Morphology with Zernike polynomials: the first application on *Planck*-SZ galaxy clusters

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Introduction

- **Purpose:** study the morphology of galaxy clusters from 2D projection maps to infer, as possible, their dynamical state

- **Method:** analytic modelling using ***Zernike polynomials***

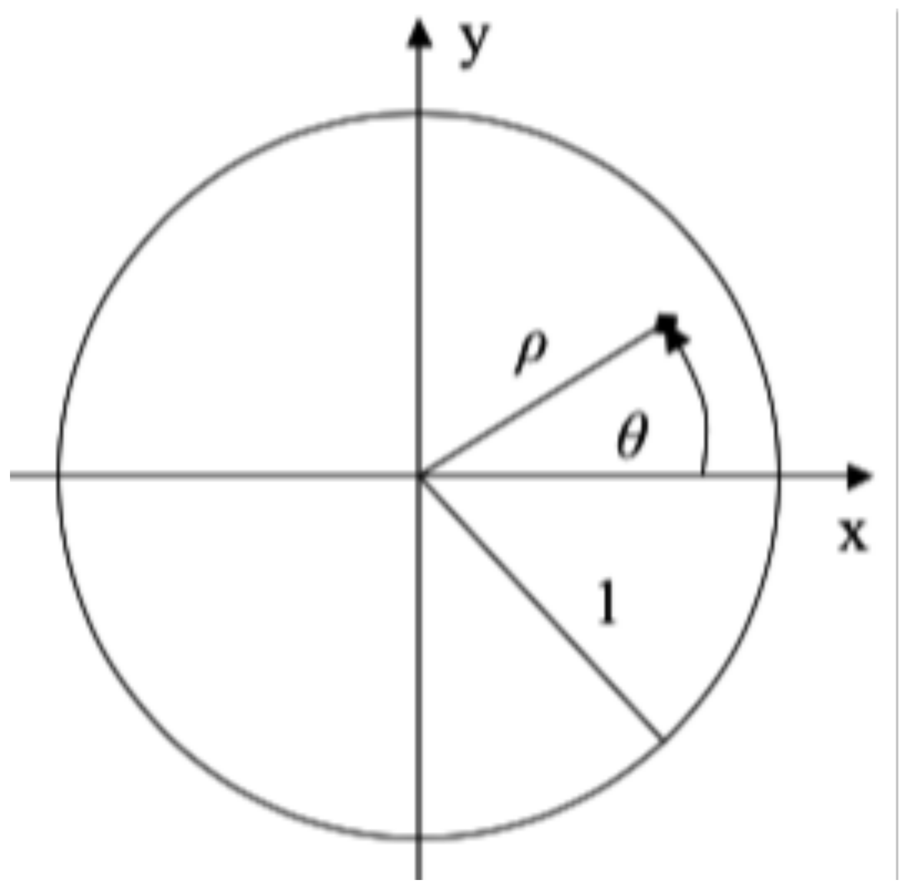
Why?

ZPs are a ***complete*** and ***orthogonal*** set of functions defined over a unit circle, useful for modelling functions in circular domains

Common applications of ZPs in several fields:

- adaptive optics *(see e.g. Noll R. G., 1976, J. Opt. Soc. Am.,66,207; Rigaut F. et al.,1991, A&A, 250, 280)*
- image analysis and pattern recognition *(see e.g. Teague M. R., 1980, J. Opt. Soc. Am., 70, 920)*
- ophthalmology, optometry, medicine *(see e.g. Liang J., Williams D. R.,1997,J. Opt. Soc. Am. A,14,2873; Tahmasbi A., et al., 201, Comput. Biol. Med., 41,726; Alizadeh E., et al., 2016, Integr. Biol., 8, 1183)*

Zernike polynomials: definition



(Noll R. G., 1976, J. Opt. Soc. Am., 66, 207)

$$\mathbf{Z_n^m}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\cos(\mathbf{m}\theta)$$

$$\mathbf{Z_n^{-m}}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\sin(\mathbf{m}\theta)$$

normalization — radial term

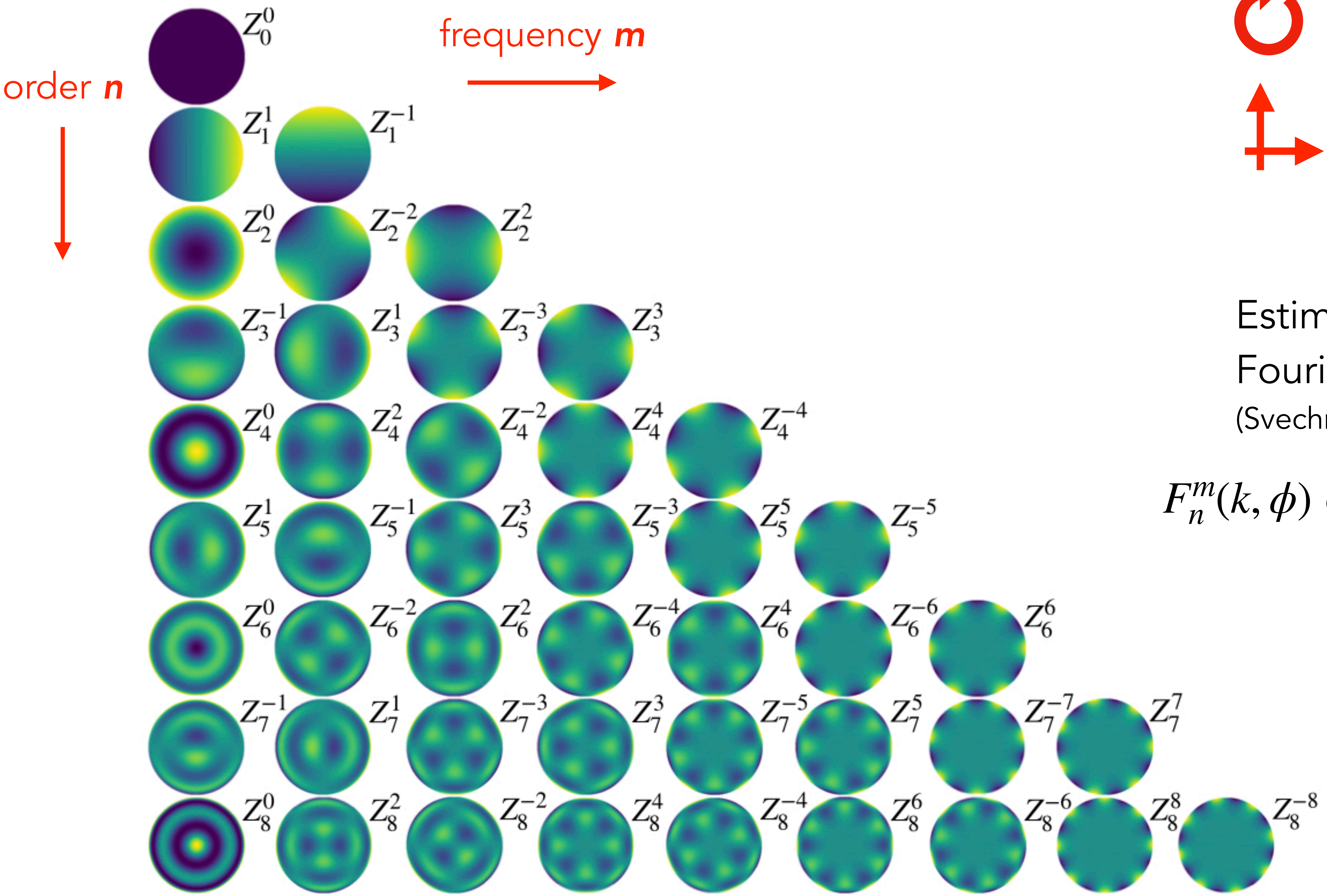
order ***n*** and frequency ***m***: ∈ N, $m \leqslant n$, $n-m=even$

- orthogonality: $\int_0^{2\pi}\int_0^1 Z_n^m(\rho,\theta)\, Z_{n'}^{m'}(\rho,\theta)\,\rho d\rho d\theta = \pi\,\delta_{nn'}\,\delta_{mm'}$
- linear expansion: an arbitrary function $\phi(\rho,\theta)$ on a unit circle can be expressed as a weighted sum of ZPs

$$\phi(\rho,\theta) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} c_{nm}\, Z_n^m(\rho,\theta)$$

orthogonality → no overlap when adding further therms

Zernike polynomials: definition



m=0: no angular dependence, continuous circular symmetry

m≠0: axial symmetry/antisymmetry

Estimate the **spatial resolution** from Fourier transform:

(Svechnikov M., et al., 2015, Opt. Express, 23, 14677)

$$F_n^m(k, \phi) \propto J_{n+1}(2\pi k), \quad k = \text{spatial frequencies} \left[\frac{1}{R}\right]$$

↓

Bessel function:

negligibly if $2\pi k < n + 1$

$$R_{max} \sim \frac{2\pi}{(n+1)}$$

Zernike fitting: validation of the method on mock y-maps

Data set: 324 mock *y*-maps of galaxy clusters from ***The Three Hundred Project*** *(Cui W. et al., 2018, MNRAS, 480, 2898*, see Weiguang Cui talk on Wednesday 🎙) at 3 redshifts (z=0, 0.45, 1.03) and different angular resolution, up to 5 arcmin.

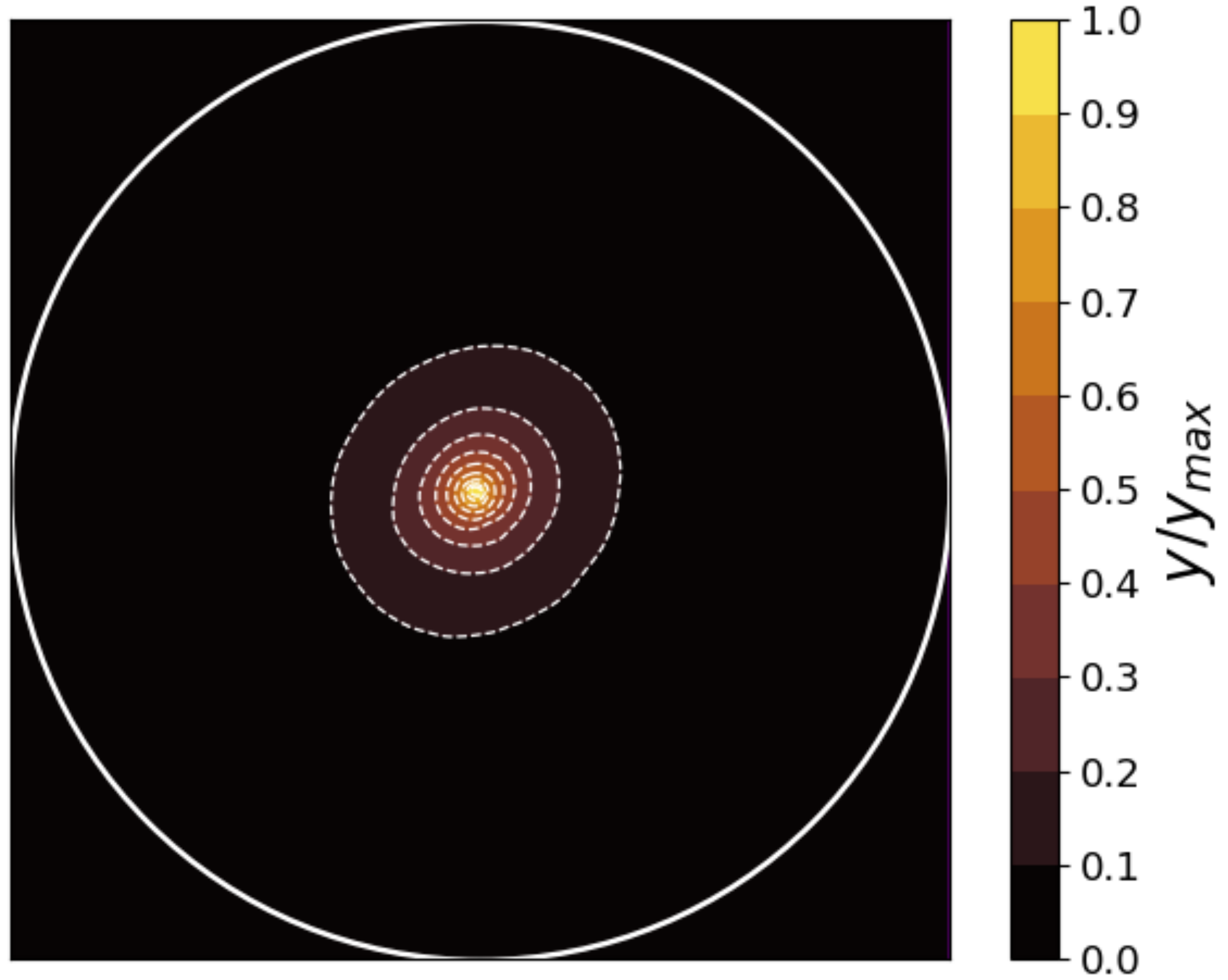
Each *y*-map is modelled with 45 ZPs up to the order *n*=8, within a circular aperture with radius R_{500} (spatial resolution $\sim 0.5R_{500}$)

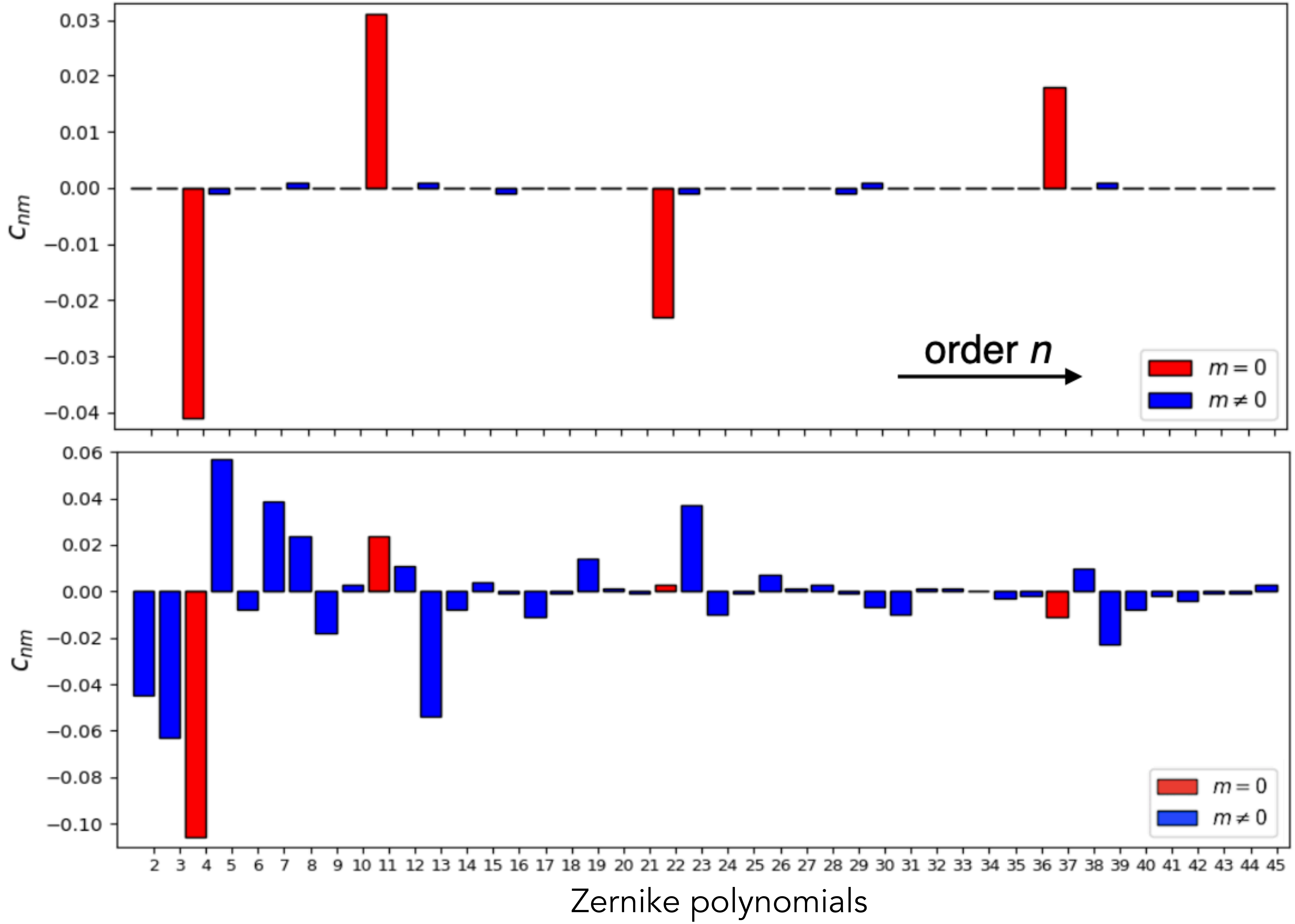
$$y = \sum_{n=0}^{8} \sum_{m=0}^{n} c_{nm} Z_n^m \longrightarrow c_{nm} = \frac{\sum y \times Z_n^m}{\pi (R_{500})^2}$$

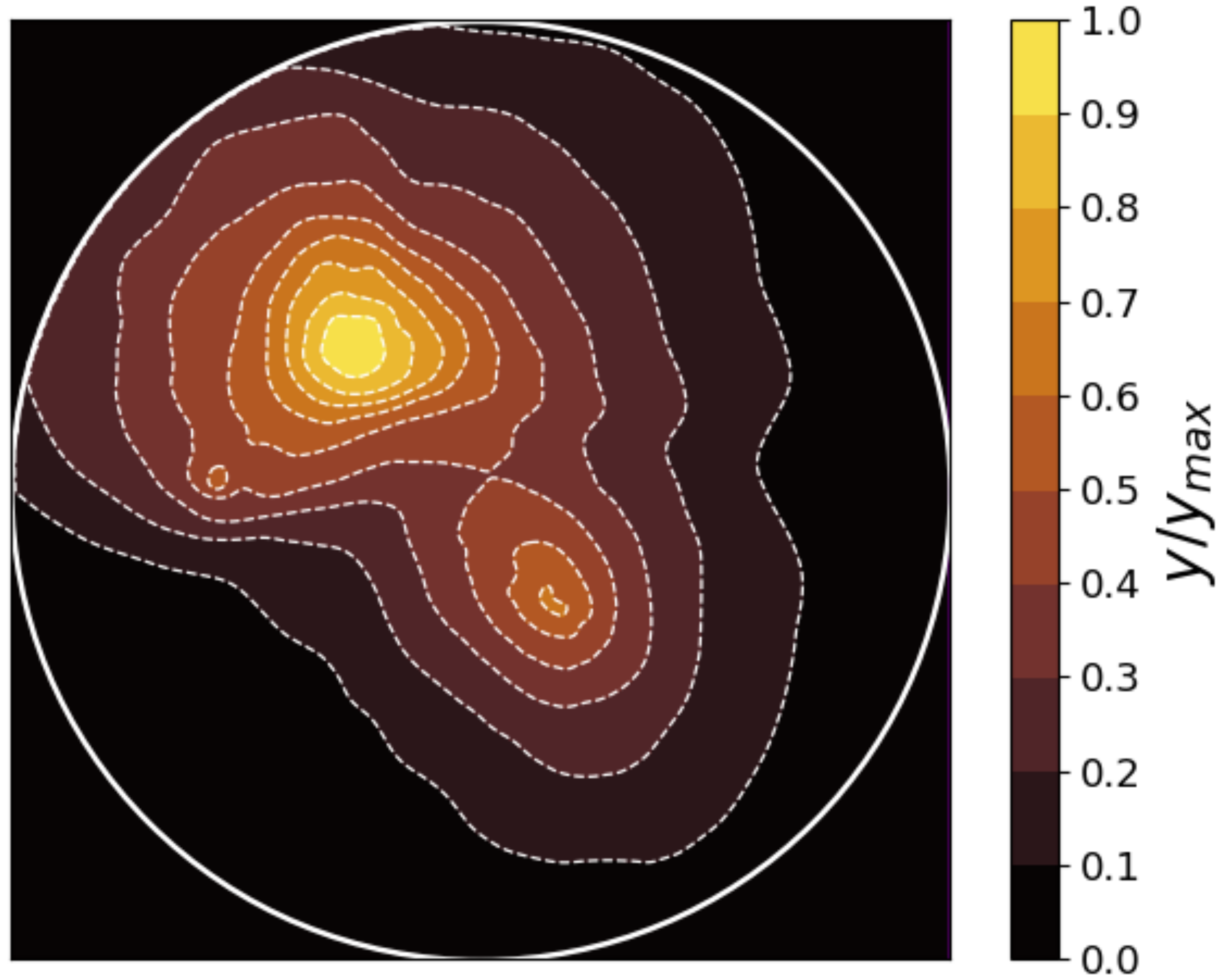
Capalbo V., et al., 2021, *MNRAS*, 503, 6155

Zernike fitting: validation of the method on mock y-maps

(*y*-map, ang. res=5.2″)

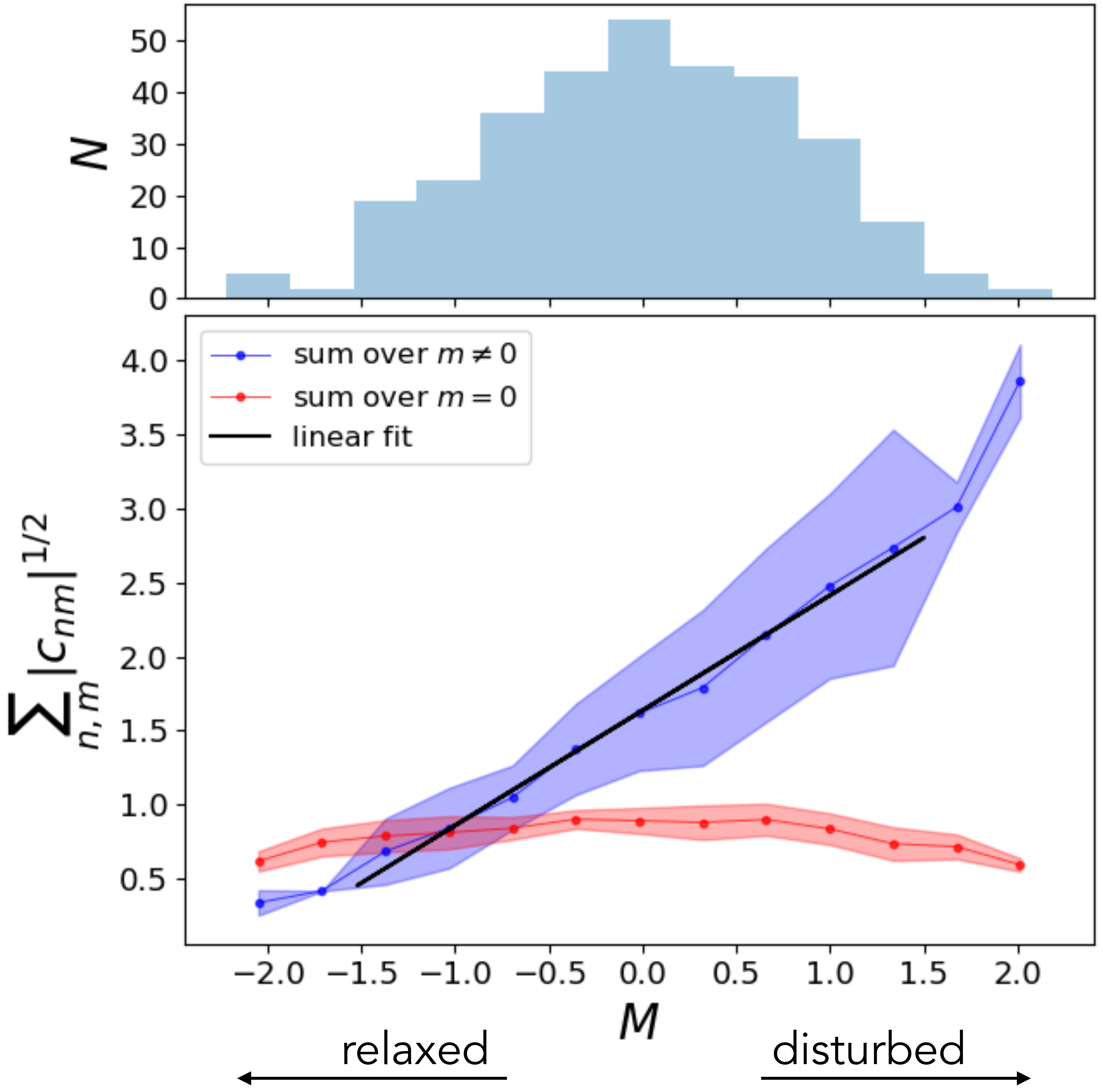






Zernike fitting: validation of the method on mock y-maps

✓ Define a single parameter from the Zernike fitting and correlate with other common morphological/ dynamical state indicators



$$C = \sum_{n,m\neq0} |c_{nm}|^{1/2}$$

M is a combination of
(De Luca F., et al., 2021, MNRAS, 504, 5383)

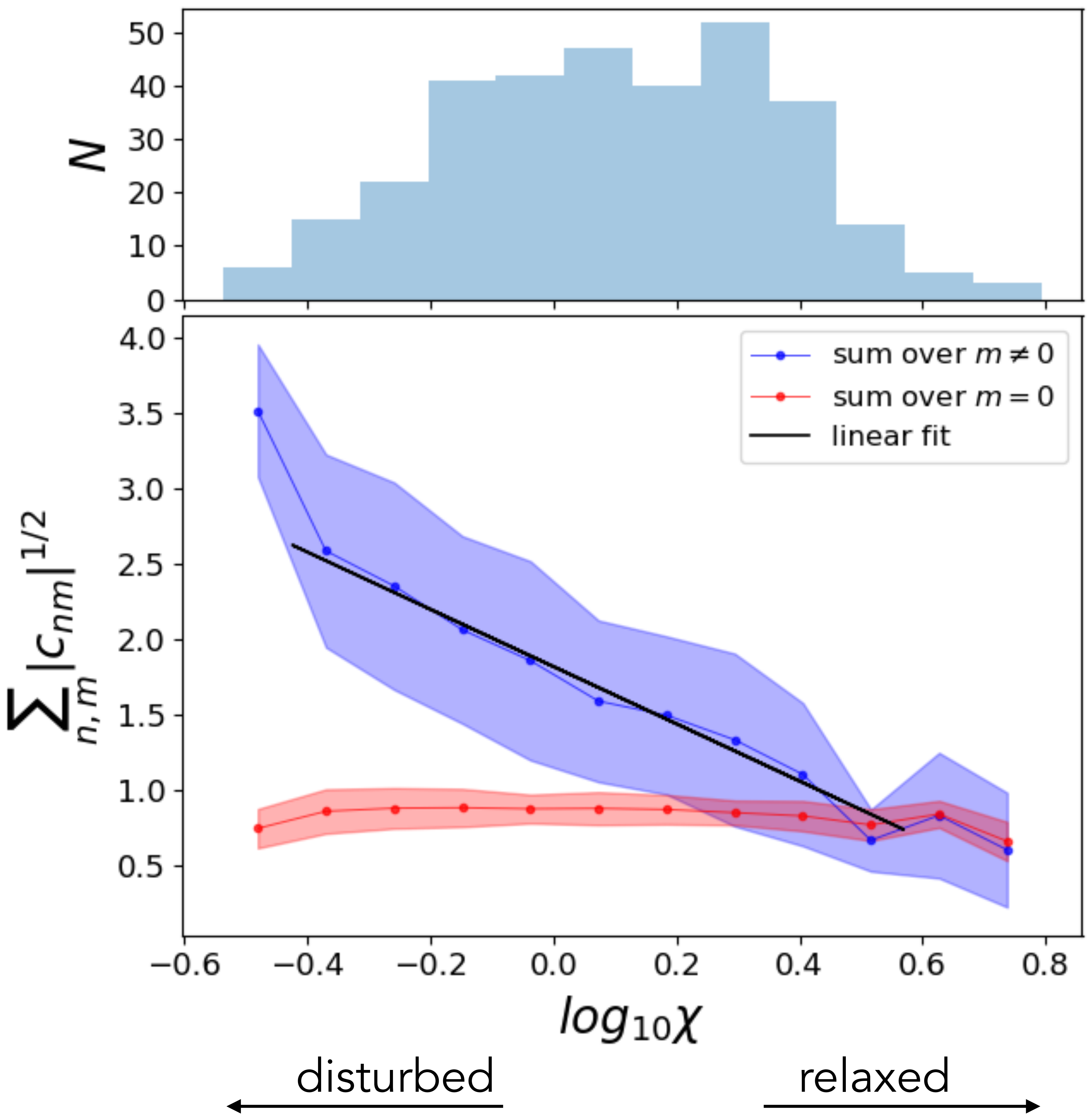
- *Asymmetry parameter (A)*
- *Light concentration parameter (c)*
- *Power ratio parameter (P)*
- *Centroid shift parameter (w)*
- *Gaussian fit parameter (G)*
- *Strip parameter (S)*

- best linear fit —— $C = aM + b$
  $\boldsymbol{C = (0.78 \pm 0.04)M + (1.64 \pm 0.03)}$
- Pearson correlation coefficient *r* with *M*:
  $\boldsymbol{r = 0.78}$
- Spearman correlation coefficient r_S with single params:

Parameter	r_s
A	0.69
c	-0.85
P	0.56
w	0.61
G	-0.25
S	0.61

Zernike fitting: validation of the method on mock y-maps

✓ Define a single parameter from the Zernike fitting and correlate with other common morphological/ dynamical state indicators



$$C = \sum_{n,m \neq 0} |c_{nm}|^{1/2}$$

χ is a combination of 3D dynamical indicators
(De Luca F., et al., 2021, MNRAS, 504, 5383)

- *Centre-of-mass offset*
- *Fraction of mass in sub-halos*

- best linear fit —— $C = -a\log_{10}\chi + b$

  $\boldsymbol{C = (-1.90 \pm 0.14)\log_{10}\chi + (1.82 \pm 0.04)}$

- Pearson correlation coefficient *r* with χ:

  $\boldsymbol{r = -0.62}$

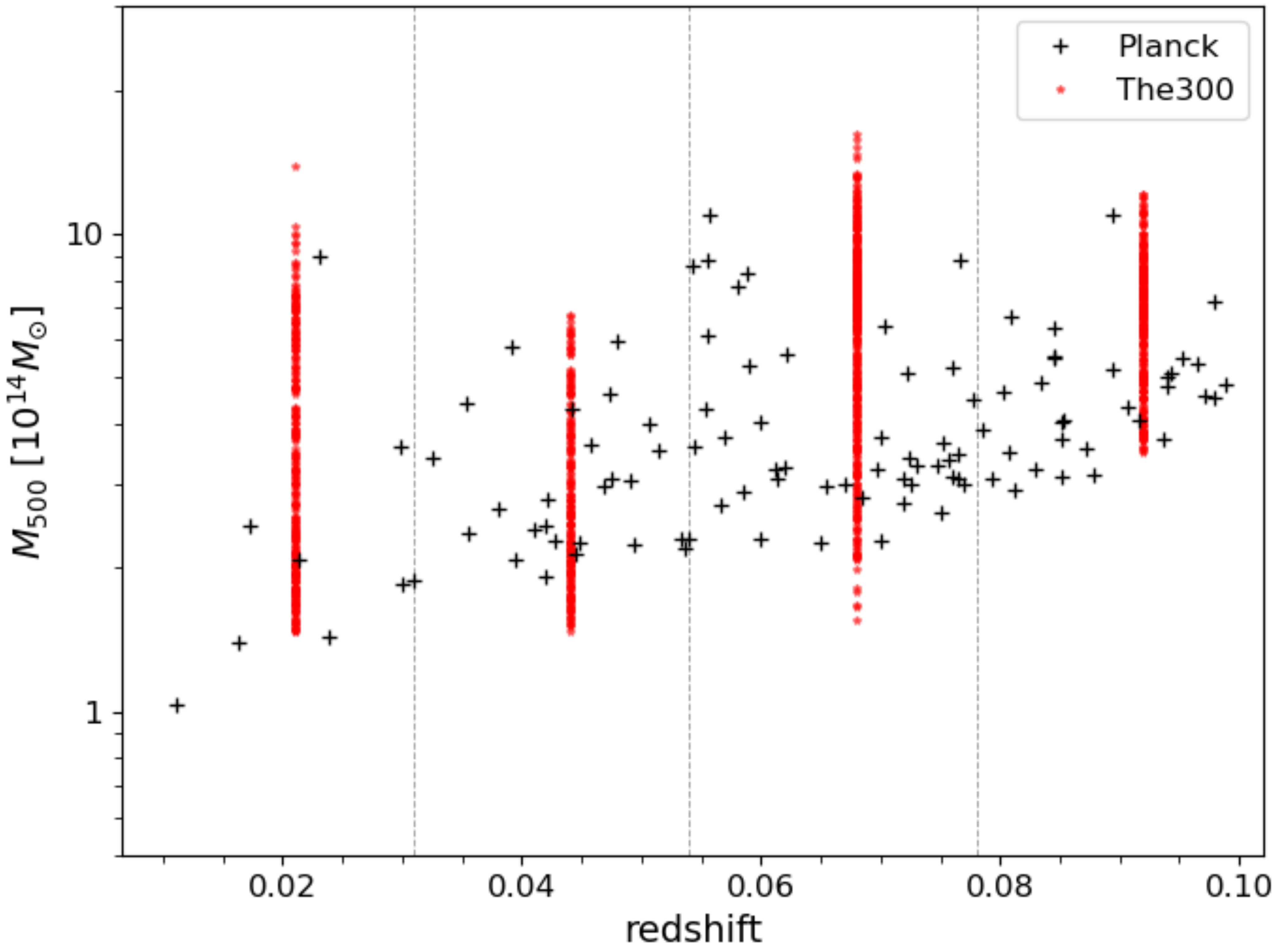
Zernike fitting: application on *Planck*-SZ selected clusters

Data set:

- the cosmology sample at ***z*<0.1**, i.e. clusters detected with **SNR>6** (**109 clusters**)
- public *y*-maps realized with MILCA and NILC methods (**angular resolution=10 arcmin, side-length=2**R_{500})
  (Planck Collab. XXVII 2016, Planck Collab. XXII 2016)

+

- mock *Planck y*-maps realized for *The300* clusters at 4 redshift snapshots (de Andres D., et al., 2022, *Nat Astron* 6, 1325–1331, see Daniel de Andres talk on Tuesday)

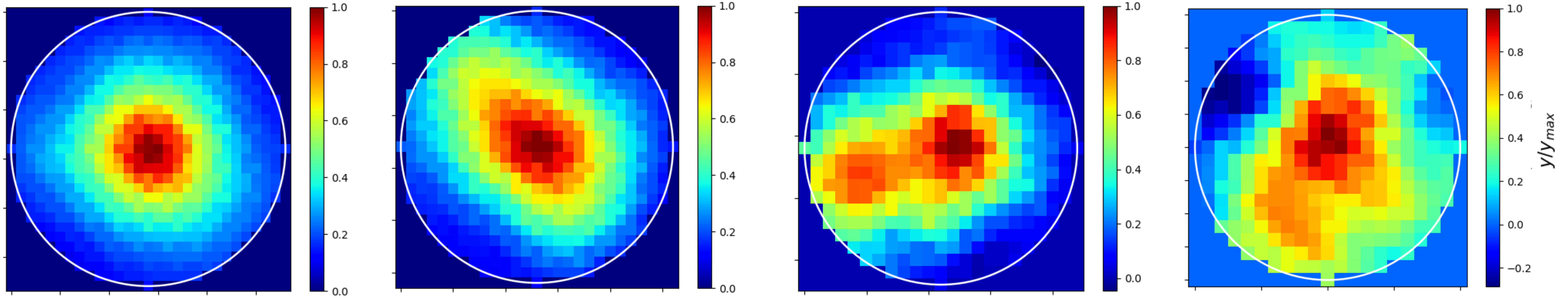


➡ in each redshift bin we select *The300* with SNR_{min}(Planck) < SNR< SNR_{max}(Planck)

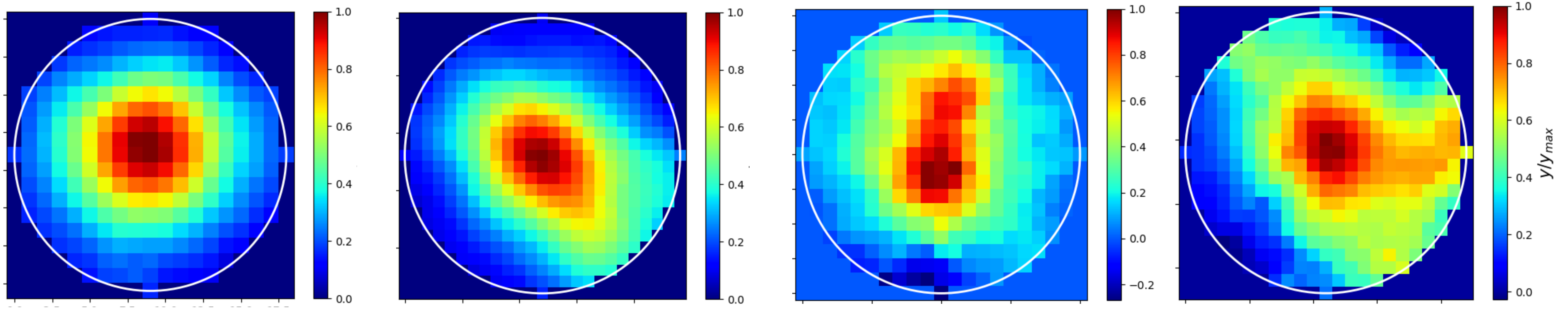
* **Capalbo V., et al., in prep.**

Zernike fitting: application on *Planck*-SZ selected clusters

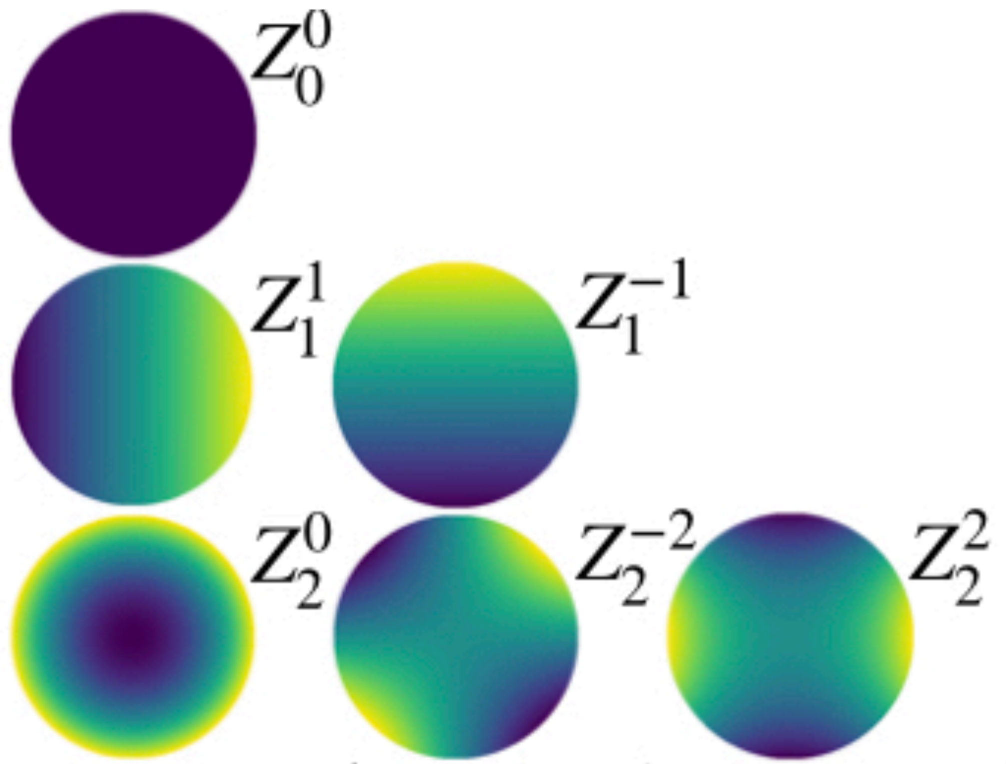
Planck y-maps



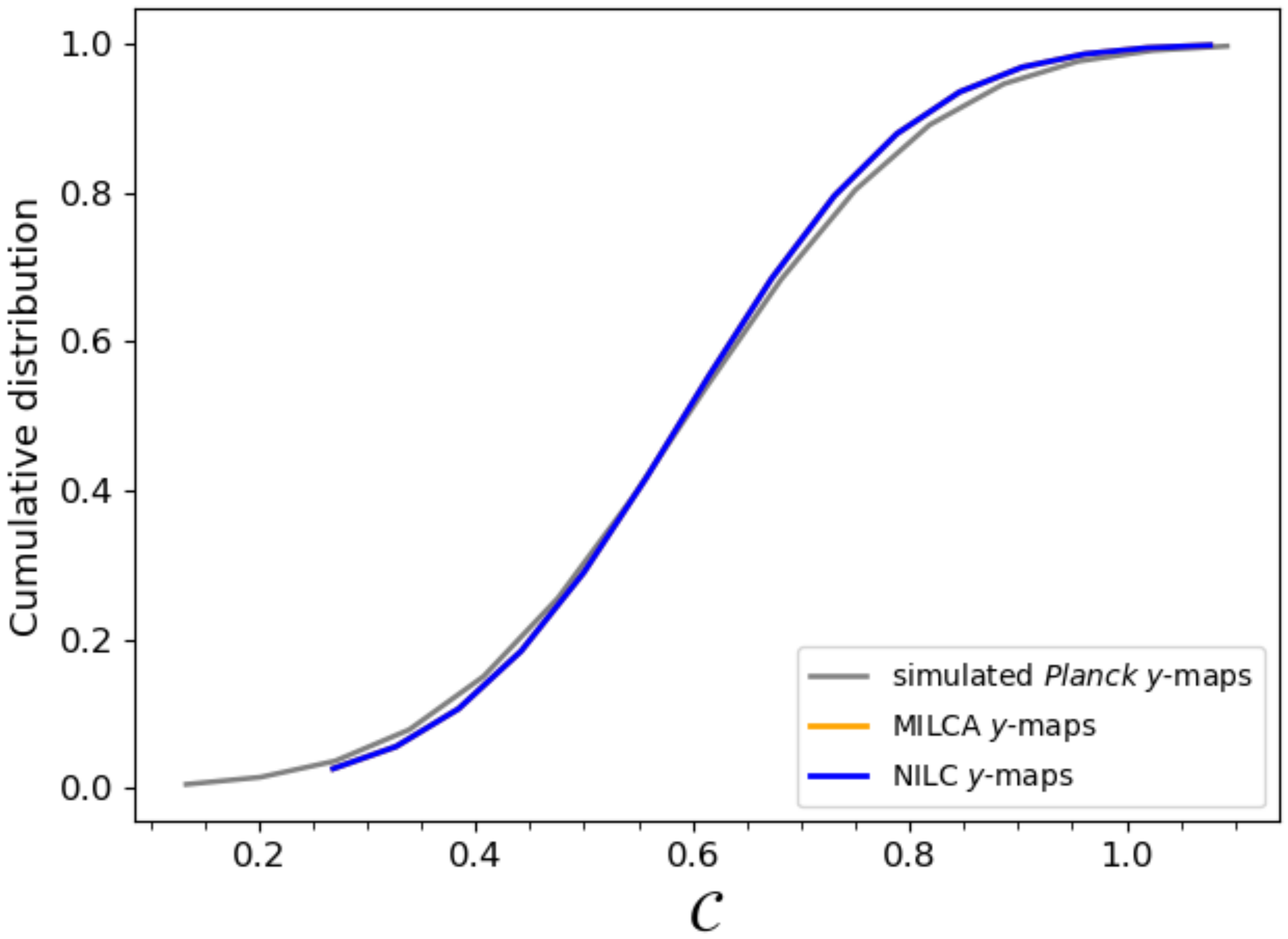
Planck-like



Zernike fitting: application on *Planck*-SZ selected clusters

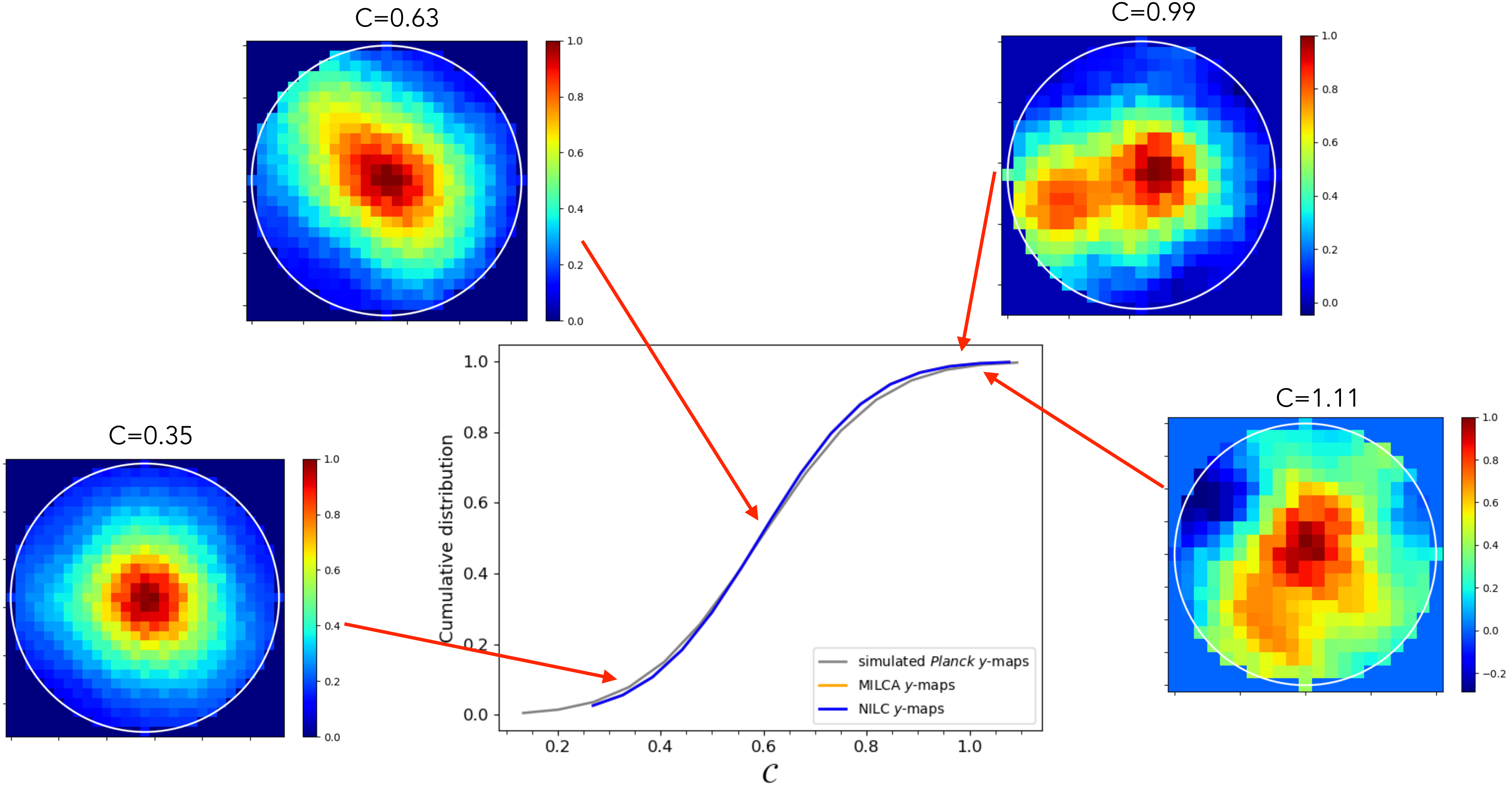


→ Spatial resolution of the Zernike modeling $\sim 2R_{500}$



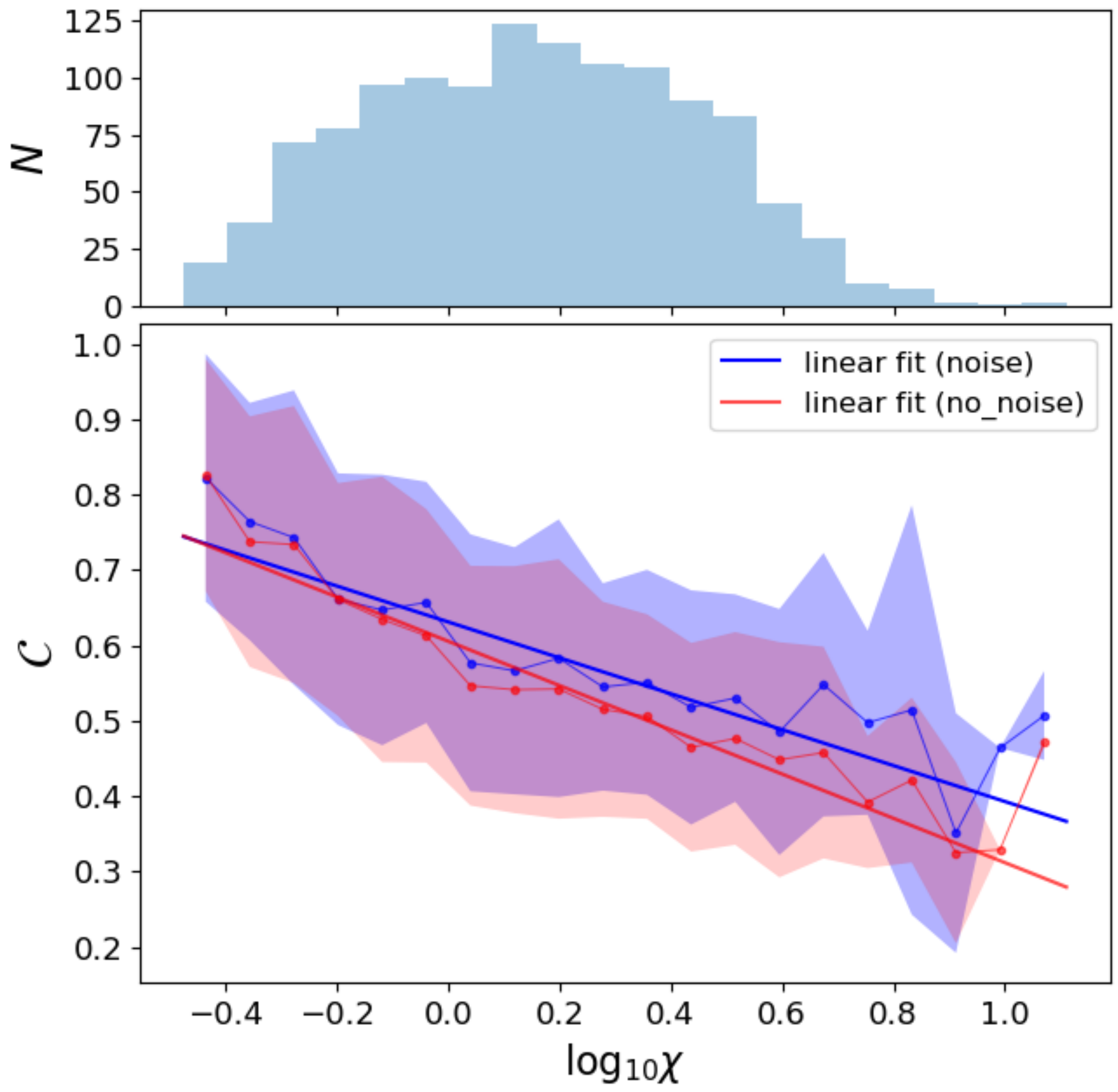
✓ mock sample well representative of the real *Planck* sample

Zernike fitting: application on *Planck*-SZ selected clusters



Zernike fitting: application on *Planck*-SZ selected clusters

✓ *The300* dynamical state is defined with χ



- best linear fit (noise) —— $C = -a\,\chi + b$

  $\boldsymbol{C = (-0.24 \pm 0.02)\,\chi + (0.63 \pm 0.01)}$

- Pearson correlation coefficient *r* with χ:

  $\boldsymbol{r = -0.38}$

- best linear fit (no_noise) —— $C = -a\,\chi + b$

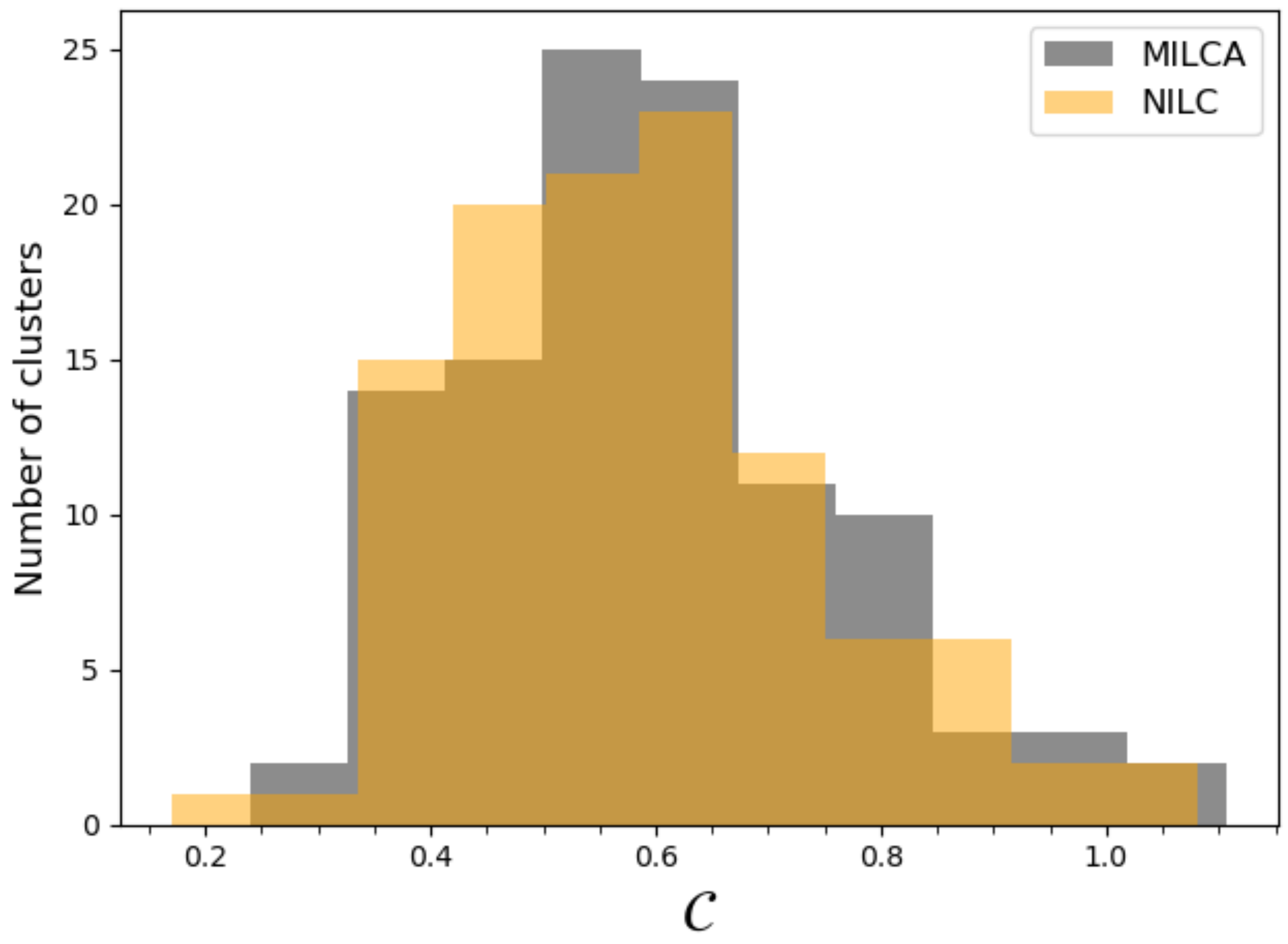
  $\boldsymbol{C = (-0.29 \pm 0.02)\,\chi + (0.61 \pm 0.01)}$

- Pearson correlation coefficient *r* with χ:

  $\boldsymbol{r = -0.47}$

Zernike fitting: application on *Planck*-SZ selected clusters

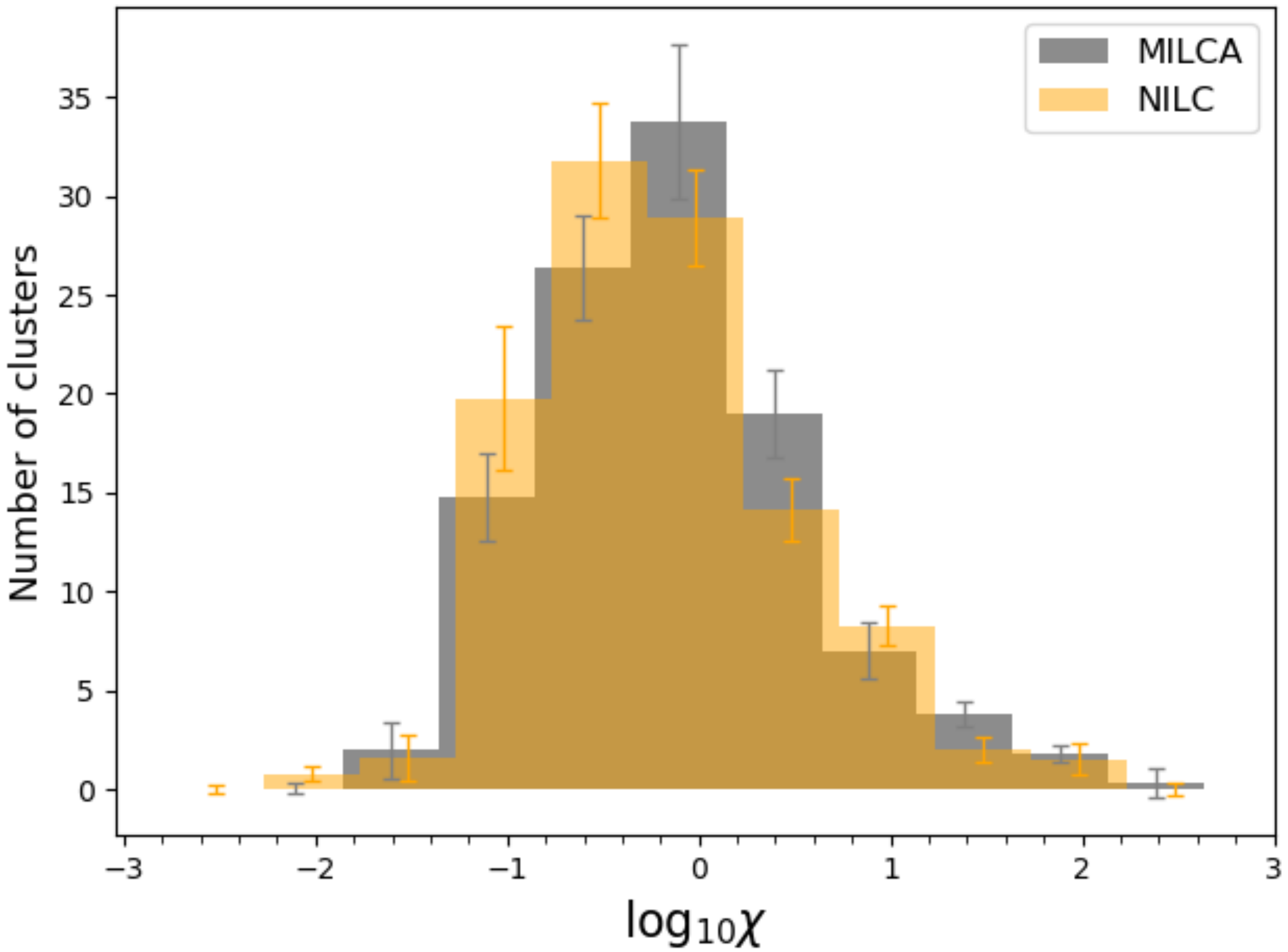
Morphology of *Planck* *y*-maps



Convert with *The300* 'calibration'

$$C = (-0.24 \pm 0.02)\,\chi + (0.63 \pm 0.01)$$

Dynamical state of *Planck* clusters



25-35% of relaxed clusters ($\log_{10}\chi > 0$)

Conclusions

- Zernike polynomials are valuable tools to study the morphology of galaxy clusters maps
- A single parameter, defined from the Zernike fitting, has a good correlation with common morphological parameters and 3D dynamical indicators
- The method is flexible: fast computation, change the number of polynomials based on the resolution you need in the modeling

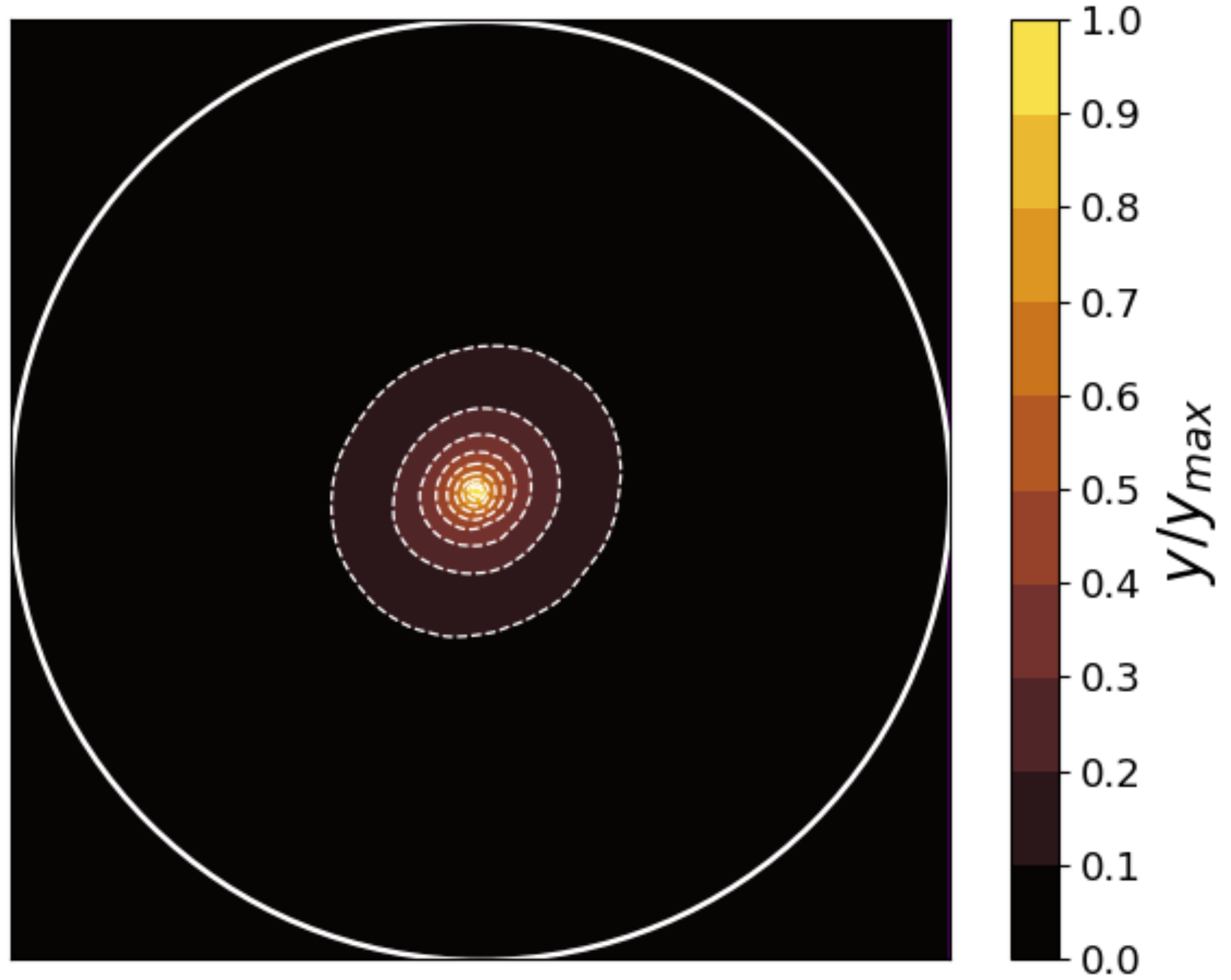
Next:

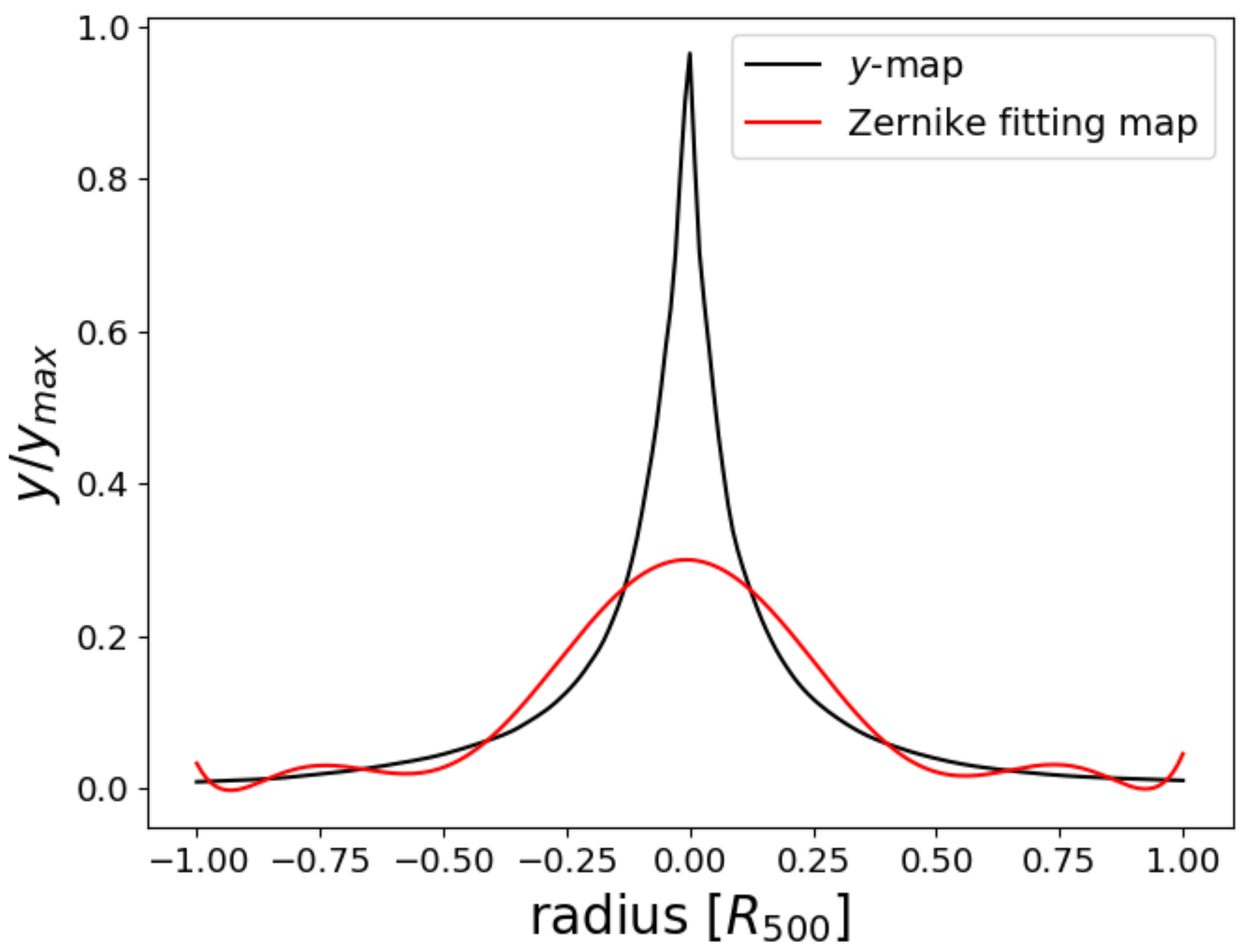
- Finalize the Zernike analysis on *Planck*-SZ clusters (comparison with other works that studied *Planck* clusters morphology at different wavelengths)
- Move to higher resolution data: ACT, SPT, NIKA2 …
- Apply the Zernike fitting on X-ray maps ⟶ (work in progress, Ferragamo A., et al., in prep.)

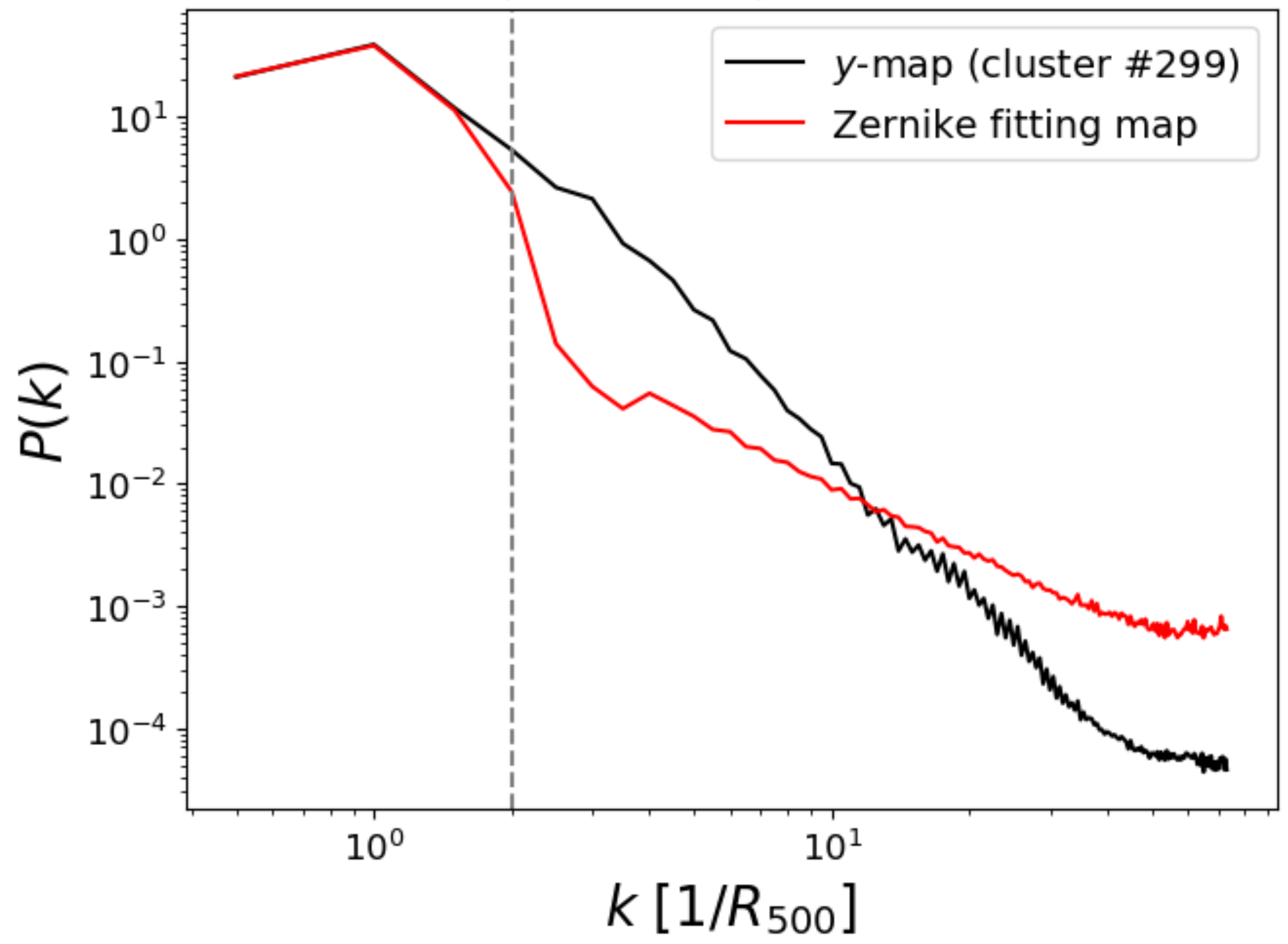
extra slides

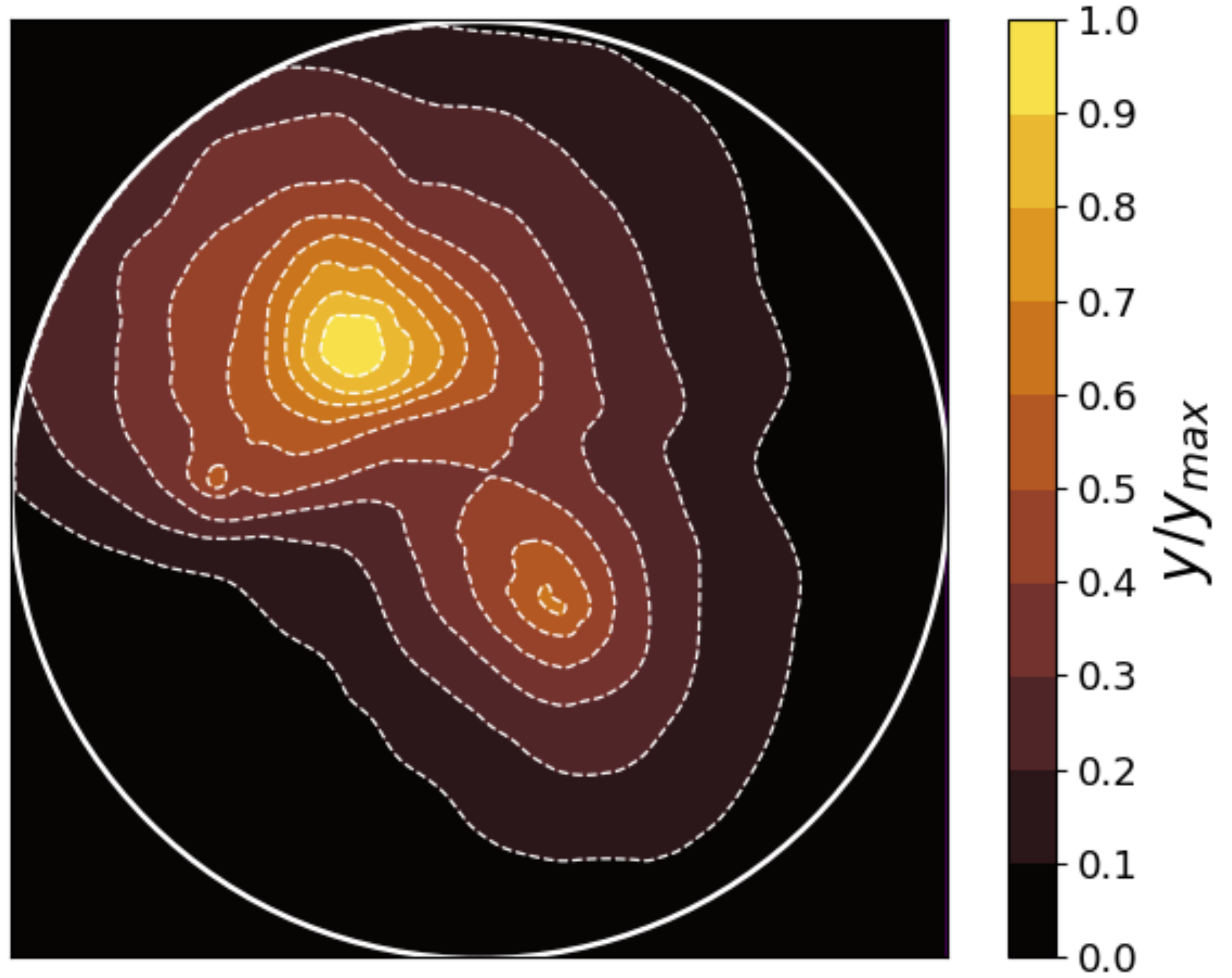
Mock y-maps for The300

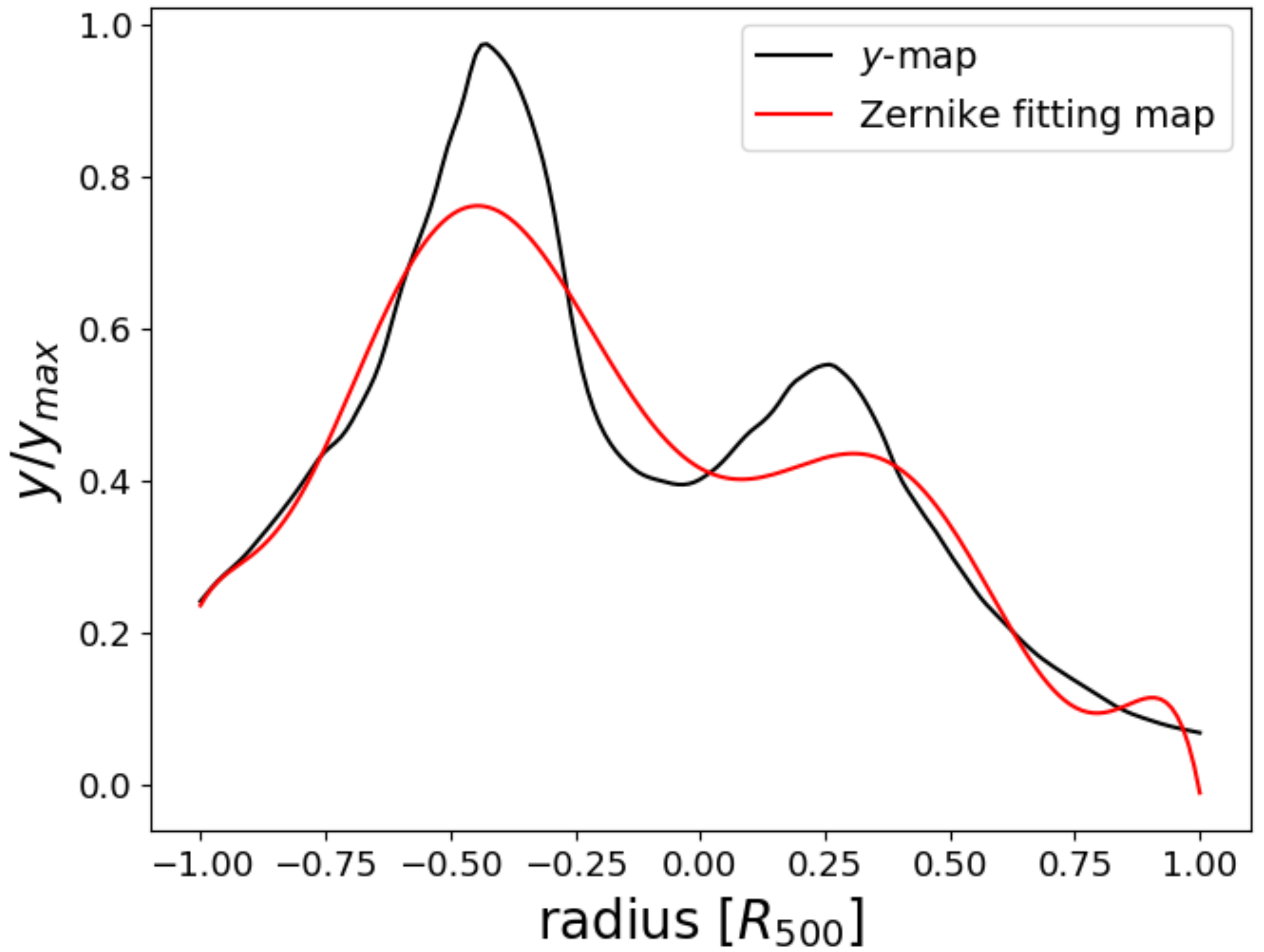
(*y*-maps, ang. res=5.2″) | sections | power spectra

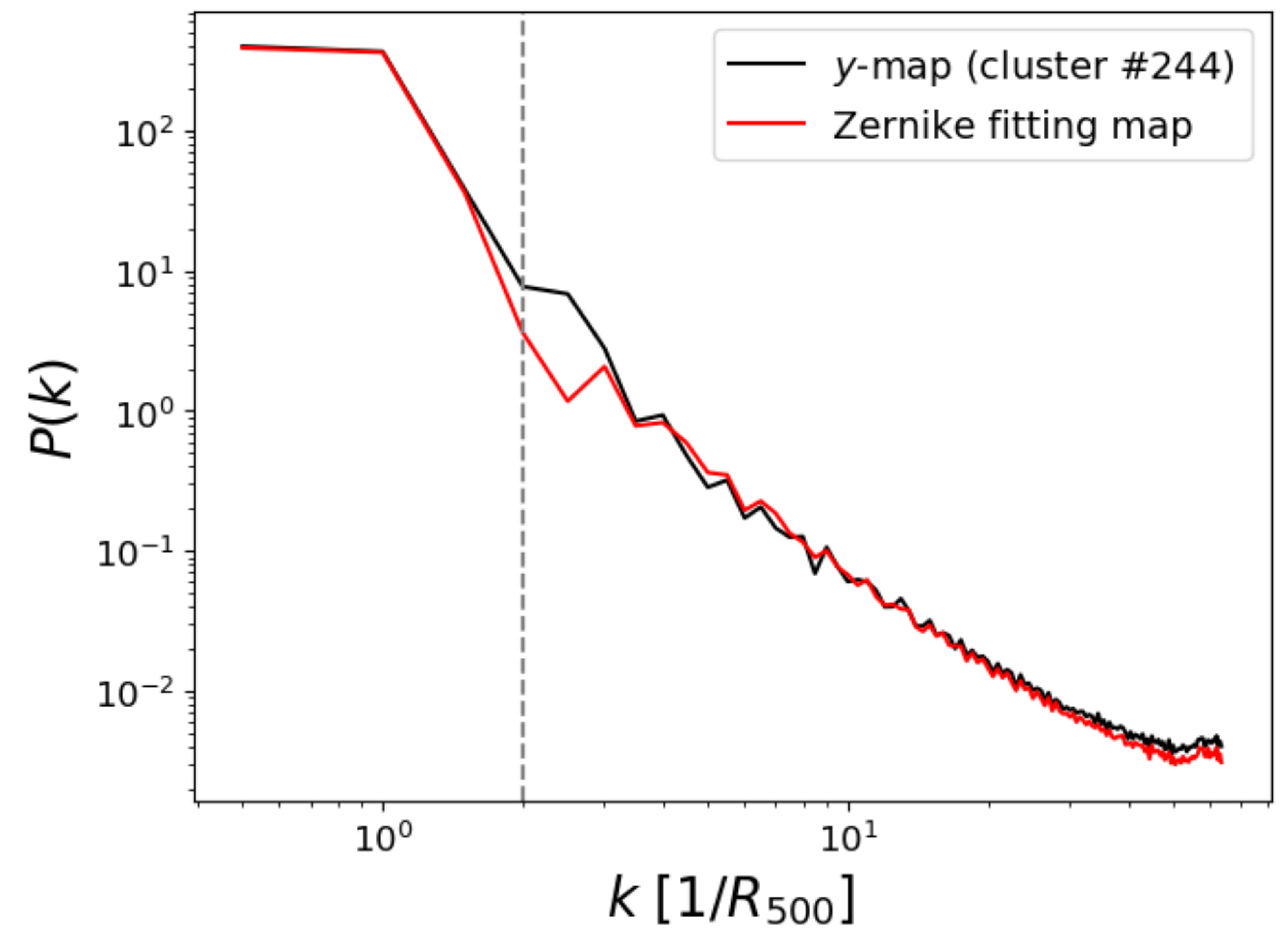


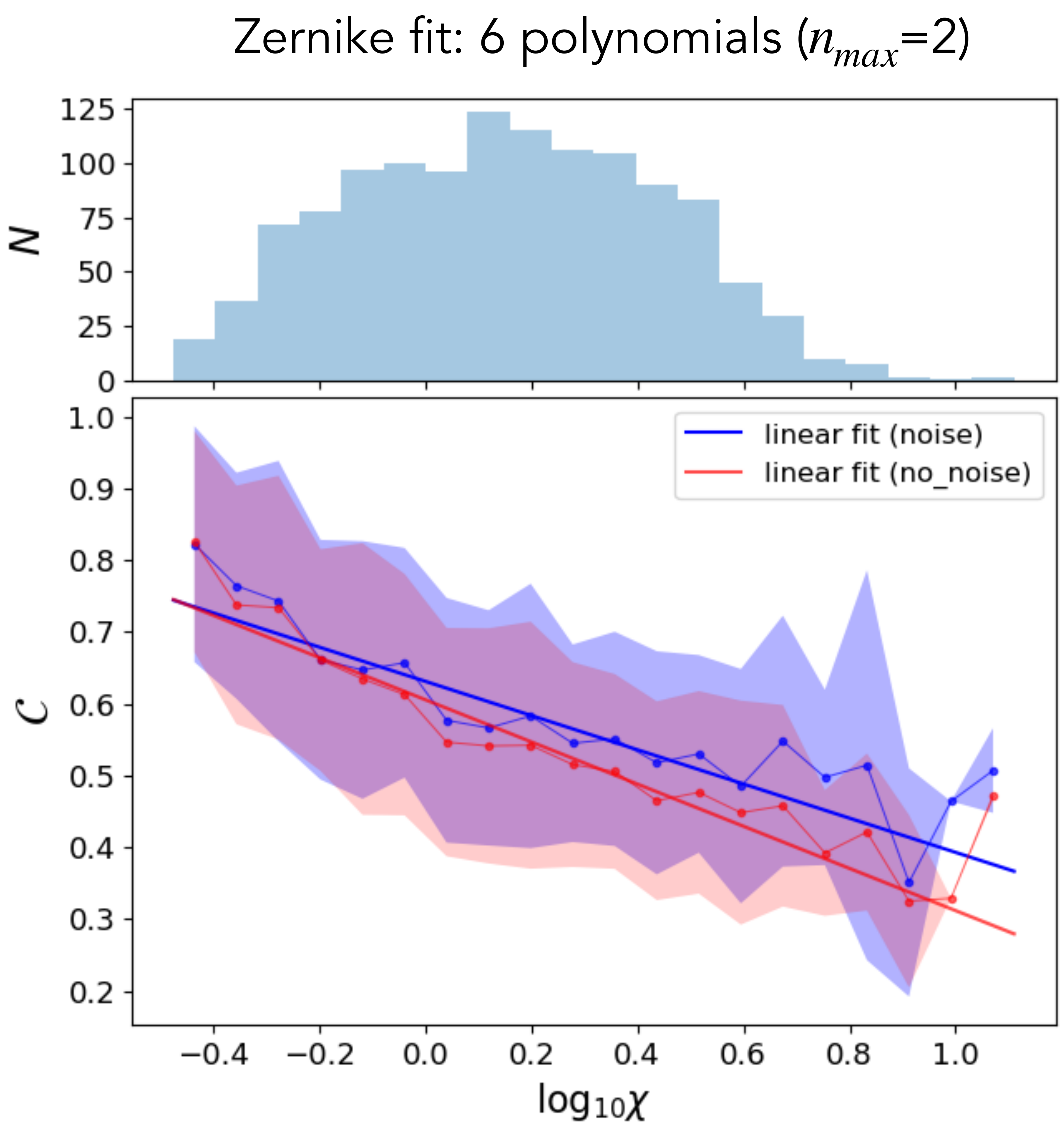


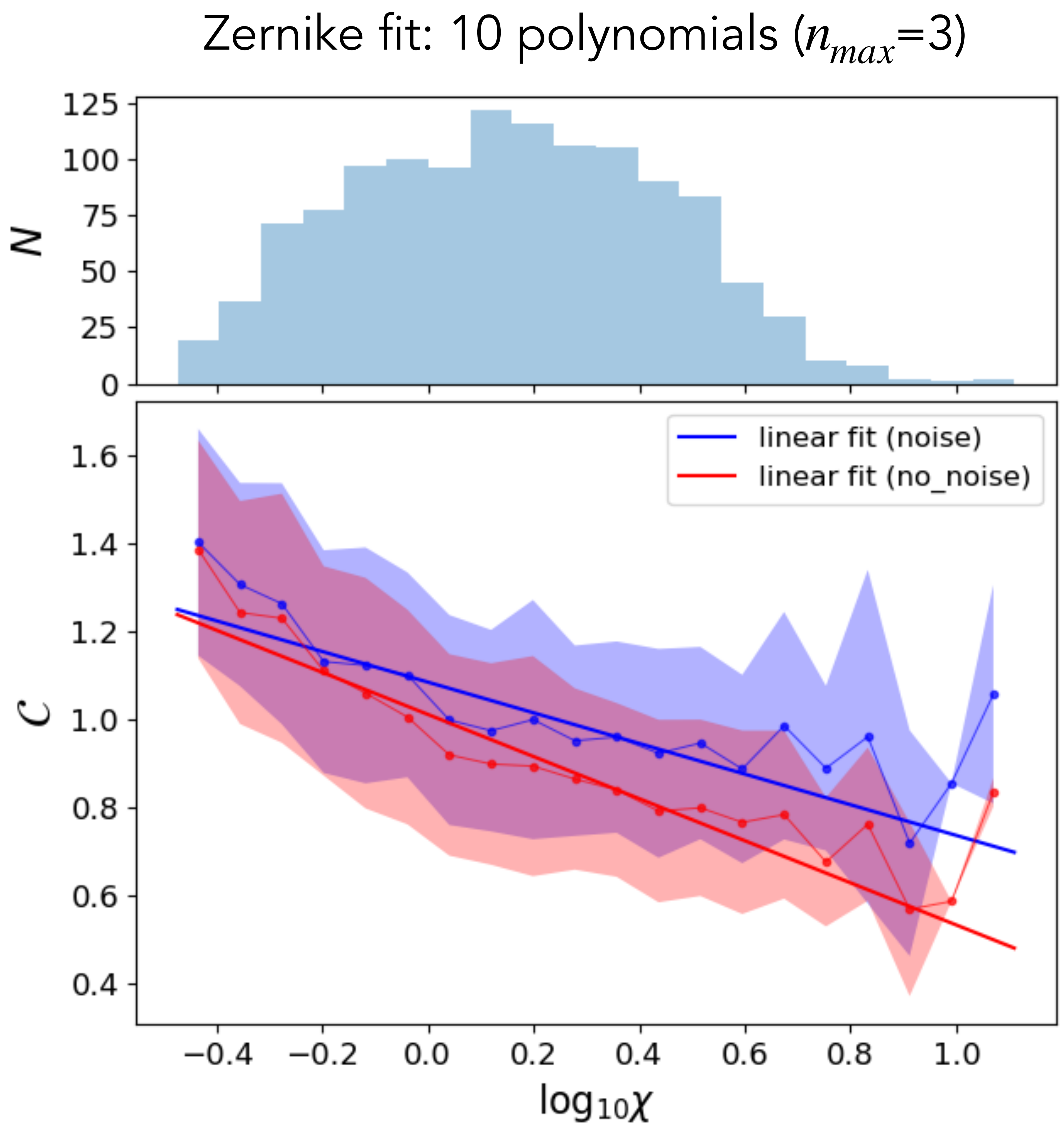












- Pearson correlation coefficient *r* with χ: r **= -0.38**
- Pearson correlation coefficient *r* with χ: r **= -0.47**

- Pearson correlation coefficient *r* with χ: r **= -0.37**
- Pearson correlation coefficient *r* with χ: r **= -0.50**