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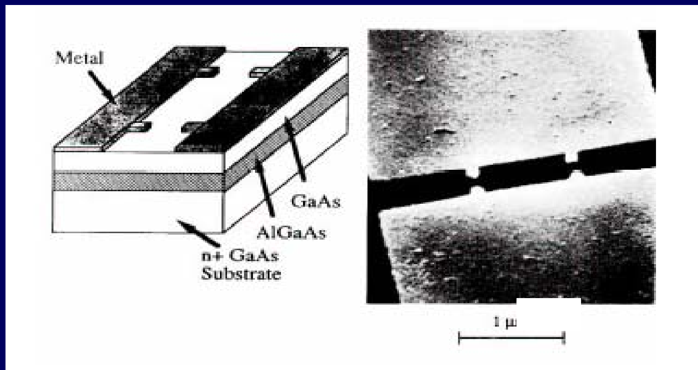
Interaction Blockade and Vortices - ultracold, trapped atoms and quantum dots

S.M. Reimann
reimann@matfys.lth.se

LUNDS UNIVERSITET

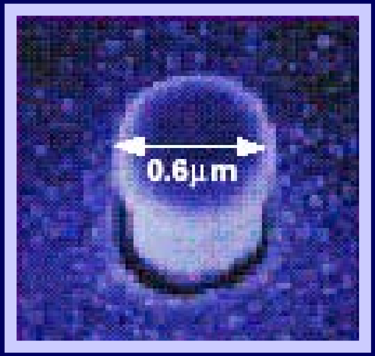
Picture ©Joseph Golden, 1969

The first quantum dot "artificial atoms"…



Meirav, Kastner and Wind, PRB '90.

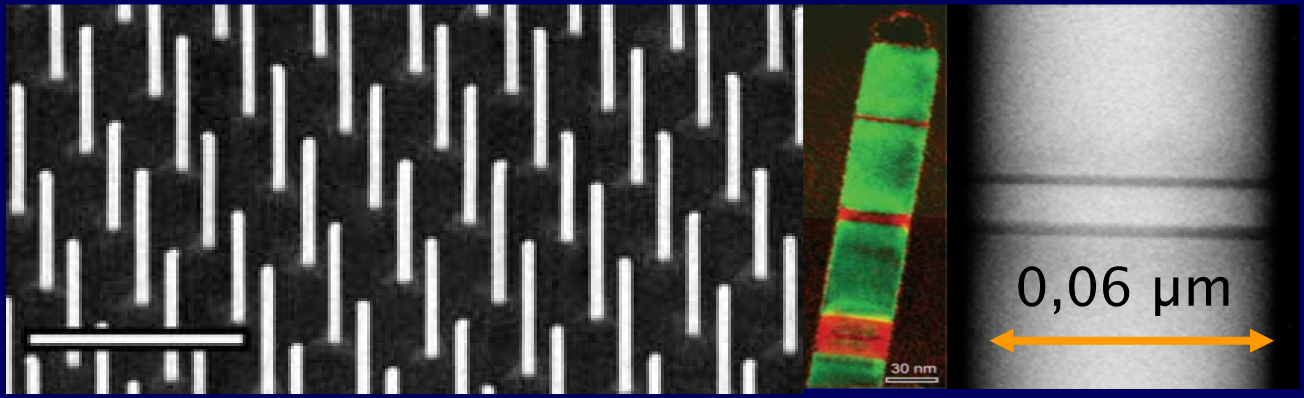
1μm



Tarucha *et al.*, PRL '96

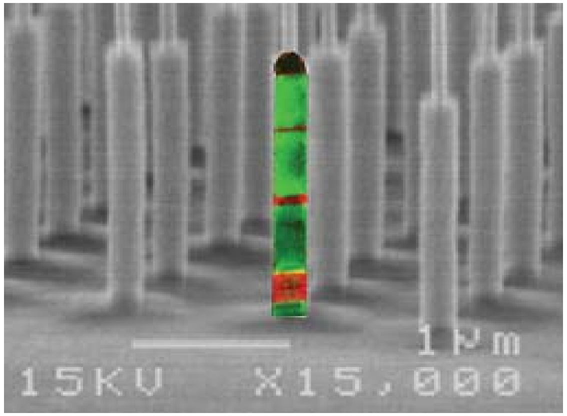
0.6μm

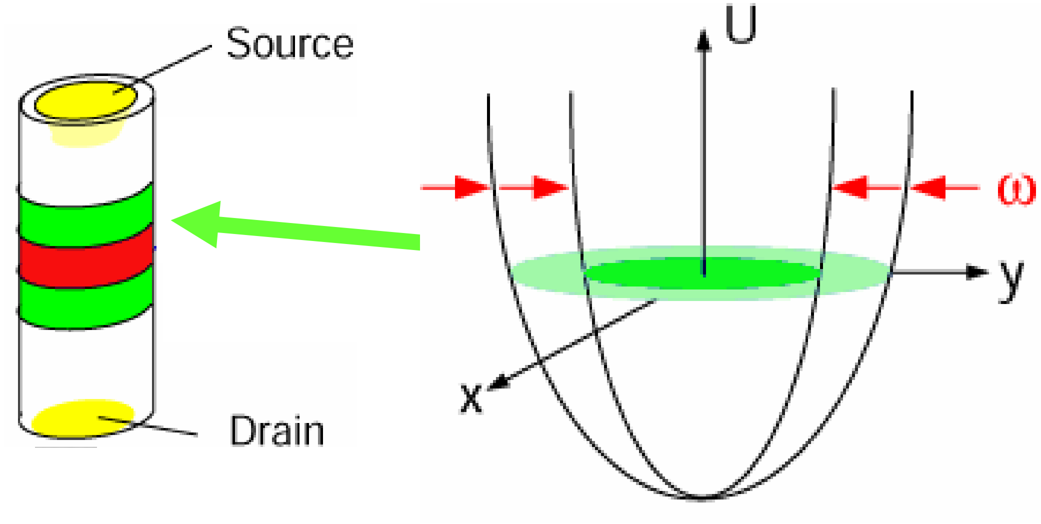
Today – smaller and more regular!



Samuelson *et al.*, Lund 2004

Many-Body Problem in a Quantum Dot





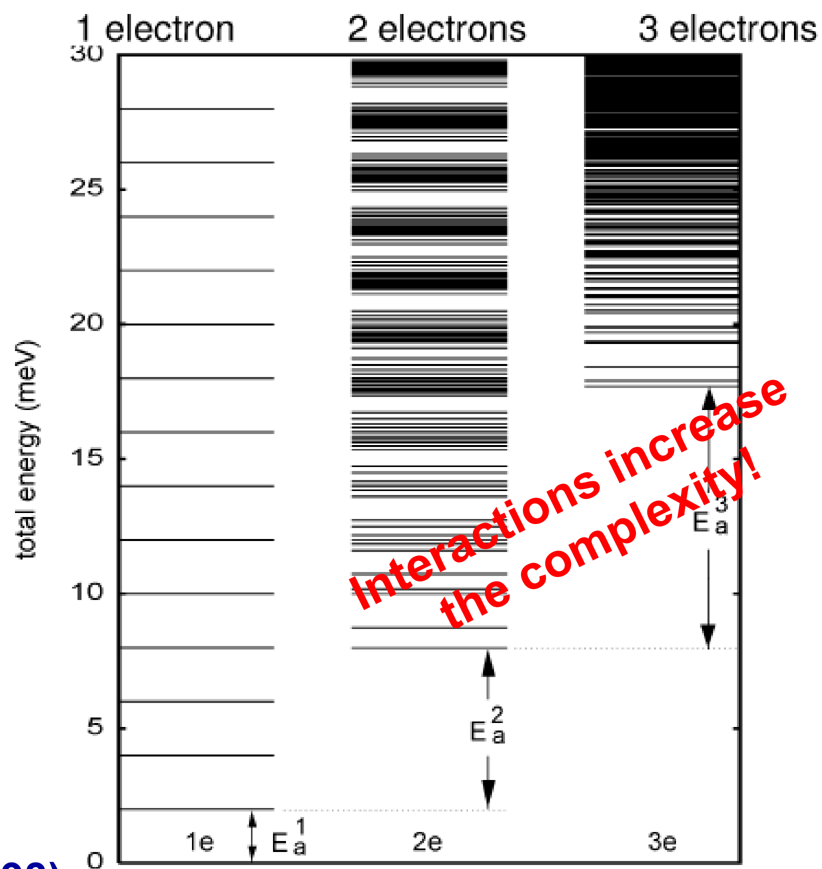
$$H = \sum_{i=1}^{N} \left\{ \frac{\boldsymbol{p}_i^2}{2m} + U(\boldsymbol{r}_i) \right\} + \frac{1}{2} \sum_{i,j=1;i\neq j}^{N} V(\boldsymbol{r}_i, \boldsymbol{r}_j)$$

For a review, see for example Reimann and Manninen, Rev. Mod. Phys. (2002)

Many-Body Problem in a Quantum Dot

$$\hat{V}_{ee} = \frac{e^2}{4\pi\epsilon} \sum_{i<j}^{N_e} \frac{1}{|\hat{\vec{r}}_i - \hat{\vec{r}}_j|}$$

Numerically exact many-body spectra for quantum dots with few electrons



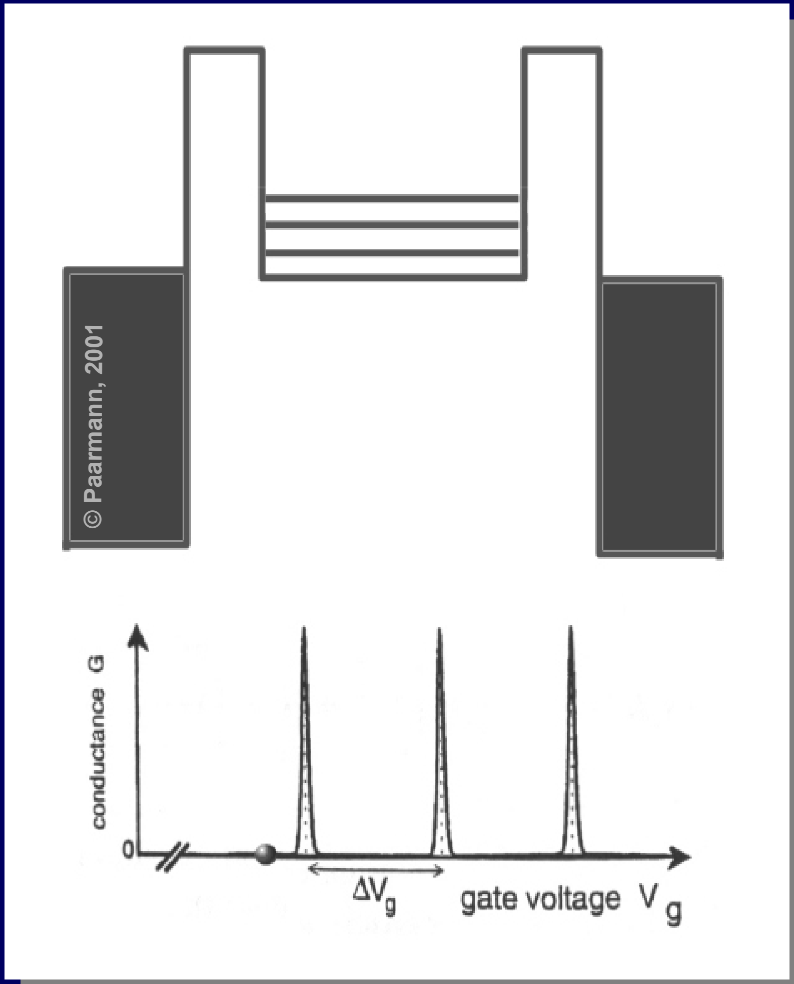
D. Pfannkuche et al., PRB (1993)

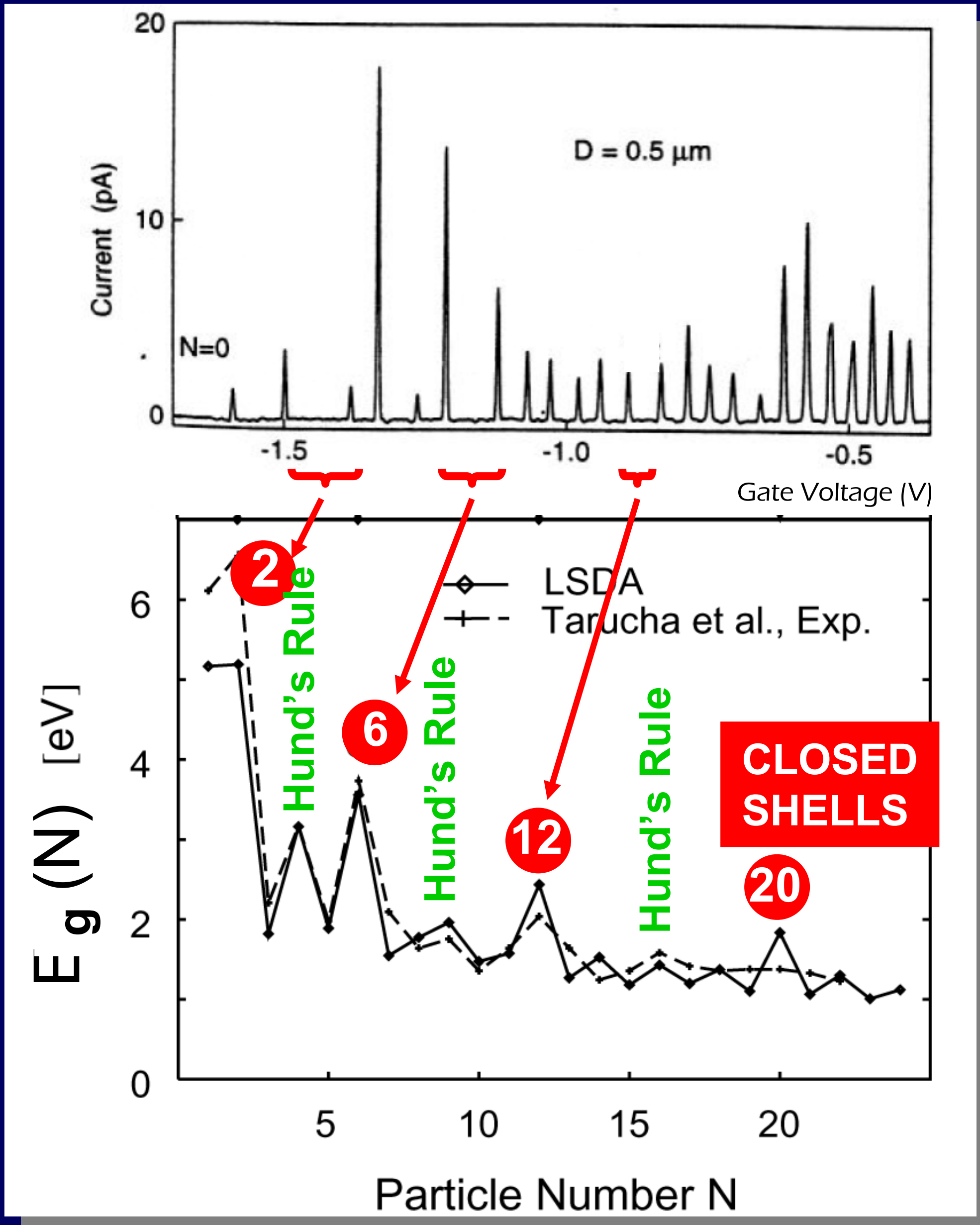
For a review, see for example Reimann and Manninen, Rev. Mod. Phys. (2002)

COULOMB BLOCKADE

in the transport through the dot structure

Tarucha et al., PRL, 1996





Gaps and Interaction Blockade

The *fundamental gap* in an N-body system:

$$E_g(N) = E(N+1) - 2E(N) + E(N-1)$$

Often approximated by density functional theory (DFT)

The *Kohn–Sham gap* is calculated from the Kohn–Sham eigenvalues of the N-body system:

$$E_g^{\mathrm{KS}} = \varepsilon_{N+1}(N) - \varepsilon_N(N)$$

HOMO-LUMO Gap

In general a very poor approximation to E_g

The exchange-correlation gap

E_g^{KS} ignores the *exchange-correlation gap*

$$\Delta_{xc} \equiv E_g - E_g^{\mathrm{KS}} = \left.\frac{\delta E_{xc}[n]}{\delta n(\mathbf{r})}\right|_{N+\eta} - \left.\frac{\delta E_{xc}[n]}{\delta n(\mathbf{r})}\right|_{N-\eta}$$

Δ_{xc} describes the gap that opens upon addition of a single particle to the system.

It disappears in the absence of interactions.

Δ_{xc} and Interaction Blockade

Usually, Coulomb blockade is modelled by a classical capacitance:

$$E_g = \Delta\varepsilon + \frac{e^2}{C}$$

Alternatively, Δ_{xc} can be associated with blockade:

$$E_g = E_g^{\mathrm{KS}} + \Delta_{xc}$$

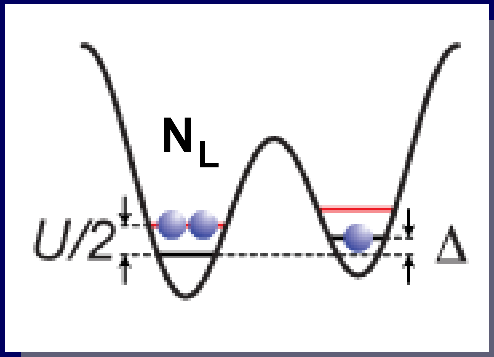
Blockade phenomena may be ubiquitous *and occur whenever there is a* Δ_{xc}

Blockade effects with cold atoms?

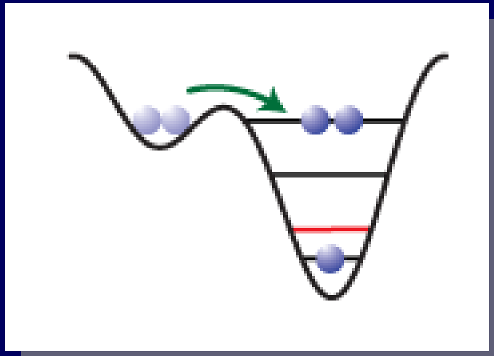
See Capelle et al. PRL, 2007

Bosonic Atoms in Optical Lattices with asymmetric wells

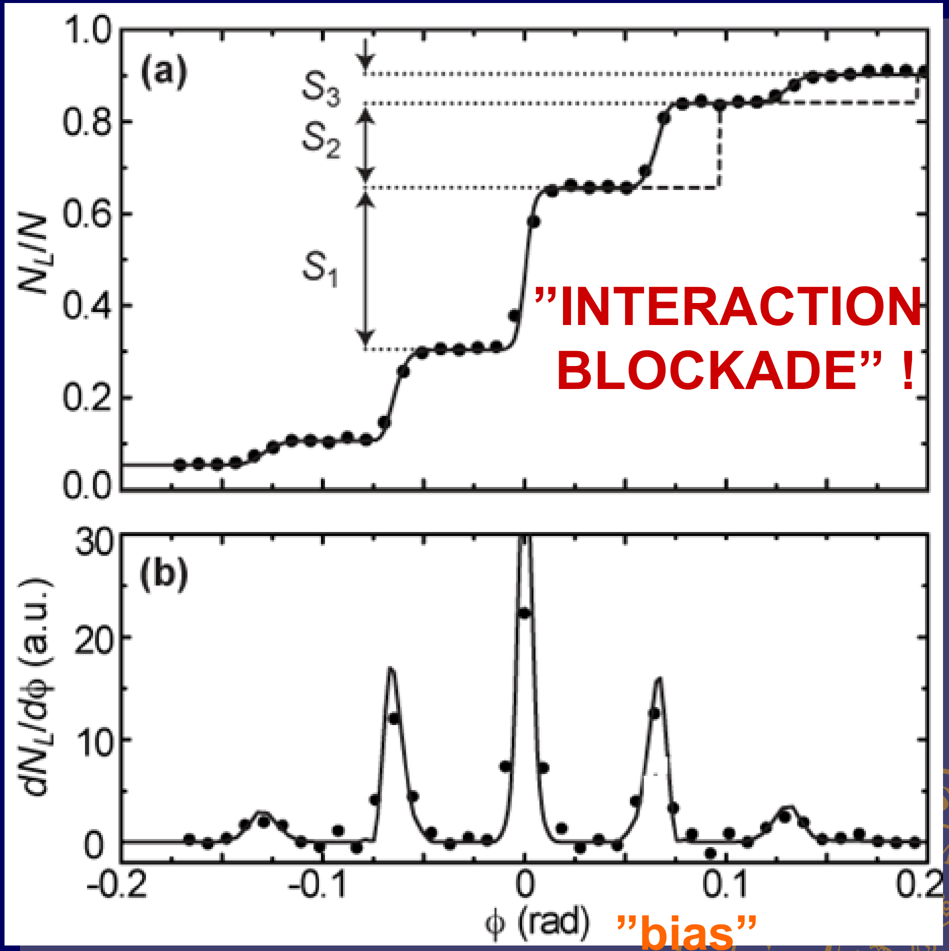
"Single Site"



Asymmetric well with "bias"



P. Cheinet *et al*., PRL, 2008



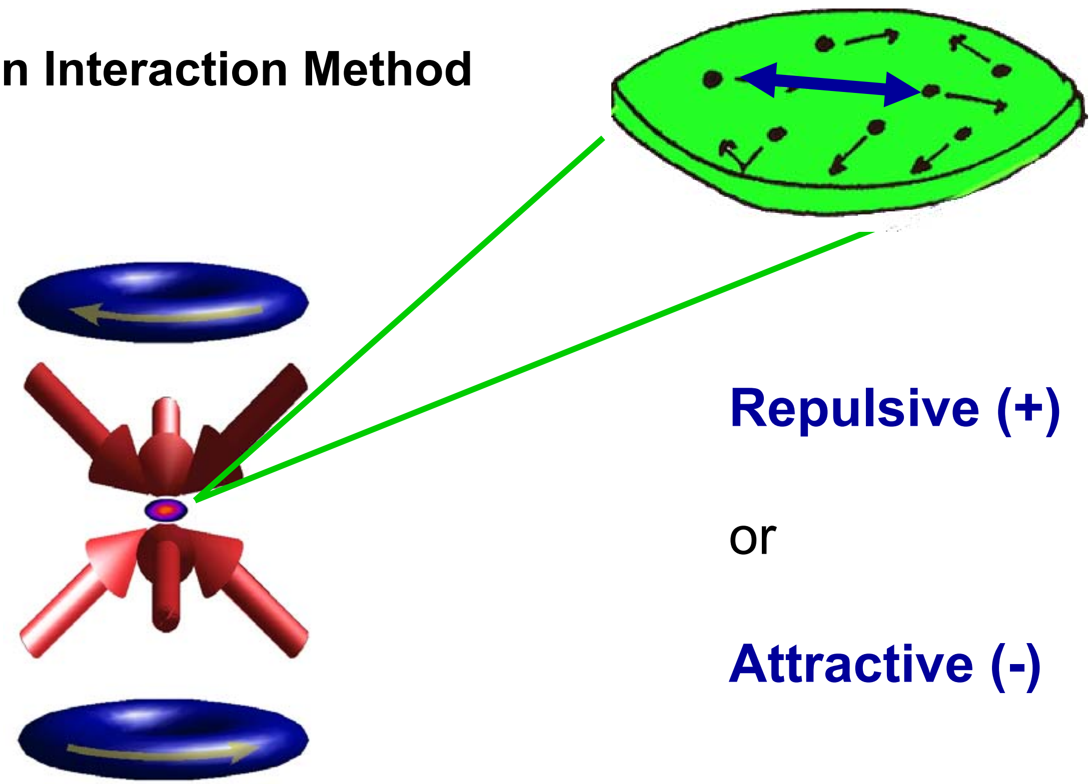
For a discussion of interaction blockade, see Capelle *et al*., PRL (2007)

"Quantum dots with atoms" – fermions with contact interactions

$$\hat{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2M}\nabla_i^2 + \frac{1}{2}M\omega_\perp^2\rho_i^2 + \frac{1}{2}M\omega_z z_i^2 \right) \pm \frac{4\pi\hbar^2 a}{2} \sum_{i,j=1,\, i\neq j}^{N} \delta(\mathbf{r}_i - \mathbf{r}_j)$$

Needs to be regularized!

Solved by
Configuration Interaction Method

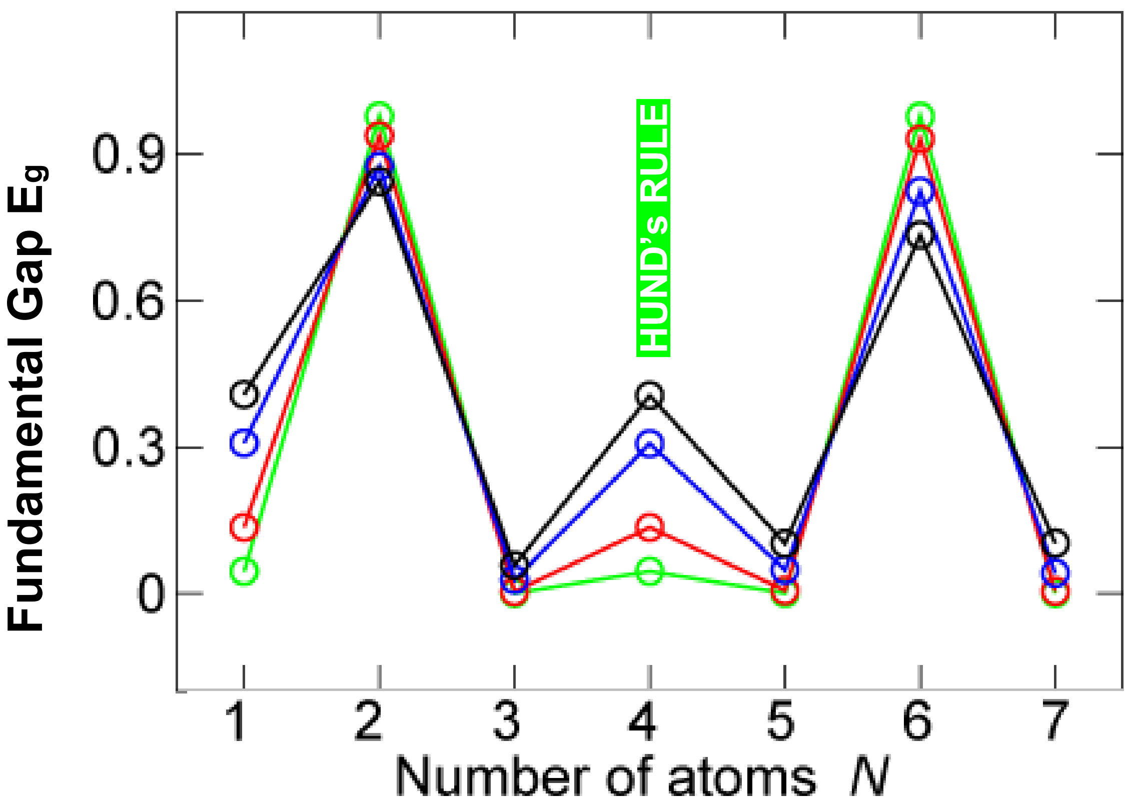


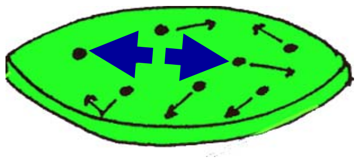
Repulsive (+)

or

Attractive (-)

"Quantum dots with atoms" ...
... shell structure and Hund's rules ...





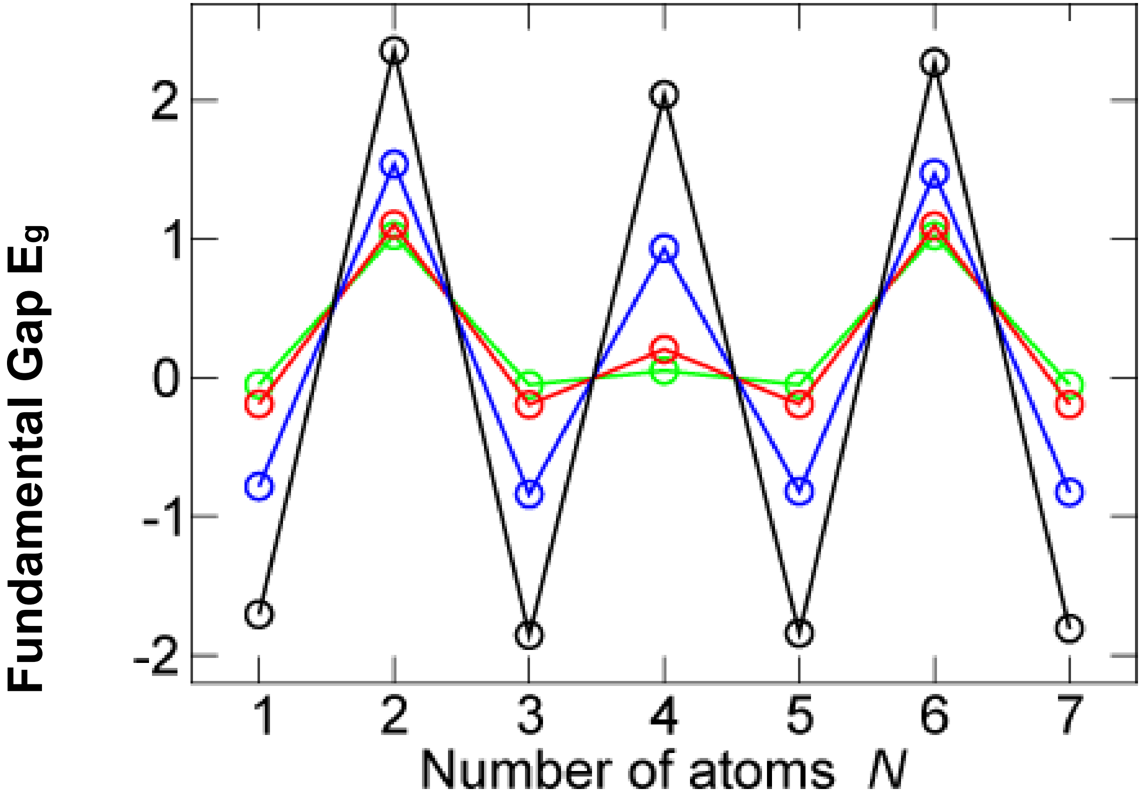
g = 0.3
1.0
3.0
5.0

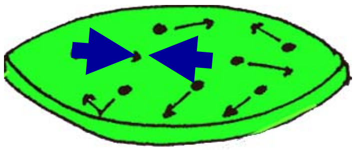
repulsive interactions

M. Rontani, J. Armstrong *et al*., recent results

... and pairing: Odd-even oscillations in "blockade spectra"?

Seniority model gives a similar result!



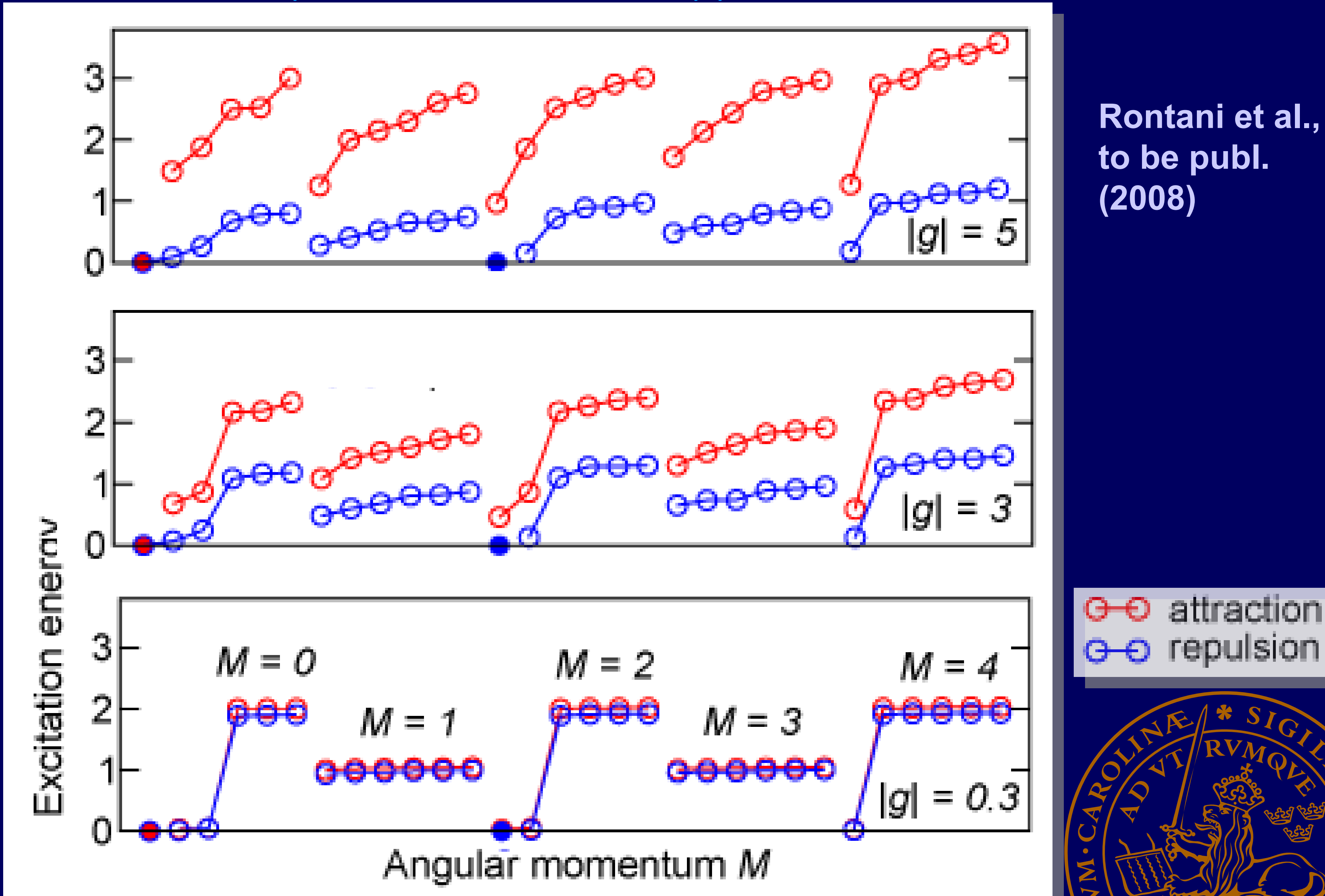


g = -0.3
-1.0
-3.0
-5.0

attractive interactions

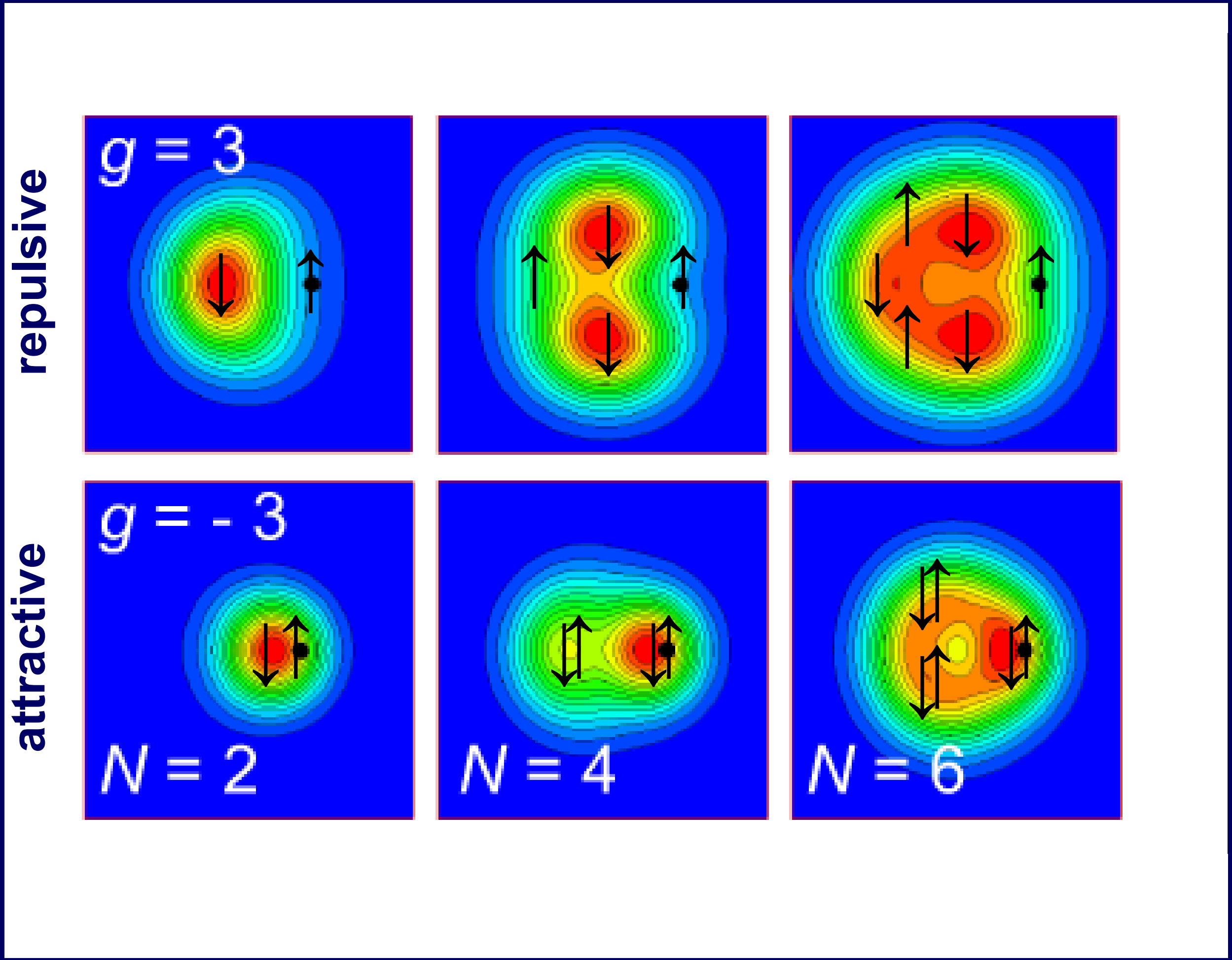
M. Rontani, J. Armstrong *et al.*, recent results

Excitation spectra for N=8 trapped fermions



Rontani et al., to be publ. (2008)

Pairing in "atomic" quantum dots with attractive interactions



Conditional Probabilities – fix one particle, look at probability to find the others

Work in progress!

M. Rontani, J. Armstrong *et al*., recent results

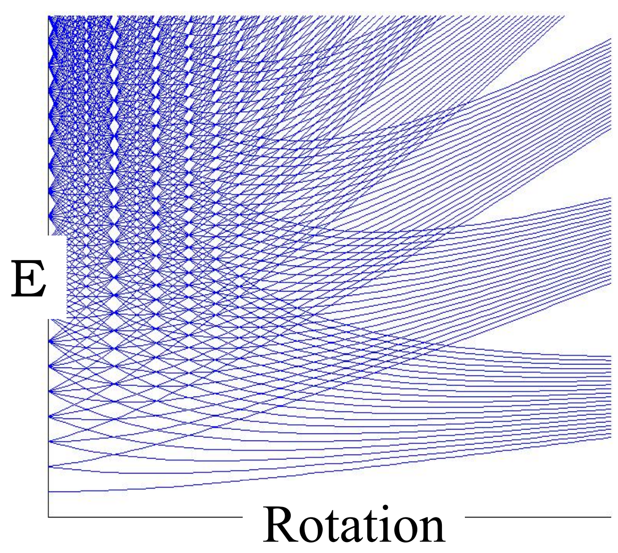
Diagonalisation in the Lowest Landau Level

$$\hat{H} = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2M}\nabla_i^2 + \frac{1}{2}M\omega_\perp^2\rho_i^2 + \frac{1}{2}M\omega_z z_i^2\right) + \frac{e^2}{4\pi\epsilon}\sum_{i<j}^{N_e}\frac{1}{|\hat{\vec{r}}_i - \hat{\vec{r}}_j|} \quad \text{-}L\cdot\Omega$$

Diagonalize for fixed particle number N and total angular momentum L

Lowest Landau Level: no radial nodes, non-negative angular momentum m

basis states $\Phi_{0m}(\rho,\varphi,z) \propto (\rho)^{|m|} e^{-\rho^2/2} e^{im\varphi}\phi_0(z)$



ROTATION $\hat{=}$ MAGNETIC FIELD

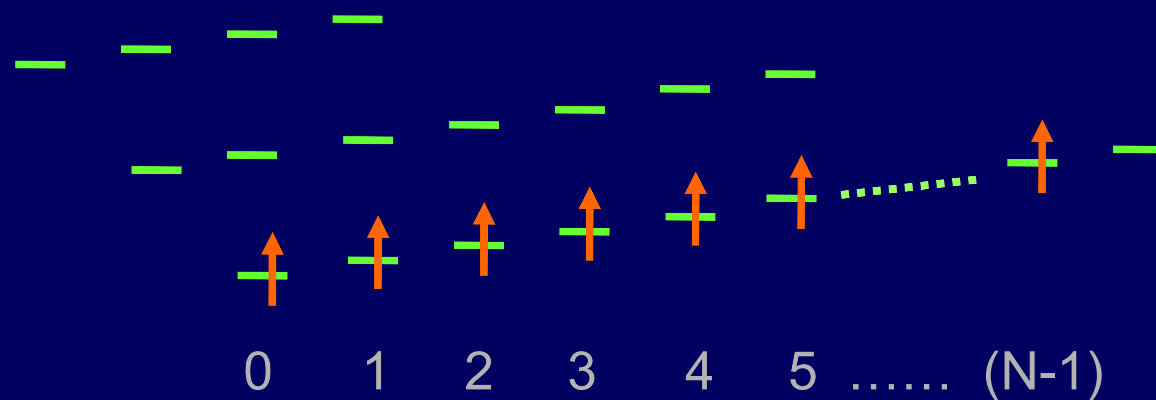
$$L = \sum_{i=1}^{N} m_i$$

total angular momentum
is still a good quantum number
SPIN is not considered yet

Repulsive Interactions

Fermion ground state

$L_{MDD} = N(N-1)/2$



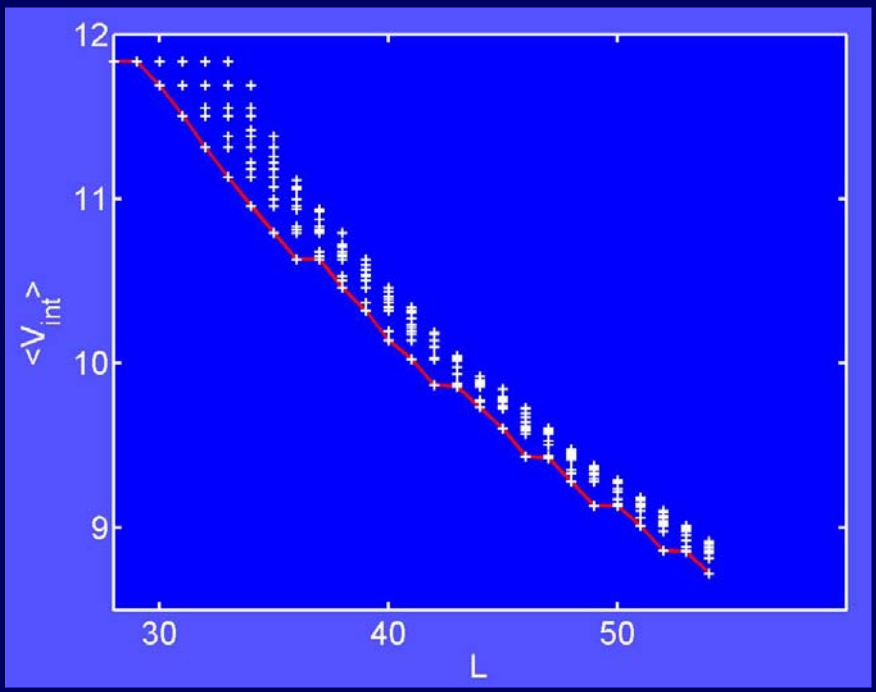
Boson ground state

$L_{BEC} = 0$

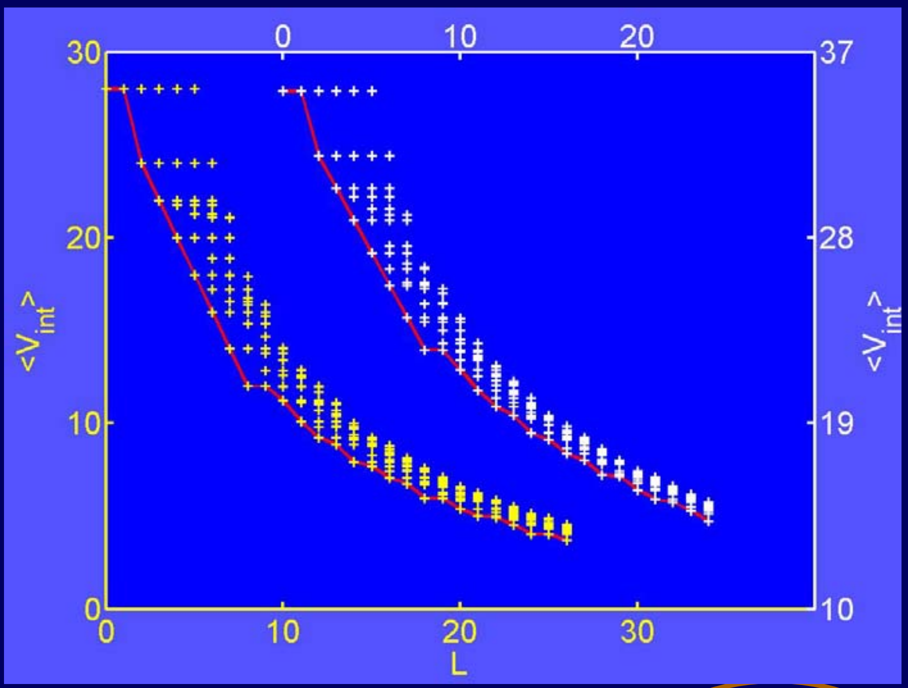
"YRAST" SPECTRUM, here for 8 particles

In Lowest Landau Level (LLL): $\langle V_{int} \rangle = E - \hbar\omega(N+L)$

FERMIONS



BOSONS



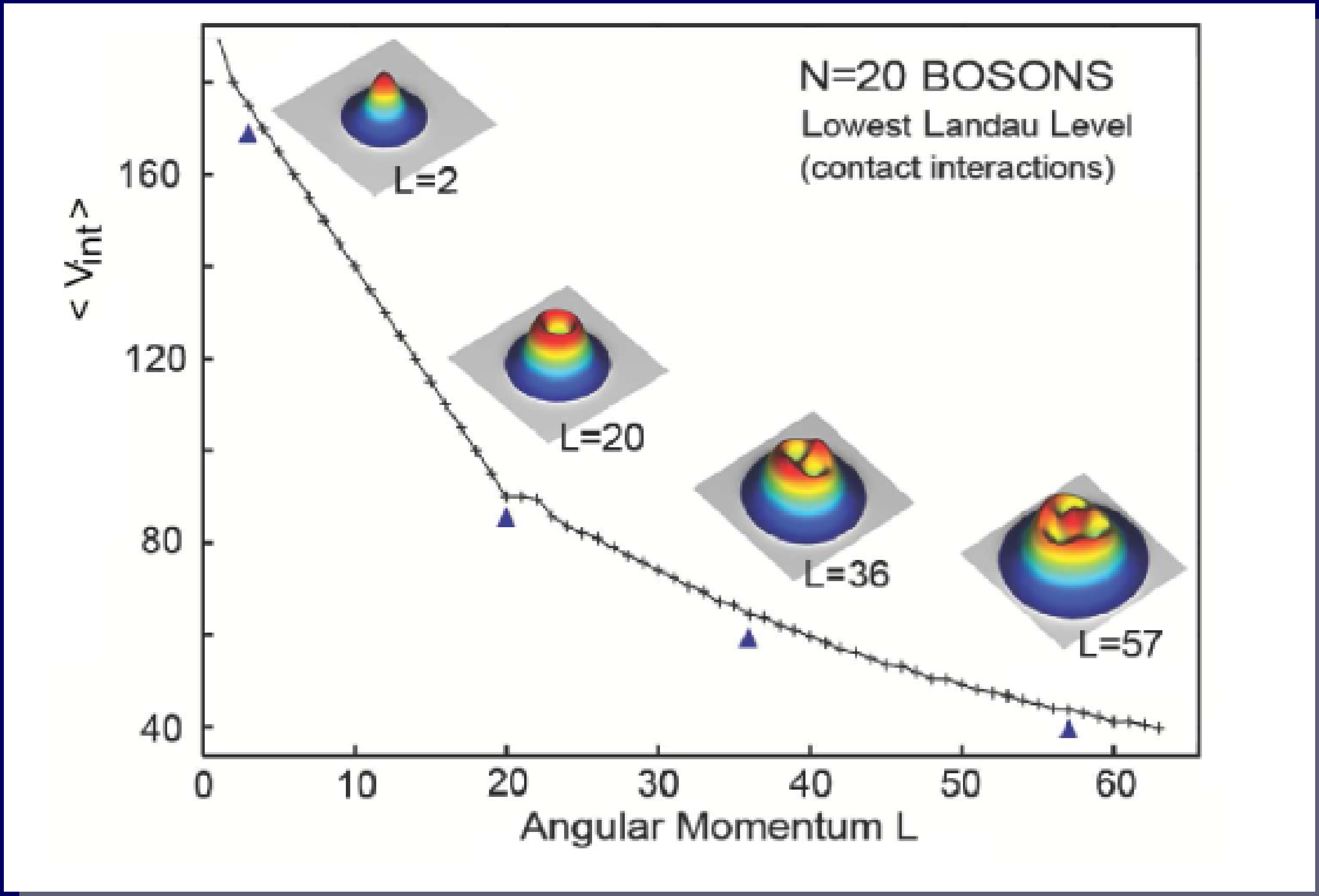
$$L_{fer} = L_{bos} + \frac{N(N-1)}{2} = L_{bos} + 28$$

"+" short-range

"+" coulomb

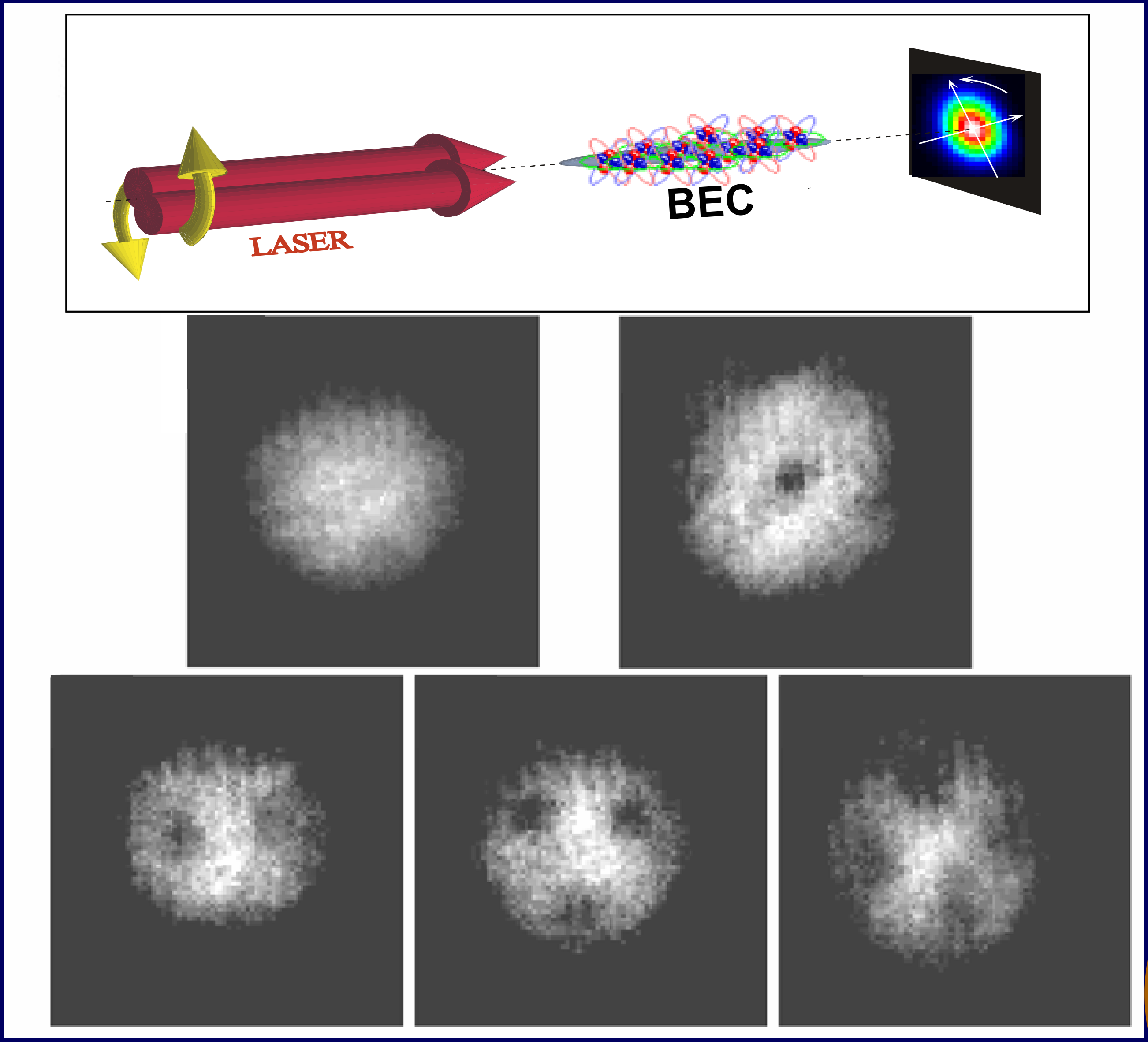
Pair Correlations for trapped bosons

Conditional Probabilities – fix one particle, look at probability to find the others



J. Christensson *et al*., Few Body Phys. (2008)

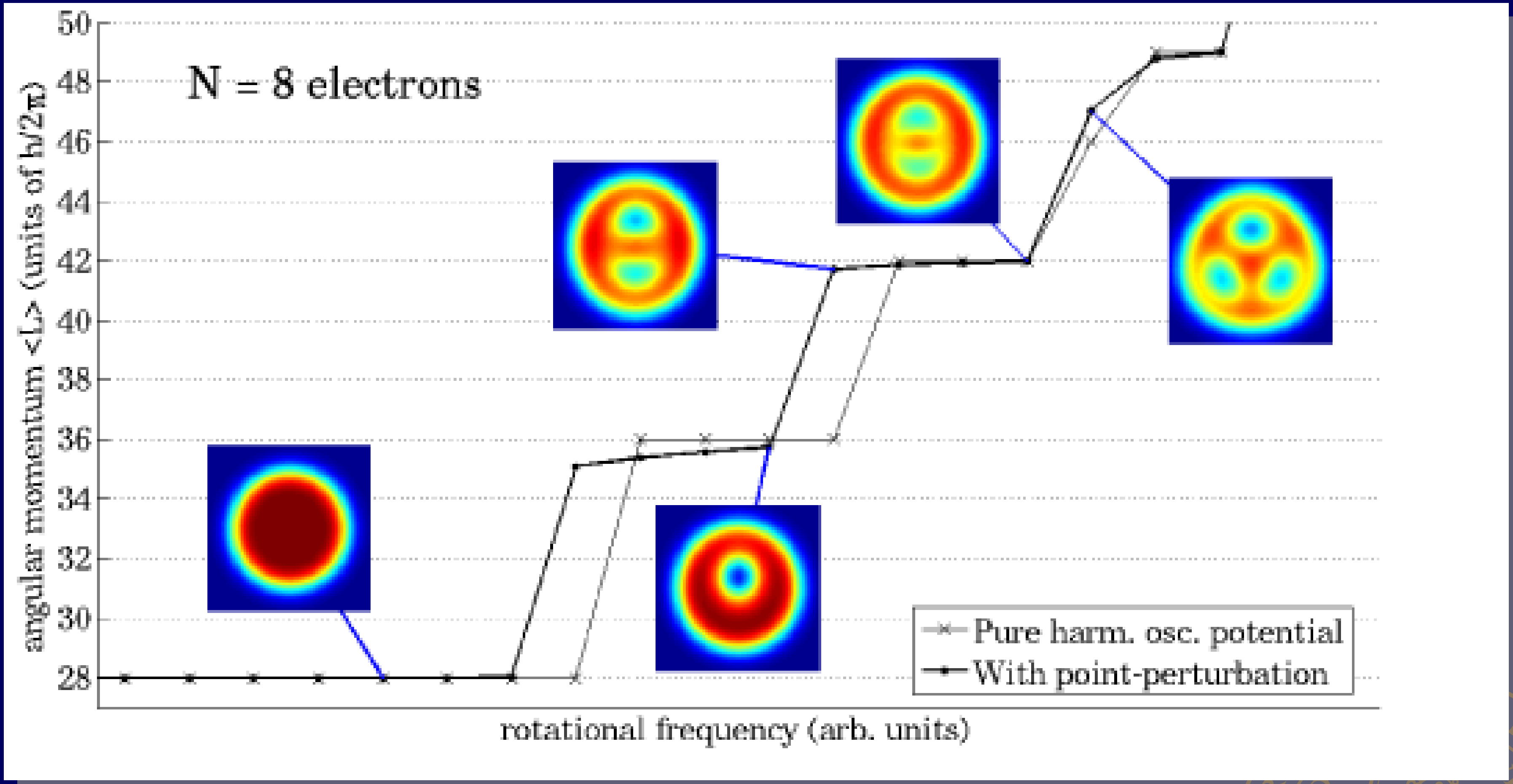
Vortices in BEC's that are set rotating:



Madison et al., PRL, 2000

Parabolic trap with N=8 electrons in LLL

Added point perturbation V = α δ(r-a) breaking the rotational symmetry, shown are the densities in the perturbed system



J. Christensson *et al*., Few Body Phys. (2008)

Parabolic trap with N=8 electrons in LLL

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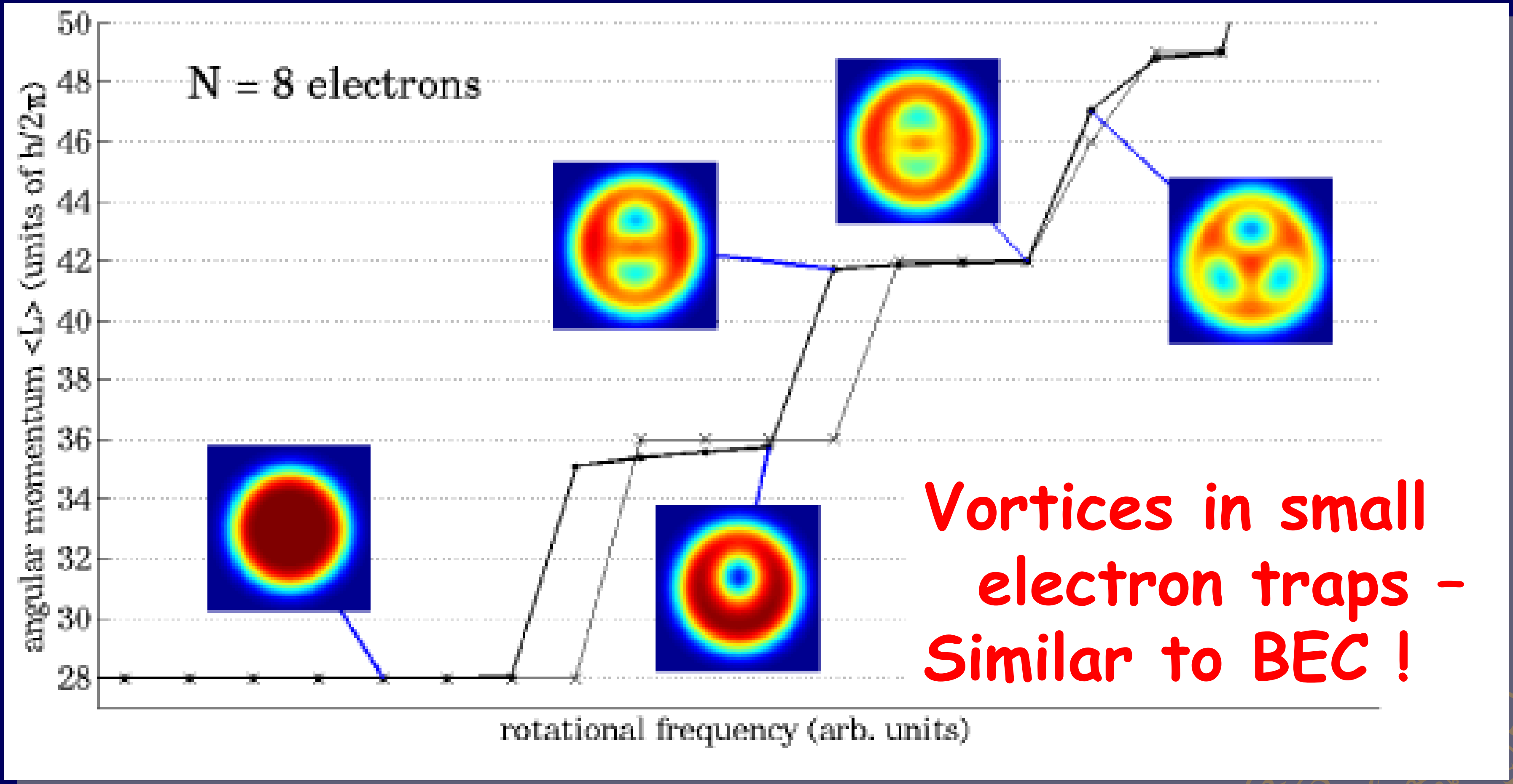
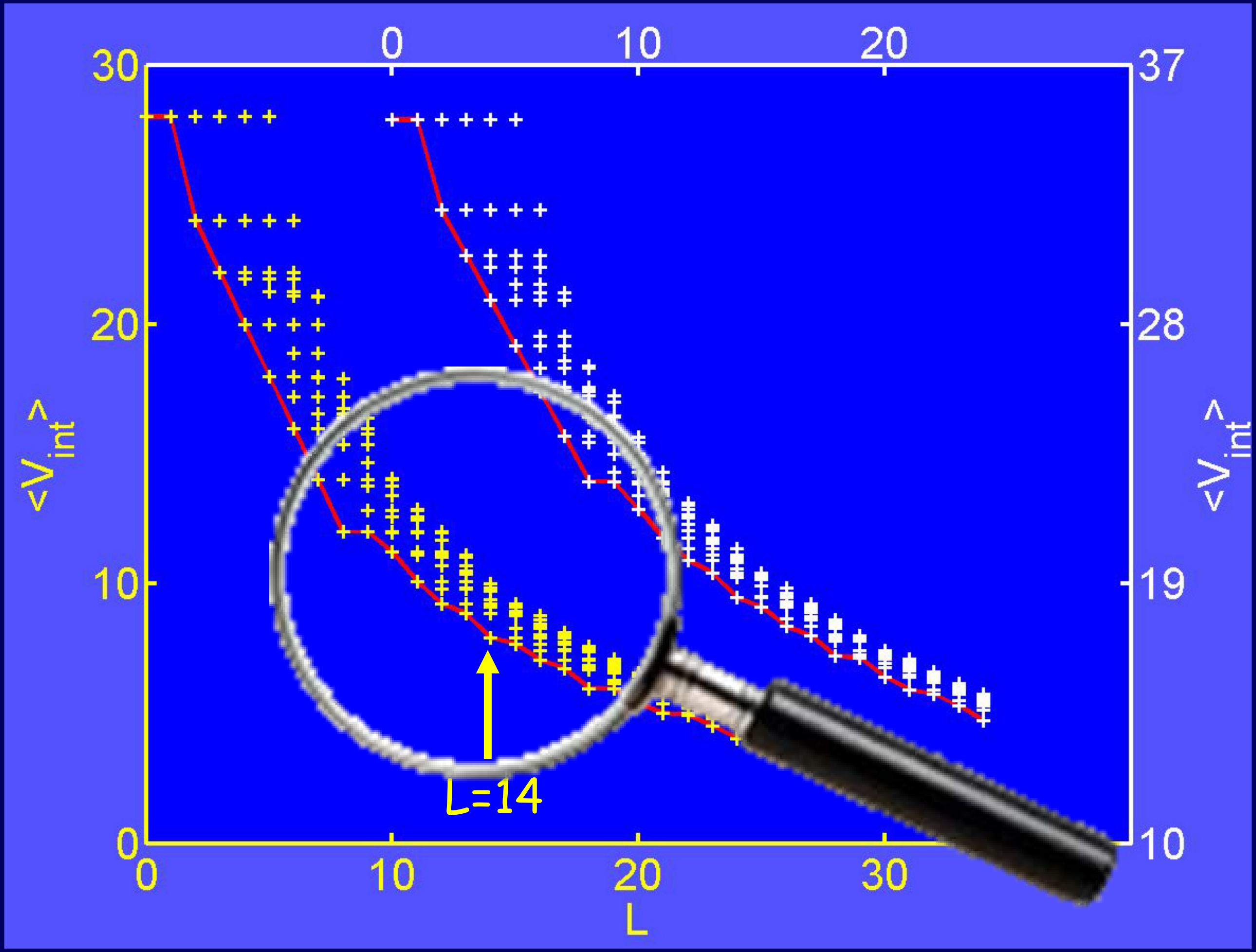


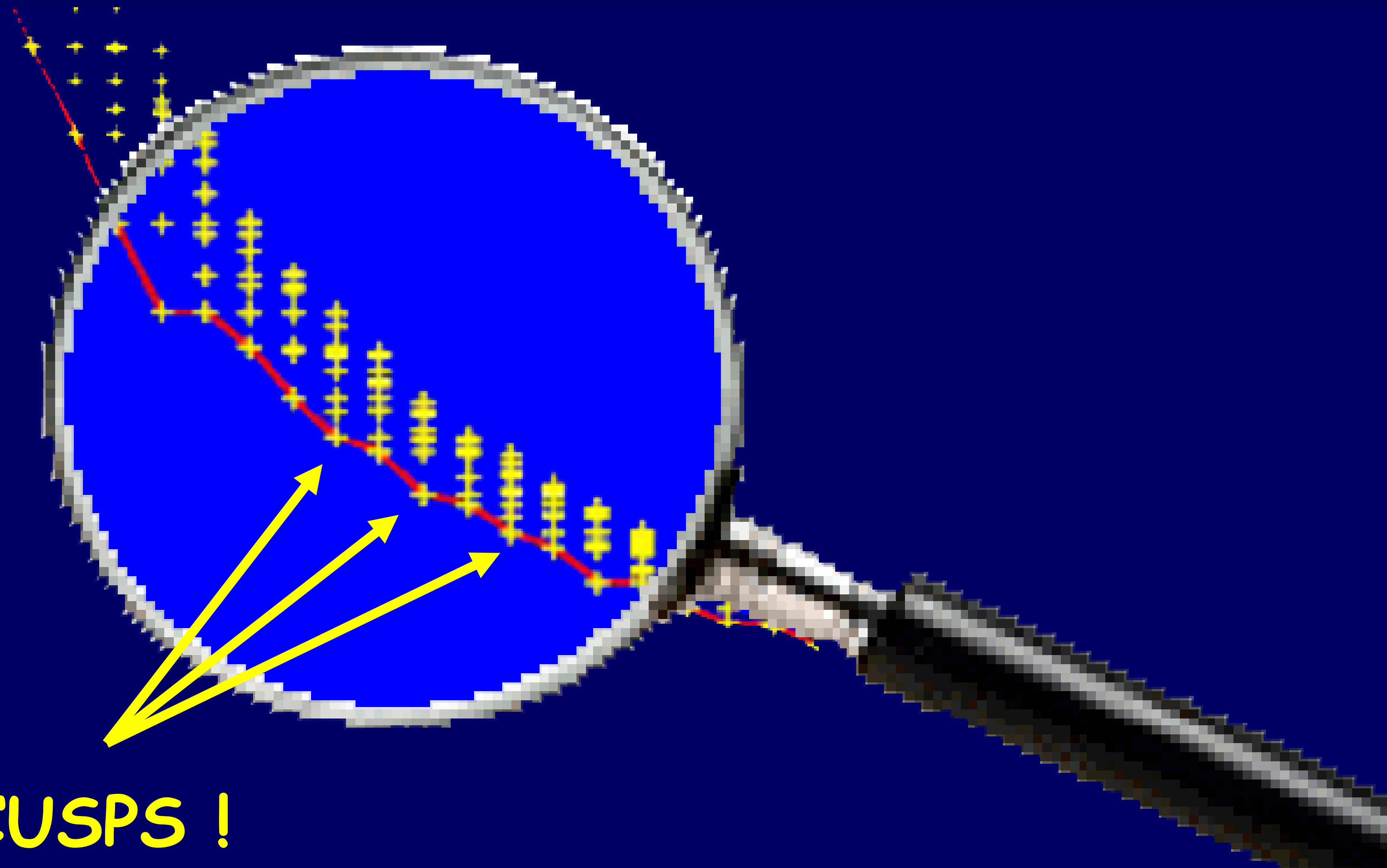
Vortices in small electron traps – Similar to BEC !

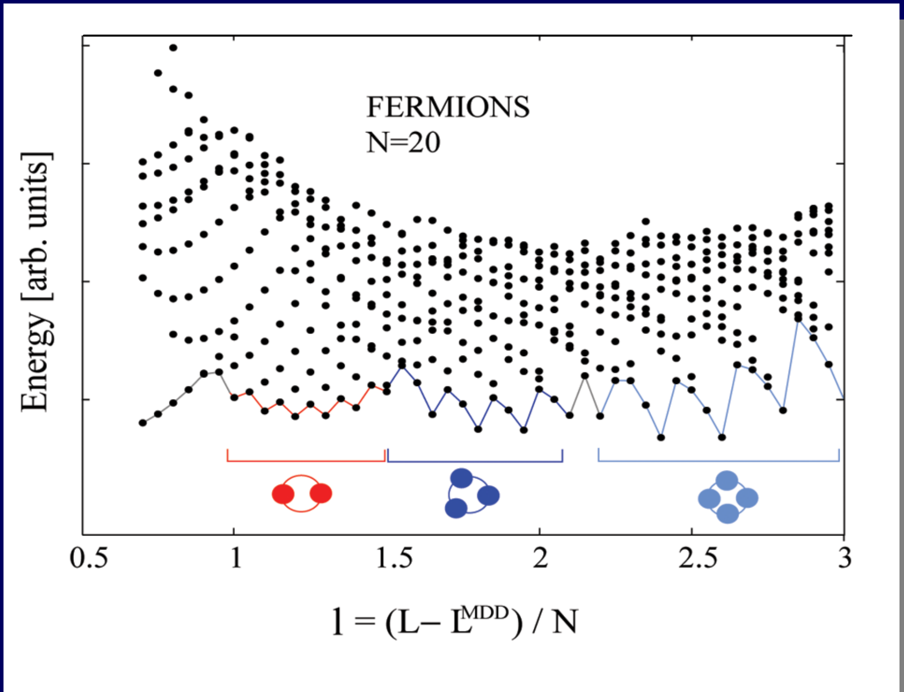
J. Christensson *et al*., Few Body Phys. (2008)

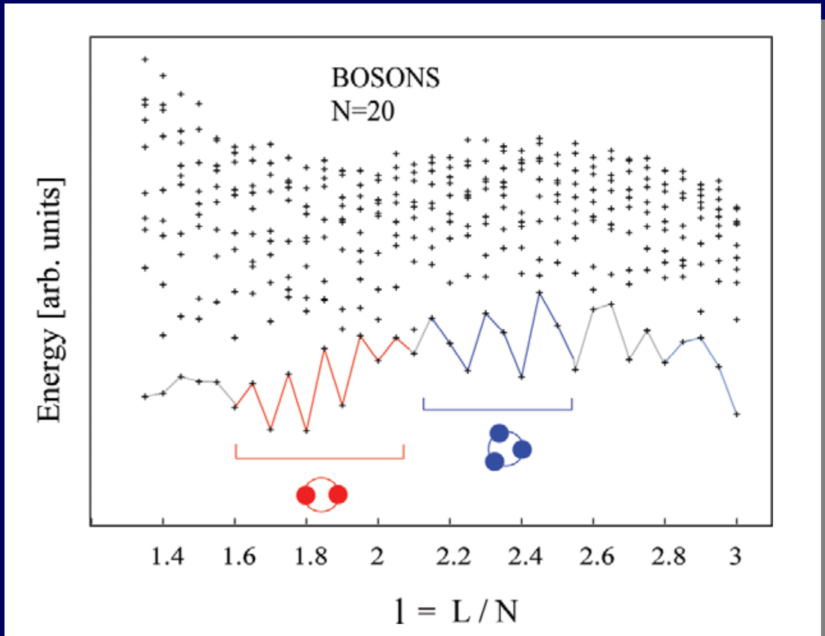


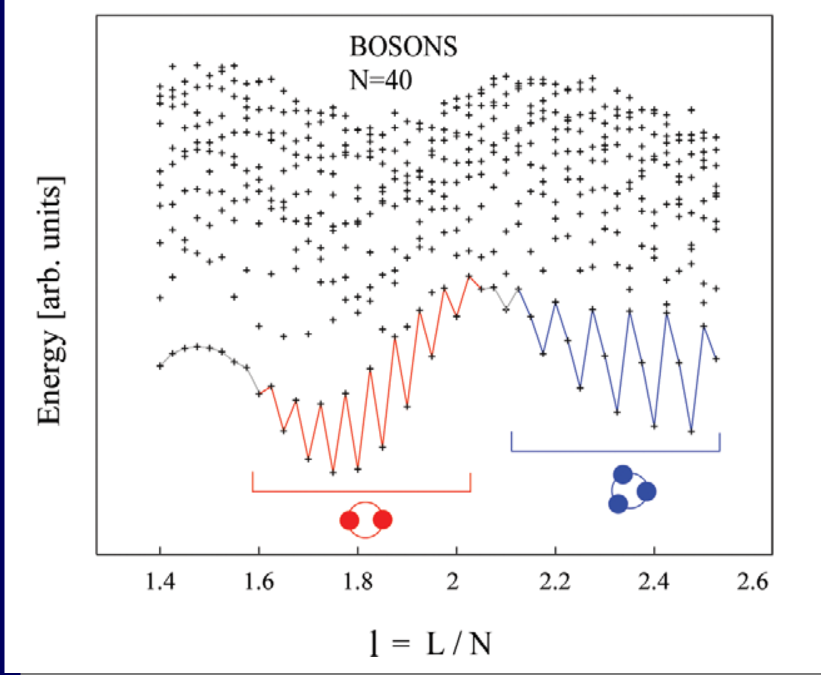
0
10
20
30
20
10
0
37
28
19
10
0
10
20
30
L
<Vint>
<Vint>
L=14

Bosons, two vortices







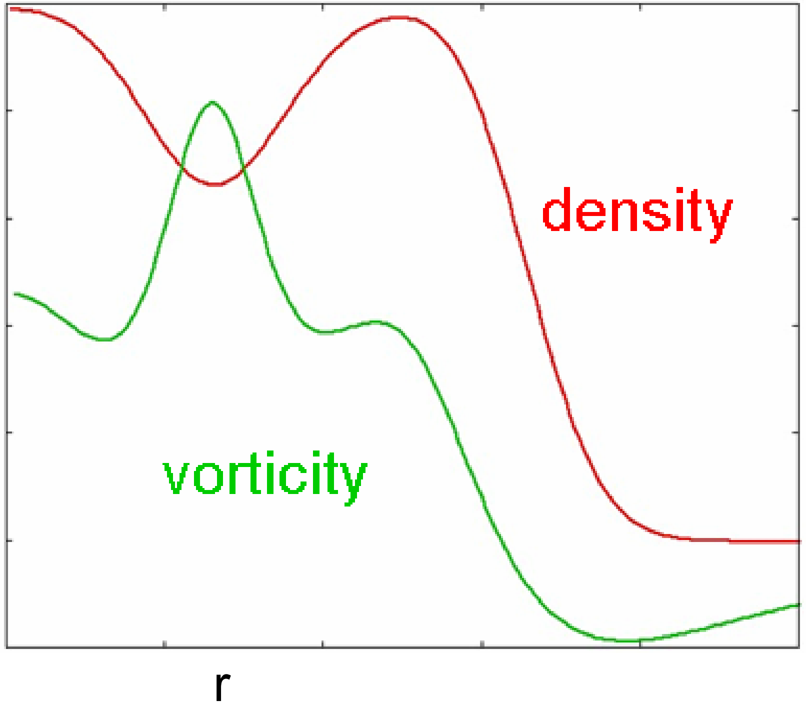


FERMION AND BOSON SPECTRA COMPARED

(smooth background subtracted)

Cusps due to vortex formation at small L, close to MDD

FERMIONS



N = 20, L = 224
(second excited state, 2 vortices)

Density: $n(r)$

Current density: $\vec{j}(r)$

Velocity field: $\vec{j}(r)/n(r)$

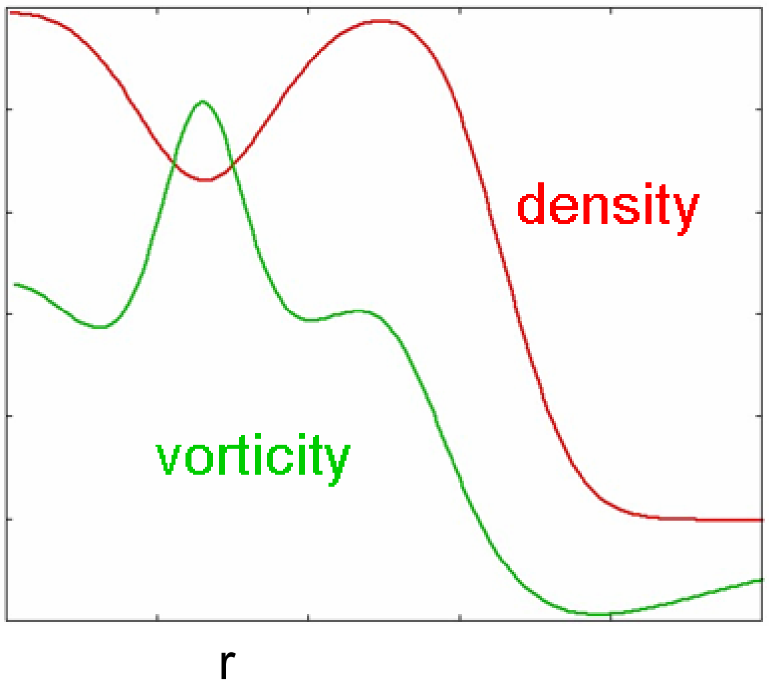
Vorticity:

$$\vec{\nu}(r) = \nabla \times \left(\frac{\vec{j}(r)}{n(r)}\right)$$

(only z-component $\nu_z(r) \neq 0$)

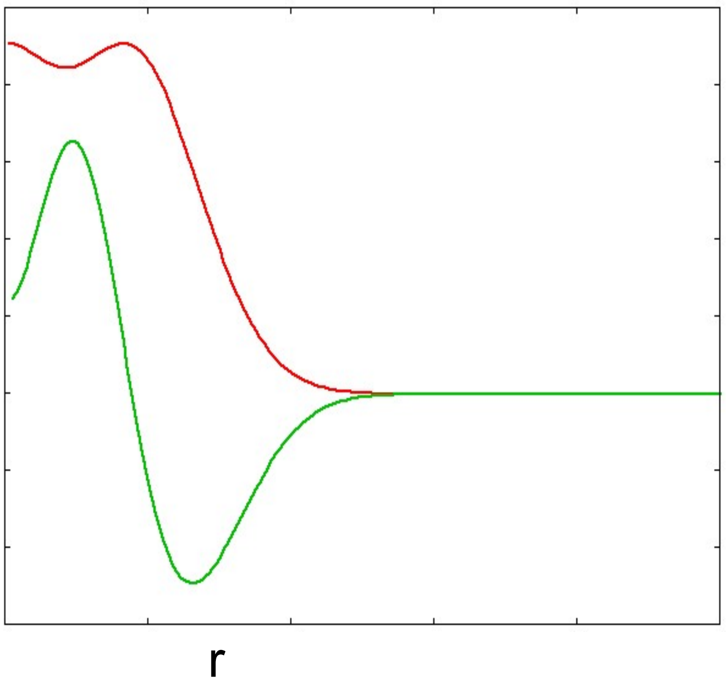
Reimann *et al*., PRA, '06

FERMIONS



N = 20, L = 224
second excited state,
2 vortices

BOSONS



N = 20, L = 34
ground state,
2 vortices

Reimann *et al*., PRA, ’06

Diagonalisation in the Lowest Landau Level

$$\hat{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2M} \nabla_i^2 + \frac{1}{2} M \omega_\perp^2 \rho_i^2 + \frac{1}{2} M \omega_z z_i^2 \right) + \frac{4\pi\hbar^2 a}{2} \sum_{i,j=1,\, i \neq j}^{N} \delta\left(\mathbf{r}_i - \mathbf{r}_j\right) \quad \text{-L}\cdot\Omega$$

two components: $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$

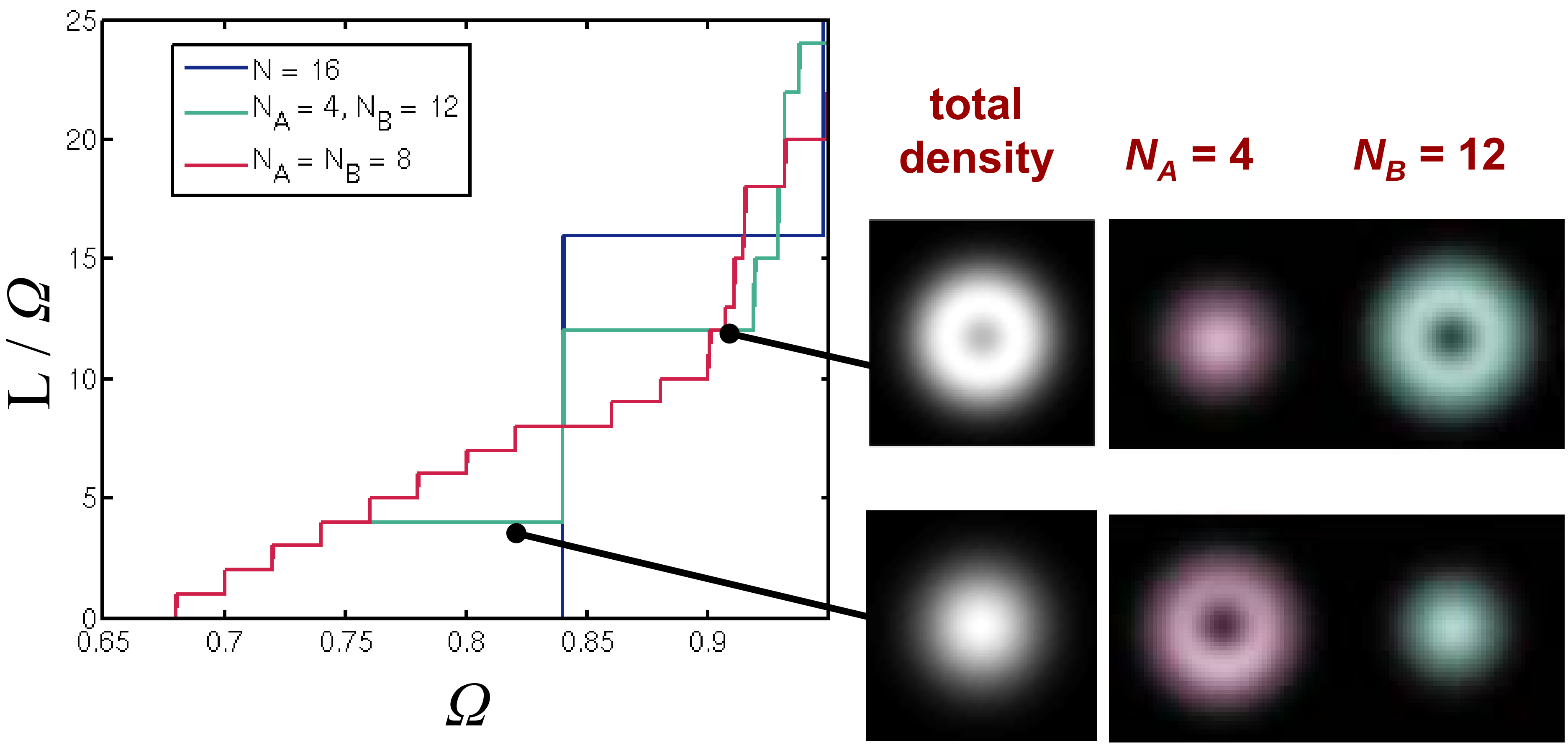
”equivalent” but distinguishable (similar to isospin; here more likely to be different hyperfine states)

equal masses $M_A = M_B$

equal scattering lengths $a_{AA} = a_{BB} = a_{AB}$

Coreless Vortices in rotating Bose gases

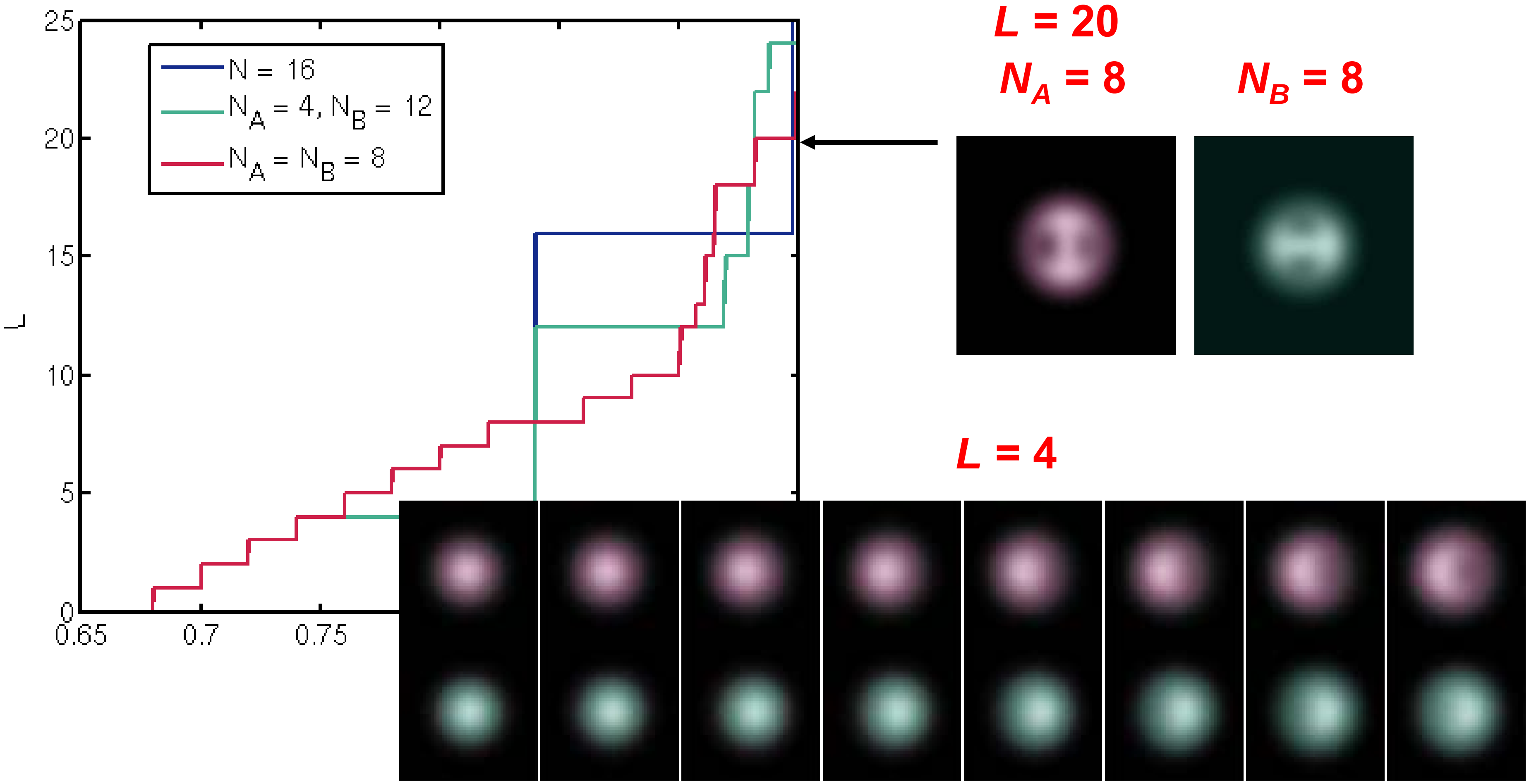
see also Kasamatsu *et al*, (2005); Bargi et al., 2007, 2008



Coreless Vortices in rotating Bose gases

see also Kasamatsu *et al*, (2005), Bargi et al., 2007

Equal populations:



SUMMARY

Many analogies between quantum dots, and cold, trapped atoms!

- ”Interaction blockade” with atoms
- Fermions in traps – from Hund’s rules to pairing
- Vortices may form with bosons AND (repulsive) fermions
  (for example, in a rotating atom trap, or in a quantum dot at strong fields)

This also applies to two-component systems!
(Coreless Vortices in quantum dots with spin)

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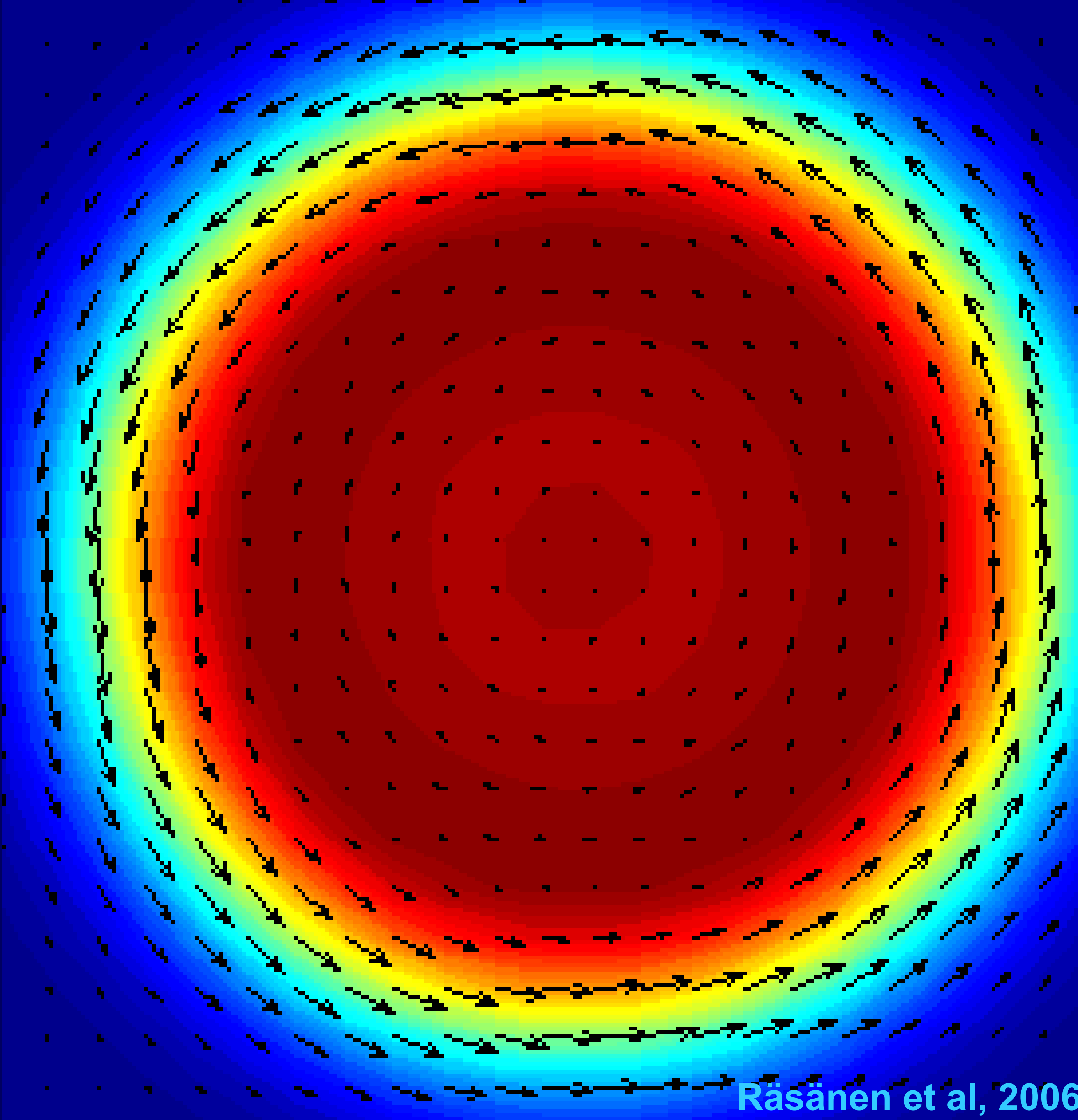

Räsänen et al, 2006

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