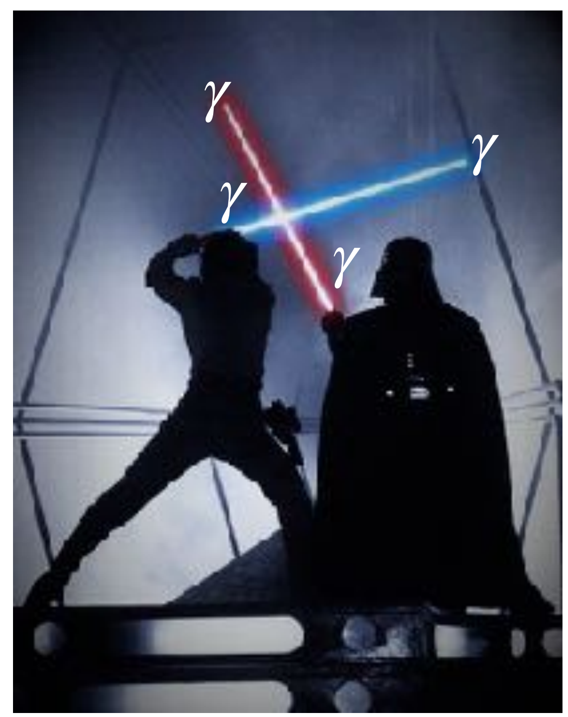
$$\mathscr{L}_{D=8} = 4c_1\left(\vec{E}^2 - \vec{B}^2\right)^2 + 16c_2\left(\vec{E}\vec{B}\right)^2 + 8c_3\left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\vec{B}\right)$$

Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \dots$$

This Lagrangian describes the effective theory of light at low energies (UV, visible, IR, microwaves, radio) at the leading order beyond the Maxwell approximation

This is the effective theory underlying the physics of lightsabers

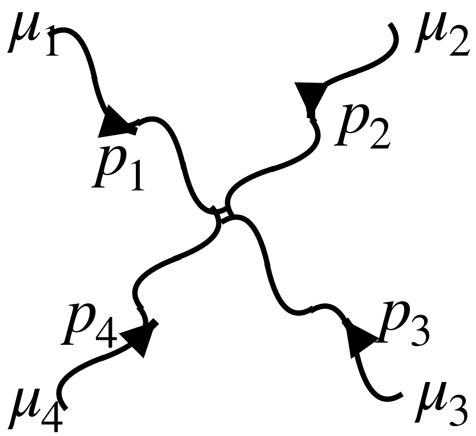


In its validity regime, it is also appropriate to describe the entire textbook electrodynamics, plus vacuum birefringence, photon-photon scattering at low energies, and more

Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \dots$$

This Lagrangian defines a completely healthy and consistent quantum field theory with quartic (and possibly higher-point) self-interactions between photons.



$$\frac{32ic_1}{\Lambda^4}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1p_2\eta^{\mu_1\mu_2}\right)\left(p_3^{\mu_4}p_4^{\mu_3} - p_3p_4\eta^{\mu_3\mu_4}\right) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4)$$
$$+\frac{32ic_2}{\Lambda^4}(\dots) + \frac{32ic_3}{\Lambda^4}(\dots)$$

Scattering amplitudes can be calculated in a systematic expansion in $1/\Lambda^4$. E.g.

$$\mathscr{M}(\gamma^+\gamma^+\gamma^+\gamma^+) = 8\frac{c_1 - c_2 + ic_3}{\Lambda^4}\left[s^2 + t^2 + u^2\right]$$

$$\mathscr{M}(\gamma^+\gamma^+\gamma^-\gamma^-) = 8\frac{c_1 + c_2}{\Lambda^4}s^2$$

$$\mathscr{M}(\gamma^-\gamma^-\gamma^-\gamma^-) = 8\frac{c_1 - c_2 - ic_3}{\Lambda^4}\left[s^2 + t^2 + u^2\right]$$

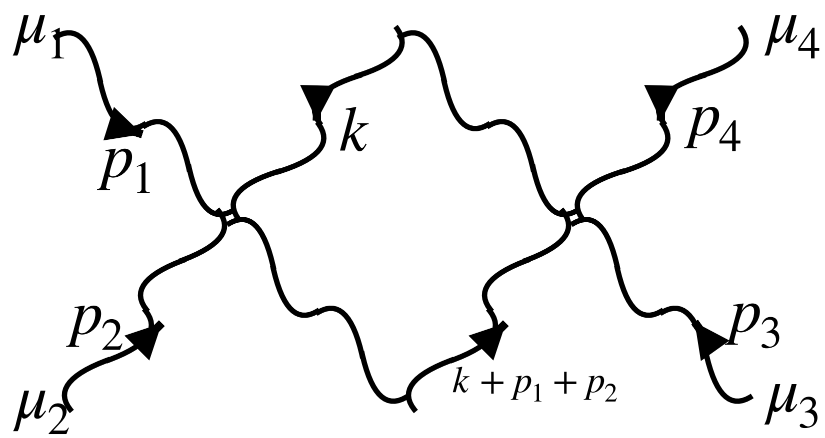
$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_3)^2$$
$$u = (p_1 + p_4)^2$$

Note that a non-zero c_3 violates parity!

Euler-Heisenberg EFT

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \dots$$

This is a healthy QFT, so we can calculate loop corrections



$$\mathcal{M} \sim \frac{1}{\epsilon}\frac{c_i c_j}{\Lambda^8}E^8 + \dots$$

dim. reg. **e.g. s^4 or $s^2 t^2$**

Dimensional analysis shows that we cannot absorb this divergence into the coefficients c_i in the Lagrangian

Instead we have to add new counterterms to the Lagragnian:

$$\Delta\mathcal{L}_{\text{eff}} = d_1(\partial_\alpha F_{\mu\nu}\partial_\alpha F_{\mu\nu})^2 + \dots$$

This EFT is not renormalizable in the usual sense of this word, because you need an infinite number of counterterms to cancel all loop divergences in the theory

But it renormalizable in another sense: if at given order you include all terms allowed by symmetries, then all divergences can be canceled at this order

Euler-Heisenberg EFT

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \dots$$

Scattered comments:

- This is the effective theory of light at low energies (UV, visible, IR, microwaves, radio) at the leading non-trivial order

- The quartic photon interaction terms in this EFT lead to non-linear field equations for the electromagnetic field. Thus, electrodynamics is really non-linear, and the superposition principle they taught you in school is not exactly true! Of course, the effect is tiny in typical engineer problems, cause they deal with energies far below Λ

- One potentially observable effect of the D=8 terms is the so-called vacuum birefringence, that is rotation of light polarization propagating in vacuum in strong magnetic field. This effect was possibly observed in 2016 in a neutron star light.

- Another potentially observable effect is light-by-light scattering. This has been routinely observed in colliders, however at higher energies where this EFT is no longer valid.

UV completion of Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \dots$$

Suppose, Jedi measure experimentally the coefficients c_i/Λ^4 but they do not have means to reach the energy scale Λ

Can they deduce what is the fundamental theory underlying this EFT ?

The answer is no in general. However they can do the following exercise:

1. **Hypothesize a theory for which, below a certain mass scale Λ, the only degrees of freedom are those of the photon**

2. **Perform the <u>matching</u> between the UV theory and the EFT, that is integrate out all particles heavier than Λ and calculate c_i in terms of the parameters of the UV theory**

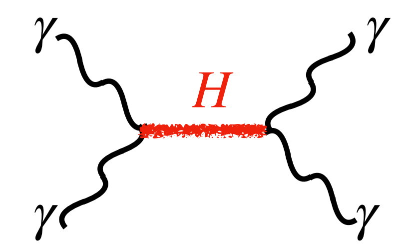
3. **Verify if the predicted pattern of c_i agrees with the one measured experimentally**

UV completion of Euler-Heisenberg EFT

$$\mathscr{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \ldots$$

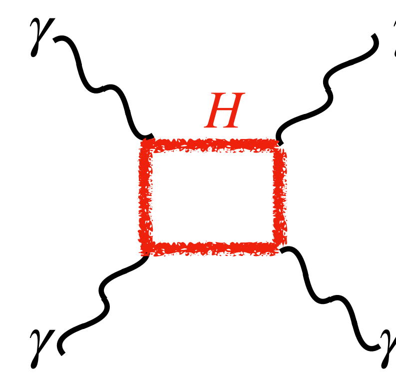
$$c_i \sim e^2$$

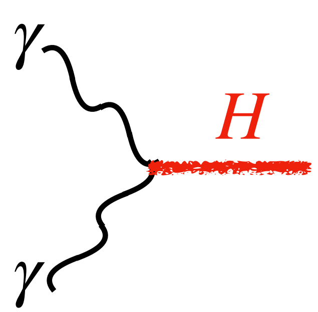
if this Wilson coefficient is generated at tree level



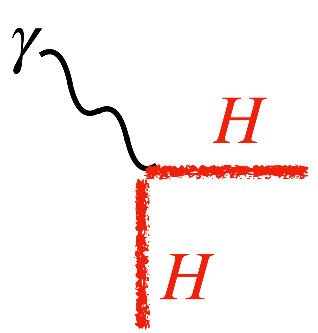
$$c_i \sim \frac{e^4}{16\pi^2}$$

if this Wilson coefficient is generated at 1-loop level





Doesn't exist in QED !



Does exist in QED !

Thus, <u>in QED</u>

$$\frac{c_i}{\Lambda^4} \sim \frac{e^4}{16\pi^2 m_e^4} = \frac{\alpha^2}{m_e^4}$$

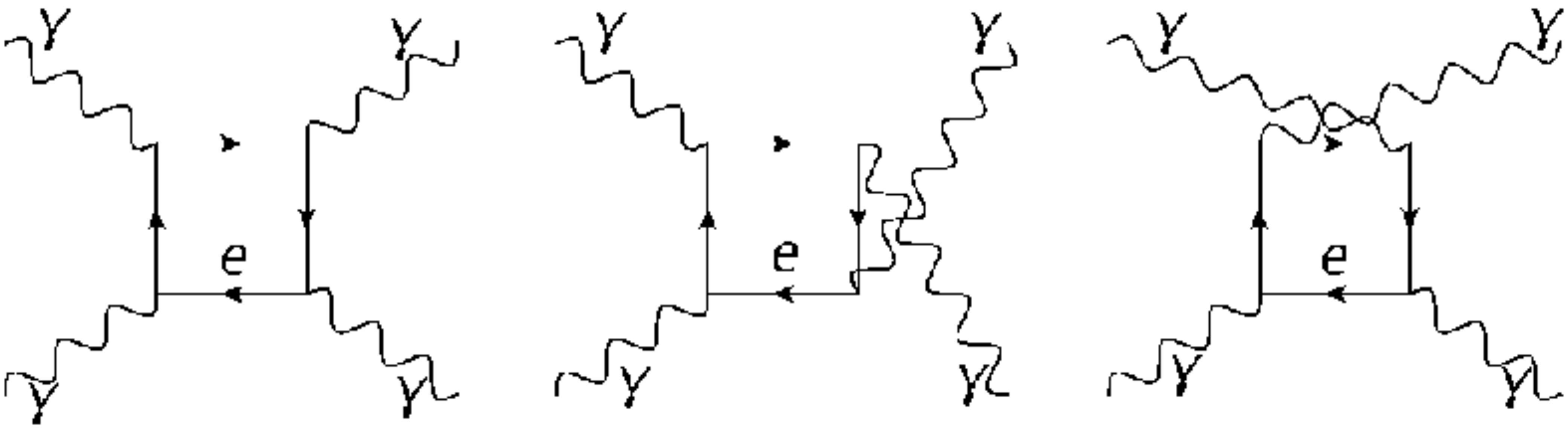
Moreover, c_3=0 in QED, due to parity conservation

UV completion of Euler-Heisenberg EFT

QED UV completion

$$\mathscr{L}_{\mathrm{UV}} \supset i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m_e\bar{\psi}\psi + eA_{\mu}\bar{\psi}\gamma^{\mu}\psi$$

In this example, the UV completion of our effective theory is a renormalizable theory, which could in principle be valid to very high energy scales



+reversed fermion line

$$\mathscr{M}_{\mathrm{QED}}(\gamma^+\gamma^+\gamma^+\gamma^+) = -\frac{\alpha^2}{15m_e^4}\left[s^2+t^2+u^2\right] + \mathcal{O}(m_e^{-6})$$

$$\mathscr{M}_{\mathrm{QED}}(\gamma^+\gamma^+\gamma^-\gamma^-) = \frac{11\alpha^2}{45m_e^4}s^2 + \mathcal{O}(m_e^{-6})$$

$$\mathscr{M}_{\mathrm{QED}}(\gamma^-\gamma^-\gamma^-\gamma^-) = -\frac{\alpha^2}{15m_e^4}\left[s^2+t^2+u^2\right] + \mathcal{O}(m_e^{-6})$$

$$\mathscr{M}_{\mathrm{EFT}}(\gamma^+\gamma^+\gamma^+\gamma^+) = 8\frac{c_1-c_2+ic_3}{\Lambda^4}\left[s^2+t^2+u^2\right]$$

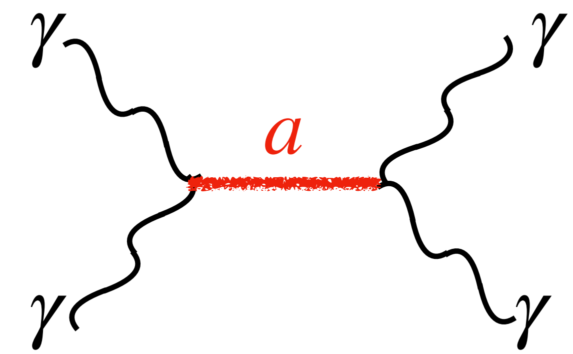
$$\mathscr{M}_{\mathrm{EFT}}(\gamma^+\gamma^+\gamma^-\gamma^-) = 8\frac{c_1+c_2}{\Lambda^4}s^2$$

$$\mathscr{M}_{\mathrm{EFT}}(\gamma^-\gamma^-\gamma^-\gamma^-) = 8\frac{c_1-c_2-ic_3}{\Lambda^4}\left[s^2+t^2+u^2\right]$$

Thus, integrating out the electron at one-loop level yields:

$$\frac{c_1}{\Lambda^4} = \frac{\alpha^2}{90m_e^4}, \qquad \frac{c_2}{\Lambda^4} = \frac{7\alpha^2}{360m_e^4}, \qquad \frac{c_3}{\Lambda^4} = 0$$

UV completion of Euler-Heisenberg EFT



ALP UV completion

$$\mathscr{L}_{\mathrm{UV}} \supset \frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{f_a}\left\{ gF_{\mu\nu}F_{\mu\nu} + \tilde{g}F_{\mu\nu}\tilde{F}_{\mu\nu} \right\}$$

Integrating out the axion at tree-level:

$$\frac{c_1}{\Lambda^4} = \frac{g^2}{2f_a^2 m_a^2}, \qquad \frac{c_2}{\Lambda^4} = \frac{\tilde{g}^2}{2f_a^2 m_a^2}, \qquad \frac{c_3}{\Lambda^4} = \frac{g\tilde{g}}{f_a^2 m_a^2}$$

Note that

$$\Lambda = \sqrt{f_a m_a}$$

In this example, the usual power counting, $\Lambda \sim m_a$, is disrupted, because the UV completion of an effective theory is itself an effective theory and contains other mass parameters than m_a

UV completion of Euler-Heisenberg EFT

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{\Lambda^4}\left\{c_1(F_{\mu\nu}F_{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + c_3(F_{\mu\nu}F_{\mu\nu})(F_{\alpha\beta}\tilde{F}_{\alpha\beta})\right\} + \ldots$$

- In the absence of new physics, the ordinary QED is the UV completion of this EFT, in which case the cutoff Λ can be identified with $2m_e$. However, in the presence of light axions or light milli-charged particles, this may no longer be the case.
- However, I'm not aware of a systematic experimental measurement of c_1, c_2, c_3. A future such measurement will be a non-trivial result, as some unknown light particles could in principle contribute to it, along with the electron and other SM charged particles

Euler-Heisenberg EFT

Summary and lessons learned

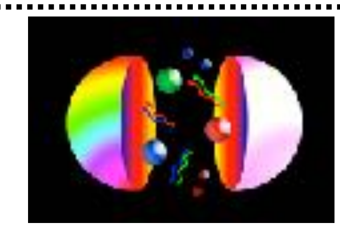
- Symmetries of a low-energy system often determine the structure of the effective theory at leading orders, up to a few unknown numerical parameters
- The EFT Lagrangian can be used for perturbative calculations of low-energy scattering amplitudes. But it is also a useful tool to work out subtle effects of classical field configurations
- A difference between this EFT and a renormalizable QFT is that counterterms of order $1/\Lambda^n$, also with n>4, are generated at loop level, thus these higher-order terms have to be added to the Lagrangian if we require precision beyond the $1/\Lambda^4$ order
- Precision measurement of the parameters of the Euler-Heisenberg EFT would give us information about its UV completion, which could possibly lead to surprises

100 TeV ???

100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

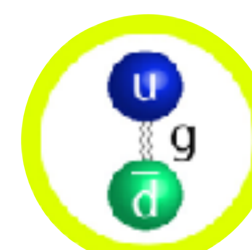
5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c

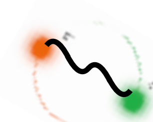


1 GeV γ, ν_i, e, μ + hadrons

100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e



γ, ν_i

Illustration #2

SM EFT

SMEFT

Rules of the game

- **Quantum Mechanics + Poincaré invariance = relativistic Quantum Field Theory**
- **Degrees of freedom: those of the SM (gluons, photon, W, Z, 6 quarks, 3 charged leptons, 3 neutrinos, Higgs doublet)**
- **SU(3)xSU(2)xU(1) gauge invariance**
- **Spontaneous breaking of SU(3)xSU(2)xU(1) down to SU(3)xU(1)**
- **Validity regime up to energies smaller than the cutoff scale Λ**
- **No other mass scale in the EFT except for Λ and Higgs mass parameter μ_H**

Starting from these principles, we will build an EFT in systematic expansion in 1/Λ

$$\mathscr{L}_{\text{eff}} = \Lambda^2 \mathscr{L}_{D=2} + \Lambda \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

For E << Λ each consecutive term is more suppressed, therefore the expansion makes sense

Standard Model

The most general Lagrangian up to D=4 consistent with these principles is just the SM Lagrangian

$$\mathscr{L}_{D=2} = \frac{\mu_H^2}{\Lambda^2} H^\dagger H$$

Unsolved mystery why the suppression by μ_H^2/Λ^2, which is called the hierarchy problem

$$\mathscr{L}_{D=3} = 0$$

$$\mathscr{L}_{D=4} = -\frac{1}{4} \sum_{V \in B, W^i, G^a} V_{\mu\nu} V^{\mu\nu} - \frac{g_S^2}{32\pi^2} \theta\, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc} V^b_\mu V^c_\nu$$

$$+ \sum_{f \in q,u,d,l,e} i \bar{f} \gamma^\mu D_\mu f$$

$$D_\mu f = \partial_\mu f - i g_s G^a_\mu T^a f - i g_L W^i_\mu \frac{\sigma^i}{2} f - i g_Y B_\mu Y f$$

$$- \left(\bar{u} Y_u q H + \bar{d} Y_d H^\dagger q + \bar{e} Y_e H^\dagger l + \mathrm{h.c.} \right)$$

$$+ D_\mu H^\dagger D^\mu H - \lambda (H^\dagger H)^2$$

$$\mathscr{L}_{\mathrm{SM}} \equiv \Lambda^2 \mathscr{L}_{D=2} + \Lambda^0 \mathscr{L}_{D=4}$$

19 physical parameters, most of them measured with a good precision, θ very well constrained

SMEFT

The dominant paradigm is that everything is EFT, and so the SM is a part of an EFT called SMEFT

SMEFT obeys the same fundamental principles as the SM, except that we don't truncate the expansion at D=4

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

Known SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

At each order we should include a <u>complete</u> and <u>non-redundant</u> set of operators eventually subject to some additional global symmetries

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.} \to c_{ij}\frac{\text{v}^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$

$$H \to \begin{pmatrix} 0 \\ \text{v}/\sqrt{2} \end{pmatrix}$$

$$L_i \to \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$$

- At dimension 5, the only operators one can construct are the so-called Weinberg operators, which break the lepton number

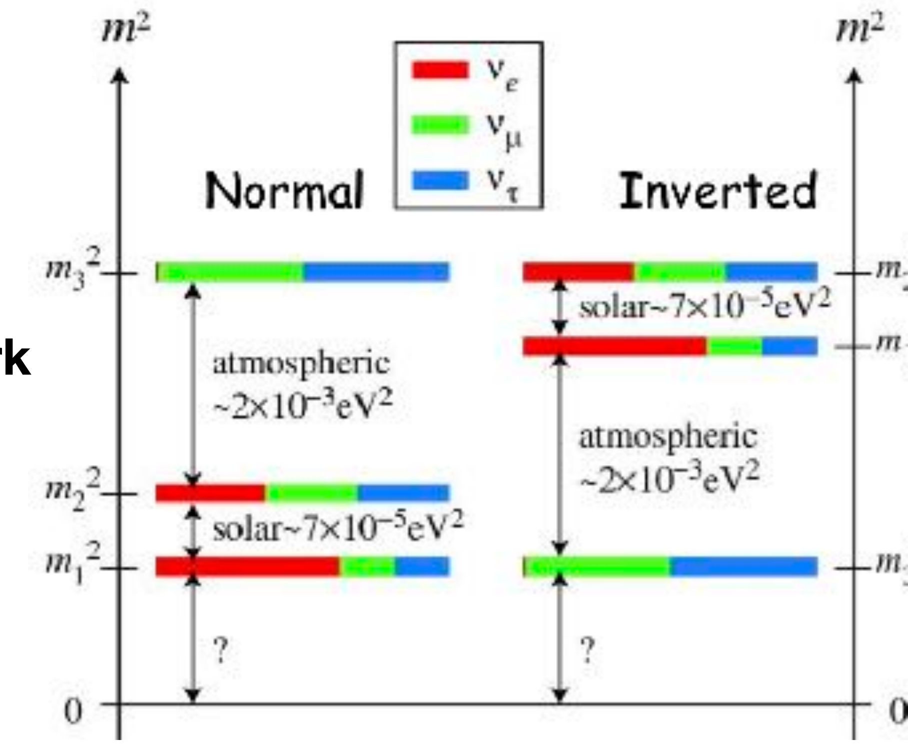
- After electroweak symmetry breaking they give rise to Majorana mass terms for the SM (left-handed) neutrinos

- Neutrino oscillation experiments strongly suggest that these operators are present (unless neutrino masses are of the Dirac type)

This is a huge success of SMEFT: corrections to the SM Lagrangian predicted at the leading order in the EFT expansion, are indeed observed in experiment!

SMEFT at dimension-5

$$\mathscr{L}_{\mathrm{SMEFT}} \supset c_{ij}\frac{\mathrm{v}^2}{\Lambda}\nu_i\nu_j + \mathrm{h.c.}$$



Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (while the lightest neutrino may even be massless)

It follows that $\Lambda / c_{ij} \sim 10^{15}$ GeV

One problem now:

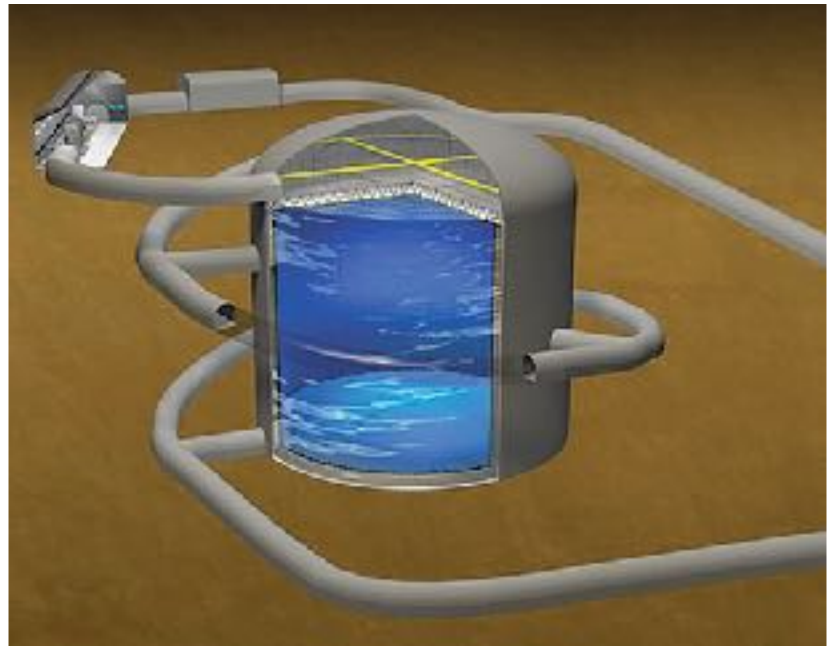
$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

Dimension-5 terms affect only neutrino physics, no other effects are observable

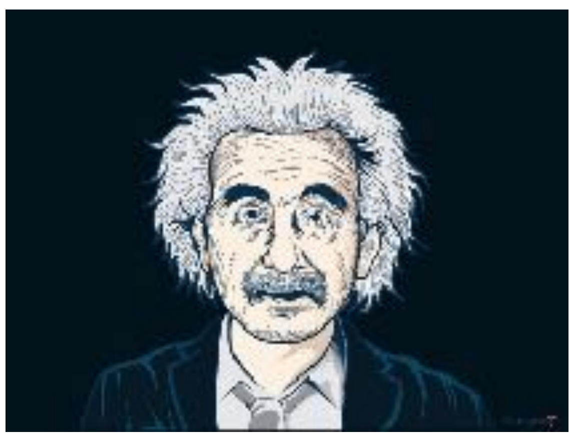
If $\Lambda \sim 10^{15}$ GeV then most dimension-6 and higher terms are too suppressed to be observable

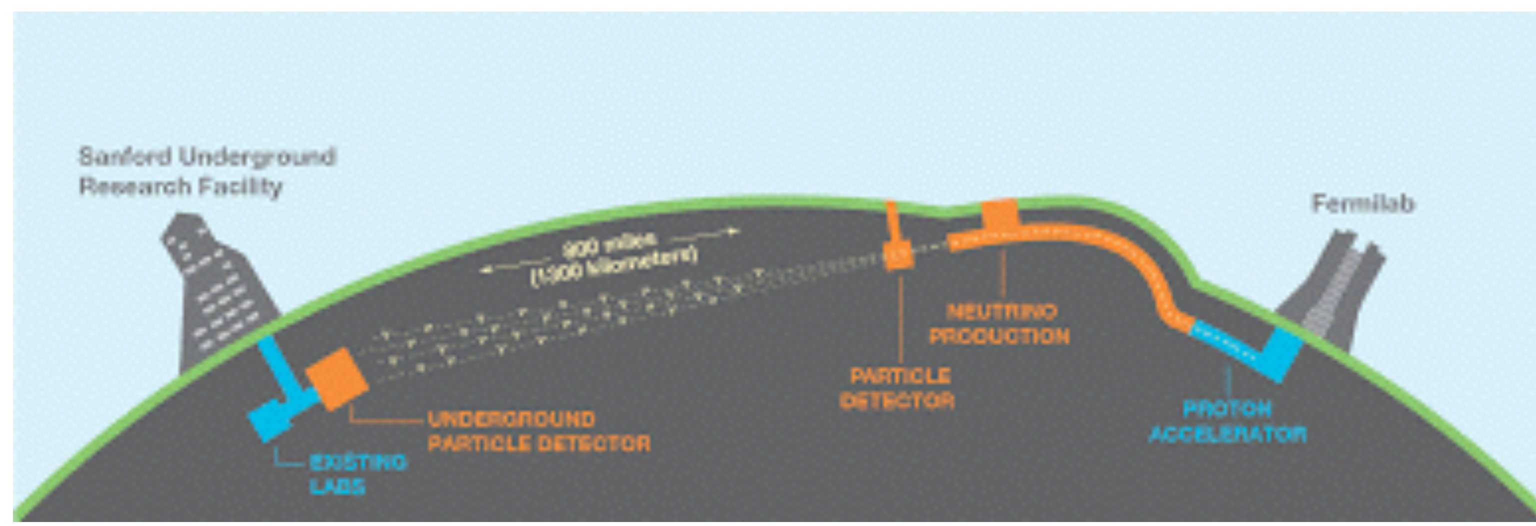
If this is really the correct expansion, then we will never see any other effects of higher-dimensional operators, except possibly of baryon-number violating ones :/

Career opportunities



?





SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

If this is really the correct expansion, then we will never see any other effects of higher-dimensional operators, except possibly of baryon-number violating ones :/

Another possibility, however is that Λ is much lower, but the UV sector (heavy neutrinos) couples very weekly to the SM degrees of freedom

Moreover, it is possible that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate <u>lepton number</u> L.
If we assume that the mass scale of new particles with L-violating interactions is Λ_L,
and there is also L-conserving new physics at the scale $\Lambda \ll \Lambda_L$, then the expansion is

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

This is our working assumption, not because it is strongly motivated by data but because the alternative is too depressing

SMEFT at dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

If $\Lambda << \Lambda_L$, leading non-SM effects in collider and precision physics may come from dimension-6 operators

There is 2499 of baryon-number-conserving D=6 operators, and another handful of B-violating operators

SMEFT at dimension-6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Bosonic CP-odd

$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Yukawa

$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I\bar\sigma_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J\tilde H^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operat the corresponding complex conjugate operator is implicitly included.

$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar e^c)(e^c\sigma_\mu\bar e^c)$	$O_{\ell e}$	$(\bar\ell\bar\sigma_\mu\ell)(e^c\sigma_\mu\bar e^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar u^c)(u^c\sigma_\mu\bar u^c)$	$O_{\ell u}$	$(\bar\ell\bar\sigma_\mu\ell)(u^c\sigma_\mu\bar u^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar d^c)(d^c\sigma_\mu\bar d^c)$	$O_{\ell d}$	$(\bar\ell\bar\sigma_\mu\ell)(d^c\sigma_\mu\bar d^c)$
O_{eu}	$(e^c\sigma_\mu\bar e^c)(u^c\sigma_\mu\bar u^c)$	O_{eq}	$(e^c\sigma_\mu\bar e^c)(\bar q\bar\sigma_\mu q)$
O_{ed}	$(e^c\sigma_\mu\bar e^c)(d^c\sigma_\mu\bar d^c)$	O_{qu}	$(\bar q\bar\sigma_\mu q)(u^c\sigma_\mu\bar u^c)$
O_{ud}	$(u^c\sigma_\mu\bar u^c)(d^c\sigma_\mu\bar d^c)$	O'_{qu}	$(\bar q\bar\sigma_\mu T^a q)(u^c\sigma_\mu T^a\bar u^c)$
O'_{ud}	$(u^c\sigma_\mu T^a\bar u^c)(d^c\sigma_\mu T^a\bar d^c)$	O_{qd}	$(\bar q\bar\sigma_\mu q)(d^c\sigma_\mu\bar d^c)$
		O'_{qd}	$(\bar q\bar\sigma_\mu T^a q)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar LL)(\bar LL)$		$(\bar LR)(\bar LR)$	
$O_{\ell\ell}$	$\eta(\bar\ell\bar\sigma_\mu\ell)(\bar\ell\bar\sigma_\mu\ell)$	O_{quqd}	$(u^cq^j)\epsilon_{jk}(d^cq^k)$
O_{qq}	$\eta(\bar q\bar\sigma_\mu q)(\bar q\bar\sigma_\mu q)$	O'_{quqd}	$(u^cT^aq^j)\epsilon_{jk}(d^cT^aq^k)$
O'_{qq}	$\eta(\bar q\bar\sigma_\mu\sigma^i q)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^cq^k)$
$O_{\ell q}$	$(\bar\ell\bar\sigma_\mu\ell)(\bar q\bar\sigma_\mu q)$	$O'_{\ell equ}$	$(e^c\bar\sigma_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar\sigma^{\mu\nu}q^k)$
$O'_{\ell q}$	$(\bar\ell\bar\sigma_\mu\sigma^i\ell)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell edq}$	$(\bar\ell\bar e^c)(d^cq)$

Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

$$O_{duq} = (d^cu^c)(\bar Q\bar\ell)$$
$$O_{qqu} = (qq)(\bar u^c\bar e^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^cu^c)(u^ce^c)$$

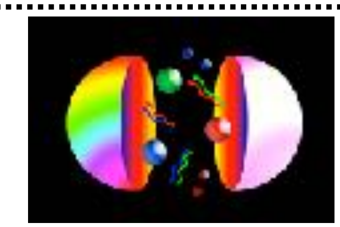

$|H|^2 B_{\mu\nu} B_{\mu\nu}$
$|H|^2 G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu} \widetilde{W}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu} W^a_{\mu\nu}$
$|H|^2 B_{\mu\nu} B_{\mu\nu}$
$|H|^6$
$|H|^2 G^a_{\mu\nu} G^a_{\mu\nu}$

100 TeV ???

100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

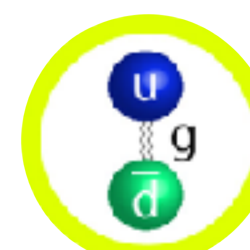
5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c



1 GeV γ, ν_i, e, μ + hadrons

100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e

γ, ν_i

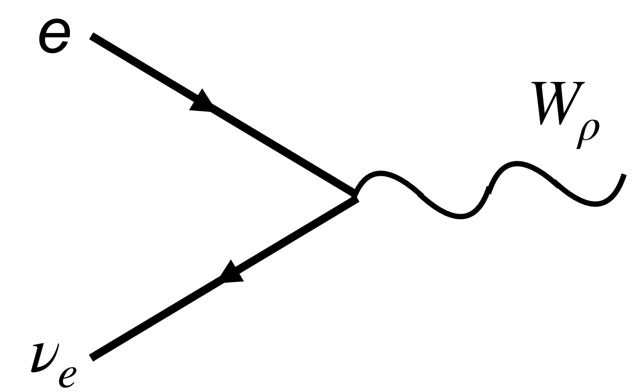
Illustration #3

Weak EFT

Muon decay in the SM

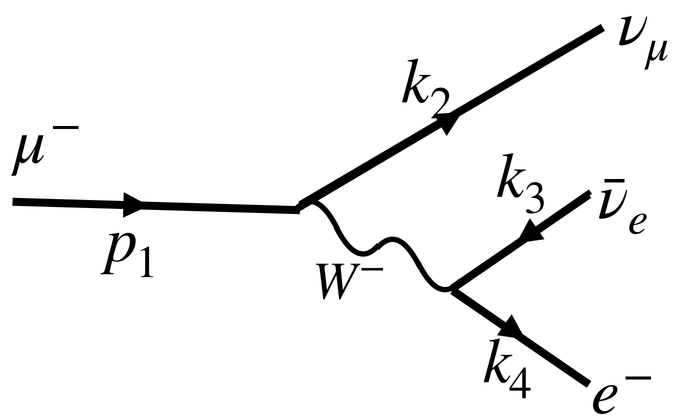
In the SM, weak interactions are mediated by W and Z bosons:

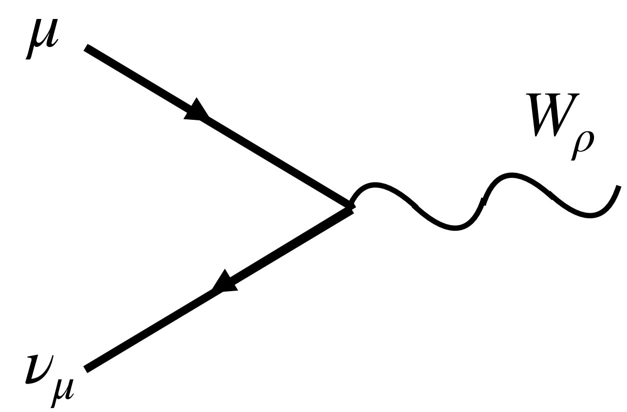
$$\mathscr{L}_{\rm SM} \supset \frac{g_L}{\sqrt{2}}\left[\bar{\nu}_L\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]W_\rho^+ + \text{h.c.}$$



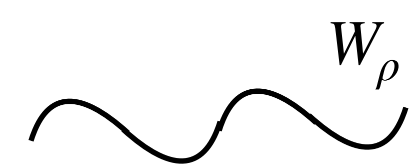
$$i\frac{g_L}{\sqrt{2}}\gamma_\rho P_L$$

In this theory, calculate muon decay





$$i\frac{g_L}{\sqrt{2}}\gamma_\rho P_L$$



$$-i\frac{\eta_{\mu\nu}}{p^2 - m_W^2}$$

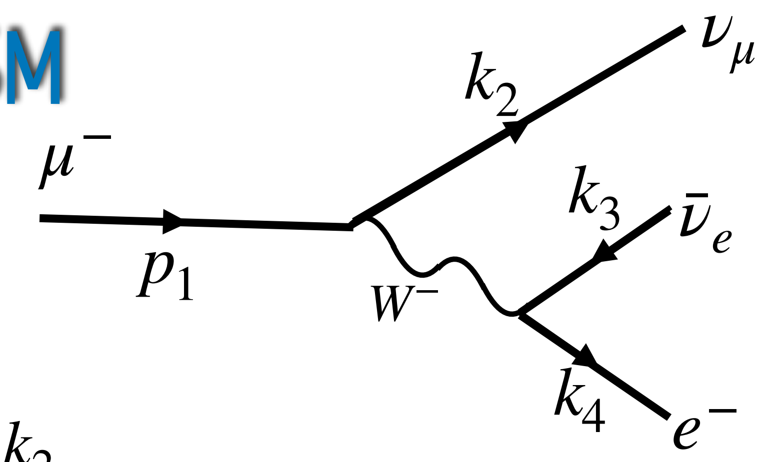
Tree-level amplitude:

$$\mathscr{M} = \frac{g_L^2}{2}\bar{u}(k_2)\gamma_\rho P_L u(p_1)\frac{1}{q^2 - m_W^2}\bar{u}(k_4)\gamma_\rho P_L \mathrm{v}(k_3)$$

$$q = p_1 - k_2$$

u(p) and v(p) are spinor wave functions for particles and anti-particles

Muon decay in the SM



Tree-level amplitude:

$$\mathscr{M} = \frac{g_L^2}{2} \bar{u}(k_2)\gamma_\rho P_L u(p_1) \frac{1}{q^2 - m_W^2} \bar{u}(k_4)\gamma_\rho P_L \mathrm{v}(k_3)$$

$$q = p_1 - k_2$$

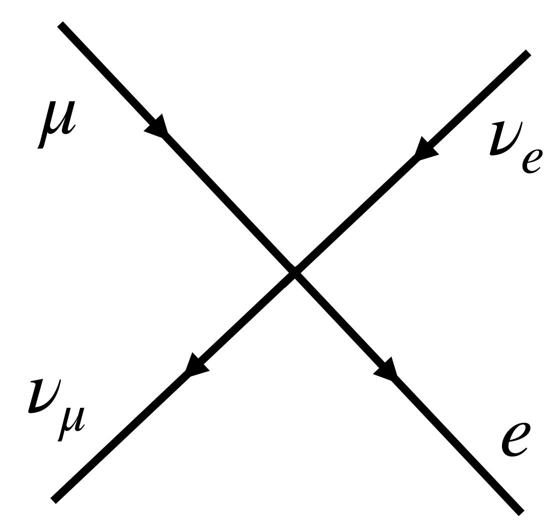
But kinematics of muon decay puts the constraint $q^2 \lesssim m_\mu^2 \ll m_W^2$

For all practical purpose one can thus approximate

$$\mathscr{M} = -\frac{g_L^2}{2m_W^2} \bar{u}(k_2)\gamma_\rho P_L u(p_1) \bar{u}(k_4)\gamma_\rho P_L \mathrm{v}(k_3) + \mathcal{O}(q^2/m_W^4)$$

This approximate amplitude can be equally well obtained from the <u>effective</u> Lagrangian

$$\mathscr{L}_{\mathrm{eff}} \supset -\frac{g_L^2}{2m_W^2} (\bar{\nu}_\mu \gamma_\rho \mu_L)(\bar{e}_L \gamma_\rho \nu_L) + \mathrm{h.c.}$$



$$-i\frac{g_L^2}{2m_W^2}[\gamma_\rho P_L][\gamma_\rho P_L]$$

EFT below the electroweak scale

- Many interesting particle physics processes, like muon decay, meson decays and oscillations, beta transitions, neutrino scattering on nuclei, EDMs, etc., occur with characteristic energy far below the electroweak scale (E << 100 GeV)

- At these energies W, Z, and also Higgs and top do not propagate, and can be integrated out from the theory, in order to simplify it but also to improve its convergence

- The weak and Higgs interactions mediated by those force carriers in the SMEFT are mimicked in the WEFT by contact interactions between light degrees of freedom: leptons, quarks, etc.

- The resulting EFT is called here the Weak EFT, or the **WEFT** in short (also names LEFT and WET exist in the literature)

WEFT

Rules of the game

- Quantum Mechanics + Poincaré invariance = relativistic Quantum Field Theory
- Degrees of freedom: gluons, photon, 5 quarks, 3 charged leptons, 3 neutrinos
- $SU(3)xU(1)_{em}$ gauge invariance
- Validity regime up to energies smaller than the cutoff scale $\Lambda = m_W$
- No other mass scale in the EFT except for Λ and particles' masses

Starting from these principles, we will build an EFT in systematic expansion in 1/Λ

$$\mathscr{L}_{\text{eff}} = \Lambda \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

For E << Λ each consecutive term is more suppressed, therefore the expansion makes sense

WEFT

D=3

$\nu\nu + \text{h.c.}$	
$\mathcal{O}_\nu$	$(\nu_{Lp}^T C \nu_{Lr})$

D=5

$(\nu\nu)X + \text{h.c.}$	
$\mathcal{O}_{\nu\gamma}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr}) F_{\mu\nu}$

$(\overline{L}R)X + \text{h.c.}$	
$\mathcal{O}_{e\gamma}$	$\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{u\gamma}$	$\bar{u}_{Lp}\sigma^{\mu\nu}u_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{d\gamma}$	$\bar{d}_{Lp}\sigma^{\mu\nu}d_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{uG}$	$\bar{u}_{Lp}\sigma^{\mu\nu}T^A u_{Rr}\, G^A_{\mu\nu}$
$\mathcal{O}_{dG}$	$\bar{d}_{Lp}\sigma^{\mu\nu}T^A d_{Rr}\, G^A_{\mu\nu}$

X^3	
$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$

D=6

$(\overline{L}L)(\overline{L}L)$	
$\mathcal{O}_{\nu\nu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{\nu}_{Ls}\gamma_\mu\nu_{Lt})$
$\mathcal{O}_{ee}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu e}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu u}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{\nu d}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{\nu edu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt}) + \text{h.c.}$
$\mathcal{O}_{uu}^{V,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{dd}^{V,LL}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V1,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V8,LL}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Ls}\gamma_\mu T^A d_{Lt})$

$(\overline{R}R)(\overline{R}R)$	
$\mathcal{O}_{ee}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{uu}^{V,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{dd}^{V,RR}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V1,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$

$(\overline{L}L)(\overline{R}R)$	
$\mathcal{O}_{\nu e}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{ee}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu u}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{\nu d}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{eu}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ue}^{V,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{de}^{V,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu edu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$
$\mathcal{O}_{uu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{uu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{ud}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{du}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{du}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{dd}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{dd}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{uddu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$
$\mathcal{O}_{uddu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A u_{Rt}) + \text{h.c.}$

$(\overline{L}R)(\overline{L}R) + \text{h.c.}$	
$\mathcal{O}_{ee}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{e}_{Ls}e_{Rt})$
$\mathcal{O}_{eu}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{u}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{ed}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}d_{Rt})$
$\mathcal{O}_{\nu edu}^{S,RR}$	$(\bar{\nu}_{Lp}e_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{\nu edu}^{T,RR}$	$(\bar{\nu}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{uu}^{S1,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{uu}^{S8,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{u}_{Ls}T^A u_{Rt})$
$\mathcal{O}_{ud}^{S1,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{ud}^{S8,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
$\mathcal{O}_{dd}^{S1,RR}$	$(\bar{d}_{Lp}d_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{dd}^{S8,RR}$	$(\bar{d}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
$\mathcal{O}_{uddu}^{S1,RR}$	$(\bar{u}_{Lp}d_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{uddu}^{S8,RR}$	$(\bar{u}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A u_{Rt})$

$(\overline{L}R)(\overline{R}L) + \text{h.c.}$	
$\mathcal{O}_{eu}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Rs}u_{Lt})$
$\mathcal{O}_{ed}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Rs}d_{Lt})$
$\mathcal{O}_{\nu edu}^{S,RL}$	$(\bar{\nu}_{Lp}e_{Rr})(\bar{d}_{Rs}u_{Lt})$

We focus on a small subset of those

Jenkins et al [arXiv:1709.04486]

Charged current interactions in SM

Consider low-energy interactions between light quarks and leptons

Starting point:
$$\mathscr{L}_{\rm SM} \supset - W_\rho^+(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left\{ \left[\bar\nu_L\gamma_\rho e_L + V_{ud}\bar u_L\gamma_\rho d_L\right] W_\rho^+ + {\rm h.c.}\right\}$$

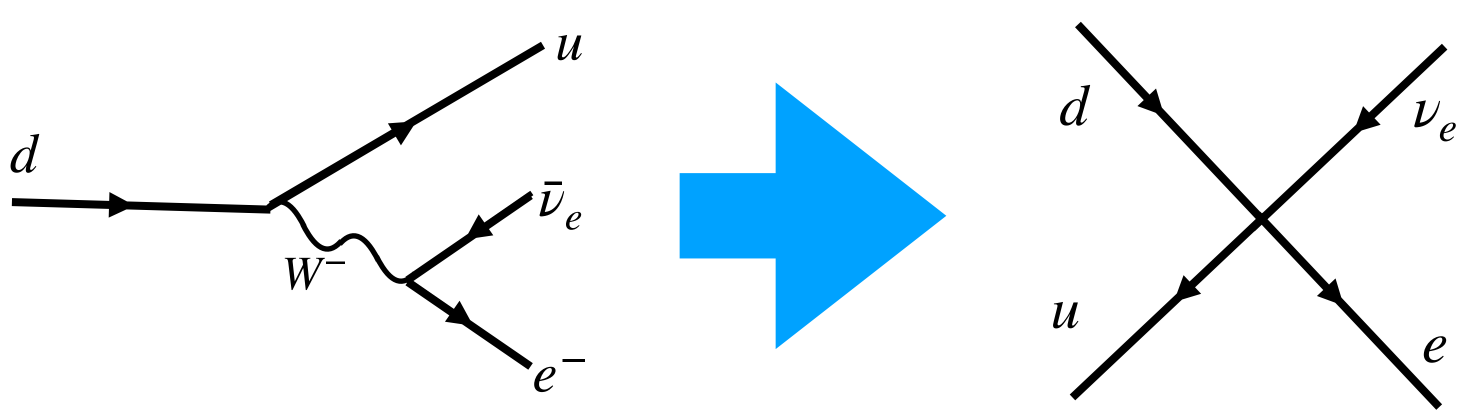
SM CKM element

e.o.m:
$$-(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar\nu_L\gamma_\rho e_L + V_{ud}\bar u_L\gamma_\rho d_L\right] = 0$$

solution:
$$W_\rho^- = \frac{g_L}{\sqrt{2}}(\Box - m_W^2)^{-1}\left[\bar\nu_L\gamma_\rho e_L + V_{ud}\bar u_L\gamma_\rho d_L\right] \approx -\frac{g_L}{\sqrt{2}m_W^2}\left[\bar\nu_L\gamma_\rho e_L + V_{ud}\bar u_L\gamma_\rho d_L\right]$$

Leading effective 4-fermion interactions:
$$\mathscr{L}_{\rm WEFT} \supset -\frac{g_L^2}{2m_W^2}\left[\bar e_L\gamma_\rho \nu_L + V_{ud}\bar d_L\gamma_\rho u_L\right]\left[\bar\nu_L\gamma_\rho e_L + V_{ud}\bar u_L\gamma_\rho d_L\right] \supset -\frac{2V_{ud}}{{\rm v}^2}(\bar e_L\gamma_\rho\nu_L)(\bar u_L\gamma_\rho d_L)$$

$${\rm v} \equiv \frac{2m_W}{g_L} \approx 246~{\rm GeV}$$



Charged current interactions in SMEFT and WEFT

Three new elements appear in SMEFT regarding the charged currents:

1. Coupling strength of W to quarks and leptons can be modified compared to the SM

2. W boson can now couple to right-handed quarks as well

3. Charged currents are mediated not only by W, but also by new contact interactions

1

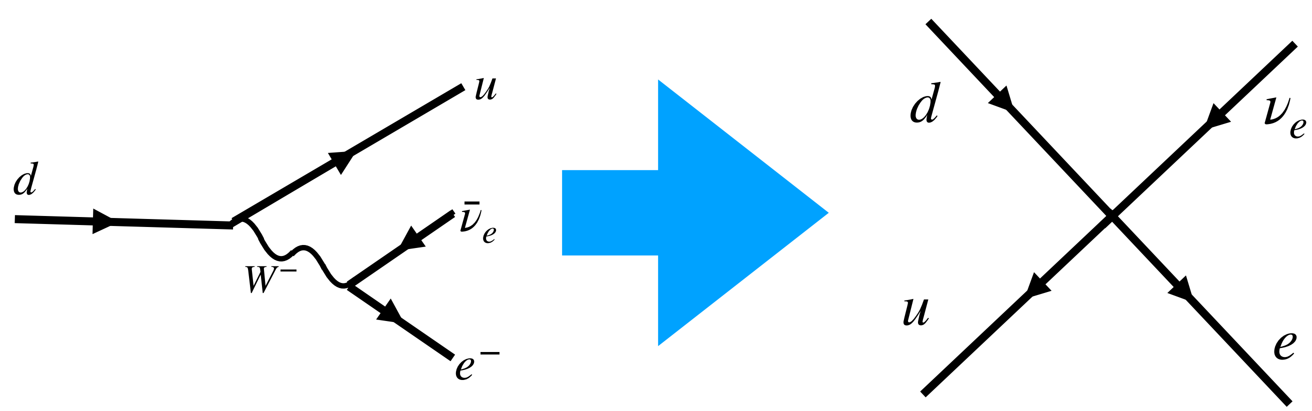
$$\mathscr{L}_{\text{SMEFT}} \supset \frac{c_{HQ}}{\Lambda^2} H^\dagger \sigma^a D_\mu H (\bar{Q}\sigma^a \gamma_\mu Q) + \frac{c_{HL}}{\Lambda^2} H^\dagger \sigma^a D_\mu H (\bar{L}\sigma^a \gamma_\mu L)$$

After Higgs gets a VEV

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{g_L}{\sqrt{2}} \left[1 + \frac{c_{HL} \mathrm{v}^2}{\Lambda^2}\right] W_\rho^+ \bar{\nu}_L \gamma_\rho e_L + V_{ud} \frac{g_L}{\sqrt{2}} \left[1 + \frac{c_{HQ} \mathrm{v}^2}{\Lambda^2}\right] W_\rho^+ \bar{u}_L \gamma_\rho d_L + \mathrm{h.c.}$$

Integrating out W boson

$$\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2} \left[1 + \frac{c_{HL} \mathrm{v}^2}{\Lambda^2}\right] \left[1 + \frac{c_{HQ} \mathrm{v}^2}{\Lambda^2}\right] (\bar{e}_L \gamma_\rho \nu_L)(\bar{u}_L \gamma_\rho d_L) \approx -\frac{2V_{ud}}{\mathrm{v}^2} \left[1 + \left(c_{HL} + c_{HQ}\right) \frac{\mathrm{v}^2}{\Lambda^2}\right] (\bar{e}_L \gamma_\rho \nu_L)(\bar{u}_L \gamma_\rho d_L)$$



Same effective interaction as in WEFT derived from SM but with a different Wilson coefficient

Charged current interactions in SMEFT and WEFT

Three new elements appear in SMEFT regarding the charged currents:

1. **Coupling strength of W to quarks and leptons can be modified compared to the SM**
2. **W boson can now couple to right-handed quarks as well**
3. **Charged currents are mediated not only by W, but also by new contact interactions**

2

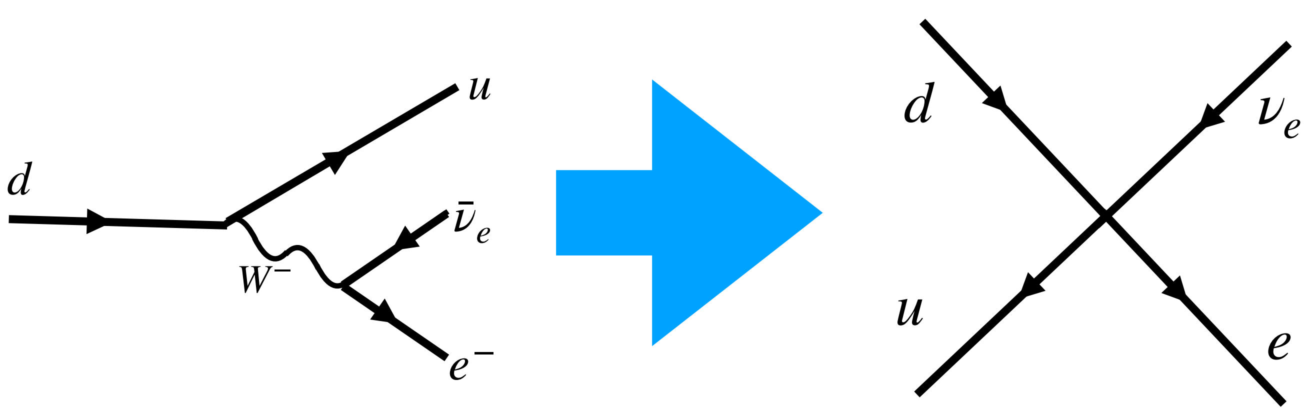
$$\mathscr{L}_{\mathrm{SMEFT}} \supset \frac{c_{Hud}}{\Lambda^2} H^T D_\mu H (\bar{u}_R \gamma_\mu d_R)$$

After Higgs gets a VEV

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \frac{g_L}{\sqrt{2}} \left[1 + \frac{c_{HL} \mathrm{v}^2}{\Lambda^2}\right] W_\rho^+ \bar{\nu}_L \gamma_\rho e_L + \frac{g_L}{\sqrt{2}} \left[\frac{c_{Hud} \mathrm{v}^2}{2\Lambda^2}\right] W_\rho^+ \bar{u}_R \gamma_\rho d_R + \mathrm{h.c.}$$

Integrating out W boson

$$\mathscr{L}_{\mathrm{WEFT}} \supset -\frac{2}{\mathrm{v}^2} \left[1 + \frac{c_{HL} \mathrm{v}^2}{\Lambda^2}\right] \left[\frac{c_{Hud} \mathrm{v}^2}{2\Lambda^2}\right] (\bar{e}_L \gamma_\rho \nu_L)(\bar{u}_R \gamma_\rho d_R) \approx -\frac{2V_{ud}}{\mathrm{v}^2} \left[\frac{c_{Hud} \mathrm{v}^2}{2V_{ud}\Lambda^2}\right] (\bar{e}_L \gamma_\rho \nu_L)(\bar{u}_R \gamma_\rho d_R)$$



Different effective interaction than in WEFT derived from SM

Charged current interactions in SMEFT and WEFT

Three new elements appear in SMEFT regarding the charged currents:

1. Coupling strength of W to quarks and leptons can be modified compared to the SM

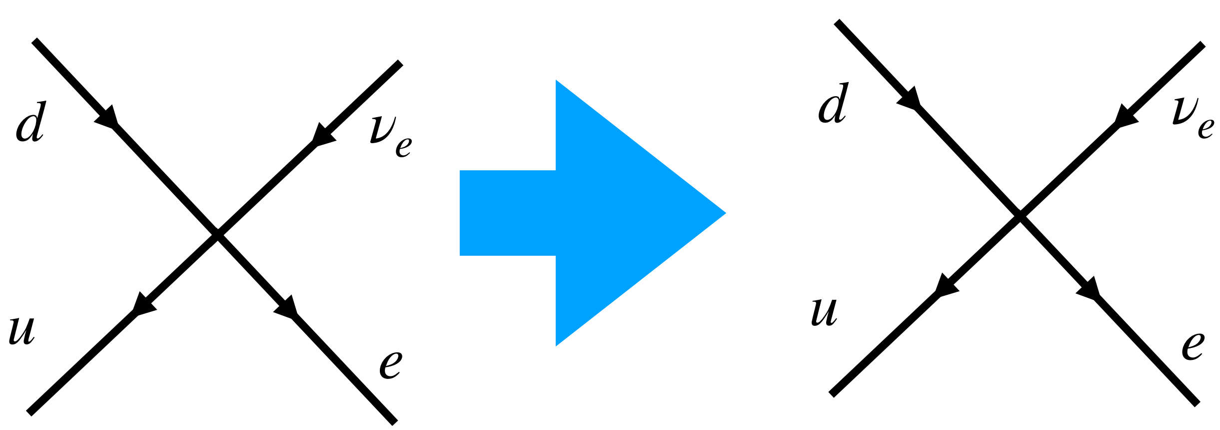
2. W boson can now couple to right-handed quarks as well

3. Charged currents are mediated not only by W, but also by new contact interactions

3

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{c_{LQ}^{(3)}}{\Lambda^2}(\bar{Q}\sigma^a\gamma_\mu Q)(\bar{L}\sigma^a\gamma_\mu L) + \frac{c_{LeQu}^{(3)}}{\Lambda^2}(\bar{e}_R\sigma_{\mu\nu}L)(\bar{u}_R\sigma_{\mu\nu}Q) + \frac{c_{LeQu}}{\Lambda^2}(\bar{e}_R L)(\bar{u}_R Q) + \frac{c_{LedQ}}{\Lambda^2}(\bar{L}e_R)(\bar{d}_R Q)$$

$$\mathscr{L}_{\text{WEFT}} \supset \frac{2V_{ud}}{\mathrm{v}^2}\left\{ \frac{c_{LQ}^{(3)}\mathrm{v}^2}{\Lambda^2}(\bar{e}_L\gamma_\rho\nu_L)(\bar{u}_L\gamma_\rho d_L) + \frac{c_{LeQu}^{(3)}\mathrm{v}^2}{2\Lambda^2}(\bar{e}_R\sigma_{\mu\nu}\nu_L)(\bar{u}_R\sigma_{\mu\nu}d_L) + \frac{c_{LeQu}\mathrm{v}^2}{2\Lambda^2}(\bar{e}_R\nu_L)(\bar{u}_R d_L) + \frac{c_{LedQ}\mathrm{v}^2}{2V_{ud}\Lambda^2}(\bar{\nu}_L e_R)(\bar{d}_R u_L) \right\}$$



First terms is same effective interaction as in WEFT derived from SM but with a different Wilson coefficient

The last 3 terms are Different effective interaction than in WEFT derived from SM

Charged current in WEFT

To summarise, starting from the SMEFT and integrating out W in the quark-lepton charged current sector, we arrived at the following interactions

CKM element

Left-handed neutrino

$$\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\text{v}^2}\Big\{ (1+\epsilon_L)\ \bar{e}_L\gamma_\mu\nu_L \cdot \bar{u}_L\gamma^\mu d_L$$

V-A

$$+\epsilon_R\,\bar{e}_L\gamma_\mu\nu_L \cdot \bar{u}_R\gamma^\mu d_R$$

V+A

Normalization scale, set by Fermi constant

$$\text{v} = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx 246\ \text{GeV}$$

$$+\epsilon_T\frac{1}{4}\bar{e}_R\sigma_{\mu\nu}\nu_L \cdot \bar{u}_R\sigma^{\mu\nu}d_L$$

Tensor

$$+\epsilon_S\frac{1}{2}\bar{e}_R\nu_L \cdot \bar{u}d$$

Scalar

Pseudo-scalar

$$-\epsilon_P\frac{1}{2}\bar{e}_R\nu_L \cdot \bar{u}\gamma_5 d\Big\} + \text{h.c.}$$

WEFT leads to a simpler description at low energies: number of parameters reduced wrt to the SMEFT

Translation from low-to-high energy EFT

The EFT below the weak scale (WEFT)
can be matched to the EFT above the weak scale (SMEFT)

$$
\begin{aligned}
\mathscr{L}_{\text{SMEFT}} \supset\ & c_{HQ} H^\dagger \sigma^a D_\mu H (\bar{Q}\sigma^a \gamma_\mu Q) + c_{HL} H^\dagger \sigma^a D_\mu H (\bar{L}\sigma^a \gamma_\mu L) \\
& + c_{Hud} H^T D_\mu H (\bar{u}_R \gamma_\mu d_R) \\
& + c^{(3)}_{LQ} (\bar{Q}\sigma^a \gamma_\mu Q)(\bar{L}\sigma^a \gamma_\mu L) + c^{(3)}_{LeQu} (\bar{e}_R \sigma_{\mu\nu} L)(\bar{u}_R \sigma_{\mu\nu} Q) \\
& + c_{LeQu} (\bar{e}_R L)(\bar{u}_R Q) + c_{LedQ} (\bar{L} e_R)(\bar{d}_R Q)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2} \Big\{ & (1+\epsilon_L)\ \bar{e}_L \gamma_\mu \nu_L \cdot \bar{u}_L \gamma^\mu d_L \\
& + \epsilon_R\, \bar{e}_R \gamma_\mu \nu_L \cdot \bar{u}_R \gamma^\mu d_R \\
& + \epsilon_T \frac{1}{4} \bar{e}_R \sigma_{\mu\nu} \nu_L \cdot \bar{u}_R \sigma^{\mu\nu} d_L \\
& + \epsilon_S \frac{1}{2} \bar{e}_R \nu_L \cdot \bar{u} d \\
& - \epsilon_P \frac{1}{2} \bar{e}_R \nu_L \cdot \bar{u} \gamma_5 d \Big\} + \mathrm{h.c.}
\end{aligned}
$$

At the scale m_W, WEFT parameters ϵ_X map to dimension-6 operators in the SMEFT

$$\epsilon_L = \left[c_{HL} + c_{HQ} - c^{(3)}_{LQ}\right] \frac{\mathrm{v}^2}{\Lambda^2} + \dots$$

more subtle…

$$\epsilon_R = \frac{1}{2V_{ud}} c_{Hud} \frac{\mathrm{v}^2}{\Lambda^2}$$

$$\epsilon_S = -\frac{1}{2V_{ud}} \left[V_{ud} c_{LeQu} + c^*_{LedQ}\right] \frac{\mathrm{v}^2}{\Lambda^2}$$

$$\epsilon_T = -2 c^{(3)}_{LeQu} \frac{\mathrm{v}^2}{\Lambda^2}$$

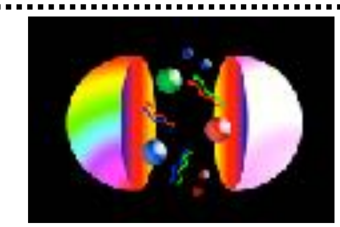
$$\epsilon_P = -\frac{1}{2V_{ud}} \left[V_{ud} c_{LeQu} - c^*_{LedQ}\right] \frac{\mathrm{v}^2}{\Lambda^2}$$

100 TeV ???

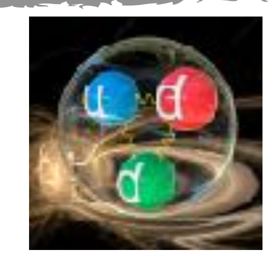
100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

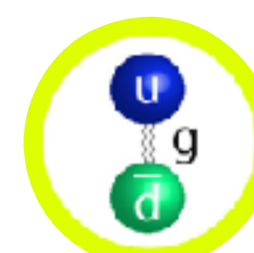
2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c



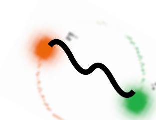
1 GeV γ, ν_i, e, μ + hadrons



100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e



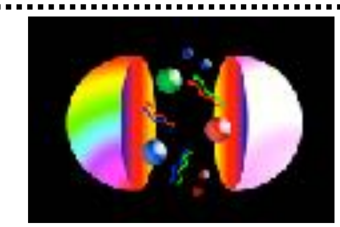
γ, ν_i

100 TeV ???

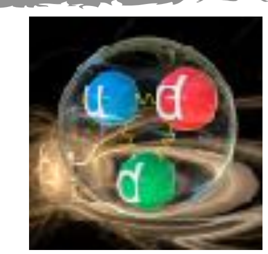
100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

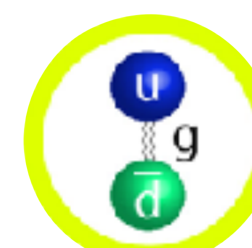
2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c



1 GeV γ, ν_i, e, μ + hadrons



100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e

γ, ν_i

Illustration #4

Fermi EFT

From WEFT to Fermi EFT

- At a scale of order 2 GeV the quarks of WEFT becomes strongly interacting
- Below that scale, the useful degrees of freedom are no longer quarks but hadrons: baryons and mesons
- We have to switch our EFT description to take into account the new degrees of freedom (and the lack of quarks)
- I focus on a special sector of the that EFT, which describes beta transitions involving nucleons (protons and neutrons) and leptons
- I call this the Fermi EFT

From WEFT to Fermi EFT

Let us take only SM-derived interactions for the start:

$$\mathscr{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2}(\bar{e}_L\gamma_\rho\nu_L)(\bar{u}_L\gamma_\rho d_L) + \mathrm{h.c.}$$

This interaction leads to beta decays, in particular to the neutron decay

$$d \to u e^- \bar{\nu}_e \quad \Rightarrow \quad n \to p e^- \bar{\nu}_e$$

Amplitude for the latter process is

$$\begin{aligned}
M(n \to pe^-\bar{\nu}_e) &= -\frac{2V_{ud}}{\mathrm{v}^2}\langle pe^-\bar{\nu}_e\,|\,(\bar{e}_L\gamma_\rho\nu_L)(\bar{u}_L\gamma_\rho d_L)\,|\,n> \\
&= -\frac{2V_{ud}}{\mathrm{v}^2}\langle e^-\bar{\nu}_e\,|\,(\bar{e}_L\gamma_\rho\nu_L)\,|\,0> \langle p\,|\,(\bar{u}_L\gamma_\rho d_L)\,|\,n> \\
&= -\frac{2V_{ud}}{\mathrm{v}^2}\Big(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\Big)\langle p\,|\,(\bar{u}_L\gamma_\rho d_L)\,|\,n> \\
&= -\frac{V_{ud}}{\mathrm{v}^2}\Big(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\Big)\Big\{\langle p\,|\,(\bar{u}\gamma_\rho d)\,|\,n> - \langle p\,|\,(\bar{u}\gamma_\rho\gamma_5 d)\,|\,n>\Big\}
\end{aligned}$$

$$P_L \equiv \frac{1-\gamma_5}{2}$$

where u(p), v(p) are the usual spinor wave functions for particle and antiparticles

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{\mathrm{v}^2} \left(\bar{u}(p_e) \gamma_\rho P_L \mathrm{v}(p_\nu) \right) \left\{ \langle p | (\bar{u}\gamma_\rho d) | n > - \langle p | (\bar{u}\gamma_\rho \gamma_5 d) | n > \right\}$$

Due to strong QCD interaction, the quark matrix element cannot be calculated perturbatively

However, with the input from dimensional analysis and QCD (approximate) symmetries they can be reduced to a few unknowns, which can be subsequently calculated on the lattice or using phenomenological models

Lorentz invariance + Parity of QCD implies

$$q \equiv p_n - p_p$$

$$\langle p | (\bar{u}\gamma_\rho d) | n > = \bar{u}(p_p) \left[g_V(q^2) \gamma_\rho + \frac{\tilde{g}_{TV}(q^2)}{2m_n} \sigma_{\rho\nu} q^\nu + \frac{\tilde{g}_S(q^2)}{2m_n} q_\rho \right] u(p_n)$$

$$\langle p | (\bar{u}\gamma_\rho \gamma_5 d) | n > = \bar{u}(p_p) \left[g_A(q^2) \gamma_\rho + \frac{\tilde{g}_{TA}(q^2)}{2m_n} \sigma_{\rho\nu} q^\nu + \frac{\tilde{g}_P(q^2)}{2m_n} q_\rho \right] \gamma_5 u(p_n)$$

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{v^2}\big(\bar{u}(p_e)\gamma_\rho P_L v(p_\nu)\big)\left\{ \langle p\,|\,(\bar{u}\gamma_\rho d)\,|\,n > - \langle p\,|\,(\bar{u}\gamma_\rho\gamma_5 d)\,|\,n > \right\}$$

For beta decay processes, and especially for neutron decay, recoil is much smaller than nucleon mass. Therefore at the leading order one can approximate

$$\langle p\,|\,(\bar{u}\gamma_\rho d)\,|\,n > = g_V \bar{u}(p_p)\gamma_\rho u(p_n) + \mathcal{O}(q)$$
$$\langle p\,|\,(\bar{u}\gamma_\rho\gamma_5 d)\,|\,n > = g_A \bar{u}(p_p)\gamma_\rho\gamma_5 u(p_n) + \mathcal{O}(q)$$

$$q \equiv p_n - p_p$$

where g_V=$g_V(0)$ and g_A=$g_A(0)$ are now numbers, called the vector and axial charges

Furthermore, in the isospin symmetric g_V=1, because the quark current is the isospin current One can prove that departures of g_V from one are second order in isospin breaking, thus tiny

All in all

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{v^2}\big(\bar{u}(p_e)\gamma_\rho P_L v(p_\nu)\big)\left\{ \bar{u}(p_p)\gamma_\rho u(p_n) - g_A \bar{u}(p_p)\gamma_\rho\gamma_5 u(p_n) + \mathcal{O}(q) \right\}$$

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{v^2}\left(\bar{u}(p_e)\gamma_\rho P_L v(p_\nu)\right)\left\{ \bar{u}(p_p)\gamma_\rho u(p_n) - g_A \bar{u}(p_p)\gamma_\rho \gamma_5 u(p_n) + \mathcal{O}(q) \right\}$$

$$\mathcal{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{v^2}(\bar{e}_L \gamma_\rho \nu_e)(\bar{u}_L \gamma_\rho d_L) + \mathrm{h.c.}$$

Matching

$$\mathcal{L}_{\mathrm{Fermi}} \supset -\frac{V_{ud}}{v^2}(\bar{e}_L \gamma_\rho \nu_e)\left\{ (\bar{p}\gamma_\rho n) - g_A(\bar{p}\gamma_\rho \gamma_5 n) \right\} + \mathrm{h.c.} + \mathcal{O}\left(\frac{q}{m_n}\right)$$

as our n→p e *v* amplitude can be obtained from this effective Lagrangian

The non-perturbative parameter g_A appearing in this matching has to be calculated on the lattice or measured in experiment

Lattice	**Experiment**
$g_A = 1.271 \pm 0.013$	$g_A = 1.27536 \pm 0.00041$

Fermi EFT

Now let's take into account non-SM interactions in WEFT

$$\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\text{v}^2}\Big\{ (1+\epsilon_L)\ \bar{e}_L\gamma_\mu\nu_L\cdot\bar{u}_L\gamma^\mu d_L + \epsilon_R\,\bar{e}_R\gamma_\mu\nu_L\cdot\bar{u}_R\gamma^\mu d_R + \epsilon_S\frac{1}{2}\bar{e}_R\nu_L\cdot\bar{u}d + \epsilon_T\frac{1}{4}\bar{e}_R\sigma_{\mu\nu}\nu_L\cdot\bar{u}_R\sigma^{\mu\nu}d_L - \epsilon_P\frac{1}{2}\bar{e}_R\nu_L\cdot\bar{u}\gamma_5 d\Big\} + \text{h.c.}$$

At nucleon scale, we then get more general set of interactions

$$\mathscr{L}_{\text{Fermi}} \supset -C_V^+\bar{p}\gamma^\mu n\,\bar{e}_L\gamma_\mu\nu_L - C_A^+\bar{p}\gamma^\mu\gamma_5 n\,\bar{e}_L\gamma_\mu\nu_L - C_S^+\bar{p}n\,\bar{e}_R\nu_L - \frac{1}{2}C_T^+\bar{p}\sigma^{\mu\nu}n\,\bar{e}_R\sigma_{\mu\nu}\nu_L + C_P^+\bar{p}\gamma_5 n\,\bar{e}_R\nu_L + \text{h.c.}$$

T.D. Lee and C.N. Yang (1956)

Short-distance radiative corrections

Dictionary is obtained along the same lines as in SM case

$$C_V^+ = \frac{V_{ud}}{\text{v}^2}\sqrt{1+\Delta_R^V}\left(1+\epsilon_L+\epsilon_R\right)$$

$\Delta_R^V = 0.02467(22)$ Seng et al 1807.10197

$$C_A^+ = -\frac{V_{ud}}{\text{v}^2}g_A\sqrt{1+\Delta_R^A}\left(1+\epsilon_L-\epsilon_R\right)$$

Flag'19: $g_A = 1.251 \pm 0.033$

$\Delta_R^A - \Delta_R^V = 4.07(8)\times 10^{-3}$ Hayen 2010.07262

$$C_T^+ = \frac{V_{ud}}{\text{v}^2}g_T\epsilon_T$$

$$C_S^+ = \frac{V_{ud}}{\text{v}^2}g_S\epsilon_S$$

$g_S = 1.02 \pm 0.10,\quad g_T = 0.989 \pm 0.034$ Gupta et al 1806.09006

$$C_P^+ = \frac{V_{ud}}{\text{v}^2}g_P\epsilon_P$$

$g_P = 349 \pm 9$ Gonzalez-Alonso et al 1803.08732

Down the rabbit hole

$$
\begin{aligned}
\mathscr{L}_{\text{Fermi}} \supset & -C_V^+ \bar{p}\gamma^\mu n\, \bar{e}_L\gamma_\mu\nu_L \\
& -C_A^+ \bar{p}\gamma^\mu\gamma_5 n\, \bar{e}_L\gamma_\mu\nu_L \\
& -C_S^+ \bar{p}n\, \bar{e}_R\nu_L \\
& -\frac{1}{2}C_T^+ \bar{p}\sigma^{\mu\nu} n\, \bar{e}_R\sigma_{\mu\nu}\nu_L \\
& +C_P^+ \bar{p}\gamma_5 n\, \bar{e}_R\nu_L \qquad\qquad +\mathrm{h.c.}
\end{aligned}
$$

This is a relativistic Lagrangian, and may not be most convenient to use for non-relativistic processes

In neutron decay the momentum transfer is much smaller then the nucleon mass, due to the tiny mass splitting between neutron and proton.

It is thus convenient to change variables in the Lagrangian, and use non-relativistic version of the neutron and proton quantum fields

$$
N \to \frac{e^{-im_N t}}{\sqrt{2}}\left(1 + i\frac{\boldsymbol{\sigma}\cdot\nabla}{2m_N}\right)\psi_N + \mathcal{O}(\nabla^2), \qquad N = p, n
$$

In these variables, and expanding in powers of ∇, the Lagrangian simplifies

$$
\mathscr{L}_{\text{Fermi}}^{\text{NR}} \supset -(\bar{\psi}_p\psi_n)\left[C_V^+\bar{e}_L\nu_L + C_S^+\bar{e}_R\nu_L\right] - (\bar{\psi}_p\sigma^k\psi_n)\left[C_A^+\bar{e}_L\sigma^k\nu_L + C_T^+\bar{e}_R\sigma^k\nu_L\right] + \mathcal{O}(\nabla/m_n)
$$

It is clear that pseudoscalar couplings do not affect neutron decay at leading order

Non-relativistic Fermi EFT

$$\mathcal{L}^{\mathrm{NR}}_{\mathrm{Fermi}} \supset -(\bar{\psi}_p \psi_n)\left[C_V^+ \bar{e}_L \nu_L + C_S^+ \bar{e}_R \nu_L\right] - \sum_{k=1}^{3} (\bar{\psi}_p \sigma^k \psi_n)\left[C_A^+ \bar{e}_L \sigma^k \nu_L + C_T^+ \bar{e}_R \sigma^k \nu_L\right] + \mathcal{O}(\nabla/m_n)$$

This Lagrangian can also describe beta decays of nuclei: $N \to N' e^- \bar{\nu}$

$$\mathcal{M} = -\mathcal{M}_F\left[C_V^+(\bar{x}_3 y_4) + C_S^+(y_3 y_4)\right] - \sum_{k=1}^{3} \mathcal{M}^k_{\mathrm{GT}}\left[C_A^+(\bar{x}_3 \sigma^k y_4) + C_T^+(y_3 \sigma^k y_4)\right]$$

where the Fermi and Gamow-Teller matrix elements are

$$\mathcal{M}_{\mathrm{F}} \equiv \langle \mathcal{N}' | \bar{\psi}_p \psi_n | \mathcal{N} \rangle \qquad \mathcal{M}^k_{\mathrm{GT}} \equiv \langle \mathcal{N}' | \bar{\psi}_p \sigma^k \psi_n | \mathcal{N} \rangle$$

Fermi transitions

Calculable from group theory in the isospin limit

Gamow-Teller transitions

Difficult to calculate from first principles

The use of non-relativistic EFT allows one to reduce the problem of calculating amplitudes for allowed beta transitions of nuclei to calculating two nuclear matrix elements

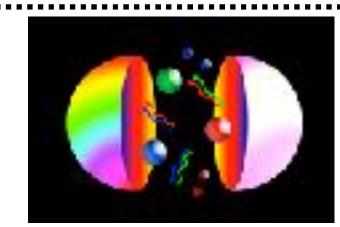
Forbidden transitions correspond to higher order terms in the non-relativistic expansion

100 TeV **???**

100 GeV **SMEFT**

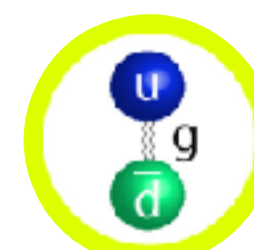
5 GeV **WEFT**

2 GeV **WEFT-4**



1 GeV **Fermi EFT**

100 MeV **Chiral Perturbation Theory**



1 MeV **effective QED**

Euler-Heisenberg EFT


BSM
SMEFT
WEFT
Fermi
ChPT
Euler
NR Fermi