## Fluctuations in Galaxy Clusters at millimeternavelengths

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# Fluctuations in Galaxy Cluster at millimeter wavelengths 

## How to quantify fluctuations?

1. Via variance (or standard deviation)
a. Could be in surface brightness or in deprojected profiles
b. E.g. Zhuravleva+ $(2013,2023)$, Ueda+ (2018), Hofmann+ (2016)

2. Via a power (or amplitude) spectrum
a. Again, either in surface brightness or deprojected

Figures from Hofmann+ (2016)


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b. E.g. Churazov+ (2012), Zhurvleva+ (2015), Khatri \& Gaspari (2016)

k, $\mathrm{kpc}^{-1}$


Figures from Zhuravleva+ (2015)

## Pros and cons

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3. Via variance (or standard deviation)
a. Fairly intuitive
b. Potentially unclear which scales are being probed
4. Via a power (or amplitude) spectrum
a. Explicitly denotes which scales are being probed
b. Requires attention to (many) details
c. Can you just take a FFT?


## What can we get from fluctuations?

- Mach number(s)

$$
\mathcal{M}_{3 \mathrm{D}} \approx 4 A_{\rho}\left(k_{\mathrm{peak}}\right) \approx 2.4 A_{P}\left(k_{\text {peak }}\right)
$$

- Supported by many theoretical studies
- Assumes that fluctuations are predominantly caused by turbulence
- or at least quasi-turbulence

$$
\sigma_{\ln \bar{P}}^{2}=\ln \left[1+b^{2} \gamma^{2} \mathcal{M}^{4}\right]
$$

E.g. Mohaptra+ (2021a)

## And why might we care about Mach numbers?

- Hydrostatic bias is a concern, which we think that is dominated by (quasi)-turbulence.

$$
\begin{gathered}
b_{\mathcal{M}}=\frac{-\gamma \mathcal{M}_{3 \mathrm{D}}^{2}}{3} \frac{d \ln P_{\mathrm{NT}}}{d \ln P_{\mathrm{th}}}\left(1+\frac{\gamma \mathcal{M}_{3 \mathrm{D}}^{2}}{3} \frac{d \ln P_{\mathrm{NT}}}{d \ln P_{\mathrm{th}}}\right)^{-1} \\
\frac{\mathrm{~d} \ln P_{\mathrm{nt}} / \mathrm{d} \ln r}{\mathrm{~d} \ln P / \mathrm{d} \ln r}=1+2 \frac{\mathrm{~d} \ln \mathrm{Ma}_{3 \mathrm{~d}} / \mathrm{d} \ln \mathrm{r}}{\mathrm{~d} \ln P / \mathrm{d} \ln r}
\end{gathered}
$$

(Khatri \& Gaspari 2016)

## Theoretical underpinnings of fluctuation analyses

- Want to understand scales of processes involved and modes of conduction, e.g.
- Injection scale
- Damping scales
- We expect that something like turbulence will dominate the cascading fluctuations.


Adapted from Gaspari+ 2014

## Methodology - overview

Part 1: obtain a $\delta y / y$ or $\delta S / S$ map

1. Fit a surface brightness profile to the cluster
a. Parametric or non-parametric model?
b. Which centroid?
c. Elliptical or circular?
d. Nuisance parameters (e.g. background or DC offset)?
2. Subtract that model
3. Divide the residual by the (ICM-only) model

Part 2: calculate power spectra

1. We adopt the delta-variance method in Arevalo+ (2012)
a. It handles an arbitrary mask well; this allows for flexible regions and masking (mostly of point sources)
b. Need to worry about biases.
c. noise/ de-biasing considered, cf. Mark Bishop's talk
2. Choose region(s)
3. Correct for PSF smoothing
4. Deproject to $\delta P / P$ or $\delta \rho / \rho$

## MUSTANG-2 and XMM observations

## Zwicky 3146

|  | MUSTANG-2 | XMM (uncleaned) |
| :--- | :--- | :--- |
| On-source time | $22.7 \mathrm{hrs}(82 \mathrm{ks})$ | $56.5+64.9+122.8 \mathrm{ks}$ |

- $\sim 8 \times 10^{14} \mathrm{M}_{\odot}$
- Relaxed, cool-core cluster with a mini-halo
- Sloshing noted as far back as Forman (2002) evident in our residuals



## MUSTANG-2 and XMM observations

Our choice of annuli is motivated primarily by the radially-varying noise profile in the MUSTANG-2 data.

We also expect fluctuations to vary with radius (as non-thermal pressure support varies with radius)

XMM PSF and exposure also vary radially.



## MUSTANG-2 and XMM observations

Physical scale (kpc)
$10^{2}$



- From the 2D spectra, Rings 2 and 3 are non-significant or marginally significant

Physical scale (kpc) $10^{2}$



## Deprojecting spectra






## Formalism:

$$
\begin{aligned}
& \quad P_{2 \mathrm{D}}\left(k_{\theta}\right)=\int P_{3 \mathrm{D}}(\mathbf{k})\left|\tilde{W}\left(k_{z}\right)\right|^{2} d k_{z} \\
& P_{2 \mathrm{D}}\left(k_{\theta}\right) \approx P_{3 \mathrm{D}}(\mathbf{k}) \int\left|\tilde{W}\left(k_{z}\right)\right|^{2} d k_{z} \\
& \text { where }
\end{aligned}
$$

$$
\begin{aligned}
W_{\mathrm{SZ}}(\theta, z) & \equiv \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{e}} c^{2}} \frac{\bar{P}(\theta, z)}{\bar{y}(\theta)} \\
W_{\mathrm{X}}(\theta, z) & \equiv \frac{\bar{\epsilon}(\theta, z)}{\bar{S}(\theta)}
\end{aligned}
$$

and

$$
N(\theta) \equiv \int\left|\tilde{W}\left(k_{z}\right)\right|^{2} d k_{z}
$$

## Thermodynamic fluctuations

- Again, for SZ and X-ray, Rings 2 and 3 are at most marginally significant.
- We prefer not to make inferences on these regions.
- Amplitudes of $\log _{10}\left(\mathrm{~A}_{\text {peak }}\right) \sim$ (-1 $\pm 0.3$ ) are $\sim$ common; cf.
Churazov+ (2012),
Zhuravleva+ (2015), Khatri \& Gaspari (2016) Ettori's slides)



## Inferences

- We predominantly focus on Ring 1
- ~large turbulent velocities inferred (for being "relaxed")
- consistent between SZ and X-ray ( P and n )
- perhaps seen elsewhere (cf. Ettori's talk)

|  |  | $\mathcal{M}_{3 \mathrm{D}, \text { peak }}$ | $\mathcal{M}_{3 \mathrm{D}, \text { int }}$ |
| :--- | :---: | :---: | :---: |
| Ring 1 | $\delta \rho / \rho$ | $0.53 \pm 0.01$ | 0.32 |
|  | $\delta P / P$ | $0.69 \pm 0.19$ | 0.80 |
| Ring 2 | $\delta \rho / \rho$ | $0.43 \pm 0.14$ | 0.38 |



## Inferences

- Can infer a hydrostatic bias
- We predominantly focus on Ring 1
- ~large turbulent velocities inferred (for being "relaxed")
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b_{\mathcal{M}}=\frac{-\gamma \mathcal{M}_{3 \mathrm{D}}^{2}}{3} \frac{d \ln P_{\mathrm{NT}}}{d \ln P_{\mathrm{th}}}\left(1+\frac{\gamma \mathcal{M}_{3 \mathrm{D}}^{2}}{3} \frac{d \ln P_{\mathrm{NT}}}{d \ln P_{\mathrm{th}}}\right)^{-1}
$$

$$
\frac{\mathrm{d} \ln P_{\mathrm{nt}} / \mathrm{d} \ln r}{\mathrm{~d} \ln P / \mathrm{d} \ln r}=1+2 \frac{\mathrm{~d} \ln \mathrm{Ma}_{3 \mathrm{~d}} / \mathrm{d} \ln \mathrm{r}}{\mathrm{~d} \ln P / \mathrm{d} \ln r}
$$

but we need to have a (logarithmic) pressure and Mach slope - we have this between Rings 1 and 2

$$
-b_{\mathcal{M}}=0.16 \pm 0.04
$$

This is somewhat interesting, but this this is only for the ~inner 100"

## Application to SPT clusters




## Application to SPT clusters

- 60 clusters w/ XMM
- $z>0.2$
- $M_{500}>3 \mathrm{e} 14 \mathrm{M}_{\odot}$
- Median mass: 6.35e14
- Median redshift: 0.45


Pilot investigations with two clusters:

## SPT-CLJ0232-4421:

$$
z=0.28 ; M_{500}=9.54 e 14
$$

~relaxed
SPT-CLJ0638-5358:

$$
z=0.23 ; M_{500}=9.42 \mathrm{e} 14
$$

~disturbed


## Preliminary results

SPT-CLJ0232-4421:

- Mach numbers $\sim 0.7$ from X-ray; ~0.24 from SZ


## SPT-CLJ0638-5358:

- Mach numbers 0.8 to 1.3 from X-ray and $\sim 0.7$ from SZ


SPT-CLJ0232-4421
Physical scale (kpc)



## Preliminary results

## SPT-CLJ0232-4421:

- Lovisari+ (2017) and Hudson+ (2010) conclude that it is relaxed and a weak cool-core, respectively
- Parekh+ (2021) and Kale+ (2019) find evidence for disturbance in X-ray and noting radio relic(s), respectively



## SPT-CLJ0638-5358:

- Botteon, Gastaldello, \& Brunetti (2018) find a shock ~2' SW of the cluster core but did not find an evident SB jump to the NE.
- They find $M \sim 1.7$ via $S B$ and temperature.



## Ongoing work

- Finalizing the choice of ring radii
- Balance observational/statistical output with physical motivations
- By extension, investigating fluctuations by sector/slice
- Worse statistics, but perhaps informative
- Qualify if not quantify the impact choice of center and ellipticity
- To quote Zhuravleva+ (2015): "Going beyond this simple spherically-symmetric model implies that we believe that the underlying cluster potential is more intricate. It is not clear to what degree of complexity of the model we should go. There is always a danger that some of the structures unrelated to the cluster gravitational potential are removed."
- See I. Bartalucci's talk (peak vs. centroid ~ negligible beyond $\sim .2 \mathrm{R}_{500}$ )
- Reconcile Mach numbers with "filling factors". Or perhaps "aperture dilution" is a better term.


## Conclusions

- mm observations are now players in the "game" of surface brightness (thermodynamic) fluctuations of the ICM.
- If we want to infer a hydrostatic bias at $R_{500}$ via this method, then we need much deeper observations (in X-ray or SZ - deeper than we have*)
- The canonical benefits of SZ are fantastic here: redshift independence + SB doesn't drop as quickly with radius.
- Here, SPT-3G is among the most promising near-term advancements
- If we want to more information on the spectral shape, we high angular resolution is imperative


## *for most clusters

## CENTER FOR

GREEN BANK OBSERVATORY

## ASTROPHYSICS

HARVARD \& SMITHSONIAN

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Appendix / Backup slides

## Deprojection check

Assuming a 3D spectrum with spectral index of $\alpha=3$ (convention $P_{k} \propto k^{-\alpha}$ ), we find that "at worst" a $20 \%$ bias is imparted; i.e. we underestimate $A_{3 D}$ by 20\%


## Beam / PSF bias



Figure B1. Bias $\bar{P} / P$ in the normalization of the recovered spectrum for a pure power-law power spectrum, as a function of slope for different dimensions of the problem (red - one dimension; blue - two dimensions; and black - three dimensions).

Physical scale (kpc) $10^{2}$

## Beam / PSF bias

Arevalo+ (2012) outlined how to calculate the bias in their method; adding (as in multiplying by) Gaussians is "relatively" trivial.

However, like the single power law bias, you still need to know (or at least guess) at the underlying spectral index.

$$
\begin{aligned}
V_{k_{r}} & =\int P_{\mathrm{u}}(k)\left[\sum_{i=1}^{N} \sum_{j=1}^{N} c_{i} c_{j} e^{-k^{2} / k_{i}^{2}} e^{-k^{2} / k_{j}^{2}}\right]\left[2 \epsilon\left(\frac{k}{k_{r}}\right)^{2} e^{-\left(k / k_{r}\right)^{2}}\right]^{2} d^{n} k \\
& =\sum_{i=1}^{N} \sum_{j=1}^{N} 4 \epsilon^{2} \int P_{\mathrm{u}}(k) c_{i} c_{j} e^{-k^{2} /\left(x_{i} k_{r}\right)^{2}} e^{-k^{2} /\left(x_{j} k_{r}\right)^{2}}\left(\frac{k}{k_{r}}\right)^{4} e^{-2\left(k / k_{r}\right)^{2}} d^{n} k
\end{aligned}
$$

and via the same variable recasting, we derive a new bias formulation:

$$
\frac{\tilde{P}}{\tilde{P}\left(k_{i}\right)}=2^{\alpha / 2}\left[\sum_{i=1}^{N} \sum_{j=1}^{N} \cos _{j}\left(\frac{2 x_{x}^{2} x_{3}^{2}+x_{i}^{2}+x_{j}^{2}}{2 x_{i}^{2} x_{j}^{2}}\right)^{n / 2+2-\alpha / 2}\right] \frac{\Gamma(n / 2+2-\alpha / 2)}{\Gamma(n / 2+2)} .
$$

Physical scale (kpc)
$10^{2}$

## Radial uncertainties

Inferred minimum uncertainties as a function of circular aperture radius.


## SPT-CLJ0638-5358 slices

SW and NE slices (i.e. 2 and 4) show larger fluctuations than slices 1 and 3.



SPT-CLJ0638-5358
Physical scale (kpc)


