Some topics in supersymmetric models

and concrete implementations in SUSY-codes

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Tentative plan

Seminar 1:
1. Short review of motivations for supersymmetry

2. Ingredients and construction of supersymmetric Lagrangians

3. Essential of Minimal Supersymmetric Standard Model (MSSM)

4. SUSY-breaking issues: spontaneous, explicit, problems

5. Different popular SUSY-breaking models
(minimal SUGRA, GMSB, AMSB, ...)

6. Concrete implementation in SUSY-code “SuSpect”

Seminar 2:
7. Basics of renormalization group (RG) techniques

8. RGE in MSSM: linking grand unification scale with electroweak scale

9. Electroweak symmetry breaking issues in MSSM

10. Concrete implementation in SUSY-code “SuSpect”

11. Some applications: constraining SUSY model parameter space

’top-down’ versus ’bottom-up’ reconstruction of parameters

(could be seminar 3)

1. Supersymmetry: Motivations

Supersymmetry: Poincaré + Fermions $\leftrightarrow$ Bosons symmetry:

$$Q|F\rangle = |B\rangle, \quad Q|B\rangle = |F\rangle$$

numerous *independent* motivations +unexpected bonus

•Super-Poincaré: the largest possible symmetry (in 4-dim):

basic algebra (schematically):

$$\{Q, Q^\dagger\} \propto P_\mu; \quad [Q, P_\mu] = 0$$

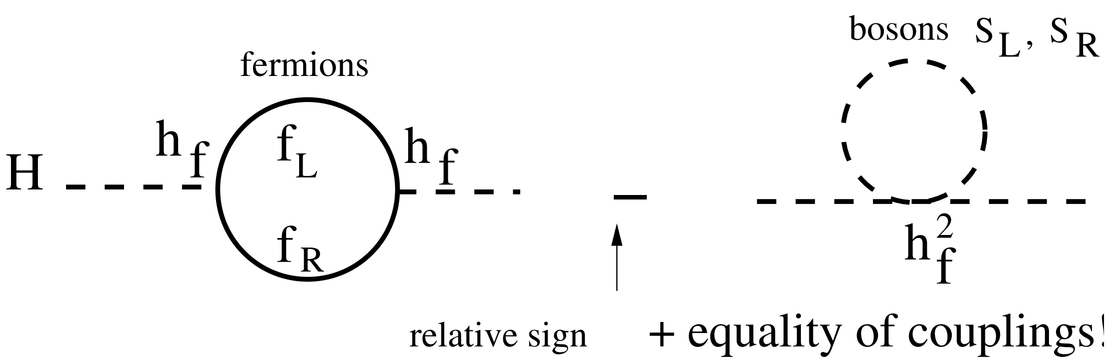
“square-root" of translation: escape of 60’s no-go theorems (Coleman-Mandula) for enlarged space-time+internal symmetries $[space - time\ sym] \otimes internal\ sym]$

-if made a local symmetry, necessary ingredient of a quantum gravity $\rightarrow$ Supergravity etc

The “hierarchy" or naturalness problem

radiative corrections to Higgs mass: $\delta m^2_{Higgs} \propto M^2_{GUT,Planck}$??

Stabilized!



$$\delta m_H^2 = \frac{N_c h_f^2}{16\pi^2}\left[-2M_{Pl}^2 + 3m_f^2 \ln\frac{M_{Pl}^2}{m_f^2} + 2M_{Pl}^2 - 2m_s^2 \ln\frac{M_{Pl}^2}{m_s^2}\right]$$

Moreover even the $\ln$ terms cancel if m_f, m_s arise

from sym. breaking ($m_f \sim h_f v = m_s$) (another graph then)

exact SUSY $\rightarrow$ equality of masses AND couplings.

Broken SUSY: $m_f \neq m_s \rightarrow \ln$ terms survive: “fine-tuning" pb

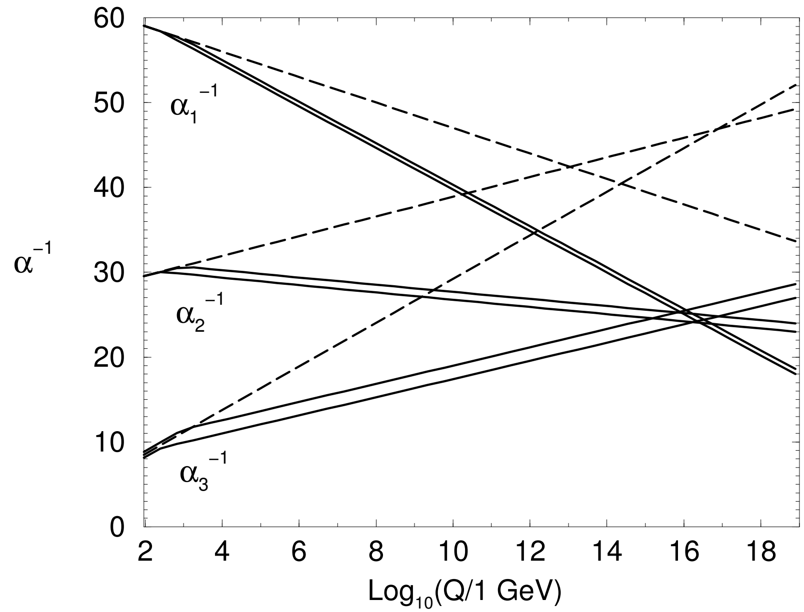
$\rightarrow$ acceptable IF $m_{sparticles} \lesssim \mathcal{O}$ (1 TeV)

NB origin of the large rad. corr. $\propto m_t^4 \ln[..]$ to MSSM H mass

+Unexpected bonus (not original motivations but welcome)

•Grand Unification *consistent* with Proton lifetime limits

-Due to SUSY particle threshold
+ SUSY Renor Group Evol.
(totally excluded in SM)



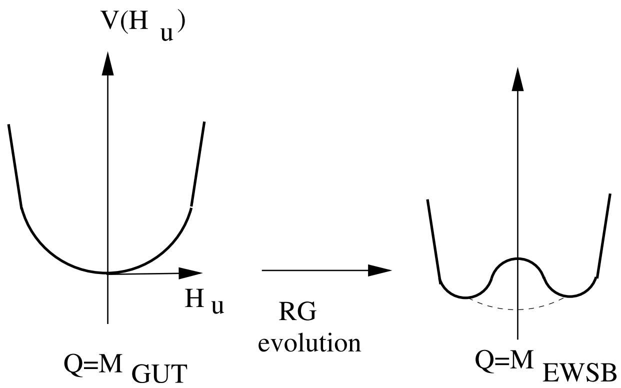
-Unification scale $M_{GUT} > 10^{16}$: large enough to escape Proton decay limits (Superkamiokande) $\sim 1.9\ 10^{33}$ years

-However, $1 - 2\%$ mismatch $1 - \alpha_S(M_{GUT})/\alpha_1(M_{GUT})$: hoped to be explained by GUT scale threshold corrections...

(but dim 5 operators can disturb this “conventional wisdom"!)

Another unexpected bonus..

• *Radiative* electro-weak sym. breaking: “mexican hat" scalar potential induced by Renormalization Group (RG) evolution: GUT $\rightarrow$ low energy



$m^2_{H_u}(E) < 0$ by RG evolution $E_{GUT} \rightarrow E_{EWSB}$ ($\propto m_t^2$)
made possible thanks to the large value of m_{top}!
(does not explain why m_{top} is large, though)

NB more on this RGE/EWSB issues in seminar 2

Yet another unexpected bonus...

•Very plausible candidate to Dark Matter (neutralino LSP)
present strong indication that $\sim$ 10% of mass in universe is neutral, weakly interacting cold DM

But, problem: SUSY has to be broken: what's the right model? :<...
To date: NO consistent model of spontaneous (or dynamical) SUSY-breaking! (breaking has to be in a "hidden" sector)

$\rightarrow$ proliferation of SUSY-breaking (arbitrary) parameters:
All possible gauge-invariant interactions between quite many (s)particles.. IF no more theoretical prejudices applied

2. Basics of supersymmetric gauge theories

•Supersymmetric extensions of SM follow the rules of (super)gauge theories:

based on two set of fields with specific gauge+susy transformations:

-Chiral fields: left-handed fermions + scalar partners
-Vector fields: vector gauge bosons + fermion (majorana) partners

-Right handed fermions: from charge conjugate representation of chiral fields: $(\psi_R)^c = (\psi^c)_L$

-Higgs field: described by chiral fields: $\Leftrightarrow$ fermion partners

A bit of supersymmetric formalism

Basic ingredients: 2-components spinors χ_α, $\bar{\psi}^{\dot{\alpha}}$ $\quad \alpha, \dot{\alpha} = 1, 2$

makes supersymmetric properties more manifest

may be contracted to form Lorentz-invariants:

$(\psi\chi) \equiv \psi^\alpha \chi_\alpha \equiv \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta$, $\epsilon_{\alpha\beta}$ antisymmetric: $\epsilon_{12} = -\epsilon_{21} = 1$

Standard Dirac spinor (4-component object):

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi}_D \equiv \Psi^\dagger \gamma_0 = (\psi^\alpha, \ \bar{\chi}_{\dot{\alpha}}), \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}$$

$\rightarrow$ standard (Dirac) contraction e.g. $\bar{\Psi}_D \Psi_D = \psi\chi + h.c.$ etc

Majorana: $\Psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$ i.e. such that $\Psi_M^c = \Psi_M$

Note $(\Psi_D)_L = \frac{1}{2}(1 - \gamma_5)\Psi_D = \chi_\alpha$, $(\Psi_D)_R = \frac{1}{2}(1 + \gamma_5)\Psi_D = \bar{\psi}^{\dot{\alpha}}$

Superspace formalism

Convenient: describe boson+ fermion by same “superfield":
in addition to usual space coordinate x_μ, introduce new anticommuting spinor variables θ_α, $\theta_{\dot\alpha}$
$\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha \to (\theta_\alpha)^2 = 0$ but $\theta\theta \equiv \theta^\alpha \epsilon_{\alpha\beta} \theta^\beta \neq 0$!

e.g chiral superfield (irreducible SUSY representation):
$\Phi(x, \theta, \bar\theta = 0) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$
where ϕ scalar, ψ fermion, F scalar (auxiliary) fields
-Expansion stops at θ^2 due to anticommuting properties of θ

-F “scalar" has dim $[m]^2$ and NO kinetic term
($\Leftrightarrow$ function of other fields from its eq. of motion):
F assures (off-shell) matching of boson vs fermion d^0 freedom

Supersymmetric transformation of fields

Supersymmetry transformation = translation in superspace parameterized in terms of infinitesimal (Grassman) ζ

-SUSY generators expressed as derivative operators

$Q_\alpha = -i\partial_\theta + \sigma^\mu \bar{\theta}\partial_\mu$ (analog of $P_\mu \to i\partial_\mu$)

where extra terms originates from $\{Q, Q^\dagger\} \propto P_\mu$

Components of chiral field transform as

$$\delta\phi = \sqrt{2}\zeta\psi, \qquad \delta F = -i\sqrt{2}\zeta\sigma^\mu\partial_\mu\psi$$

$$\delta\psi = -i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu\phi + \sqrt{2}\zeta F$$

Note F transforms as total derivative:

a basic ingredient for SUSY-invariant Lagrangians

Vector Superfield (are hermitian)

Similarly the vector superfield reads in the simplest gauge choice (so called Wess-Zumino):

$$V(x,\theta,\bar{\theta}) = -(\theta\sigma^{\mu}\bar{\theta})V_{\mu} + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

where V_{μ} usual vector field, λ its Majorana fermion partner, D auxiliary (scalar), with appropriate SUSY-transformations.

Again, auxiliary field D transforms as total derivative

There is also a chiral superfield, derived from V, generalizing "gauge field strength":

$$W_{\alpha}(x,\theta,\bar{\theta}) = -i\lambda_{\alpha} + (\theta\sigma_{\mu\nu})^{\alpha}F^{\mu\nu} + \theta^{\alpha}D - \theta^2(\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\bar{\lambda})^{\alpha}$$

tranforms like usual $V_{\mu\nu}$ under gauge symmetry

$\rightarrow$ building blocks to construct SUSY-invariant Lagrangian.

Supersymmetric Lagrangian

Armed with this formalism, "straightforward" to construct SUSY- and gauge-invariant Lagrangians

$\mathcal{L}_{SUSY} = \frac{1}{4g^2}(Tr[W^\alpha W_\alpha]_F + h.c) + \sum_i [\bar{\Phi} e^{(gV)} \Phi]_D + [W(\Phi)]_F$

where $[\cdots]_{F,D}$ means appropriate "projection" (θ^2, $\theta^2\bar{\theta}^2$ coefficients resp.) that transform as total derivative.

$W(\Phi)$ superpotential = dim-3 gauge-invariant polynomial function of chiral field Φ:

$$W(\Phi) = c_i \Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_j\Phi_k$$

Scalar potential:

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2}\sum_a (D^a)^2$$

$$F_i^* = \frac{\partial W(\Phi)}{\partial \Phi_i}, \qquad D^a = -g\sum_i(\phi_i^* T^a \phi_i)$$

3. Minimal Supersymmetric Standard Model (MSSM) in short

Table 1: Chiral Supermultiplet of MSSM

(s)particles		spin 0	spin 1/2	$SU(3)_c, SU(2)_L, U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, 1/6)$
(x 3 families)	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -2/3)$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, 1/3)$
sleptons, leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -1/2)$
(x 3 families)	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, 1)$
Higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(1, 2, 1/2)$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(1, 2, -1/2)$

Table 2: Vector Supermultiplet of MSSM

(s)particles	spin 1/2	spin 1	$SU(3)_c, SU(2)_L, U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(8, 1, 0)$
Winos, W boson	$\tilde{W}^\pm, \tilde{W}^0$	$W^\pm W^0$	$(1, 3, 0)$
Binos, B boson	$\tilde{B}$	B	$(1, 1, 0)$

MSSM Superpotential (R-parity conserving!)

$$W = \sum_{i,j=gen} -Y^u_{ij}\, \hat{u}_{Ri} \hat{H}_u . \hat{Q}_j + Y^d_{ij}\, \hat{d}_{Ri} \hat{H}_d . \hat{Q}_j + Y^l_{ij}\, \hat{l}_{Ri} \hat{H}_d . \hat{L}_j + \mu \hat{H}_u . \hat{H}_d \,,$$

$\mathcal{L}_{SUSY}$ = kin. terms (SUSY +gauge) +F^2, D^2 terms $\propto \partial_{\phi_i} W$, etc

- $\hat{H}_u$, $\hat{Q}$ etc superfield: contain both fermion+boson

Note at this (exact supersymmetric SM) stage:

-$m_{fermions} = m_{bosons}$? Yes, before EWSB, but most masses zero!

-quartic couplings determined by gauge couplings

-equality of fermion and boson couplings:

essential for cancellation of all quadratic UV div.

$\Rightarrow$ only logarithmic div (wave fction and gauge cpling renormalization, superpotential $W(\Phi)$ NOT renormalized)

-Only new parameter: μ

Clearly unrealistic! must introduce supersymmetry breaking...

Digression: R-parity and its violation business

In MSSM, Higgs superfields H_u, H_d have same quantum numbers as leptons: $\rightarrow$ SUSY+gauge-inv allow mixing: $\mu^i L_i H_u, \lambda^{ijk} L_i L_j \bar{e}_k$ etc $\rightarrow$ L-violation *+ ν-mass contributions !!*
similarly trilinear quark terms allowed: $\bar{u}\bar{d}\bar{d} \rightarrow B$-violation

Some couplings very constrained by rare decays, P decay, etc, but not all

$\rightarrow$ introduce discrete symmetry: R-parity (Fayet 1976)
$R = (-1)^{2s+3B+L}$
$\rightarrow R_P$(matter fermions) $= +1$, R_P(all spartners) $= -1$
ensure that superpartners produced by pairs
lightest R_P-odd partner (LSP) stable (DM candidate)

Rk: R_P is discrete version of $U(1)$ R-sym in extended models

General (arbitrary) parameters of “soft" SUSY-breaking:

soft SUSY-breaking = that do not reintroduce quadratic UV divergences

•Mass Terms for Gluinos, Winos and Binos:

$$-\mathcal{L}_{\rm gaugino} = \frac{1}{2}\left[M_1 \tilde{B}\tilde{B} + M_2 \sum_{a=1}^{3} \tilde{W}^a \tilde{W}_a + M_3 \sum_{a=1}^{8} \tilde{G}^a \tilde{G}_a \;+\; \text{h.c.}\right]$$

minimal SUGRA universality: $M_1(E_{GUT}) = M_2(E_{GUT}) = M_3(E_{GUT}) \equiv m_{1/2}$

•Mass terms for sfermions:

$$-\mathcal{L}_{\rm sfermions} = \sum_{i=gen} m^2_{\tilde{Q}i} \tilde{Q}^\dagger_i \tilde{Q}_i + m^2_{\tilde{L}i} \tilde{L}^\dagger_i \tilde{L}_i + m^2_{\tilde{u}i} |\tilde{u}_{R_i}|^2 + m^2_{\tilde{d}i} |\tilde{d}_{R_i}|^2 + m^2_{\tilde{l}i} |\tilde{l}_{R_i}|^2$$

mSUGRA universality: $m_{\tilde{Q}i}(E_{GUT}) = \cdots = m_{\tilde{l}i}(E_{GUT}) \equiv m_0$

Mass and bilinear terms for Higgs scalars:

$$-\mathcal{L}_{\text{Higgs}} = m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d + B\mu(H_u.H_d + \text{h.c.})$$

mSUGRA universality: $m^2_{H_u}(E_{GUT}) = m^2_{H_d}(E_{GUT}) \equiv m_0^2$

•Finally, some trilinear interactions between scalars (sfermions and Higgs bosons):

$$-\mathcal{L}_{\text{tril.}} = \sum_{i,j=gen} \left[-A^u_{ij} Y^u_{ij} \tilde{u}_{R_i} H_u.\tilde{Q}_j + A^d_{ij} Y^d_{ij} \tilde{d}_{R_i} H_d.\tilde{Q}_j + A^l_{ij} Y^l_{ij} \tilde{l}_{R_i} H_d.\tilde{L}_j \; + \; \text{h.c.} \right]$$

mSUGRA universality: $A^u_{ij}(E_{GUT}) = A^d_{ij}(E_{GUT}) = A^l_{ij}(E_{GUT}) \equiv A_0 \delta_{ij}$

Sparticle spectrum: • 5 Higgs scalars: $h, H, H^\pm, A$

•2 Charginos: $\tilde{\chi}^\pm_{1,2}$; 4 neutralinos $\tilde{\chi}^0_{1-4}$, 1 gluino $\tilde{g}$

•Numerous sfermions: sleptons ($\tilde{e}, \tilde{\mu}, \tilde{\nu}_e$,.. $\tilde{\tau}_{1,2}$),

squarks: $(\tilde{u}, \tilde{d}, ..\tilde{b}_{1,2}, \tilde{t}_{1,2})$

From Lagrangian to concrete masses and couplings

1) Sfermion sector:

$$W_{superpotential} = -Y_u \, \hat{u}_R \hat{H}_u.\hat{Q} + \cdots + \mu \hat{H}_u.\hat{H}_d \ ,$$

$\rightarrow \mathcal{L}^{scalar}_{SUSY} \sim |\frac{\partial W}{\partial \phi}|^2 + \sum_i g_i^2 (\phi T_i \phi)^2, \quad \phi = \tilde{Q}, \tilde{H}_{u,d}, \tilde{u}_R$

(F and D terms respectively)

thus generate $\phi\phi$, $\phi\phi\phi$, $\phi\phi\phi\phi$ terms

$\rightarrow$ AFTER EWSB: $H_u \rightarrow v_u + .., H_d \rightarrow v_d + .., v_u = v \sin\beta, v_d = v \cos\beta$

give $\tilde{u}\tilde{u}$ ($\tilde{t}\tilde{t}$ etc) mass terms and $\tilde{u}\tilde{u}H$ couplings:

from $|F|^2$: $Y_t^2 H_u H_u Q_L^* Q_L \rightarrow$ (EWSB) $\rightarrow [m_t^2, Y_t^2 v_u]$

same for $Y_t^2 H_u H_u t_R^* t_R$

$\mu \, Y_t t_R^* Q_L H_d \rightarrow$ (EWSB) $\rightarrow [\mu Y_t v_d, \mu Y_t]$

from $|D|^2$: $g^2 (Q_L^* T_3 Q_L)(H_u H_u, H_d H_d) \rightarrow$ (EWSB) $\rightarrow g^2 [v_u^{(2)}, v_d^{(2)}]$

same for $g'^2 (t_R^* y t_R)(H_u H_u, H_d H_d) \rightarrow g'^2 [v_u^{(2)}, v_d^{(2)}]$

Now add soft-breaking terms:

direct mass terms: $-m^2_{t_L}\tilde{Q}^*_L Q_L - m^2_{t_R}\tilde{t}^*_R\tilde{t}_R$

trilinear: $-A_tY_t\tilde{u}_RH_u\tilde{Q}_L \to$ (EWSB) $\to [A_tY_t, A_tY_tv_u]$

All terms combine to give sfermion mass matrix:

$$M^2_{\tilde{f}} \equiv \begin{pmatrix} m^2_{LL} & m^2_{LR} \\ m^2_{LR} & m^2_{RR} \end{pmatrix}$$

$$\begin{aligned} m^2_{LL} &= m^2_{\tilde{f}_L} + (T^3_f - e_f s^2_W)M^2_Z\cos 2\beta + m^2_f \\ m^2_{LR} &= m_f(A_f - \mu r_f)\,, \quad (r_b = r_\tau = 1/r_t = \tan\beta) \\ m^2_{RR} &= m^2_{\tilde{f}_R} + e_f s^2_W M^2_Z \cos 2\beta + m^2_f \end{aligned}$$

$$\tan 2\theta_f = \frac{2m^2_{LR}}{m^2_{LL} - m^2_{RR}}$$

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_f & \sin\theta_f \\ -\sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

2. Chargino and Neutralino sector

NB tracing origin: (soft breaking; susy; D-term after EWSB)

Charginos:

$$M_C = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{pmatrix}$$

Neutralinos: in the $(-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis:

$$M_N = \begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

Diagonalized by real (by convention) matrix N_{ij}

e.g. LSP $\chi_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}_3 + N_{13}\tilde{H}_d + N_{14}\tilde{H}_u$

3. Higgs sector

$$V_{Higgs} = (m_{H_d}^2 + \mu^2)H_d^\dagger H_d + (m_{H_u}^2 + \mu^2)H_u^\dagger H_u + B\mu\, H_u \cdot H_d$$
$$+ \frac{g_1^2 + g_2^2}{8}(H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{g_2^2}{2}(H_d^\dagger H_u)(H_u^\dagger H_d)$$

pseudoscalar A: $\bar{m}_A^2(Q) \equiv \frac{B\mu}{\sin\beta\cos\beta} = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2$

$$\mathcal{M}^S(q^2) = \begin{pmatrix} \bar{M}_Z^2 \cos\beta^2 + \bar{M}_A^2 \sin^2\beta - S_{11}(q^2) & -\frac{1}{2}(\bar{M}_Z^2 + \bar{M}_A^2)\sin 2\beta - S_{12}(q^2) \\ -\frac{1}{2}(\bar{M}_Z^2 + \bar{M}_A^2)\sin 2\beta - S_{12}(q^2) & \bar{M}_Z^2 \sin^2\beta + \bar{M}_A^2 \cos^2\beta - S_{22}(q^2) \end{pmatrix}$$

$\rightarrow m_h, m_H$ neutral eigenstates ($S_{ij}(q^2)$ rad.corr. from self-energies)

Approx. : $m_h^2 \sim m_h^{2,tree} + \frac{3gm_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4}\right]$

where $X_t = A_t - \mu\cot\beta$, $M_S^2 \simeq m_{\tilde{t}_1} m_{\tilde{t}_2}$

4. How to break supersymmetry?

Why is it so difficult to break SUSY *spontaneously*?

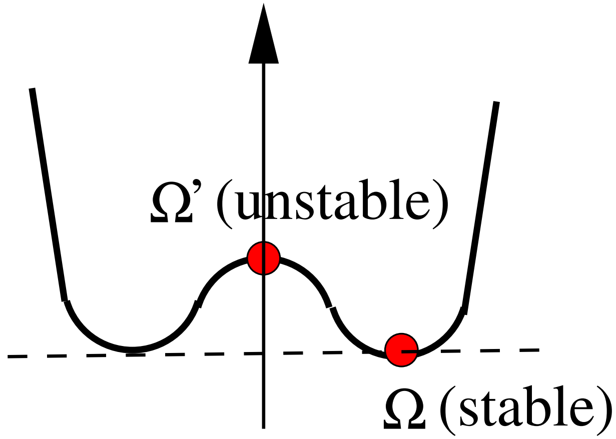
SUSY algebra involves the Hamiltonian: $H = P_0 = \sum Q_\alpha^2 \geq 0$

$\rightarrow$ expect (in global SUSY)

$\langle H \rangle_{\Omega\ supersymmetric} = 0;$

$\langle H \rangle_{\Omega'\ non-supersymmetric} > 0$

$V \sim \frac{1}{2}\sum(F^2 + D^2) > 0$



from SUSY-transformation (schematically):

$\delta\psi \sim (\sigma^\mu \partial_\mu \phi + F)\zeta,$ $\delta\lambda \sim (\sigma^\mu \sigma^\nu V_{\mu\nu} + D)\zeta$

$\langle F \rangle$and/or$\langle D \rangle \neq 0 \leftrightarrow \langle \delta\psi \rangle$and/or$\langle \delta\lambda \rangle \neq 0$ spont. breaking with sfermion ψ or gaugino λ Goldstone fermion resp.

(Analogy with usual SSB: $\delta\phi_2 = \theta\phi_1$, so $\langle \phi_1 \rangle \neq 0 \rightarrow \langle \delta\phi_2 \rangle \neq 0$)

Only way to get spontaneous SUSY-breaking:

look for models where $F_i = 0$ and/or $D^a = 0$ cannot be *simultaneously* satisfied for *any* field values.

Toy models do exist, but turn to be both

-contrived and exceptional situations

-phenomenologically unrealistic
(can't match SM gauge etc structure and/or strongly already excluded e.g due to sparticle mass limits)

Toy models of spontaneous SUSY-breaking

-O'Raifeartaigh: (F-term breaking) superpotential W:

$W = \lambda \Phi_3(\Phi_1^2 - M^2) + m\Phi_2\Phi_3$ such that

$V = \sum_i |F_i|^2 = |m\phi_1|^2 + |\lambda(\phi_1^2 - M^2)|^2 + |m\phi_2 + 2\lambda\phi_3\phi_1|^2$

immediate that the first two terms can't be *both* zero $\rightarrow$ SSSB.

More precisely if $|m|^2 > 2|\lambda^2 M^2|$ global min at $\phi_1 = \phi_2 = 0$;

$\rightarrow \langle F_3 \rangle \neq 0$: *flat* direction along ϕ_3 (so-called "moduli" field)

SUSY-breaking manifests as fermion ψ_1 mass m

while ϕ_1^+, ϕ_1^- mass $m^2 \pm 2\lambda^2 M^2$.

However note the sum rule (a generic feature):

$m^2_{\phi_1^-} + m^2_{\phi_1^+} = 2m^2_{\psi_1}$ just like exact SUSY...

Clearly excluded in MSSM!

D-term spontaneous SUSY-breaking

Fayet-Iliopoulos model: for $U(1)$ gauge symmetry

$V = |mQ|^2 + |m\bar{Q}|^2 + \frac{1}{8}|Q^\dagger Q - \bar{Q}^\dagger \bar{Q} + 2\kappa_{FI}|^2$, $Q, \bar{Q}$ chiral Sfields.

Linear term in κ_{FI} allows SSSB (for $m^2 > \kappa_{FI}/2$):

only OK for $U(1)$ (non-abelian sym: no invariant linear term!).

-maybe possible for extra $U(1)$ beyond SM: Z' models

(Still, not sufficient for realistic MSSM spectrum)

-Note D-term and F-term present in MSSM:

some $m_F \neq m_B$ amount triggered by EWSB...

(e.g. in sfermions mass terms) but not consistent alone

(tachyonic and/or obviously excluded) sfermion masses

typically $\rightarrow$ MSSM really needs soft terms!

$\rightarrow$ SUSY-breaking in hidden sector, communicated to SM

5. Generic features of hidden sector SUSY-breaking

Analogy with EWSB in SM: parameterized by $\langle v \rangle$

EWSB sector	Mediating interactions (= Yukawa couplings)	Observable sector
$h \to \langle v \rangle$	h, q, l	q, l

“Hidden" SUSY-breaking sector	Mediating interactions	Observable sector
$Z \to \langle F \rangle$	Z, Q, L	Q, L

SUSY-breaking parameterized by $\langle F \rangle$ of dim $[m]^2$

3 popular patterns: gravity-, gauge-, and anomaly-mediated

Actually all appear in a complete Supergravity picture!

Distinction arise from assumption on dominant mechanisms

Gravity-mediated susy breaking (minimal SUperGRAvity)

Start from Supergravity with “Kähler potential" $K(\phi, \phi^*)$
(Non-renormalizable terms)$\rightarrow$ suppressed by $1/M_{Planck}$
$\rightarrow$ soft terms of order $\sim \langle F \rangle / M_{Planck}$ when $Z \rightarrow \langle F \rangle$:

$$c_{ij} \; \frac{Z^\dagger Z}{M^2_{Planck}} \; \phi_i^* \phi_j \rightarrow m_0^2 \text{ scalar masses}$$
$$c_a \; \frac{Z}{M_{Planck}} \; \lambda_a \lambda_a \rightarrow m_{1/2} \text{ gaugino masses}$$
$$c_{ijk} \; \frac{Z}{M_{Planck}} \; \phi_i \phi_j \phi_k \rightarrow A_0 \text{ trilinear terms}$$

$F \sim M_{weak} M_{Planck} \sim [10^{10} GeV]^2$: high scale SUSY-breaking
(but $\langle F \rangle$ may also be triggered by gaugino condensation)
-Caution: famous universality in mSUGRA comes from minimal assumptions on Kähler and Super potential
(i.e *separable* hidden/visible $K(\phi, \phi^*)$, $W(\phi)$ contributions)

Non-universal terms are there in more general scenario...

Practical implementation of minimal SUGRA

–Unification of the gaugino [bino, wino and gluino] masses:

$$M_1(M_{\rm GUT}) = M_2(M_{\rm GUT}) = M_3(M_{\rm GUT}) \equiv m_{1/2}$$

–Universal scalar [i.e. sfermion and Higgs boson] masses [i is the generation index]:

$$\begin{aligned} M_{\tilde{Q}_i}(M_{\rm GUT}) &= M_{\tilde{u}_{Ri}}(M_{\rm GUT}) = M_{\tilde{d}_{Ri}}(M_{\rm GUT}) = M_{\tilde{L}_i}(M_{\rm GUT}) = M_{\tilde{l}_{Ri}}(M_{\rm GUT}) \\ &= M_{H_u}(M_{\rm GUT}) = M_{H_d}(M_{\rm GUT}) \equiv m_0 \end{aligned}$$

–Universal trilinear couplings:

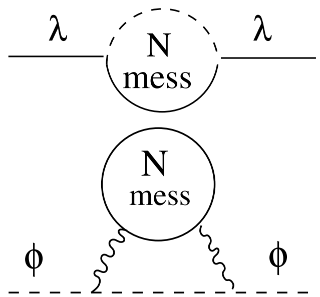
$$A^u_{ij}(M_{\rm GUT}) = A^d_{ij}(M_{\rm GUT}) = A^l_{ij}(M_{\rm GUT}) \equiv A_0\,\delta_{ij}$$

Apart from $m_{1/2}, m_0$ and A_0, input are μ and $\tan\beta = v_u/v_d$ after consistent EWSB requirement.

Gauge-mediated SUSY-breaking (GMSB)

Add N “messenger" $\hat{Q}, \hat{L}$ heavy (S)fields with mass M_{mess} and singlet $\hat{S}$: $\lambda \hat{S}(\hat{Q}\hat{Q} + \hat{L}\hat{L})$ with SUSY-breaking vev $\langle F \rangle$ that couple to SM gauge fields

$$M_\lambda^i \sim N \frac{g_i^2}{16\pi^2} \frac{\langle F \rangle}{M_{mess}}$$



$$m_\phi^2 \sim N [\frac{g_i^2}{16\pi^2}]^2 \; [\frac{\langle F \rangle}{M_{mess}}]^2$$

Trilinear terms $A_i(M_{mess}) \sim 0$ (2-loop; but much suppressed)

choose $M_{mess} \ll M_{Planck}$:

$\frac{F}{M_{mess}} \gg \frac{F}{M_{Planck}} \rightarrow$ gravity-mediated contributions negligible

Scalar masses determined by gauge quantum nbs:

solve SUSY flavor pb

Low scale SUSY breaking $F \sim M_{mess}^2$, $\sqrt{F} \sim 10^4$ GeV

but 10^4GeV $\lesssim M_{mess} \lesssim 10^{14}$ GeV possible

NB LSP can be (very light) gravitino: $M_{3/2} \sim \langle F \rangle / M_{Planck}$

Practical implementation of GMSB

messenger scale $M_{\rm mes} = \lambda\langle S\rangle$,

$$\begin{aligned} M_i(M_{mes}) &= \frac{\alpha_i(M_{\rm mes})}{4\pi}\Lambda\, g\left(\frac{\Lambda}{M_{\rm mes}}\right)\sum_m N_R^i(m) \\ m_s^2(M_{mes}) &= 2\Lambda^2 f\left(\frac{\Lambda}{M_{\rm mes}}\right)\sum_{m,G}\left[\frac{\alpha_i(M_{mes})}{4\pi}\right]^2 N_R^G(m) C_R^G(s) \\ A_i(M_{mes}) &\simeq 0 \end{aligned}$$

where $\Lambda = F_S/S$, $G = {\rm U(1), SU(2), SU(3)}$,

$$\mathcal{NC}(\tilde{Q}) = \frac{1}{16\pi^2}\left[\left(\frac{n_{\hat{l}}}{100} + \frac{n_{\hat{q}}}{150}\right)\alpha_1^2 + \frac{3n_{\hat{l}}}{4}\alpha_2^2 + \frac{4n_{\hat{q}}}{3}\alpha_3^2\right]$$

and similar expressions for U, D, E, L, H_u, H_d

$$\begin{aligned} g(x) &= \frac{1}{x^2}[(1+x)\log(1+x) + (1-x)\log(1-x)] \\ f(x) &= \frac{1+x}{x^2}\left[\log(1+x) - 2{\rm Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}{\rm Li}_2\left(\frac{2x}{1+x}\right)\right] + (x \leftrightarrow -x) \end{aligned}$$

NB intermediate scale $Q_{EWSB} \ll M_{mes} \ll Q_{GUT}$ for RGE

Anomaly-mediated SUSY-breaking (AMSB)

The anomaly (symmetry breaking at quantum level) of the (super)conformal symmetry induces soft SUSY breaking!
NB was always present; but assumed sub-dominant (loop-suppressed) in standard “mSUGRA"

gauginos: $M_i \sim b_i \frac{g_i^2}{16\pi^2} M_{3/2}$ b_i(RGE) $= (33/5, 1, -3)$

squarks, sleptons: $(m^2)^i_j \sim (\dot{\gamma})^i_j [\frac{M_{3/2}}{16\pi^2}]^2$; also $A_i \sim \frac{M_{3/2}}{16\pi^2}$
γ^i_j standard RGE anomalous mass dimensions
e.g. $\gamma_Q = -Y_u^\dagger Y_u - Y_d^\dagger Y_d + \sum_i c_i g_i^2$
Almost flavor blind!
But generally tachyonic $\tilde{l}_L$, $\tilde{l}_R \rightarrow$ add a m_0 term by hand...
however some recent criticisms (e.g. Dine+Seiberg ’07)

perhaps more consistent “m_0" terms will soon emerge??..

Practical implementation of AMSB

$$M_a = \frac{\beta_{g_a}}{g_a} m_{3/2} \, ,$$
$$A_i = \frac{\beta_{Y_i}}{Y_i} m_{3/2}$$
$$m_i^2 = -\frac{1}{4}\left(\Sigma_a \frac{\partial\gamma_i}{\partial g_a}\beta_{g_a} + \Sigma_k \frac{\partial\gamma_i}{\partial Y_k}\beta_{Y_k}\right) m_{3/2}^2$$

NB RG invariant Eqs $\rightarrow$ valid at any scale!

$$m_{\tilde{S}_i}^2 = c_{S_i} m_0^2 - \frac{1}{4}\left(\Sigma_a \frac{\partial\gamma_i}{\partial g_a}\beta_{g_a} + \Sigma_k \frac{\partial\gamma_i}{\partial Y_k}\beta_{Y_k}\right) m_{3/2}^2 + \mathrm{D\,terms.}$$

where ad hoc c_{S_i} correct for tachyonic sfermions...

model param.: $m_0, m_{3/2}, \tan\beta, \mathrm{sign}(\mu), c_{S_i}$

E.g. pheno. "minimal" AMSB model: $c_Q = c_{u_R} = \cdots = 1$

-"gaugino assisted" AMSB where gauge and gaugino fields are in bulk (extra-D):

$c_Q = 21/10, c_{u_R} = 8/5, c_{d_R} = 7/5, c_L = 9/10, c_e = 3/5, c_{H_u} = 9/10 c_{H_d}$

Ingredients of spectrum calculation in MSSM

–for example SuSpect 2.41 (A. Djouadi, JLK, G. Moultaka)

•Low energy input $\alpha(M_Z), \alpha_S(M_Z)$, $M_t^{\text{pôle}}$, $M_\tau^{\text{pôle}}$, $m_b^{\overline{\rm MS}}(m_b)$; $\tan\beta(M_Z)$

via radiative corrections $\Rightarrow g_{1,2,3}^{\overline{\rm DR}}(M_Z), Y_\tau^{\overline{\rm DR}}(M_Z), Y_b^{\overline{\rm DR}}(M_Z), Y_t^{\overline{\rm DR}}(M_Z)$

•Choice of SUSY-breaking model (mSUGRA, GMSB, AMSB,..)

Fixes initial condition at high energy (mSUGRA: $m_0, m_{1/2}, A_0$, sign(μ), etc...).

•Evolution of parameters by RGE down to $M_{\rm EWSB} \sim \mathcal{O}(100GeV - fewTeVs)$

•Control of EWSB consistency (convergence of μ, no CCB minima, etc...)

•Diagonalisation of mass mixing matrices and pole mass calculation (Including Rad. Corrections for Higgses, sfermions, gauginos)

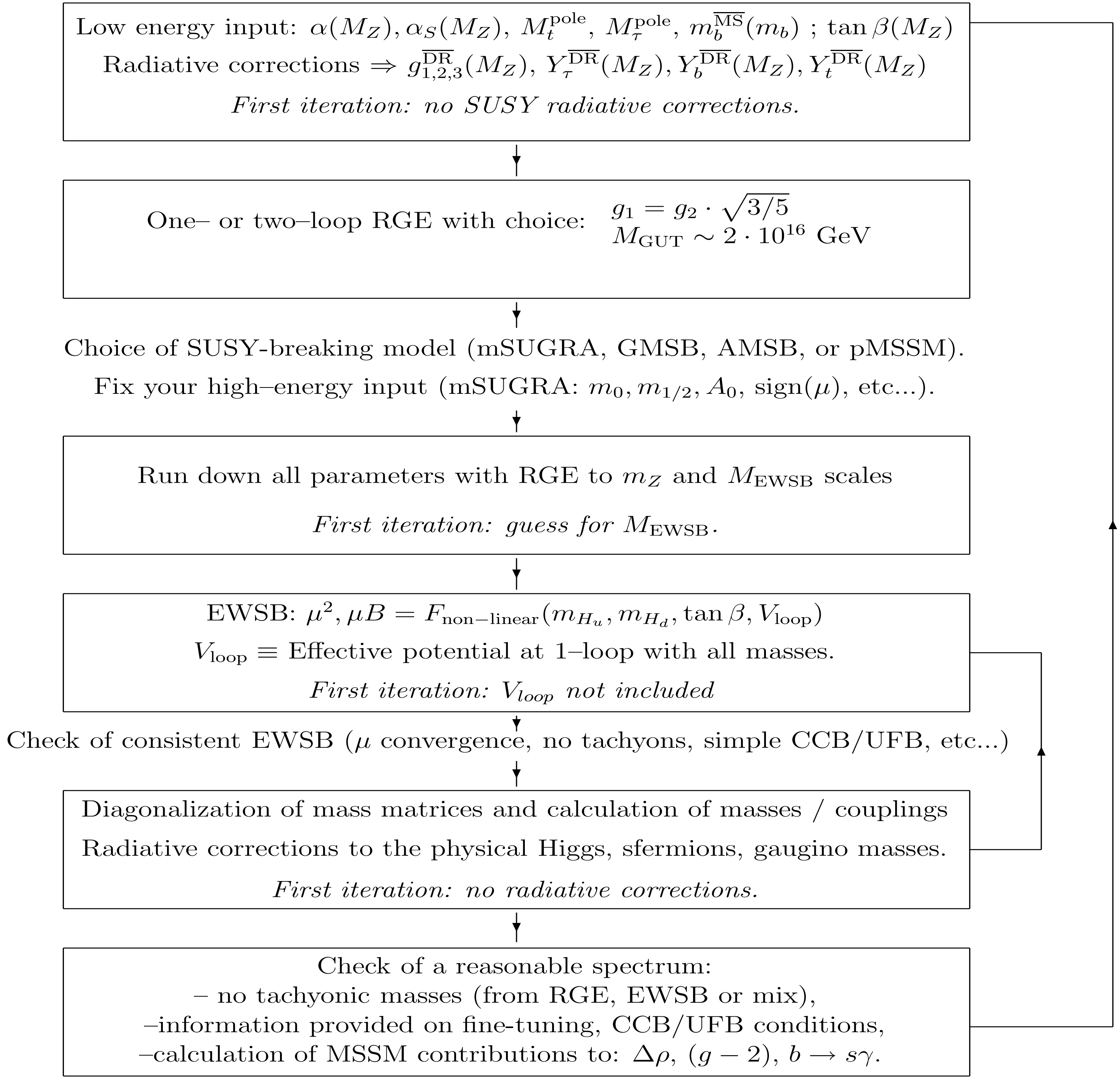


Figure 1: Iterative algorithm for the calculation of the SUSY particle spectrum in `SuSpect` *from the choice of input (first step) to the check of the spectrum (last step). The steps*

```
# SUSY Les Houches Accord 2.0 - example input file for SUSPECT ver >= 2.4
Block MODSEL  # Select model (with the second parameter):
#             General MSSM (arbitrary soft terms) at low scale input:    0
#             SUGRA (!includes non-univ. soft terms, def. in block EXTPAR):  1
#             GMSB                                                    : 2
#             AMSB                                                   :  3
#             Bottom-up RGE for general MSSM input at EWSB scale: -1
#             (a specific SuSpect option)
    1    1   #  mSUGRA
Block MINPAR  # specific model input parameters
#   input for SUGRA models (! comment (#) all other (GMSB,AMSB) lines):
     1    100.    # m0
     2    250.    # m1%2
     5   -100.      # A0
     3    10.    # tanbeta(MZ)
     4      1.0   # sign(mu)
#   input for GMSB models (! comment (#) all other (mSUGRA,AMSB) lines):
#     1    100.d3  # Lambda_susy
#     2    200.d3  # Lambda_mess
#     3    10     # tanbeta(MZ)
#     4    1.      # sign(MU)
#     5    1       # Nl_mes
#     6    1       # Nq_mes
#   input for AMSB models (! comment (#) all other (mSUGRA,GMSB) lines):
#     1    450.    # m0
#     2    60.d3   # M_3%2 gravitino mass
#     3    10.     # tanbeta(MZ)
#     4    1.      # sign(MU)
#     5    1.      # cQ  : weight of m0 for Q_L (3rd gen.) doublet
#     6    1.      # cuR : weight of m0 for u_R
#     7    1.      # cdR : weight of m0 for d_R
#     8    1.      # cL  : weight of m0 for L (1st, 2d gen.) doublet
#     9    1.      # ceR : weight of m0 for e_R (1st, 2d gen.)
#     10   1.      # cHu : weight of m0 for Hu
#     11   1.      # cHd : weight of m0 for Hd
#
```

```
BLOCK MASS  # Mass Spectrum
# PDG code           mass       particle
        24     8.04539688E+01   # W+
        25     1.09190354E+02   # h
        35     3.77796787E+02   # H
        36     3.77333195E+02   # A
        37     3.86150989E+02   # H+
         5     4.70844921E+00   # b pole mass calculated from mb(mb)_MSbar
   1000001     5.31790822E+02   # ~d_L
   2000001     5.07639109E+02   # ~d_R
   1000002     5.25910490E+02   # ~u_L
   2000002     5.07991280E+02   # ~u_R
   1000003     5.31790822E+02   # ~s_L
   2000003     5.07639109E+02   # ~s_R
   1000004     5.25910490E+02   # ~c_L
   2000004     5.07991280E+02   # ~c_R
   1000005     4.79548812E+02   # ~b_1
   2000005     5.08805010E+02   # ~b_2
   1000006     3.69707369E+02   # ~t_1
   2000006     5.48452898E+02   # ~t_2
   1000011     2.00833753E+02   # ~e_L
   2000011     1.42868597E+02   # ~e_R
   1000012     1.84912666E+02   # ~nu_eL
   1000013     2.00833753E+02   # ~mu_L
   2000013     1.42868597E+02   # ~mu_R
   1000014     1.84912666E+02   # ~nu_muL
   1000015     1.34029547E+02   # ~tau_1
   2000015     2.04454656E+02   # ~tau_2
   1000016     1.84031067E+02   # ~nu_tauL
   1000021     5.69612737E+02   # ~g
   1000022     9.77422056E+01   # ~chi_10
   1000023     1.79527580E+02   # ~chi_20
   1000025    -3.40585906E+02   # ~chi_30
   1000035     3.62250249E+02   # ~chi_40
   1000024     1.78737633E+02   # ~chi_1+
   1000037     3.62315788E+02   # ~chi_2+
```

Seminar 2:

7. Basics of renormalization group (RG) techniques

8. RGE in MSSM: linking grand unification scale with electroweak scale

9. Concrete Rad. Corr. implementation in SUSY-code "SuSpect"

10. Electroweak symmetry breaking issues in MSSM

11. Some applications: constraining SUSY model parameter space

'top-down' versus 'bottom-up' reconstruction of parameters

(could be seminar 3)

7. Basics of Renormalization Group (RG) technics

renormalizable theory: *finite* parts of counterterms arbitrary
RG expresses how a change in these finite parts is exactly compensated by appropriate change in masses, couplings, fields, so that physical quantities are invariant

in infinitesimal form: $Q\frac{d}{dQ}G_0^N = 0$
where Q scale, G_0^N N-points bare Green fction

basic (homogeneous) RGE:
$[Q\frac{\partial}{\partial Q} + \beta(g)\frac{\partial}{\partial g} - m\gamma_m(g)\frac{\partial}{\partial m}]G^N = 0$ ($\overline{MS}$ scheme)
$\beta(g) \equiv d\,g/d\ln Q$ drives running of coupling
anom. mass dim $\gamma_m(g) \equiv -d\,\ln m/d\ln Q$ drives running mass
only depend on g (NB in SUSY $\overline{MS} \rightarrow \overline{DR}$ to preserve susy)

Some useful results in dim.reg.

From $g_0 \equiv Z_g(g)\, g = Q^{\epsilon/2}[g + \sum_{j=1}^{\infty} \frac{d_j(g)}{\epsilon^j}]$ (in dim.reg.)

Applying RG-inv of g_0, i.e. $Q\frac{d}{dQ}\, g_0 \equiv 0$, expanding in ϵ powers, one finds basic properties:

$\beta(g) + \frac{1}{2}(1 - g\frac{\partial}{\partial g})d_1(g) = 0$

i.e. only simple poles determine $\beta(g)$ at any pert. order

• All higher order pole coefficients determined only from $d_1(g)$ by recurrence relations

Once renormalized, coefficient of poles in ϵ are related to coefficients of Logarithmic terms

$\rightarrow$ RG can resum leading Logs, sub-leading logs, etc

$g^2(Q_1) = \frac{g^2(Q_2)}{1+b_0 g^2(Q_1)\ln(Q_2/Q_1)}\,, \quad m(Q_1) = m(Q_2)[1 + b_0 g^2(Q_1)\ln(Q_2/Q_1)]^{-\frac{\gamma_0}{2b_0}}$

8. RGE for MSSM

Many interactions, many particles $\rightarrow$ non-linear coupled equations (specially scalar sector)

-gauge cplings: $\frac{d}{dt} g_i = b_i g_i^3 + \cdots$ $t \equiv \ln Q$, $b_i = (33/5, 1, -3)/(16\pi^2)$

-gaugino masses: $\frac{d}{dt} \ln M_i = 2 b_i g_i^2 + \cdots$ $\longrightarrow \frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}$

-Yukawa couplings:

$\frac{d}{dt} Y_t = \frac{Y_t}{16\pi^2}(6Y_t^2 + 2Y_b^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2)$

-scalars: e.g. $m_{H_u}^2$:

$\frac{d}{dt} m_{H_u}^2 =$

$\frac{1}{16\pi^2}[6Y_t^2(m_{H_u}^2 + m_{t_R}^2 + Y_t^2 A_t^2) + \frac{3}{5} g_1^2 Tr[y m^2] - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2]$

similar expressions for $m_Q^2, m_u^2, m_d^2, m_L^2, ..$

NB Suspect (and SoftSusy, Spheno,..) include full 2-loop RGE

Solving RGE concretely

Highly non-linear coupled differential eqs...

Moreover, many different scales + different boundary conditions

$Q = m_Z, \;\; Q = Q_{EWSB}, \;\; Q = Q_{GUT}$

Actually “analytic” solutions exists, but only at 1-loop level (in semi-analytic, iterative form: Auberson +Moultaka ’99, Kazakov +Moultaka ’2000)

Otherwise use numerical method: Runge-Kutta (as implemented in most codes):

4th order i.e. error $\mathcal{O}(h^5)$, h finite diff. step

(adaptative steps: goes faster if smoothly varying parameters)

Top-down RGE: concrete mSUGRA (SPS1a) example

par.	GUT scale input(GeV)	EWSB scale output
M_1	250	101.5
M_2	250	191.6
M_3	250	586.6
$m^2_{H_d}$	100	$(179.9)^2$
$m^2_{H_u}$	100	$-(358.1)^2$
μ	determined from EWSB	356.9
m_{e_R}	100	136
m_{Q1_L}	100	545.8
m_{Q3_L}	100	497
m_{u_R}	100	527.8
m_{t_R}	100	421.5
m_{b_R}	100	522.4
$-A_t$	100	494.5
$-A_b$	100	795.2

Renormalization Group “bottom-up" evolution

•Once parameters determined at Q_{EWSB} scale, evolve them to GUT scale

RGE evidently invertible, but to evolve MSSM parameters from EWSB scale UP to GUT scale, while matching low-energy (gauge+yukawa) data is not staightforward.

+ Care to be taken: $Tr[ym^2] \neq 0$ may increase error propagations!

NB public bottom-up RGE option now installed in (new) SuSpect ver $\geq$ 2.40

Bottom-up RG evolution with error propagations

par.	input(GeV)	GUT output	$\Delta M_3 = \mp 1\%$	$\Delta m_{H_u} = \mp 1\%$	$\Delta m_{Q3_L} = \mp 1\%$
M_1	101.5	250.004	negl.	negl.	negl.
M_2	191.6	249.998	" "	" "	" "
M_3	586.6	249.999	± 2.2	" "	" "
$m^2_{H_d}$	$(179.9)^2$	$(100.004)^2$	$(100.6)^2$– $(99.4)^2$	$(100.7)^2$– $(99.2)^2$	$(101.2)^2$– $(98.7)^2$
$m^2_{H_u}$	$-(358.1)^2$	$(100.017)^2$	$(132.6)^2$– $(48.4)^2$	$(64.9)^2$– $(124.4)^2$	$(63.7)^2$– $(126.4)^2$
(μ)	356.9	353			
m_{e_R}	136	99.998	100–99.9	98.4–101.6	96.8–103.1
m_{Q1_L}	545.8	100.001	121–72	99.7–100.3	99.1–100.8
m_{Q3_L}	497	100.005	131–52	94.6–104.6	55.2–130.4
m_{u_R}	527.8	99.997	121–72	101–99	101.8–98.1
m_{t_R}	421.5	100.006	140–14	90.6–107.5	81.9–115.3
m_{b_R}	522.4	99.997	122–72	99.4–100.6	98.5–101.5
$-A_t$	494.5	100.009	$111 - -89$	" "	" "
$-A_b$	795.2	100.002	$106 - -94$	" "	" "

From RG running masses to pole masses

Ideal picture: $M^{pole} = \bar{m}(Q)\left[1 + \sum_{i,p} c_{ip} g_i^p(Q)\right]|_{Q=M^{pole}}$

$m(Q)$ contains LL,NLL, etc; $g_i(Q)$ couplings and p pert. order

c_i finite (non-log) parts of loop self-energy contributions

in QCD: only one coupling g_S: in $\overline{MS}$ scheme, step fction

threshold corrections manageable in principle

But in MSSM many different mass scales, different couplings... + mixing -> ambiguous threshold scales

Not yet fully satisfactory treatment of threshold corrections

Different approaches exist, in Suspect practically:

-include finite "threshold" corrections (to masses, couplings) explicitly (at $Q = Q_{EWSB}$ (unique, $\neq M_i^{pole}$))

-Run full MSSM RGE (beyond (unique) scale $\sim Q_{EWSB}$)

-Improvements should be possible in principle.

9. Practical implementation of threshold corrections

"SM" input: $G_F, M_Z, \alpha_{\rm em}^{-1}(M_Z)^{\overline{MS}}, \alpha_S(M_Z)$

$m_z^2 = M_Z^2 + \mathrm{Re}\,\Pi_{ZZ}^T(M_Z^2) = \frac{1}{2}(g_1^2 + g_2^2)\,v^2$

(NB $m_W, \sin^2\theta_W$ derived from this)

$\tan\beta(m_Z) \equiv v_u/v_d, \qquad Y_i = \frac{m_i}{v\,\sin\beta}$

$\bar{m}_f = M_f + \Sigma_f^{\overline{DR}}(M_f)$

$\bar{m}_t = M_t + \Sigma_t(M_t) + (\Delta m_t)^{2-\mathrm{loop,\,QCD}}$

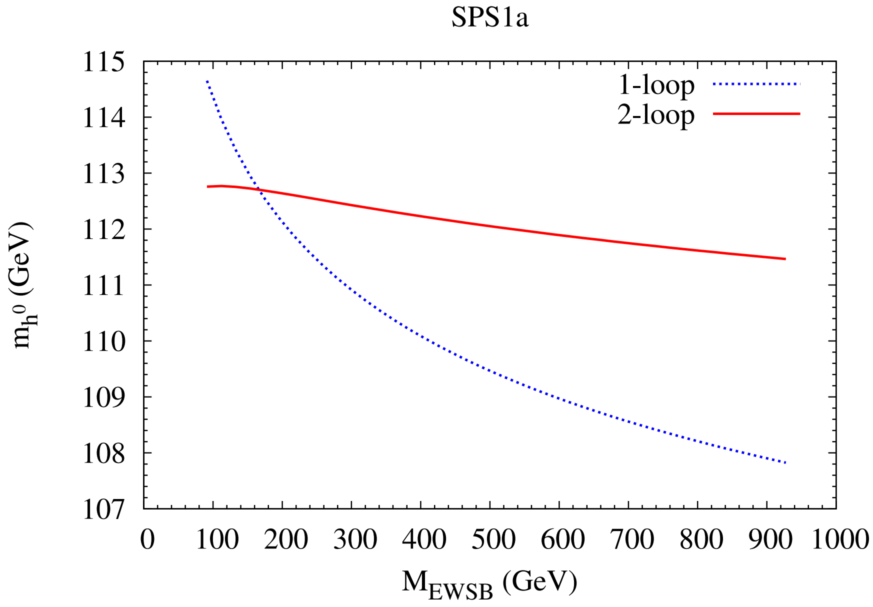
$\widehat{m}_b \equiv m_b(M_Z)^{\overline{DR}}_{\rm MSSM} = \frac{\bar{m}_b}{1-\Delta_b}$

Δ_b MSSM corrections (resums $(\alpha_S \tan\beta)^n$ dependence)

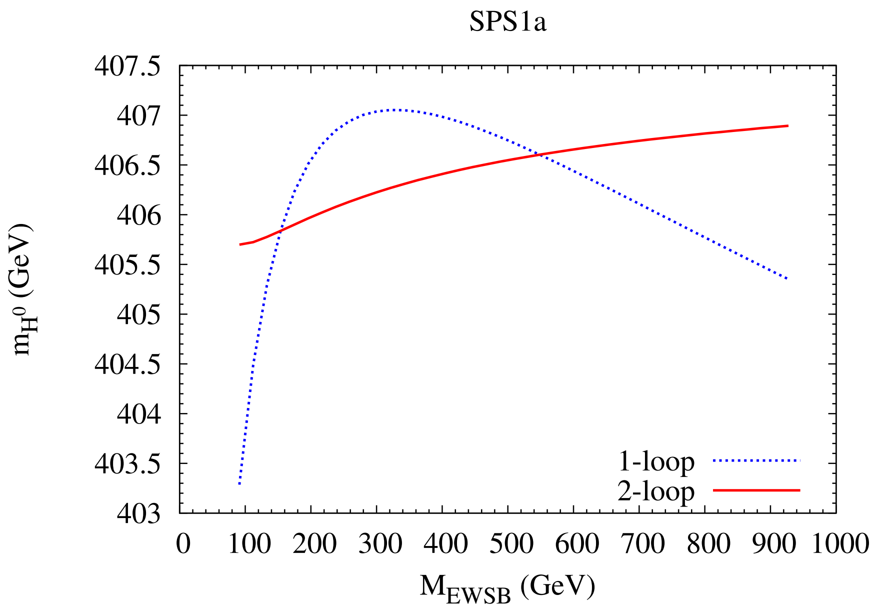
- All sparticles ($\neq$ Higgses):

$m_{\tilde{f},\tilde{\chi}} = M_{\tilde{f},\tilde{\chi}} + \Sigma_{\tilde{f},\tilde{\chi}}^{\text{full 1-loop},\overline{DR}}(Q_{EWSB})$

- Higgses: $M_{h,H,A}$ incl. $\Sigma_h^{\text{full 1-loop+lead 2-loop},\overline{DR}}(Q_{EWSB})$



m_h (above) and m_H (below) as a function of Q_{EWSB}

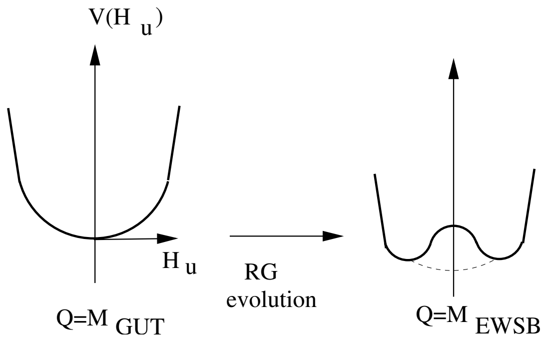


10. Electroweak sym. breaking issues in MSSM

Unescapable constraint: *consistent* electro-weak symmetry breaking (EWSB) $SU(2)_L \times U(1)_Y \to U(1)_{em}$

In MSSM: *produced* by RG evolution of $m^2_{H_u}(E)$, $m^2_{H_d}(E)$:

$\sim$ OK if $m^2_{H_u}(E) < 0$ by RG evolution $E_{GUT} \to E_{EWSB}$ ($\propto m_t^2$)



AND $|\mu|, B$ *determined* by minimization of the scalar potential:

$2\mu^2 = \tan(2\beta)(\hat{m}^2_{H_u} \tan\beta - \hat{m}^2_{H_d} cot\beta - M_Z^2)$

$2B\,\mu = \sin 2\beta\, (\hat{m}^2_{H_u} + \hat{m}^2_{H_d} + 2\mu^2)$

$\tan\beta \equiv \frac{v_u}{v_d}$, $\qquad \hat{m}^2_{H_i} = m^2_{H_i} + \partial_{v_i} V^{eff}_{loop}(m_{sparticles}, \mu)$

$\to$ needs iteration for μ (but converges fastly)

μ very sensitive to rad. corr., m_t,.. via RGE:

$$\frac{d\,(m^2_{H_u})}{d\,\ln E} \propto m_t^2(m^2_{H_u} + ...)$$

and $\mu^2 \sim -m^2_{H_u} - m^2_Z/2$ (for $\tan\beta \gg 1$),

•μ enters everywhere in MSSM spectrum:
Higgses, $\tilde{\chi}^{\pm}, \tilde{\chi}^0$ (via Higgsinos $\tilde{H}_u, \tilde{H}_d$), $\tilde{q}, \tilde{l}$ (via mixing)

Also: "CCB" minima (Charge and/or Color breaking)
deeper than electroweak min. can appear
(CCB domains to exclude e.g if trilin. cpling A_i too large)

Alternative input in general MSSM

In some case (e.g. general MSSM, non-universal Higgs model) convenient to use EWSB consistency constraint to change input/output parameters:

e.g. Very convenient: $m_A(pole)$ and $\mu(Q_{EWSB})$ as input:

Other relevant parameters determined as

$\bar{m}_A^2 = m_A^2(pole) + \Pi_{AA}$(1 loop) -tadpole +2-loop

$$m_{H_u}^2 = (\cos^2\beta - \sin^2\beta)\bar{m}_Z^2/2 + \cos^2\beta\bar{m}_A^2 - \mu^2 - \partial_{v_u^2}V_{loop}^{eff}$$
$$m_{H_d}^2 = (\sin^2\beta - \cos^2\beta)\bar{m}_Z^2/2 + \sin^2\beta\bar{m}_A^2 - \mu^2 - \partial_{v_d^2}V_{loop}^{eff}$$

NB also needs iteration since involves $V_{loop}^{eff}(m_{sparticles})$

Other possibilities: e.g. μ, $B\mu$ (GUT) input $\rightarrow$ determine $\tan\beta$