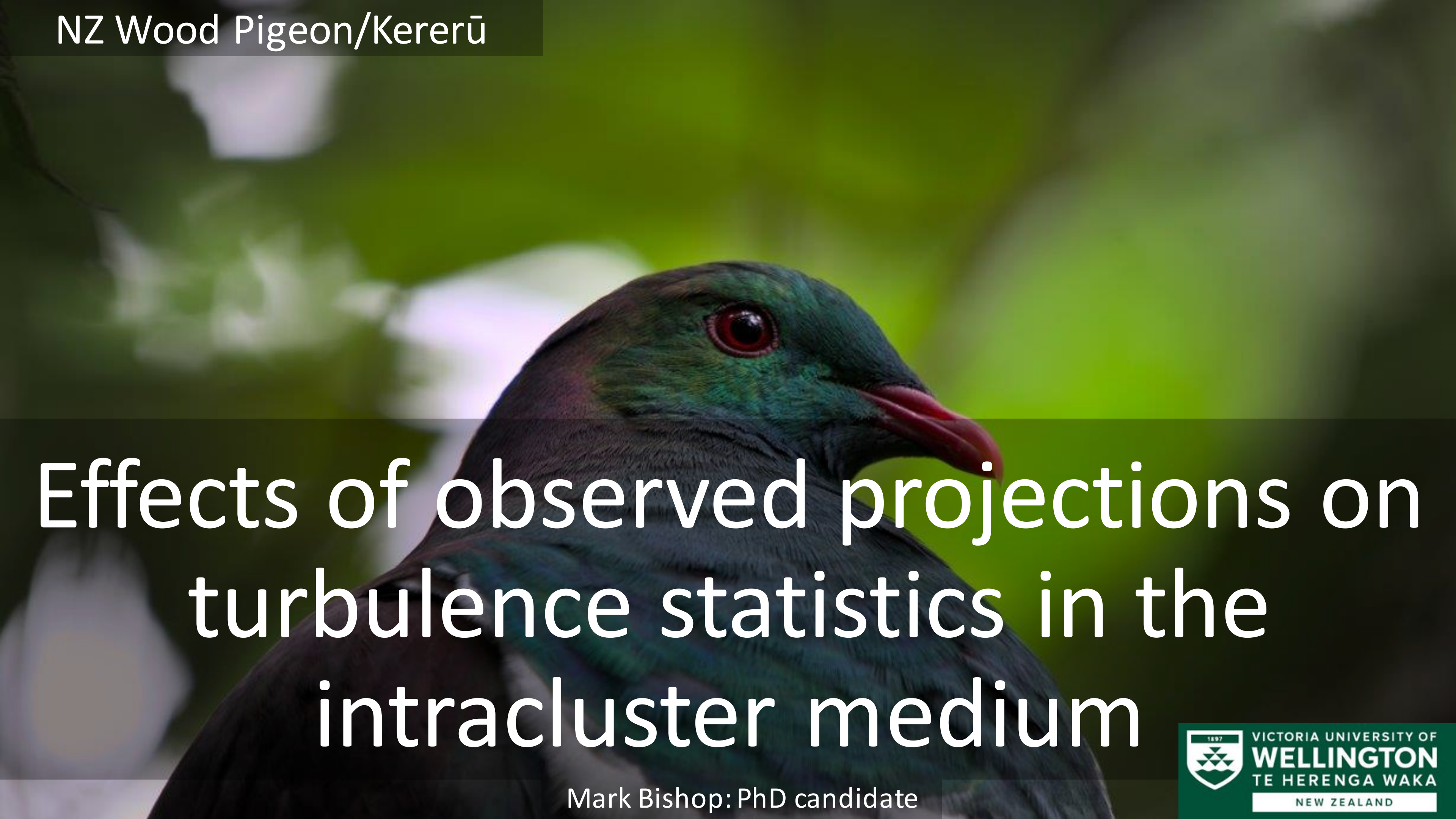


Effects of observed projections on turbulence statistics in the intracluster medium

Mark Bishop: PhD candidate

VICTORIA UNIVERSITY OF WELLINGTON TE HERENGA WAKA NEW ZEALAND

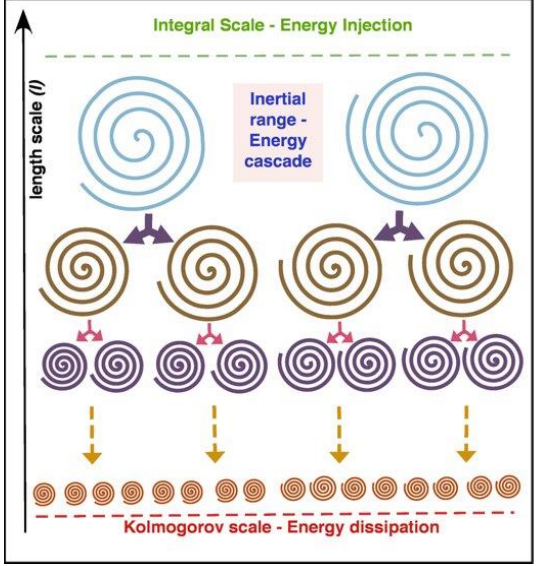
Turbulence

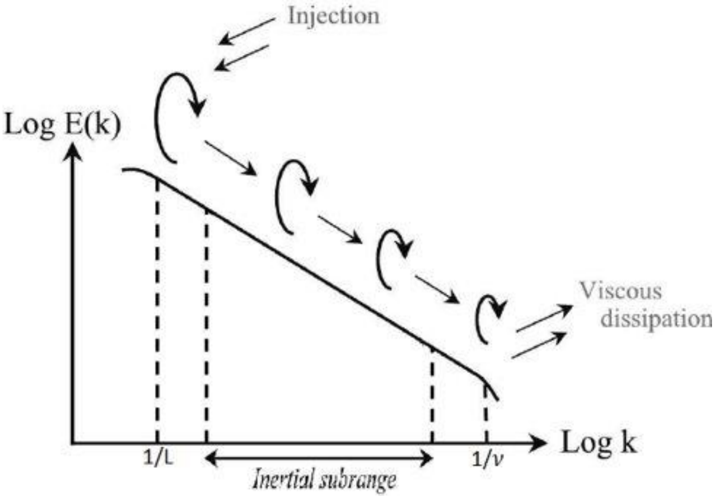
- Why are we interested?
  - Deviations from hydrostatic equilibrium (via turbulence) produces bias in mass estimates.
  - Acts to mix and transfer properties like mass, momentum and energy
  - Potential mechanism for transferring the energy into heating via processes like AGN feedback (as solutions to problems like the cooling flow).

Energy cascade

- Chaotic/random, requires statistical techniques to analyze (like the power spectrum)
- But has statistical structure over scales

- Most of the energy lies in the large scales, shown as a peak in the power spectrum coinciding with the energy injection
- Power law inertial range describing the energy cascade; the transfer of energy from the large scales to small scales
- Dissipation scale where viscous effects become dominant





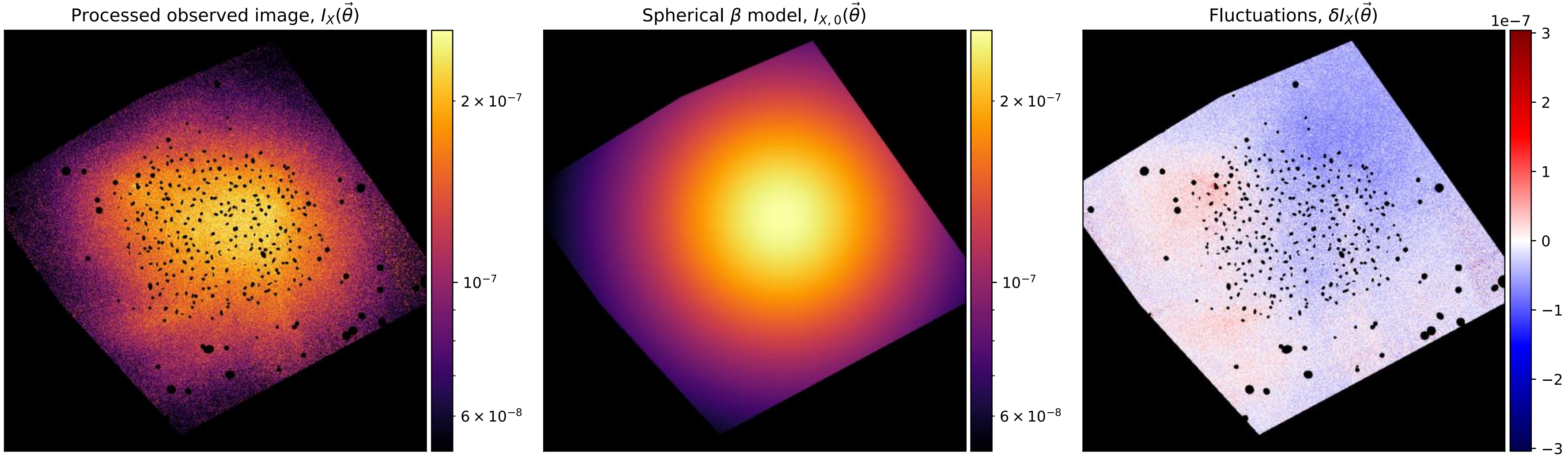
X-ray & SZ Observations

$$I_X(\vec{\theta}) \propto \int n_i(\vec{\theta}, \ell) n_e(\vec{\theta}, \ell) \Lambda(T) d\ell$$

$$Y_{SZ}(\vec{\theta}) \propto \int n_e(\vec{\theta}, \ell) T_e(\vec{\theta}, \ell) d\ell$$

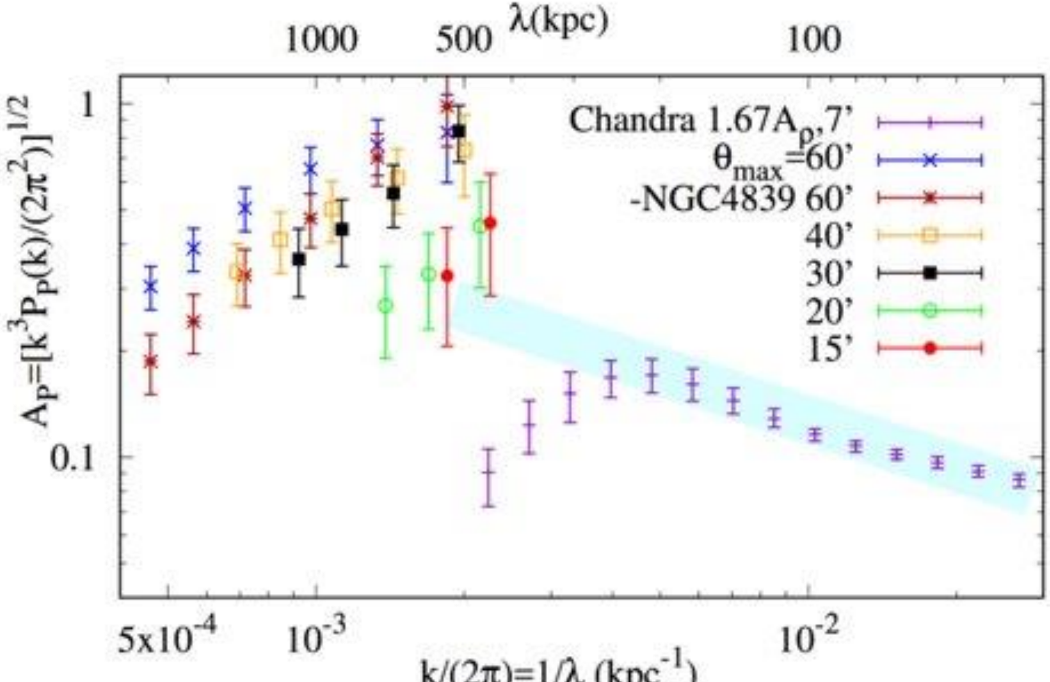
Reynolds decomposition

$$I_X - I_{X,0} = \delta I_X$$

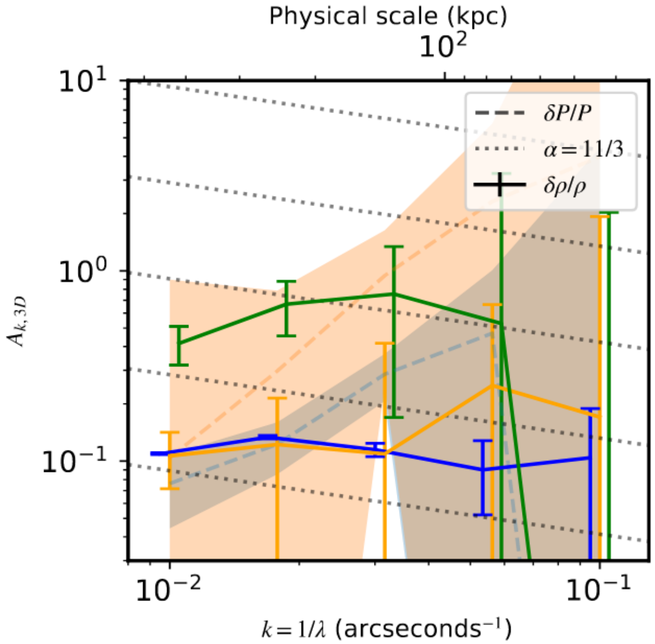


- Not many studies involving SZ effect.
- Constrained by instrument resolution, starting to be able to probe turbulence scales.

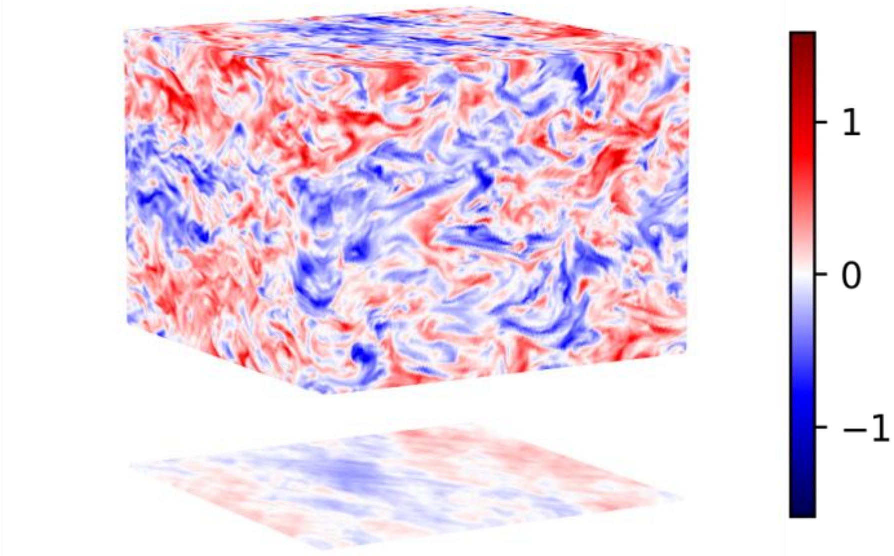
Coma: Khatri & Gaspari 2016

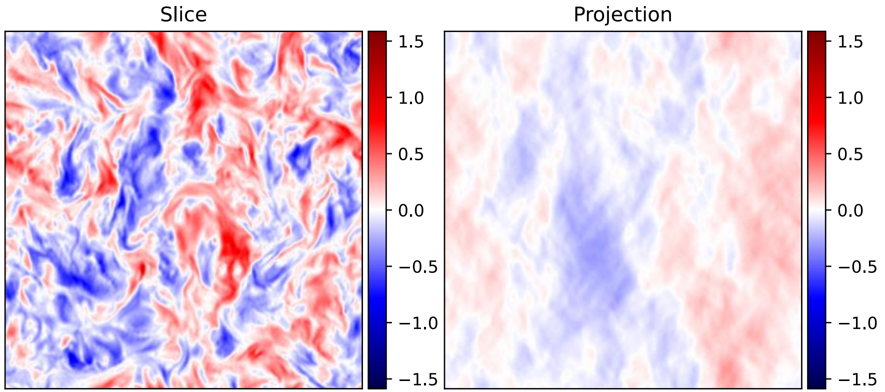


Zwicky 3146: Romero et al. 2023
Talking tomorrow



Projected Information





Incompressible MHD Simulation

Projection-slice theorem

- The projection-slice theorem states that the Fourier transform of a projection is the same as a slice of the Fourier transform of the original data.

$$E_{2D}^{\mathrm{modal}}(k_x, k_y) = E_{3D}^{\mathrm{modal}}(k_x, k_y, 0)$$

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- For projections with an emissivity factor:

$$E_{2D}^{\mathrm{modal}}(k_x, k_y) = \int E_{3D}^{\mathrm{modal}}(k_x, k_y, k_z) E_{EM}^{\mathrm{modal}}(k_x, k_y, k_z) dk_z$$

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- With some approximations....

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- With some approximations....

Typically treated as a wavenumber independent scaling factor

$$E_{2D}^{\mathrm{modal}}(k_x, k_y) \approx E_{3D}^{\mathrm{modal}}(k_x, k_y, 0) \int E_{EM}^{\mathrm{modal}}(k_x, k_y, k_z) dk_z$$

Power spectrum estimation

- Fourier methods

Using Fourier transforms

Fourier wavevector decomposition

Power spectrum estimation

- Fourier methods

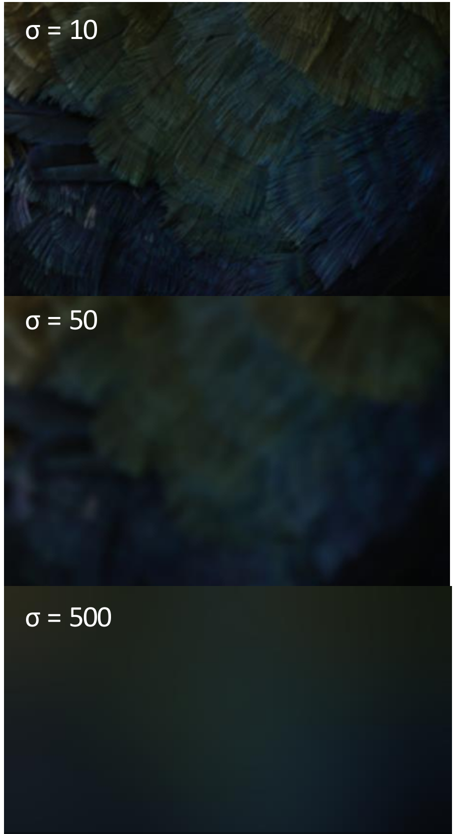
Using Fourier transforms

Fourier wavevector decomposition

- Arevalo et al. 2012 method

Using Gaussian convolutions

Scale space decomposition



Takahe Feathers

Power spectrum estimation

- Fourier methods

Using Fourier transforms

Fourier wavevector decomposition

- Arevalo et al. 2012 method

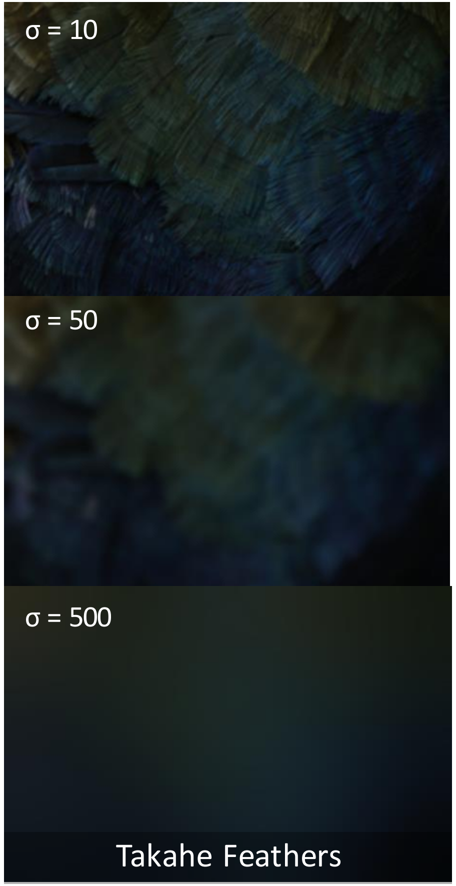
Using Gaussian convolutions

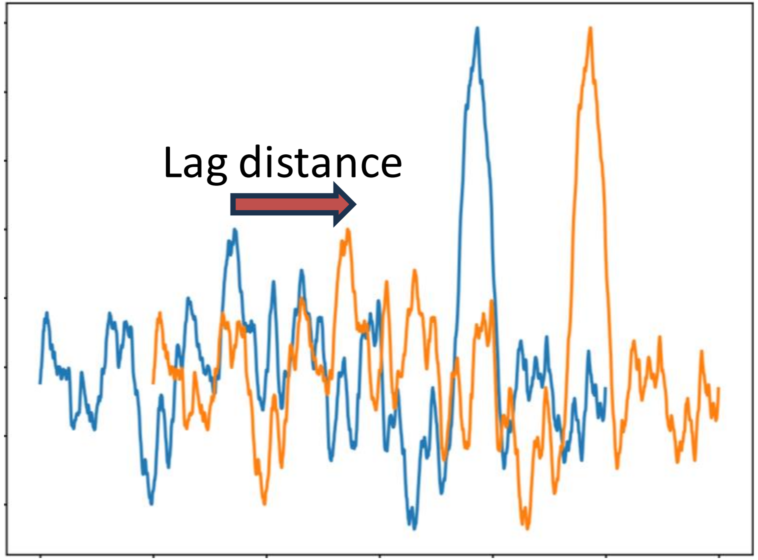
Scale space decomposition

- Equivalent structure function

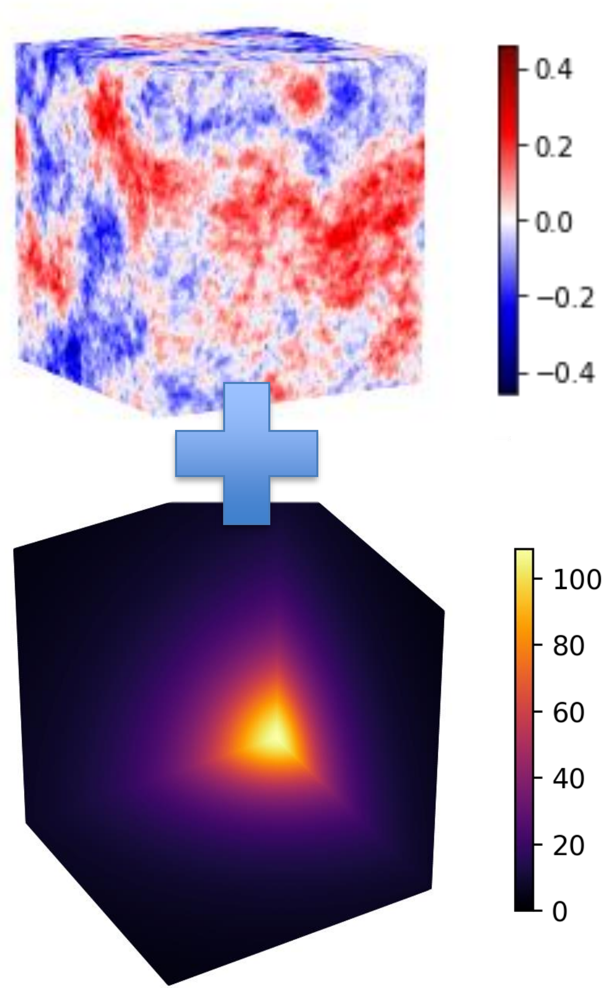
Configuration space calculation

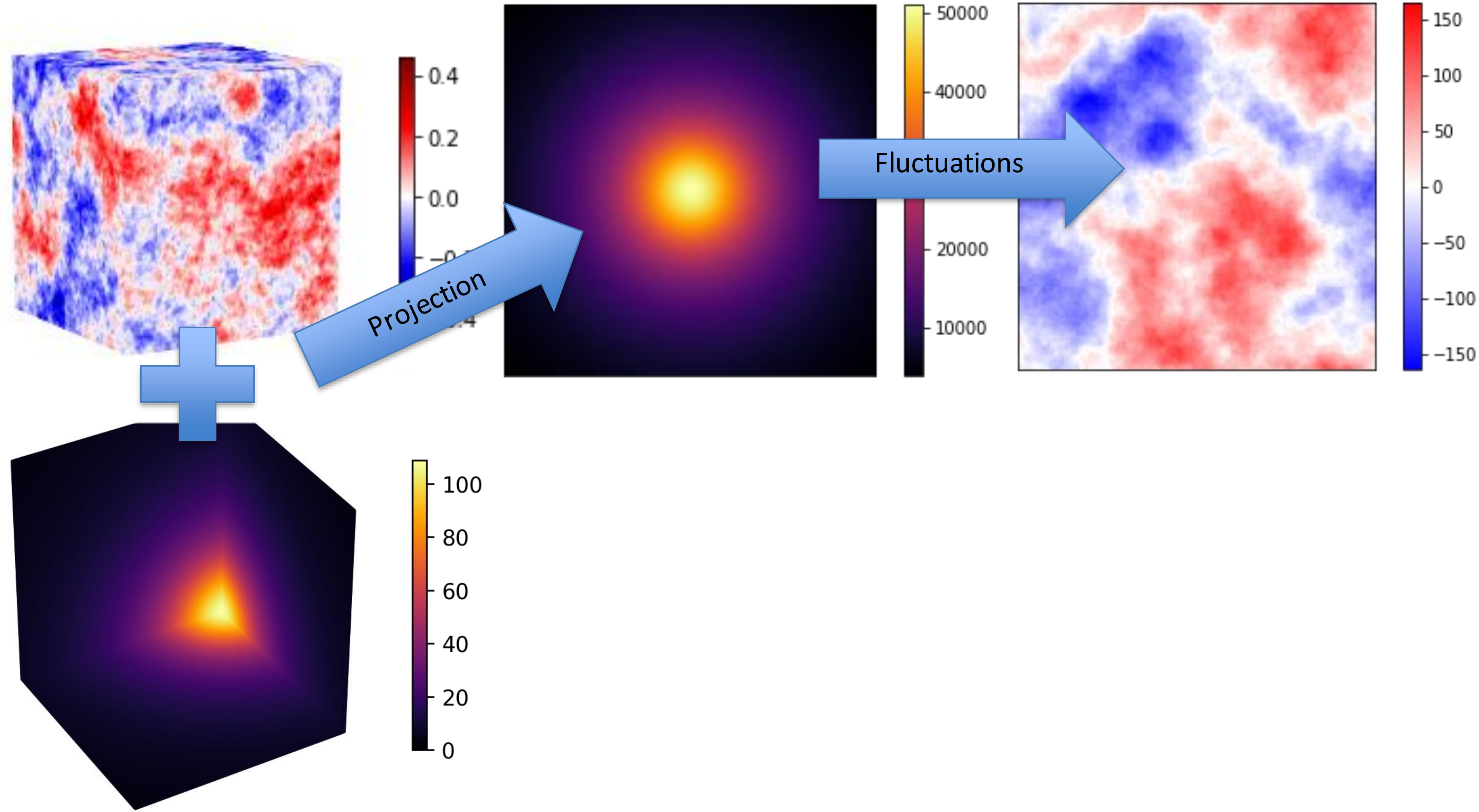
Crude approximation converts it to an effective power spectrum





Tūī

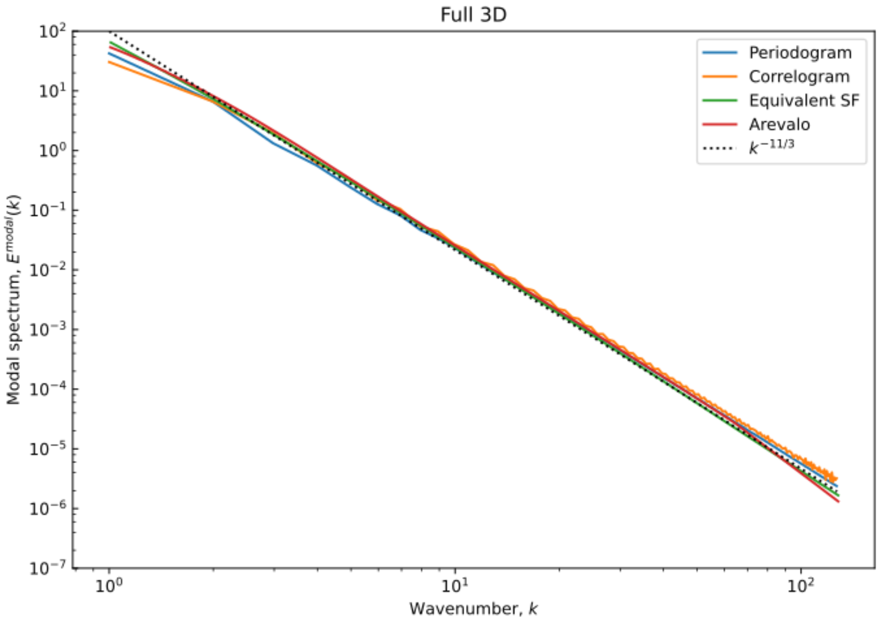

0.4
0.2
0.0
−0.2
−0.4
100
80
60
40
20
0


0.4
0.2
0.0
Projection
50000
40000
20000
10000
Fluctuations
150
100
50
0
−50
−100
−150
100
80
60
40
20
0

Methods & Projections

- Just the fluctuations; constant emissivity & no mean profile

3D Synthetic data



Methods & Projections

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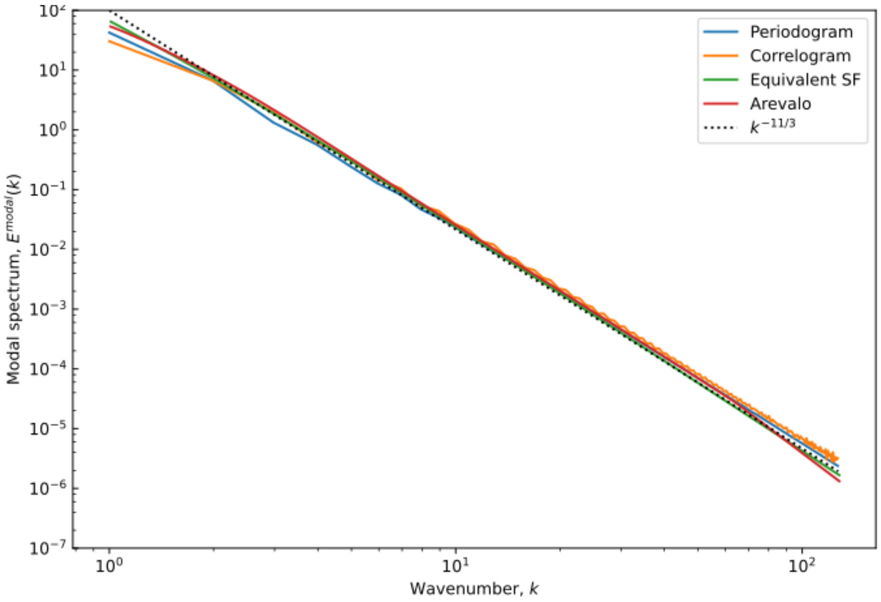
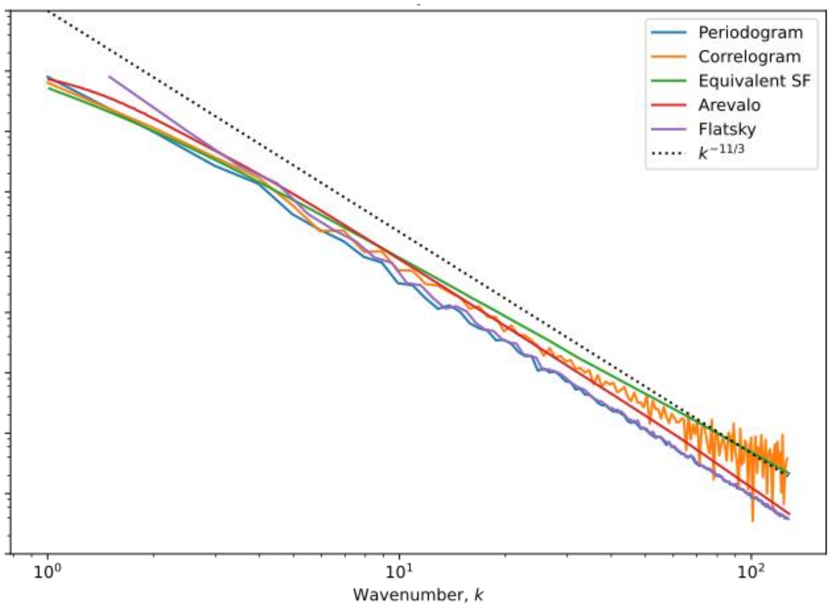
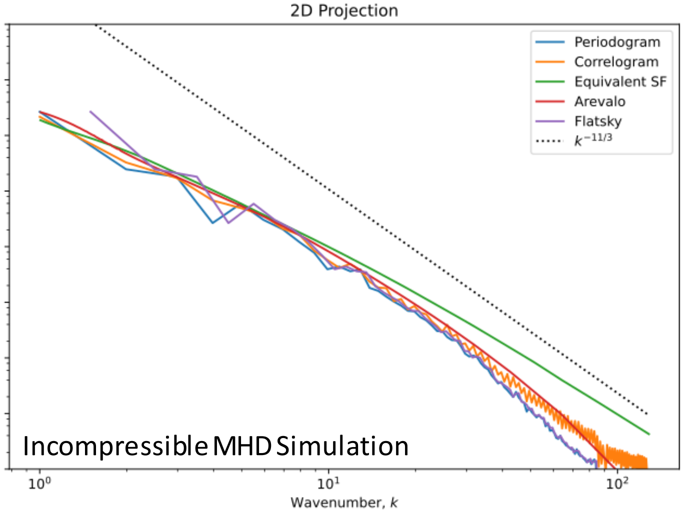


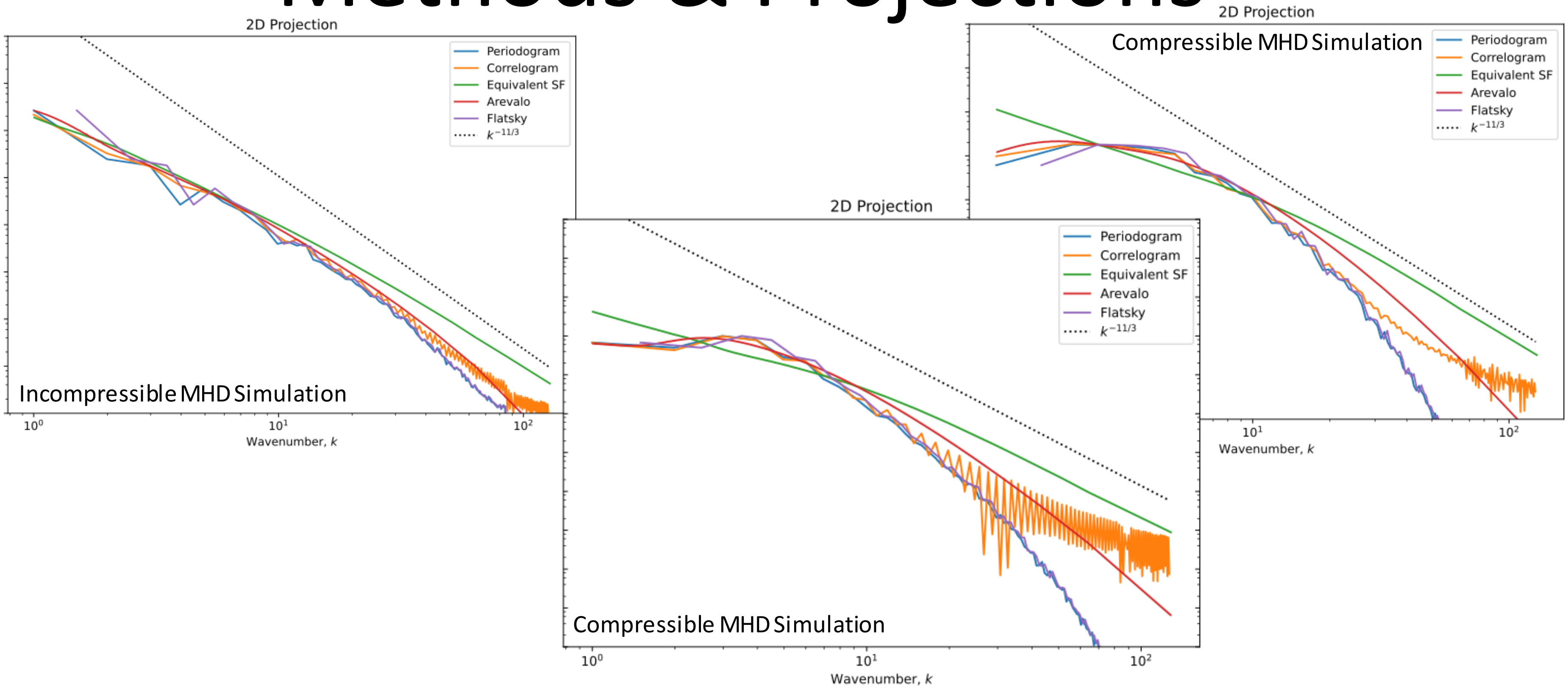
3D->2D projected synthetic data



Methods & Projections

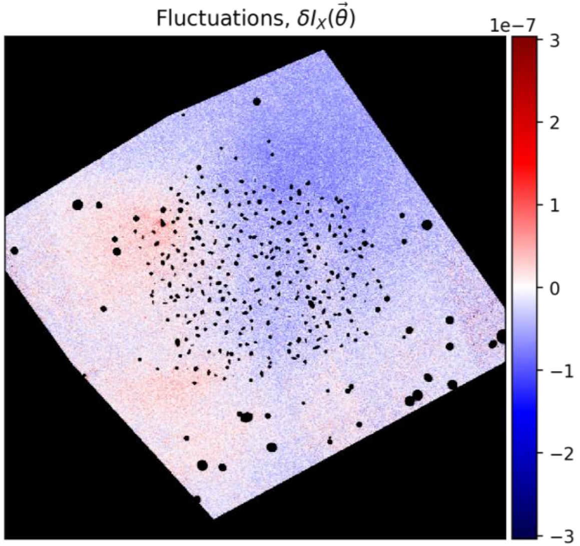


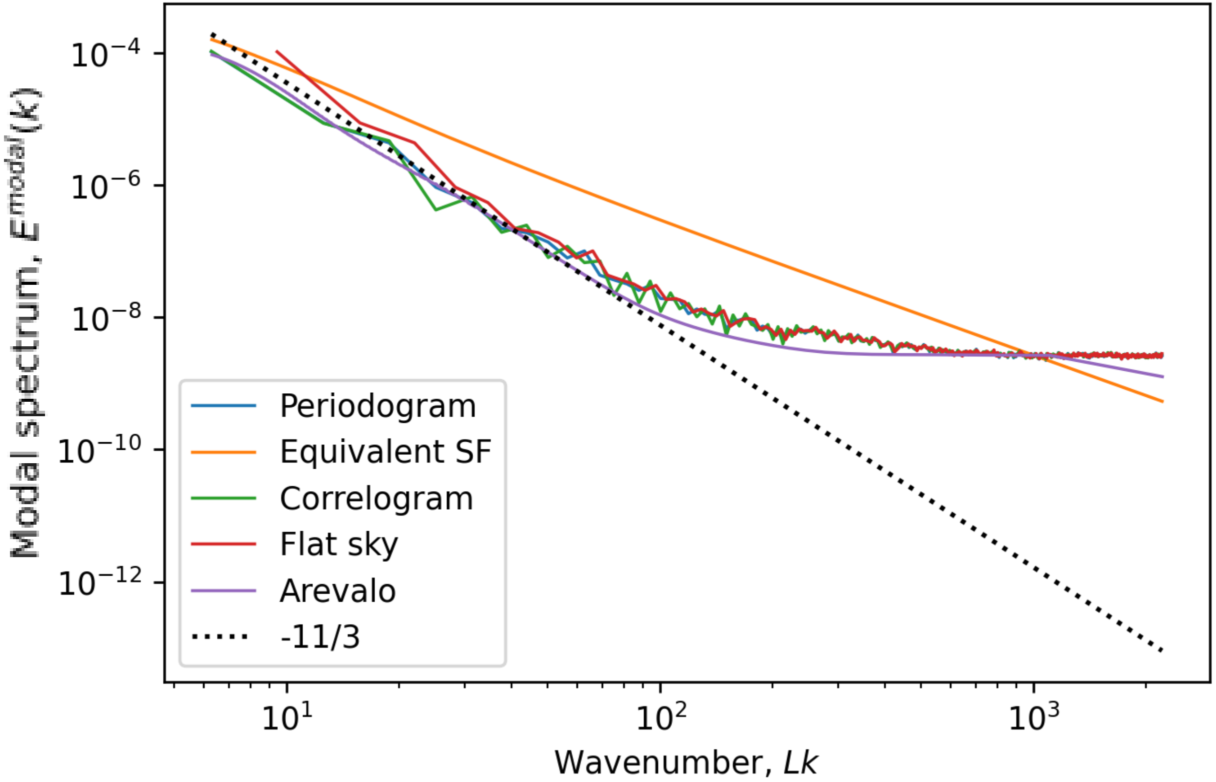
Methods & Projections



Projections with noise

- Real observations have noise
- White noise – constant in Fourier space





Noise removal techniques

- Churazov et al. 2012 method (X-ray)

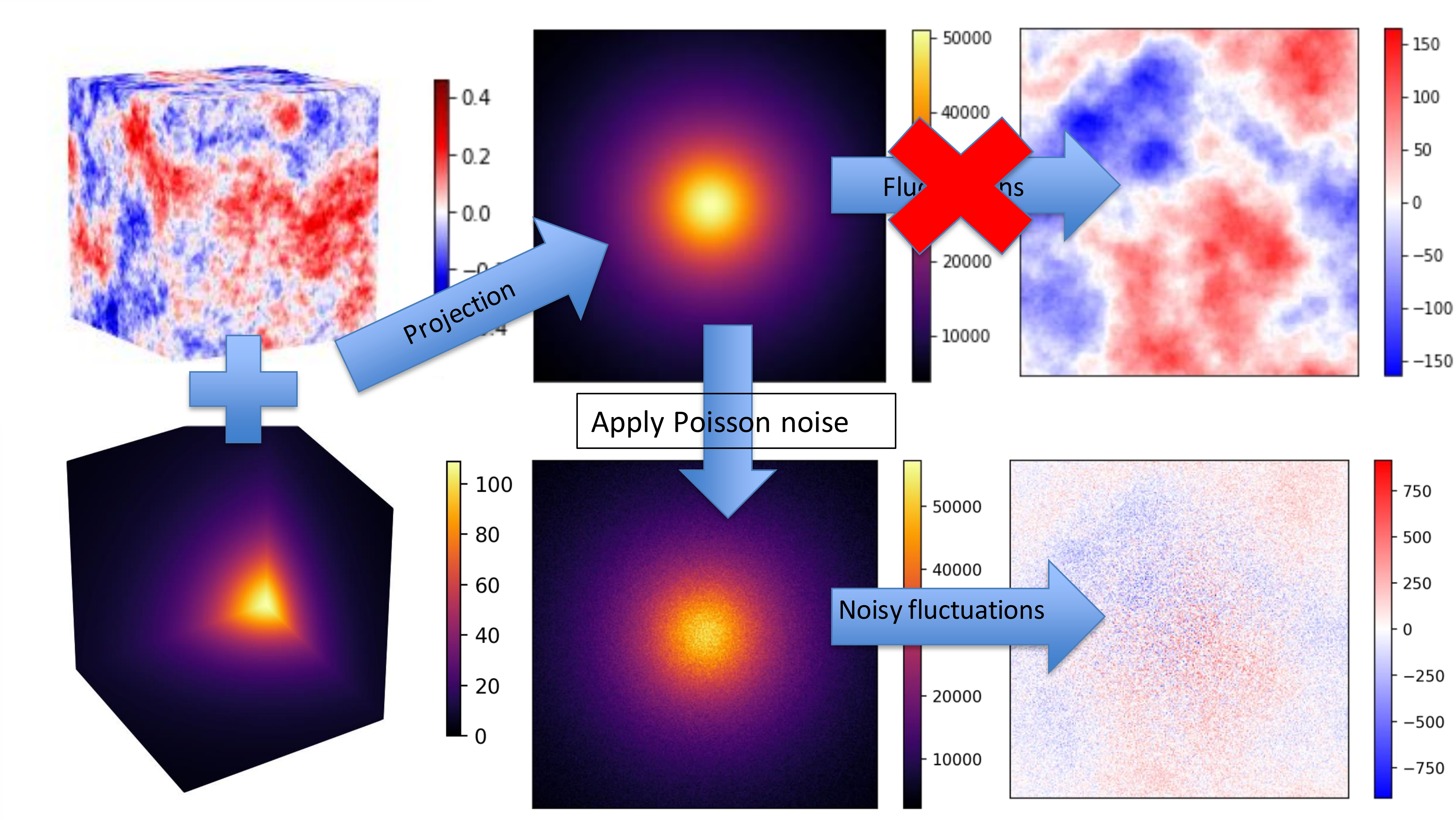
Averaging (in Fourier space) over artificially generated Poisson noise realizations

- Cross spectrum

Cross spectrum from two independent observations

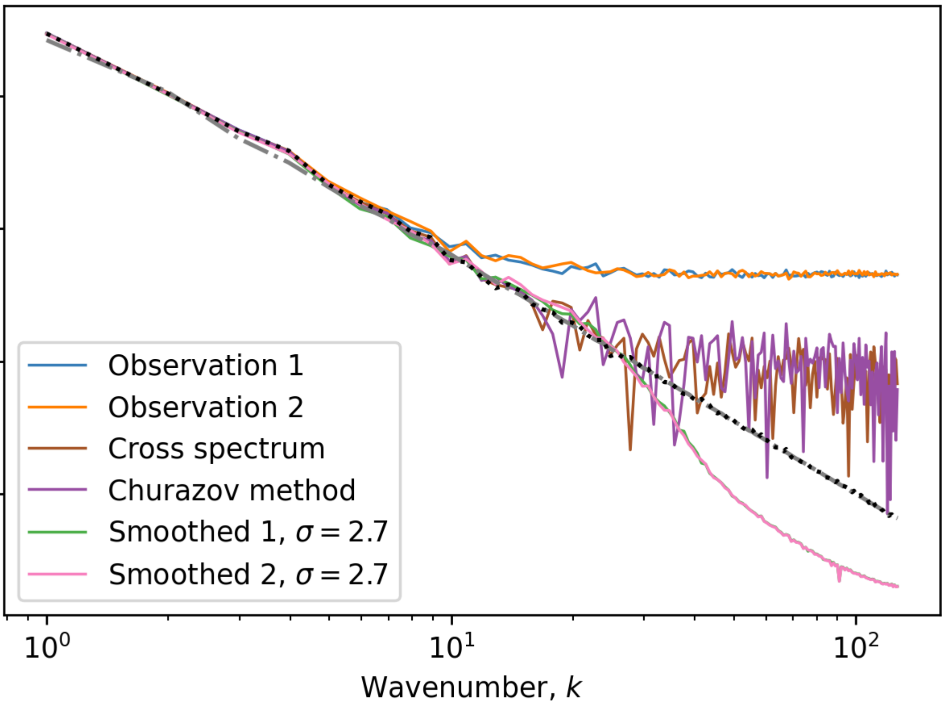
- Smoothing

Applying simple Gaussian smoothing

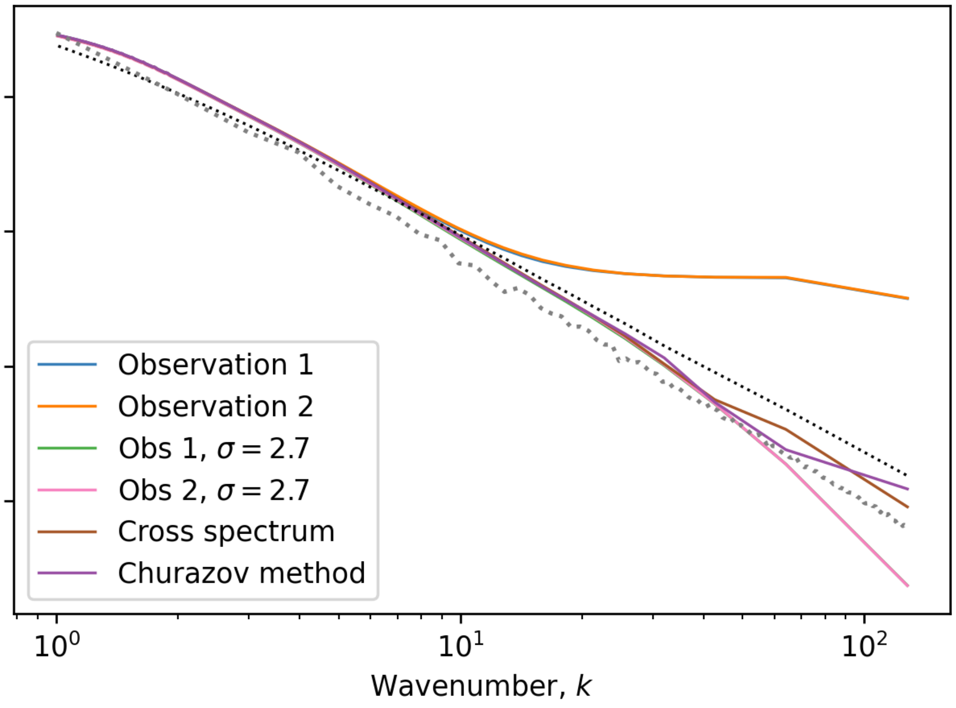


Projection
Apply Poisson noise
Noisy fluctuations

Synthetic data - Periodogram



Synthetic data - Arevalo



Summary

- Shown the different power spectrum estimation methods introduce differences when using projected data at small scales
- Started to examine different spectra techniques with different noise reduction methods
- Next steps, work on introducing exposure and masks.
- Properly quantifying the bias for different spectra using Gaussian fields and turbulence simulations.


Thanks for listening!
Questions?
NZ Fantail/Pīwakawaka

The backup slides

Statistics -Fourier methods

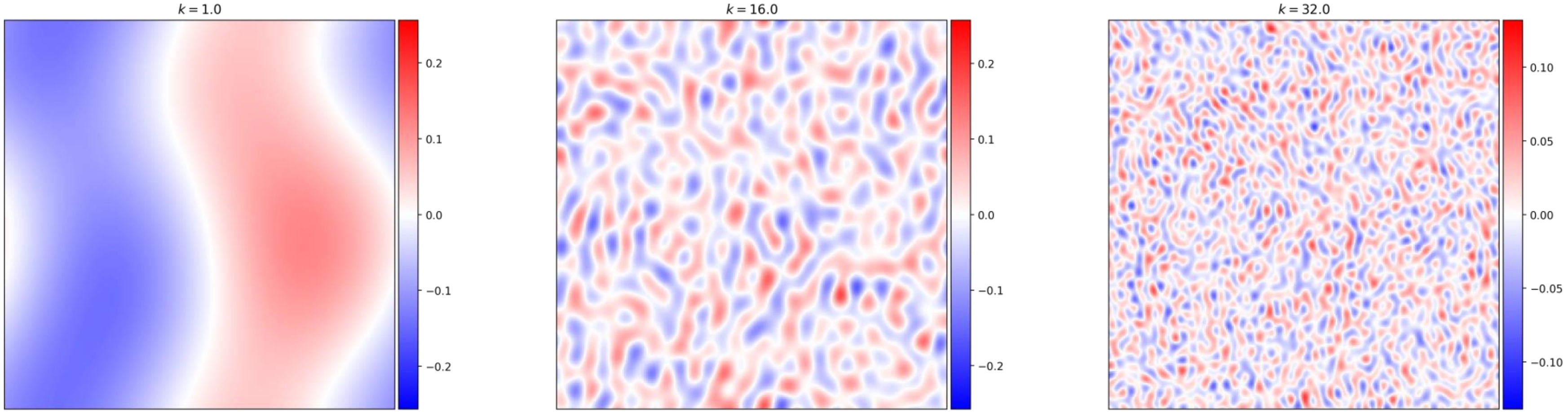
- Do different techniques add any bias?

Fourier space

Wavenumber decompositions

$$E^{\mathrm{modal}}(k) = \sum_{k \leq |\vec{k}| < k+dk} \left| \int I(\vec{x}) e^{-i\vec{k}\cdot\vec{x}} d^D\vec{x} \right|^2$$

$$E^{\mathrm{modal}}(k) = \sum_{k \leq |\vec{k}| < k+dk} \int R(\vec{r}) e^{-i\vec{k}\cdot\vec{r}} d^D\vec{r}$$



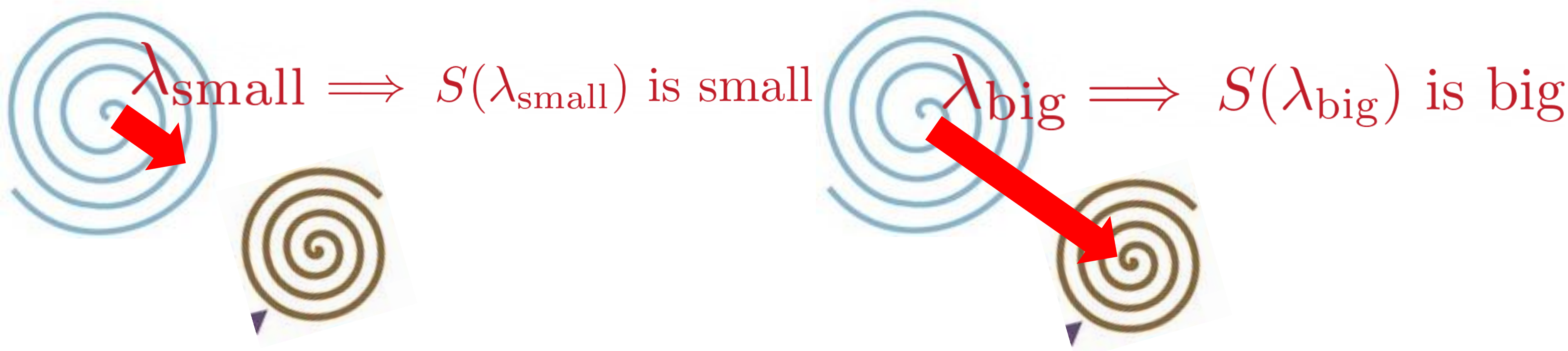
Statistics –Equivalent structure function

- Do different techniques add any bias?

Common turbulence analysis method

Lag space

$$S(\lambda) = \left\langle \left(I(\vec{x} + \vec{\lambda}) - I(\vec{x}) \right)^2 \right\rangle$$



Statistics –Equivalent structure function

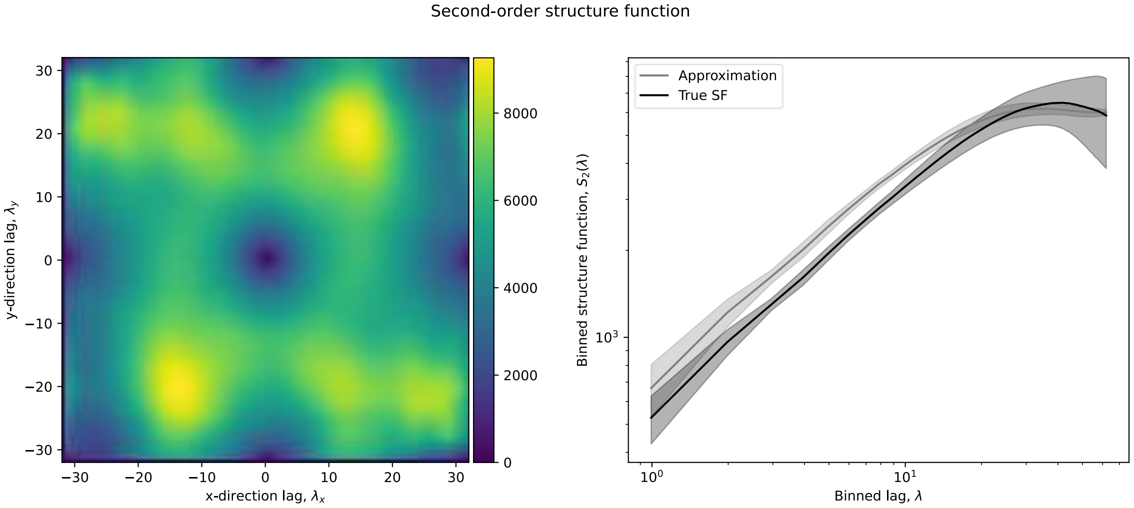
- Do different techniques add any bias?

Common turbulence analysis method

Lag space

$$S(\lambda) = \left\langle \left(I(\vec{x} + \vec{\lambda}) - I(\vec{x}) \right)^2 \right\rangle$$

$$E^{\mathrm{modal}}(k) \approx \frac{S(1/k)}{2\pi k^2}$$



Statistics –Arevalo method

- Do different techniques add any bias?

Convolving with a Gaussian obtains a blurred image with scales greater than or equal to the Gaussian stdev.

$$\text{all scales}$$

Taking the difference of two Gaussian blurred images with close scales provides an image with ONLY scales in between.

$$\text{scales} \gtrsim \sigma = 3$$

$$\text{scales} \gtrsim \sigma = 10$$

Statistics –Arevalo method

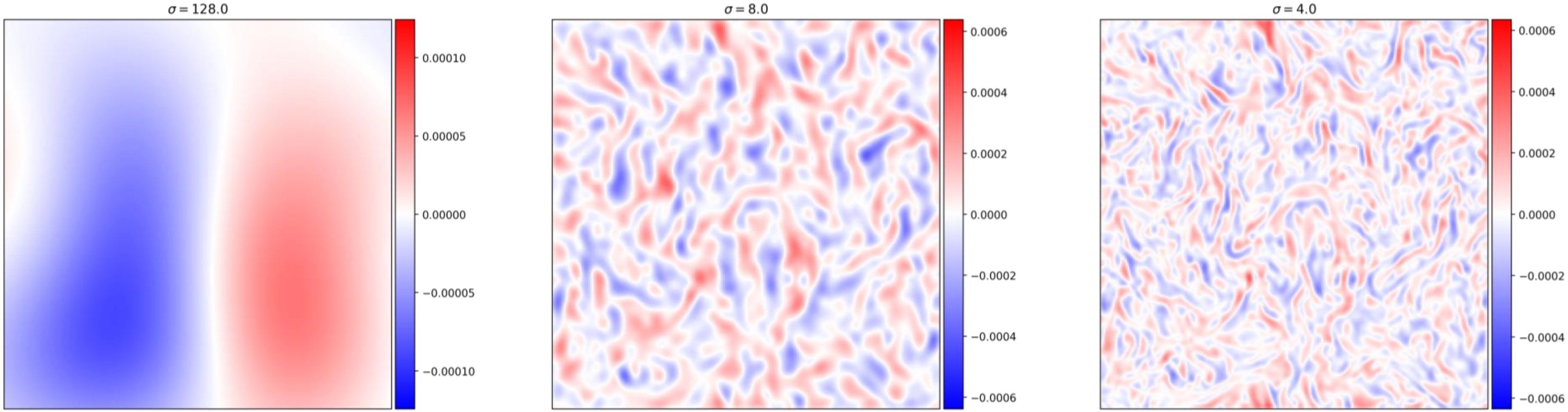
- Do different techniques add any bias?

Scale space decomposition

Deals with masks and exposure maps

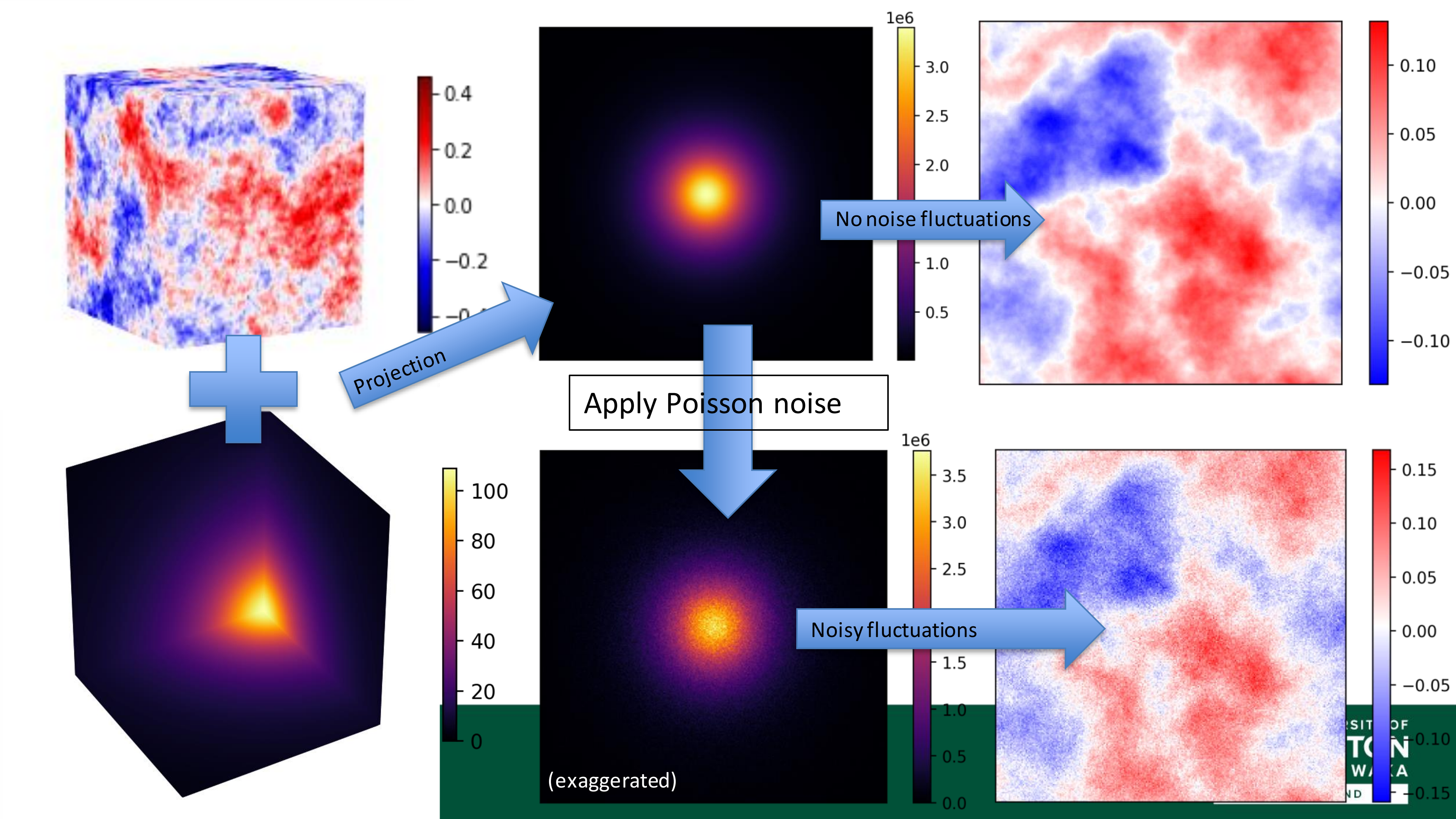
$$E^{\mathrm{modal}}(k_\sigma) = \frac{\int |(I * f)(\vec{x})|^2 d^D \vec{x}}{\int |\hat{f}(\vec{k})|^2 d^D \vec{k}}$$

$$f(x) = \frac{1}{\sqrt{2\pi^2\sigma_1^2}} e^{-x^2/2\sigma_1^2} - \frac{1}{\sqrt{2\pi^2\sigma_2^2}} e^{-x^2/2\sigma_2^2}$$

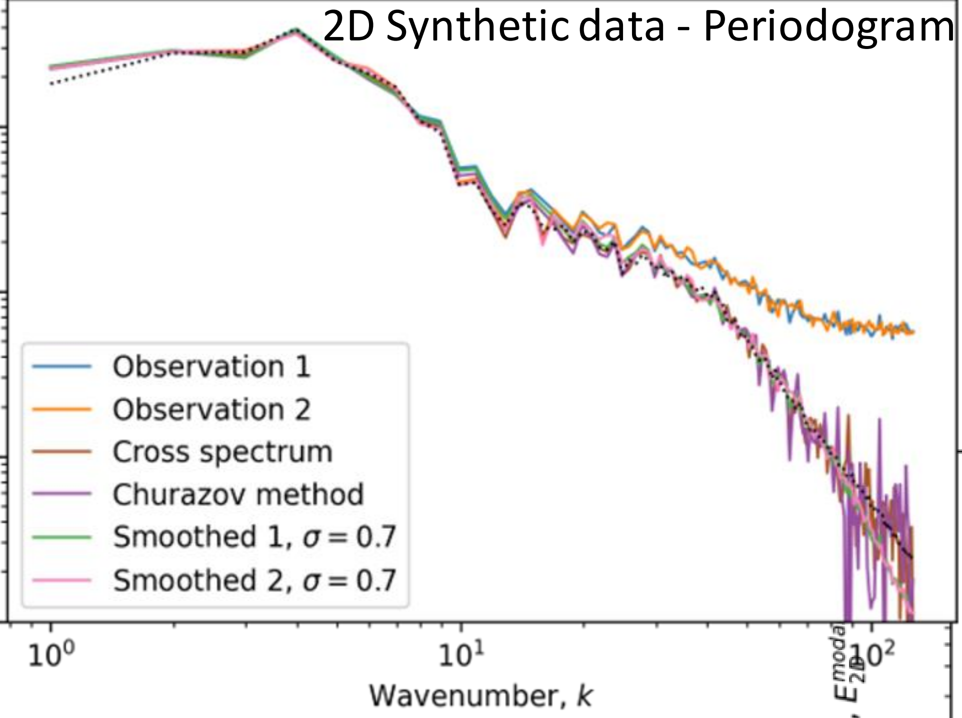


Porjected

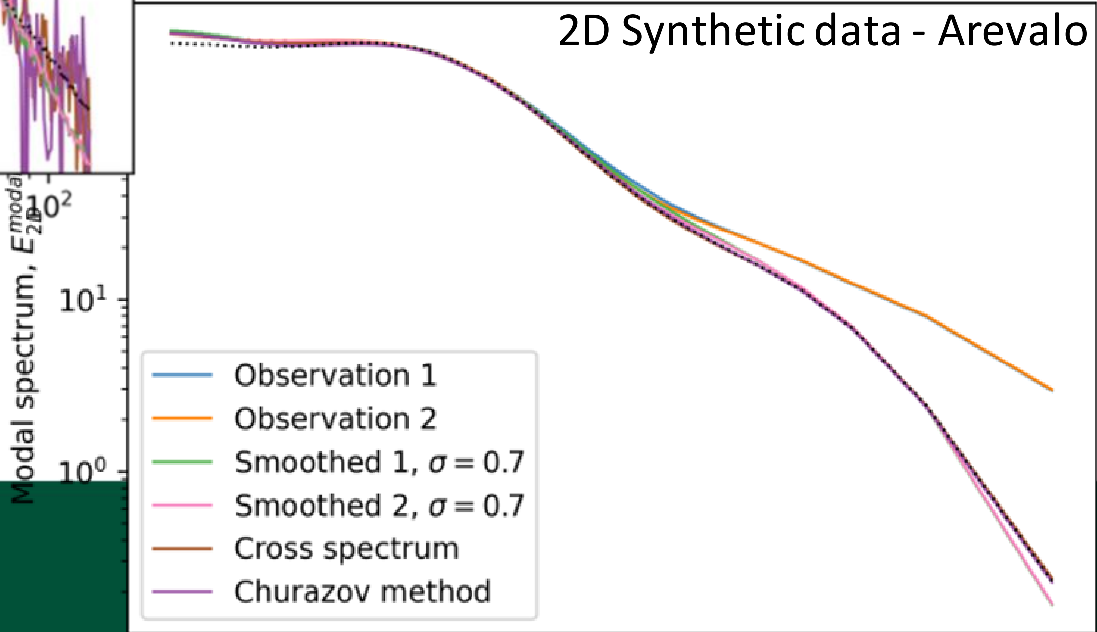
- [Brunt & Mac Low 2004; Compressible Supersonic turbulence]
- [Mohapatra et al. 2002; Velocity SF]


Projection
Apply Poisson noise
No noise fluctuations
Noisy fluctuations
(exaggerated)
1e6
1e6

2D Synthetic data - Periodogram



2D Synthetic data - Arevalo



- You found the secret slide =)

CAPITAL THINKING.
GLOBALLY MINDED.
MAI I TE IHO KI TE PAE