Virtual states, halos and resonances in three-body atomic and nuclear systems

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OUTLINE

Physics of weakly bound three-body systems

Two neutron weakly bound three-body halo nuclei/
Atomic weakly bound three-body systems

Scales of n-n-c system: contact interaction

Classification scheme

Threshold conditions for excited Efimov states

n-^{19}C scattering and Efimov physics

Summary and outlook

Physics of weakly bound three-body systems

Charateristic phenomena three-body systems:

Thomas collapse (1935) and Efimov effect (1970)

$r_o \rightarrow 0$ $|a| \rightarrow \infty$

??? infinitely many weakly bound states

$|a|/r_o \rightarrow \infty$

Thomas-Efimov effect!

S.K. Adhikari, A. Delfino, T. Frederico, I.D. Goldman, and L. Tomio, Phys. Rev. A **37**, 3666 (1988).

One three-body scale is necessary to represent short-range physics !!!!

A. S. Jensen, K. Riisager, D. V. Fedorov, and E. Garrido, Rev. Mod. Phys. **76**, 215 (2004).

E. Braaten, H.-W. Hammer, Phys. Rep. **428**, 259 (2006)

Zero-range 3-boson equation: Thomas-Efimov effect

Skorniakov and Ter-Martirosian equations (1956)

$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm\sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}\vec{y}^2}} \int d^3x \left(\frac{1}{\epsilon_3 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} - \frac{1}{1 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} \right) \chi(\vec{x})$$

$$\epsilon_3 = E_3 / \mu^2_{(3)} \qquad \epsilon_2 = E_2 / \mu^2_{(3)} \qquad \mu^2_{(3)} = 1$$

Adhikari,TF,Goldman, PRL74 (1995) 487

Thomas collapse: $\mu^2_{(3)} \rightarrow \infty$

Efimov effect: $E_2 \rightarrow 0$

$$\epsilon_2 = E_2 / \mu^2_{(3)}$$

$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)}(\pm\sqrt{\epsilon_2})$$

Scaling limit & limit cycle

$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)}(\pm\sqrt{\epsilon_2})$$

$$\xi \equiv \pm\sqrt{\epsilon_2} = \pm\left(E_2\,\epsilon_3^{(N)}/E_3^{(N)}\right)^{1/2}$$

$$\frac{E_3^{(N+1)}}{E_3^{(N)}} = \lim_{N\to\infty} \frac{\epsilon_3^{(N+1)}(\xi)}{\epsilon_3^{(N)}} = \mathcal{F}\left(\pm\sqrt{\frac{E_2}{E_3^{(N)}}}\right)$$

$$\mathcal{F}(0) = 1/515$$

Scaling function

Efimov 1970

Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

Limit cycle:
Mohr et al Ann.Phys. 321 (2006)225

Scaling functions: Correlation between observables

$$O(E, E_3, E_2) = (E_3)^{\eta} \mathcal{A}(\sqrt{E/E_3}, \sqrt{E_2/E_3})$$

Scaling function

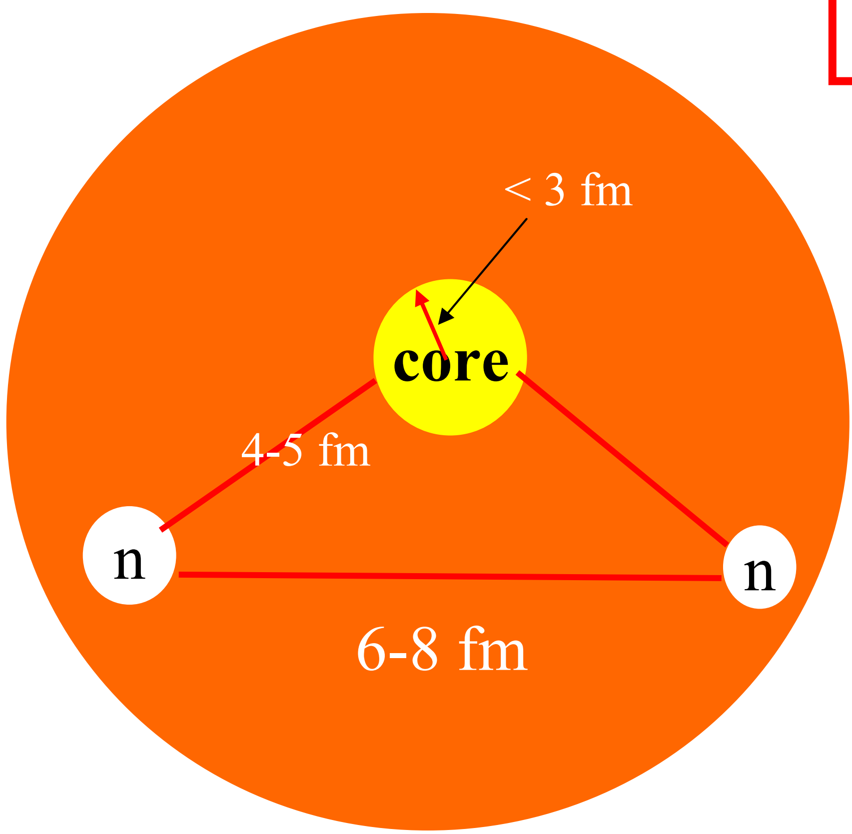
Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

Limit cycle:
Mohr et al Ann.Phys. 321 (2006)225

Correlation between S-wave observables

- Phillips plot: triton B.E. versus doublet scattering length
- 2nd order n-d polarization observables versus triton B.E.
- Trapped atomic trimer B.E. versus recombination rate

Two neutron weakly bound three-body halo nuclei



core-neutron-neutron halo nuclei

^{11}Li ^{14}Be ^{20}C

Binding energy ~ MeV or < MeV

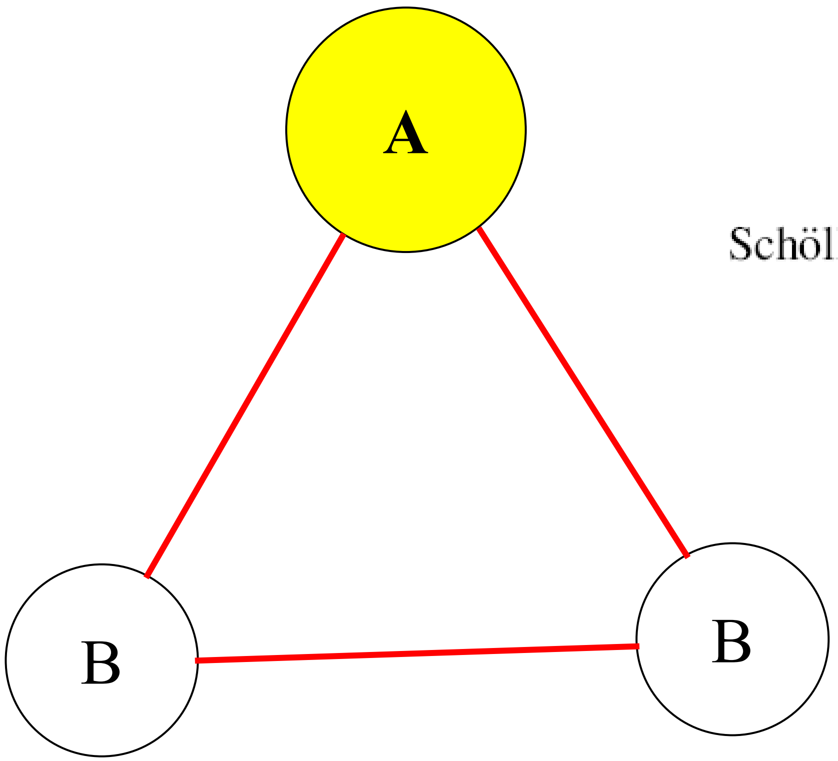
R_{nn}(Exp) ~ 6 - 8 fm (^{11}Li)

F. M. Marqués et al. Phys. Rev. C **64**, 061301 (2001)

M. Petrascu et al. Nucl. Phys. A **738**, 503 (2004)

Tanihata et al., Phys. Rev. Lett. **55**, 2676 (1985)

Atomic weakly bound three-body systems



Schöllkopf, W., Toennies, J. P.: Science **266**, 1345 (1994)

dimer $R_{^4He-^4He} \sim 50$ A

A-B-B weakly bound molecules

ultra-low binding ~ mK or < mK

$^{133}Cs_3$ (trapped ultracold gas near a Feshbach resonance)

$^4He_3$ $^4He_2 - ^7Li$ $^4He_2 - ^6Li$ $^4He_2 - ^{23}Na$

Scales of n-n-c system: contact interaction

E_{nn} Energy of the bound/virtual nn system

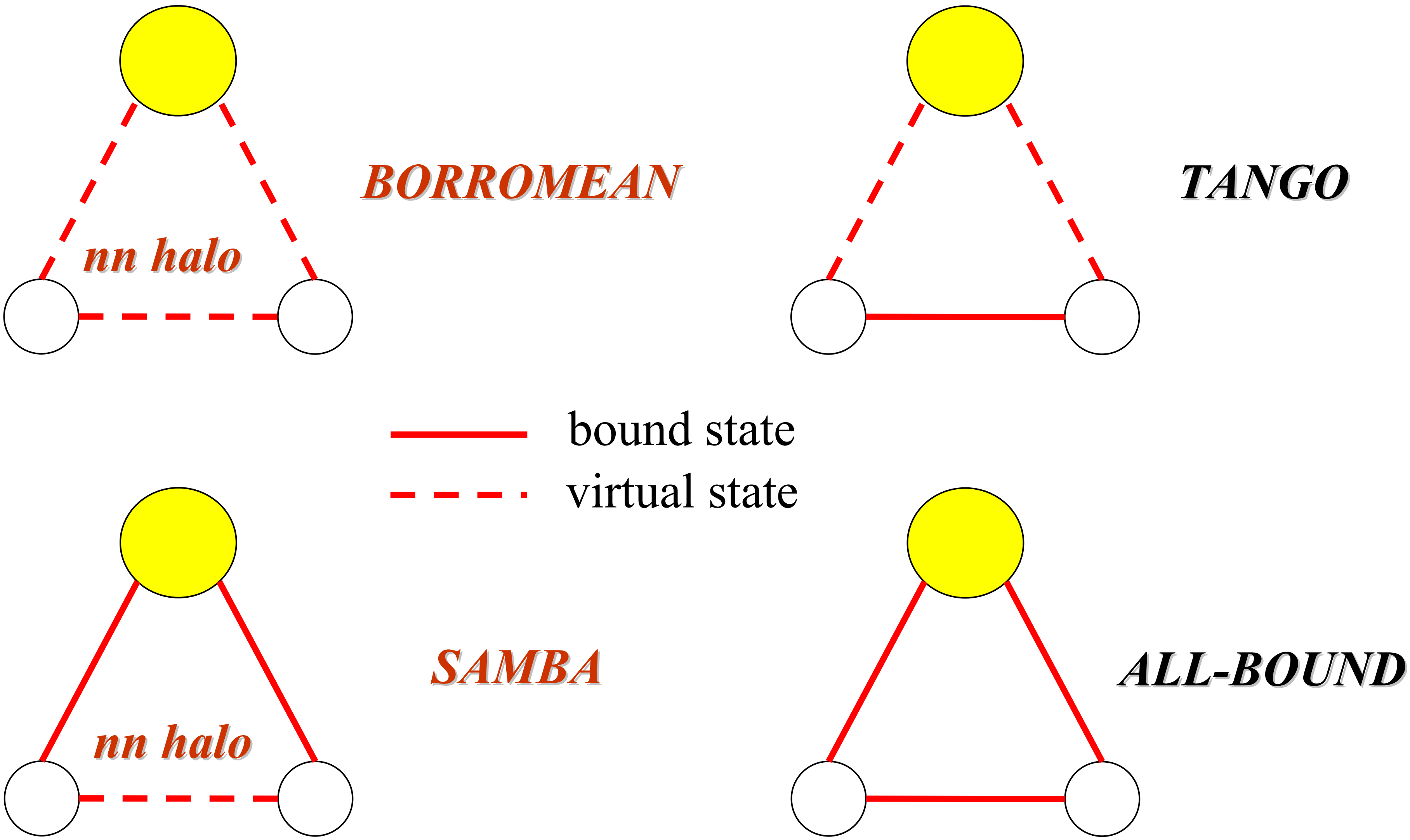
E_{nc} Energy of the bound/virtual nc system

$B_N = \left| E_3^{(N)} \right|$ Energy of the Nth state of the nnc system

A = mass of the core

Classification scheme

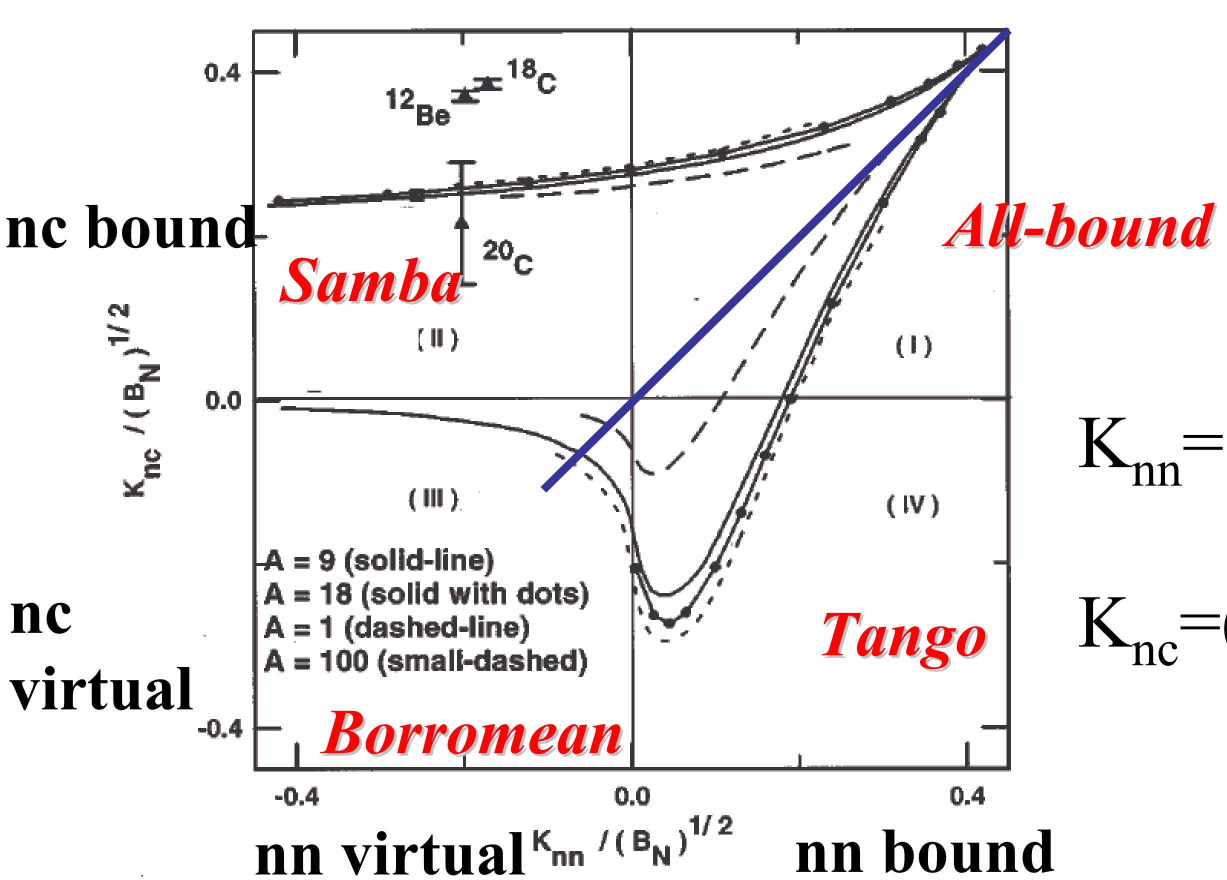
Yamashita, Tomio and T. F. Nucl. Phys. A 735, 40 (2004)



A.S. Jensen, K. Riisager, D.V. Fedorov, E. Garrido, Europhys. Lett. 61 (2003) 320.
F. Robicheaux, Phys. Rev. A 60 (1999) 1706.

Threshold for an excited Efimov state: *Halo-nuclei*

Critical condition for an excited (N+1)-th Efimov state above the N-th one:



$K_{nn}=(B_{nn})^{1/2}$

$K_{nc}=(B_{nc})^{1/2}$

Amorim,TF,Tomio PRC56(1997)2378

D. L. Canham and H.-W. Hammer, *Universal properties and structure of halo nuclei*, arXiv:0807.3258.

Threshold for an excited Efimov state: Weakly bound molecules

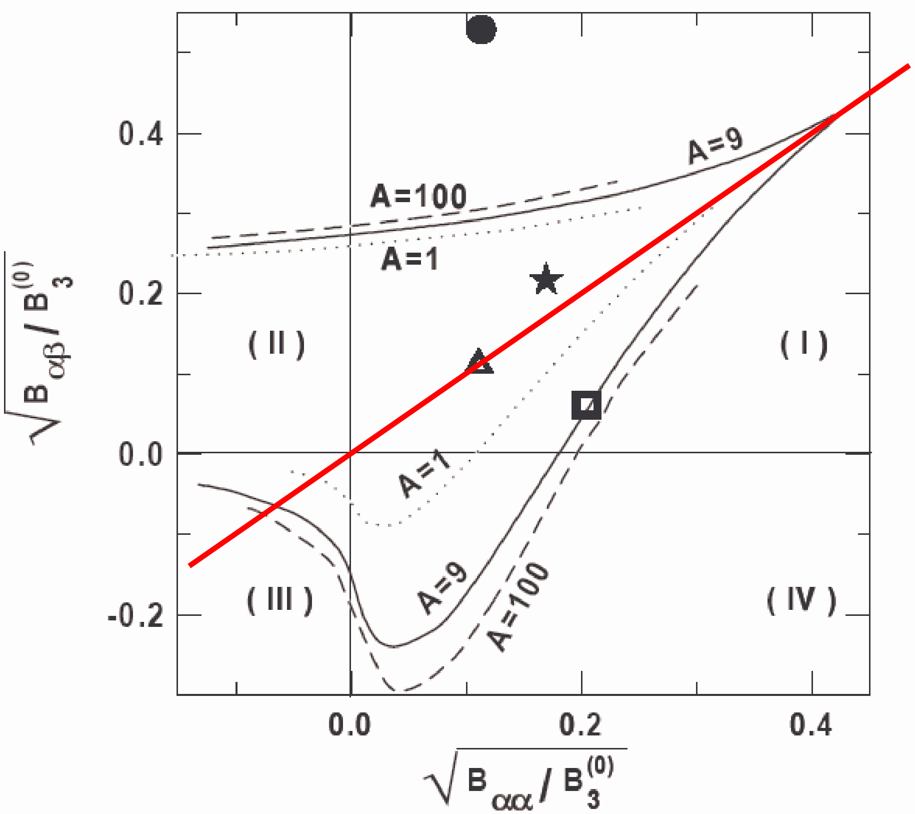


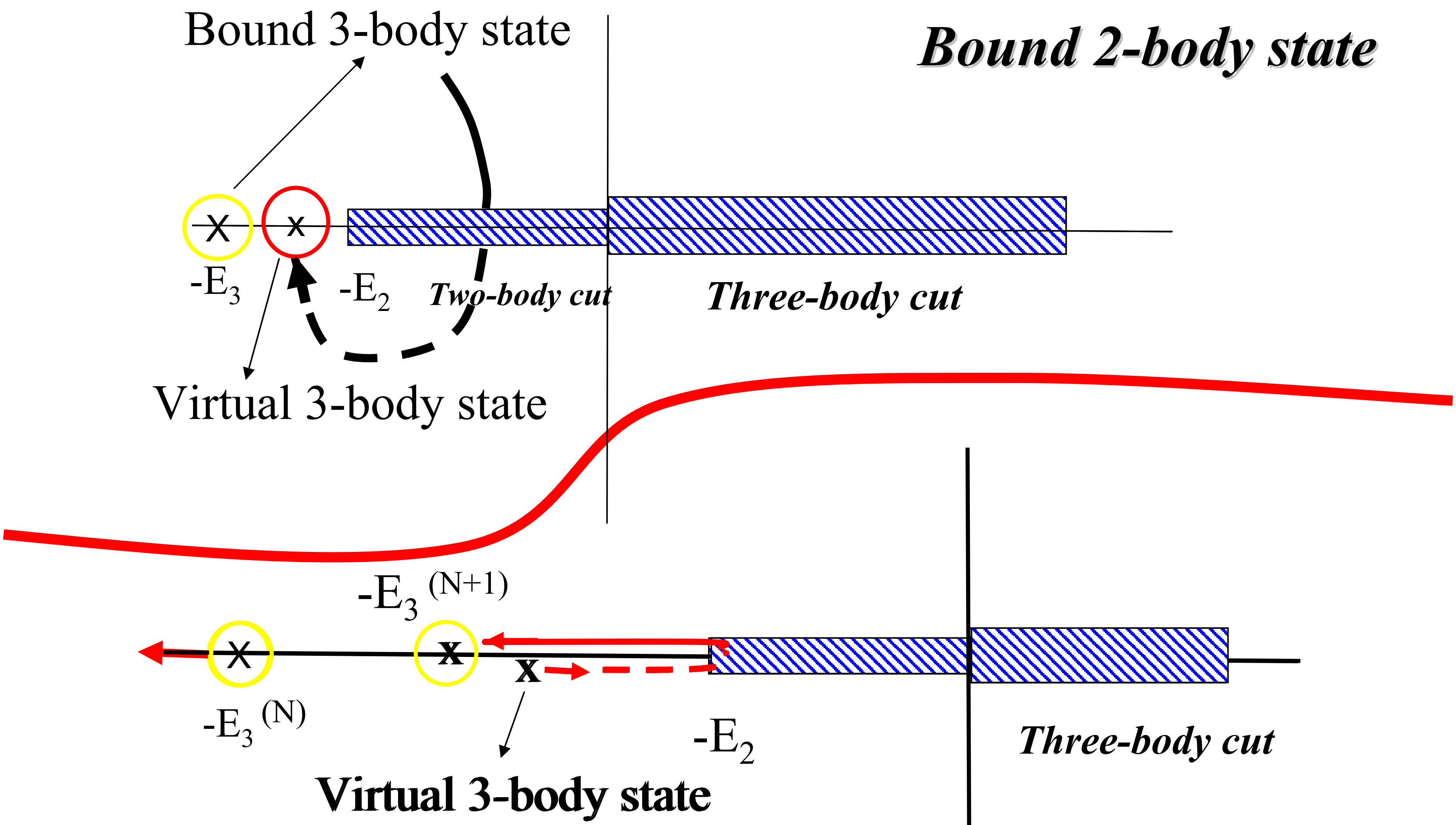
FIG. 1. The scaling approach for the three-body system $\alpha-\alpha-\beta$. The coordinates of the systems with $\alpha \equiv {}^4$He and $\beta = {}^4$He, ^{7}Li, ^{6}Li and ^{23}Na are respectively represented by a triangle, a star, a square and a full circle.

$$K_{aa}=(B_{aa})^{1/2}$$

$$K_{ab}=(B_{ab})^{1/2}$$

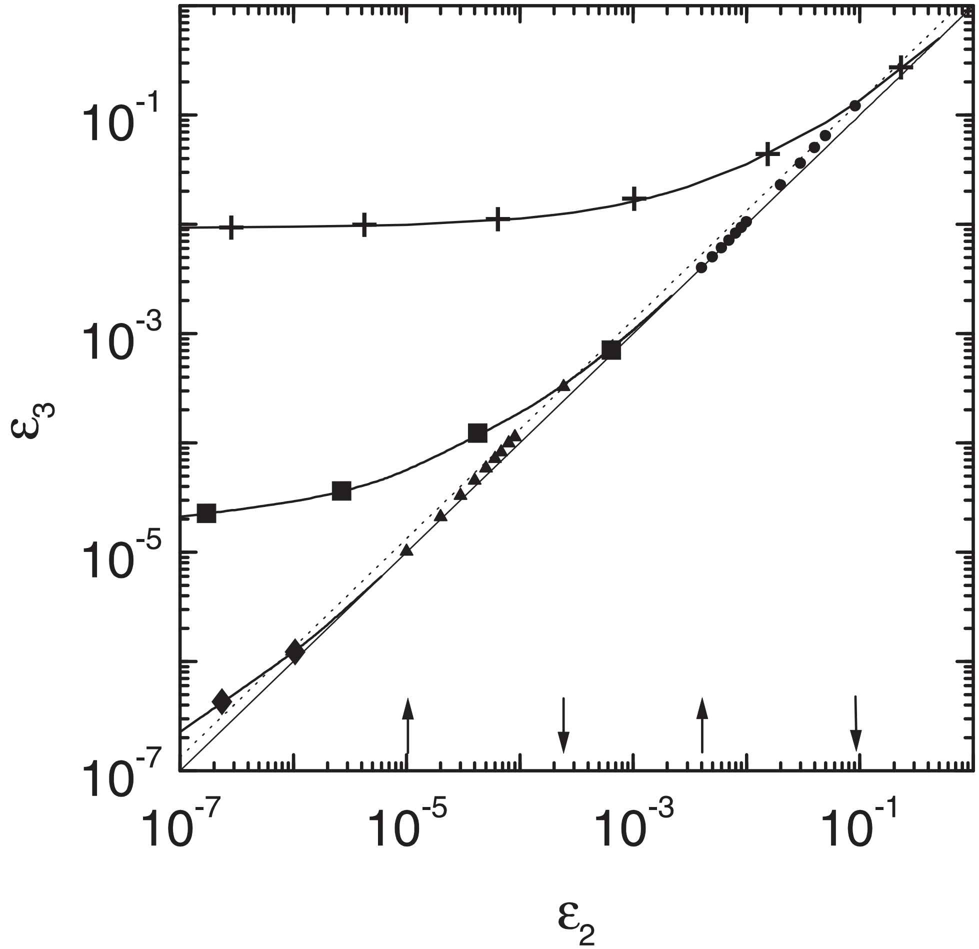
Delfino et al., JCP 113 (2000) 7874

Three-bosons: analytic structure & Efimov state trajectory



S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).

Efimov States – Bound and virtual states (3 identical bosons)



Bound-states (lines with simbols)

With Plus - Fundamental state

With squares – 1st. excited

With diamonds – 2nd. excited

Virtual-states (just simbols)

circles – 1st. Excited state

triangles – 2nd. Excited state

ε_2 bound

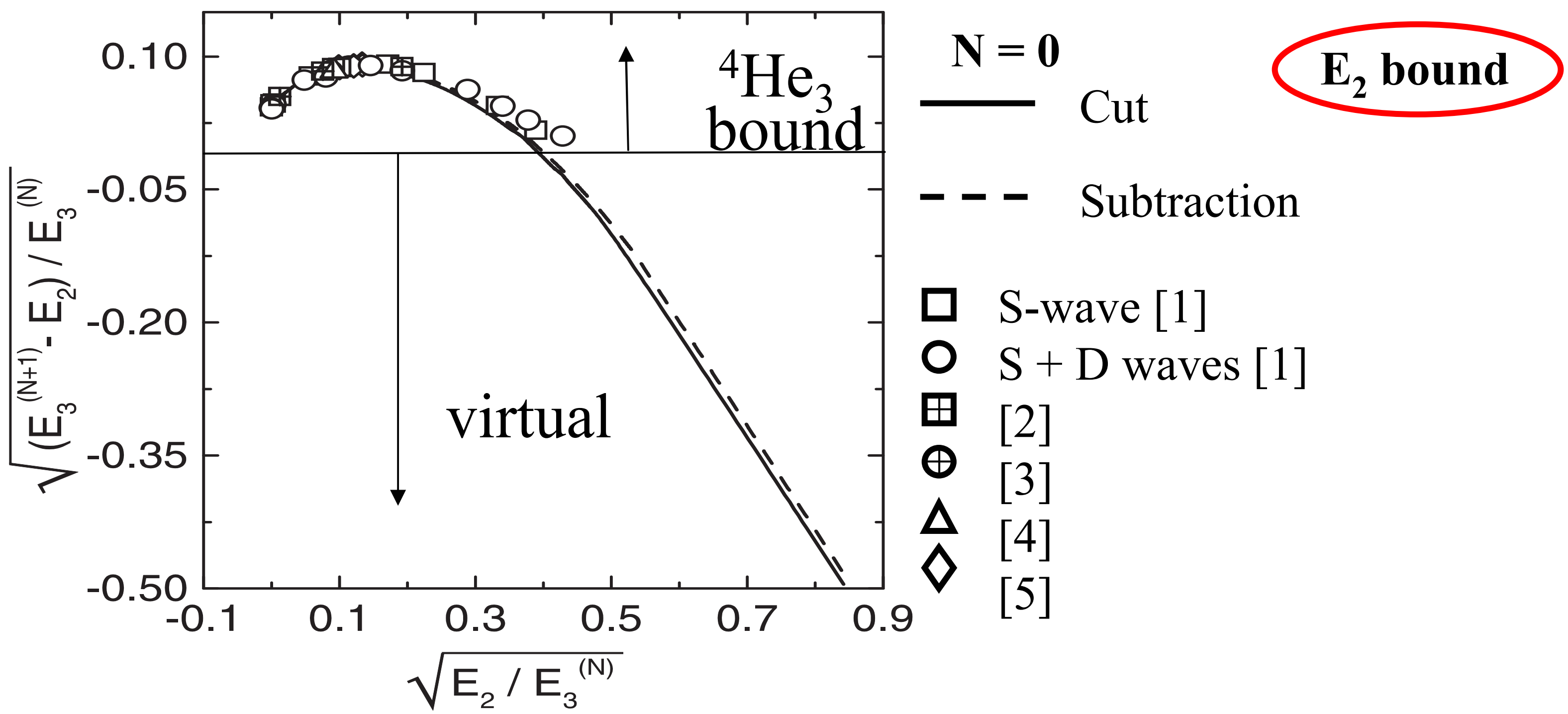
↓ the virtual state starts (dotted line) $\varepsilon_3 = \frac{4}{3}\varepsilon_2$

↑ Virtual-state turns to an excited state (continuous line) $\varepsilon_3 = \varepsilon_2$

S. K. Adhikari, A. C. Fonseca and LT *Phys. Rev.* **C27**, 1826 (1983).

Efimov States – Bound and virtual states (3 identical bosons)

T. Frederico, LT, A. Delfino and E. A. Amorim. *Phys. Rev.* **A60**, R9 (1999).



[1] Th. Cornelius e W. Glöckle. *J. Chem. Phys.* **85**, 1 (1996).
[2] S. Huber. *Phys. Rev.* **A31**, 3981 (1985).
[3] P. Barletta e A. Kievsky. *Phys. Rev.* **A64**, 042514 (2001).
[4] D. V. Fedorov e A. S. Jensen. *J. Phys.* **A34**, 6003 (2001).
[5] E. A. Kolganova, A. K. Motovilov e S. A. Sofianos. *Phys. Rev.* **A56**, R1686 (1997).

Range correction: Thogersen, Fedorov, Jensen PRA78(2008)020501(R)

Range correction: qualitative discussion

$$f(k)=(-a^{-1}+r_0k^2/2-ik)^{-1}$$

Bound state: $k=i\,(E_2)^{1/2} \rightarrow -a^{-1}-r_0E_2/2+(E_2)^{1/2}=0$

$$f(k)\sim(-E_2^{1/2}-ik)^{-1}\,(1+r_0E_2^{1/2})$$

Effective interaction in 3-boson eq. **increases**!
→ Critical value of a to allow an Efimov state **decreases**!

Virtual state: $k=-i\,(E_2)^{1/2} \rightarrow -a^{-1}-r_0E_2/2-(E_2)^{1/2}=0$

$$f(k)\sim(E_2^{1/2}-ik)^{-1}\,(1-r_0E_2^{1/2}/2)$$

Effective interaction in 3-boson eq. **decreases**!
→ Critical value of a to allow an Efimov state **increases**!

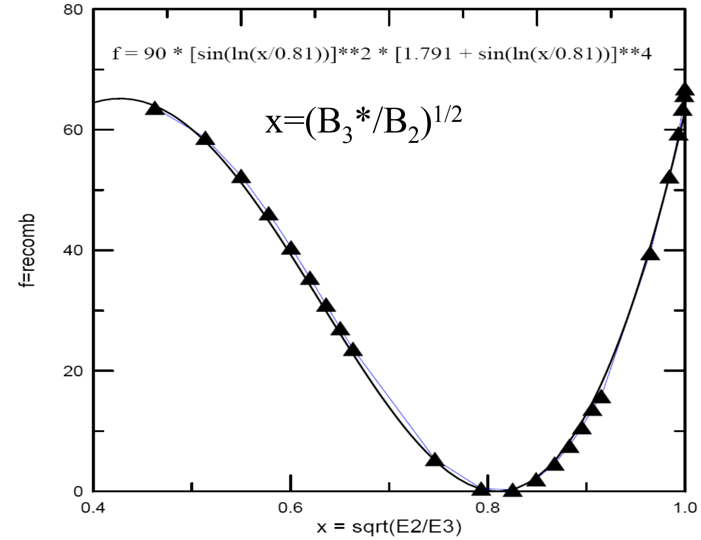
Three-atom recombination & trimer energy

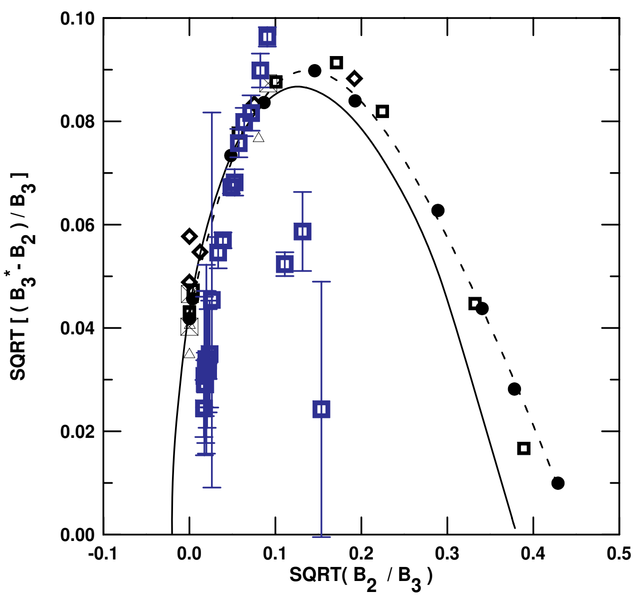
A+A+A→A+D

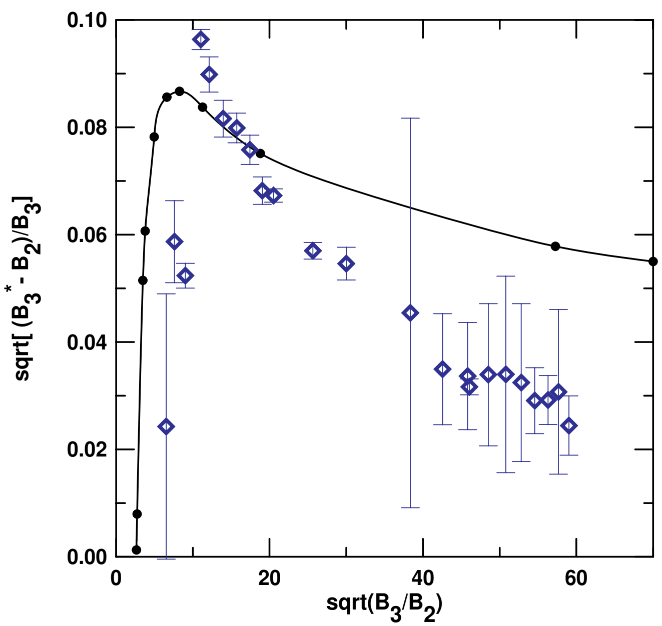
3-body recombination of Cesium ultracold gas data near Feshbach resonance v.s. scattering length

T. Kraemer et al, Nature **440**, 315 (2006)

Resonance @ T~0 and a=-898a_0 →B_3 =1.31mK

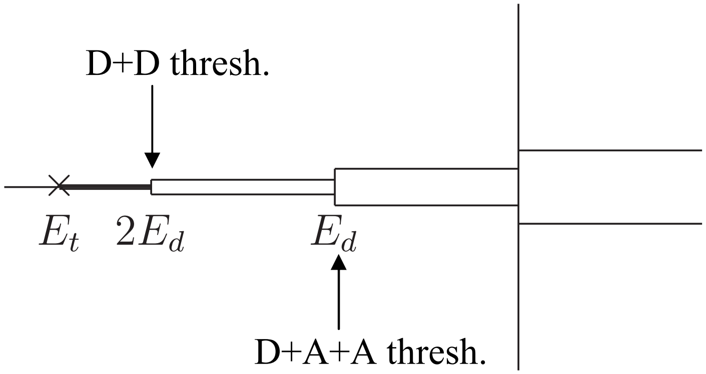






PRA**68**, 033406 (2003)
Trimer formation in D+D

Efimov states & D+D recombination



How many Efimov states between D+D and D+A+A thresholds (Ed→0)?

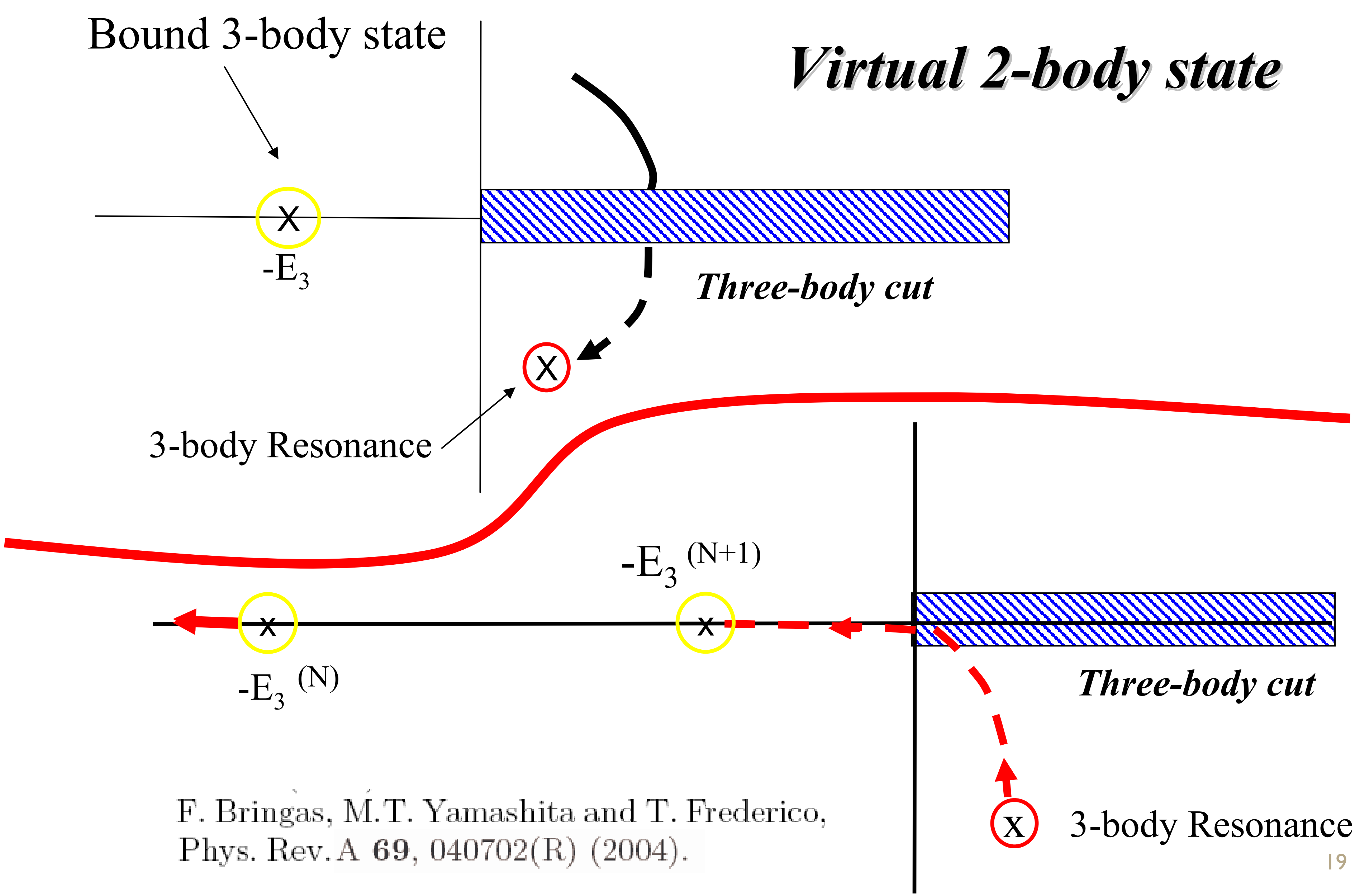
For Et = 6.9Ed → Et*=Ed & Et > 6.9Ed → |Et*| > |Ed|

→At most one Efimov state appears in the range: 2|Ed| < |Et| <|Ed| !!!!
(Range corrections hardly destroy that ~ 30%!)

→ dimer+dimer recombination with trapped ultracold cesium?

F. Ferlaino, S. Knoop, M. Mark, M. Berninger, H. Schbel, H.-C. Ngerl, R. Grimm Phys. Rev. Lett. **101**, 023201 (2008).

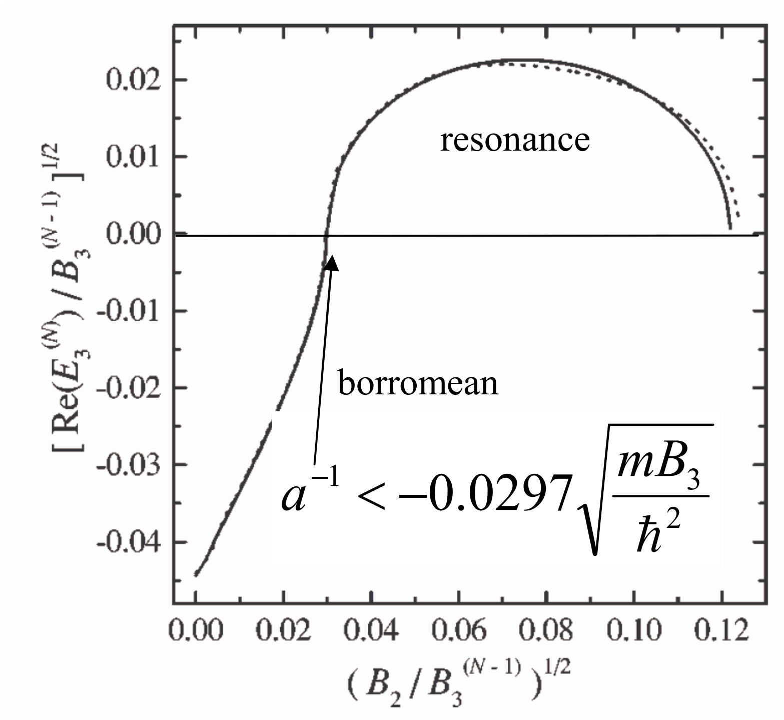
Borromean configuration: analytic structure & Efimov state trajectory

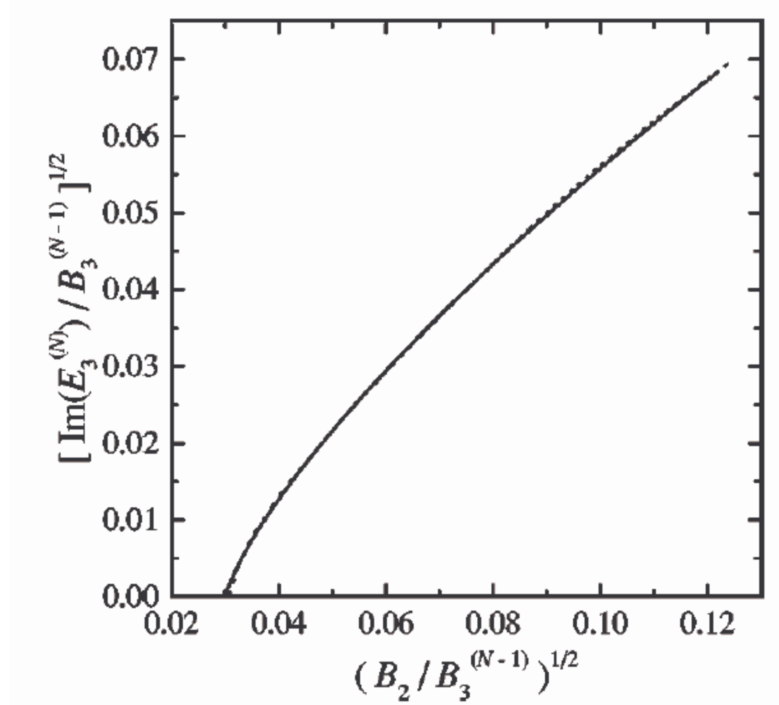


F. Bringas, M.T. Yamashita and T. Frederico,
Phys. Rev. A **69**, 040702(R) (2004).

Efimov state trajectory: borromean case

S-wave three-boson resonance





F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

Evidence of continuum resonances in recombination of ultracold Cs atoms

T. Kraemer et al, Nature **440**, 315 (2006)

Evidence of continuum resonances in ultracold cesium gas

T. Kraemer et al, Nature **440**, 315 (2006)

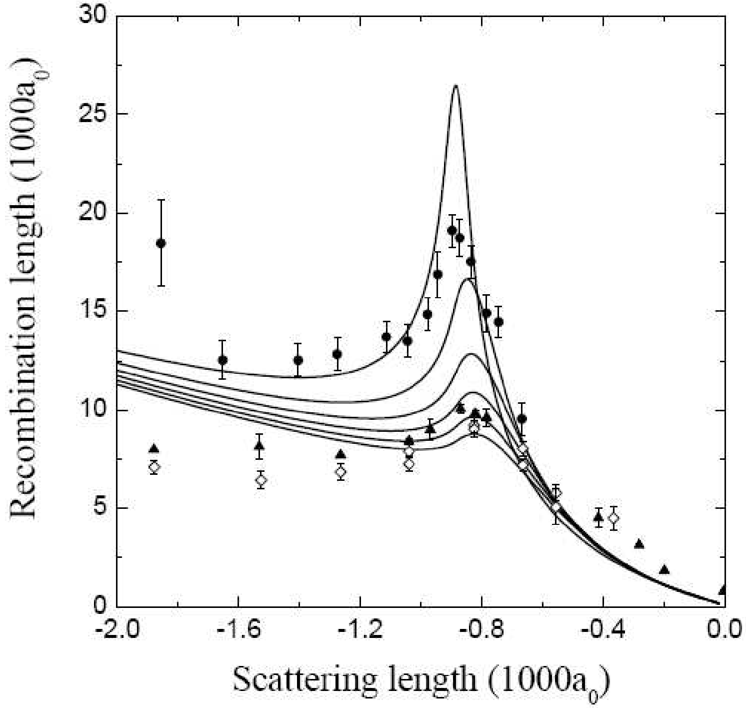
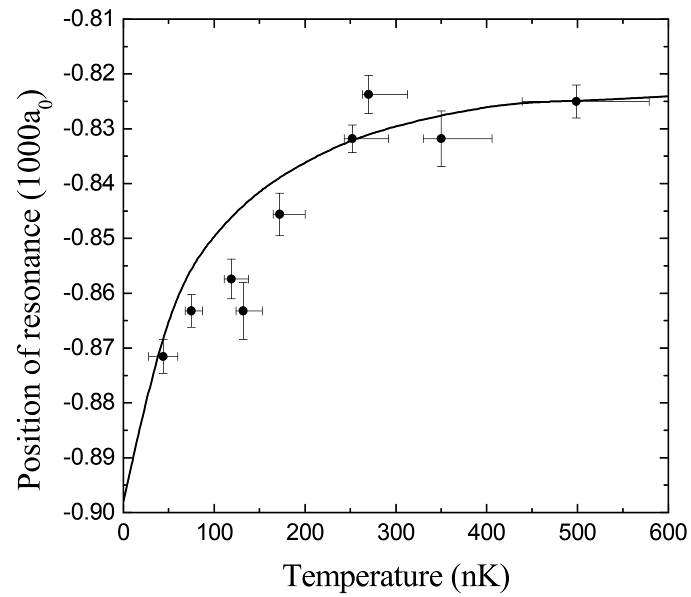


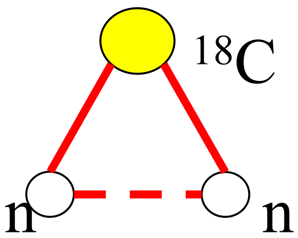
FIG. 2: Recombination length ($\rho_3 = [2m/(\sqrt{3}\hbar)\langle L_3\rangle_T]$) in the cesium trapped gas as a function of the scattering length and temperature. The solid curves from up to bottom are the theoretical results for T = 10 nK, 100 nK, 200 nK, 300 nK, 400 nK and 500 nK. The symbols are the experimental results for T = 10 nK (full circles), 200 nK (full triangles) and 250 nK (open diamonds) from Ref. [4].

Position of the maximum of the recombination length as a function of the temperature. Experimental data from B. Engeser et al., *in preparation*.

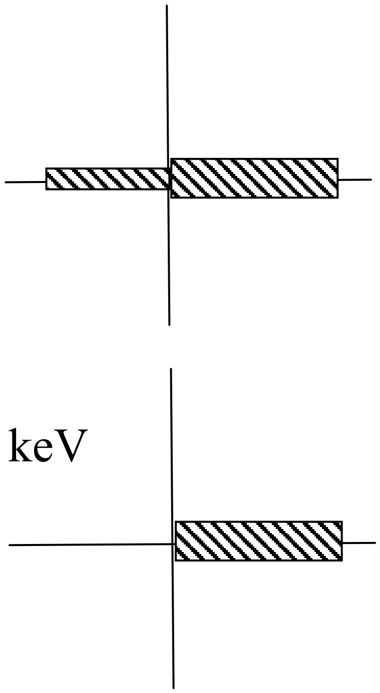


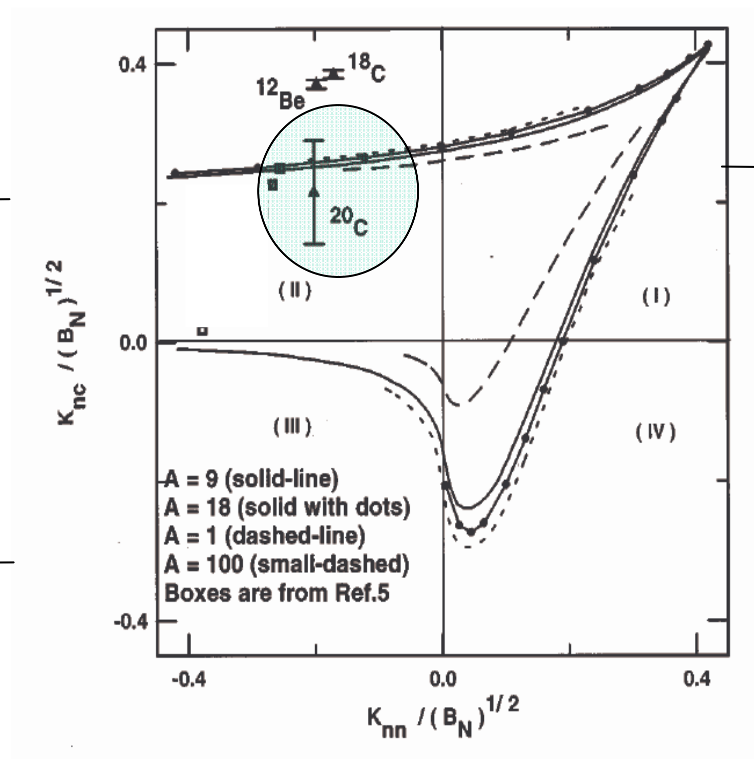
M.T. Yamashita et al. / Physics Letters A 363 (2007) 468

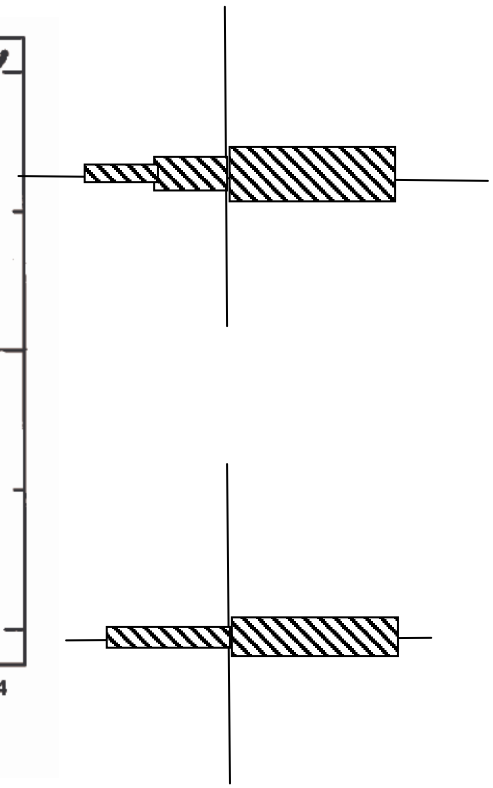
Threshold for an excited Efimov state and trajectory: ^{20}C



$E_3 = 3.5$ MeV
$E_{nc} = 160 \pm 110$ keV







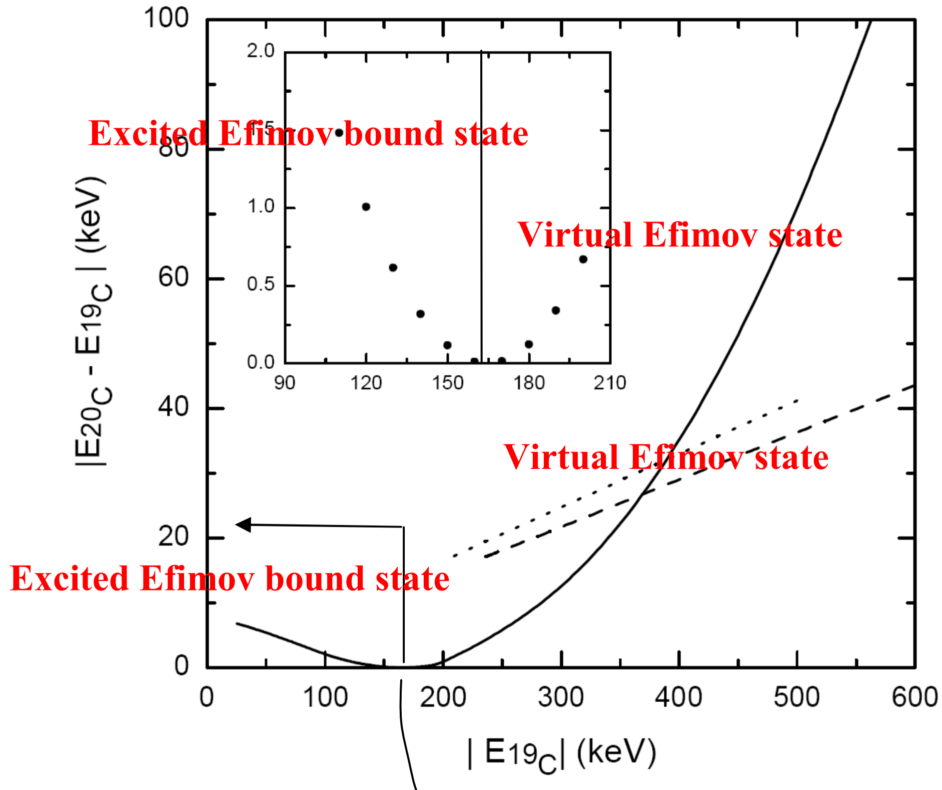
^{20}C can have a continuum resonance or virtual Efimov state?

Arora, Mazumdar, Bhasin PRC69 (2004)061301(R) Mazumdar, Rau, Bhasin PRL97(2006)062503

Efimov state→resonance of n+^{19}C by changing Knc

Threshold for an excited Efimov state and trajectory: ^{20}C

Yamashita, Frederico,Tomio,
PRL99 (2007)269201 Comment on"Efimov States and their Fano resonances..."
& PLB660(2008)339



Efimov state in ^{20}C goes to a virtual state for $|E_{19C}| > 165keV$!

Critical value: $|E_{19C}|$=165keV

n-^{19}C scattering and Efimov physics

➡ Pole in s-wave kcot(δ) for n-d system ! Well known ~ 50 years

Delves' 60, Van oers & Seagrave' 67, Girard & Fuda' 78

$$k\cot\delta_0 = -A + Bk^2 - \frac{C}{1+Dk^2},$$

The existence of the triton virtual state was found on the basis of the effective range expansion.

➡ Universal property!

The atom-dimer (three-boson) scattering length is approximately given in Bratten and Hammer (Phys. Rep. 428 (2006) 259):

$$a_{AD} = (1.46 - 2.15\tan[s_0 ln(a\Lambda_*) + 0.09])a\ ,$$

where $s_0 = 1.00624$.

2-body scatt. lengths:
a_B for a_{AD}=infinity
a_0 for a_{AD}=0

$$\frac{a_B}{a_0} = \exp\left(\frac{\pi/2 - 0.59654}{s_0}\right)$$

n-^{19}C scattering and Efimov physics

Brief view…

$$\chi_n(\vec{q}) \equiv (2\pi)^3\delta(\vec{q}-\vec{k}_i) + 4\pi\frac{h_n(\vec{q};\mathcal{E}(k_i))}{q^2-k_i^2-\mathrm{i}\epsilon},$$

$$h_n^\ell(q;\mathcal{E}_i) = \mathcal{V}^\ell(q,k_i;\mathcal{E}_i) + \frac{2}{\pi}\int_0^\infty dk\,k^2\,\frac{\mathcal{V}^\ell(q,k;\mathcal{E}_i)\,h_n^\ell(k;\mathcal{E}_i)}{k^2-k_i^2-\mathrm{i}\epsilon}.$$

$$\mathcal{V}^\ell(q,k;\mathcal{E}) \equiv \pi\frac{(A+1)}{A+2}\left[K_2^\ell(q,k;\mathcal{E}) + \int_0^\infty dk' k'^2 K_1^\ell(q,k';\mathcal{E})\tau_{nn}(k';\mathcal{E})K_1^\ell(k,k';\mathcal{E})\right]$$

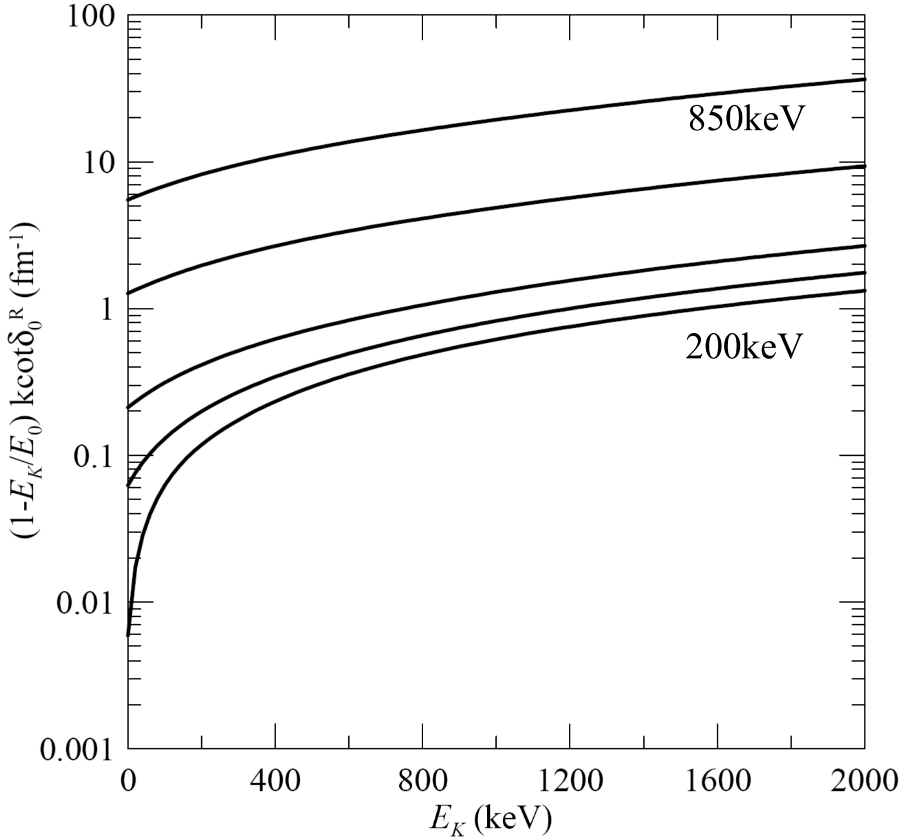
$$\tau_{nn}(q;\mathcal{E}) \equiv \frac{-2}{\pi}\left[\sqrt{|\epsilon_{nn}|} + \sqrt{\frac{A+2}{4A}q^2-\mathcal{E}}\right]^{-1},$$

$$\bar{\tau}_{nc}(q;\mathcal{E}) \equiv \frac{-1}{\pi}\left(\frac{A+1}{2A}\right)^{\frac{3}{2}}\left(\sqrt{|\epsilon_{nc}|} + \sqrt{\frac{(A+2)q^2}{2(A+1)}-\mathcal{E}}\right),$$

$$K_{i=1,2}^\ell(q,k;\mathcal{E}) \equiv G_i^\ell(q,k;\mathcal{E}) - \delta_{\ell 0}\, G_i^\ell(q,k;-\mu^2),$$

$$G_i^\ell(q,k;\mathcal{E}) = \int_{-1}^{1} dy \frac{P_\ell(y)}{\mathcal{E} - \frac{A+1}{A+A^{i-1}}q^2 - \frac{A+1}{2A}k^2 - \frac{kqy}{A^{i-1}} + \mathrm{i}\epsilon}.$$

n-^{19}C scattering and Efimov physics



$$k\cot\delta_0^R = \frac{-a^{-1}_{n-{}^{19}C} + \beta\, E + \gamma\, E^2}{1 - E/E_0},$$

$\vert E_{^{19}C}\vert$(keV)	$(a_{n-{}^{19}C})^{-1}$ (fm^{-1})	β (fm.keV)$^{-1}$	γ (fm.keV^2)$^{-1}$	E_0 (keV)
200	-0.591 10^{-2}	5.685 10^{-4}	4.673 10^{-8}	1442.745
400	-0.624 10^{-1}	6.743 10^{-4}	8.821 10^{-8}	823.887
600	-2.118 10^{-1}	9.337 10^{-4}	1.464 10^{-7}	451.398
800	-1.268	3.11 10^{-3}	4.424 10^{-7}	114.976
850	-5.510	1.201 10^{-2}	1.641 10^{-6}	28.845

n-^{19}C scattering and Efimov physics

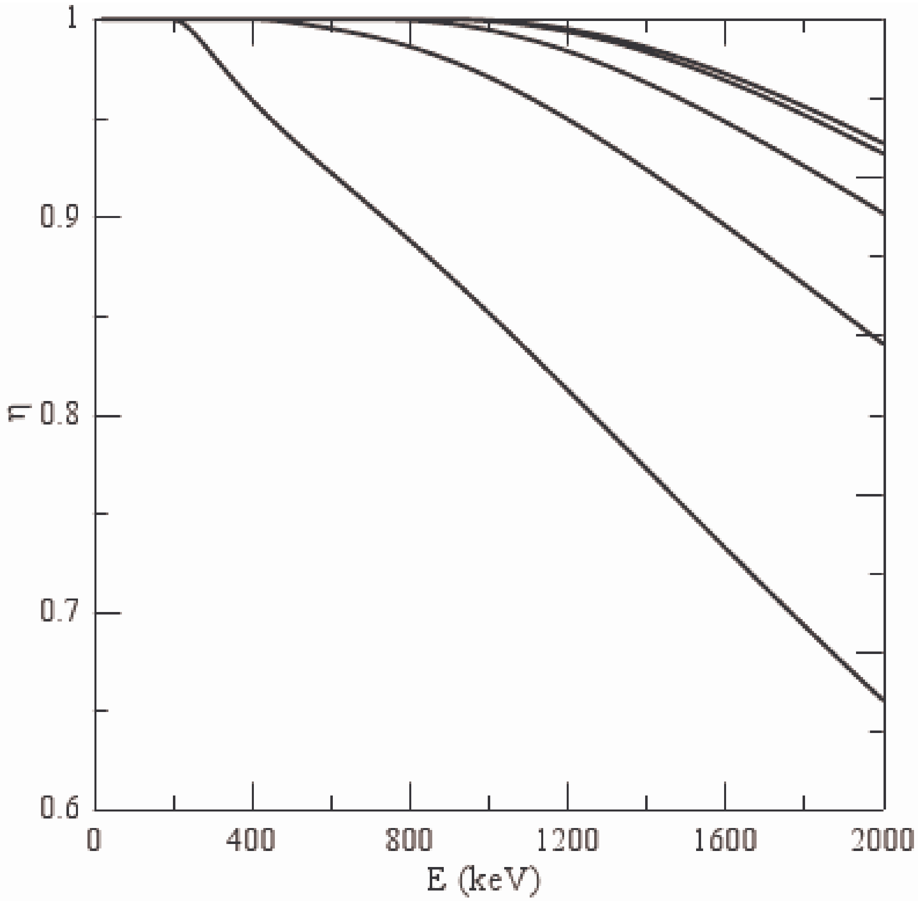
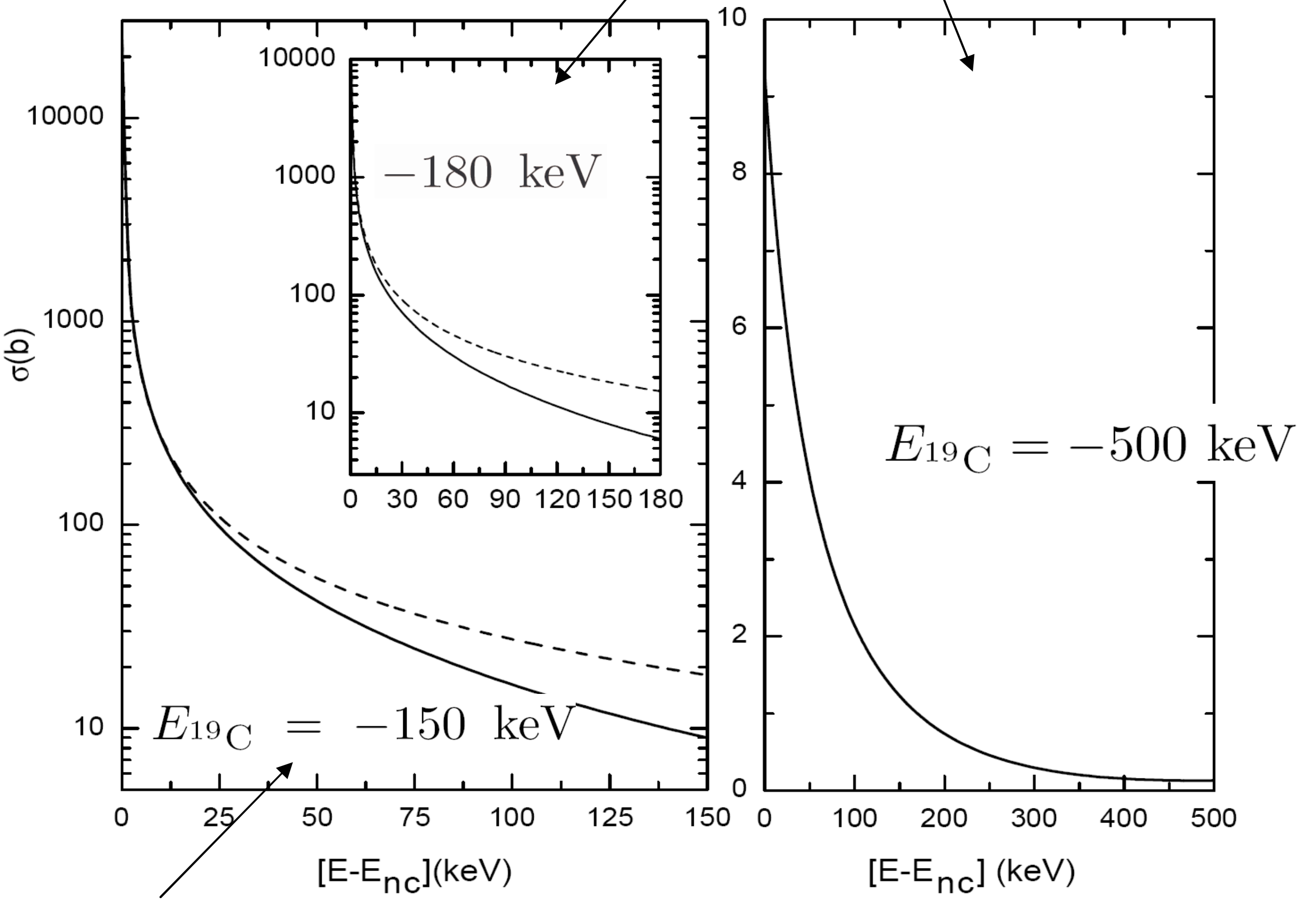


Fig. 4. s−wave absorption parameter as a function of the CM kinetic energy. From left to right the curves corresponds to the following ^{19}C energies: 200, 400, 600, 800 and 850 keV.

n-^{19}C scattering and Efimov physics

^{20}C has a virtual Efimov state



^{20}C has an excited bound Efimov state

n-^{19}C scattering and Efimov physics

$$s_0 \sim 1.12 \text{ for } A = 18$$

$$a_0/a_B \simeq 0.42$$

Numerical solution of the scattering eq.s ~ 0.44

Summary and outlook

- Weakly bound & large systems: **few scales regime** in halo nuclei, molecules, trapped atoms
- Zero-range model: classification of weakly-bound systems
  threshold conditions for excited states and resonances
  (evidence of the trajectory of resonance in ultra-cold atoms)
- n-n-c systems:
  Borromean configuration: **Efimov state→ resonance**
  All-bound, Samba, Tango (at least one subsystem is bound): **Efimov state→ virtual state**
- ^{20}C **Efimov state→ virtual state** $\mathbf{E_{19C} > 165}$ **keV**
- n+^{19}C scattering: **pole in the s-wave phase-shift**
- Exploration of universality in scattering, breakup of halo nuclei and large molecules
- Resonances with different mass ratios in Borromean configurations?
- Trapped atoms with a_{AD}=0 decoupling of atom-dimer condensates? if a_{AD} =*infinity?*
- Four-boson excited states, resonances & scattering, formation of trimers in traps?
- Evidence for a four-boson scale?