Adam Falkowski

Les Houches lectures on EFT

Lectures given at the 4th International Workshop on Searches for a Neutron Electric Dipole Moment

14-19 February 2021

Timetable

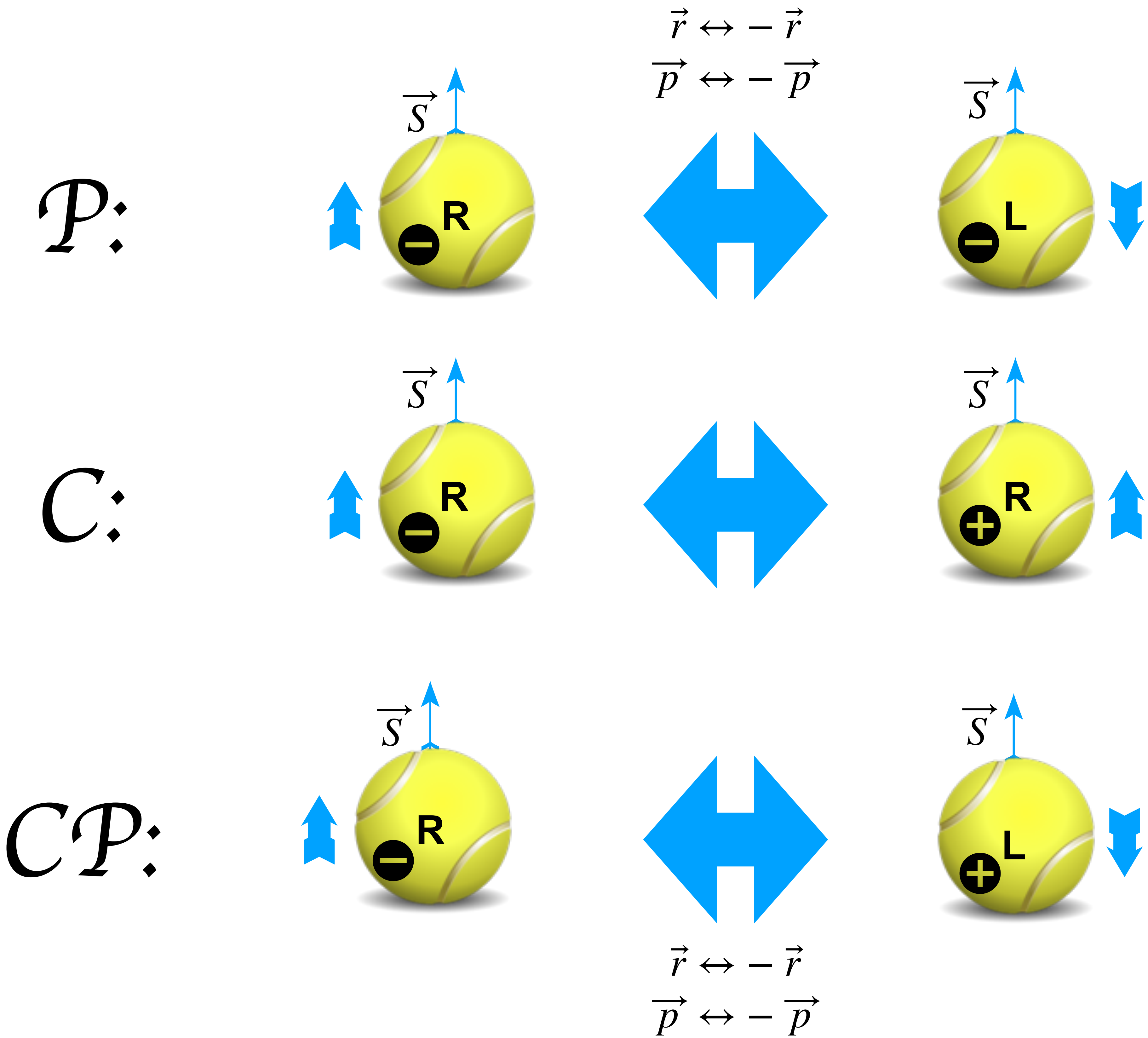
2 lectures on EFT, at a fairly elementary level

- Lecture 1 (Tuesday 10:30-12:00): Philosophy and Landscape of EFTs
- Lecture 2 (Thursday 10:30-12:00): CP-violation in EFT

Lecture 2

CP violation in EFT

What is CP



$\vec{r} \leftrightarrow -\vec{r}$
$\vec{p} \leftrightarrow -\vec{p}$
$\mathcal{P}$:
$\vec{S}$
R
L
C:
R
CP:
R
L
$\vec{r} \leftrightarrow -\vec{r}$
$\vec{p} \leftrightarrow -\vec{p}$

What is CP

CP relates $\mathscr{M}(A \to B)$ **and** $\mathscr{M}(\bar{A} \to \bar{B})$

CPT relates $\mathscr{M}(A \to B)$ **and** $\mathscr{M}(\bar{B} \to \bar{A})$

CPT is conserved in any relativistic and unitary QFT

Note that CPT implies in particular $\mathscr{M}(A \to A) = \mathscr{M}(\bar{A} \to \bar{A})$

If A is a one-particle state

$$\mathscr{M}(A \to A) = p^2 - m_A^2 + i m_A \Gamma_A$$

$$\mathscr{M}(\bar{A} \to \bar{A}) = p^2 - m_{\bar{A}}^2 + i m_{\bar{A}} \Gamma_{\bar{A}}$$

It follows $m_A = m_{\bar{A}}$ $\qquad$ $\Gamma_A = \Gamma_{\bar{A}}$

that is the masses and <u>total</u> lifetimes of particles and its antiparticle are the same

It follows

$$\sum_B \int d\Pi_n |\mathscr{M}(A \to B)|^2 = \sum_{\bar{B}} \int d\Pi_n |\mathscr{M}(\bar{A} \to \bar{B})|^2$$

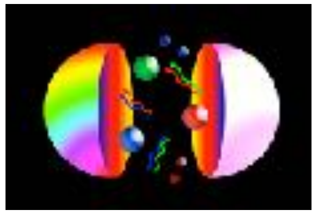
However, for a particular B it is possible that $|\mathscr{M}(A \to B)|^2 \neq |\mathscr{M}(\bar{A} \to \bar{B})|^2$

if CP is not conserved

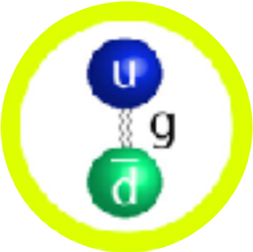
100 TeV ??? 

100 GeV **SMEFT** 

5 GeV **WEFT** 

2 GeV **WEFT-4** 

1 GeV **Fermi EFT** 

100 MeV **Chiral Perturbation Theory** 

1 MeV **effective QED** 

Euler-Heisenberg EFT 

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

Known SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

Question of the day: ***what are the sources of CP violation in the SMEFT Lagrangian ?***

CP on spin-0 scalars

annihilates particle / **creates antiparticle**

$$\Phi = \int \frac{d^3k}{(2\pi)^3 2E_k}\left[a(k)e^{-ikx} + b^\dagger(k)e^{ikx}\right]$$

$$\Phi^\dagger = \int \frac{d^3k}{(2\pi)^3 2E_k}\left[b(k)e^{-ikx} + a^\dagger(k)e^{ikx}\right]$$

annihilates antiparticle / **creates antiparticle**

$\boxed{\Phi \leftrightarrow \Phi^\dagger}$ **exchanges particles with antiparticles, thus it describes charge conjugation C**

For parity even scalar this is also the action of CP , for pseudoscalar there is a minus sign

2-component fermions

4-component Dirac fermion $\Psi_a \quad a = 1\ldots 4$ **describes a <u>pair</u> of spin 1/2 fermions**

Convenient for P and C conserving theories, like QED and QCD
Extremely inconvenient when Majorana fermions are involved,
or when discrete symmetries are discussed

Split the Dirac fermion into halves:

$$\Psi_a = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^c_{\dot{\alpha}} \end{pmatrix} \qquad \begin{matrix} \alpha = 1,2 \\ \dot{\alpha} = 1,2 \end{matrix}$$

The 2 halves transform independently under the Lorentz symmetry.
The Lorentz algebra is equivalent to SU(2)xSU(2):

- **upper 2-component spinor** ψ **transforms under the first SU(2),**
- **lower 2-component spinor** $\bar{\psi}^c$ **transforms under the second SU(2)**

Thus the 2-component spinors are fundamental building blocks

In the 2-component language:

Dirac mass: $\mathscr{L} = m\,\psi^c\psi + m\,\bar{\psi}\bar{\psi}^c$

Majorana mass: $\mathscr{L} = M\,\psi\psi + M\,\bar{\psi}\bar{\psi}$

By convention, I'll be always working in the basis where the masses are real

2-component fermions

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At high energies, E>>m,
ψ **describes spin 1/2 particle with negative helicity (left-handed),**
$\bar{\psi}^c$ **describes spin 1/2 particle with positive helicity (right-handed).**

$\boxed{\psi \leftrightarrow \bar{\psi}^c}$ **exchanges left and right, thus it describes parity operation P**

2-component fermions

$$\Psi = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(k,h)u(k,h)e^{-ikx} + b^\dagger(k,h)v(k,h)e^{ikx} \right] \quad u = \begin{pmatrix} x \\ \bar{y} \end{pmatrix} \qquad v = \begin{pmatrix} y \\ \bar{x} \end{pmatrix}$$

4-component spinor wave functions

The same in terms of 2-component spinor

annihilates particle — **creates antiparticle**

$$\psi = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(k,h)x(k,h)e^{-ikx} + b^\dagger(k,h)y(k,h)e^{ikx} \right]$$

$$\psi^c = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[b(k,h)x(k,h)e^{-ikx} + a^\dagger(k,h)y(k,h)e^{ikx} \right]$$

annihilates antiparticle — **creates particle**

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^c \end{pmatrix}$$

$$\boxed{\psi \leftrightarrow \psi^c}$$

exchanges particles with antiparticles, thus it describes charge conjugation C

CP on spin-1/2 fermions

P: $\boxed{\psi \leftrightarrow \bar{\psi}^c}$

C: $\boxed{\psi \leftrightarrow \psi^c}$

CP: $\boxed{\psi \leftrightarrow \bar{\psi}}$ $\boxed{\psi^c \leftrightarrow \bar{\psi}^c}$

The same in the language of Dirac spinors: $\Psi_L \leftrightarrow \bar{\Psi}_L, \qquad \Psi_R \leftrightarrow \bar{\Psi}_R$

Example of Yukawa interactions:

$$\mathscr{L} = y\, h\, \psi^c \psi + \bar{y}\, h\, \bar{\psi}\bar{\psi}^c$$

Dirac notation:

$$\mathrm{Re}[y]\, h\, \bar{\Psi}\Psi - i\,\mathrm{Im}[y]\, h\, \bar{\Psi}\gamma_5\Psi$$

CP transformation, assuming the real scalar *h* is CP-even:*

$$\mathrm{CP}[\mathscr{L}] = y\, h\, \bar{\psi}^c \bar{\psi} + \bar{y}\, h\, \psi \psi^c$$

$\mathrm{CP}[\mathscr{L}] = \mathscr{L}$ **only if the Yukawa coupling *y* is real**

CP is violated if *y* is complex (in the basis where masses are real)

*** I'm glossing here over the signs, to avoid discussing how spinor indices are contracted**

CP on spin-1/2 fermions

P: $\boxed{\psi \leftrightarrow \bar{\psi}^c}$

C: $\boxed{\psi \leftrightarrow \psi^c}$

CP: $\boxed{\psi \leftrightarrow \bar{\psi}}$ $\boxed{\psi^c \leftrightarrow \bar{\psi}^c}$

Example of Fermi interactions:

Dirac notation:

$$-C_V^+ \bar{p}\gamma^\mu n\, \bar{e}_L\gamma_\mu\nu_L - \bar{C}_V^+ \bar{n}\gamma^\mu p\, \bar{\nu}_L\gamma_\mu e_L$$

$$\mathscr{L} = -C_V^+\left(\bar{p}\bar{\sigma}^\mu n + p^c\sigma^\mu\bar{n}^c\right)\bar{e}\bar{\sigma}_\mu\nu - \bar{C}_V^+\left(\bar{n}\bar{\sigma}^\mu p + n^c\sigma^\mu\bar{p}^c\right)\bar{\nu}\bar{\sigma}_\mu e$$

$$= -2C_V^+\big((\bar{p}\bar{e})(n\nu) - (\bar{n}^c\bar{e})(p^c\nu)\big) - 2\bar{C}_V^+\big((pe)(\bar{n}\bar{\nu}) - (n^c e)(\bar{p}^c\bar{\nu})\big)$$

CP transformation:

$$\mathrm{CP}[\mathscr{L}] = -2C_V^+\big((pe)(\bar{n}\bar{\nu}) - (n^c e)(\bar{p}^c\bar{\nu})\big) - 2\bar{C}_V^+\big((\bar{p}\bar{e})(n\nu) - (\bar{n}^c\bar{e})(p^c\nu)\big)$$

$$\mathrm{CP}[\mathscr{L}] = \mathscr{L}$$

only if the coupling C_V^+ is real

CP is violated if the coupling C_V^+ is real

$$\sigma^0 = \bar{\sigma}^0 = \mathbf{1} \qquad \sigma^i = -\bar{\sigma}^i = [\vec{\sigma}]_i$$

CP on spin-1 vectors

CP violation is often associated with phases in the Lagrangian, but not always !

Consider CP acting on spin-1 vector fields $A_\mu = (A_0, A_i)$**:**

P: $A_0 \to A_0, \quad A_i \to -A_i$ **C:** $A_\mu \to -A_\mu$ ➡ **CP:** $A_0 \to -A_0, \quad A_i \to A_i$

It follows the field strength transforms as $F_{0i} = \partial_0 A_i - \partial_i A_0 \to F_{0i}, \qquad F_{ij} = \partial_i A_j - \partial_j A_i \to -F_{ij},$

Then $$F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} = 2\epsilon^{ijk}F_{0i}F_{jk} \to -2\epsilon^{ijk}F_{0i}F_{jk} = -F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Thus e.g. the Higgs interaction $\mathscr{L} \supset \tilde{c}_{\gamma\gamma}\frac{h}{v}F_{\mu\nu}\tilde{F}^{\mu\nu}$ **is CP odd, even though it has no complex phase**

Similarly one can show that e.g. the cubic gauge coupling:

$$\mathscr{L} \supset \frac{\tilde{c}_{3G}}{\Lambda^2} f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} \tilde{G}^c_{\rho\mu}$$

is CP odd, even though it has no complex phase

Weinberg operator

CP on spin 1/2 and spin 1 interactions

Another example of dipole interactions

$$\mathscr{L} = d\,\psi^c \sigma^{\mu\nu} \psi F_{\mu\nu} + \bar{d}\,\bar{\psi}\bar{\sigma}^{\mu\nu}\bar{\psi}^c F_{\mu\nu}$$

P: $A_0 \to A_0, \quad A_i \to -A_i$ **C:** $A_\mu \to -A_\mu$ ⇒ **CP:** $A_0 \to -A_0, \quad A_i \to A_i$

Looking only at one part of the expression but the result the same for all components of $F_{\mu\nu}$:

$$\mathscr{L} \supset 2d\,\psi^c \sigma^{0i} \psi\, F_{0i} + 2\bar{d}\,\bar{\psi}\bar{\sigma}^{0i}\bar{\psi}^c\, F_{0i} \to 2d\,\bar{\psi}^c \sigma^{0i} \bar{\psi}\, F_{0i} + 2\bar{d}\,\psi\bar{\sigma}^{0i}\psi^c\, F_{0i}$$
$$= 2d\,\bar{\psi}^c \bar{\sigma}^{0i} \bar{\psi}\, F_{0i} + 2\bar{d}\,\psi\bar{\sigma}^{0i}\psi^c\, F_{0i}$$

$\mathrm{CP}[\mathscr{L}] = \mathscr{L}$ **only if dipole couplings *d* are real**

CP is violated if *d* is complex

Real part of d corresponds to anomalous **magnetic** moment of fermion ψ (CP conserving)

Imaginary part of d corresponds to anomalous **electric** moment of fermion ψ (CP violating)

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu\right] \qquad \bar{\sigma}_{\mu\nu} = \frac{i}{2}\left[\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu\right] \qquad \sigma^0 = \bar{\sigma}^0 = \mathbf{1} \qquad \sigma^i = -\bar{\sigma}^i = [\vec{\sigma}]_i$$

CP violation at D=4

Standard Model

The most general Lagrangian up to D=4 consistent with these principles is just the SM Lagrangian

$$\mathscr{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} - \frac{g_S^2}{32\pi^2}\theta G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$+ \sum_{f\in q,u,d,l,e} i\bar{f}\gamma^\mu D_\mu f$$

$$-\left(\bar{u}Y_u qH + \bar{d}Y_d H^\dagger q + \bar{e}Y_e H^\dagger l + \mathrm{h.c.}\right)$$

$$+D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2$$

Yukawa couplings are complex in general, thus they can provide source of CP violation

$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc} V^b_\mu V^c_\nu$ $\quad$ $D_\mu f = \partial_\mu f - i g_s G^a_\mu T^a f - i g_L W^i_\mu \frac{\sigma^i}{2} f - i g_Y B_\mu Y f$

CP violation in the Standard Model

- After redefining away all the phases, 2 sources of CP violation remain in the D <= 4 part of the SMEFT

- One is the phase in the CKM matrix, describing charged current interactions between W and left-handed quarks.
  The effects of this phase have been observed in the B-meson, D-meson, and kaon systems. The value of this phase seems to be generic, that is it is consistent with order random one phase in the quark Yukawa couplings

- The other is a combination of the θ parameter and the phase of the determinant of the quark matrix. This phase should lead to an EDM of the neutron and composite nuclei. The effects of this phase have not been observed so far and we have only stringent limits. It is a mystery why this phase, unlike the former one, does not take a generic value

CP violation at D=5

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{D=5} = \sum_{\alpha,\beta=1}^{3}\left[\frac{c_{\alpha\beta}}{\Lambda_L}(L_\alpha H)(L_\beta H) + \frac{\bar{c}_{\alpha\beta}}{\Lambda_L}(H^\dagger \bar{L}_\alpha)(H^\dagger \bar{L}_\beta)\right]$$

$$L_\alpha \to \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}$$

$$= \left(1 + \frac{h}{\mathrm{v}}\right)^2 \frac{\mathrm{v}^2}{\Lambda_L} \sum_{\alpha,\beta=1}^{3}\left[c_{\alpha\beta}\nu_\alpha\nu_\beta + \bar{c}_{\alpha\beta}\bar{\nu}_\alpha\bar{\nu}_\beta\right]$$

$$\mathrm{CP}[\mathscr{L}_{D=5}] = \left(1 + \frac{h}{\mathrm{v}}\right)^2 \frac{\mathrm{v}^2}{\Lambda_L}\left[c_{\alpha\beta}\bar{\nu}_\alpha\bar{\nu}_\beta + \bar{c}_{\alpha\beta}\nu_\alpha\nu_\beta\right]$$

Neutrino masses can violate CP if they are complex!

SMEFT at dimension-5

QFT it is awkward to work with complex and off-diagonal masses, so we usually diagonalize the mass matrix and remove the phases by field redefinition

$$\mathscr{L}_{D=5} \supset \frac{\mathrm{v}^2}{\Lambda_L} \sum_{\alpha,\beta=1}^{3} \left[c_{\alpha\beta} \nu_\alpha \nu_\beta + \bar{c}_{\alpha\beta} \bar{\nu}_\alpha \bar{\nu}_\beta \right]$$

Unitary PMNS matrix

Rotate $\nu_\alpha \to U_{\alpha j} \nu_j$

$$c_{\alpha\beta} \nu_\alpha \nu_\beta \to U_{\alpha i} c_{\alpha\beta} U_{\beta j} \nu_i \nu_j$$

Choose $U^T c U = -\,\mathrm{diag}(c_1, c_2, c_3)$

$$\mathscr{L}_{D=5} \supset -\frac{\mathrm{v}^2}{\Lambda_L} \sum_{i=1}^{3} \left[c_i \nu_i \nu_i + \bar{c}_i \bar{\nu}_i \bar{\nu}_i \right]$$

$$m_{\nu_i} = c_i \frac{\mathrm{v}^2}{\Lambda_L} = |c_i| e^{i\phi_i} \frac{\mathrm{v}^2}{\Lambda_L}$$

Rephase $\nu_i \to P_i \nu_i, \quad P_i = e^{-i\phi_i}$

$$\mathscr{L}_{D=5} \supset -\sum_{i=1}^{3} m_{\nu_i} \left[\nu_i \nu_i + \bar{\nu}_i \bar{\nu}_i \right]$$

Masses are now real and all traces of CP violation vanish…

SMEFT at dimension-5

… not so fast

SMEFT Lagrangian at D=4 contains the CC interactions between leptons and W

$$\mathscr{L}_{D=5} \supset -\sum_{i=1}^{3} m_{\nu_i}\left[\nu_i\nu_i + \bar{\nu}_i\bar{\nu}_i\right]$$

$$\mathscr{L}_{D=4} \supset \frac{g_L}{\sqrt{2}} W_\mu^- \sum_{\alpha=1}^{3} \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_\alpha + \mathrm{h.c.}$$

$$\nu_\alpha \to \sum_{j=1}^{3} U_{\alpha j} P_j \nu_j \qquad P_i = e^{-i\phi_i}$$

$$\mathscr{L}_{D=4} \to \frac{g_L}{\sqrt{2}} W_\mu^- \sum_{\alpha,j=1}^{3} U_{\alpha j} P_j \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_j + \mathrm{h.c.}$$

CP-violating if U or P are complex

CP violation migrated from the neutrino mass matrix to charged-current interactions of leptons

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta}s_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$P_i = e^{i\phi}\begin{pmatrix} e^{i\alpha/2} & e^{i\beta/2} & 1 \end{pmatrix}$$

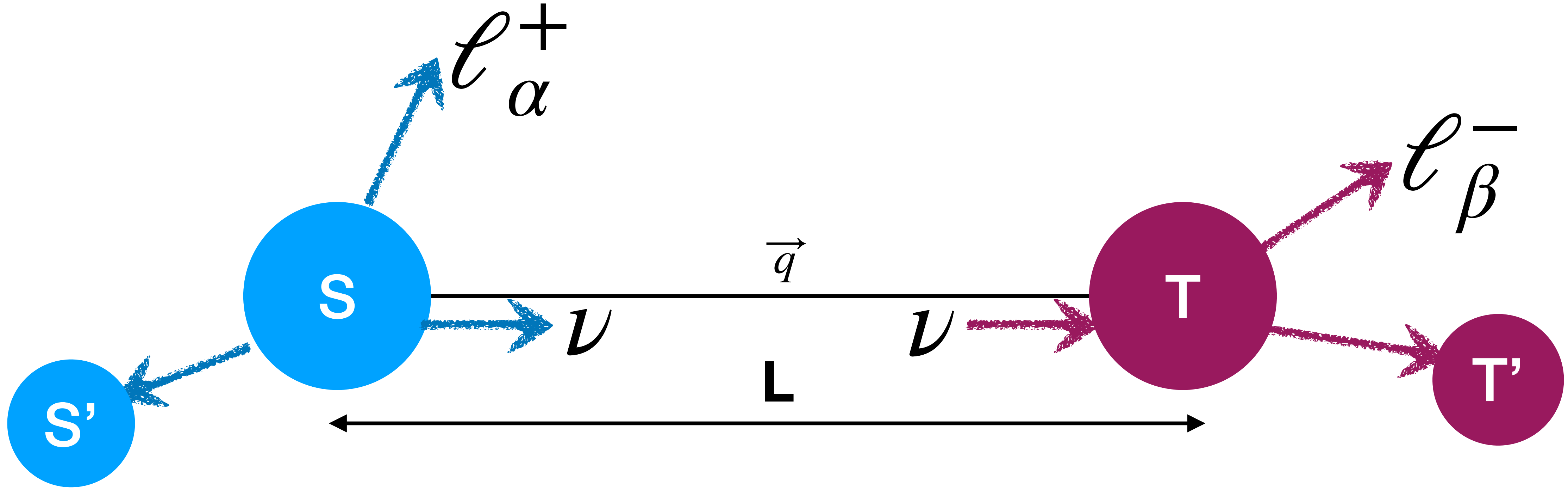
PMNS matrix U is totally analogous, to the CKM matrix for quarks (though numerically it is very different)

These are qualitatively new phases compared to the quark sector

The phase δ is called the Dirac phase

They are called the Majorana phases

Neutrino oscillations in QFT



$$\mathcal{M}(ST \to S'e_\alpha T'e_\beta) = \sum_{k=1}^{3} \frac{\mathcal{M}(S \to S'e_\alpha \nu_k)\mathcal{M}(\nu_k T \to Te_\beta)}{q^2 - m_k^2 + i\epsilon} \equiv \sum_{k=1}^{3} \frac{\mathcal{M}^P_{\alpha k}\mathcal{M}^D_{\beta k}}{q^2 - m_k^2 + i\epsilon}$$

Oscillation probability

$$\boxed{P(\nu_\alpha \to \nu_\beta) = \frac{\sum_{k,l=1}^{3} \exp\left(-i\frac{L(m^2_{\nu_k} - m^2_{\nu_l})}{2E_\nu}\right)\int d\Pi_P \mathcal{M}^P_{\alpha k}\bar{\mathcal{M}}^P_{\alpha l}\int d\Pi_D \mathcal{M}^D_{\beta k}\bar{\mathcal{M}}^D_{\beta l}}{\sum_{k,l=1}^{3}\int d\Pi_P |\mathcal{M}^P_{\alpha k}|^2 \int d\Pi_D |\mathcal{M}^D_{\beta l}|^2}}$$

Neutrino Oscillations

$$P(\nu_\alpha \to \nu_\beta) = \frac{\sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_{\nu_k}^2 - m_{\nu_l}^2)}{2E_\nu}\right) \int d\Pi_P \mathcal{M}_{\alpha k}^{P} \mathcal{M}_{\alpha l}^{P*} \int d\Pi_D \mathcal{M}_{\beta k}^{D} \mathcal{M}_{\beta l}^{D*}}{\sum_{k,l=1}^{3} \int d\Pi_P |\mathcal{M}_{\alpha k}^{P}|^2 \int d\Pi_D |\mathcal{M}_{\beta l}^{D}|^2}$$

$$\mathscr{L}_{D=4} \to \frac{g_L}{\sqrt{2}} \sum_{\alpha,k=1}^{3} \left\{ W_\mu^- U_{\alpha k} P_k \,(\bar{\ell}_\alpha \bar{\sigma}_\mu \nu_k) + W_\mu^+ U_{\alpha k}^* P_k^* \,(\bar{\nu}_k \bar{\sigma}_\mu \ell_\alpha) \right\}$$

Neutrino production: $\mathcal{M}_{\alpha k}^{P} \sim U_{\alpha k}^* P_k^*$ $\qquad$ $\mathcal{M}_{\alpha l}^{P*} \sim U_{\alpha l} P_l$

Neutrino detection: $\mathcal{M}_{\beta k}^{D} \sim U_{\beta k} P_k$ $\qquad$ $\mathcal{M}_{\beta l}^{D*} \sim U_{\beta l}^* P_l^*$

$$P(\nu_\alpha \to \nu_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k}^* U_{\alpha l} U_{\beta k} U_{\beta l}^*$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k} U_{\alpha l}^* U_{\beta k}^* U_{\beta l}$$

$$\Delta_{kl}^2 \equiv m_{\nu_k}^2 - m_{\nu_l}^2$$

The Majorana phases cancel out in neutrino oscillations

CP violation in Neutrino Oscillations

$$P(\nu_\alpha \to \nu_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U^*_{\alpha k} U_{\alpha l} U_{\beta k} U^*_{\beta l}$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U_{\alpha k} U^*_{\alpha l} U^*_{\beta k} U_{\beta l}$$

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 2i \sum_{k,l=1}^{3} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} \mathrm{Im}[U^*_{\alpha k} U_{\alpha l} U_{\beta k} U^*_{\beta l}]$$

In the usual parametrization of the PMNS matrix, for $\alpha \neq \beta$

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \pm\, s_{12} s_{13} s_{23} c_{12} c^2_{13} c_{23} \sin \delta \left[\sin\left(\frac{\Delta^2_{21} L}{2E_\nu}\right) - \sin\left(\frac{\Delta^2_{31} L}{2E_\nu}\right) + \sin\left(\frac{\Delta^2_{32} L}{2E_\nu}\right) \right]$$

while for $\alpha = \beta$ **,** $P(\nu_\alpha \to \nu_\alpha) - P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 0$ **by CPT**

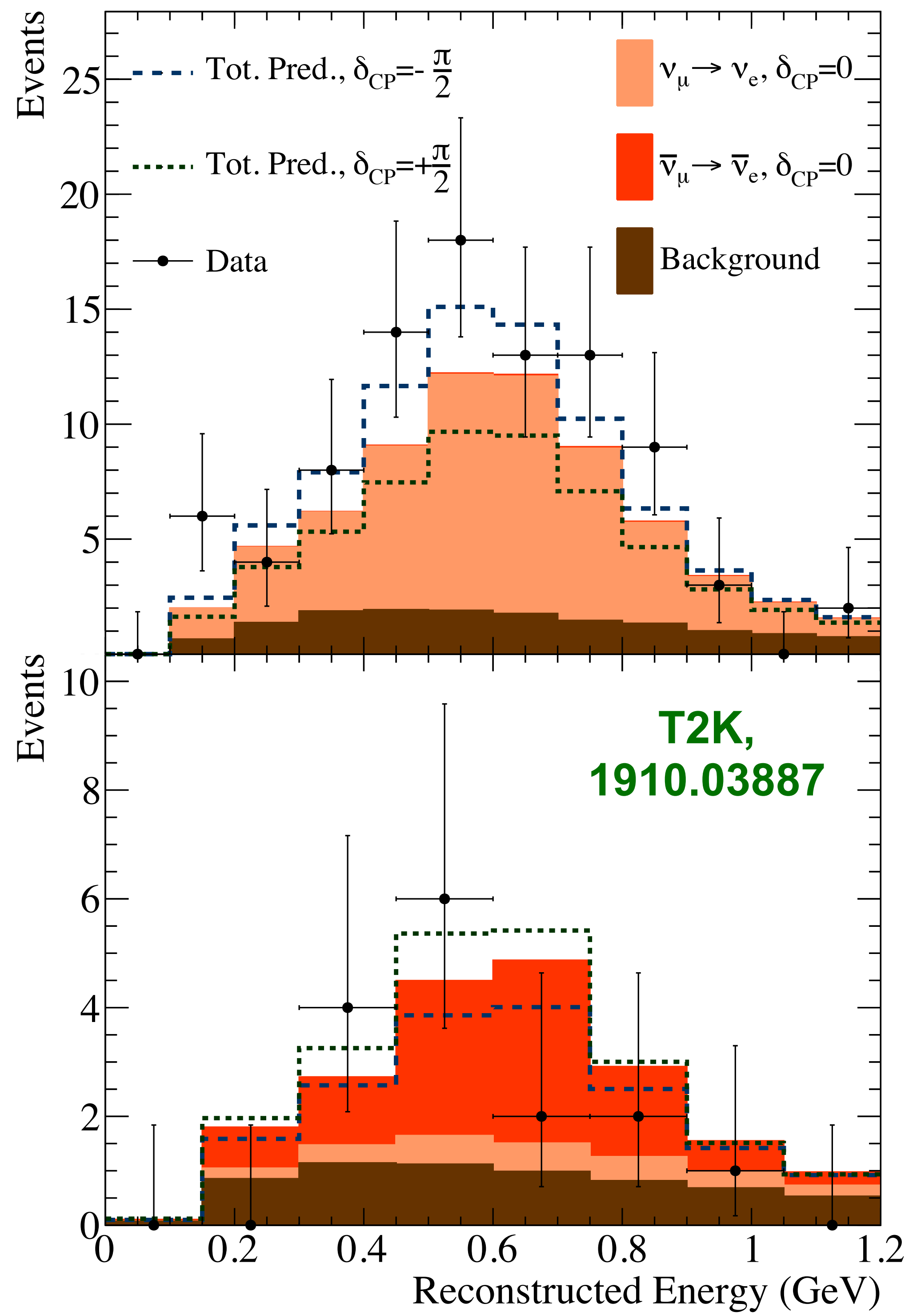
CP violation is hard…

- **At least 3 neutrinos must exist in nature**
- **All 3 mixing angles have to be non-trivial**
- **All 3 mass splittings have to be non-zero**
- **The Dirac phase needs to be different from 0 and from π**

Fortunately, it seems that these conditions are fulfilled in the real world, barring confirmation about the Dirac phase…

CP violation in Neutrino Oscillations

Some mild preference for $\delta \sim -\pi/2$



Best fit

$$\delta = -1.89^{+0.70}_{-0.58}$$

Another triumph of SMEFT?
As expected by power counting arguments, CP violation first observed at D=4, then at D=5…

Interlude: why is CP violation hard

CP violating amplitudes:

$$\mathscr{M}(A \to B) = re^{i\phi} \qquad \mathscr{M}(\bar{A} \to \bar{B}) = re^{-i\phi}$$

where r is real and φ is a CP violating phase in the Lagrangian

Rates:

$$R(A \to B) \sim |\mathscr{M}(A \to B)|^2 = r^2$$
$$R(\bar{A} \to \bar{B}) \sim |\mathscr{M}(\bar{A} \to \bar{B})|^2 = r^2$$

In this simple example, CP violation present at the amplitude level is invisible at the level of observables

Interlude: why is CP violation hard

CP violating amplitudes:

$$\mathscr{M}(A \to B) = r_1 e^{i\phi_1} + r_2 e^{i\phi_2} \qquad \mathscr{M}(\bar{A} \to \bar{B}) = r_1 e^{-i\phi_1} + r_2 e^{-i\phi_2}$$

where r_1 and r_2 are real and φ_1 and φ_2 are CP violating phases in the Lagrangian

Rates:

$$R(A \to B) \sim |\mathscr{M}(A \to B)|^2 = r_1^2 + r_2^2 + 2r_1 r_2 \cos(\phi_1 - \phi_2)$$
$$R(\bar{A} \to \bar{B}) \sim |\mathscr{M}(\bar{A} \to \bar{B})|^2 = r_1^2 + r_2^2 + 2r_1 r_2 \cos(\phi_1 - \phi_2)$$

In this less simple example, CP violation present at the amplitude level is invisible at the level of observables

Interlude: why is CP violation hard

CP violating amplitudes:

$$\mathscr{M}(A \to B) = r_1 e^{i\eta_1} e^{i\phi_1} + r_2 e^{i\eta_2} e^{i\phi_2} \qquad \mathscr{M}(\bar{A} \to \bar{B}) = r_1 e^{i\eta_1} e^{-i\phi_1} + r_2 e^{i\eta_2} e^{-i\phi_2}$$

where r_1 and r_2 are real,
φ_1 and φ_2 are CP violating (weak) phases in the Lagrangian,
and η_1 and η_2 are CP conserving (strong) phases from the dynamics

Rates:

$$R(A \to B) \sim |\mathscr{M}(A \to B)|^2 = r_1^2 + r_2^2 + 2r_1 r_2 \cos(\phi_1 - \phi_2 + \eta_1 - \eta_2)$$

$$R(\bar{A} \to \bar{B}) \sim |\mathscr{M}(\bar{A} \to \bar{B})|^2 = r_1^2 + r_2^2 + 2r_1 r_2 \cos(\phi_1 - \phi_2 - \eta_1 + \eta_2)$$

In this much less simple example, CP violation present at the amplitude level finally survives at the level of observables

$$R(A \to B) - R(\bar{A} \to \bar{B}) = 4 r_1 r_2 \sin(\eta_2 - \eta_1) \sin(\phi_1 - \phi_2)$$

Interlude: why is CP violation hard

$$\mathscr{M}(A \to B) = r_1 e^{i\eta_1} e^{i\phi_1} + r_2 e^{i\eta_2} e^{i\phi_2} \qquad \mathscr{M}(\bar{A} \to \bar{B}) = r_1 e^{i\eta_1} e^{-i\phi_1} + r_2 e^{i\eta_2} e^{-i\phi_2}$$

$$R(A \to B) - R(\bar{A} \to \bar{B}) = 4 r_1 r_2 \sin(\eta_2 - \eta_1) \sin(\phi_1 - \phi_2)$$

In neutrino oscillations,
the interfering amplitudes come from contributions of different mass eigenstates
the weak phase comes the PMNS matrix,
and the strong phases come from neutrino propagation in coordinate space

In addition: $P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu) = X \quad \Longrightarrow \quad P(\nu_e \to \nu_\tau) - P(\bar{\nu}_e \to \bar{\nu}_\tau) = -X$

by conservation of probability… that’s why at least 3 neutrinos have to involved

More generally, strong phases can also come from loops, and from the width

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{g_L}{\sqrt{2}} W^-_\mu \sum_{\alpha,j=1}^{3} U_{\alpha j} P_j \, \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_j + \text{h.c.}$$

CP-violating if U or P are complex

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta}s_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$P_i = e^{i\phi} \begin{pmatrix} e^{i\alpha/2} & e^{i\beta/2} & 1 \end{pmatrix}$$

PMNS matrix U is totally analogous,
to the CKM matrix for quarks
(though numerically it is very different)

These are qualitatively new phases
compared to the quark sector

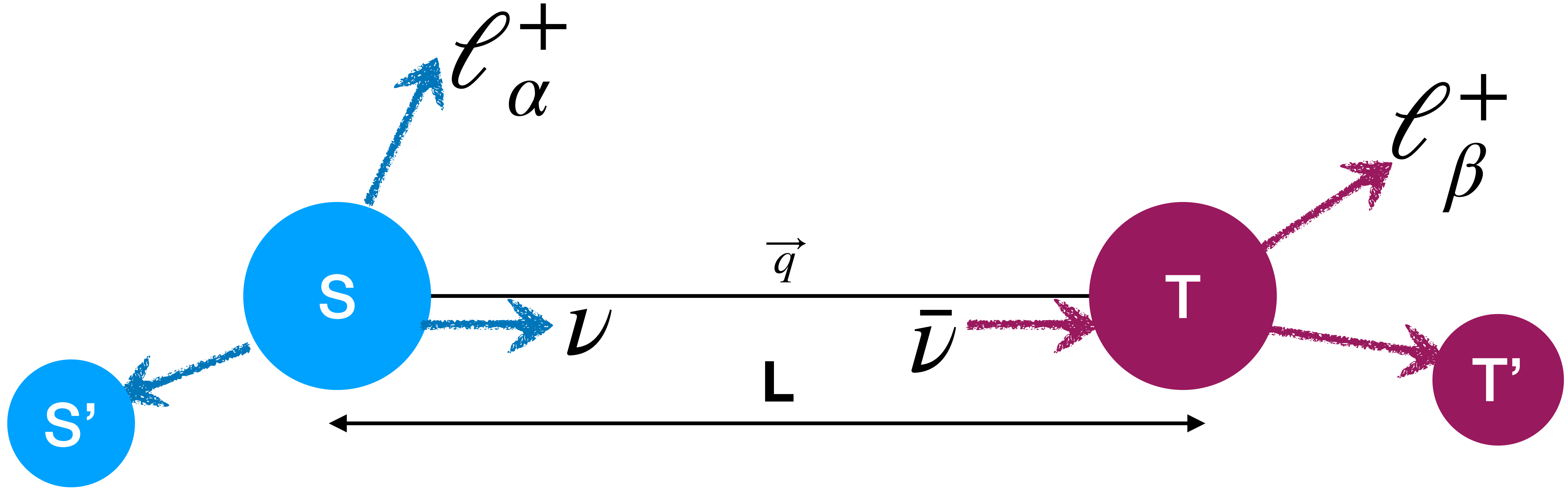
The phase δ is called the Dirac phase

They are called the Majorana phases

Almost there

Are these physical ???

Neutrino antineutrino oscillations



Transition rate

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} \frac{m_{\nu_k} m_{\nu_l}}{E_\nu^2} \exp\left(-i\frac{L(m_{\nu_k}^2 - m_{\nu_l}^2)}{2E_\nu}\right) \int d\Pi_P \mathcal{M}_{\alpha k}^P \bar{\mathcal{M}}_{\alpha l}^P \int d\Pi_D \mathcal{M}_{\beta k}^D \bar{\mathcal{M}}_{\beta l}^D$$

Neutrino Antineutrino Oscillations

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} \frac{m_{\nu_k} m_{\nu_l}}{E_\nu^2} \exp\left(-i \frac{L(m_{\nu_k}^2 - m_{\nu_l}^2)}{2E_\nu}\right) \int d\Pi_P \mathscr{M}_{\alpha k}^P \bar{\mathscr{M}}_{\alpha l}^P \int d\Pi_D \mathscr{M}_{\beta k}^D \bar{\mathscr{M}}_{\beta l}^D$$

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \frac{g_L}{\sqrt{2}} \sum_{\alpha,k=1}^{3} \left\{ W_\mu^- U_{\alpha k} P_k (\bar{\ell}_\alpha \bar{\sigma}_\mu \nu_k) + W_\mu^+ U_{\alpha k}^* P_k^* (\bar{\nu}_k \bar{\sigma}_\mu \ell_\alpha) \right\}$$

Neutrino production: $\mathscr{M}_{\alpha k}^P \sim U_{\alpha k}^* P_k^*$ $\qquad \mathscr{M}_{\alpha l}^{P*} \sim U_{\alpha l} P_l$

Anti-Neutrino detection: $\mathscr{M}_{\beta k}^D \sim U_{\beta k}^* P_k^*$ $\qquad \mathscr{M}_{\beta l}^{D*} \sim U_{\beta l} P_l$

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k}^* U_{\alpha l} U_{\beta k}^* U_{\beta l} (P_k^*)^2 (P_l)^2$$

$$\Delta_{kl}^2 \equiv m_{\nu_k}^2 - m_{\nu_l}^2$$

$$R(\bar{\nu}_\alpha \to \nu_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k} U_{\alpha l}^* U_{\beta k} U_{\beta l}^* (P_k)^2 (P_l^*)^2$$

Majorana phases don’t cancel out!

Neutrino-Antineutrino Oscillations

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U^*_{\alpha k} U_{\alpha l} U^*_{\beta k} U_{\beta l} (P^*_k)^2 (P_l)^2$$

$$R(\bar{\nu}_\alpha \to \nu_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U_{\alpha k} U^*_{\alpha l} U_{\beta k} U^*_{\beta l} (P_k)^2 (P^*_l)^2$$

Take the limit for $s_{13} \to 0$ simplicity

$$R(\nu_e \to \bar{\nu}_\mu) - R(\bar{\nu}_e \to \nu_\mu) \sim m_{\nu_1} m_{\nu_2} c^2_{12} s^2_{12} \sin(\alpha - \beta) \sin\left(\frac{\Delta^2_{21} L}{2E_\nu}\right)$$

Majorana phases control CP violation in neutrino-antineutrino oscillations
The effect occurs even in the 2-neutrino oscillation limit

Unfortunately, the effect is very suppressed by the small neutrino masses, and may never be observed…

De Gouvea et al
hep-ph/0211394

CP violation at D= 6

SMEFT at dimension-6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Bosonic CP-odd	
$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I\bar\sigma_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J \tilde H^\dagger D_\mu H$

Dipole	
$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operato the corresponding complex conjugate operator is implicitly included.

$(\bar RR)(\bar RR)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar e^c)(e^c\sigma_\mu\bar e^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar u^c)(u^c\sigma_\mu\bar u^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar d^c)(d^c\sigma_\mu\bar d^c)$
O_{eu}	$(e^c\sigma_\mu\bar e^c)(u^c\sigma_\mu\bar u^c)$
O_{ed}	$(e^c\sigma_\mu\bar e^c)(d^c\sigma_\mu\bar d^c)$
O_{ud}	$(u^c\sigma_\mu\bar u^c)(d^c\sigma_\mu\bar d^c)$
O'_{ud}	$(u^c\sigma_\mu T^a\bar u^c)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar LL)(\bar RR)$	
$O_{\ell e}$	$(\bar\ell\bar\sigma_\mu\ell)(e^c\sigma_\mu\bar e^c)$
$O_{\ell u}$	$(\bar\ell\bar\sigma_\mu\ell)(u^c\sigma_\mu\bar u^c)$
$O_{\ell d}$	$(\bar\ell\bar\sigma_\mu\ell)(d^c\sigma_\mu\bar d^c)$
O_{eq}	$(e^c\sigma_\mu\bar e^c)(\bar q\bar\sigma_\mu q)$
O_{qu}	$(\bar q\bar\sigma_\mu q)(u^c\sigma_\mu\bar u^c)$
O'_{qu}	$(\bar q\bar\sigma_\mu T^a q)(u^c\sigma_\mu T^a\bar u^c)$
O_{qd}	$(\bar q\bar\sigma_\mu q)(d^c\sigma_\mu\bar d^c)$
O'_{qd}	$(\bar q\bar\sigma_\mu T^a q)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar LL)(\bar LL)$	
$O_{\ell\ell}$	$\eta(\bar\ell\bar\sigma_\mu\ell)(\bar\ell\bar\sigma_\mu\ell)$
O_{qq}	$\eta(\bar q\bar\sigma_\mu q)(\bar q\bar\sigma_\mu q)$
O'_{qq}	$\eta(\bar q\bar\sigma_\mu\sigma^i q)(\bar q\bar\sigma_\mu\sigma^i q)$
$O_{\ell q}$	$(\bar\ell\bar\sigma_\mu\ell)(\bar q\bar\sigma_\mu q)$
$O'_{\ell q}$	$(\bar\ell\bar\sigma_\mu\sigma^i\ell)(\bar q\bar\sigma_\mu\sigma^i q)$

$(\bar LR)(\bar LR)$	
O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^c q^k)$
$O'_{\ell equ}$	$(e^c\bar\sigma_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar\sigma^{\mu\nu}q^k)$
$O_{\ell edq}$	$(\bar\ell\bar e^c)(d^c q)$

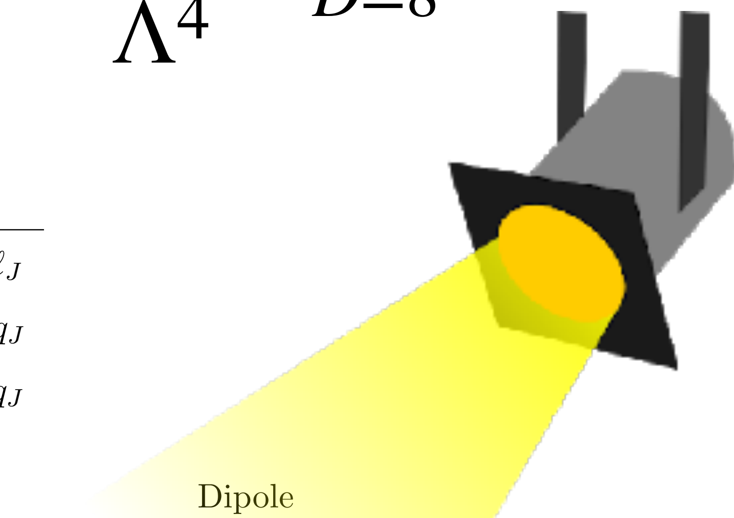
Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

$$O_{duq} = (d^c u^c)(\bar Q\bar\ell)$$
$$O_{qqu} = (qq)(\bar u^c\bar e^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^c u^c)(u^c e^c)$$

SMEFT at dimension-6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$



Yukawa

$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \tilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Bosonic CP-even

O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$		
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$		
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$		
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Vertex		Dipole	
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$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$\boxed{[O^\dagger_{eB}]_{IJ}}$	$\boxed{e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}}$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\tilde H^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\tilde H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\tilde H^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J \tilde H^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

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$$O_{duq} = (d^c u^c)(\bar Q \bar\ell)$$
$$O_{qqu} = (qq)(\bar u^c \bar e^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^c u^c)(u^c e^c)$$

CP violation in leptonic dipoles

Electron's EDM in the Lagrangian:

$$\mathscr{L} \supset \frac{i}{2} d_e \left[e^c \sigma_{\mu\nu} e - \bar{e} \bar{\sigma}_{\mu\nu} \bar{e}^c \right] F_{\mu\nu}$$

$$\mathscr{L} \supset -\frac{i}{2} d_e \bar{e} \sigma_{\mu\nu} \gamma_5 e F_{\mu\nu}$$

Dipoles in the SMEFT Lagrangian:

$$\mathscr{L}_{D=6} \supset \frac{\bar{c}_{eB}}{\Lambda^2} e^c \sigma_{\mu\nu} H^\dagger L B_{\mu\nu} + \mathrm{h.c.} \to \frac{\bar{c}_{eB} \mathrm{v} \cos\theta_W}{\sqrt{2}\Lambda^2} e^c \sigma_{\mu\nu} e F_{\mu\nu} + \mathrm{h.c.}$$

Dictionary:

$$d_e = -\mathrm{Im}[c_{eB}] \frac{\mathrm{v} \cos\theta_W}{\sqrt{2}\Lambda^2}$$

ACME limit:

$$|d_e| < 1.1 \times 10^{-29} e \cdot cm = \frac{1.7 \times 10^{-16}}{\mathrm{GeV}}$$

It follows

$$\Lambda \gtrsim 10^9 \ \mathrm{GeV} \sqrt{|\mathrm{Im}[\mathrm{c_{eB}}]|}$$

CP violation in leptonic dipoles

$$\Lambda \gtrsim 10^6\ \mathrm{TeV}\sqrt{|\mathrm{Im}[c_{eB}]|}$$

The reach of electron EDM depends on the hypothesis about the Wilson coefficient c_{eB}

1 $c_{eB} \sim 1$ $\Rightarrow$ $\Lambda \gtrsim 10^6\ \mathrm{TeV}$ **5 orders of magnitude above LHC!**

2 $c_{eB} \sim \dfrac{m_e}{\mathrm{v}}$ $\Rightarrow$ $\Lambda \gtrsim 10^3\ \mathrm{TeV}$ **2 orders of magnitude above LHC!**

3 $c_{eB} \sim \dfrac{1}{16\pi^2}\dfrac{m_e}{\mathrm{v}}$ $\Rightarrow$ $\Lambda \gtrsim 10^2\ \mathrm{TeV}$ **1 order of magnitude above LHC!**

Unlikely there is new physics below 100 TeV, because CP violation seems generic in nature and electron's EDM does not violate any other symmetry than CP and chiral symmetry

EDM as a lightning rod

At D=4

$$d_n = -\,0.003(7)(20)\; e\;\mathrm{fm}\left[\theta + \arg\det M_q\right]$$

Bhattacharya et al. 2101.07230

At D=6

$$\mathcal{O}^{ij\,lm}_{1\,LR} = \bar d^m \gamma^\mu P_L u^l\, \bar u^i \gamma_\mu P_R d^j\ , \qquad \mathcal{O}^{ij\,lm}_{2\,LR} = \bar d^m_\alpha \gamma^\mu P_L u^l_\beta\, \bar u^i_\beta \gamma_\mu P_R d^j_\alpha$$

Nuclear dipoles pick up many contributions from many CP violating operators; even more when RG running is taken into account

$$\begin{aligned} d_n \;=\; & \Big((43\pm 27)\tilde C^{us\,us}_{1\,LR} + (210\pm 130)\,\tilde C^{us\,us}_{2\,LR} + (22\pm 14)\,\tilde C^{ud\,ud}_{1\,LR} + (110\pm 70)\,\tilde C^{ud\,ud}_{2\,LR} \\ & -(0.93\pm 0.05)\,\tilde c^{uu}_{\gamma u} - (4.0\pm 0.2)\,\tilde c^{dd}_{\gamma d} - (0.8\pm 0.9)\,\tilde c^{ss}_{\gamma d} \\ & -(3.9\pm 2.0)\,\tilde c^{uu}_{gu} - (16.8\pm 8.4)\,\tilde c^{dd}_{gd} \pm (320\pm 260) v^2 C_{\tilde G}\Big) \times 10^{-9}\, e\,\mathrm{fm}\,, \end{aligned}$$

$$-\frac{eQ_d}{2}\sum_{ij\in\{d,s,b\}} m_{d_j} C^{ij}_{\gamma d}\, \bar d^i_L \sigma^{\mu\nu} F_{\mu\nu} d^j_R$$

$$-\frac{eQ_u}{2}\sum_{ij\in\{u,c\}} m_{u_j} C^{ij}_{\gamma u}\, \bar u^i_L \sigma^{\mu\nu} F_{\mu\nu} u^j_R$$

Alioli et al. 1703.04751

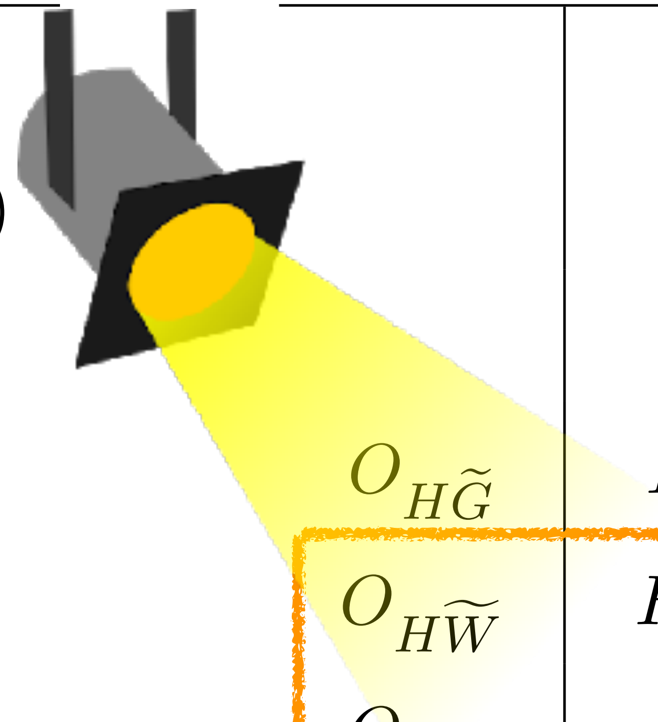
$$-\frac{g_s}{2}\sum_{ij\in\{u,c\}} m_{u_j} C^{ij}_{gu}\, \bar u^i_L \sigma^{\mu\nu} G^a_{\mu\nu} t^a u^j_R - \frac{g_s}{2}\sum_{ij\in\{d,s,b\}} m_{d_j} C^{ij}_{gd}\, \bar d^i_L \sigma^{\mu\nu} G^a_{\mu\nu} t^a d^j_R$$

$$C_{\tilde G}\frac{g_s}{3} f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} \widetilde{G}^c_{\rho\mu}$$

SMEFT at dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$



Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Yukawa

$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I\bar\sigma_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J\tilde H^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operat the corresponding complex conjugate operator is implicitly included.

$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar e^c)(e^c\sigma_\mu\bar e^c)$	$O_{\ell e}$	$(\bar\ell\bar\sigma_\mu\ell)(e^c\sigma_\mu\bar e^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar u^c)(u^c\sigma_\mu\bar u^c)$	$O_{\ell u}$	$(\bar\ell\bar\sigma_\mu\ell)(u^c\sigma_\mu\bar u^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar d^c)(d^c\sigma_\mu\bar d^c)$	$O_{\ell d}$	$(\bar\ell\bar\sigma_\mu\ell)(d^c\sigma_\mu\bar d^c)$
O_{eu}	$(e^c\sigma_\mu\bar e^c)(u^c\sigma_\mu\bar u^c)$	O_{eq}	$(e^c\sigma_\mu\bar e^c)(\bar q\bar\sigma_\mu q)$
O_{ed}	$(e^c\sigma_\mu\bar e^c)(d^c\sigma_\mu\bar d^c)$	O_{qu}	$(\bar q\bar\sigma_\mu q)(u^c\sigma_\mu\bar u^c)$
O_{ud}	$(u^c\sigma_\mu\bar u^c)(d^c\sigma_\mu\bar d^c)$	O'_{qu}	$(\bar q\bar\sigma_\mu T^a q)(u^c\sigma_\mu T^a\bar u^c)$
O'_{ud}	$(u^c\sigma_\mu T^a\bar u^c)(d^c\sigma_\mu T^a\bar d^c)$	O_{qd}	$(\bar q\bar\sigma_\mu q)(d^c\sigma_\mu\bar d^c)$
		O'_{qd}	$(\bar q\bar\sigma_\mu T^a q)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar LL)(\bar LL)$		$(\bar LR)(\bar LR)$	
$O_{\ell\ell}$	$\eta(\bar\ell\bar\sigma_\mu\ell)(\bar\ell\bar\sigma_\mu\ell)$	O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O_{qq}	$\eta(\bar q\bar\sigma_\mu q)(\bar q\bar\sigma_\mu q)$	O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
O'_{qq}	$\eta(\bar q\bar\sigma_\mu\sigma^i q)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^c q^k)$
$O_{\ell q}$	$(\bar\ell\bar\sigma_\mu\ell)(\bar q\bar\sigma_\mu q)$	$O'_{\ell equ}$	$(e^c\bar\sigma_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar\sigma^{\mu\nu}q^k)$
$O'_{\ell q}$	$(\bar\ell\bar\sigma_\mu\sigma^i\ell)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell edq}$	$(\bar\ell\bar e^c)(d^c q)$

Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

$$O_{duq} = (d^c u^c)(\bar Q\bar\ell)$$
$$O_{qqu} = (qq)(\bar u^c\bar e^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^c u^c)(u^c e^c)$$

CP violation in Higgs sector

$$\mathscr{L}_{D=6} \supset \frac{c_{H\tilde{B}}}{\Lambda^2} H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu} + \frac{c_{H\tilde{W}}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} \widetilde{W}^a_{\mu\nu} + \frac{c_{H\tilde{W}B}}{\Lambda^2} H^\dagger \sigma^a H \widetilde{W}^a_{\mu\nu} B^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$
$$\widetilde{B}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}$$

This leads to new CP violating interactions of the Higgs boson with electroweak vector bosons

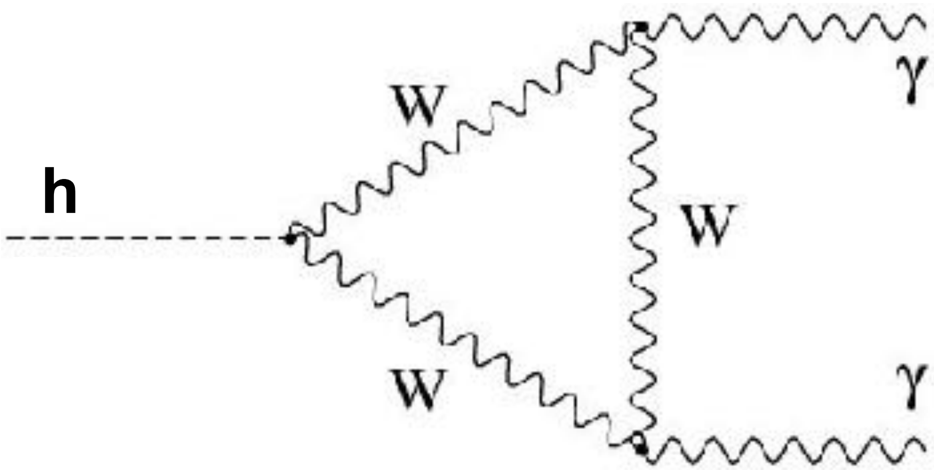
$$\mathscr{L}_{\rm SMEFT} \supset \frac{h}{\rm v}\Big[2m_W^2 W^+_\mu W^-_\mu + m_Z^2 Z_\mu Z_\mu \quad \leftarrow \textbf{SM interactions}$$

$$+ \tilde{c}_{ww} W^+_{\mu\nu} \widetilde{W}^-_{\mu\nu} + \tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu} + \tilde{c}_{z\gamma} F_{\mu\nu} \widetilde{Z}_{\mu\nu} + \tilde{c}_{zz} Z_{\mu\nu} \widetilde{Z}_{\mu\nu} \Big] \quad \leftarrow \textbf{New CP violating interactions}$$

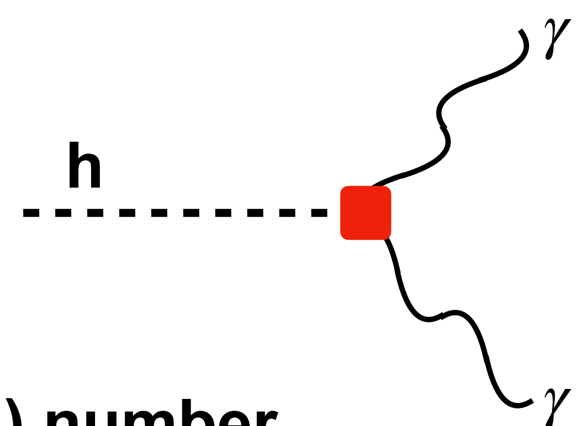
4 couplings $\tilde{c}_{\rm vv}$ from 3 Wilson coefficients $c_{H\tilde{B}}$, $c_{H\tilde{W}}$, $c_{H\tilde{W}B}$

Thus SMEFT predicts one relation between these CP violating couplings

These couplings will affect the Higgs production rates and decay width, e.g. Higgs decay to two photons



+



O(1) number

$$\Gamma(h \to \gamma\gamma) = \Gamma(h \to \gamma\gamma)_{\rm SM}\left(1 + \#16\pi^2 |\tilde{c}_{\gamma\gamma}|^2\right)$$

CP violation in Higgs sector

$$\Gamma(h \to \gamma\gamma) = \Gamma(h \to \gamma\gamma)_{\rm SM}\left(1 + \#16\pi^2 |\tilde{c}_{\gamma\gamma}|^2\right)$$

The Higgs branching ratio to photons is known at the 10% level

$$16\pi^2 |\tilde{c}_{\gamma\gamma}|^2 \lesssim 0.1 \qquad \Longrightarrow \qquad |\tilde{c}_{\gamma\gamma}| \lesssim 3 \times 10^{-2}$$

Translated to the scale of new physics

$$\mathscr{L}_{D=6} \supset \frac{c_{H\tilde{B}}}{\Lambda^2} H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu}$$

$$\tilde{c}_{\gamma\gamma} \sim c_{H\tilde{B}} \frac{\mathrm{v}^2}{\Lambda^2} \qquad \Longrightarrow \qquad \Lambda \gtrsim 1.5 \text{ TeV} \sqrt{|c_{H\tilde{B}}|}$$

Only new physics close to the TeV scale can be probed
(this is the feature of all Higgs physics, not only for CP violating Higgs couplings)

Note that this observable cannot distinguish CP-violating and CP-conserving contributions

Higgs decays to either two positive or two negative helicity photons
and the relative phase between the two is affected by the CP violating coupling.
However, polarisation of high-energy photons is very difficult (impossible?) to observe

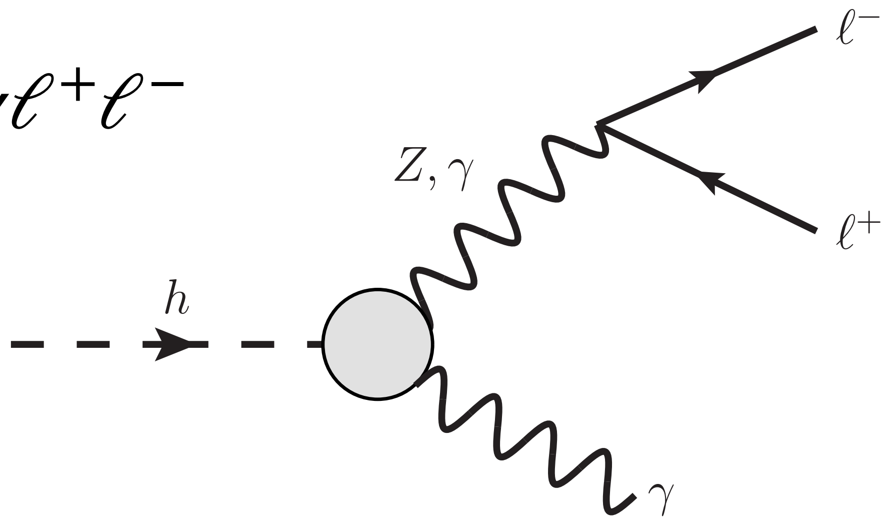
Bishara et al
1312.2955

CP violation in Higgs sector

CP violating observable can be constructed for 3- and more-body final states of Higgs decay

Process

$$h \to \gamma Z/\gamma^* \to \gamma \ell^+ \ell^-$$



$$\mathscr{L}_{\text{SMEFT}} \supset \frac{h}{\text{v}} \left[\tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu} + \tilde{c}_{z\gamma} F_{\mu\nu} \widetilde{Z}_{\mu\nu} \right]$$

All conditions for CP violation reunited:

- **weak phase due to CP violating couplings of photons and Z to the Higgs**
- **strong phase thanks to the relatively large Z width**
- **interference of different amplitudes with different weak and strong phases**
- **Polarization of Z/γ* can be probed by looking at the distribution of the lepton decay angle in the rest frame of intermediate Z/γ***

$$\frac{d\Gamma(h \to \gamma \ell^+ \ell^-)}{d\cos\theta} = (1 + \cos^2\theta) A_{\text{even}} + \cos\theta\, A_{\text{odd}}$$

$$A_{\text{odd}} \sim \frac{\Gamma_Z}{m_Z} \left(\#_1 \tilde{c}_{\gamma\gamma} + \#_2 \tilde{c}_{z\gamma} \right)$$

Chen et al. 1405.6723

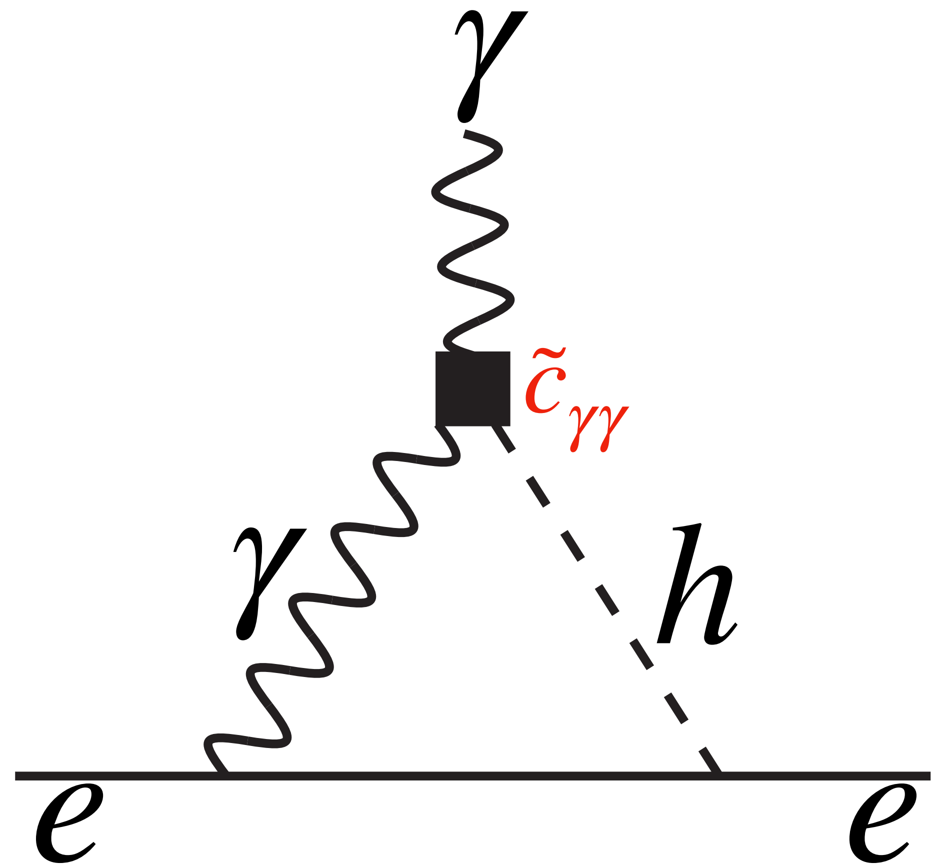
Thus CP violation can be in principle observed in the Higgs sector (also in h→ ZZ→ 4l, h→ττ, …)
However, the sensitivity remains only for new physics around the TeV scale

Have you checked EDMs?

Higgs vs EDM

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{h}{\text{v}} \tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$\mathscr{L} \subset \frac{i}{2} d_e \left[e^c \sigma_{\mu\nu} e - \bar{e} \bar{\sigma}_{\mu\nu} \bar{e}^c \right] F_{\mu\nu}$$



By dimensional analysis:
(or RG running)

$$d_e \sim \frac{\tilde{c}_{\gamma\gamma}}{16\pi^2} \frac{m_e e}{\text{v}^2}$$

ACME limit $$|d_e| < \frac{1.7 \times 10^{-16}}{\text{GeV}}$$ $\Rightarrow$ $$|\tilde{c}_{\gamma\gamma}| \lesssim 10^{-5}$$

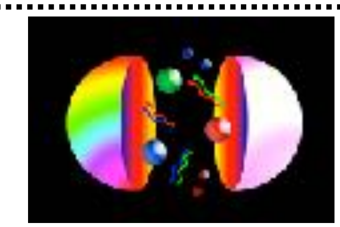
Unless conspiracy, electron EDM limit exclude CP violating Higgs coupling to photon large enough to be ever observable at the LHC

100 TeV ???

100 GeV γ,g,W,Z,ν_i,e,μ,τ + u, d, s, c, b, t + h

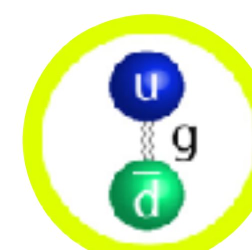
5 GeV γ,g,ν_i,e,μ,τ + u, d, s, c, b

2 GeV γ,g,ν_i,e,μ,τ + u, d, s, c



1 GeV γ,ν_i,e,μ + hadrons

100 MeV γ,ν_i,e,μ + pions and kaons



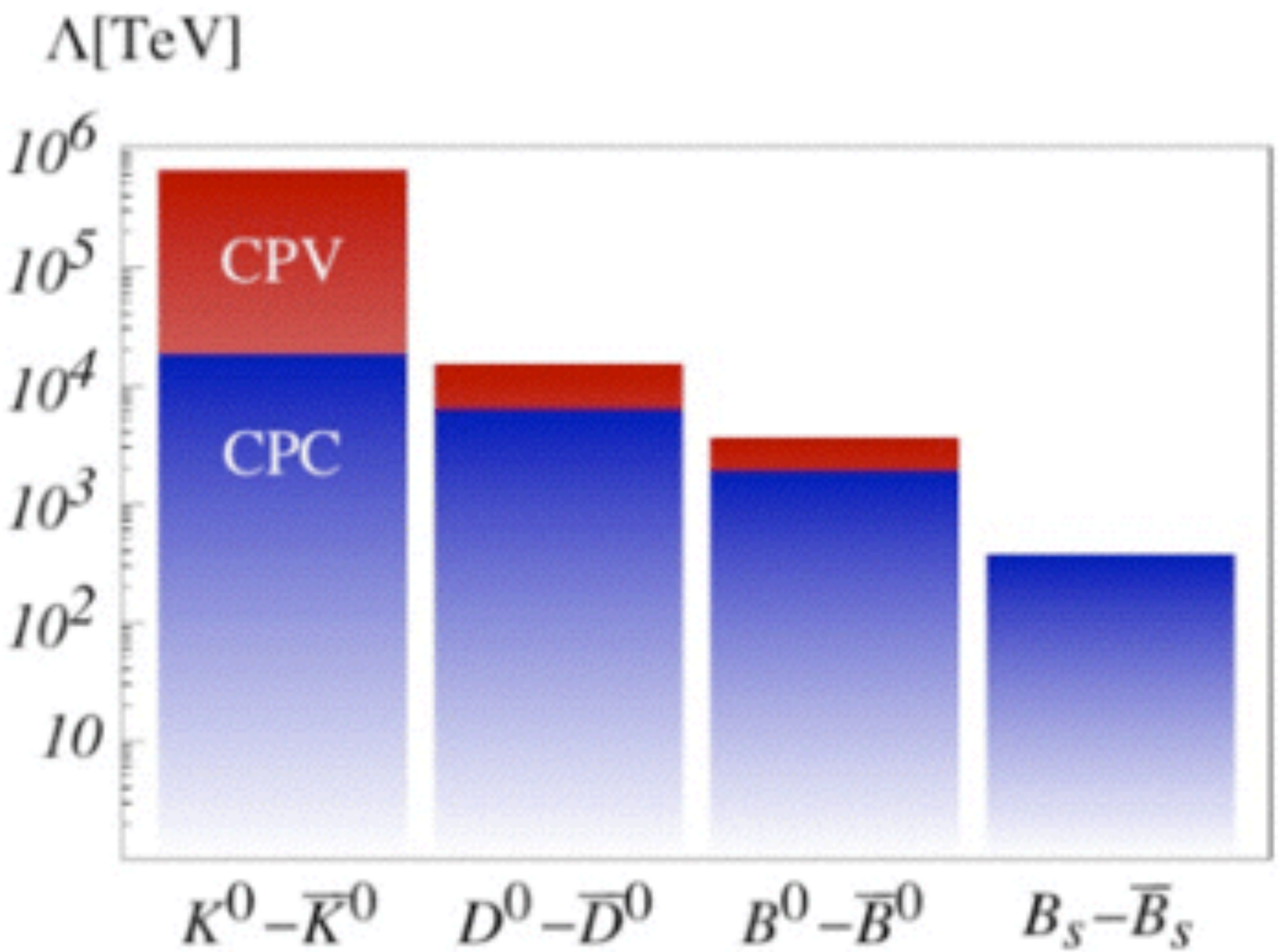
1 MeV γ,ν_i,e

γ,ν_i

CP violation in flavor physics

$$\mathscr{L}_{D=6} \supset \frac{c_1}{\Lambda^2}\left[(d^c s)(\bar{d}^c \bar{s}^c) + \mathrm{h.c.}\right] + \dots$$

One slide about 5 hour topic....



Borrowed from J. Kamenik

CP violation in nuclear physics

CKM element

$$\mathscr{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2}\Big\{ (1+\epsilon_L)\;\bar{e}_L\gamma_\mu\nu_L\cdot\bar{u}_L\gamma^\mu d_L$$ **V-A**

$$+\epsilon_R\,\bar{e}_L\gamma_\mu\nu_L\cdot\bar{u}_R\gamma^\mu d_R$$ **V+A**

Normalization scale, set by Fermi constant

$$\mathrm{v} = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx 246\ \mathrm{GeV}$$

$$+\epsilon_T\frac{1}{4}\bar{e}_R\sigma_{\mu\nu}\nu_L\cdot\bar{u}_R\sigma^{\mu\nu}d_L$$ **Tensor**

$$+\epsilon_S\frac{1}{2}\bar{e}_R\nu_L\cdot\bar{u}d$$ **Scalar**

Pseudo-scalar

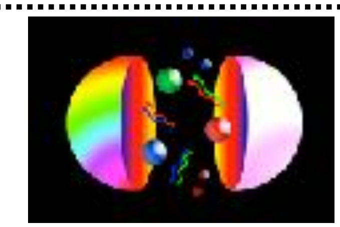
$$-\epsilon_P\frac{1}{2}\bar{e}_R\nu_L\cdot\bar{u}\gamma_5 d\Big\} + \mathrm{h.c.}$$

100 TeV ???

100 GeV $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h

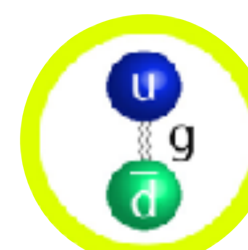
5 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

2 GeV $\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c

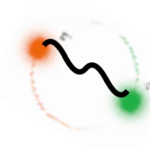


1 GeV γ, ν_i, e, μ + hadrons

100 MeV γ, ν_i, e, μ + pions and kaons



1 MeV γ, ν_i, e



γ, ν_i

Illustration #4

Fermi EFT

From WEFT to Fermi EFT

- At a scale of order 2 GeV the quarks of WEFT becomes strongly interacting
- Below that scale, the useful degrees of freedom are no longer quarks but hadrons: baryons and mesons
- We have to switch our EFT description to take into account the new degrees of freedom (and the lack of quarks)
- I focus on a special sector of the that EFT, which describes beta transitions involving nucleons (protons and neutrons) and leptons
- I call this the Fermi EFT

From WEFT to Fermi EFT

Let us take only SM-derived interactions for the start:

$$\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\text{v}^2}(\bar{e}_L\gamma_\rho\nu_L)(\bar{u}_L\gamma_\rho d_L) + \text{h.c.}$$

This interaction leads to beta decays, in particular to the neutron decay

$$d \to u e^- \bar{\nu} \quad \Rightarrow \quad n \to p e^- \bar{\nu}$$

Amplitude for the latter process is

$$\begin{aligned}
M(n \to p e^- \bar{\nu}_e) &= -\frac{2V_{ud}}{\text{v}^2}\langle p e^- \bar{\nu}_e | (\bar{e}_L\gamma_\rho\nu_L)(\bar{u}_L\gamma_\rho d_L) | n > \\
&= -\frac{2V_{ud}}{\text{v}^2}\langle e^- \bar{\nu}_e | (\bar{e}_L\gamma_\rho\nu_L) | 0 > \langle p | (\bar{u}_L\gamma_\rho d_L) | n > \\
&= -\frac{2V_{ud}}{\text{v}^2}\big(\bar{u}(p_e)\gamma_\rho P_L \text{v}(p_\nu)\big)\langle p | (\bar{u}_L\gamma_\rho d_L) | n > \\
&= -\frac{V_{ud}}{\text{v}^2}\big(\bar{u}(p_e)\gamma_\rho P_L \text{v}(p_\nu)\big)\Big\{ \langle p | (\bar{u}\gamma_\rho d) | n > - \langle p | (\bar{u}\gamma_\rho\gamma_5 d) | n > \Big\}
\end{aligned}$$

$$P_L \equiv \frac{1-\gamma_5}{2}$$

where u(p), v(p) are the usual spinor wave functions for particle and antiparticles

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{\mathrm{v}^2}\Big(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\Big)\Big\{ \langle p\,|\,(\bar{u}\gamma_\rho d)\,|\,n > - \langle p\,|\,(\bar{u}\gamma_\rho\gamma_5 d)\,|\,n > \Big\}$$

Due to strong QCD interaction, the quark matrix element cannot be calculated perturbatively

However, with the input from dimensional analysis and QCD (approximate) symmetries they can be reduced to a few unknowns, which can be subsequently calculated on the lattice or using phenomenological models

Lorentz invariance + Parity of QCD implies

$$q \equiv p_n - p_p$$

$$\langle p\,|\,(\bar{u}\gamma_\rho d)\,|\,n > = \bar{u}(p_p)\Big[g_V(q^2)\gamma_\rho + \frac{\tilde{g}_{TV}(q^2)}{2m_n}\sigma_{\rho\nu}q^\nu + \frac{\tilde{g}_S(q^2)}{2m_n}q_\rho\Big]u(p_n)$$

$$\langle p\,|\,(\bar{u}\gamma_\rho\gamma_5 d)\,|\,n > = \bar{u}(p_p)\Big[g_A(q^2)\gamma_\rho + \frac{\tilde{g}_{TA}(q^2)}{2m_n}\sigma_{\rho\nu}q^\nu + \frac{\tilde{g}_P(q^2)}{2m_n}q_\rho\Big]\gamma_5 u(p_n)$$

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{\mathrm{v}^2}\left(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\right)\left\{ \langle p \,|\, (\bar{u}\gamma_\rho d) \,|\, n > - \langle p \,|\, (\bar{u}\gamma_\rho\gamma_5 d) \,|\, n > \right\}$$

For beta decay processes, and especially for neutron decay, recoil is much smaller than nucleon mass. Therefore at the leading order one can approximate

$$\langle p \,|\, (\bar{u}\gamma_\rho d) \,|\, n > = g_V \bar{u}(p_p)\gamma_\rho u(p_n) + \mathcal{O}(q)$$

$$\langle p \,|\, (\bar{u}\gamma_\rho\gamma_5 d) \,|\, n > = g_A \bar{u}(p_p)\gamma_\rho\gamma_5 u(p_n) + \mathcal{O}(q)$$

$$q \equiv p_n - p_p$$

where g_V=g_V(0) and g_A=g_A(0) are now numbers, called the vector and axial charges

Furthermore, in the isospin symmetric g_V=1, because the quark current is the isospin current One can prove that departures of g_V from one are second order in isospin breaking, thus tiny

All in all

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{\mathrm{v}^2}\left(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\right)\left\{ \bar{u}(p_p)\gamma_\rho u(p_n) - g_A \bar{u}(p_p)\gamma_\rho\gamma_5 u(p_n) + \mathcal{O}(q) \right\}$$

Fermi EFT

$$M(n \to p e^- \bar{\nu}_e) = -\frac{V_{ud}}{\mathrm{v}^2}\left(\bar{u}(p_e)\gamma_\rho P_L \mathrm{v}(p_\nu)\right)\left\{\bar{u}(p_p)\gamma_\rho u(p_n) - g_A \bar{u}(p_p)\gamma_\rho\gamma_5 u(p_n) + \mathcal{O}(q)\right\}$$

$$\mathscr{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2}(\bar{e}_L\gamma_\rho\nu_e)(\bar{u}_L\gamma_\rho d_L) + \mathrm{h.c.}$$

Matching

$$\mathscr{L}_{\mathrm{Fermi}} \supset -\frac{V_{ud}}{\mathrm{v}^2}(\bar{e}_L\gamma_\rho\nu_e)\left\{(\bar{p}\gamma_\rho n) - g_A(\bar{p}\gamma_\rho\gamma_5 n)\right\} + \mathrm{h.c.} + \mathcal{O}\left(\frac{q}{m_n}\right)$$

as our n→p e ν amplitude can be obtained from this effective Lagrangian

The non-perturbative parameter g_A appearing in this matching has to be calculated on the lattice or measured in experiment

Lattice	**Experiment**
$g_A = 1.271 \pm 0.013$	$g_A = 1.27536 \pm 0.00041$

Fermi EFT

Now let's take into account non-SM interactions in WEFT

$$\mathscr{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{\text{v}^2}\Big\{ (1+\epsilon_L)\ \bar{e}_L\gamma_\mu\nu_L \cdot \bar{u}_L\gamma^\mu d_L + \epsilon_R\, \bar{e}_R\gamma_\mu\nu_L \cdot \bar{u}_R\gamma^\mu d_R + \epsilon_S \frac{1}{2}\bar{e}_R\nu_L \cdot \bar{u}d + \epsilon_T \frac{1}{4}\bar{e}_R\sigma_{\mu\nu}\nu_L \cdot \bar{u}_R\sigma^{\mu\nu}d_L - \epsilon_P \frac{1}{2}\bar{e}_R\nu_L \cdot \bar{u}\gamma_5 d \Big\} + \text{h.c.}$$

At nucleon scale, we then get more general set of interactions

$$\mathscr{L}_{\text{Fermi}} \supset -C_V^+ \bar{p}\gamma^\mu n\, \bar{e}_L\gamma_\mu\nu_L - C_A^+ \bar{p}\gamma^\mu\gamma_5 n\, \bar{e}_L\gamma_\mu\nu_L - C_S^+ \bar{p}n\, \bar{e}_R\nu_L - \frac{1}{2}C_T^+ \bar{p}\sigma^{\mu\nu}n\, \bar{e}_R\sigma_{\mu\nu}\nu_L + C_P^+ \bar{p}\gamma_5 n\, \bar{e}_R\nu_L \quad + \text{h.c.}$$

T.D. Lee and C.N. Yang (1956)

Short-distance radiative corrections

Dictionary is obtained along the same lines as in SM case

$$C_V^+ = \frac{V_{ud}}{\text{v}^2}\sqrt{1+\Delta_R^V}\left(1+\epsilon_L+\epsilon_R\right) \qquad \Delta_R^V = 0.02467(22)$$

Seng et al 1807.10197

$$C_A^+ = -\frac{V_{ud}}{\text{v}^2} g_A \sqrt{1+\Delta_R^A}\left(1+\epsilon_L-\epsilon_R\right) \qquad g_A = 1.251 \pm 0.033 \qquad \Delta_R^A - \Delta_R^V = 4.07(8)\times 10^{-3}$$

Flag'19

Hayen 2010.07262

$$C_T^+ = \frac{V_{ud}}{\text{v}^2} g_T \epsilon_T$$

$$C_S^+ = \frac{V_{ud}}{\text{v}^2} g_S \epsilon_S$$

$$g_S = 1.02 \pm 0.10, \quad g_T = 0.989 \pm 0.034$$

Gupta et al 1806.09006

$$C_P^+ = \frac{V_{ud}}{\text{v}^2} g_P \epsilon_P \qquad g_P = 349 \pm 9$$

Gonzalez-Alonso et al 1803.08732

Down the rabbit hole

$$
\begin{aligned}
\mathscr{L}_{\text{Fermi}} \supset & -C_V^+ \bar{p}\gamma^\mu n\, \bar{e}_L \gamma_\mu \nu_L \\
& -C_A^+ \bar{p}\gamma^\mu \gamma_5 n\, \bar{e}_L \gamma_\mu \nu_L \\
& -C_S^+ \bar{p} n\, \bar{e}_R \nu_L \\
& -\frac{1}{2} C_T^+ \bar{p}\sigma^{\mu\nu} n\, \bar{e}_R \sigma_{\mu\nu} \nu_L \\
& +C_P^+ \bar{p}\gamma_5 n\, \bar{e}_R \nu_L \qquad + \mathrm{h.c.}
\end{aligned}
$$

This is a relativistic Lagrangian, and may not be most convenient to use for non-relativistic processes

In neutron decay the momentum transfer is much smaller then the nucleon mass, due to the tiny mass splitting between neutron and proton.

It is thus convenient to change variables in the Lagrangian, and use non-relativistic version of the neutron and proton quantum fields

$$
N \to \frac{e^{-im_N t}}{\sqrt{2}} \left(1 + i \frac{\boldsymbol{\sigma} \cdot \nabla}{2m_N} \right) \psi_N + \mathcal{O}(\nabla^2), \qquad N = p, n
$$

In these variables, and expanding in powers of ∇, the Lagrangian simplifies

$$
\mathscr{L}_{\text{Fermi}}^{\text{NR}} \supset -(\bar{\psi}_p \psi_n) \left[C_V^+ \bar{e}_L \nu_L + C_S^+ \bar{e}_R \nu_L \right] - (\bar{\psi}_p \sigma^k \psi_n) \left[C_A^+ \bar{e}_L \sigma^k \nu_L + C_T^+ \bar{e}_R \sigma^k \nu_L \right] + \mathcal{O}(\nabla / m_n)
$$

It is clear that pseudoscalar couplings do not affect neutron decay at leading order

Non-relativistic Fermi EFT

$$\mathscr{L}_{\text{Fermi}}^{\text{NR}} \supset -(\bar{\psi}_p \psi_n)\left[C_V^+ \bar{e}_L \nu_L + C_S^+ \bar{e}_R \nu_L\right] - \sum_{k=1}^{3} (\bar{\psi}_p \sigma^k \psi_n)\left[C_A^+ \bar{e}_L \sigma^k \nu_L + C_T^+ \bar{e}_R \sigma^k \nu_L\right] + \mathcal{O}(\nabla/m_n)$$

This Lagrangian can also describe beta decays of nuclei: $N \to N' e^- \bar{\nu}$

$$\mathscr{M} = -\mathscr{M}_F\left[C_V^+(\bar{x}_3 y_4) + C_S^+(y_3 y_4)\right] - \sum_{k=1}^{3} \mathscr{M}_{\text{GT}}^k\left[C_A^+(\bar{x}_3 \sigma^k y_4) + C_T^+(y_3 \sigma^k y_4)\right]$$

where the Fermi and Gamow-Teller matrix elements are

$$\mathscr{M}_{\text{F}} \equiv \langle \mathscr{N}' | \bar{\psi}_p \psi_n | \mathscr{N} \rangle \qquad \mathscr{M}_{\text{GT}}^k \equiv \langle \mathscr{N}' | \bar{\psi}_p \sigma^k \psi_n | \mathscr{N} \rangle$$

Fermi transitions

Calculable from group theory in the isospin limit

Gamow-Teller transitions

Difficult to calculate from first principles

The use of non-relativistic EFT allows one to reduce the problem of calculating amplitudes for allowed beta transitions of nuclei to calculating two nuclear matrix elements

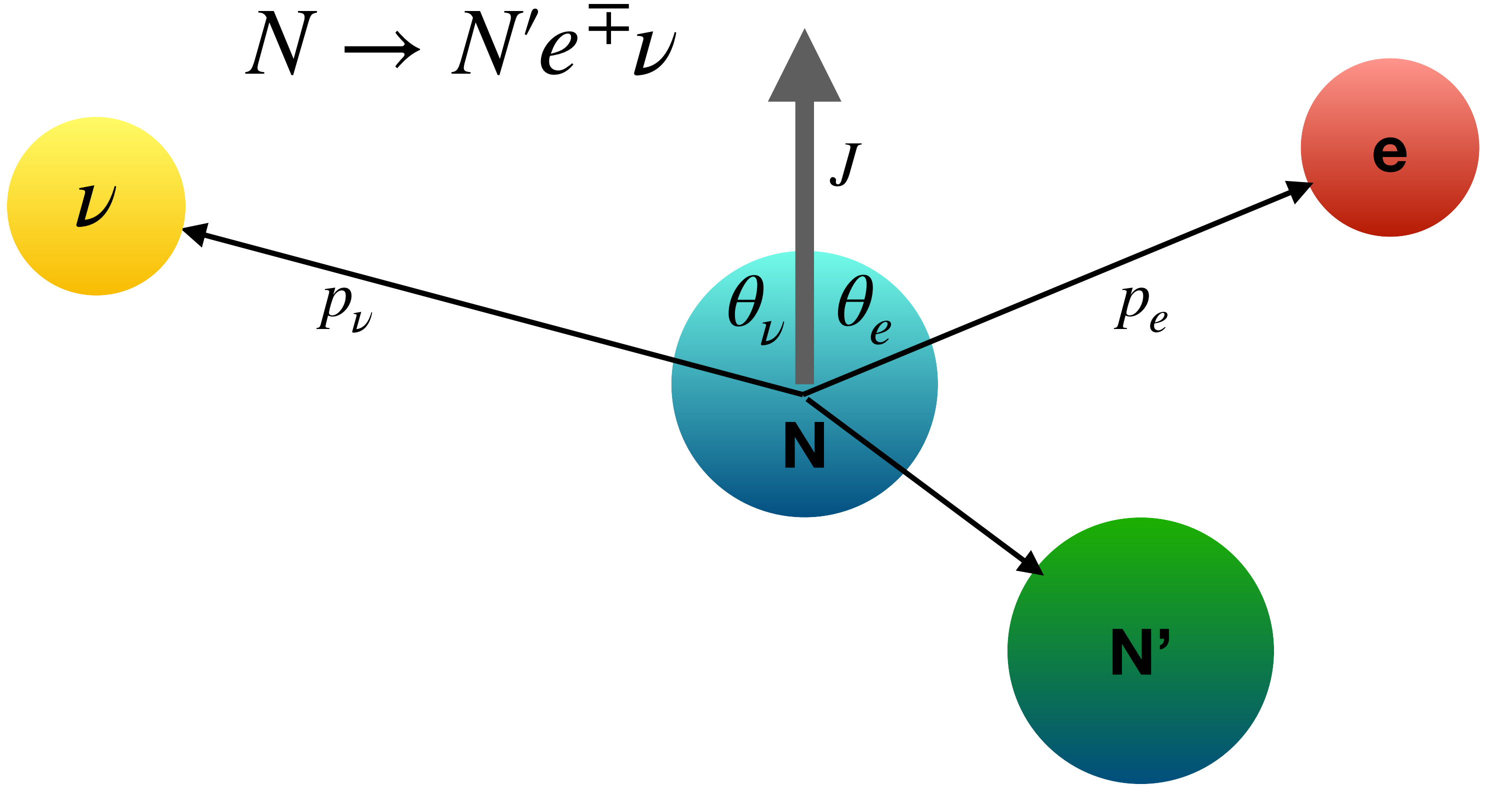
Forbidden transitions correspond to higher order terms in the non-relativistic expansion

Observables in beta decays

Effective Lagrangian describing allowed nuclear beta decays:

$$\mathscr{L}^{\mathrm{NR}}_{\mathrm{Fermi}} \supset -(\bar{\psi}_p \psi_n)\left[C_V^{+}\bar{e}_L\nu_L + C_S^{+}\bar{e}_R\nu_L\right] - \sum_{k=1}^{3}(\bar{\psi}_p\sigma^k\psi_n)\left[C_A^{+}\bar{e}_L\sigma^k\nu_L + C_T^{+}\bar{e}_R\sigma^k\nu_L\right]$$

$$N \to N' e^{\mp}\nu$$



Electron energy/momentum

$$E_e = \sqrt{p_e^2 + m_e^2}$$

Neutrino energy

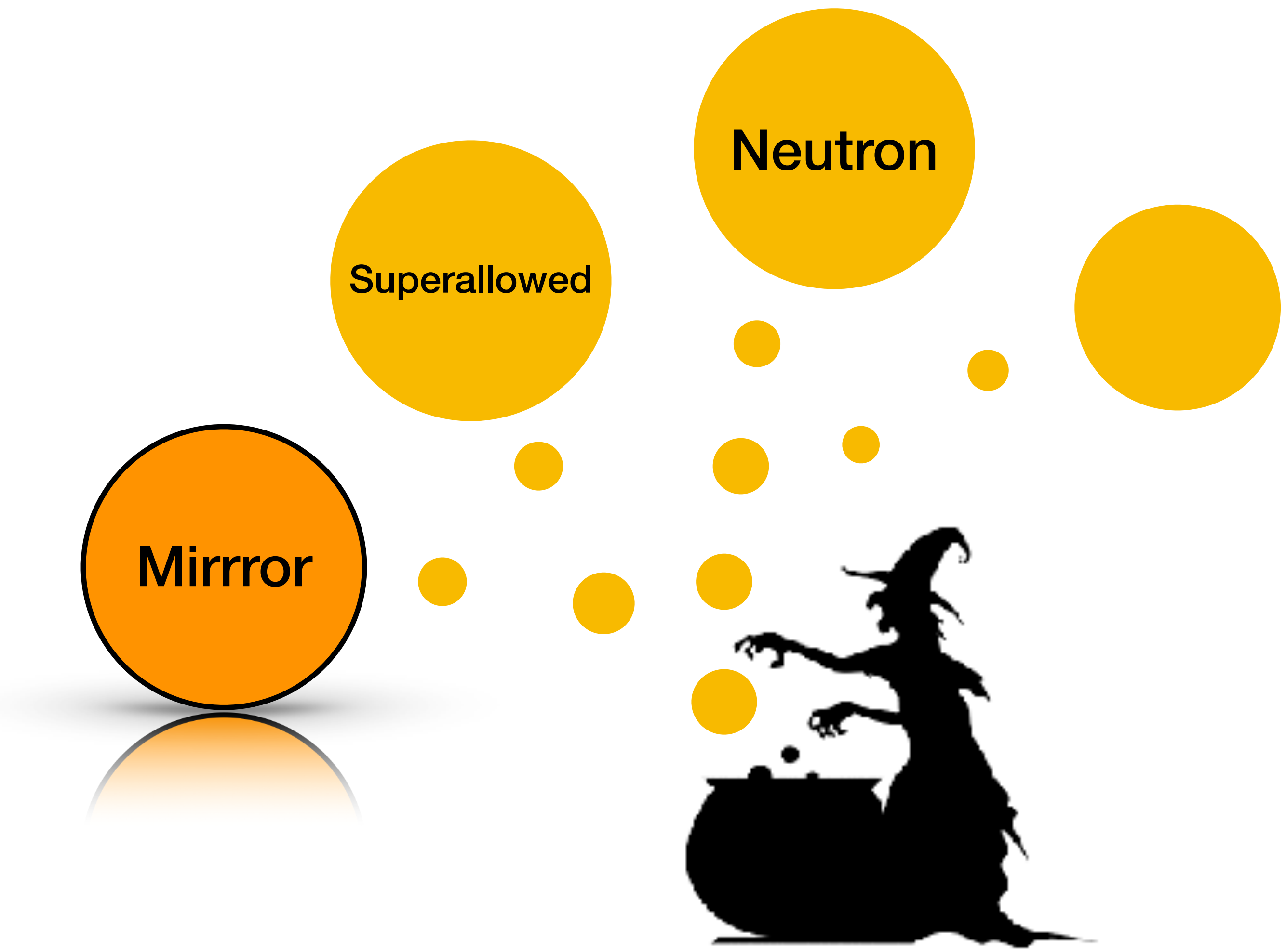
$$E_\nu = p_\nu = m_N - m_{N'} - E_e$$

Information about the Wilson coefficients can be accessed by measuring lifetimes and correlations

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = F(E_e)\left\{1 + b\frac{m_e}{E_e} + a\frac{\boldsymbol{p}_e\cdot\boldsymbol{p}_\nu}{E_e} + A\frac{\langle \boldsymbol{J}\rangle\cdot\boldsymbol{p}_e}{JE_e} + B\frac{\langle \boldsymbol{J}\rangle\cdot\boldsymbol{p}_\nu}{JE_\nu}\right.$$
$$\left. + c\frac{\boldsymbol{p}_e\cdot\boldsymbol{p}_\nu - 3(\boldsymbol{p}_e\cdot\boldsymbol{j})(\boldsymbol{p}_\nu\cdot\boldsymbol{j})}{3E_eE_\nu}\left[\frac{J(J+1) - 3(\langle \boldsymbol{J}\rangle\cdot\boldsymbol{j})^2}{J(2J-1)}\right] + D\frac{\langle \boldsymbol{J}\rangle\cdot(\boldsymbol{p}_e\times\boldsymbol{p}_\nu)}{JE_eE_\nu}\right\}$$

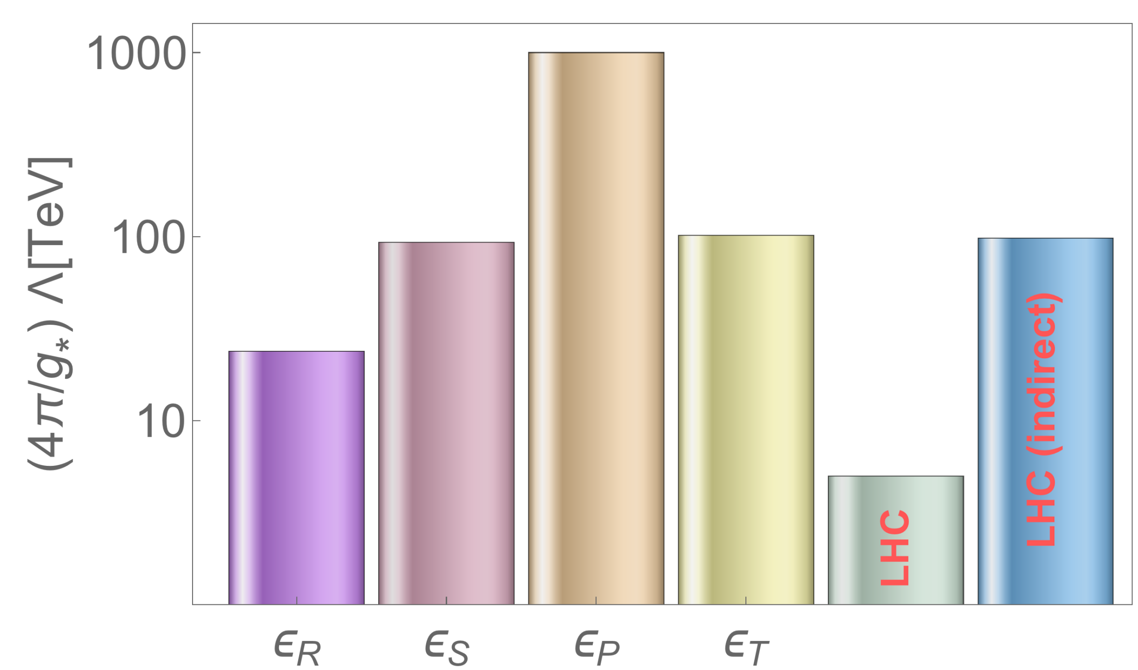
Constraints on the CP-conserving Wilson coefficients

AA, Gonzalez-Alonso, Naviliat-Cuncic
2010.13797



$$v^2 \mathrm{Re} \begin{pmatrix} C_V^+ \\ C_A^+ \\ C_S^+ \\ C_T^+ \end{pmatrix} = \begin{pmatrix} 0.98571(41) \\ -1.25707(55) \\ 0.0001(10) \\ 0.0004(12) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \hat{V}_{ud} \\ g_A \\ \epsilon_R \\ \epsilon_S \\ \epsilon_T \end{pmatrix} = \begin{pmatrix} 0.97377(41) \\ 1.251(33) \\ -0.009(13) \\ 0.0001(10) \\ 0.0005(13) \end{pmatrix}$$



CP-violating observables in beta decays

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = F(E_e)\left\{1 + b\frac{m_e}{E_e} + a\frac{\boldsymbol{p}_e\cdot\boldsymbol{p}_\nu}{E_e} + A\frac{\langle \boldsymbol{J}\rangle\cdot\boldsymbol{p}_e}{JE_e} + B\frac{\langle \boldsymbol{J}\rangle\cdot\boldsymbol{p}_\nu}{JE_\nu}\right.$$
$$\left. + c\frac{\boldsymbol{p}_e\cdot\boldsymbol{p}_\nu - 3(\boldsymbol{p}_e\cdot\boldsymbol{j})(\boldsymbol{p}_\nu\cdot\boldsymbol{j})}{3E_eE_\nu}\left[\frac{J(J+1)-3(\langle \boldsymbol{J}\rangle\cdot\boldsymbol{j})^2}{J(2J-1)}\right] + D\frac{\langle \boldsymbol{J}\rangle\cdot(\boldsymbol{p}_e\times\boldsymbol{p}_\nu)}{JE_eE_\nu}\right\}$$

The triple correlation D is CP-violating

$$D = -2r\sqrt{\frac{J}{J+1}}\frac{\mathrm{Im}\left[C_V^+\bar{C}_A^+ - C_S^+\bar{C}_T^+\right]}{|C_V^+|^2 + |C_S^+|^2 + r^2\left[|C_A^+|^2 + |C_T^+|^2\right]} \qquad r \equiv \rho C_V^+/C_A^+$$

Back to the quark level Lagrangian:

$$\mathscr{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2}\left\{ (1+\epsilon_L)\ \bar{e}_L\gamma_\mu\nu_L\cdot\bar{u}_L\gamma^\mu d_L + \epsilon_R\,\bar{e}_L\gamma_\mu\nu_L\cdot\bar{u}_R\gamma^\mu d_R + \epsilon_T\frac{1}{4}\bar{e}_R\sigma_{\mu\nu}\nu_L\cdot\bar{u}_R\sigma^{\mu\nu}d_L + \epsilon_S\frac{1}{2}\bar{e}_R\nu_L\cdot\bar{u}d\right\} + \mathrm{h.c.}$$

$$C_V^+ = \frac{V_{ud}}{\mathrm{v}^2}\sqrt{1+\Delta_R^V}(1+\epsilon_L+\epsilon_R)$$
$$C_A^+ = -\frac{V_{ud}}{\mathrm{v}^2}g_A\sqrt{1+\Delta_R^A}(1+\epsilon_L-\epsilon_R)$$
$$C_T^+ = \frac{V_{ud}}{\mathrm{v}^2}g_T\epsilon_T$$
$$C_S^+ = \frac{V_{ud}}{\mathrm{v}^2}g_S\epsilon_S$$

$$\Rightarrow \quad D = -\frac{4\rho}{1+\rho^2}\sqrt{\frac{J}{J+1}}\,\mathrm{Im}[\epsilon_R] + \mathcal{O}(\epsilon_X^2)$$

Constraints from D parameter

$$D = -\frac{4\rho}{1+\rho^2}\sqrt{\frac{J}{J+1}}\mathrm{Im}[\epsilon_R] + \mathcal{O}(\epsilon_X^2)$$

$$\mathcal{L}_{\mathrm{WEFT}} \supset -\frac{2V_{ud}}{\mathrm{v}^2}\epsilon_R\,\bar{e}_L\gamma_\mu\nu_L \cdot \bar{u}_R\gamma^\mu d_R + \mathrm{h.c.}$$

D-parameter probes the CP violating part of the V+A currents in the WEFT Lagrangian

For neutron, the current PDG combination $D_n = (-1.2 \pm 2.0) \times 10^{-4}$

$J_n = 1/2$ $\rho_n \approx -\sqrt{3}g_A \approx -2.2$ $D_n \approx 0.86\,\mathrm{Im}[\epsilon_R]$

This translates into the constraint

$$\boxed{\mathrm{Im}\,\epsilon_R = (-1.4 \pm 2.3) \times 10^{-4}}$$

Up the ladder to the SMEFT:

$$\epsilon_R = \frac{1}{2V_{ud}}c_{Hud}\frac{\mathrm{v}^2}{\Lambda^2}$$

$$\Lambda \gtrsim 10\ \mathrm{TeV}\sqrt{|c_{Hud}|}$$

Have you checked EDMs?