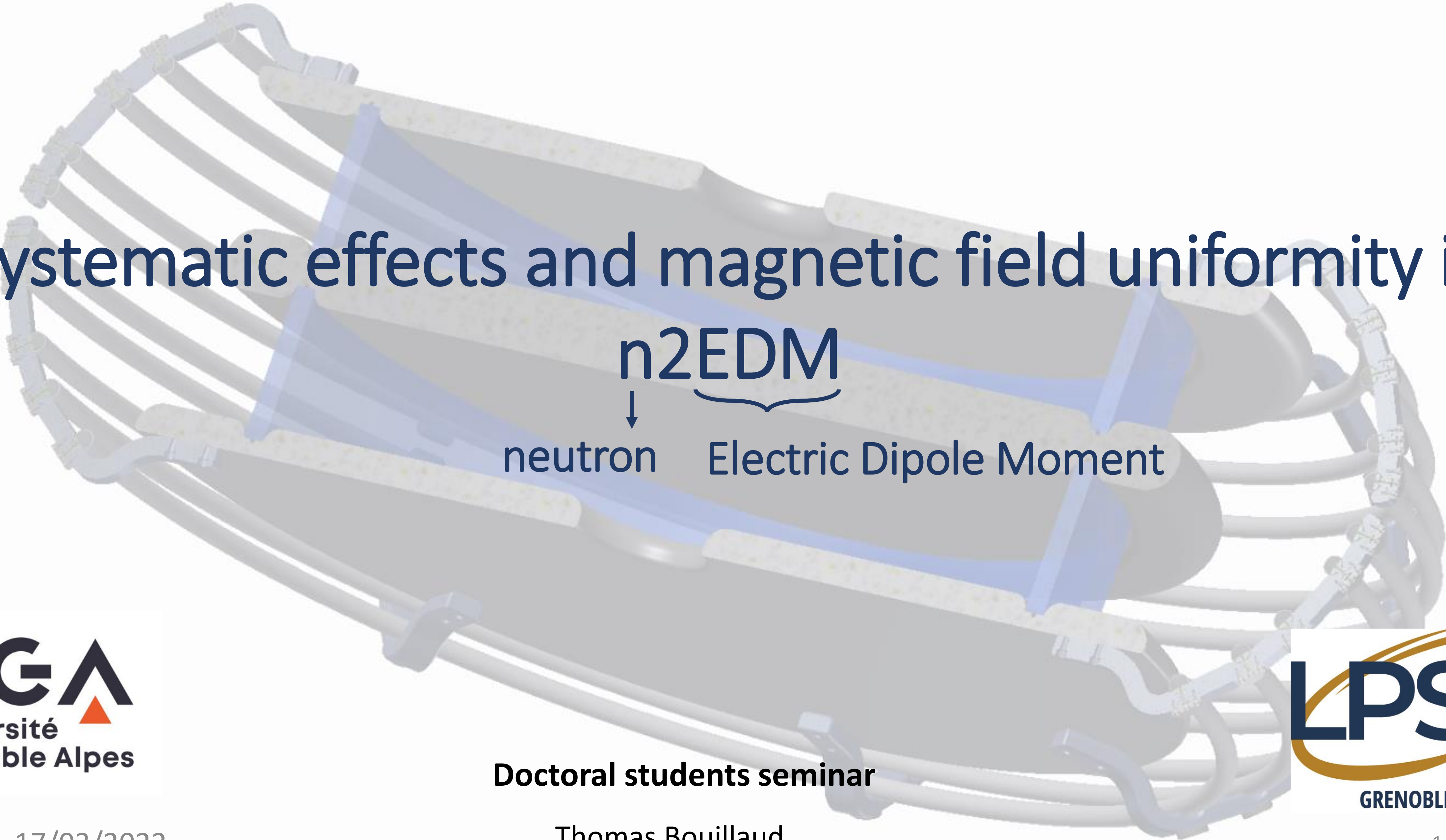


Systematic effects and magnetic field uniformity in n2EDM

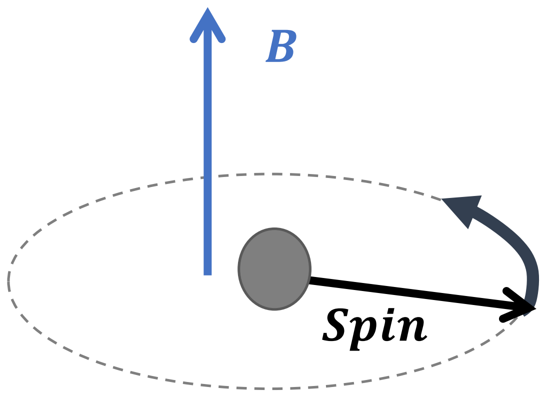
neutron — Electric Dipole Moment

Doctoral students seminar

Thomas Bouillaud

17/03/2022

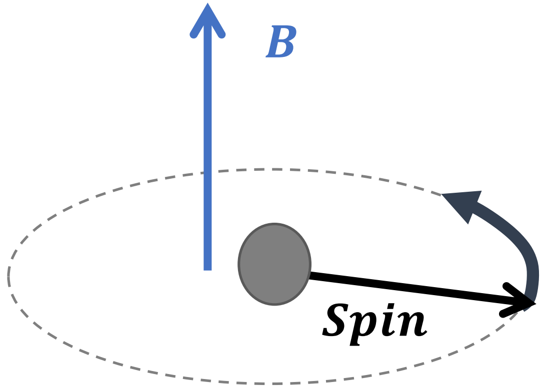
What is the electric dipole moment?



Spin ½ particle in a magnetic field

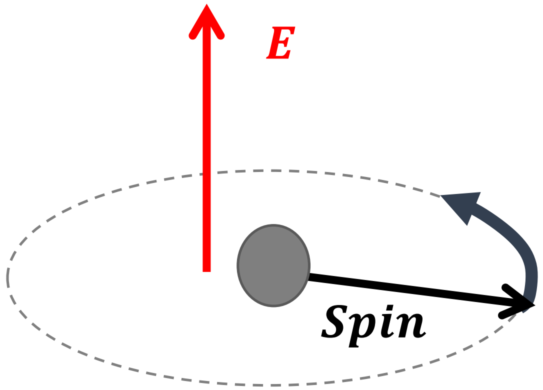
- $H = -\mu\, \boldsymbol{\sigma} \cdot \boldsymbol{B}$
- With $\boldsymbol{B} = B_0 \boldsymbol{u_z}$, precession frequency given by $\hbar 2\pi f = 2\mu\, B_0$
- Neutron in $B_0 = 1\ \mu\mathrm{T}$ → $f \approx 30\ s^{-1}$

What is the electric dipole moment?



Spin ½ particle in a magnetic field

- $H = -\mu\, \boldsymbol{\sigma} \cdot \boldsymbol{B}$
- With $\boldsymbol{B} = B_0 \boldsymbol{u_z}$, precession frequency given by $\hbar 2\pi f = 2\mu\, B_0$
- Neutron in $B_0 = 1\ \mu\mathrm{T}$ → $f \approx 30\ s^{-1}$



What about a spin ½ particle in an electric field?

- $H = -d\, \boldsymbol{\sigma} \cdot \boldsymbol{E}$
- With $\boldsymbol{E} = E_0 \boldsymbol{u_z}$, precession frequency given by $\hbar 2\pi f = 2d\, E_0$
- Neutron in $E = 1\ k\mathrm{V/cm}$ → $f < 4\ \mathrm{year}^{-1}$ (according to the current nEDM limit)

  ↔ $\boldsymbol{d_n}$ almost zero

Why measure the neutron EDM?

Cosmological motivation: explain baryon asymmetry $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10}$

Sakharov conditions for baryogenesis:

1. Non-conservation of baryonic number
2. Out-of-equilibrium thermal interactions
3. **Violation of C and CP symmetries**

Why measure the neutron EDM?

Cosmological motivation: explain baryon asymmetry $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10}$

Sakharov conditions for baryogenesis:

1. Non-conservation of baryonic number
2. Out-of-equilibrium thermal interactions
3. **Violation of C and CP symmetries**

EDMs are described by couplings that violate CP!

Violates T → violates CP by CPT

Formally: CP violating term (EM field and quark coupling)

$$\mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\,F^{\mu\nu} - \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\,F^{\mu\nu} \quad \xrightarrow{CP} \quad \mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\,F^{\mu\nu} + \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\,F^{\mu\nu}$$

$$H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} - d\,\boldsymbol{\sigma}\cdot\boldsymbol{E} \qquad\qquad H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} + d\,\boldsymbol{\sigma}\cdot\boldsymbol{E}$$

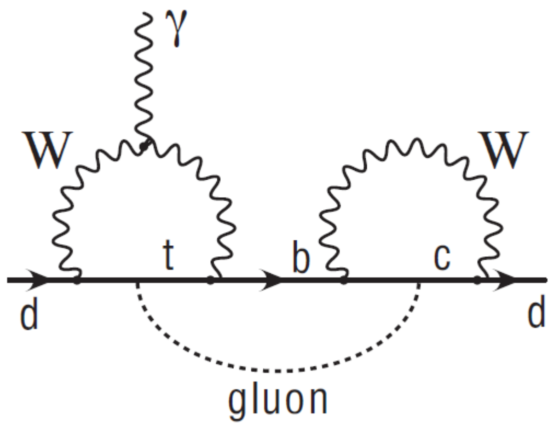
Formally: CP violating term (EM field and quark coupling)

$$\mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\,F^{\mu\nu} - \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\,F^{\mu\nu} \quad \xrightarrow{CP} \quad \mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\,F^{\mu\nu} + \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\,F^{\mu\nu}$$

$$H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} - d\,\boldsymbol{\sigma}\cdot\boldsymbol{E} \qquad\qquad H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} + d\,\boldsymbol{\sigma}\cdot\boldsymbol{E}$$

In the Standard Model: ☹

- CKM phase contribution to quark EDMs through at least 3 loops diagrams → very negligible ($d_n \sim 10^{-32} e.\mathrm{cm}$).

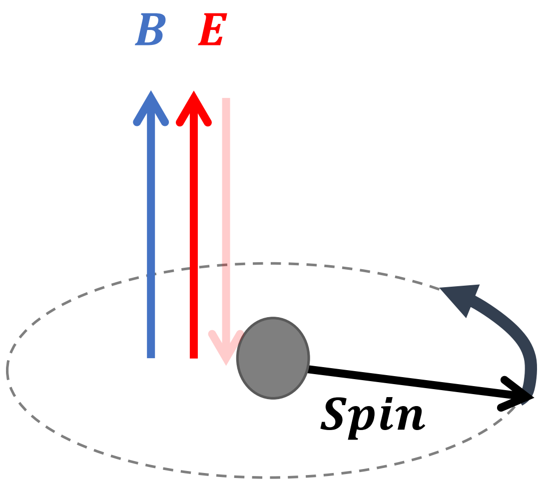


- QCD contribution $\frac{\alpha}{8\pi}\bar{\theta}\,G^{\mu\nu}\widetilde{G_{\mu\nu}}$ should generate huge EDMs ($d_n \sim 10^{-16} e.\mathrm{cm}$). : **current limit $d_n < 10^{-26} e.cm$** $\Rightarrow \bar{\theta} < 10^{-10}$(strong CP problem).

Beyond the SM:

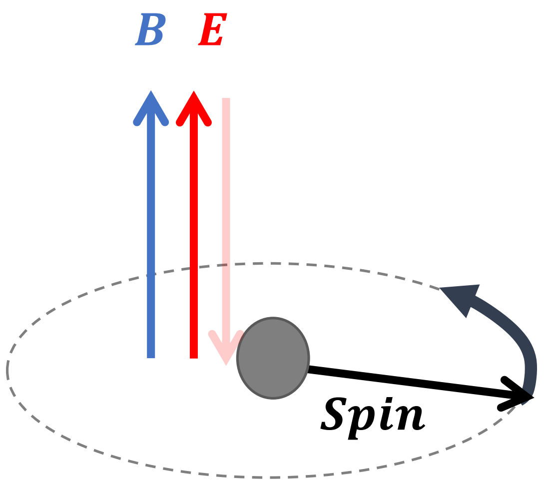
- (ex) modified Higgs-fermion Yukawa coupling $\mathcal{L} = -\frac{y_f}{\sqrt{2}}\left(\kappa_f \bar{f} f h + i\tilde{\kappa}_f\,\bar{f}\gamma_5 f h\right)$ generates EDM at 2 loops.

How do we measure the neutron EDM?



$$2\pi f = \frac{2\mu}{\hbar} B \pm \frac{2d}{\hbar} |E| \quad \Rightarrow \quad f(\uparrow\uparrow) - f(\uparrow\downarrow) = -\frac{2}{\pi\hbar} d\,|E|$$

How do we measure the neutron EDM?



$$2\pi f = \frac{2\mu}{\hbar} B \pm \frac{2d}{\hbar} |E| \quad \Rightarrow \quad f(\uparrow\uparrow) - f(\uparrow\downarrow) = -\frac{2}{\pi\hbar} d\,|E|$$

$f_{\text{larmor}} \approx 30\ s^{-1}$

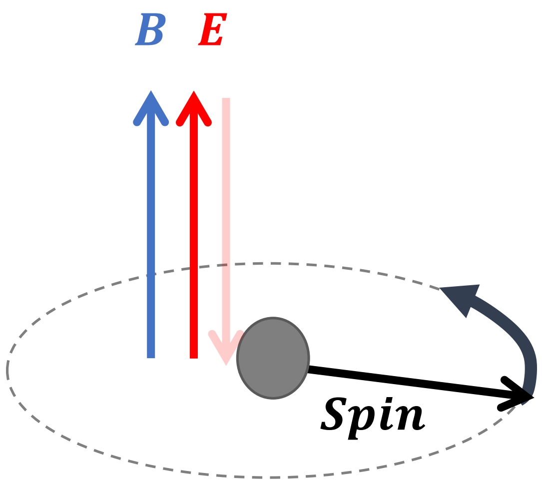
at $B = 1\mu\text{T}$

If $d_n = 10^{-26} e.\text{cm}$:

$f_{\text{elec}} \approx 2\ \text{year}^{-1}$

at $E = 15\ k\text{V}.\text{cm}^{-1}$

How do we measure the neutron EDM?



$$2\pi f = \frac{2\mu}{\hbar} B \pm \frac{2d}{\hbar} |E| \quad \Rightarrow \quad f(\uparrow\uparrow) - f(\uparrow\downarrow) = -\frac{2}{\pi\hbar} d\, |E|$$

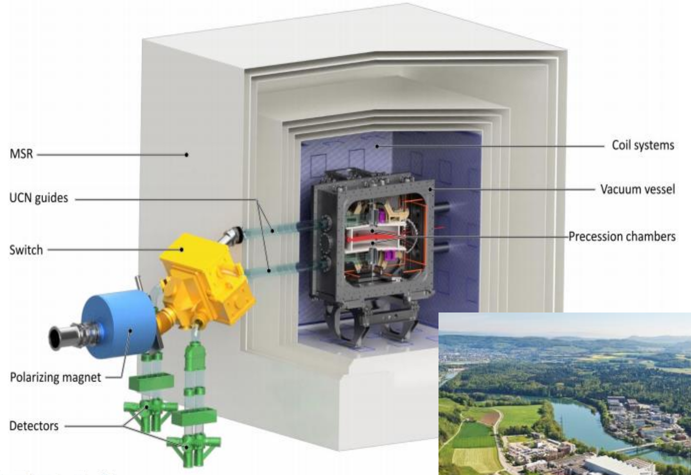
$f_{\mathrm{larmor}} \approx 30\ s^{-1}$

at $B = 1\mu\mathrm{T}$

If $d_n = 10^{-26} e.\mathrm{cm}$:

$f_{\mathrm{elec}} \approx 2\ \mathrm{year}^{-1}$

at $E = 15\ k\mathrm{V.cm}^{-1}$

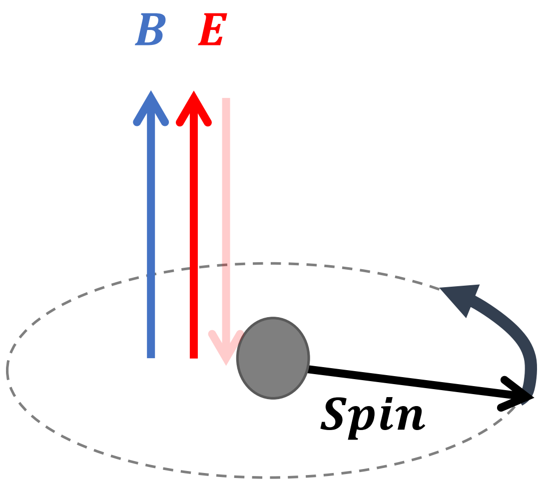


What can we do to detect something that small ?

- Maximize the interaction time → Ultra Cold Neutrons
- Maximize the statistics → Large cell volume, efficient UCN transport
- Control the magnetic field → Hg co-magnetometry, magnetic shielding (MSR, AMS), field mapping

→ Deal with systematics: false EDM, gravitational shift

How do we measure the neutron EDM?



$$2\pi f = \frac{2\mu}{\hbar} B \pm \frac{2d}{\hbar} |E| \Rightarrow f(\uparrow\uparrow) - f(\uparrow\downarrow) = -\frac{2}{\pi\hbar} d\,|E|$$

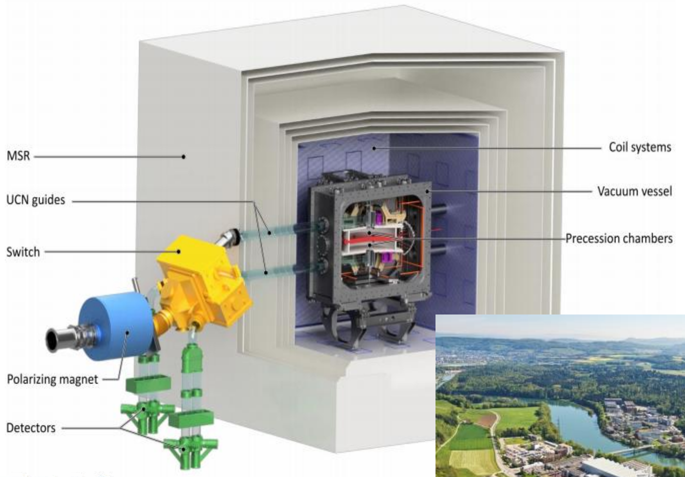
$f_{\text{larmor}} \approx 30\ s^{-1}$

at $B = 1\mu\text{T}$

If $d_n = 10^{-26} e.\text{cm}$:

$f_{\text{elec}} \approx 2\ \text{year}^{-1}$

at $E = 15\ k\text{V}.\text{cm}^{-1}$



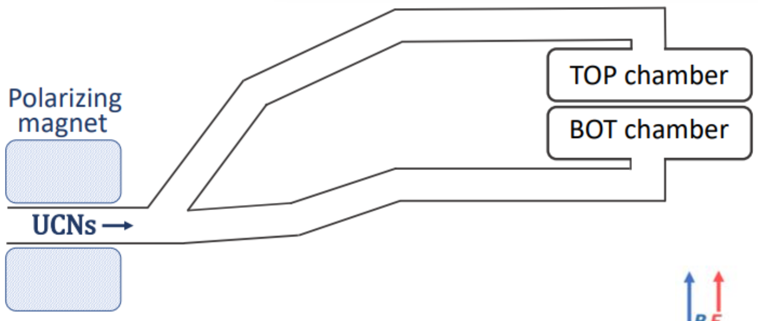
n2EDM

What can we do to detect something that small ?

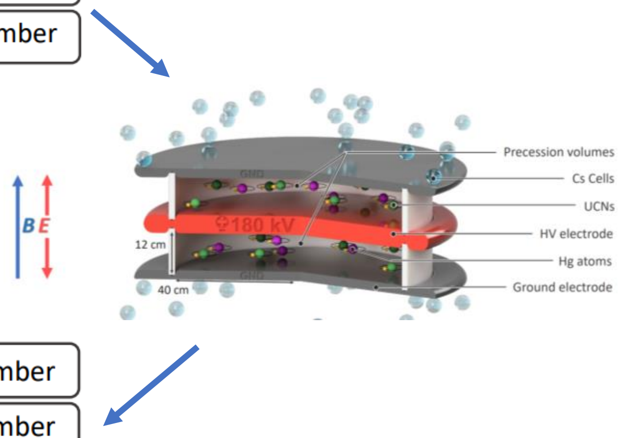
- Maximize the interaction time → Ultra Cold Neutrons
- Maximize the statistics → Large cell volume, efficient UCN transport
- **Control the magnetic field** → Hg co-magnetometry, magnetic shielding (MSR, AMS), field mapping, …

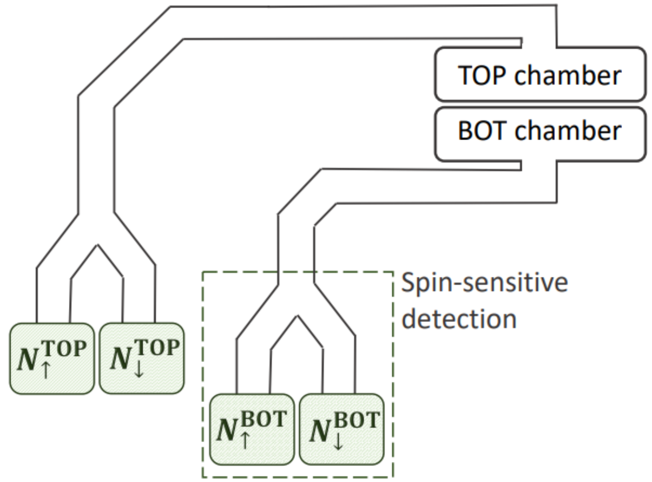
→ Deal with systematics: false EDM, gravitational shift, …

Counting spins with the Ramsey method

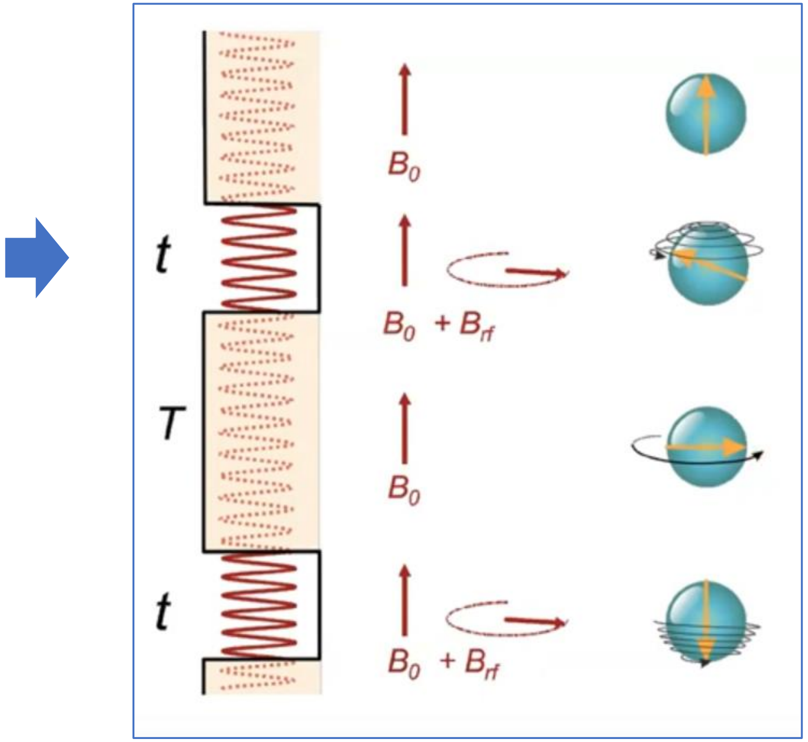


(a) Polarized UCNs fill the precession chambers

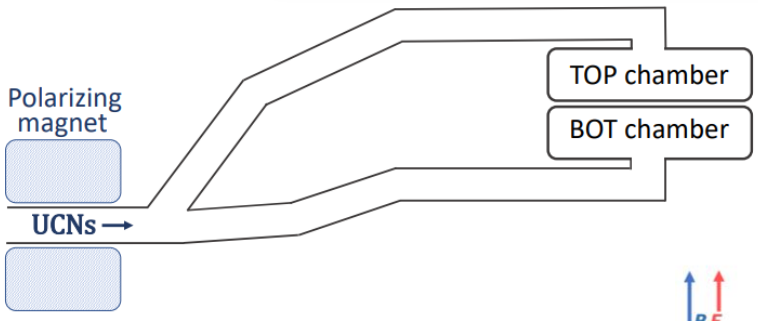




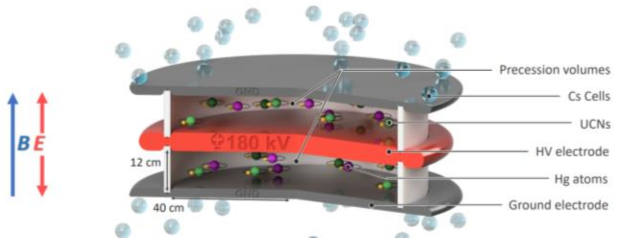
(b) UCNs are guided towards the spin-sensitive detectors.

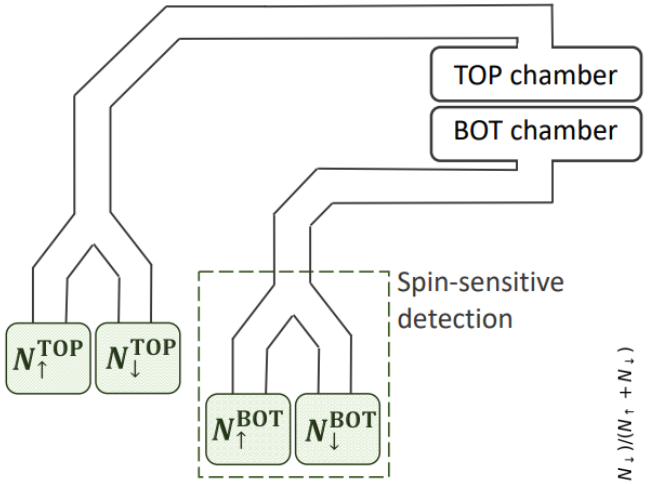


Counting spins with the Ramsey method

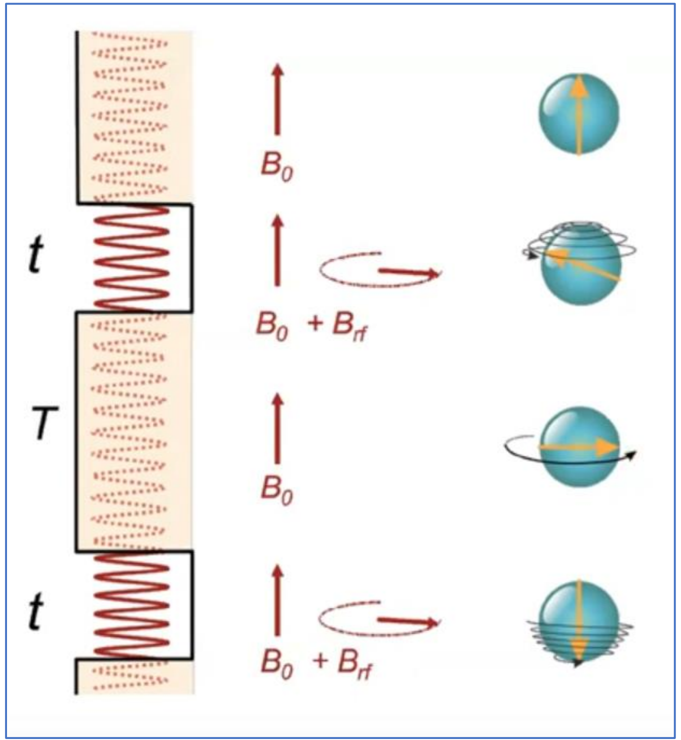


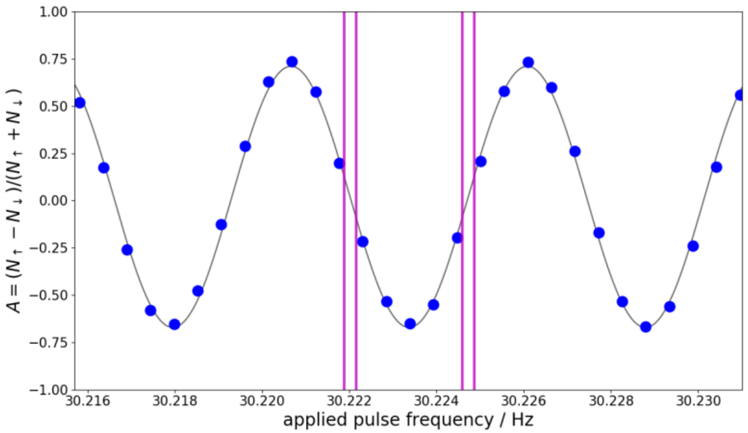
(a) Polarized UCNs fill the precession chambers





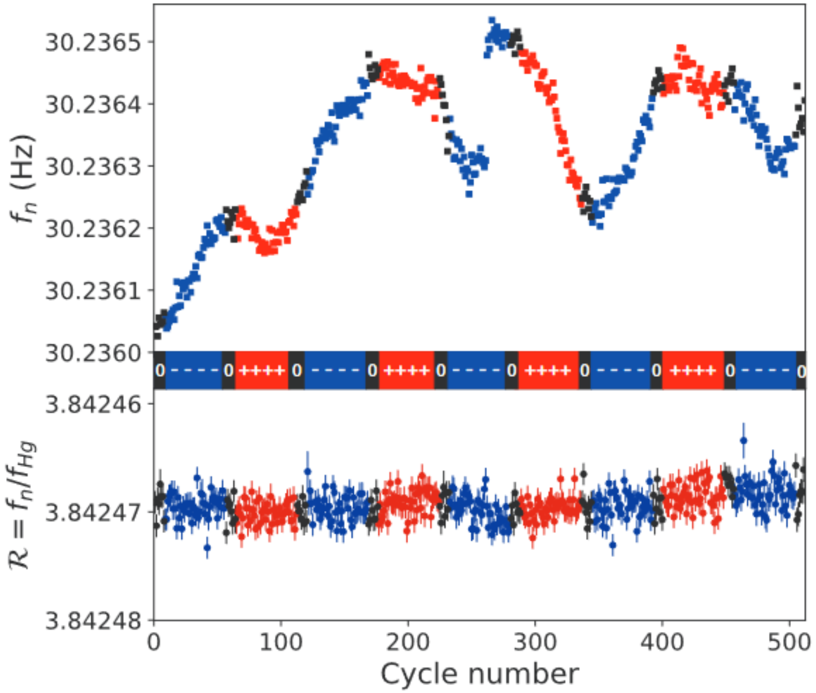
(b) UCNs are guided towards the spin-sensitive detectors.

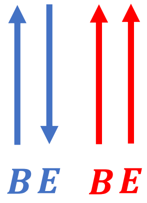




Up-down spin asymmetry A → precession frequency f

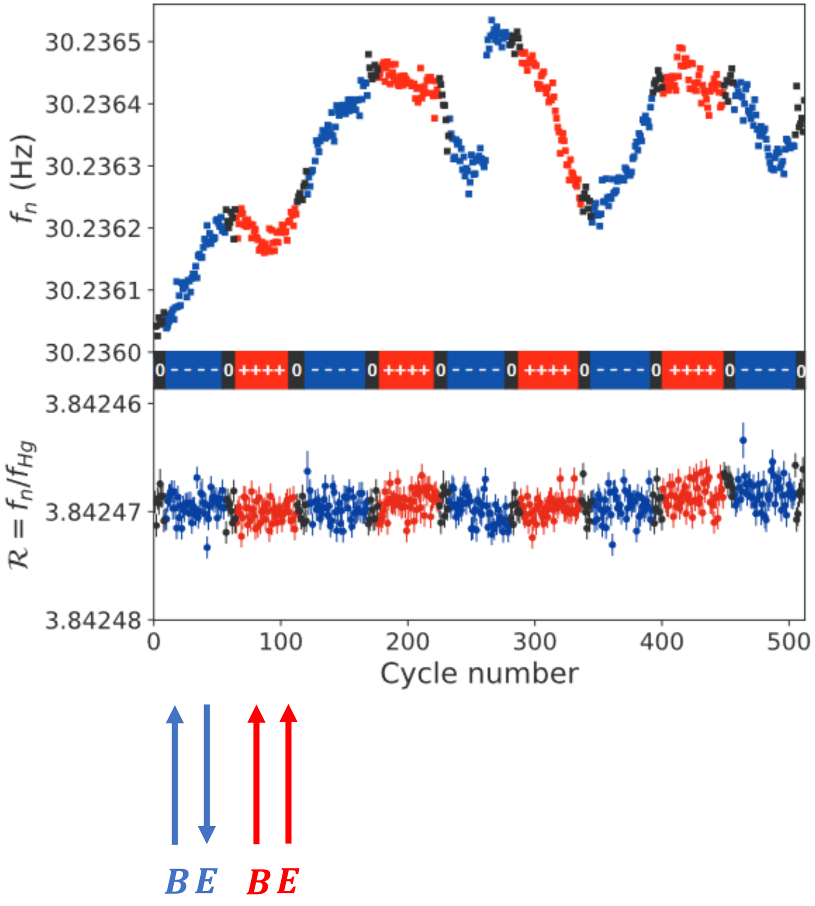
Hg co-magnetometry to compensate magnetic field fluctuations





Problem:
Uncertainty on f dominated by magnetic field fluctuations!

Hg co-magnetometry to compensate magnetic field fluctuations



Problem:
Uncertainty on f dominated by magnetic field fluctuations!

Solution:
Measure instead the ratio of mercury and neutron frequencies:

$$\mathcal{R} = \frac{f_n}{f_{Hg}} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \mp \frac{|E|}{\pi\hbar f_{Hg}} d_n$$

Contribution from EDM

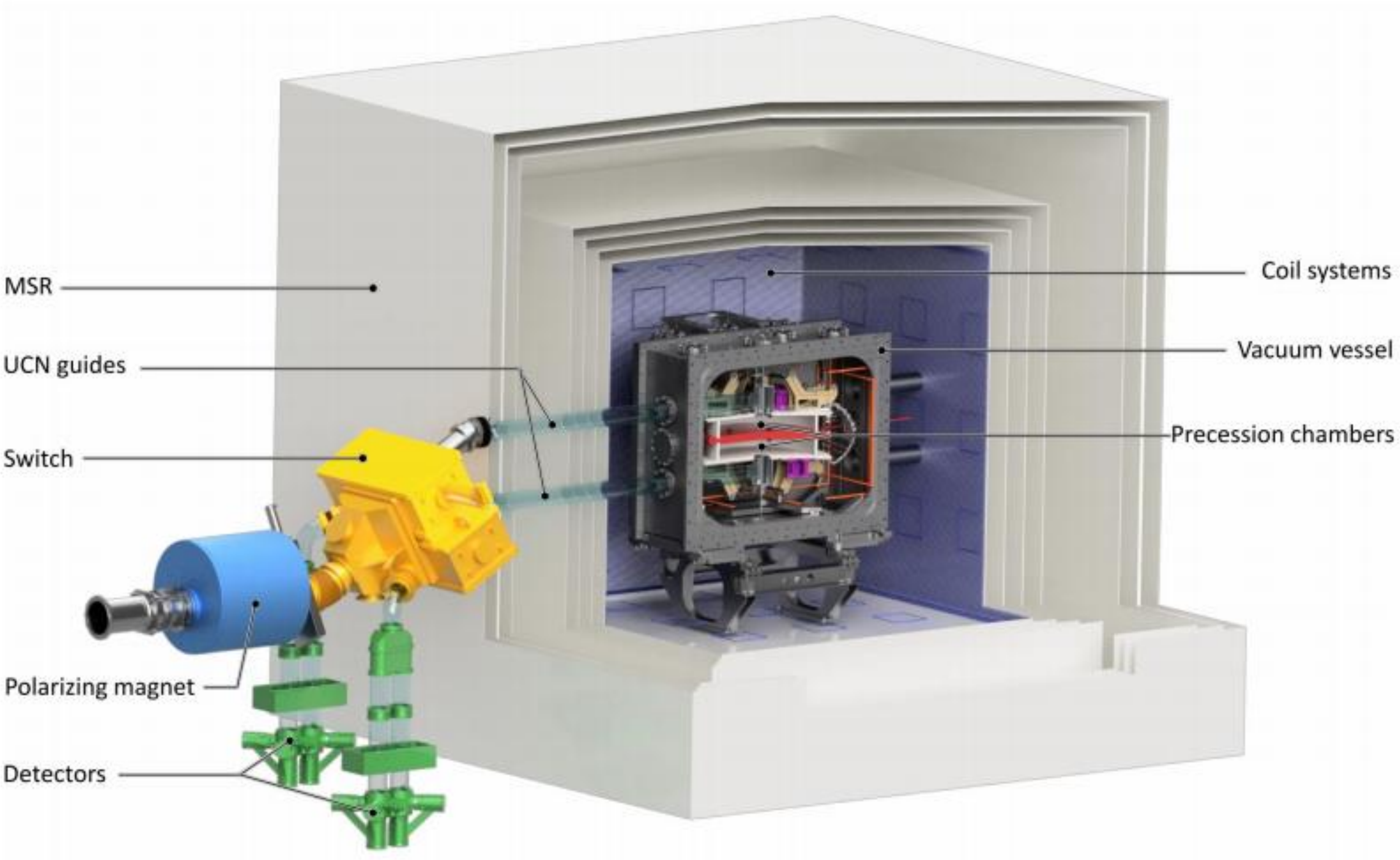
$$f_n = \left|\frac{\gamma_n}{2\pi}\right| B_0 \mp \frac{d_n}{\pi\hbar}|E|$$

No contribution from EDM!

$$f_{Hg} = \left|\frac{\gamma_{Hg}}{2\pi}\right| B_0$$

…which is free from the magnetic field fluctuations!

Overview of n2EDM



How do we parametrize the magnetic field?

Polynomial field expansion

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} G_{lm} \boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$$

How do we parametrize the magnetic field?

Polynomial field expansion

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} G_{lm} \boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$$

+

Maxwell's equations

$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{0}$

↓

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla}\Sigma(\boldsymbol{r})$$

with

$$\Delta\Sigma(r, \varphi, \theta) = 0$$

Laplace equation in spherical coordinates

How do we parametrize the magnetic field?

Polynomial field expansion

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} G_{lm} \boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$$

+

Maxwell's equations
$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{0}$

↓

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla}\Sigma(\boldsymbol{r})$$

with

$$\Delta\Sigma(r, \varphi, \theta) = 0$$

Laplace equation in spherical coordinates

→

Harmonic modes $\boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$ deduced from solutions of Laplace equation

TABLE IV. The basis of harmonic polynomials sorted by degree in cylindrical coordinates.

l	m	Π_ρ	Π_ϕ	Π_z
0	−1	$\sin\phi$	$\cos\phi$	0
0	0	0	0	1
0	1	$\cos\phi$	$-\sin\phi$	0
1	−2	$\rho\sin 2\phi$	$\rho\cos 2\phi$	0
1	−1	$z\sin\phi$	$z\cos\phi$	$\rho\sin\phi$
1	0	$-\frac{1}{2}\rho$	0	z
1	1	$z\cos\phi$	$-z\sin\phi$	$\rho\cos\phi$
1	2	$\rho\cos 2\phi$	$-\rho\sin 2\phi$	0
2	−3	$\rho^2\sin 3\phi$	$\rho^2\cos 3\phi$	0
2	−2	$2\rho z\sin 2\phi$	$2\rho z\cos 2\phi$	$\rho^2\sin 2\phi$
2	−1	$\frac{1}{4}(4z^2 - 3\rho^2)\sin\phi$	$\frac{1}{4}(4z^2 - \rho^2)\cos\phi$	$2\rho z\sin\phi$
2	0	$-\rho z$	0	$-\frac{1}{2}\rho^2 + z^2$
2	1	$\frac{1}{4}(4z^2 - 3\rho^2)\cos\phi$	$\frac{1}{4}(\rho^2 - 4z^2)\sin\phi$	$2\rho z\cos\phi$
2	2	$2\rho z\cos 2\phi$	$-2\rho z\sin 2\phi$	$\rho^2\cos 2\phi$
2	3	$\rho^2\cos 3\phi$	$-\rho^2\sin 3\phi$	0

How do we parametrize the magnetic field?

Polynomial field expansion

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} G_{lm} \boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$$

+

Maxwell's equations
$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{0}$

↓

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla}\Sigma(\boldsymbol{r})$$

with

$$\Delta\Sigma(r, \varphi, \theta) = 0$$

Laplace equation in spherical coordinates

→

Harmonic modes $\boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$ deduced from solutions of Laplace equation

TABLE IV. The basis of harmonic polynomials sorted by degree in cylindrical coordinates.

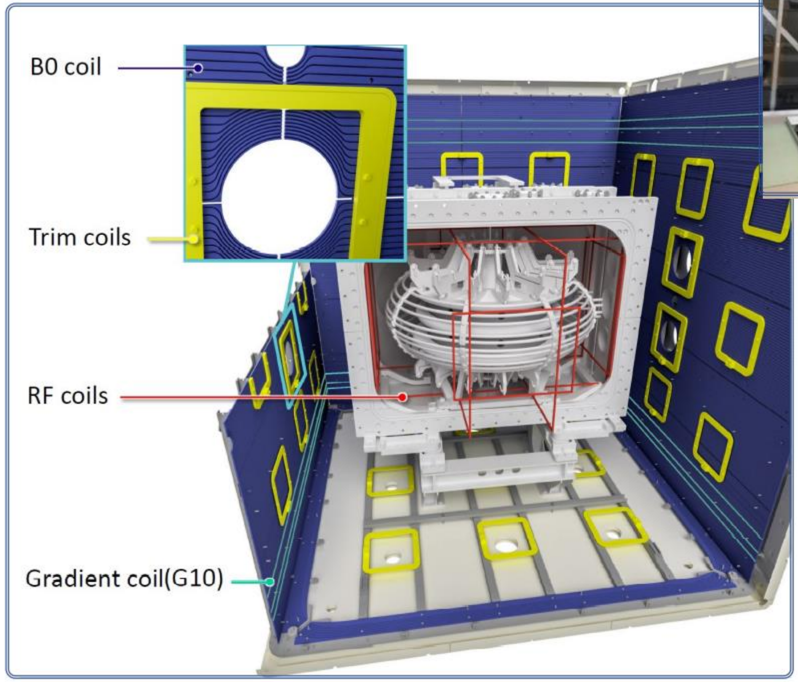
l	m	Π_ρ	Π_ϕ	Π_z
0	−1	$\sin\phi$	$\cos\phi$	0
0	0	0	0	1
0	1	$\cos\phi$	$-\sin\phi$	0
1	−2	$\rho\sin 2\phi$	$\rho\cos 2\phi$	0
1	−1	$z\sin\phi$	$z\cos\phi$	$\rho\sin\phi$
1	0	$-\frac{1}{2}\rho$	0	z
1	1	$z\cos\phi$	$-z\sin\phi$	$\rho\cos\phi$
1	2	$\rho\cos 2\phi$	$-\rho\sin 2\phi$	0
2	−3	$\rho^2\sin 3\phi$	$\rho^2\cos 3\phi$	0
2	−2	$2\rho z\sin 2\phi$	$2\rho z\cos 2\phi$	$\rho^2\sin 2\phi$
2	−1	$\frac{1}{4}(4z^2-3\rho^2)\sin\phi$	$\frac{1}{4}(4z^2-\rho^2)\cos\phi$	$2\rho z\sin\phi$
2	0	$-\rho z$	0	$-\frac{1}{2}\rho^2+z^2$
2	1	$\frac{1}{4}(4z^2-3\rho^2)\cos\phi$	$\frac{1}{4}(\rho^2-4z^2)\sin\phi$	$2\rho z\cos\phi$
2	2	$2\rho z\cos 2\phi$	$-2\rho z\sin 2\phi$	$\rho^2\cos 2\phi$
2	3	$\rho^2\cos 3\phi$	$-\rho^2\sin 3\phi$	0

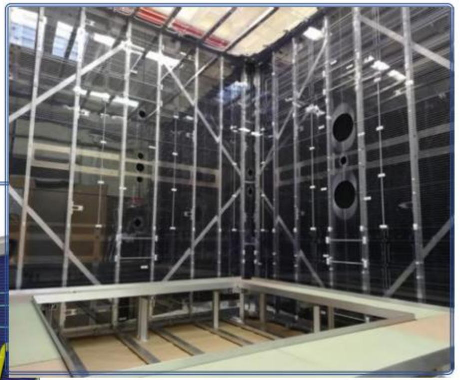
So what do we measure? The *generalized gradients* G_{lm} :

- "Online" with mercury co-magnetometry and cesium magnetometers.
- "Offline" with the mapper.

One use of field mapping: moving the B_0 coil

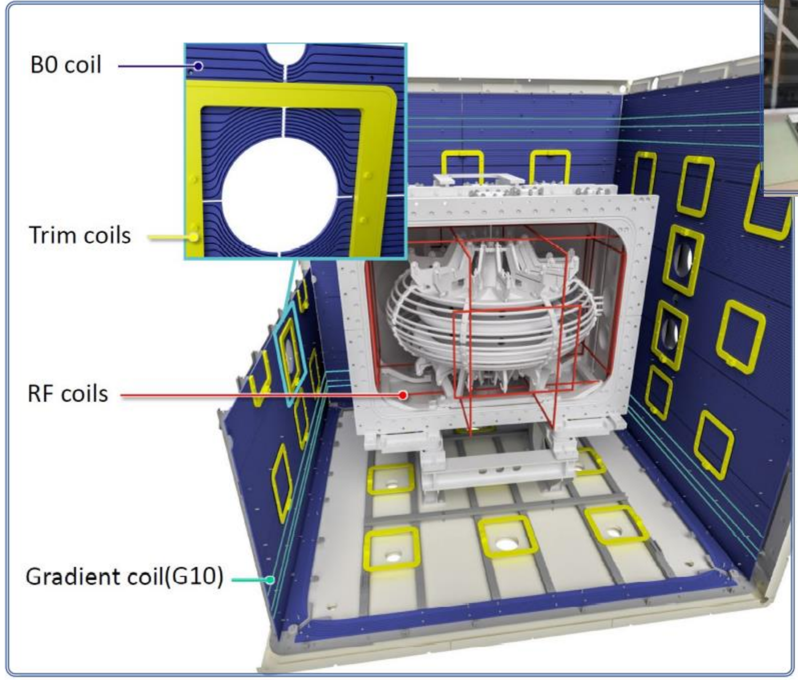
- Produce a very uniform B0 field (1μT)
- Produce specific gradients
- Hold the UCN polarisation
- Neutron spin manipulation

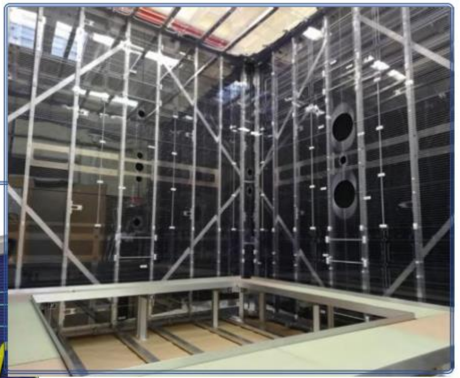


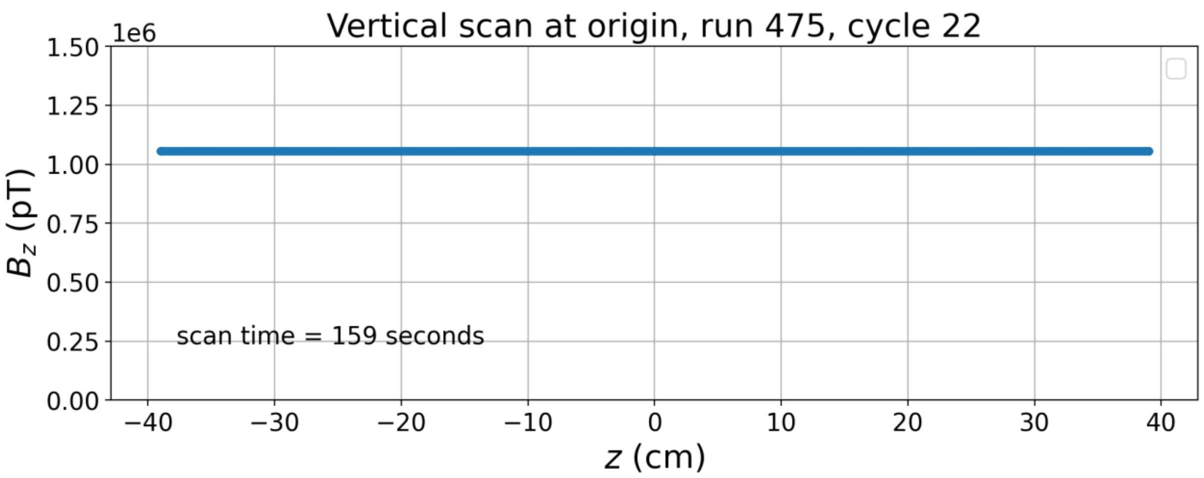


One use of field mapping: moving the B_0 coil

- Produce a very uniform B0 field (1μT)
- Produce specific gradients
- Hold the UCN polarisation
- Neutron spin manipulation

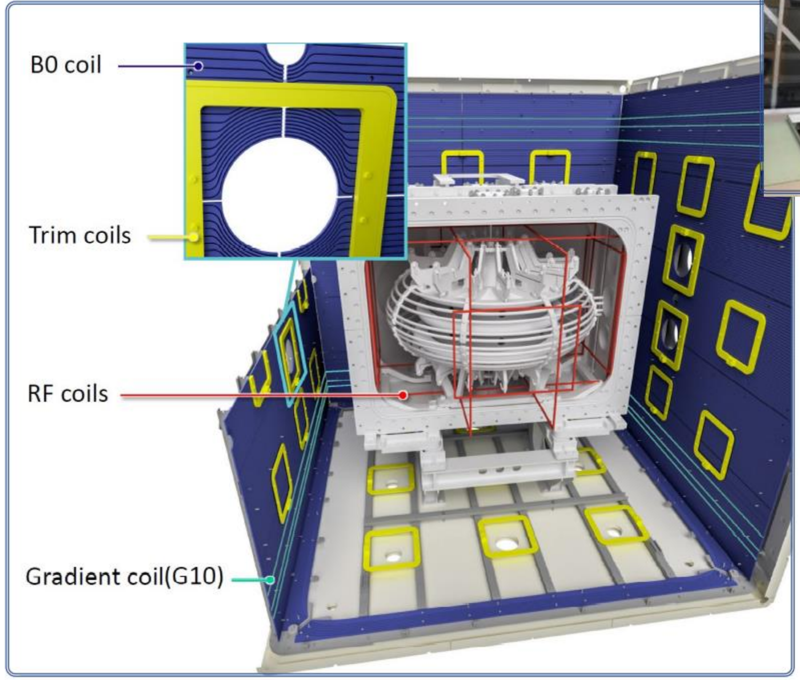


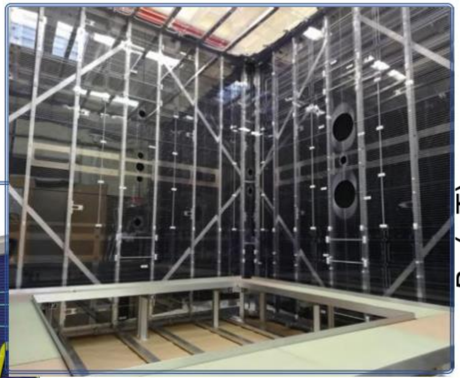


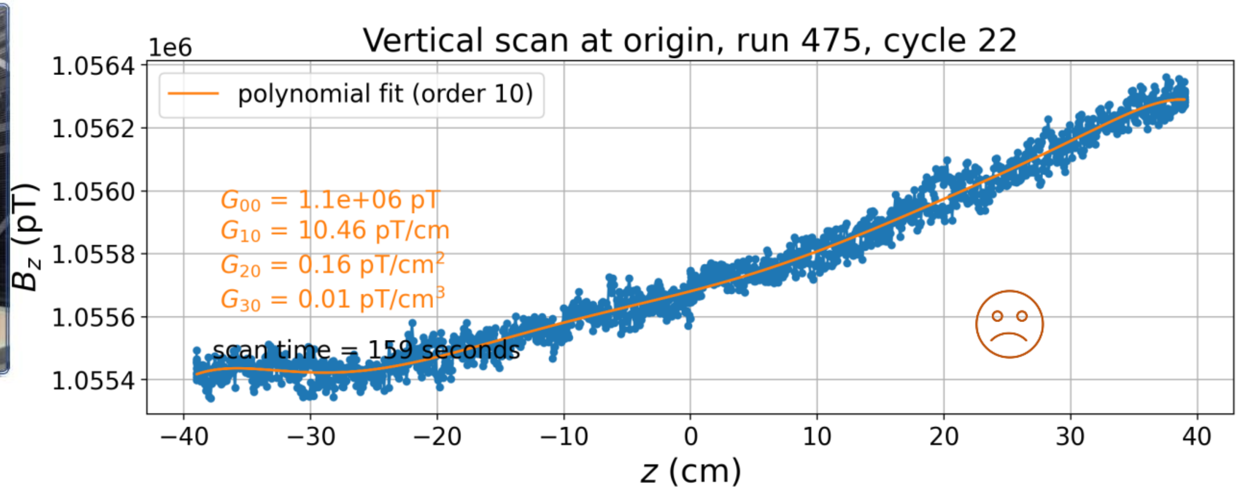


One use of field mapping: moving the B_0 coil

- Produce a very uniform B0 field (1µT)
- Produce specific gradients
- Hold the UCN polarisation
- Neutron spin manipulation

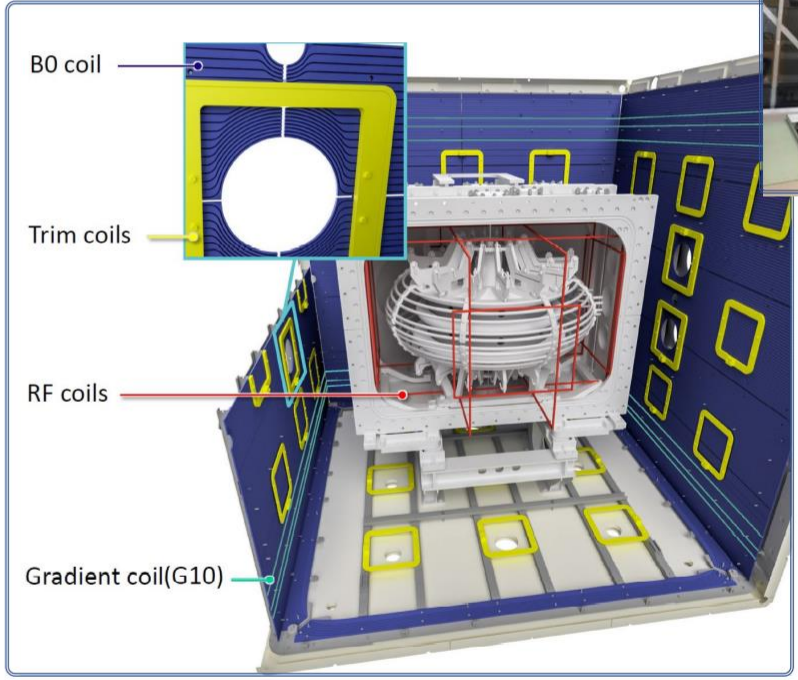


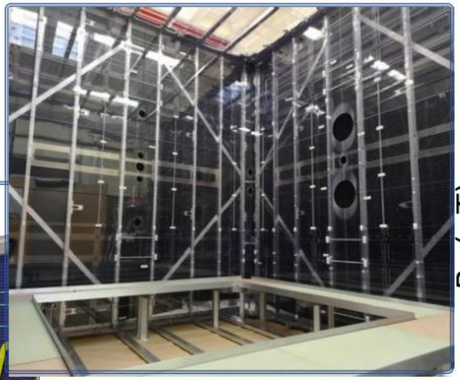


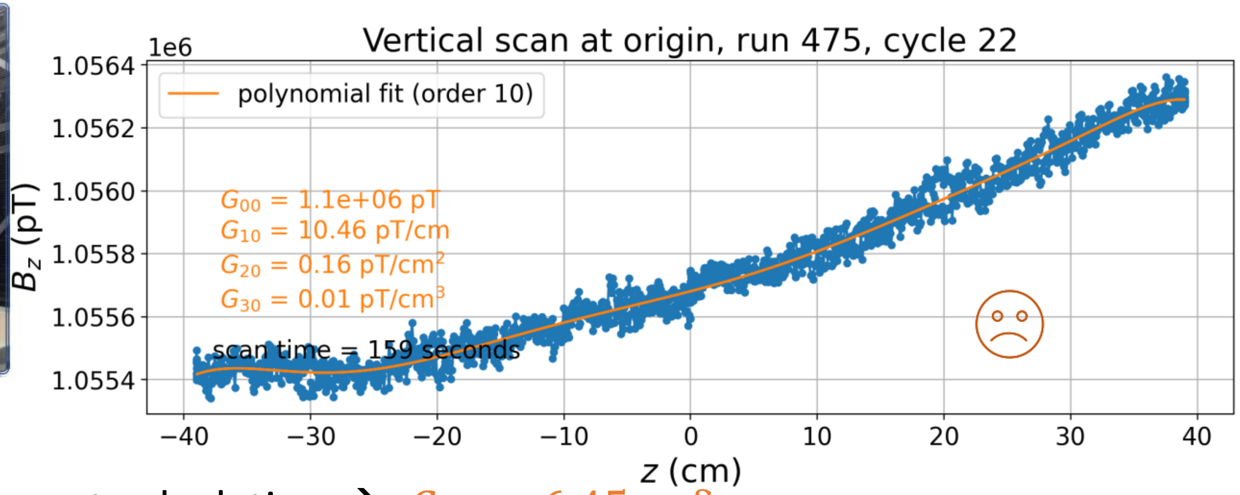


One use of field mapping: moving the B_0 coil

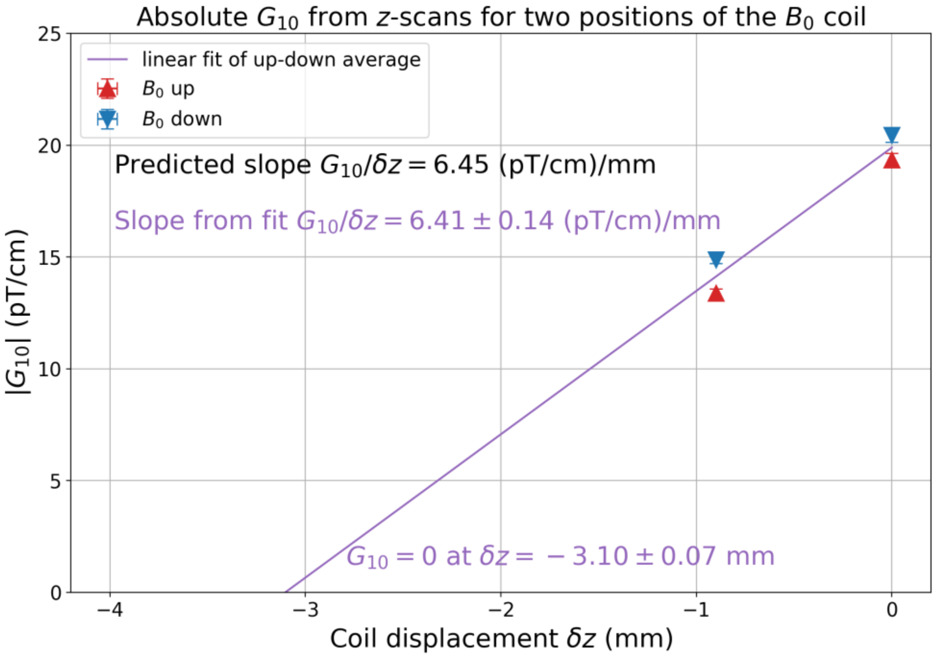
- Produce a very uniform B0 field (1μT)
- Produce specific gradients
- Hold the UCN polarisation
- Neutron spin manipulation







Finite Element calculation → $G_{10} = 6.45 \times \delta_z$



Result: we need to move the coil by 3mm!

An important systematic effect, the “false EDM”

Because the magnetic field is not perfectly uniform *and* because the mercury atoms and the neutrons do not move at the same velocity, they **do not see the same magnetic field.**
This induces extra terms in the frequency ratio that act like EDMs:

$$\mathcal{R} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \mp \frac{2|E|}{\pi\hbar|\gamma_{Hg}B_0|}\left(d_n + d_n^{\text{false}} + d_{n\leftarrow Hg}^{\text{false}} + \cdots\right)$$

False neutron EDM induced by the false mercury EDM

An important systematic effect, the “false EDM”

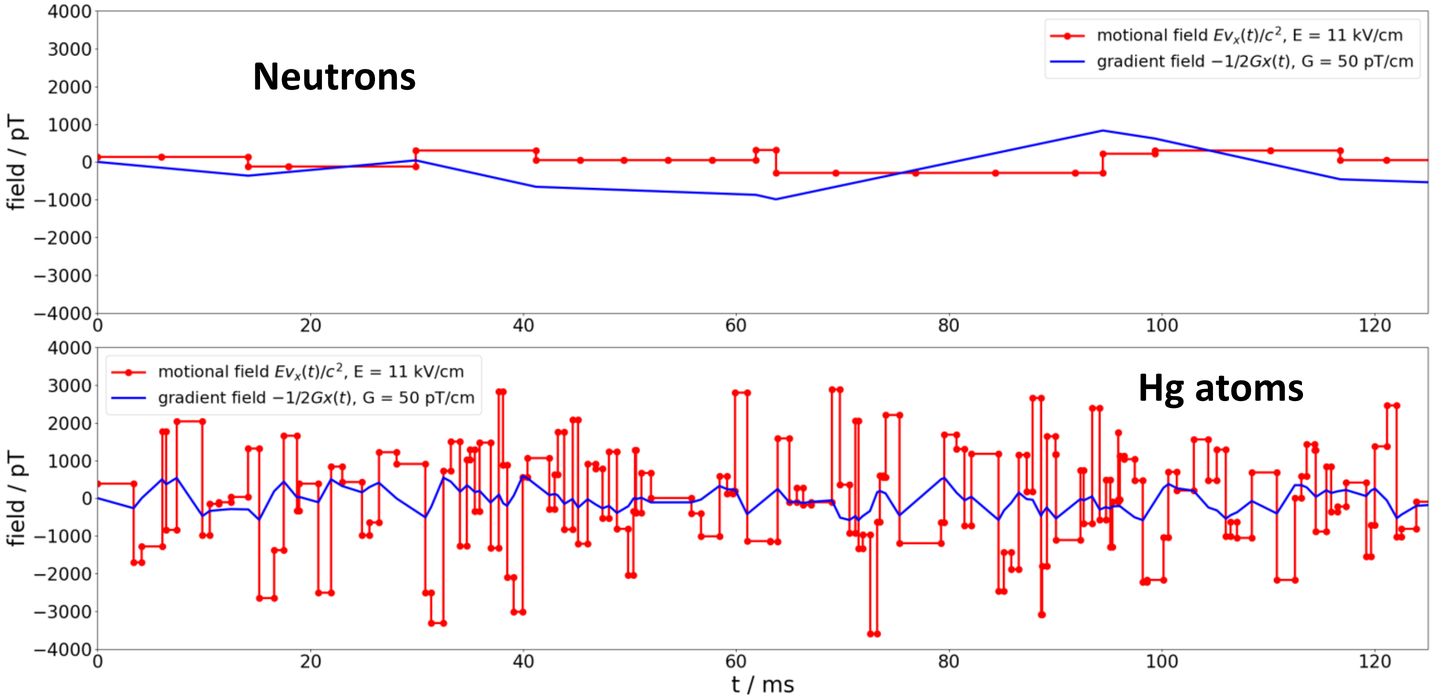
Because the magnetic field is not perfectly uniform *and* because the mercury atoms and the neutrons do not move at the same velocity, they **do not see the same magnetic field.**
This induces extra terms in the frequency ratio that act like EDMs:

$$\mathcal{R} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \mp \frac{2|E|}{\pi\hbar|\gamma_{Hg}B_0|}\left(d_n + d_n^{\text{false}} + d_{n\leftarrow Hg}^{\text{false}} + \cdots\right)$$

False neutron EDM induced by the false mercury EDM

$$b(\tau) = \left[\boldsymbol{B}_T(\boldsymbol{r}(\tau)) + \frac{E}{c^2}\times\dot{\boldsymbol{r}}(\tau)\right]\cdot\left[\boldsymbol{e}_x + i\boldsymbol{e}_y\right]$$ horizontal field fluctuations

non-uniform field + motional field



$v_n \approx 3\ m.s^{-1}$

- Greater time constants for neutrons than for Hg means dilated non-uniform field
- Motional field larger for fast Hg atoms than for slow neutrons

$v_{Hg} \approx 150\ m.s^{-1}$

An expression for the false EDM

The **false** EDM is the difference in frequency **shifts** of opposite electric field configurations

$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n|}{4|E|}\left(\delta\omega_{Hg}(-E) - \delta\omega_{Hg}(E)\right)$$

An expression for the false EDM

The **false** EDM is the difference in frequency **shifts** of opposite electric field configurations

$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n|}{4|E|}\left(\delta\omega_{Hg}(-E) - \delta\omega_{Hg}(E)\right)$$

$$b(\tau) = \left[\boldsymbol{B}_T(\boldsymbol{r}(\tau)) + \frac{\boldsymbol{E}}{c^2} \times \dot{\boldsymbol{r}}(\tau)\right] \cdot \left[\boldsymbol{e}_x + i\boldsymbol{e}_y\right]$$

where the frequency shift is given by spin relaxation theory as a function of the fluctuating transverse magnetic field

$$\delta\omega_{Hg} = \frac{\gamma_{Hg}^2}{2}\int_0^\infty d\tau\, \mathrm{Im}\left[e^{i\omega\tau}\langle b^*(0)b(\tau)\rangle\right]$$

<u>Conclusion:</u> the **combination of a non-uniform field and moving particles** generates a systematic effect

$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau \cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

How do we deal with the false EDM?

A) Estimate it

- @ $B_0 = 1\ \mu T$

because $d_{n\leftarrow Hg}^{\text{false}}$ has an analytical expression valid for **low fields:**

$$d_{n\leftarrow Hg}^{\text{false}} = -\frac{\hbar|\gamma_n \gamma_{Hg}|}{2c^2}\langle xB_x + yB_y\rangle$$

$$= \frac{\hbar|\gamma_n \gamma_{Hg}|}{8c^2}R^2\left(G_{10} - G_{30}\left(\frac{R^2}{2} - \frac{H^2}{4}\right) + \cdots\right)$$

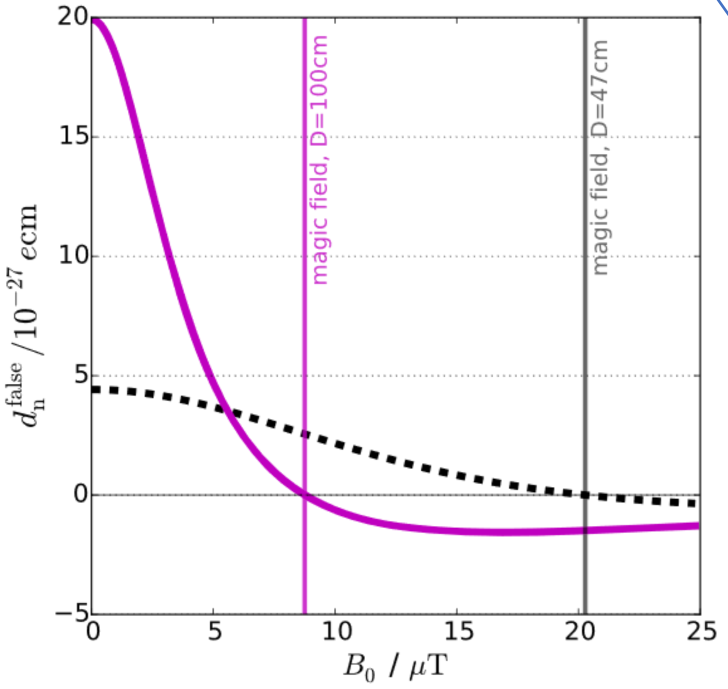
…but need to know the generalized gradients accurately

How do we deal with the false EDM?

A) Estimate it

OR

B) Suppress it



False EDM produced by a linear gradient field as a function of holding field B_0

- @ $B_0 = 1\ \mu T$

because $d_{n\leftarrow Hg}^{\text{false}}$ has an analytical expression valid for **low fields:**

$$d_{n\leftarrow Hg}^{\text{false}} = -\frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\langle xB_x + yB_y\rangle$$

$$= \frac{\hbar|\gamma_n\gamma_{Hg}|}{8c^2}R^2\left(G_{10} - G_{30}\left(\frac{R^2}{2} - \frac{H^2}{4}\right) + \cdots\right)$$

…but need to know the generalized gradients accurately.

- @ $B_m \approx 10\ \mu T$ **“magic field”**

because $d_{n\leftarrow Hg}^{\text{false}}(B_m) = 0$ for some specific field configuration

…but no analytical expression.

The magic field, take one

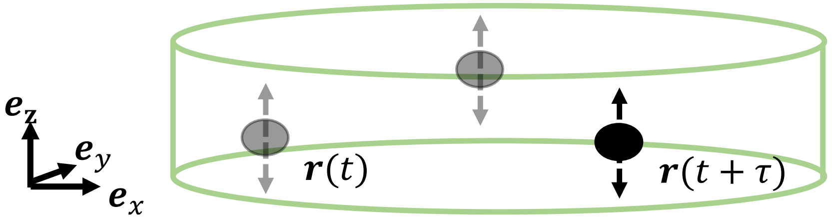
$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau\cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

1) Calculate the correlation function with a Monte-Carlo simulation for a given magnetic configuration

i. Simulate trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$ of Hg atoms

ii. Calculate polynomial pieces $\langle x(\tau)x^i(0)y^j(0)\rangle$ with the **ergodicity property**:
average over all particles ⇔ time average of one particle over infinite time

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty dt\, x(t)x(t+\tau)$$



The magic field, take one

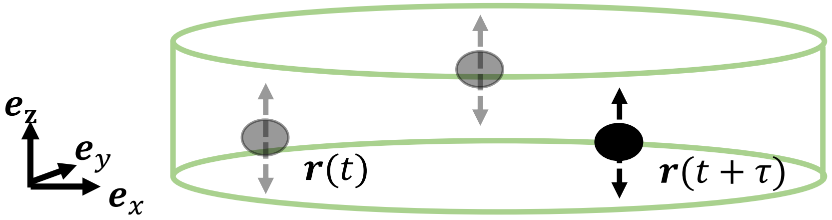
$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau \cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

1) Calculate the correlation function with a Monte-Carlo simulation for a given magnetic configuration

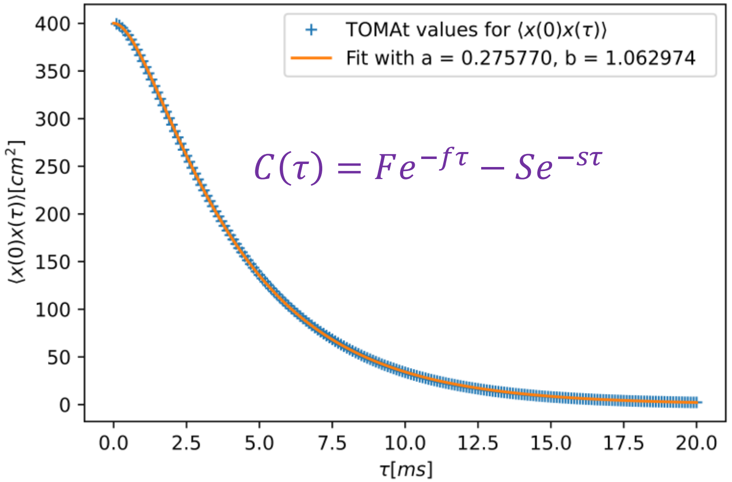
i. Simulate trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$ of Hg atoms

ii. Calculate polynomial pieces $\langle x(\tau)x^i(0)y^j(0)\rangle$ with the **ergodicity property**:
average over all particles ⇔ time average of one particle over infinite time

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty dt\, x(t)x(t+\tau)$$



2) Fit the correlation function



The magic field, take one

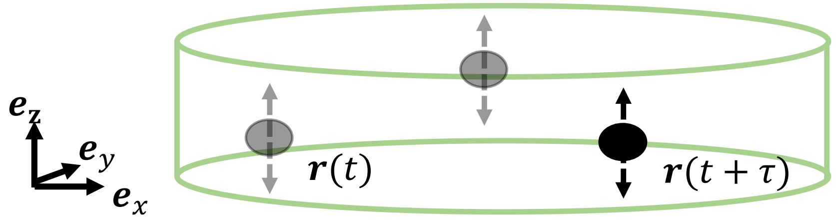
$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau \cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

1) Calculate the correlation function with a Monte-Carlo simulation for a given magnetic configuration

i. Simulate trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$ of Hg atoms

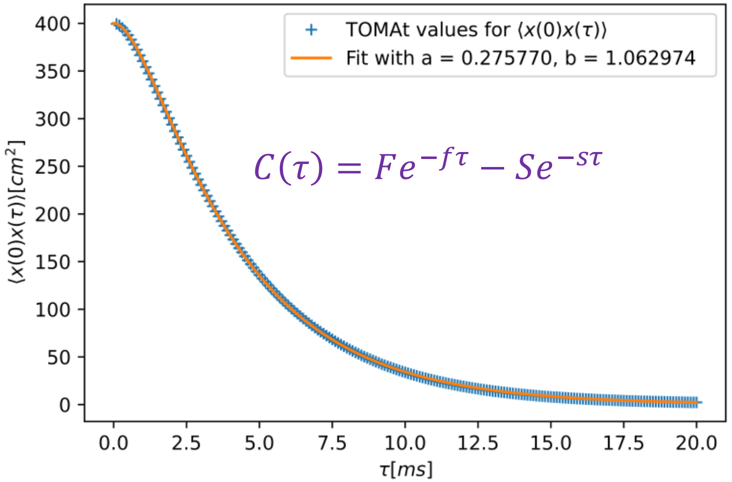
ii. Calculate polynomial pieces $\langle x(\tau)x^i(0)y^j(0)\rangle$ with the **ergodicity property**:
average over all particles ⇔ time average of one particle over infinite time

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty dt\, x(t)x(t+\tau)$$



2) Fit the correlation function

3) Calculate false EDM



The magic field, take one

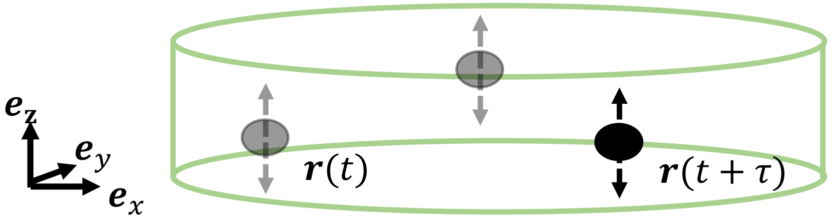
$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau\cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0) + y(\tau)B_y(0)\rangle$$

1) Calculate the correlation function with a Monte-Carlo simulation for a given magnetic configuration

i. Simulate trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$ of Hg atoms

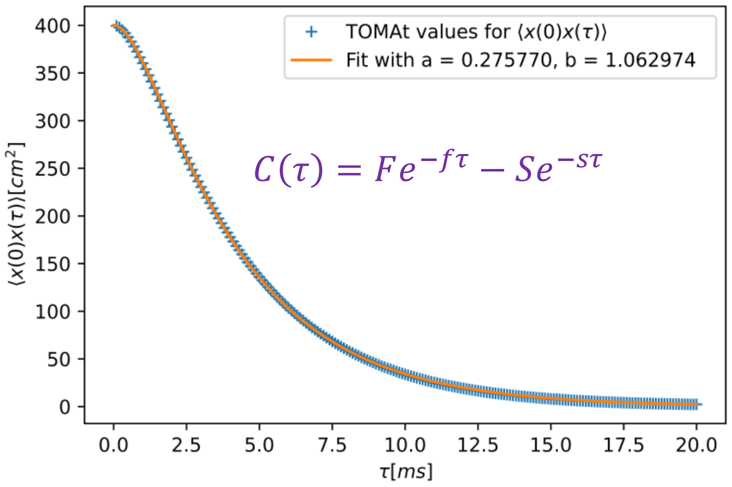
ii. Calculate polynomial pieces $\langle x(\tau)x^i(0)y^j(0)\rangle$ with the **ergodicity property**: average over all particles ⇔ time average of one particle over infinite time

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty dt\, x(t)x(t+\tau)$$



2) Fit the correlation function

3) Calculate false EDM

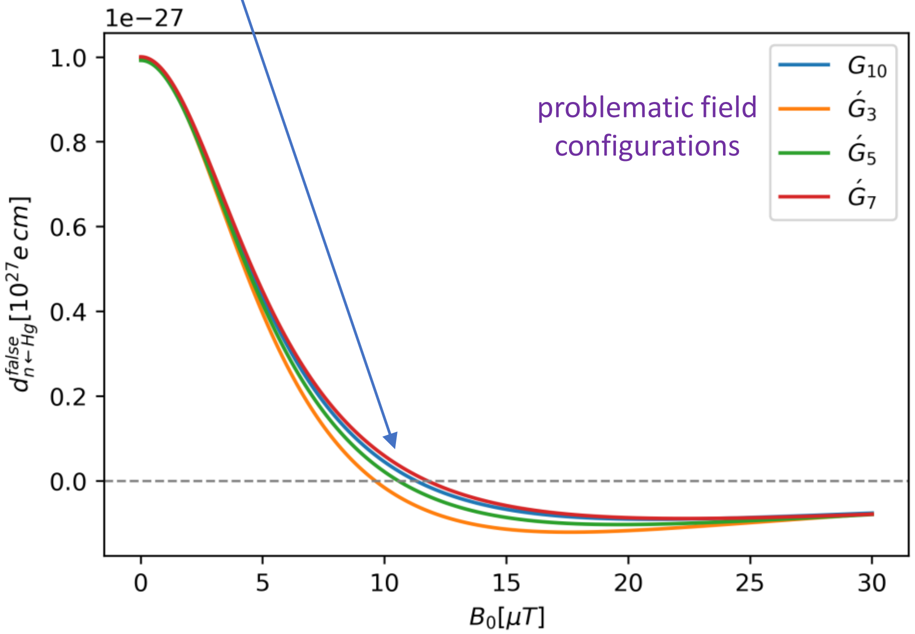


4) Set the holding field to a value that cancels the false EDM generated by this magnetic configuration

Example: the "magic" value that cancels the false EDM generated by

$$\boldsymbol{B}(x,y,z) = G_{10}\begin{pmatrix}-x/2\\ -y/2\\ z\end{pmatrix}$$

is $B_m = 11.2\ \mu\text{T}$



The magic field, take two

- 1st method is biased by the correlation function fit

- The correlation function of a signal is linked to its Power Spectral Density by the **Wiener-Khinchin theorem**

$$S_{ij}(\omega) = \int_{-\infty}^{+\infty} d\tau \, \langle x_i(0) x_j(\tau) \rangle e^{-i\omega\tau}$$

The magic field, take two

- 1st method is biased by the correlation function fit

- The correlation function of a signal is linked to its Power Spectral Density by the **Wiener-Khinchin theorem**

$$S_{ij}(\omega) = \int_{-\infty}^{+\infty} d\tau \, \langle x_i(0) x_j(\tau) \rangle e^{-i\omega\tau}$$

→ we can access the false EDM through the PSD

$$S_{ij}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left\langle \left(\int_{-\infty}^{+\infty} dt_1 \, x_i(t_1) e^{-i\omega t_1} \right)^* \left(\int_{-\infty}^{+\infty} dt_2 \, x_i(t_2) e^{-i\omega t_2} \right) \right\rangle$$

For a linear vertical gradient field:

$$d_{n\leftarrow Hg}^{\text{false}}(\omega_0) = -\frac{\hbar |\gamma_n \gamma_{Hg}|}{4\pi c^2} \text{P.V} \int_{-\infty}^{+\infty} d\omega \, \omega \, \frac{S_{xx}(\omega) + S_{yy}(\omega)}{\omega - \omega_0}$$

The magic field, take two

- 1st method is biased by the correlation function fit

- The correlation function of a signal is linked to its Power Spectral Density by the **Wiener-Khinchin theorem**

$$S_{ij}(\omega) = \int_{-\infty}^{+\infty} d\tau \left\langle x_i(0) x_j(\tau) \right\rangle e^{-i\omega\tau}$$

→ we can access the false EDM through the PSD

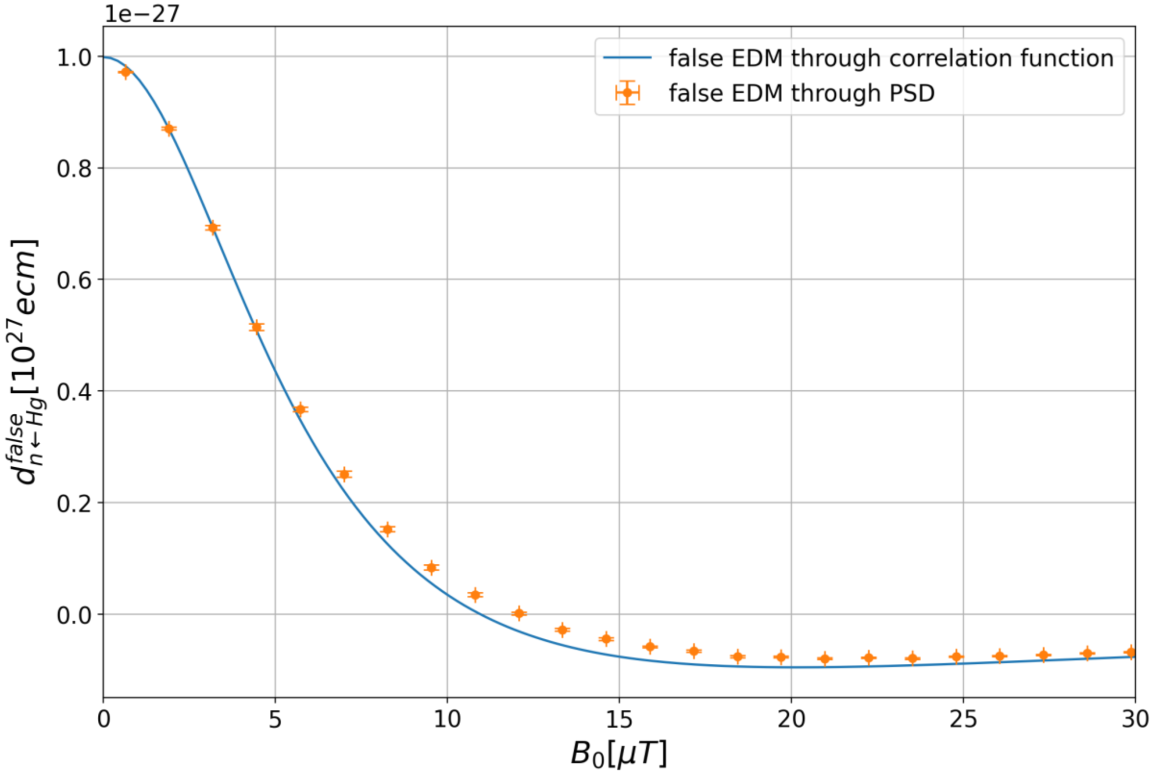
$$S_{ij}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left\langle \left(\int_{-\infty}^{+\infty} dt_1\, x_i(t_1) e^{-i\omega t_1} \right)^* \left(\int_{-\infty}^{+\infty} dt_2\, x_i(t_2) e^{-i\omega t_2} \right) \right\rangle$$

For a linear vertical gradient field:

$$d_{n\leftarrow Hg}^{\text{false}}(\omega_0) = -\frac{\hbar|\gamma_n\gamma_{Hg}|}{4\pi c^2} \text{P.V} \int_{-\infty}^{+\infty} d\omega\, \omega\, \frac{S_{xx}(\omega) + S_{yy}(\omega)}{\omega - \omega_0}$$

$$= \cdots = -\frac{\hbar|\gamma_n\gamma_{Hg}|}{4\pi c^2} \frac{1}{N_t} \sum_{n,k,l}^{N_t,N_c,N_c} \frac{\Delta t_k \Delta t_l}{\Delta T_n} \int_{-\infty}^{+\infty} d\omega\, \omega\, \frac{I_k(\omega) I_l^*(\omega)}{\omega - \omega_0}$$

numerical sum of explicit elementary integrals



The magic field, take two

- 1^{st} method is biased by the correlation function fit
- The correlation function of a signal is linked to its Power Spectral Density by the **Wiener-Khinchin theorem**

$$S_{ij}(\omega) = \int_{-\infty}^{+\infty} d\tau \, \langle x_i(0) x_j(\tau) \rangle e^{-i\omega\tau}$$

→ we can access the false EDM through the PSD

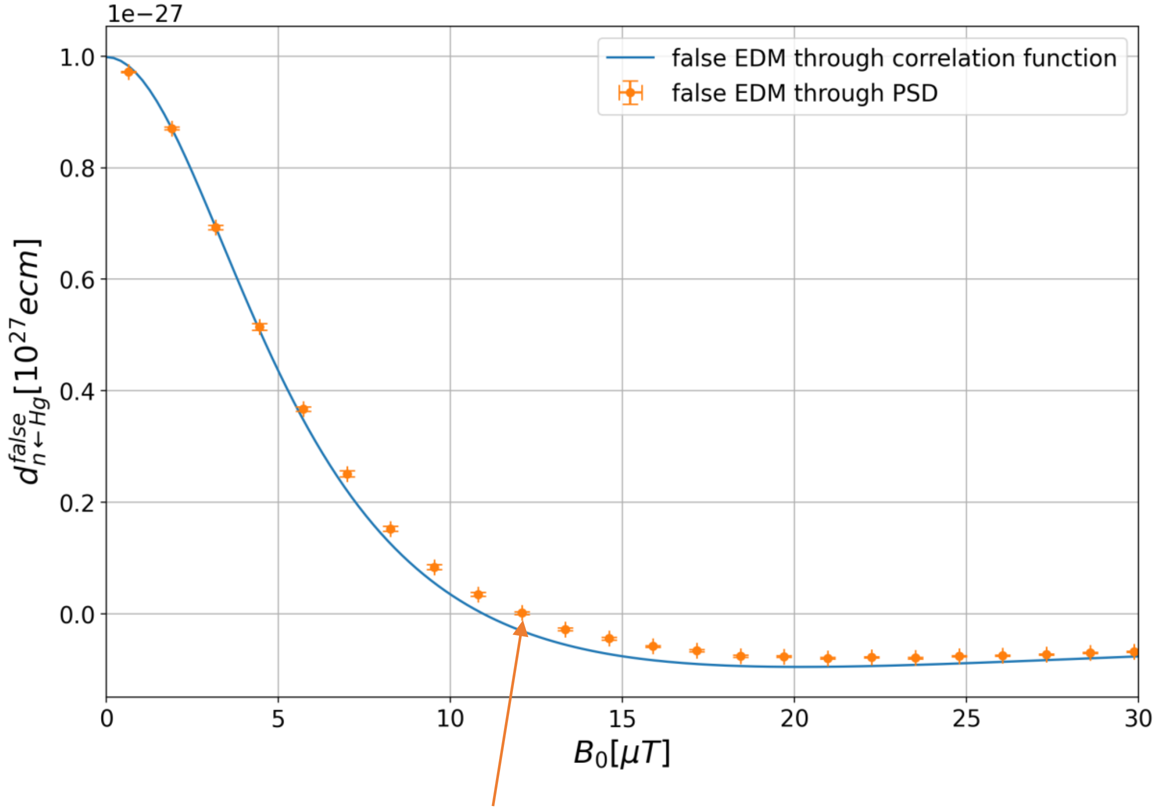
$$S_{ij}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left\langle \left(\int_{-\infty}^{+\infty} dt_1 \, x_i(t_1) e^{-i\omega t_1} \right)^* \left(\int_{-\infty}^{+\infty} dt_2 \, x_i(t_2) e^{-i\omega t_2} \right) \right\rangle$$

For a linear vertical gradient field:

$$d_{n\leftarrow Hg}^{\text{false}}(\omega_0) = -\frac{\hbar|\gamma_n\gamma_{Hg}|}{4\pi c^2} \mathrm{P.V} \int_{-\infty}^{+\infty} d\omega \, \omega \, \frac{S_{xx}(\omega) + S_{yy}(\omega)}{\omega - \omega_0}$$

$$= \cdots = -\frac{\hbar|\gamma_n\gamma_{Hg}|}{4\pi c^2} \frac{1}{N_t} \sum_{n,k,l}^{N_t,N_c,N_c} \frac{\Delta t_k \Delta t_l}{\Delta T_n} \int_{-\infty}^{+\infty} d\omega \, \omega \, \frac{I_k(\omega) I_l^*(\omega)}{\omega - \omega_0}$$

numerical sum of explicit elementary integrals



Result: magic value is slightly different

$$B_m = 12.0 \, \mu\mathrm{T}$$

Conclusion on systematics

- Non-uniformities and relativistic effects in the magnetic field generate a “false EDM”
- False EDM can be estimated at low fields or suppressed at specifically high fields (“magic fields”)
- Different challenges:
    1. Low fields: require accurate measurement of generalized gradients
    2. Magic fields: require accurate numerical estimation of magic values

       →next step: thorough comparison of numerical efficiency of two methods