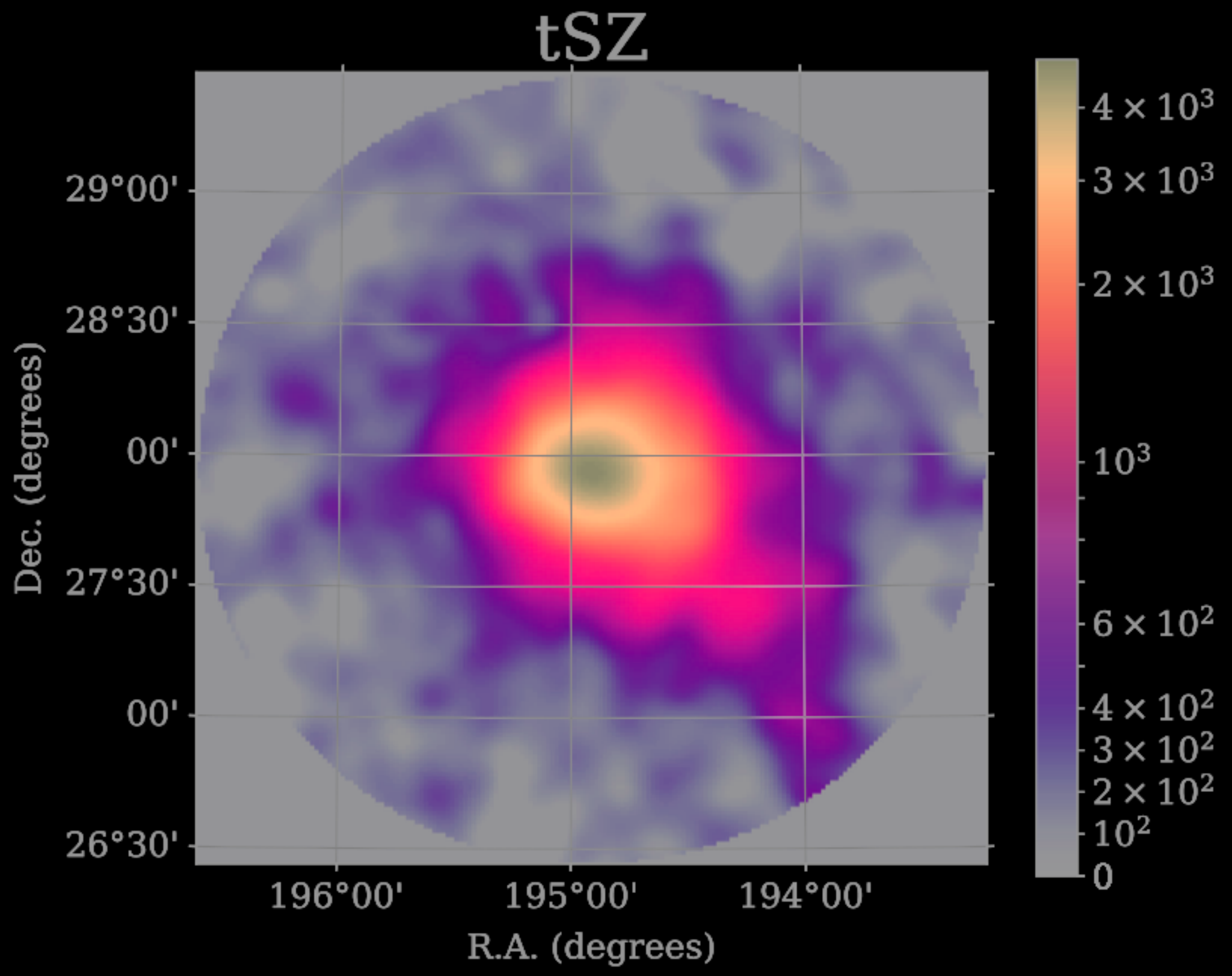
Extracting Gas Pressure Profiles of Galaxy Clusters

mm Universe 2023
LPSC Grenoble

Kang Wang, Yvette Perrott, Richard Arnold, David Huijser

VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA


Swift
Fermi
Chandra
NuSTAR
GALEX
Kepler
Hubble
Spitzer
Herschel
Planck
Spektr-R
gamma ray
X-ray
ultraviolet
visible
infrared
microwave
radio
SOFIA
Keck
SALT
Gemini
HESS
CARMA
Greenbank
VLA
IC: NASA



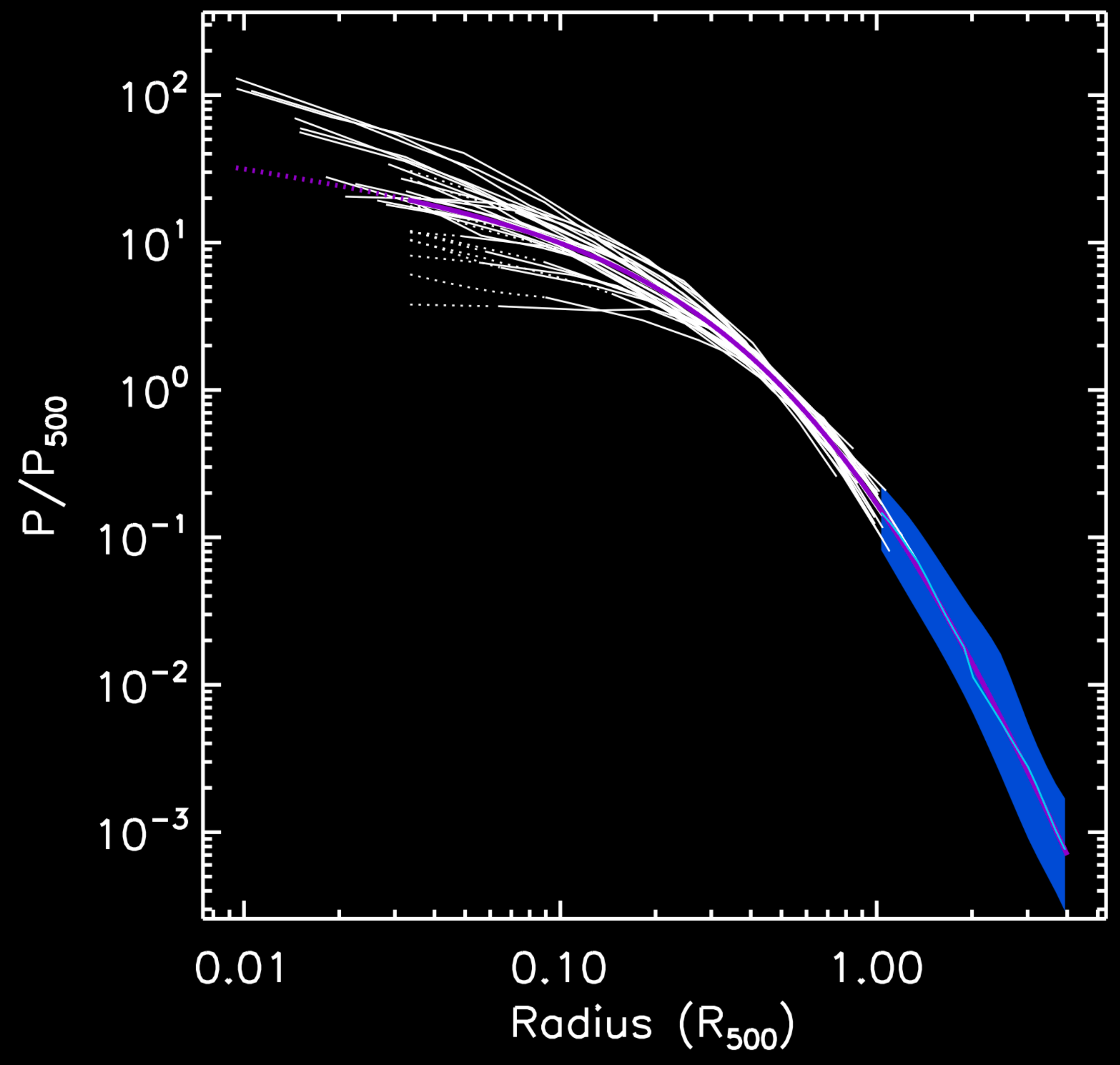
The tSZ signal of Coma cluster from Planck Legacy Archive (Adam et al., 2021)

$$y(r_{proj}) = \int \frac{\sigma_T}{m_e c^2} P(r) d\ell$$

Cluster Pressure Profiles

gNFW model

$$P(r) = \frac{P_{ei}}{\left(\frac{r}{r_s}\right)^{\gamma}\left[1 + \left(\frac{r}{r_s}\right)^{\alpha}\right]^{(\beta-\gamma)/\alpha}}$$



Arnaud et al., 2010

gNFW Model

Parameter and prior

$$y = \frac{\sigma_T}{m_e c^2} \int_{-\infty}^{\infty} \frac{P_{ei}}{\left(\frac{\sqrt{r_{proj}^2 + \ell^2}}{r_s}\right)^{\gamma} \left[1 + \left(\frac{\sqrt{r_{proj}^2 + \ell^2}}{r_s}\right)^{\alpha}\right]^{(\beta-\gamma)/\alpha}} d\ell$$

$$r_{proj} = \sqrt{(x - corefindX)^2 + (y - corefindY)^2}$$

$$corefindX \sim U(0,2.55362)\ Mpc$$

$$corefindY \sim U(0,2.55362)\ Mpc$$

$$\alpha \sim U(0,2)$$

$$\beta \sim U(1,10)$$

$$\gamma \sim U(0.001,1)$$

$$r_s \sim U(1,1.915)\ Mpc$$

$$P_{ei} \sim U(0,10^{-18})\ \frac{M_{\odot}}{Mpc \cdot s^2}$$

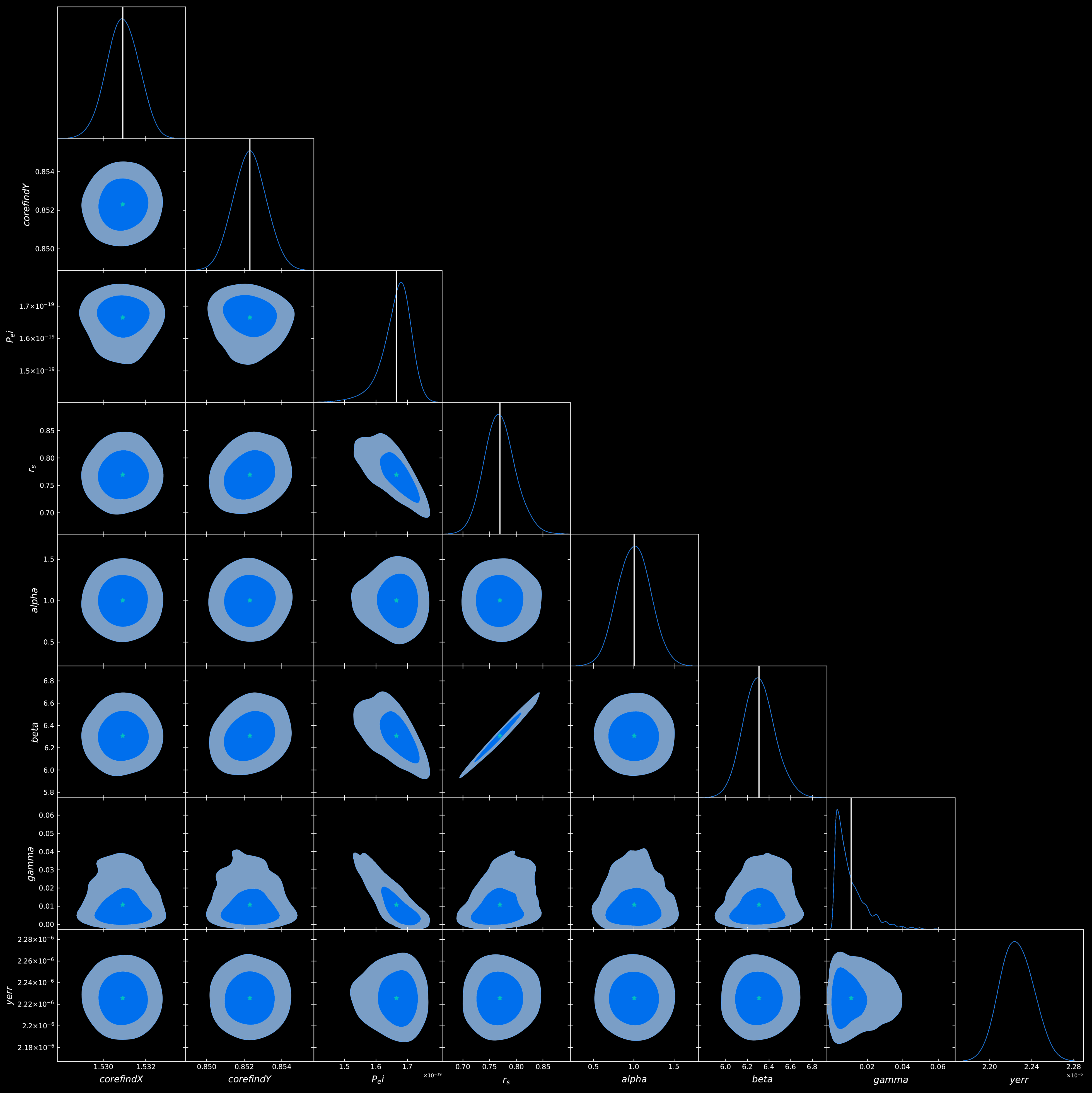
gNFW Model
Likelihood Function

$$\Pr(\mathbf{D}\,|\,\Theta, \mathbf{M}) = (2\pi)^{-N/2} \exp\left[-\chi^2/2\right] \prod_{k=1}^{N} \sigma_k^{-1}$$

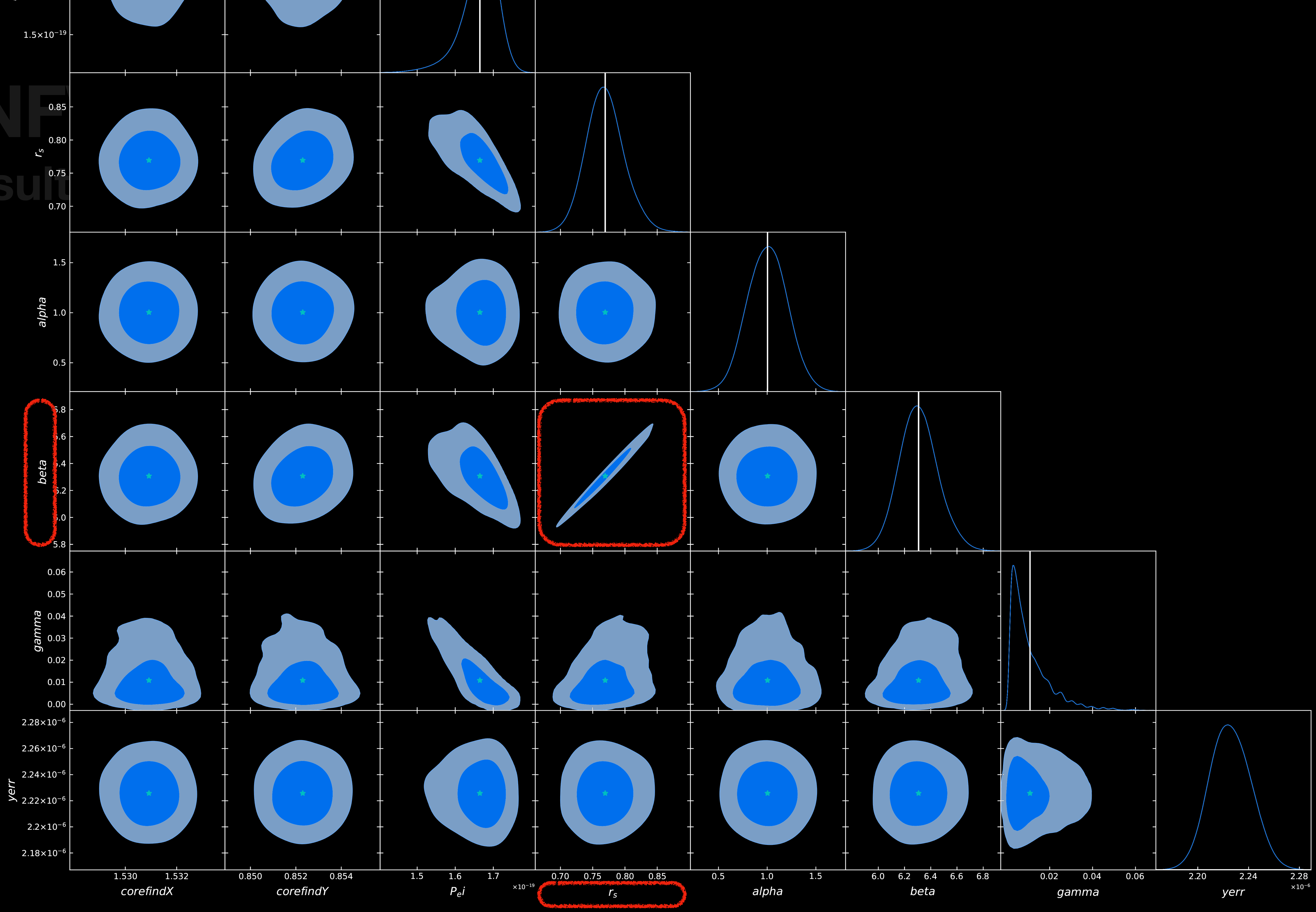
$$\ln \Pr(\mathbf{D}\,|\,\Theta, \mathbf{M}) = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\chi^2 - \sum_{k=1}^{N} \ln \sigma_k$$

$$\chi^2 = \sum_{k=1}^{N} \left(\frac{\mathbf{D}_k - \mathbf{M}_k}{\sigma_k}\right)^2$$

gNFW Model
Results



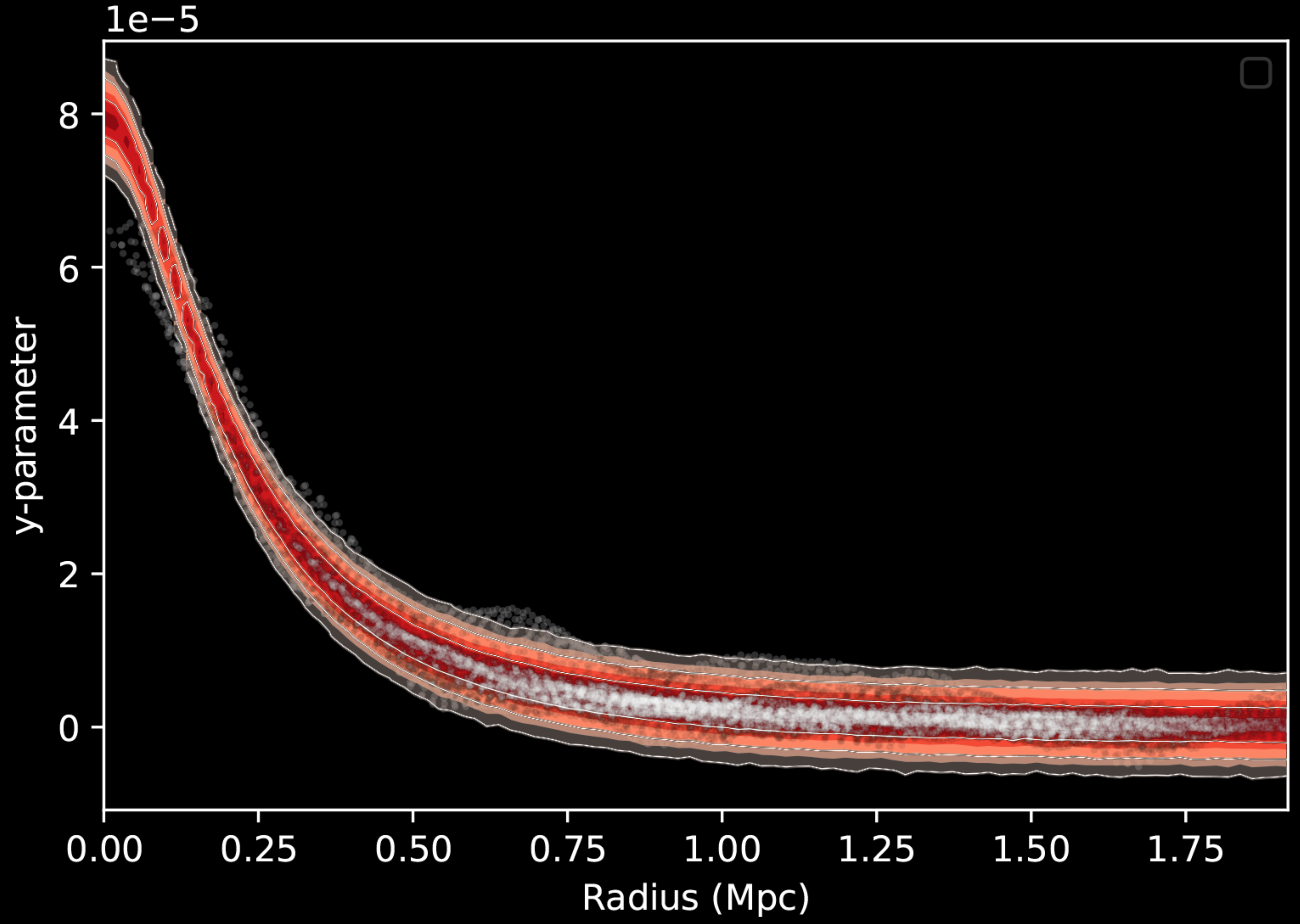
gNF
Result



1.5×10⁻¹⁹
0.85
0.80
0.75
0.70
r_s
1.5
1.0
0.5
alpha
beta
0.06
0.05
0.04
0.03
0.02
0.01
0.00
gamma
2.28×10⁻⁶
2.26×10⁻⁶
2.24×10⁻⁶
2.22×10⁻⁶
2.2×10⁻⁶
2.18×10⁻⁶
yerr
1.530
1.532
corefindX
0.850
0.852
0.854
corefindY
1.5
1.6
1.7
×10⁻¹⁹
P_ei
0.70
0.75
0.80
0.85
r_s
0.5
1.0
1.5
alpha
6.0
6.2
6.4
6.6
6.8
beta
0.02
0.04
0.06
gamma
2.20
2.24
2.28
×10⁻⁶
yerr

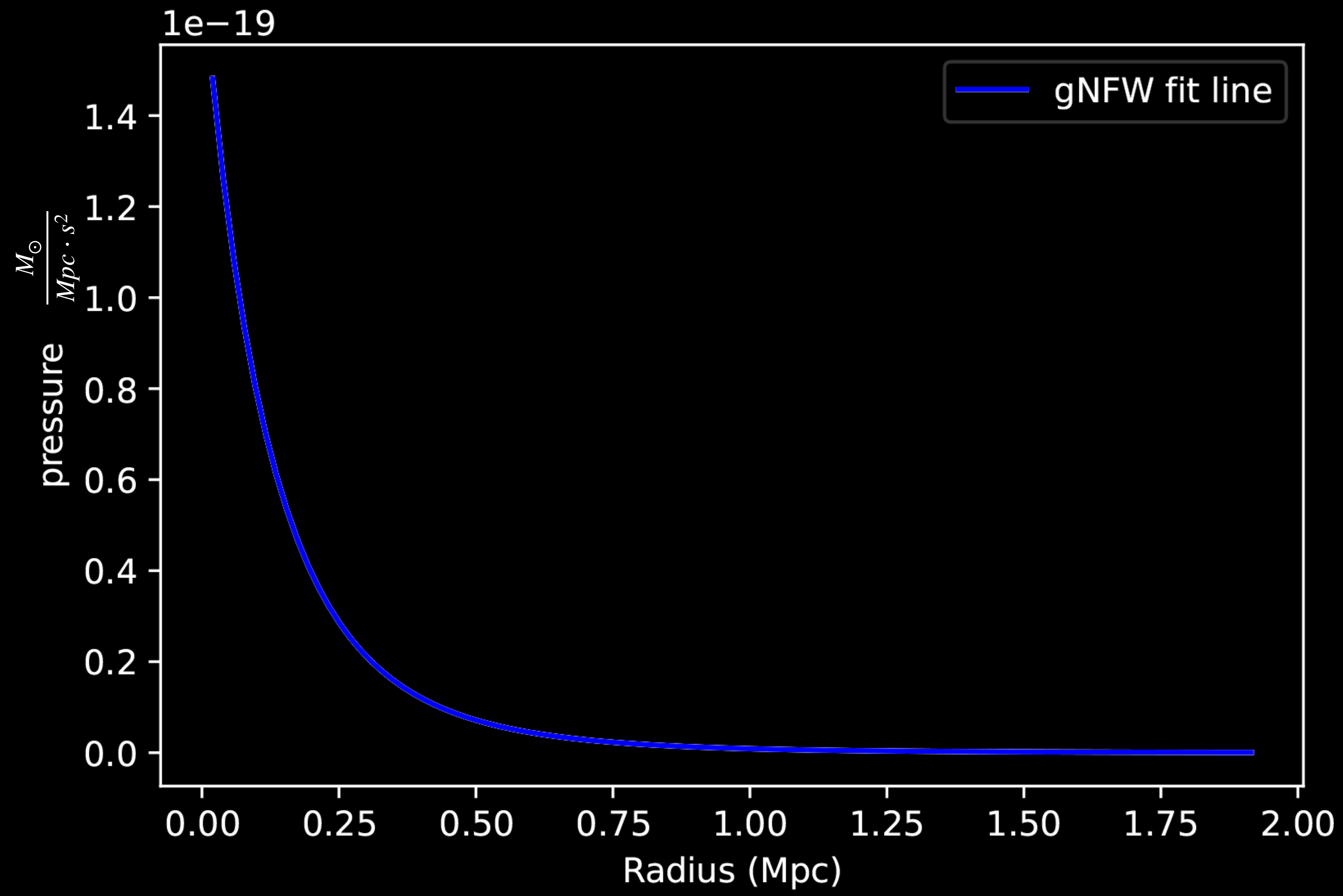
gNFW Model

Results



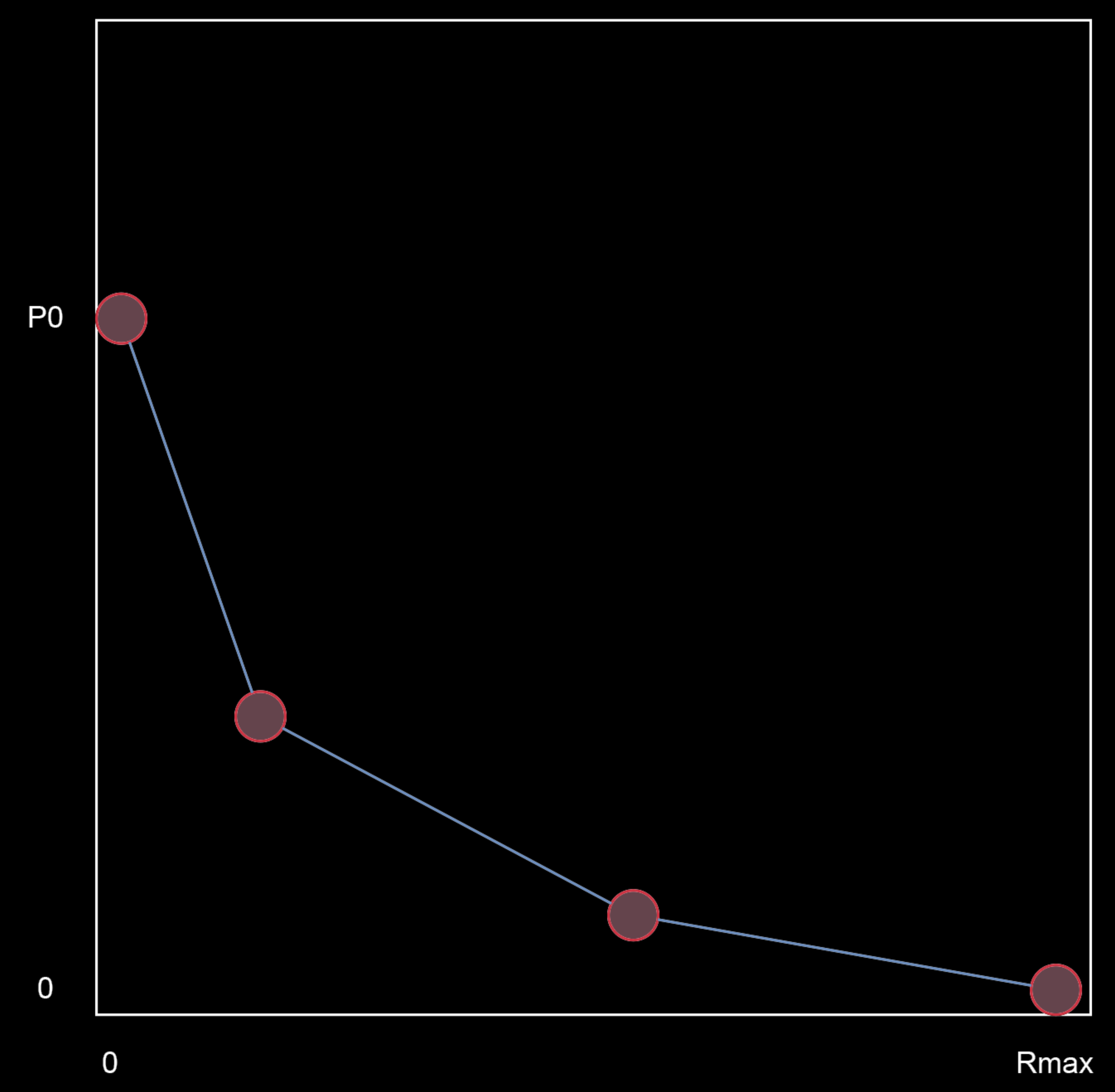
gNFW Model

Results



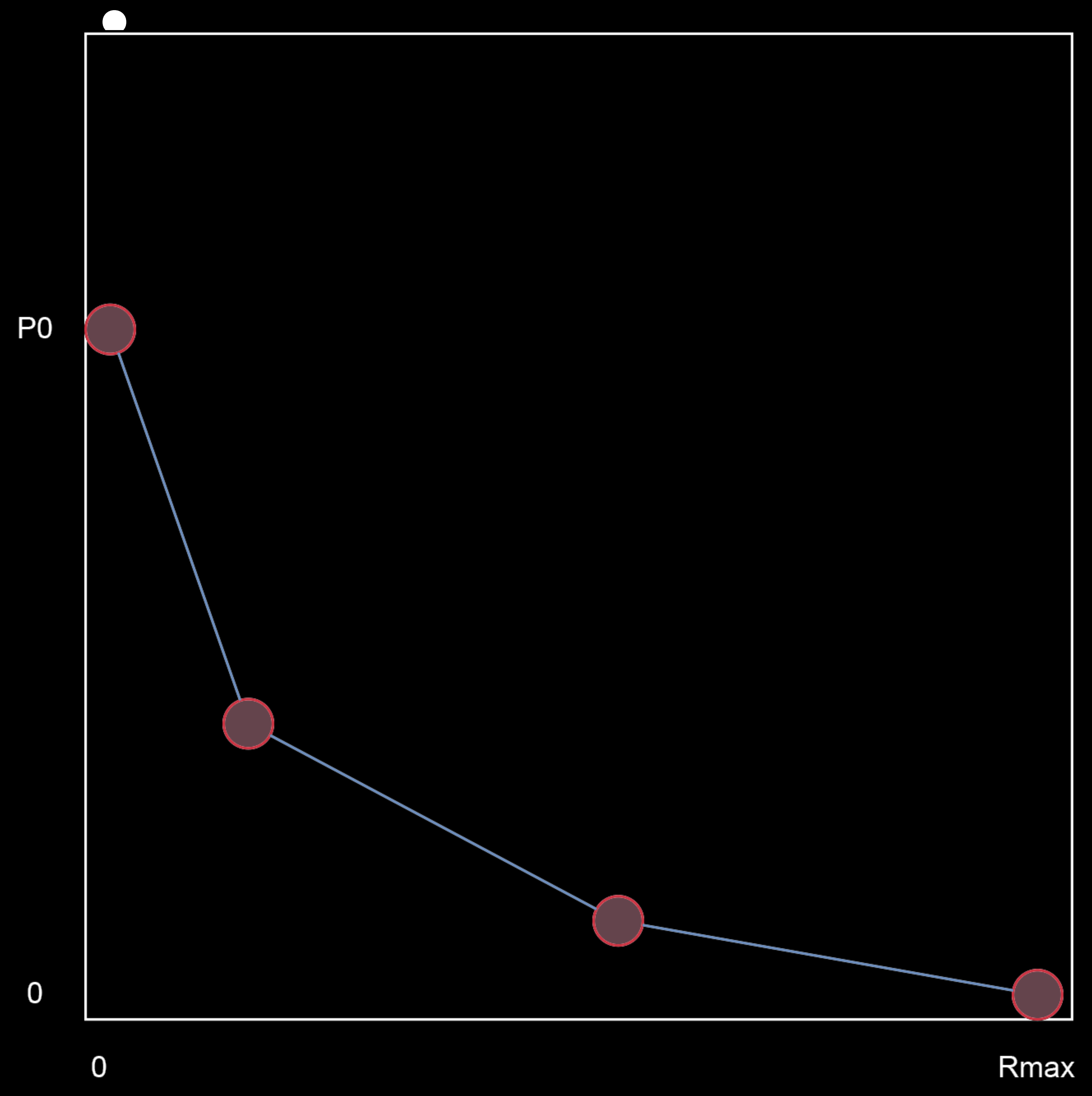
How to build a flexible model?

Nodal Model



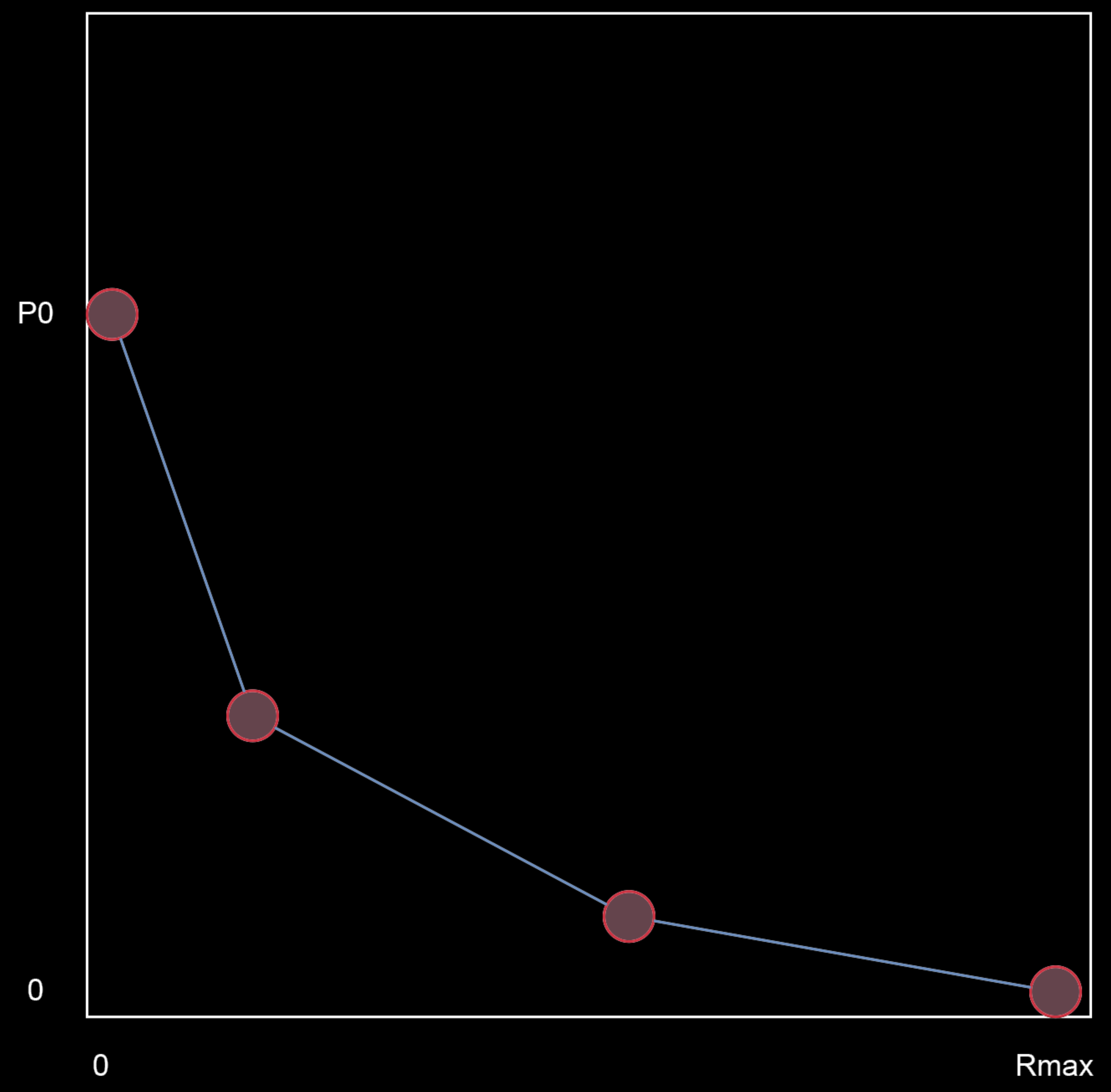
Olamaie et al., 2018

Nodal Model



$$P\left(r\right) = \left(\frac{P_0 - P_1}{r_0 - r_1}\left(r - r_0\right) + P_0\right) 1_{\left[r_0 \le r \le r_1\right]} + \left(\frac{P_1 - P_{max}}{r_1 - r_{max}}\left(r - r_1\right) + P_1\right) 1_{\left[r_1 \le r \le r_{max}\right]}$$

Nodal Model

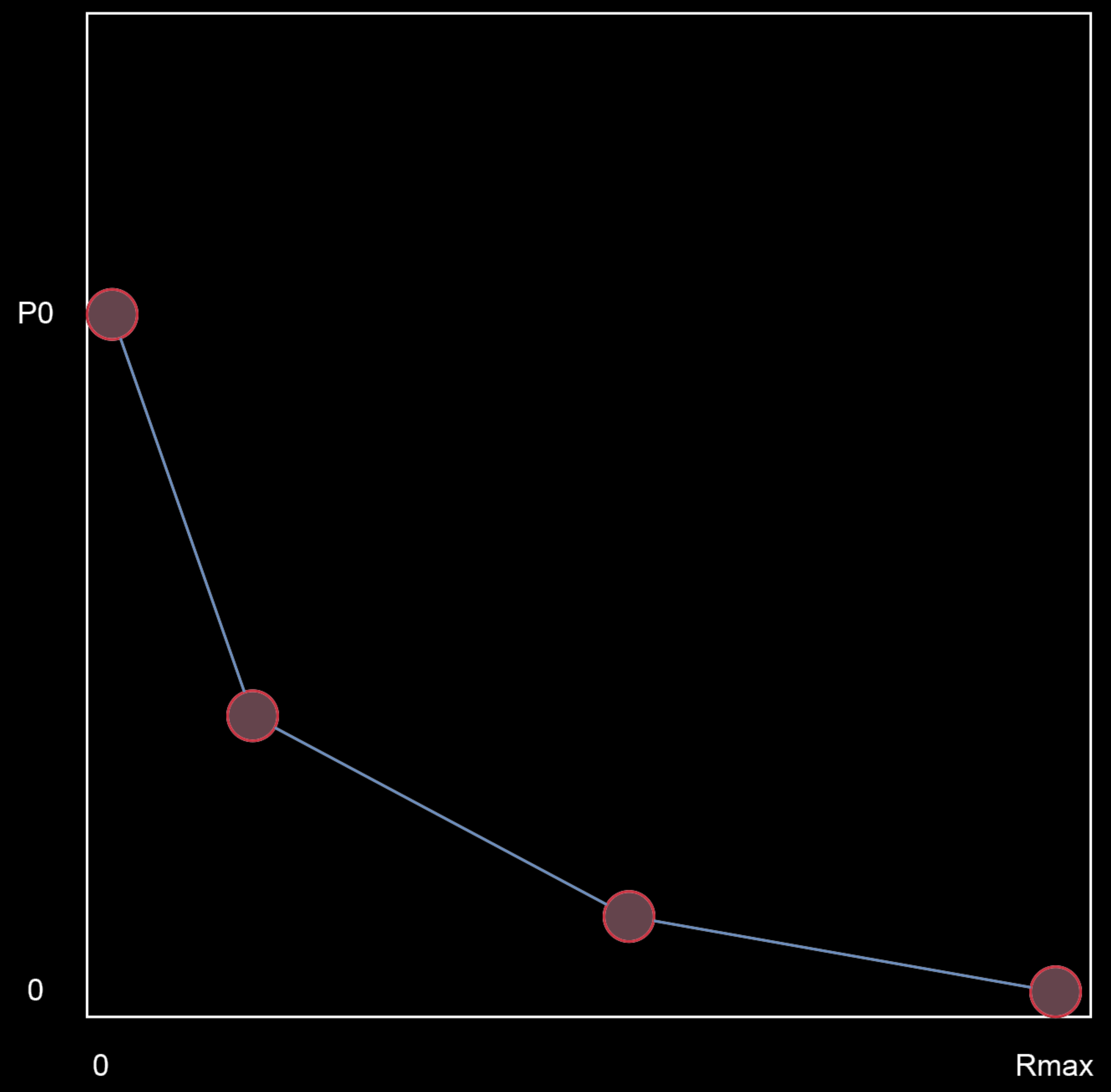


$$P(r) = \left(\frac{P_0 - P_1}{r_0 - r_1}(r - r_0) + P_0\right) 1_{[r_0 \le r \le r_1]} + \left(\frac{P_1 - P_{max}}{r_1 - r_{max}}(r - r_1) + P_1\right) 1_{[r_1 \le r \le r_{max}]}$$

$$\frac{\sigma_T}{m_e c^2} \int \left(\frac{P_0 - P_1}{r_0 - r_1}\left(\sqrt{r_p^2 + l^2} - r_0\right) + p_0\right) \mathbb{1}_{[r_0 \le \sqrt{r_p^2 + l^2} \le r_1]}$$
$$+ \left(\frac{P_1 - P_{max}}{r_1 - r_{max}}\left(\sqrt{r_p^2 + l^2} - r_1\right) + p_1\right) \mathbb{1}_{[r_1 \le \sqrt{r_p^2 + l^2} \le r_{max}]} dl$$
$$= \frac{\sigma_T}{m_e c^2} \left\{ \left[\frac{P_0 - p_1}{2(r_0 - r_1)} \left(l \left(\sqrt{l^2 + r_p^2} - 2r_1 \right) + r_p^2 \log\left(\sqrt{l^2 + r_p^2} + l \right) \right) \right]_{\sqrt{r_0^2 - r_p^2}}^{\sqrt{r_1^2 - r_p^2}} \right\}$$
$$+ \frac{\sigma_T}{m_e c^2} \left\{ \left[\frac{P_1 - P_{max}}{2(r_1 - r_{max})} \left(l \left(\sqrt{l^2 + r_p^2} - 2r_{max} \right) + r_p^2 \log\left(\sqrt{l^2 + r_p^2} + l \right) \right) \right]_{\sqrt{r_1^2 - r_p^2}}^{\sqrt{r_{max}^2 - r_p^2}} \right\}$$

Nodal Model

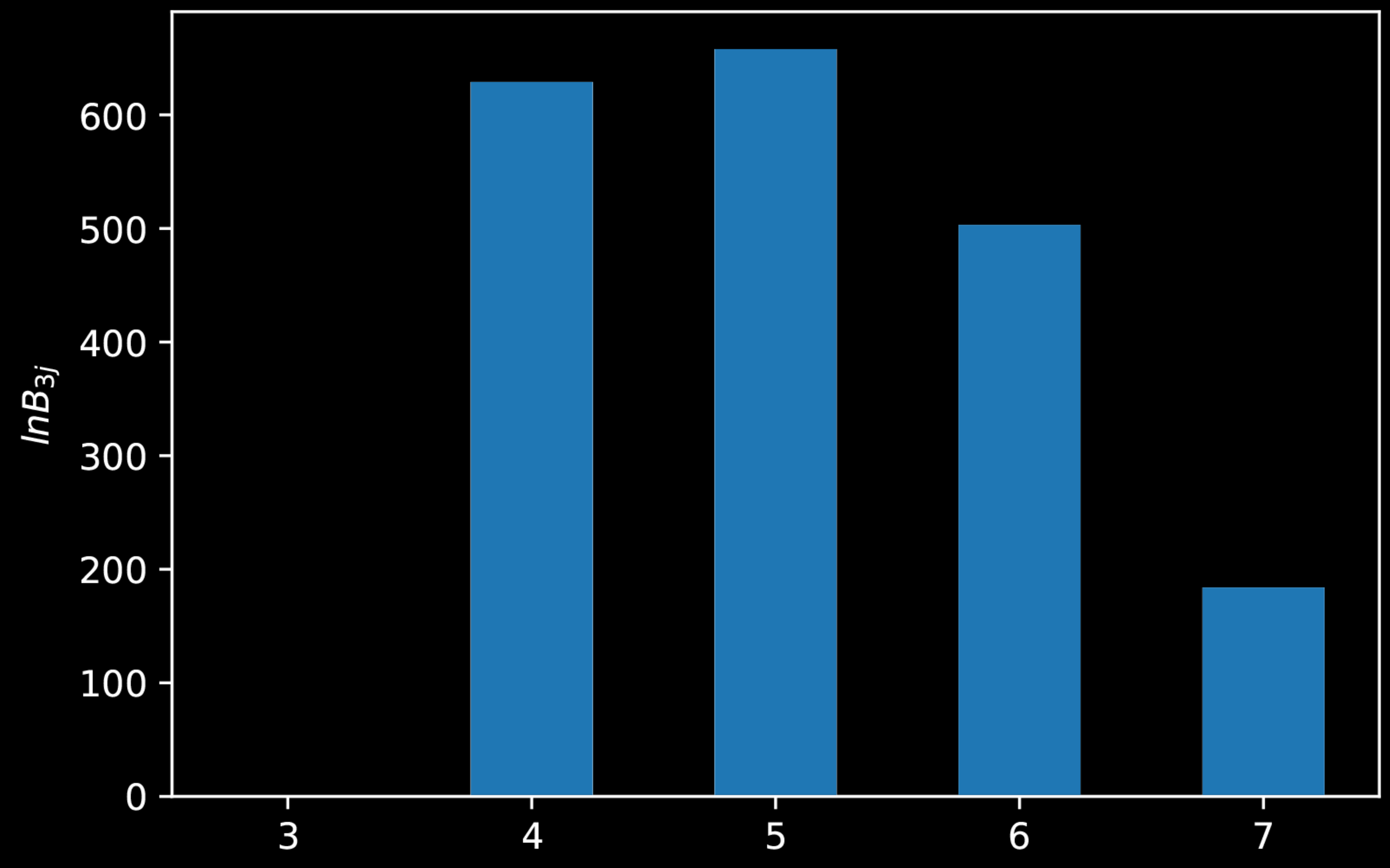
Prior



$$r_0 < r_1 < r_2 \ldots < r_{max}$$

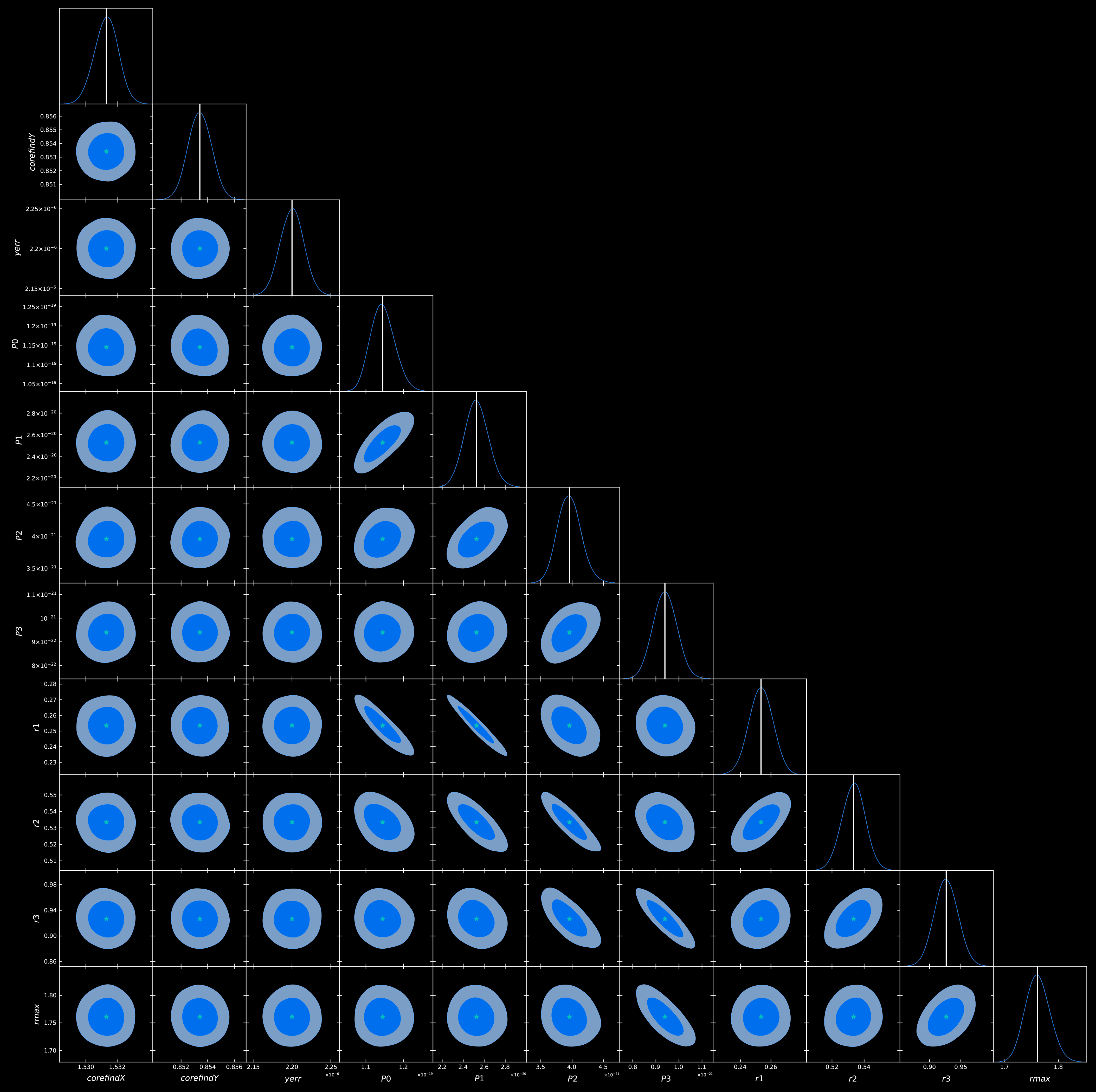
Nodal Model

Results



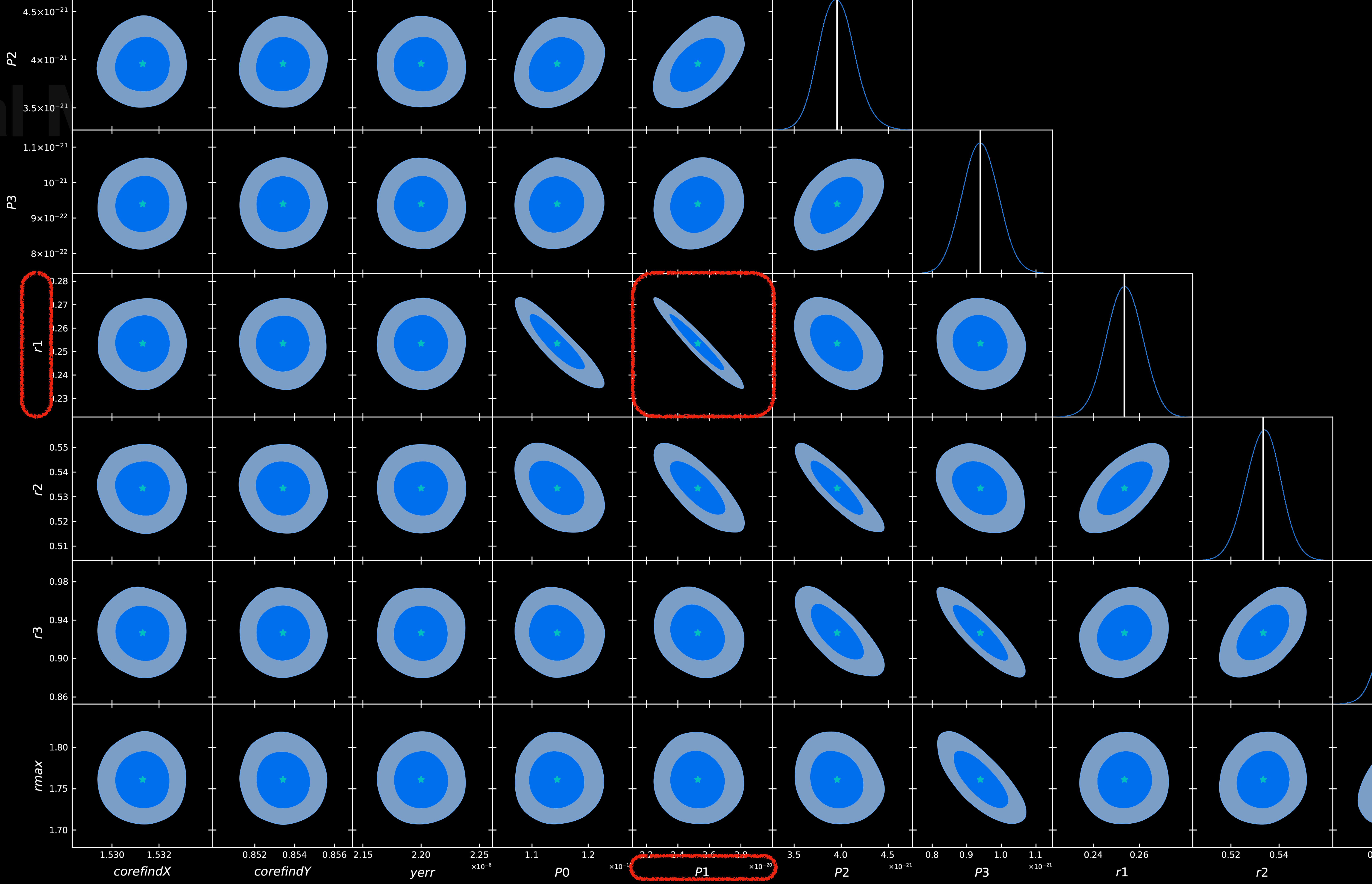
Nodal Model

Results



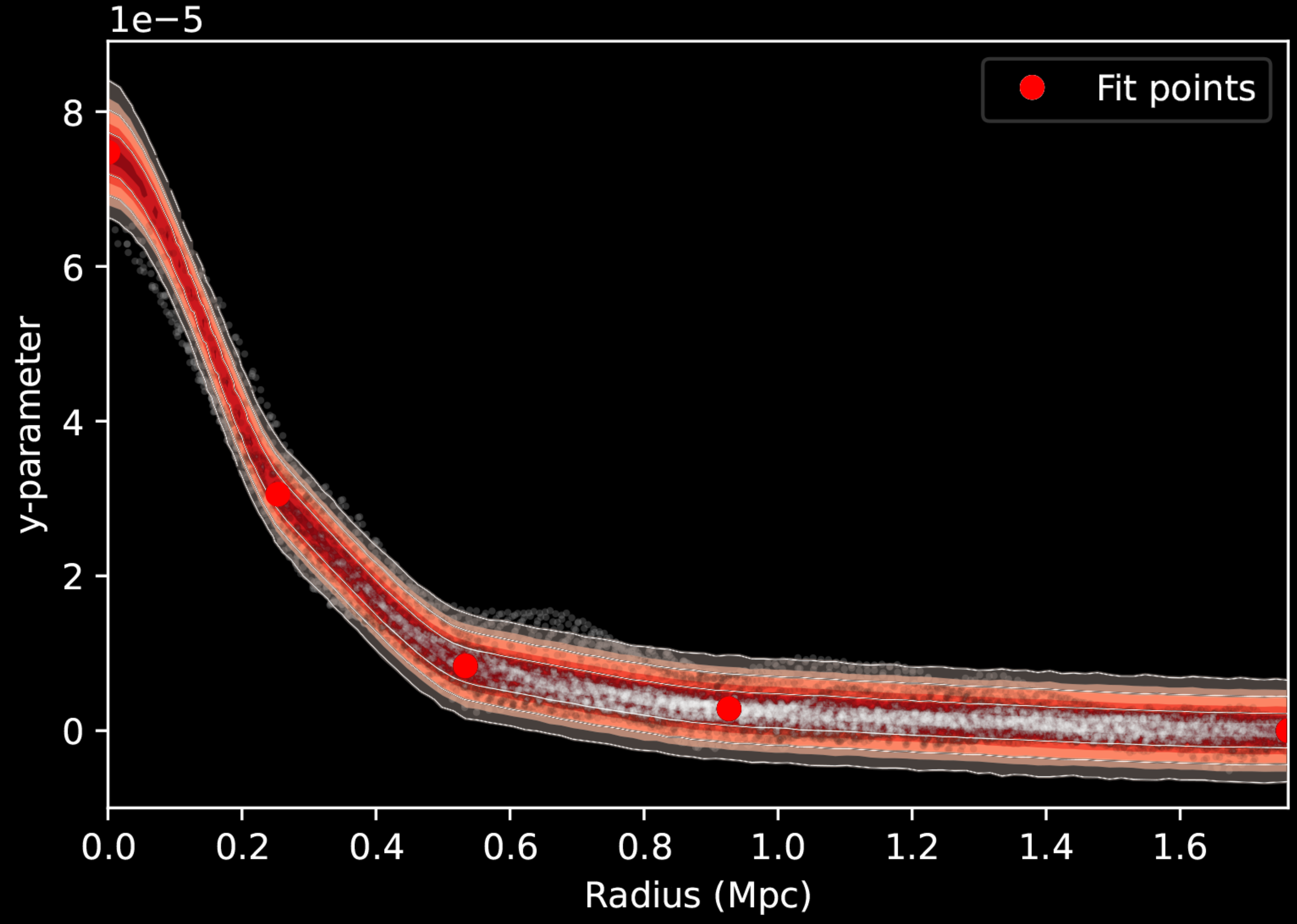
Nodal M

Results



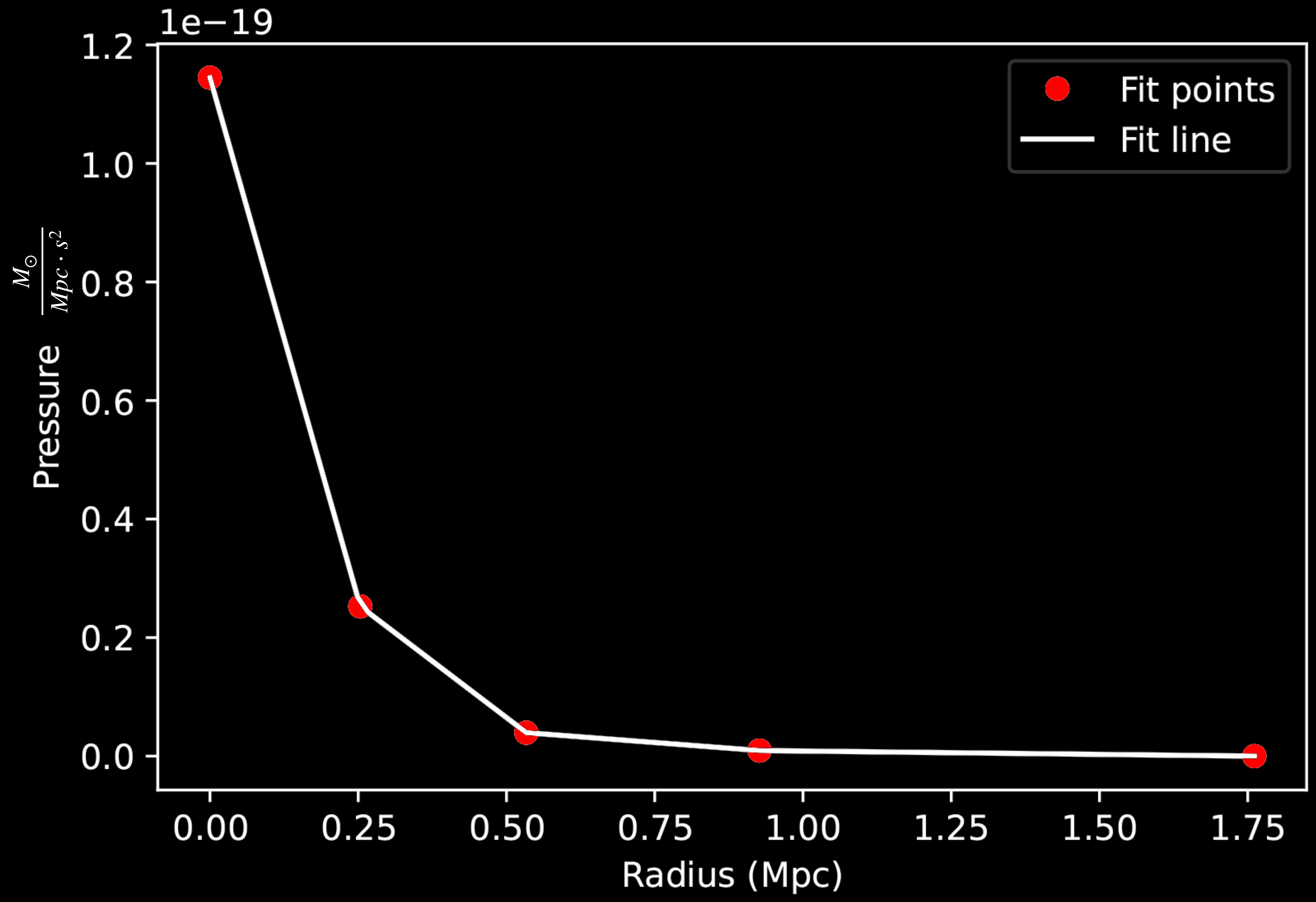
Nodal Model

Results



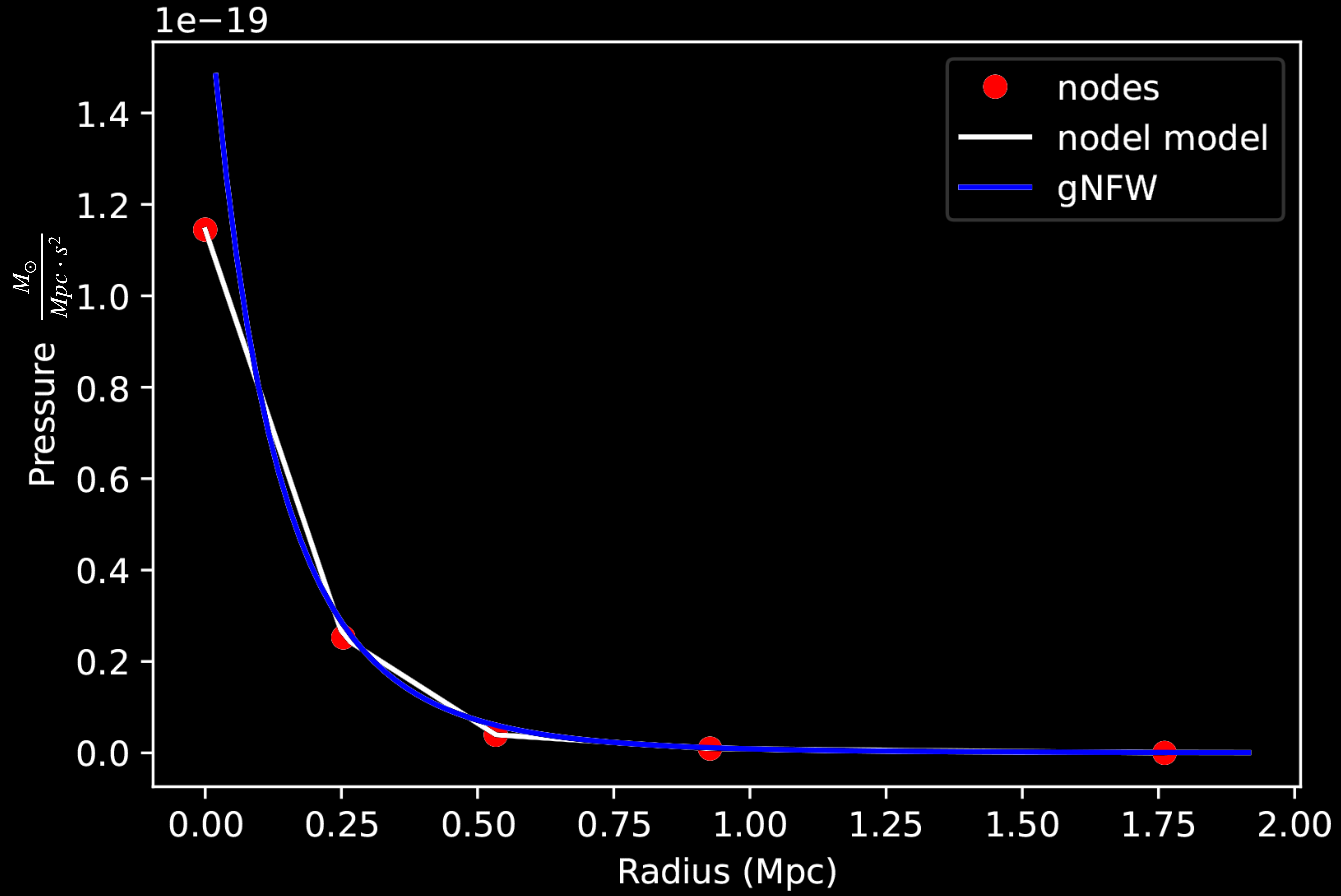
Nodal Model

Results



Nodal Model

Results



How to deal with trans-dimensional problem?

DNest

Nested Sampling vs Diffusive Nested Sampling

- Nested Sampling is limited to ascending in likelihood levels and can only explore the most recent level.
- Diffusive Nested Sampling, however, can backtrack, thereby gathering samples from lower levels.
- Unlike Nested Sampling, which halts after establishing the final level, Diffusive Nested Sampling continues exploring all levels.

Reversible Jump

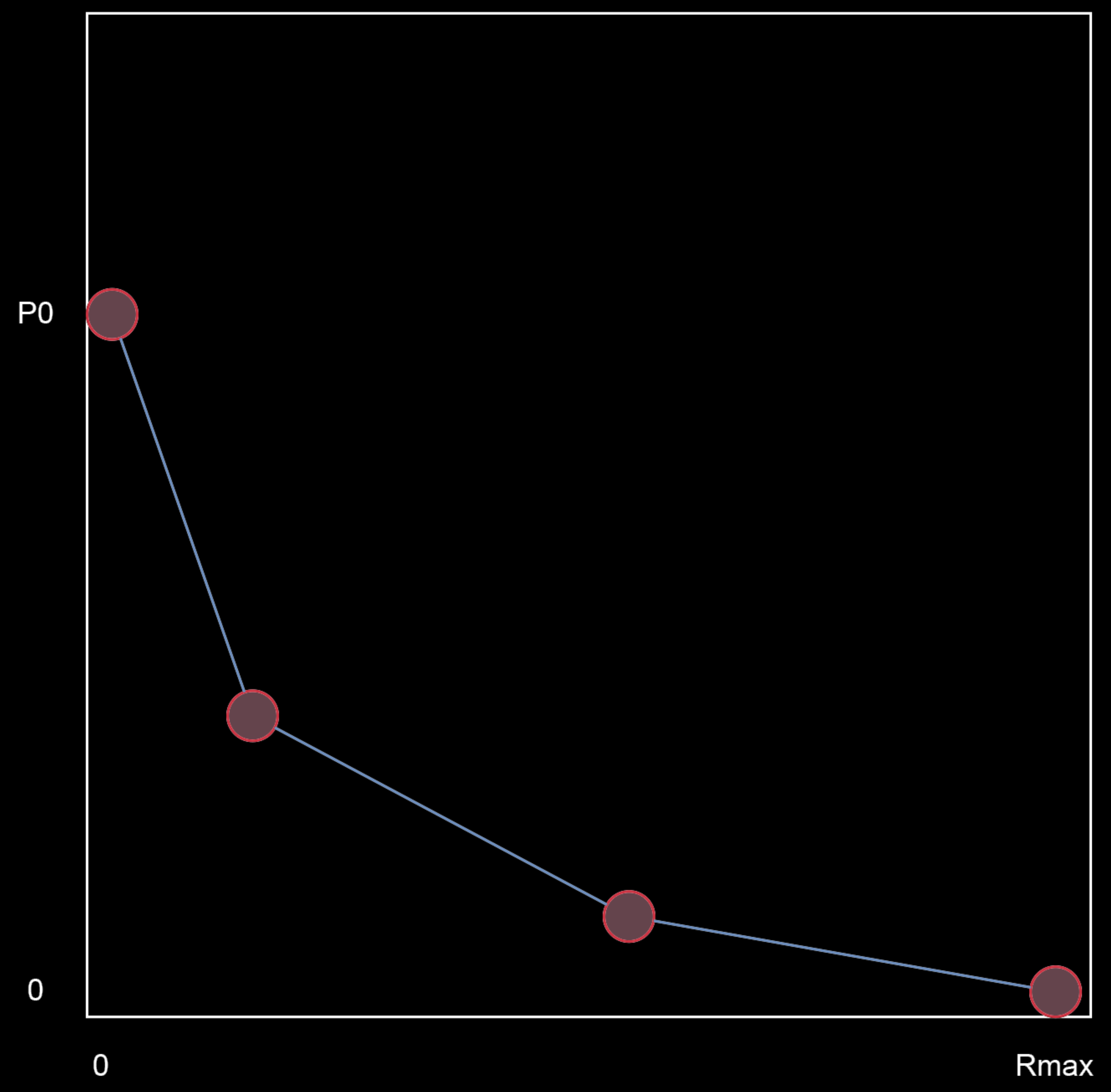
- A generalised version of the Metropolis-Hastings algorithm
- The exploration of multiple parameter spaces by reducing or increasing the number of parameters at each iteration.
    - *E.g. compare completely different models with either the same or different numbers of parameters*
    - *Or look for models with unknown numbers of objects where each additional object increases the number of parameters by the same amount.*

Conclusion

- Parametric models have degenerate problems
- The non-parametric nodel model is more flexible
- Future work: DNest + RJ

Nodal Model

Prior



$$r_0 < r_1 < r_2 \ldots < r_{max}$$

$$U_i \sim U(0,1)$$

$$x_i = -\ln U_i$$

$$d_i = \frac{x_i}{\sum_j x_j} \sim Dir$$

$$w_i = \sum_i d_i = \frac{\sum_{k=1}^{i} x_k}{\sum_{j=1}^{n} x_j} = \frac{\sum_{k=1}^{i} -\ln U_k}{\sum_{j=1}^{n} -\ln U_j}$$

$$r_i = r_0 + w_i \left(r_{\max} - r_0 \right)$$