# The Renormalization Group and its applications for physics beyond the Standard Model

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# Outline

# Introduction

## The Standard Model and beyond

# The Standard Model and beyond

Despite the remarkable success of the Standard Model to describe fundamental particles and their interactions, some great questions remain unanswered

- Dark matter
- CP violation (What origin? Matter/antimatter asymmetry?)
- Tension for the muon's $g - 2$
- Gravity ?
- Neutrino masses
- Higgs vacuum meta-stability
- Tension for several flavor observables

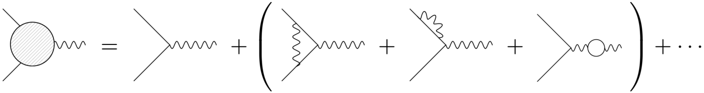
$\oplus$ Also a lot of «why»'s

- Naturalness (hierarchy & strong CP problems)
- Why 3 generations of fermions?
- Why $SU(3) \times SU(2) \times U(1)$ ?
- Origin of fermion mass patterns?
- More generally: is there a reason why the parameters of the SM take their observed values?

# The Renormalization Group

# Renormalization in perturbative QFT

- Physical quantities are computed according to a **perturbative expansion** (around the non-interacting theory)



- **Renormalization**: Get a well-behaved theory despite the presence of divergences
- **Regularization**:
  - Split divergences into a finite + infinite part
  - This decomposition is not unique: various **renormalization schemes**
  - Introduction of an arbitrary energy scale (UV cutoff $\Lambda$, renormalization scale $\mu$, ...)
- **Renormalization group equations**: Some quantities (observables, bare couplings) do not depend explicitly on $\mu$

$$\frac{d}{d\mu}\{\cdots\} = 0$$

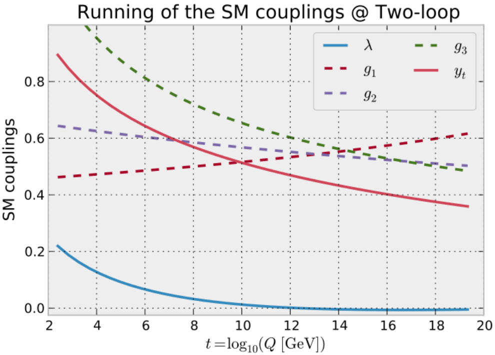
# Example: Running couplings in the Standard Model

- **Renormalization group equations**: Some quantities (observables, bare couplings) do not depend explicitly on $\mu$

$$\frac{d}{d\mu}\{\cdots\} = 0$$

- Direct consequence: the couplings of the theory acquire a dependence on $\mu$

$$\frac{dg}{d\log\mu} = \mu\frac{dg}{d\mu} \equiv \beta_g = \beta_g^{(1)} + \beta_g^{(2)} + \cdots \qquad \textbf{("Beta-function")}$$



# Example: Running couplings in the Standard Model

- Standard Model gauge group: $SU(3) \times SU(2) \times U(1)$
- Standard Model Lagrangian density:

$$\mathcal{L}_{\text{SM}} \supset \mathcal{L}_{\text{kinetic}} - y_t \bar{Q}_L \widetilde{\phi}\, t_R - \left[\mu\, \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2\right]$$

- $\beta$-functions for (some) Standard Model couplings at 1-loop in the $\overline{\text{MS}}$ scheme:

$$16\pi^2 \beta(g_1) = \frac{41}{6} g_1^3\,, \quad 16\pi^2 \beta^{(1)}(g_2) = -\frac{19}{6} g_2^3\,, \quad 16\pi^2 \beta^{(1)}(g_3) = -7 g_3^3$$

$$16\pi^2 \beta(y_t) = y_t \left(\frac{9}{2} y_t^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2\right)$$

$$16\pi^2 \beta(\lambda) = 24\lambda^2 + 12\lambda y_t^2 - 6 y_t^4 - (3 g_1^2 + 9 g_2^2)\lambda + \frac{3}{8} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{8} g_2^4$$

$$16\pi^2 \beta(\mu) = \left(12\lambda + 6 y_t^2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2\right) \mu$$

# RGEs for general (non-SUSY) gauge theories: a brief history

Two-loop results (80's):

- M.E. Machacek, M.T. Vaughn, Nucl. Phys. B222, 83 (**1983**)
- M.E. Machacek, M.T. Vaughn, Nucl. Phys. B236, 221 (**1984**)
- M.E. Machacek, M.T. Vaughn, Nucl. Phys. B249, 709 (**1985**)

Some corrections + extension to dimensionful couplings:

- M.-x. Luo, H.-w. Wang and Y. Xiao, Phys. Rev. D67 (**2003**)

Kinetic mixing:

- M. Luo, Y. Xiao, Phys. Lett. B 555, 279 (**2003**)
- R. M. Fonseca, M. Malinsky, F. Staub, Phys. Lett. B 726, 882 (**2013**)

} Implemented in SARAH and PyR@TE 2

Some corrections + correct treatment for off-diagonal W.F. renormalization:

- I. Schienbein, F. Staub, T. Steudtner, K. Svirina, Nucl. Phys. B939 (**2019**)

3-loop gauge coupling RGEs for theories based on a simple gauge group:

- A. Pickering, J. Gracey, D. Jones, Phys. Lett. B 510 (**2001**)

# RGEs for general (non-SUSY) gauge theories: recent developments

- [C. Poole, A. E. Thomsen, arXiv:1906.04625]
  - 3-loop general gauge coupling RGEs
  - Partial results at order 4-3-2 (+ full 4-loop gauge coupling RGEs in the SM)
  - Fixed the $\gamma_5$ ambiguity at order 4-3-2
- [LS, arXiv:2006.12307]
  - 2-loop RGEs for dimensionful couplings in the above new formalism

} PyR@TE 3

- [T. Steudtner, arXiv:2007.06591]
  - 3-loop RGEs in purely scalar fields theories
  - Partial 4-loop results
- [T. Steudtner, arXiv:2101.05823]
  - 3-loop RGEs in scalar-Yukawa theories (partial results)

# PyR@TE 3

Computing RGEs for general gauge theories

[LS & I. Schienbein, arXiv:2007.12700]

[LS, arXiv:2006.12307]

# PyR@TE 3: What's new

Compared to the previous version, the code was essentially rewritten (PyR@TE 2 was no longer maintained since 2017)

- Python 2 $\rightarrow$ Python 3
- Sympy 0.7 $\rightarrow$ Sympy 1.6
- Implementation of the above formalism
  $\rightarrow$ Gauge coupling RGEs up to 3-loop
- New model file syntax
- Some additional new features
- Performance drastically improved [$\mathcal{O}(100)$ to $\mathcal{O}(10\,000)$ times faster]

# Deriving the RGEs: General formalism

- Gauge group $\mathcal{G} = \prod \mathcal{G}_u$
- Weyl fermions $\psi_i \longrightarrow \Psi_i \equiv \begin{pmatrix} \psi \\ \psi^\dagger \end{pmatrix}_i$
- Real scalars $\phi_a$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\left(G^{-2}\right)_{AB} F^A_{\mu\nu} F^{B\,\mu\nu} + \frac{1}{2}(D_\mu\phi)_a (D^\mu\phi)_a + \frac{i}{2}\Psi^{\mathrm{T}} \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} D_\mu \Psi \\ &- \frac{1}{2} y_{aij}\,\Psi_i \Psi_j\,\phi_a - \frac{1}{2} m_{ij}\,\Psi_i\Psi_j \\ &- \frac{1}{2}\mu_{ab}\phi_a\phi_b - \frac{1}{3!} t_{abc}\phi_a\phi_b\phi_c - \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d\,.\end{aligned}$$

$$D_\mu \Psi_i = \partial_\mu \Psi_i - i \sum_A V^A_\mu \left(T^A_\Psi\right)_{ij} \Psi_j$$

$$D_\mu \phi_a = \partial_\mu \phi_a - i \sum_A V^A_\mu \left(T^A_\phi\right)_{ab} \phi_b$$

# Deriving the RGEs: General formalism

Perturbative expansion:

$$\text{Gauge:}\quad \beta_{AB} \equiv \frac{dG^2_{AB}}{dt} = \frac{1}{2}\sum_{\text{perm}}\sum_{\ell}\frac{1}{(4\pi)^{2\ell}}\,G^2_{AC}\,\beta^{(\ell)}_{CD}\,G^2_{DB}\,,$$

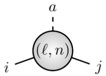
$$\text{Yukawa:}\quad \beta_{aij} \equiv \frac{dy_{aij}}{dt} = \frac{1}{2}\sum_{\text{perm}}\sum_{\ell}\frac{1}{(4\pi)^{2\ell}}\,\beta^{(\ell)}_{aij}\,,$$

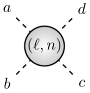
$$\text{Quartic:}\quad \beta_{abcd} \equiv \frac{d\lambda_{abcd}}{dt} = \frac{1}{4!}\sum_{\text{perm}}\sum_{\ell}\frac{1}{(4\pi)^{2\ell}}\,\beta^{(\ell)}_{abcd}$$

At fixed loop order, factorization of scheme dependence / model dependence:

$$\beta^{(\ell)}_{AB} = \sum_n \mathfrak{g}^{(\ell)}_n \; A$$


$$B\,,$$

$$\beta^{(\ell)}_{aij} = \sum_n \mathfrak{y}^{(\ell)}_n$$


, and $\beta^{(\ell)}_{abcd} = \sum_n \mathfrak{q}^{(\ell)}_n$



$$\beta^{(1)}_{abcd} = \mathfrak{q}^{(1)}_1\,(T^A_\phi T^C_\phi)_{ab}G^2_{AB}G^2_{CD}(T^B_\phi T^D_\phi)_{cd} \quad +\mathfrak{q}^{(1)}_2\,[C_2(S)]_{ae}\lambda_{ebcd} \quad +\mathfrak{q}^{(1)}_3\,\lambda_{abef}\lambda_{efcd}$$
$$+\mathfrak{q}^{(1)}_4\,[Y_2(S)]_{ae}\lambda_{ebcd} \quad +\mathfrak{q}^{(1)}_5\,\mathrm{Tr}[y_a\tilde{y}_b y_c\tilde{y}_d]$$

# PyR@TE 3: overview

**Input**

- Gauge group
- Particle content
- Lagrangian

↓

PyR@TE model file

►

**Mapping on the general formalism**

- Fill $\phi_a$ and $\Psi_i$
- Fill $G_{AB}$, $(T_\phi^A)_{ab}$ and $(T_\Psi^A)_{ij}$
- Expand Lagrangian

↓

Fill $y_{aij}$, $\lambda_{abcd}$, $t_{abc}$, $\mu_{ab}$, $m_{ij}$

►

**Compute the RGEs**

- Minimal set of linearly independent couplings

$$\tilde{\mathbf{g}} = M\mathbf{g}$$

- Compute the $\beta(\tilde{g}_i)$'s
- Infer $\beta(g_i) = (M^{-1})^{ij}\beta(\tilde{g}_j)$

**Output**

- LaTeX
- Mathematica (expressions + RGE solver)
- Python (expressions + RGE solver)

# Performance improvement compared to PyR@TE 2

| Model | Loop order | PyR@TE 2 | PyR@TE 3 |
|---|---|---|---|
| SM B-L | 1 | 114 | 1.5 |
| | 2 | 8823 | 11 |
| | 2 + 3 (gauge) | / | 23 |
| SM + complex triplet | 1 | 385 | 1.0 |
| | 2 | 59936 | 3.2 |
| | 2 + 3 (gauge) | / | 5.7 |
| SM + scalar singlet | 1 | 79 | 0.9 |
| | 2 | 5765 | 4.3 |
| | 2 + 3 (gauge) | / | 5.6 |
| SM + complex doublet | 1 | 153 | 1.2 |
| | 2 | 39666 | 6.2 |
| | 2 + 3 (gauge) | / | 9.4 |
| SM + Majorana triplet + Vectorlike doublet | 1 | 262 | 1.3 |
| | 2 | 15653 | 10.7 |
| | 2 + 3 (gauge) | / | 13.2 |

Execution time (in seconds)

# An asymptotically safe $SO(10)$ model

[A. Held, J. Kwapisz, LS – Work in progress]

# Asymptotic safety

- Originally a theory of quantum gravity (*non-perturbative renormalizability*)
- By extension, a theory of interacting gravity+matter
- **General idea:** All the couplings of the theory converge to a fixed point at (arbitrary) high energies
  - Quantum scale invariance
  - No Landau poles $\Rightarrow$ UV completion
- In the SM, asymptotic safety gives:
  - A Higgs mass of $\approx 126\,\mathrm{GeV}$ [M. Shaposhnikov, C. Wetterich, arXiv:0912.0208]
  - A top quark mass of $\approx 171\,\mathrm{GeV}$ [A. Eichhorn, A. Held, arXiv:1707.01107]
  - A possible explanation to the fermion mass hierarchy [A. Eichhorn, A. Held *et al.*, arXiv:2003.08401]

# Asymptotic safety – Gravitational contributions to the $\beta$-functions

- The impact of gravity on the RG flow of the (marginal) matter couplings is given by:

$$\beta(g_i) \longrightarrow \beta(g_i) = -f_g g_i + \beta(g_i)$$

[Below $M_P$] $$\beta(y_i) \longrightarrow \beta(y_i) = -f_y y_i + \beta(y_i)$$ [Above $M_P$]

$$\beta(\lambda_i) \longrightarrow \beta(\lambda_i) = f_\lambda \lambda_i + \beta(\lambda_i)$$

- All studies to date indicate that $f_g, f_y, f_\lambda > 0$
- Example in the SM (at 1-loop):

$$\beta_{g_1} = \frac{41}{6}\frac{g_1^3}{16\pi^2} \quad \longrightarrow \quad \beta_{g_1} = -f_g g_1 + \frac{41}{6}\frac{g_1^3}{16\pi^2}$$

Fixed points:

$$\beta_{g_1}(g_1^*) = 0 \quad \Rightarrow \quad g_1^* = 0 \quad \text{or} \quad g_1^* = 4\pi\sqrt{\frac{6 f_g}{41}}$$

# Asymptotic safety – Example: Gauge couplings in the SM

$$\beta_{g_1} = -f_g g_1 + \frac{41}{6}\frac{g_1^3}{16\pi^2} \quad \Rightarrow \quad g_1^* = 4\pi\sqrt{\frac{6f_g}{41}}$$

$$\beta_{g_2} = -f_g g_2 - \frac{19}{6}\frac{g_2^3}{16\pi^2} \quad \Rightarrow \quad g_2^* = 0$$

$$\beta_{g_3} = -f_g g_3 - 7\frac{g_3^3}{16\pi^2} \quad \Rightarrow \quad g_3^* = 0$$

($g_2^* = 0$, $g_3^* = 0$:) Asymptotic freedom

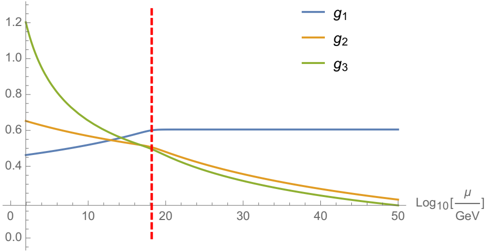



Figure: Asymptotically safe gauge couplings in the SM.

$f_g \approx .01$ matches the IR value for $g_1$

# Grand Unified Theories

- Very appealing UV-completions of the Standard Model
- First GUTs were built in the context of SUSY
- **Basic idea:** The three known interactions are unified at high energies

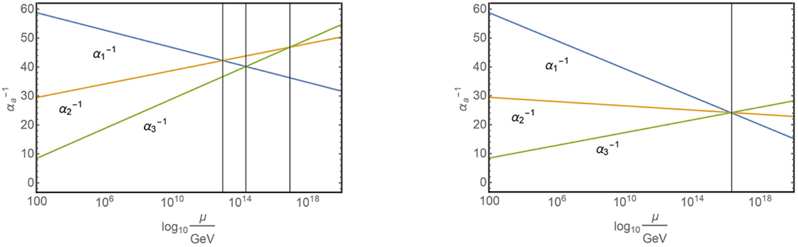



Figure: Running gauge couplings in the SM (left) and the MSSM (right)

[D. Croon *et al.*, arXiv:1903.04977]

# Grand Unified Theories – $SO(10)$

- In the language of gauge theories, the SM gauge group $SU(3) \times SU(2) \times U(1)$ is the low energy residual symmetry of a broken, larger simple gauge group
- Most promising candidates: $SU(5)$, $SO(10)$, $E_6$
- Here we focus on $SO(10)$:
  - All fermions (+ right-handed neutrino) are gathered in a single 16-dimensional multiplet
  - Anomaly-free
  - Can generate the observed fermion mass patterns
  - Provides a mechanism to generate neutrino masses
  - "Unification" of Left and Right chiralities
- **Many** viable $SO(10)$ models can be constructed depending on the scalar sector

$$\text{Scalars} \in \left\{\mathbf{10}, \mathbf{16}, \mathbf{45}, \mathbf{54}, \mathbf{120}, \mathbf{126}, \mathbf{144}, \mathbf{210}, \ldots\right\}$$

*Couples to the fermions

- Two main constraints:
  - Achieve symmetry breaking towards $SU(3) \times SU(2) \times U(1)$ (see next slide)
  - Reproduce the SM fermion mass & mixing matrices (= at least 2 **red scalars**)

# Grand Unified Theories – $SO(10)$ symmetry breaking

- The scalar content determines in which directions $SO(10)$ **can** break
- The value of the couplings in the scalar potential determines in which direction $SO(10)$ **breaks**
- The breaking chain influences (generates) the phenomenology of the model

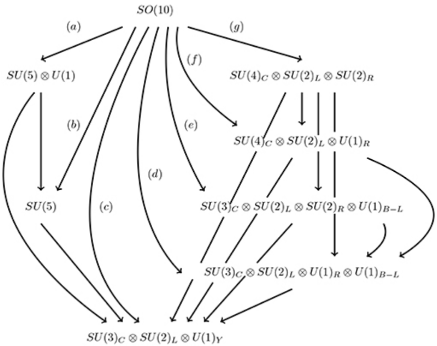



Figure: Breaking chains of $SO(10)$ down to the Standard Model

[D. Croon *et al.*, arXiv:1903.04977]

# Grand Unified Theories – How to make predictions ?

- In the coarsest approximation, the gauge and Yukawa sectors can be studied independently of the precise form of the scalar potential.
- In any other case, the scalar potential must be studied in order to make reliable predictions.
- However, with scalars as big as $\mathbf{120}$ or $\mathbf{126}$ the scalar potential gets really involved. In particular, there are a lot of free parameters for very few input...

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... Asymptotic safety comes into play:

- Requiring the existence of a fixed point at high energies (above $M_p$) can **fix the value of all the quartic couplings**
- The model chosen for this analysis contains:

$$\text{Fermions}: \ \mathbf{16}, \qquad \text{Scalars}: \ \mathbf{10} \oplus \mathbf{45} \oplus \mathbf{126}$$

- It is sometimes considered in the literature as the *minimal* non-SUSY $SO(10)$ model
- We're interested in whether asymptotic safety can provide the following information:
  - The direction of the symmetry breaking (*i.e.* the breaking pattern)
  - If possible, the scale of symmetry breaking

# An asymptotically safe $SO(10)$ model

Current status of the project:

1. The $\beta$-functions for this model were computed using a modified version of PyR@TE 3 (that will be made public in the future)
2. A code was developpped to determine the breaking chain once the numerical values for the couplings are known
3. The fixed point analysis (in the 1-fermion generation approximation) revealed very interesting features:

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3. The fixed point analysis (in the 1-fermion generation approximation) revealed very interesting features:

- The couplings involving scalars which do not couple do fermions are asymptotically free (operators $\mathbf{45}^4$, $\mathbf{45}^2\mathbf{10}^2$ and $\mathbf{45}^2\mathbf{126}^2$)
- In particular, "portal" couplings between $\mathbf{45}$ and the other scalars are $\sim 0$, and the $\mathbf{45}$ effectively decouples from the rest of the scalar sector. This allows for:
  - A clear hierarchy between the breaking scales
  - An independent study of its scalar potential and of the first breaking step
- The first step of symmetry breaking is entirely driven by the $\mathbf{45}$ (and is of Coleman-Weinberg type, *i.e.* driven by the quantum corrections to the scalar potential)
- **This is the expected behavior to break down to the SM !**

# Outlook

- An asymptotically safe $SO(10)$ model:
  - First exciting results
  - A next step is to further study the implications of asymptotic safety on the subsequent breaking steps
  - In the future, study the phenomenological implications (gauge coupling unification, scalar and fermion spectrum, proton decay...)
- PyR@TE 3:
  - New features currently in development
  - Higher order results for the $\beta$-functions expected to come soon. They will be implemented as soon as possible.
- Another ongoing project: the Multiple Point Principle in the Two Higgs Doublet model
  - This is another kind of high-scale principle, providing boundary conditions near the Planck scale
  - In the SM, it predicts (back in 1995) a Higgs boson mass of $\approx 135\,\mathrm{GeV}$
  - We're studying the implications of this principle in the THDM
  - [M. Maniatis, LS, I. Schienbein, arXiv:2001.10541]
  - [B. Herrmann, M. Maniatis, LS, I. Schienbein – Work in progress]

Thank you for your attention !

Any questions ?