Feshbach resonances in ultracold atomic gases

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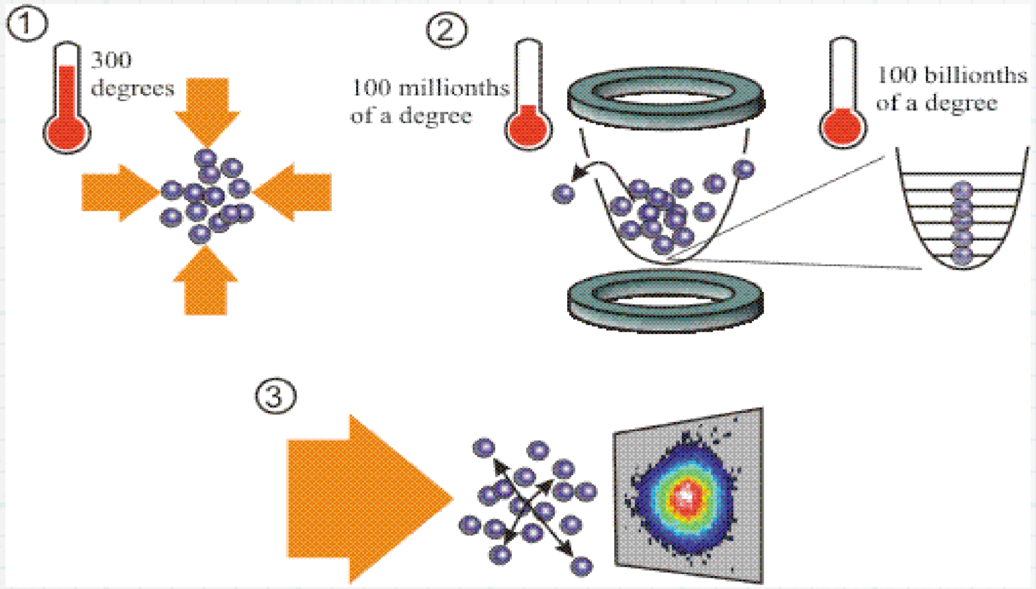
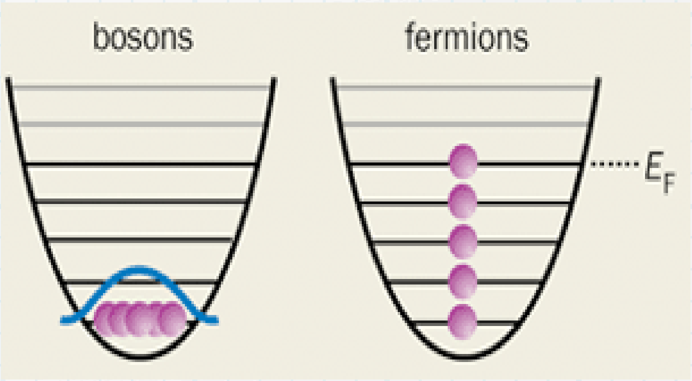
Outline

* Trapping and cooling, orders of magnitude
* Interactions, Feshbach resonance
* Many-body scattering theory à la Landau
* Medium effects & collective modes
* Polarized Fermi gas, Spin polarons <-> molecules

Trapping & cooling





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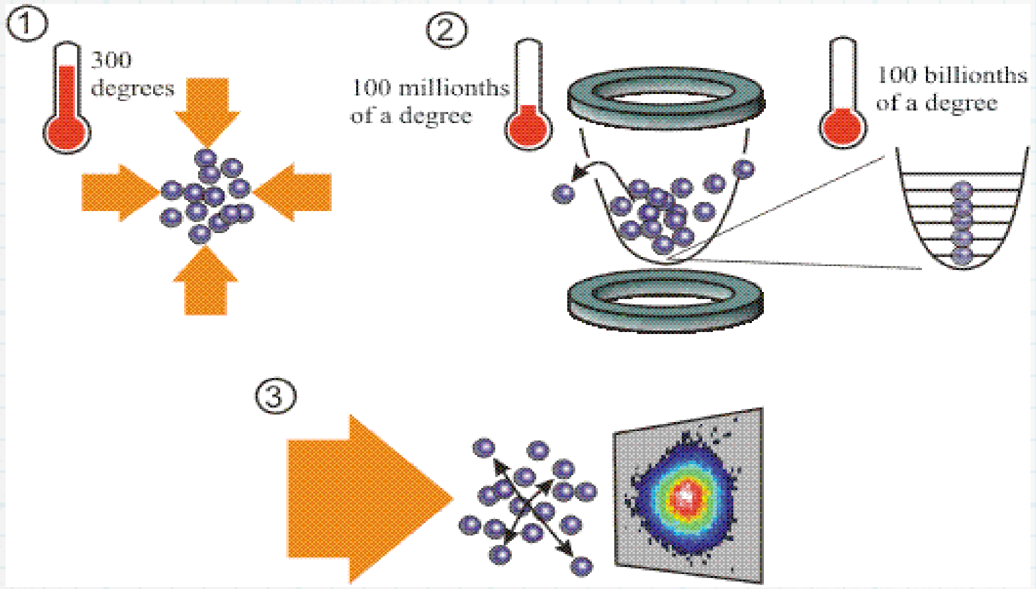
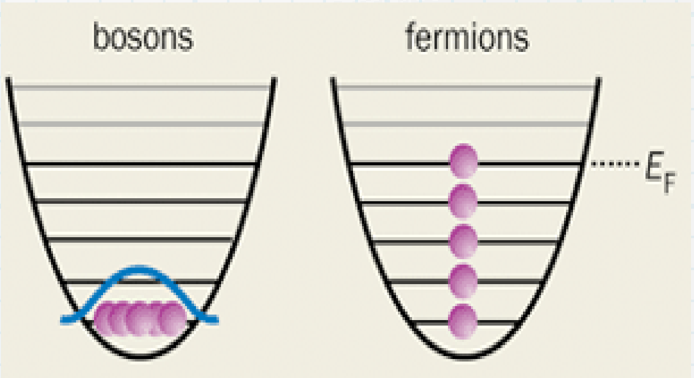
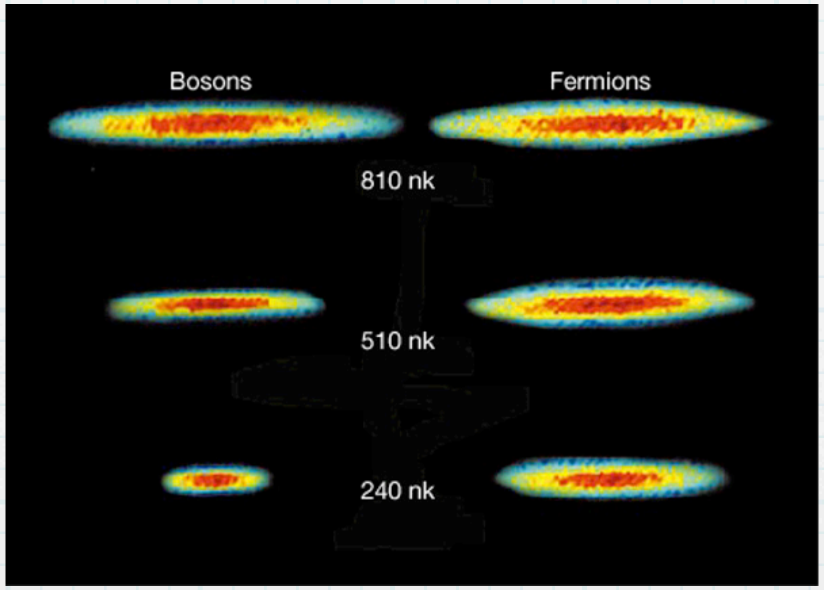




^{7}Li ^{6}Li

T/T_c=1.5



T/T_F=1.5

T/T_c=0.25

T/T_F=1.0

Truscott et al. Science 291, 2570 (2001)

Characteristics

* Dilute $n \approx 10^{12}$-$10^{15} cm^{-3}$
  Air $n \approx 10^{19} cm^{-3}$ Solids $\approx 10^{22} cm^{-3}$
* Cold $T \leq 10^{-5} K$,   record $T \approx 0.45 nK$
* $N \approx 10$-10^9
* Flexible desk-top experiment: geometry, statistics, interaction, lattice, shake, rotate..

Atom-atom interactions

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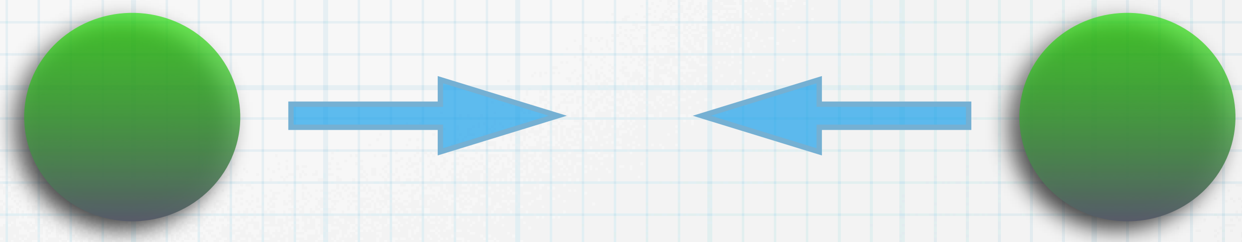
Two atoms collide:



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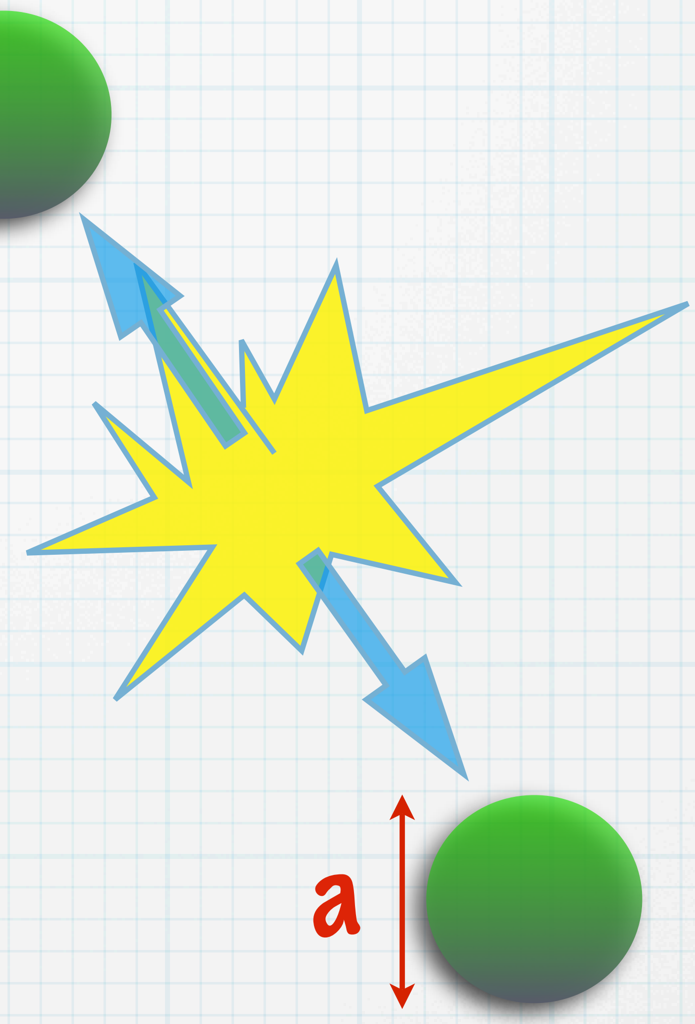

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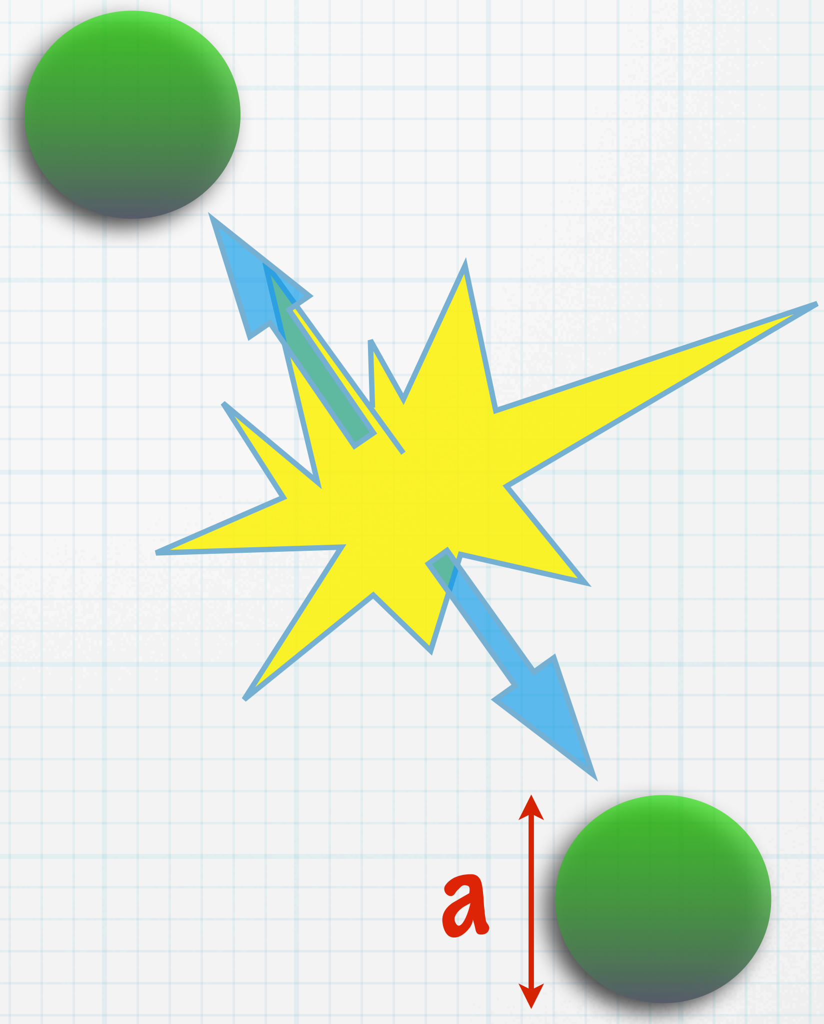
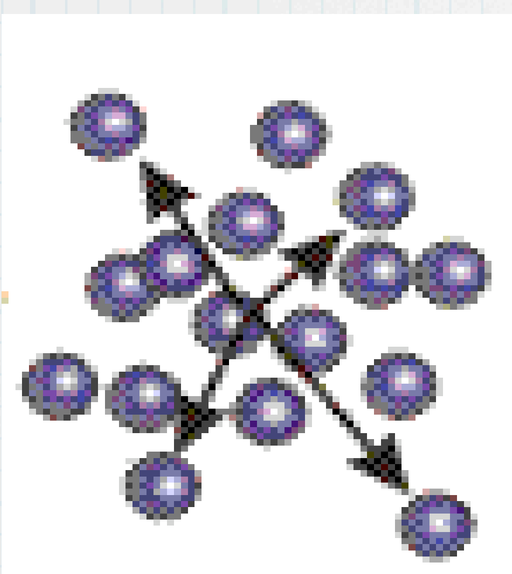
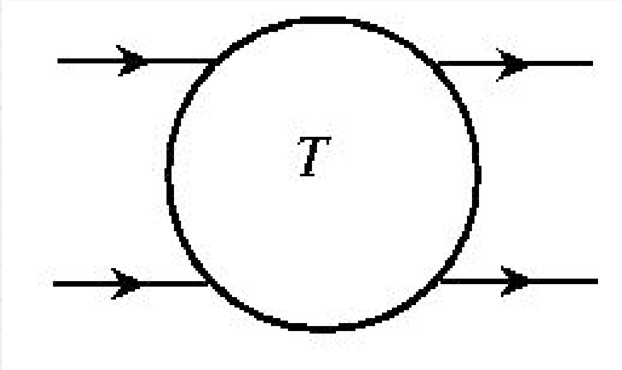


Weak Coupling: $k_F|a| \ll 1$



$$T = \frac{4\pi a}{m}$$



Hyperfine Hamiltonian: $\hat{H}_{\rm spin} = A\,\vec{I}\cdot\vec{S} + C\,S_z + D\,I_z$

$$\hat{H}_{\rm spin}|\alpha\rangle = \epsilon_\alpha|\alpha\rangle \qquad |\alpha\rangle \equiv |F, m_F\rangle \qquad \vec{F} = \vec{S} + \vec{I}$$

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Singlet potential ↘ ↙ Triplet potential

Atom-atom interaction: $V(r) = \dfrac{V_s(r) + 3\,V_t(r)}{4} + [V_t(r) - V_s(r)]\,\vec{S}_1\cdot\vec{S}_2$

Mixes hyperfine states -> scattering channels

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Singlet potential → $V_s(r)$; Triplet potential → $V_t(r)$

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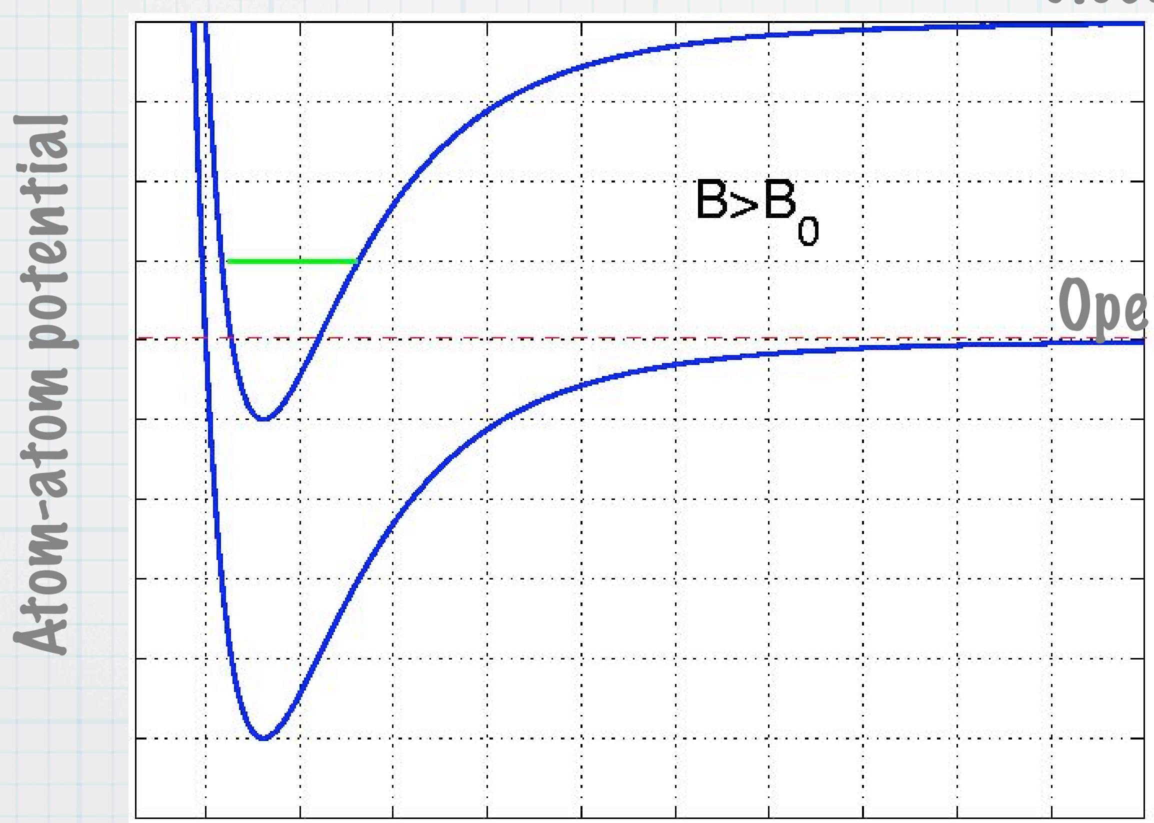

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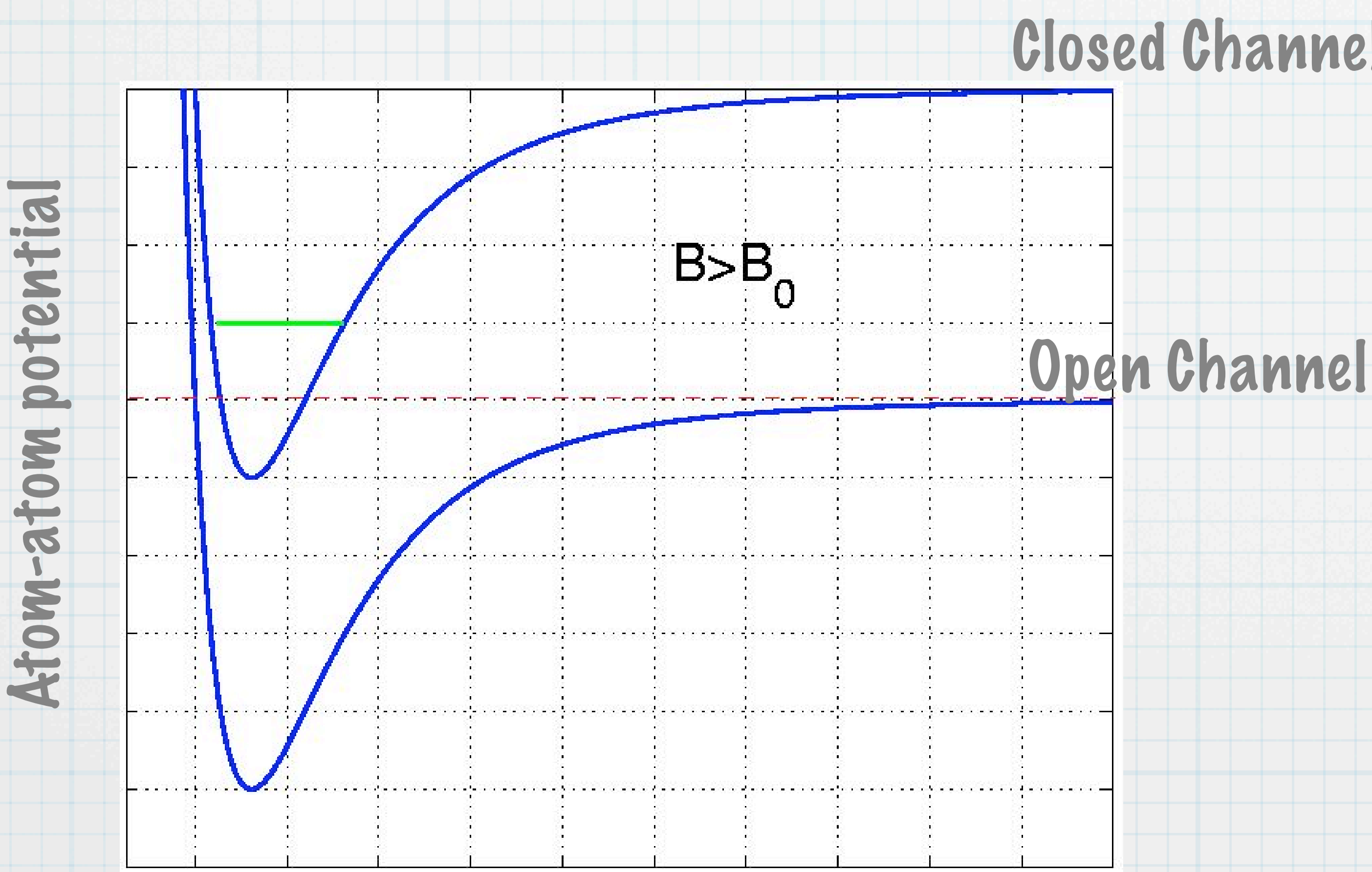

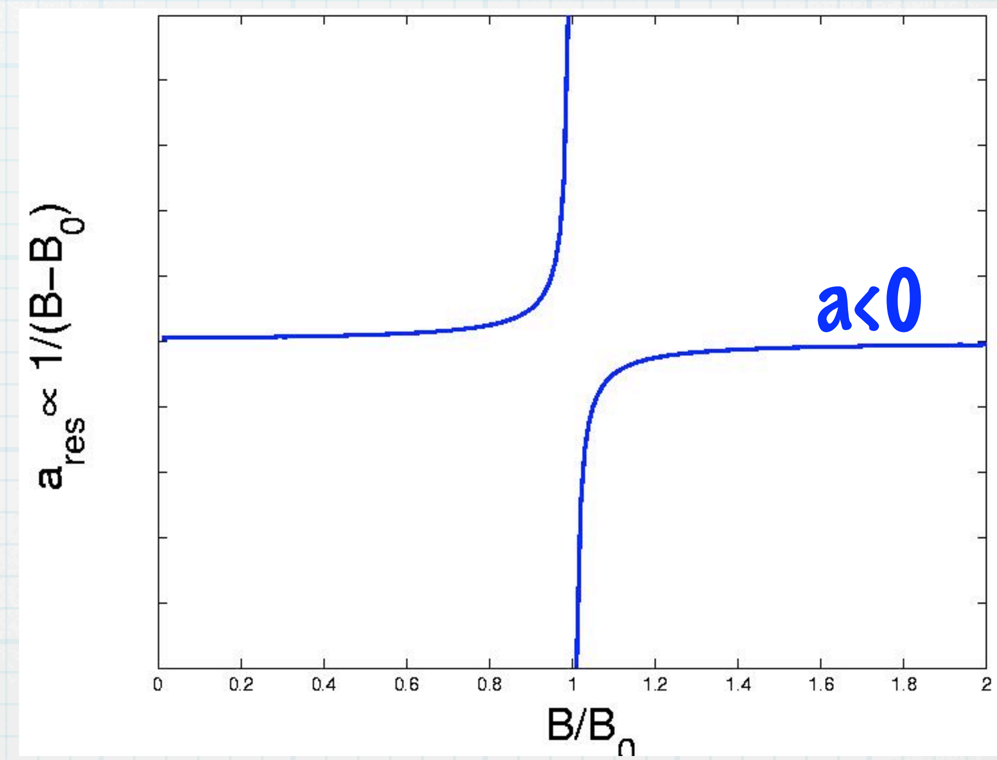

$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

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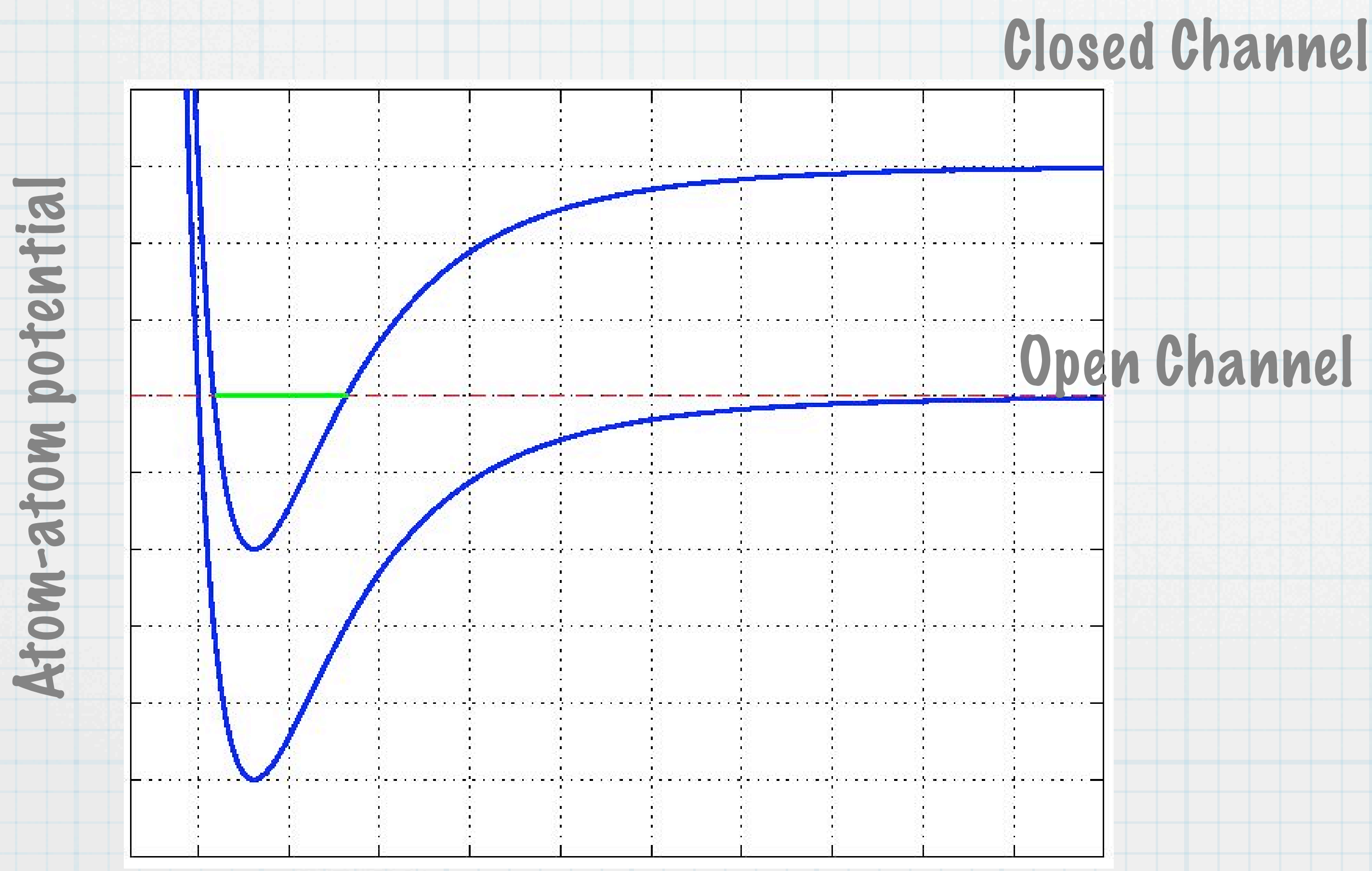

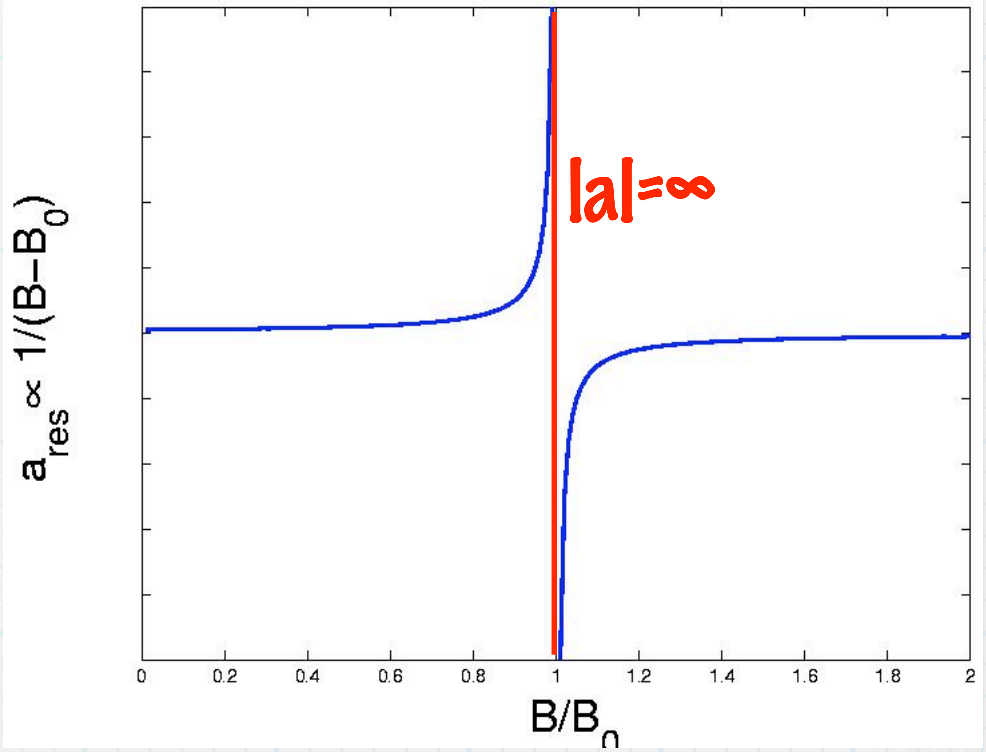

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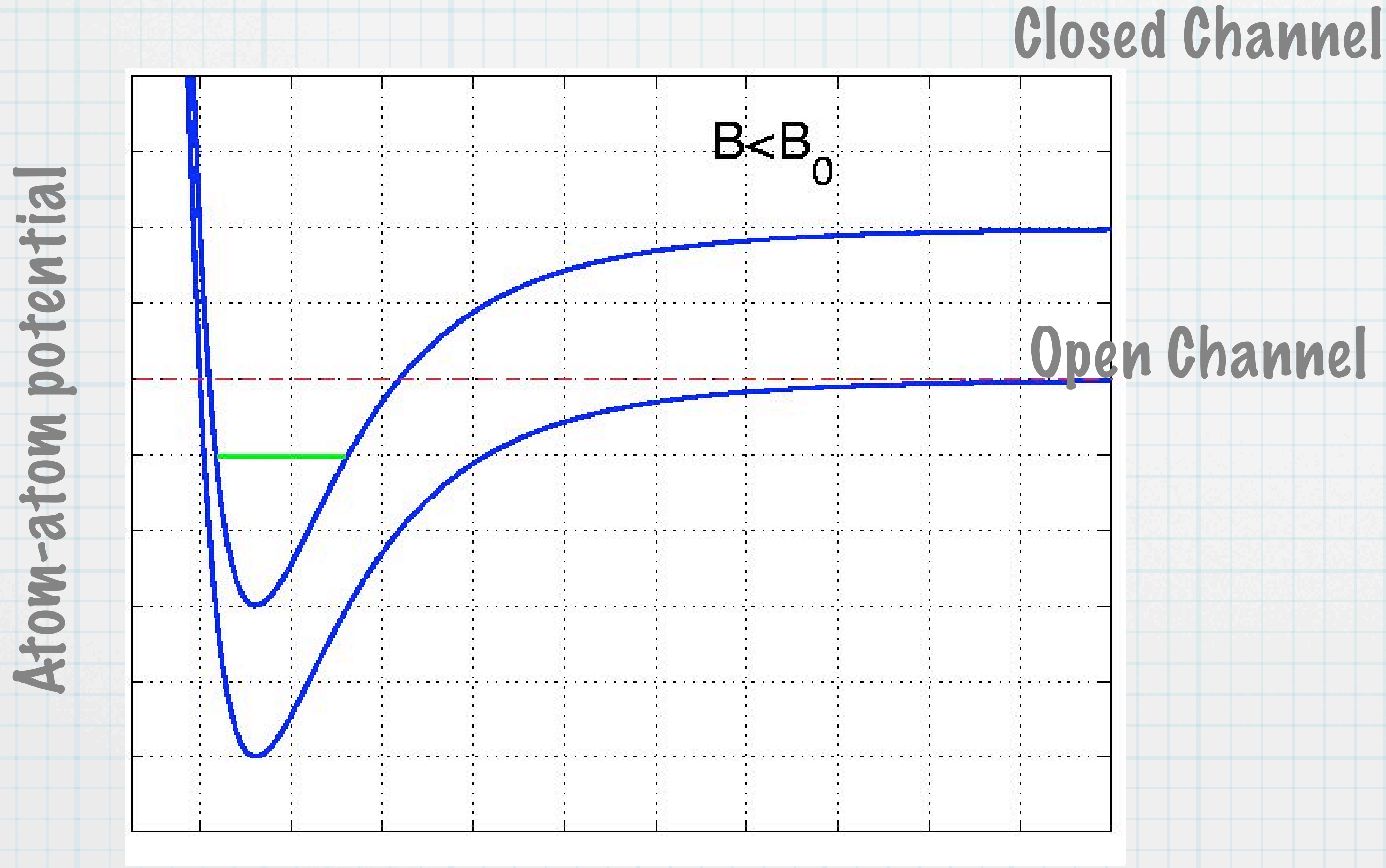

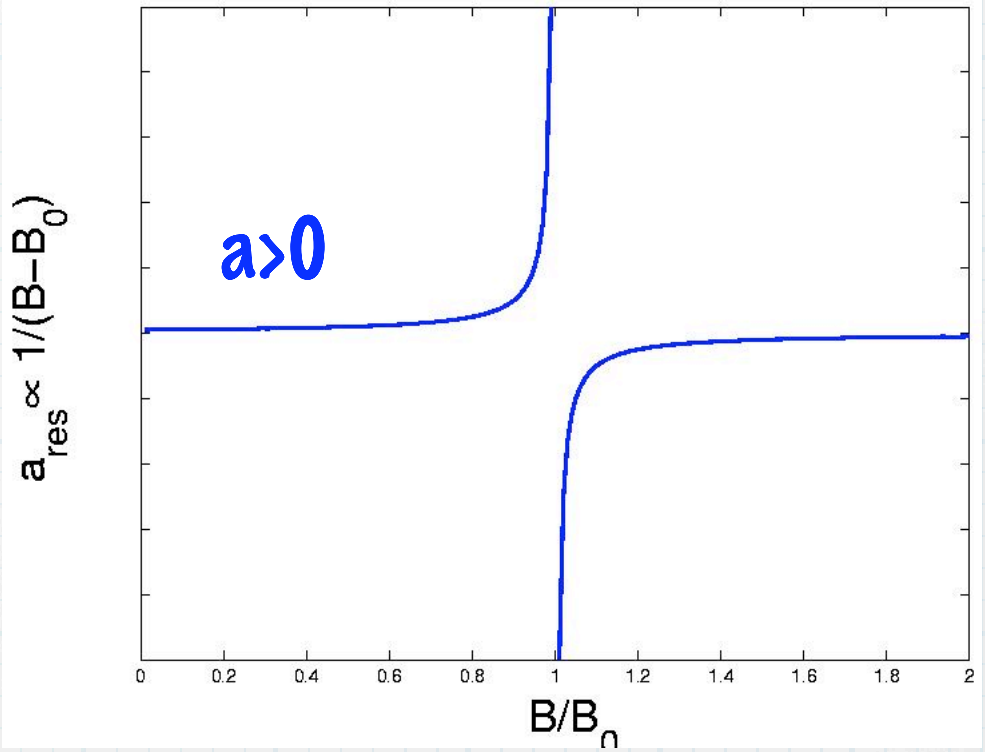

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Specific example: ^{40}K

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Electron spin 1/2

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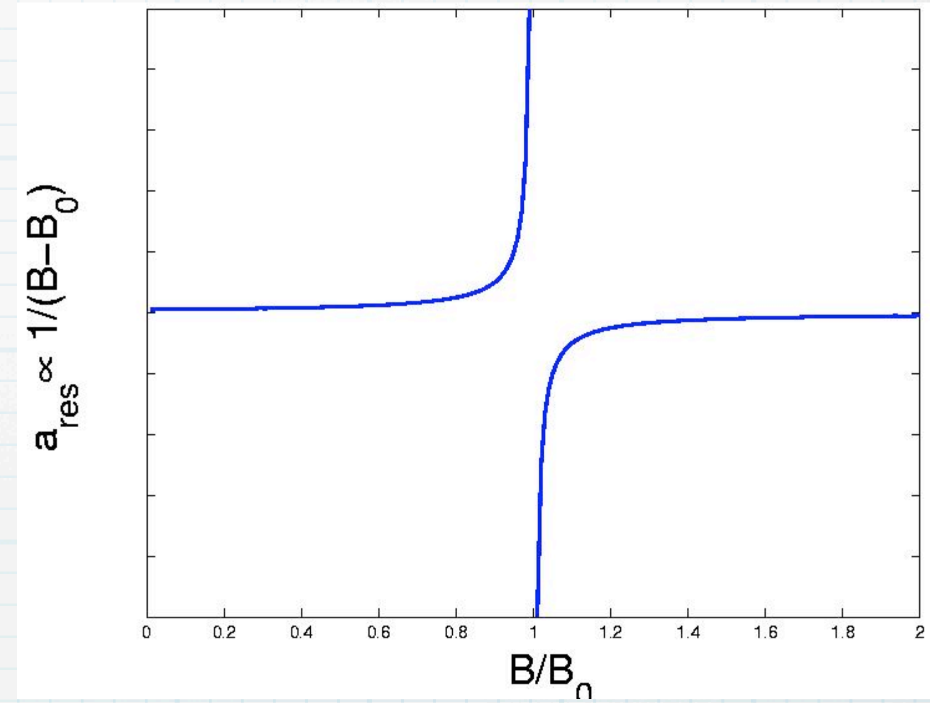
$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

$B_0 \approx 201.6G$ $a_{bg} \approx 290 a_0$ $\Delta B \approx 8G$



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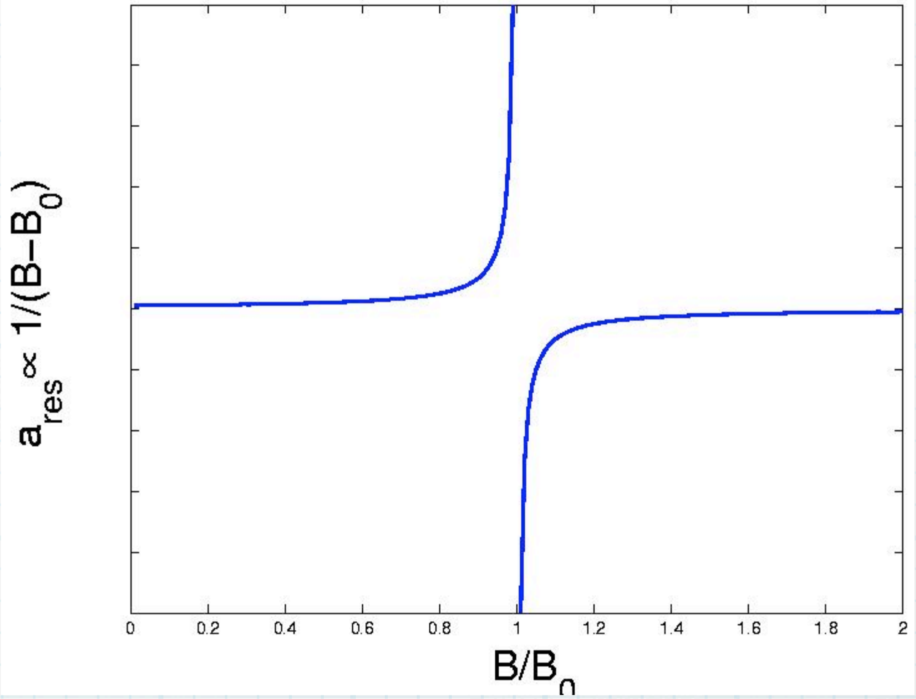

Open channel: $|\frac{9}{2}, -\frac{9}{2}\rangle \otimes |\frac{9}{2}, -\frac{7}{2}\rangle$

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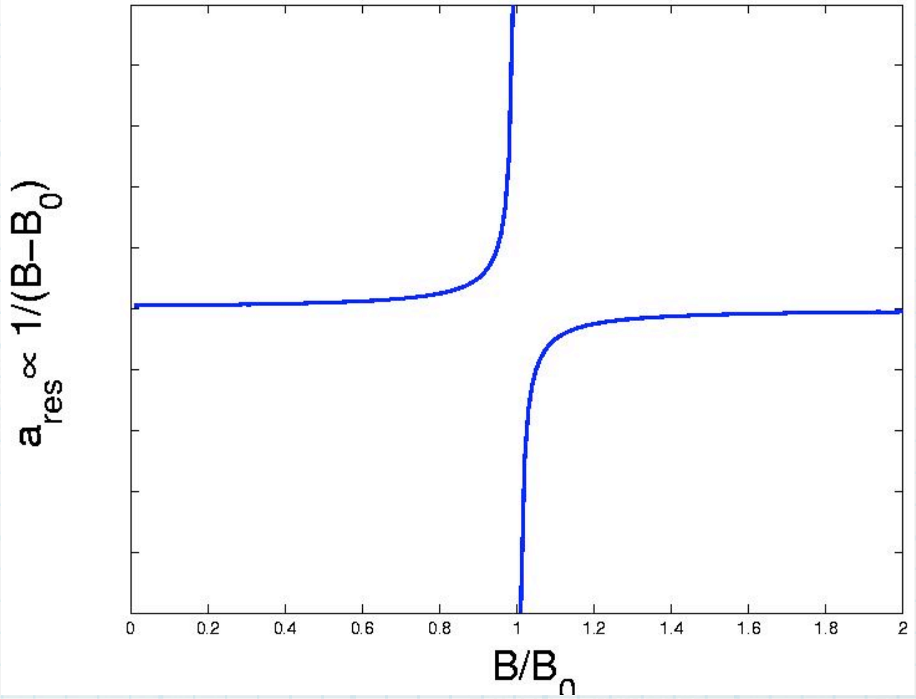

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Closed channel: $|\frac{9}{2}, -\frac{9}{2}\rangle \otimes |\frac{7}{2}, -\frac{7}{2}\rangle$

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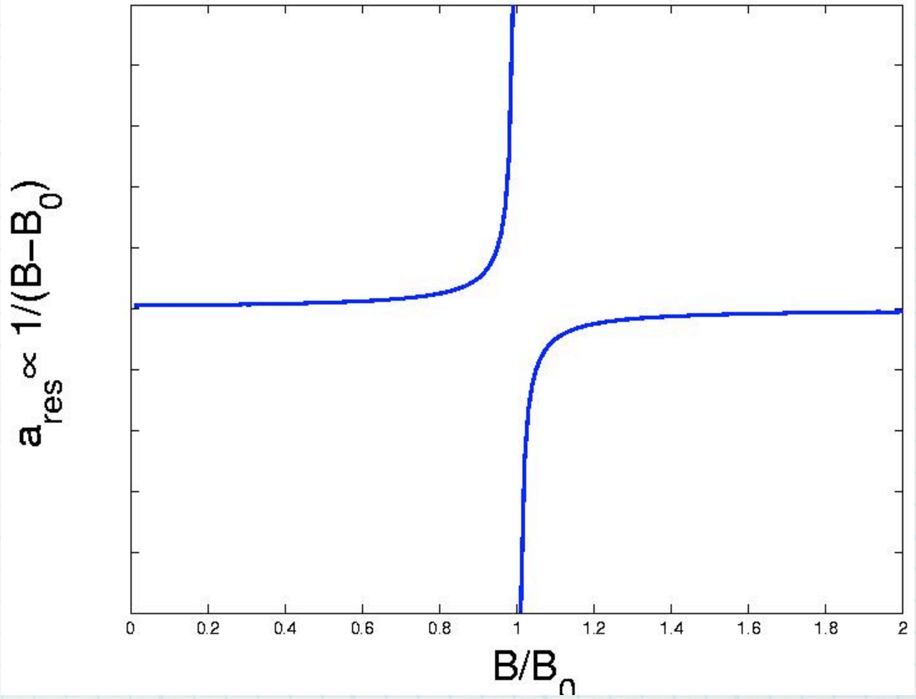

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Common state

Non-trivial medium effects

Closed channel: $|\frac{9}{2}, -\frac{9}{2}\rangle \otimes |\frac{7}{2}, -\frac{7}{2}\rangle$

Many-body Scattering

Start from Hamiltonian:

$$\hat{\Psi}_{\vec{q}}(\vec{K}) = \begin{bmatrix} \hat{a}_{\vec{K}/2+\vec{q}\alpha_4}\hat{a}_{\vec{K}/2-\vec{q}\alpha_3} \\ \hat{a}_{\vec{K}/2+\vec{q}\alpha_2}\hat{a}_{\vec{K}/2-\vec{q}\alpha_1} \end{bmatrix}$$

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Don't want to work with microscopic interaction.

Describe scattering in terms of physical observables

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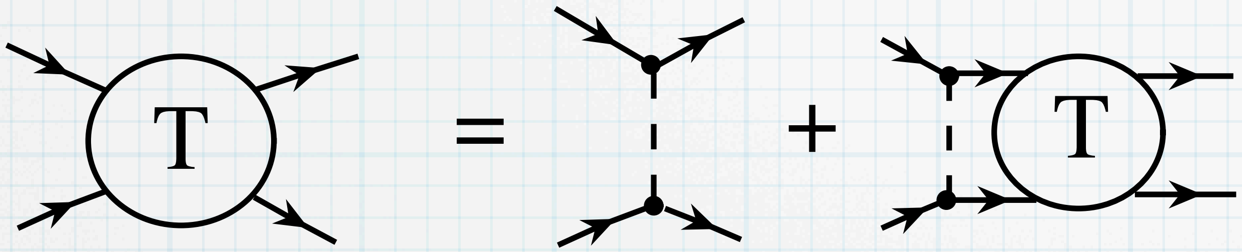
Landau Theory

Scattering - the Lippmann-Schwinger equation:

$$\begin{bmatrix} T_{cc} & T_{co} \\ T_{oc} & T_{oo} \end{bmatrix}^{-1} = \begin{bmatrix} V_{cc} & V_{co} \\ V_{oc} & V_{oo} \end{bmatrix}^{-1} - \begin{bmatrix} G_c & 0 \\ 0 & G_o \end{bmatrix}$$



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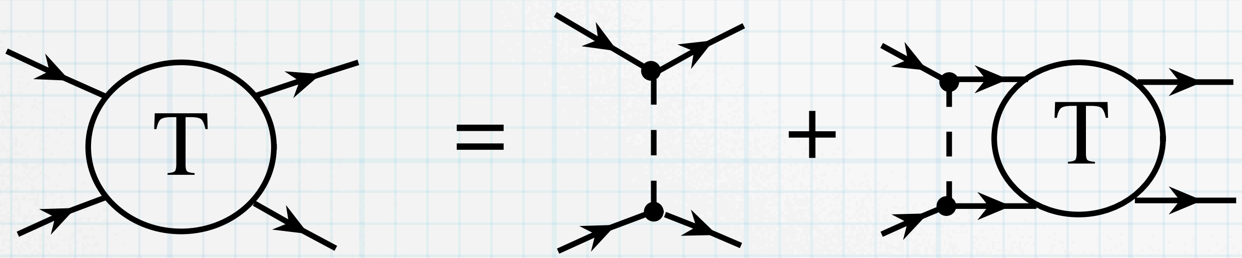
Pair propagators



$$G_o(\omega, \vec{K}, \vec{q}) = \frac{1 - f_{\alpha_1}(\frac{1}{2}\vec{K} - \vec{q}) - f_{\alpha_2}(\frac{1}{2}\mathbf{K} + \vec{q})}{\omega + i\delta - \frac{K^2}{4\,m} - \frac{q^2}{m}}$$

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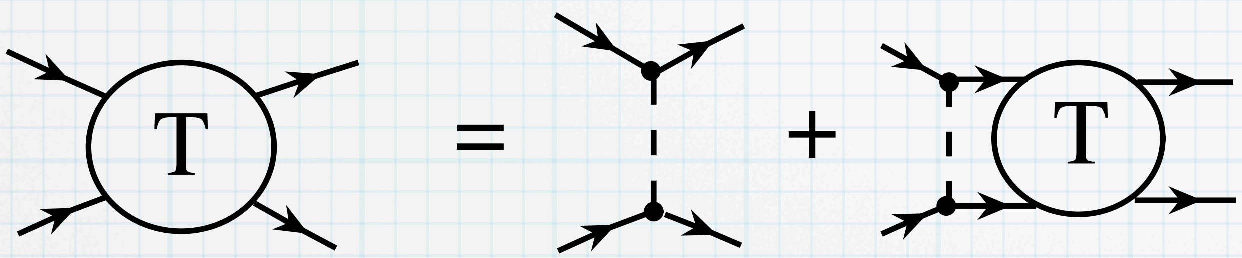

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Zero energy scattering in a vacuum:

$$\begin{bmatrix} U_{cc} & U_{co} \\ U_{oc} & U_{oo} \end{bmatrix}^{-1} = \begin{bmatrix} V_{cc} & V_{co} \\ V_{oc} & V_{oo} \end{bmatrix}^{-1} - \begin{bmatrix} G^{\rm vac} & 0 \\ 0 & G^{\rm vac} \end{bmatrix}$$

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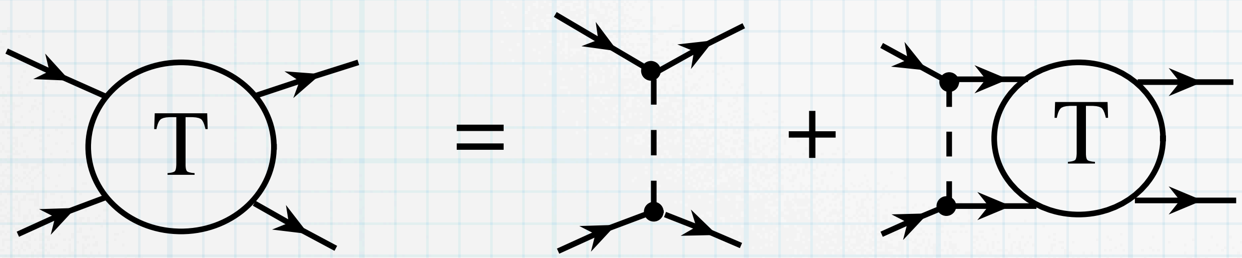

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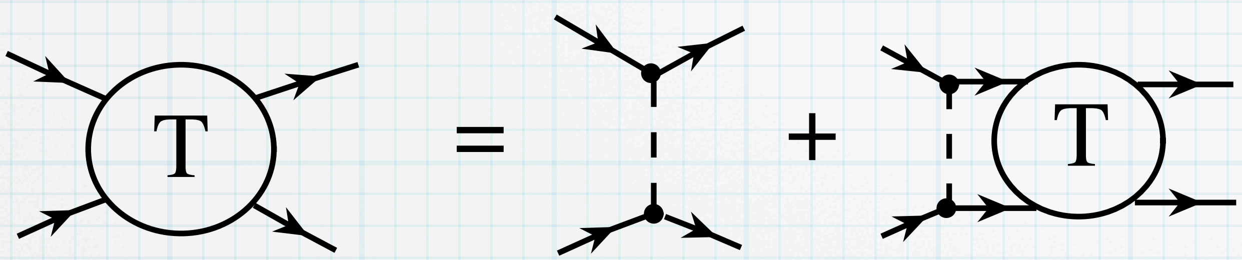

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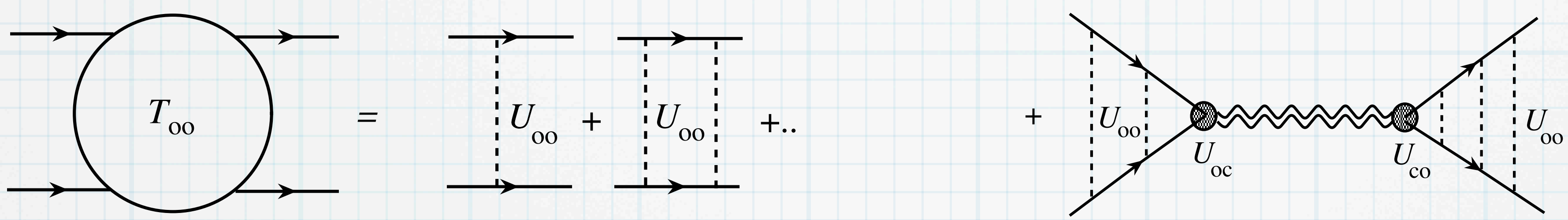
Scattering lengths

Open channel scattering:

$$T_{oo} = \frac{U_{oo}}{1 - U_{oo}\Pi_o} + \frac{U_{oc}}{1 - U_{oo}\Pi_o} D \frac{U_{co}}{1 - U_{oo}\Pi_o}$$



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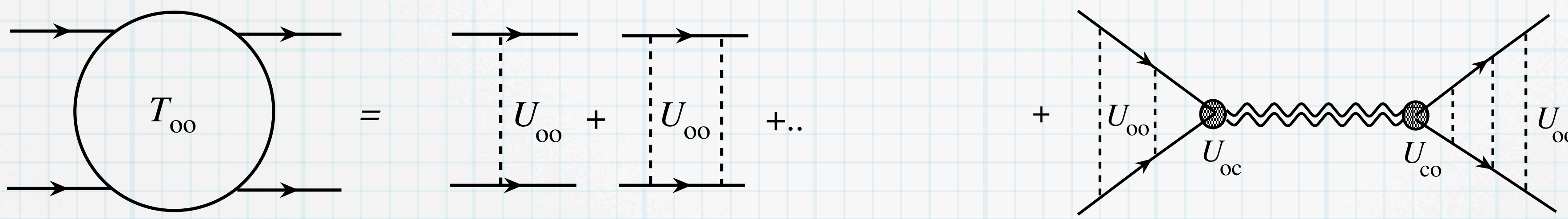
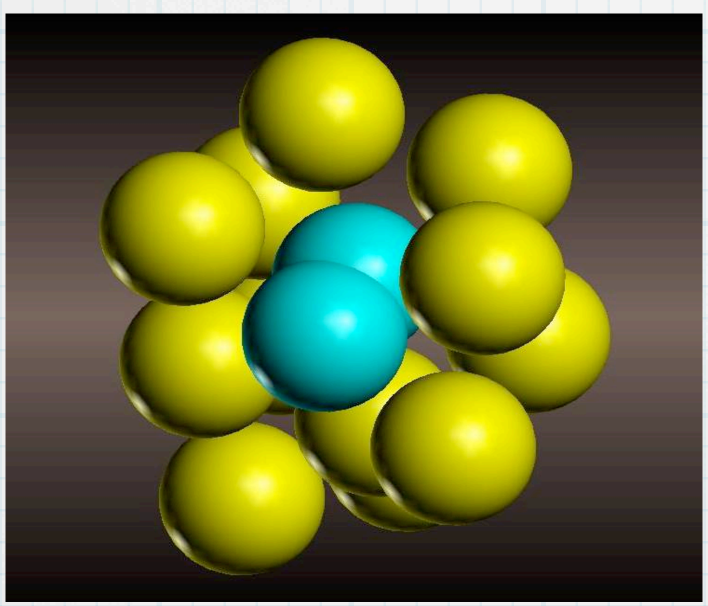


Dressed Molecule:

$$D^{-1} = \Pi_c^{-1} - U_{cc} - {U_{oc}}^2 \frac{\Pi_o}{1 - U_{oo}\Pi_o}$$



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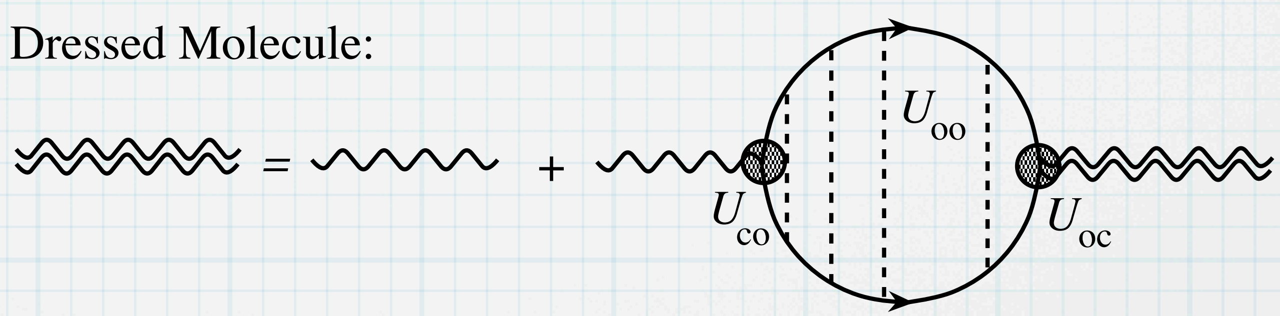
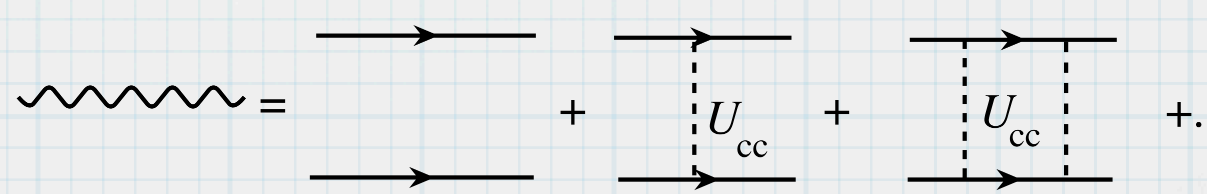


Bare Molecule in closed channel:



Feshbach resonance: $$T_{oo}^{vac} = \frac{4\pi a_{\rm bg}}{m}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

PRA 71, 052713 (2005)

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A Taylor expansion yields

$$T_{oo} = \frac{T_{\rm bg}}{\left(1 + \frac{\Delta\mu\Delta B}{\tilde{\omega} + h(\omega) - \Delta\mu(B - B_0)}\right)^{-1} - T_{\rm bg}\Pi_o^{\rm inf}(\omega)}$$

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Closed channel many-body physics

Open channel many-body physics

PRA 71, 052713 (2005)

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Closed channel many-body physics

Open channel many-body physics

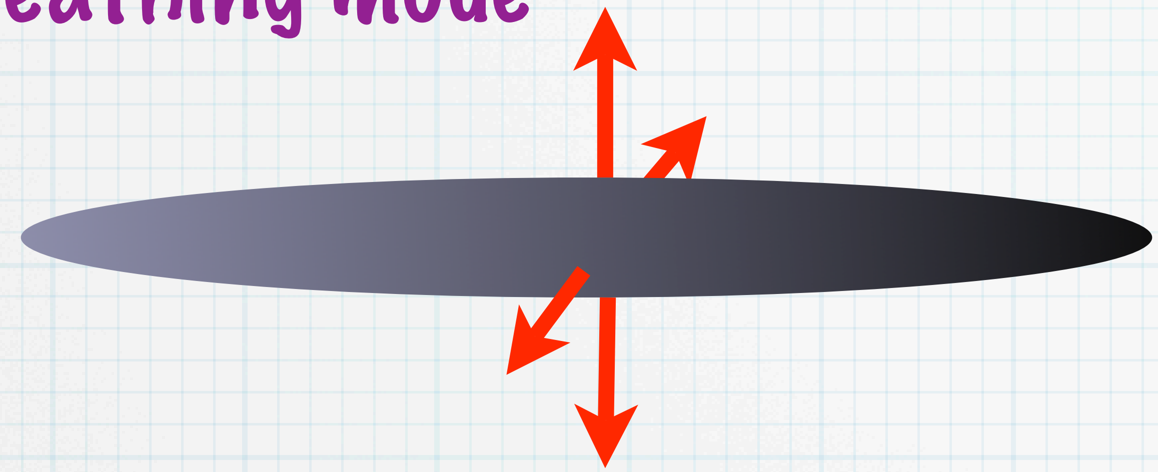
Have now expressed Feshbach scattering in a medium i terms of the physical 2-body parameters:

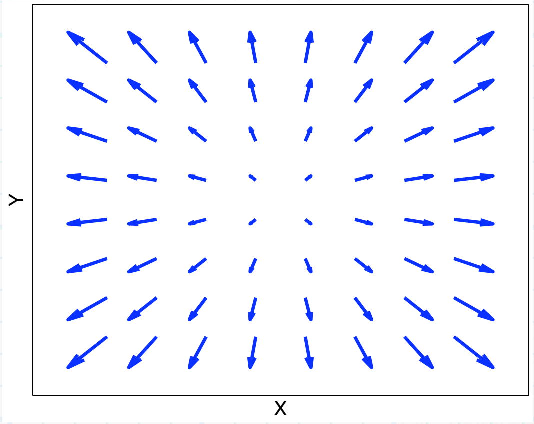
a_{bg} B_0 $\Delta\mu$ ΔB

PRA 71, 052713 (2005)

Collective Modes

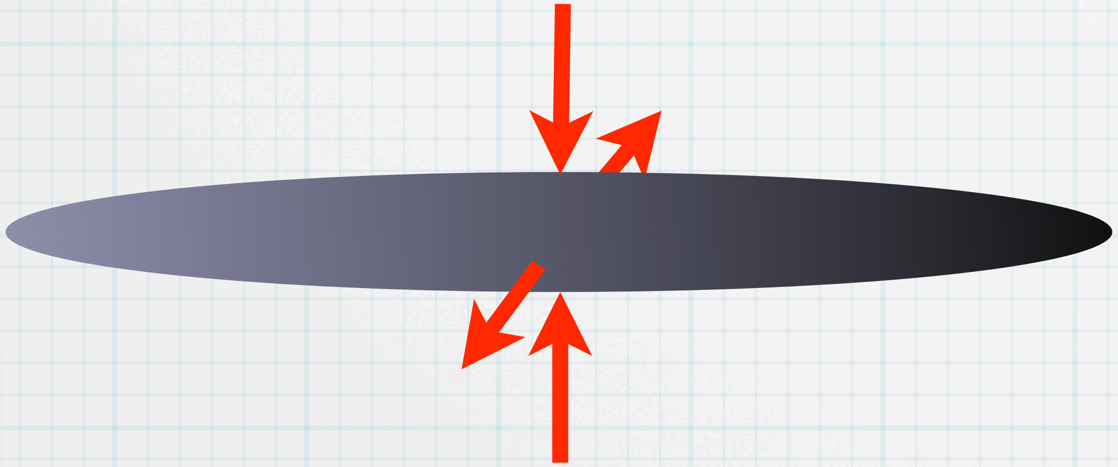
Breathing mode

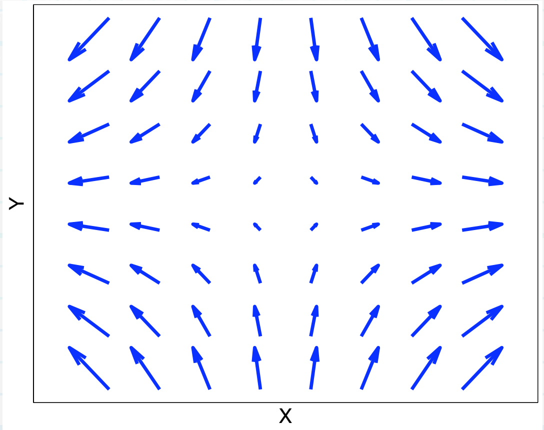




$$\mathbf{v} \propto \begin{pmatrix} x \\ y \end{pmatrix}$$

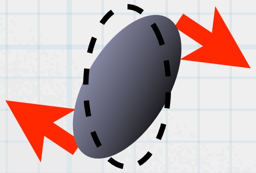
Quadrupole mode

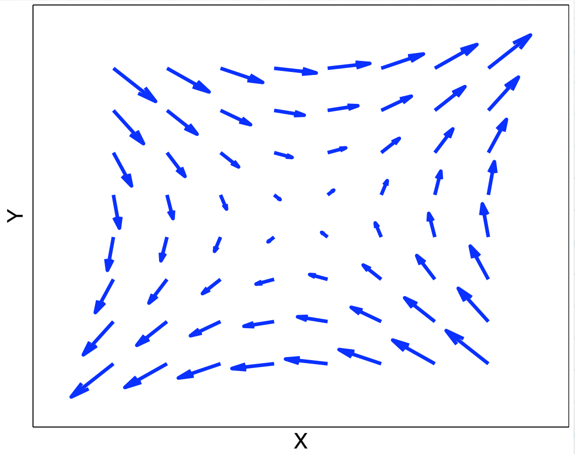




$$\mathbf{v} \propto \begin{pmatrix} x \\ -y \end{pmatrix}$$

Scissors mode



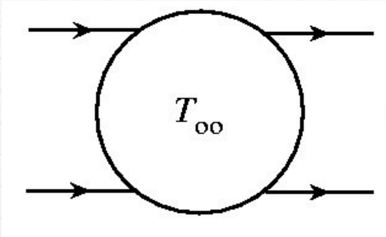


$$\mathbf{v} \propto \begin{pmatrix} y \\ x \end{pmatrix}$$

Boltzmann equation

$$\sigma = \frac{4\pi a^2}{1 + (p_r a/\hbar)^2}$$

$$\frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = -I[f]$$



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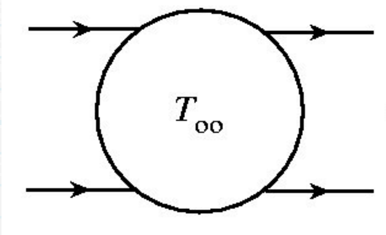

Linearize $f = f^0 + \delta f$

$$\delta f(\mathbf{r}, \mathbf{p}, t) = f^0(\mathbf{r}, \mathbf{p})[1 - f^0(\mathbf{r}, \mathbf{p})]\Phi(\mathbf{r}, \mathbf{p}, t)$$

$$I[\Phi] = \int \frac{d^3 p_1}{(2\pi\hbar)^3} \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{v} - \mathbf{v}_1| [\Phi + \Phi_1 - \Phi' - \Phi_1'] f^0 f_1^0 (1 - f^{0\prime})(1 - f_1^{0\prime})$$

Unknown

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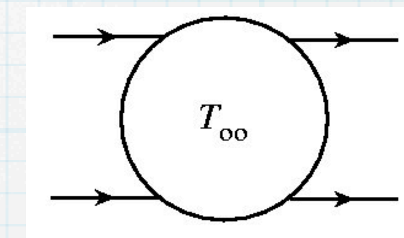

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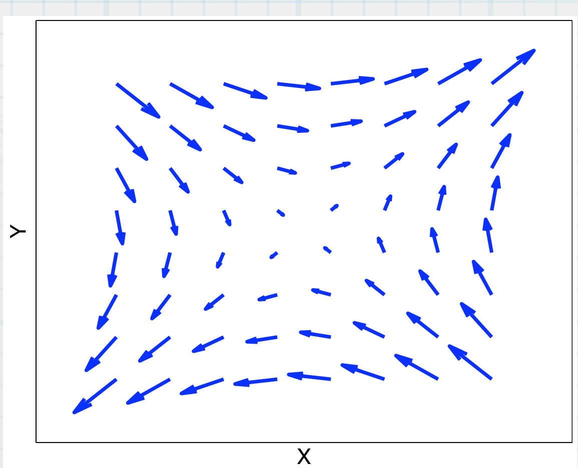

Local equilibrium

$$f(\mathbf{r}, \mathbf{p}) = f^0(\mathbf{r}, \mathbf{p} - m\mathbf{v})$$

Leads to Ansatz

$$\Phi(\mathbf{r}, \mathbf{p}, t) = (c_1 xy + c_2 x p_y + c_3 y p_x + c_4 p_x p_y) e^{-i\omega t}$$

Parameters

Mode frequency

Insert in Boltzmann Eq. and take moments: $\int d^3r d^3p (p_x p_y \ldots)$

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Root of determinant yields

$$\frac{i\omega}{\tau}(\omega^2 - \omega_h^2) + (\omega^2 - \omega_{c1}^2)(\omega^2 - \omega_{c2}^2) = 0$$

hydrodynamic frequency $\omega_h = \sqrt{\omega_x^2 + \omega_y^2}$

collisionless frequencies $\omega_{c1} = \omega_x + \omega_y$, $\omega_{c2} = |\omega_x - \omega_y|$

Viscous Relaxation Rate

$$I[\Phi] = \int \frac{d^3p_1}{(2\pi\hbar)^3} \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{v} - \mathbf{v}_1| [\Phi + \Phi_1 - \Phi' - \Phi_1'] f^0 f_1^0 (1 - f^{0\prime})(1 - f_1^{0\prime})$$

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$$\frac{1}{\tau} = \frac{(U, HU)}{(U, U)} \qquad U \propto p_x p_y \qquad H\Phi = \frac{1}{f^0(1-f^0)} I[\Phi]$$

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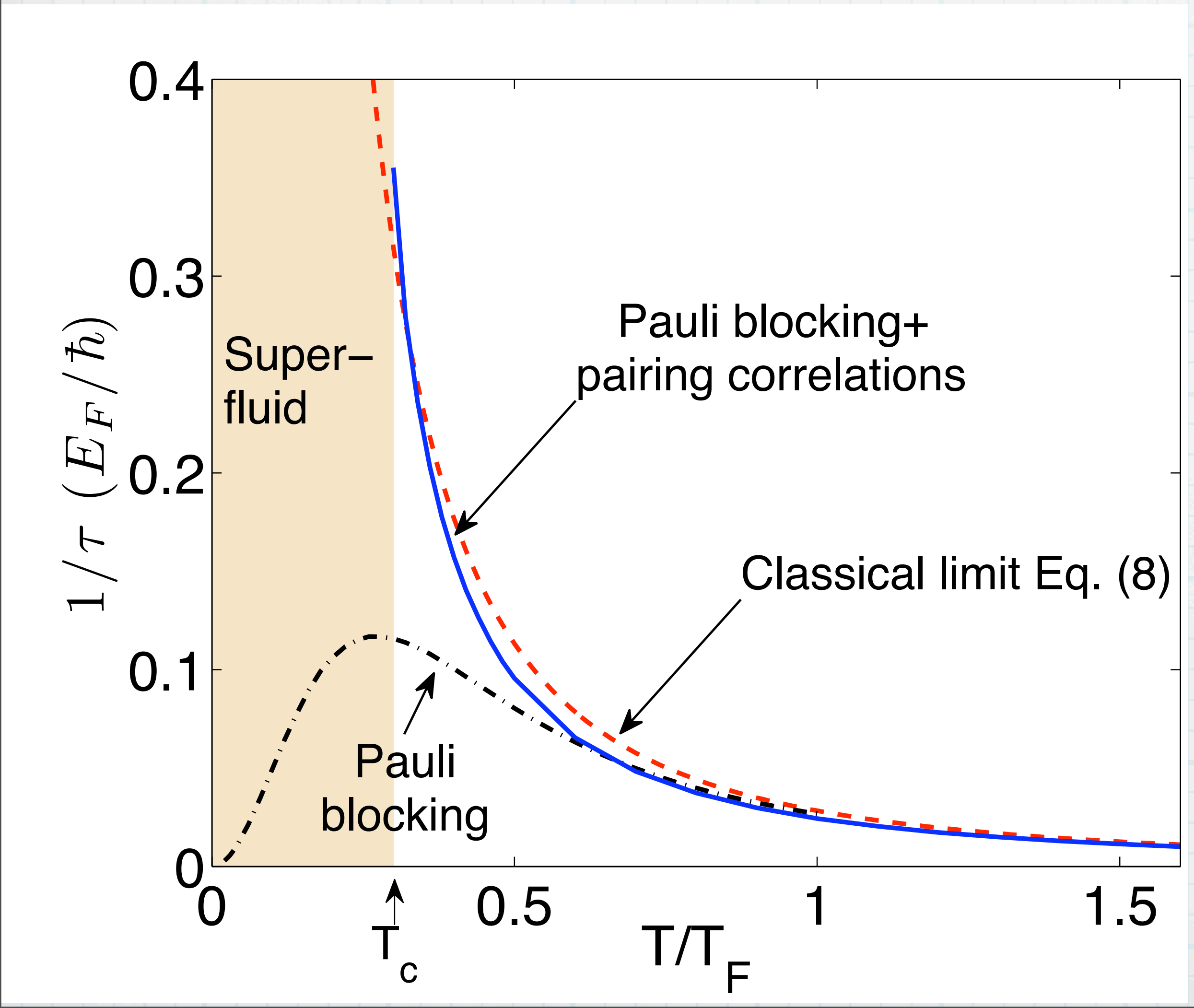
Unitary limit



$$\mathcal{T}_{\text{uni}} = \frac{4\pi\hbar^2 a}{m} \frac{1}{1 + iqa}$$

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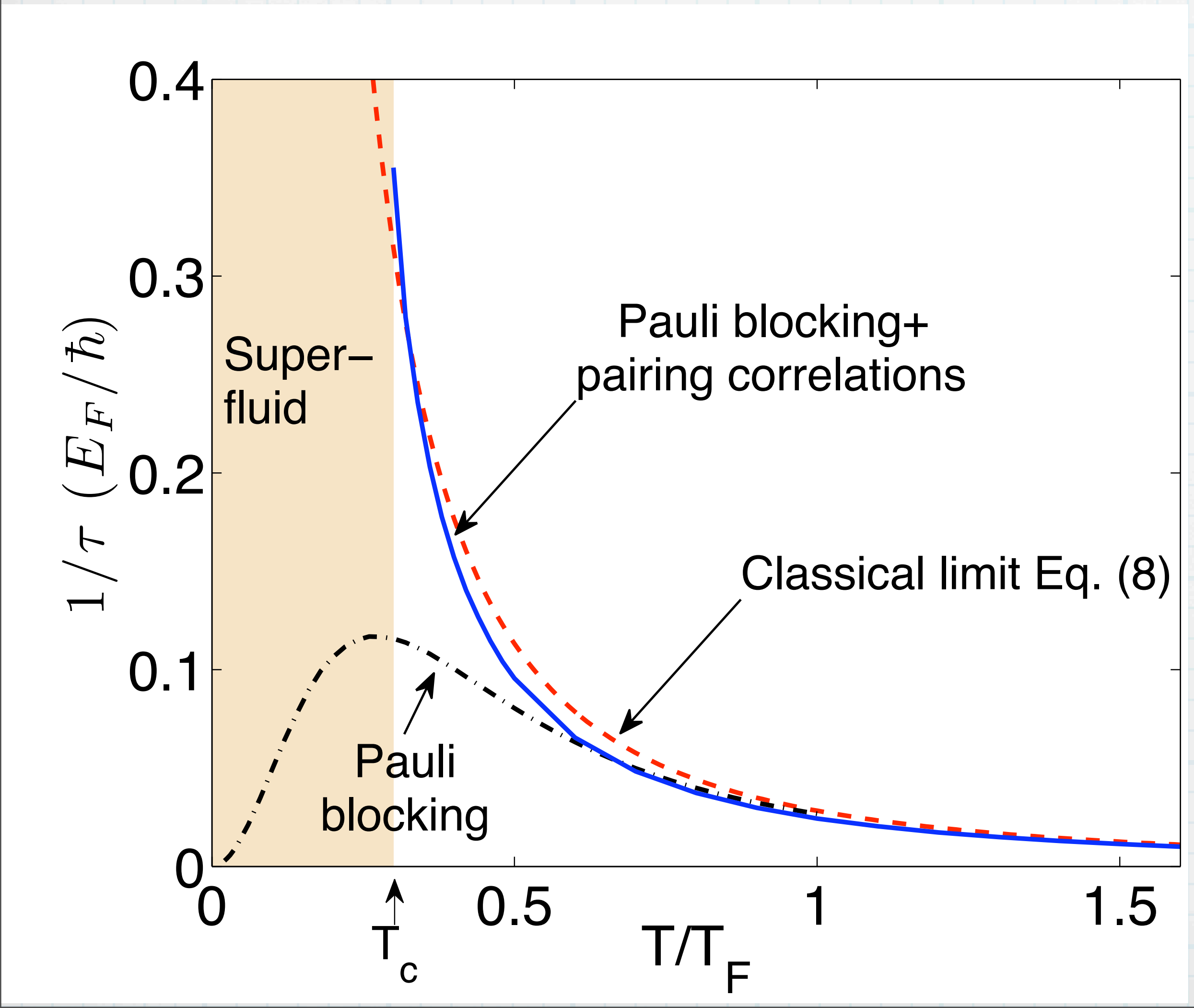

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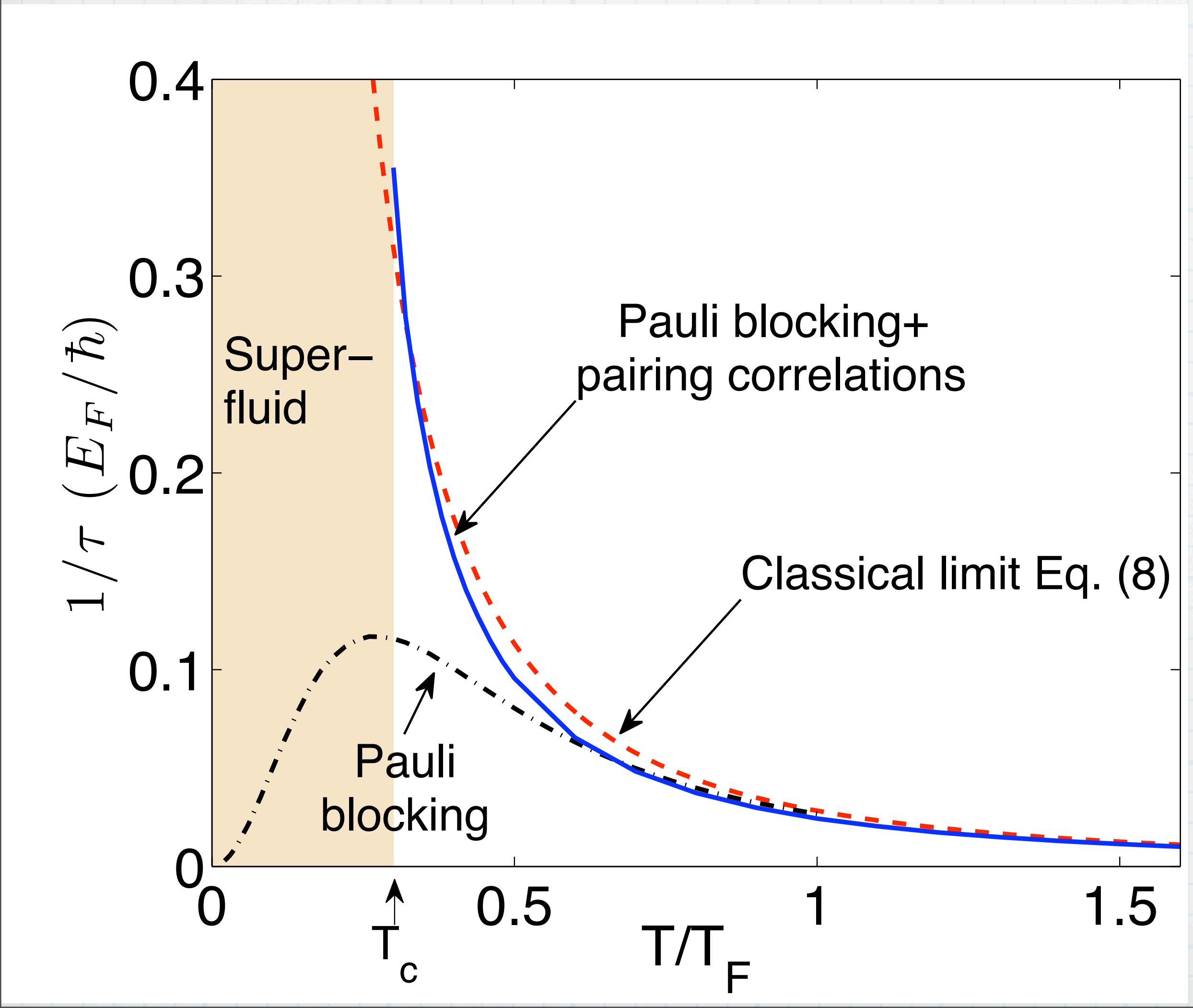

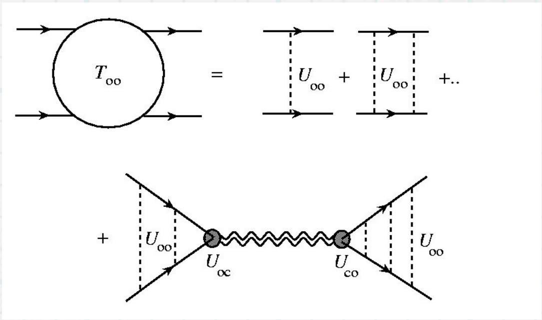

$$\mathcal{T}(\omega, \mathbf{K}) = \frac{\mathcal{T}_{\rm bg}}{\left(1 + \frac{\Delta\mu \Delta B}{\hbar\tilde{\omega} + h(\omega, \mathbf{K}) - \Delta\mu(B - B_0)}\right)^{-1} - \mathcal{T}_{\rm bg}\Pi(\omega, \mathbf{K})},$$

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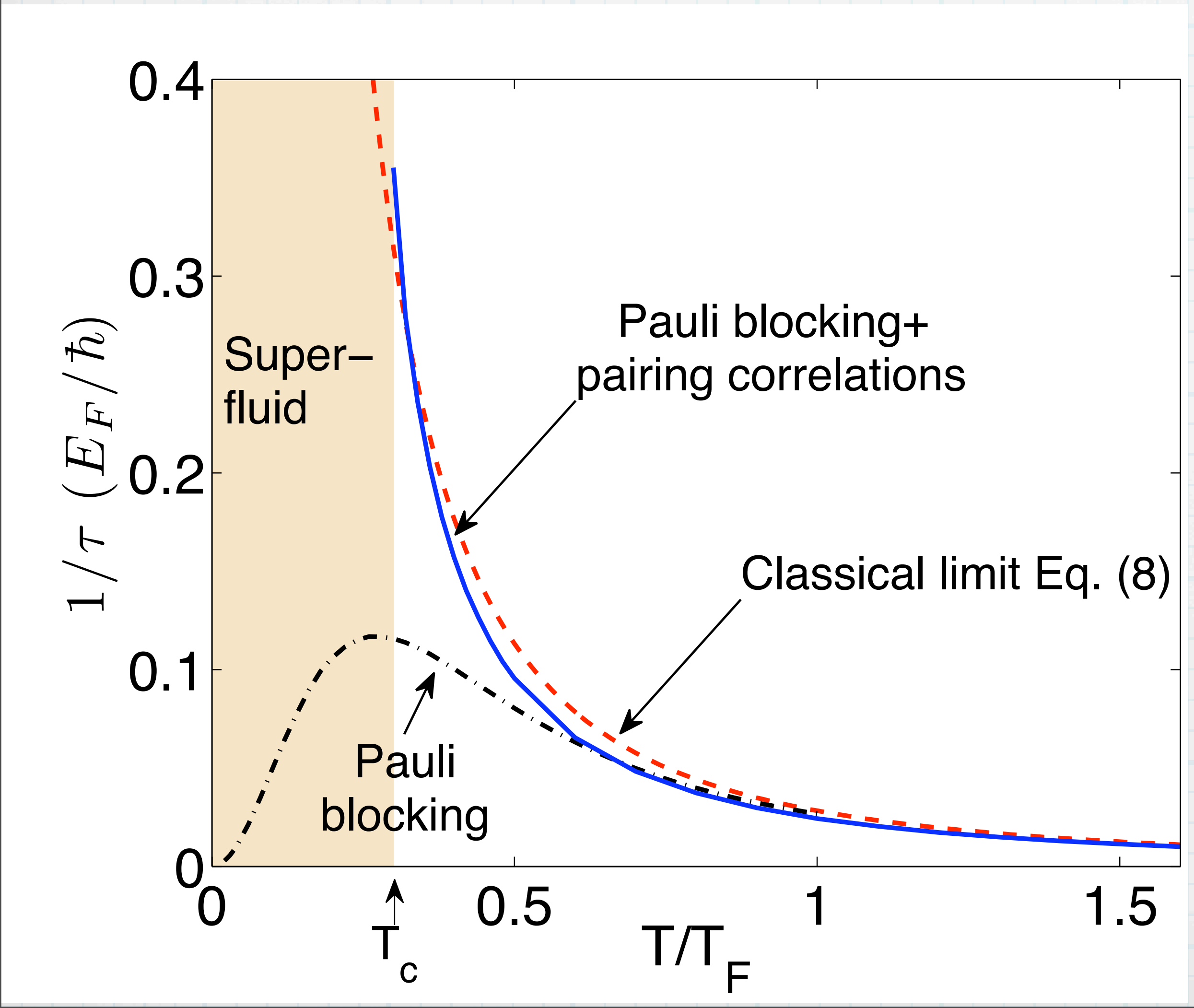

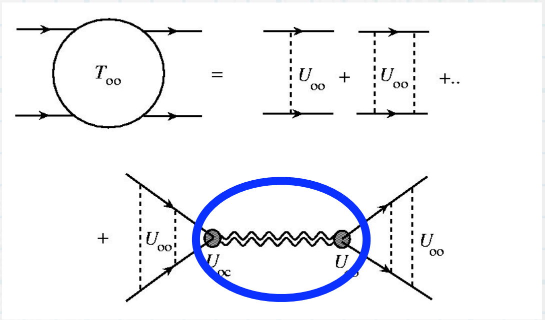

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Increased scattering rate due to Cooper instability

Viscous Relaxation Rate

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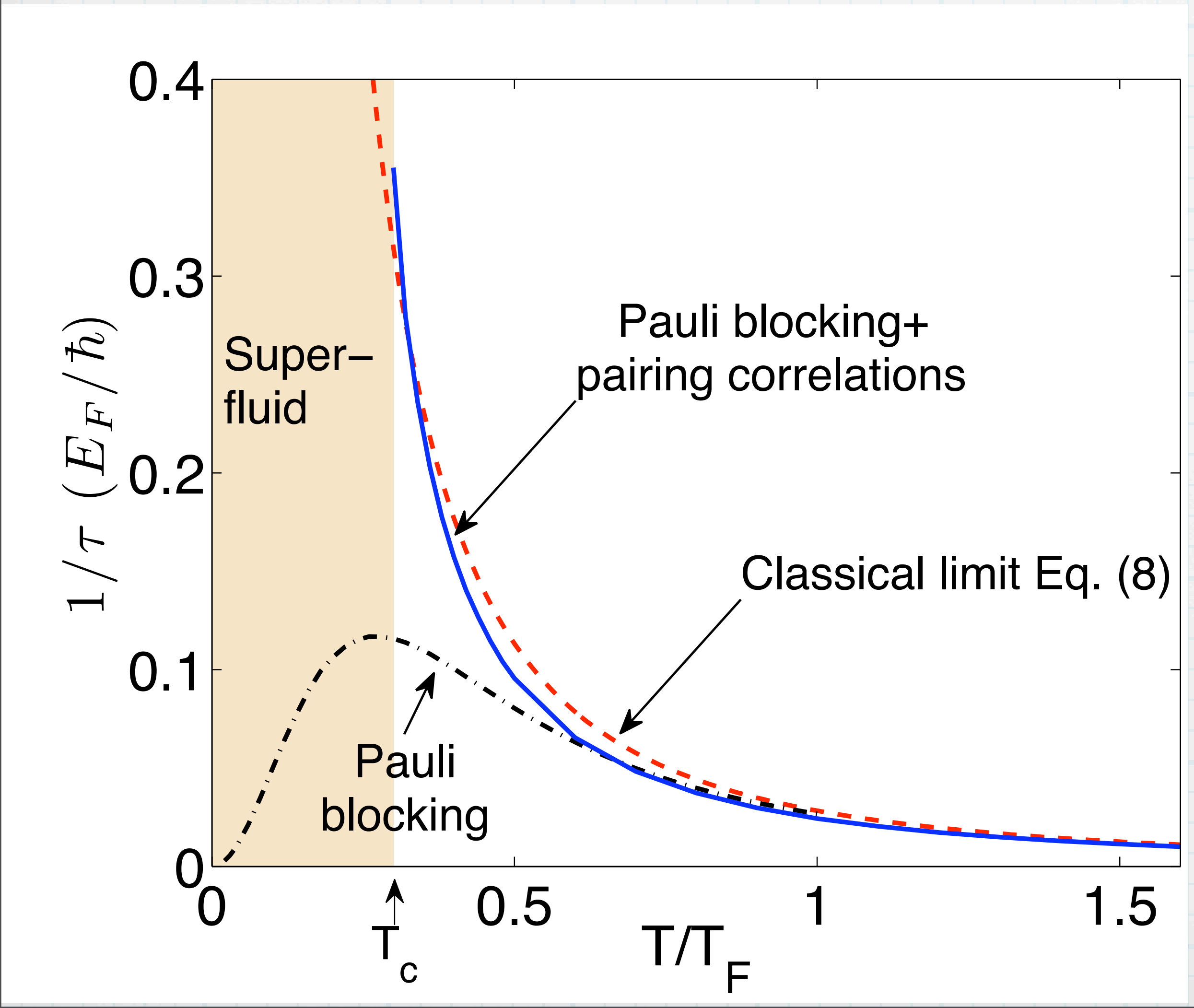
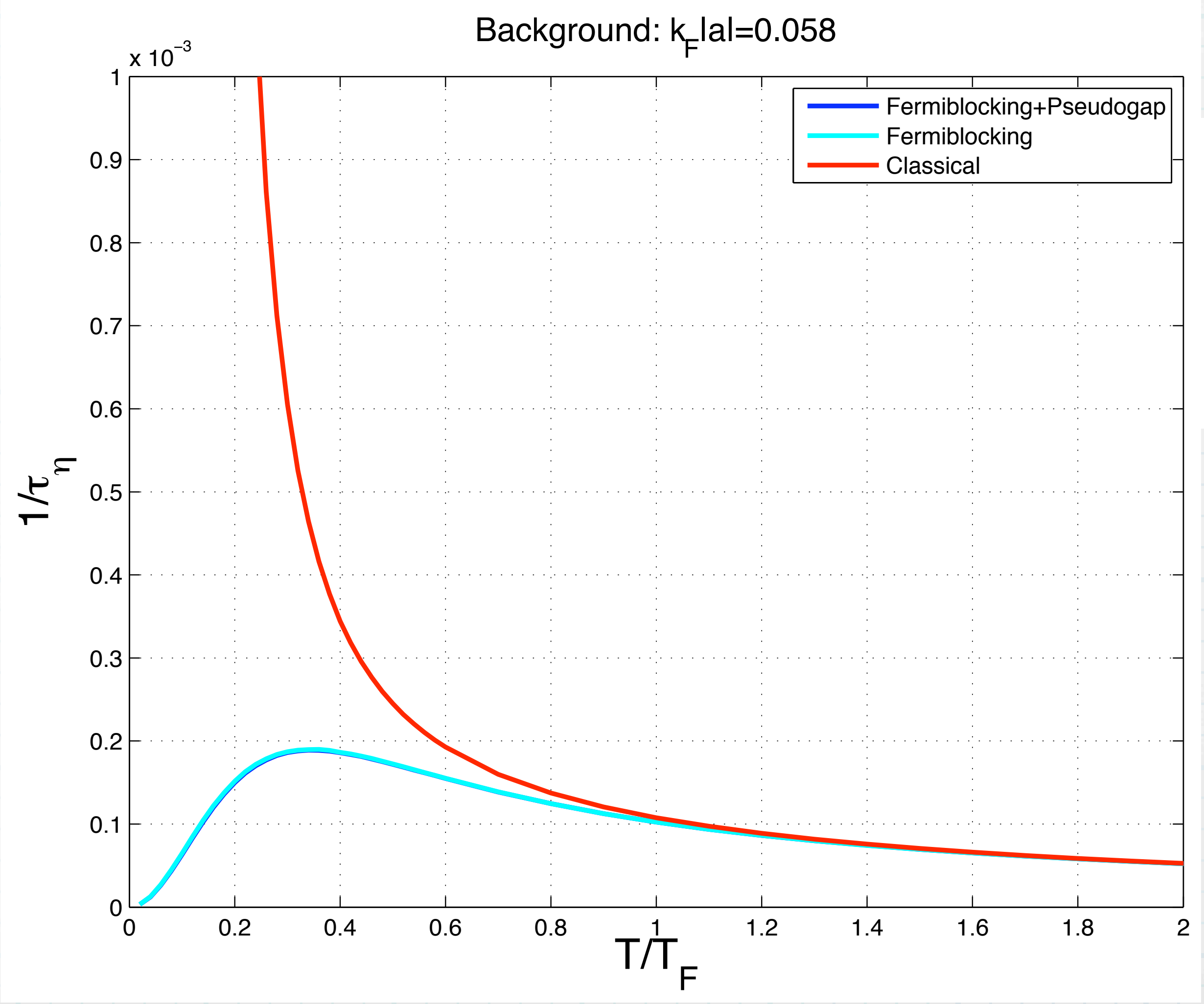
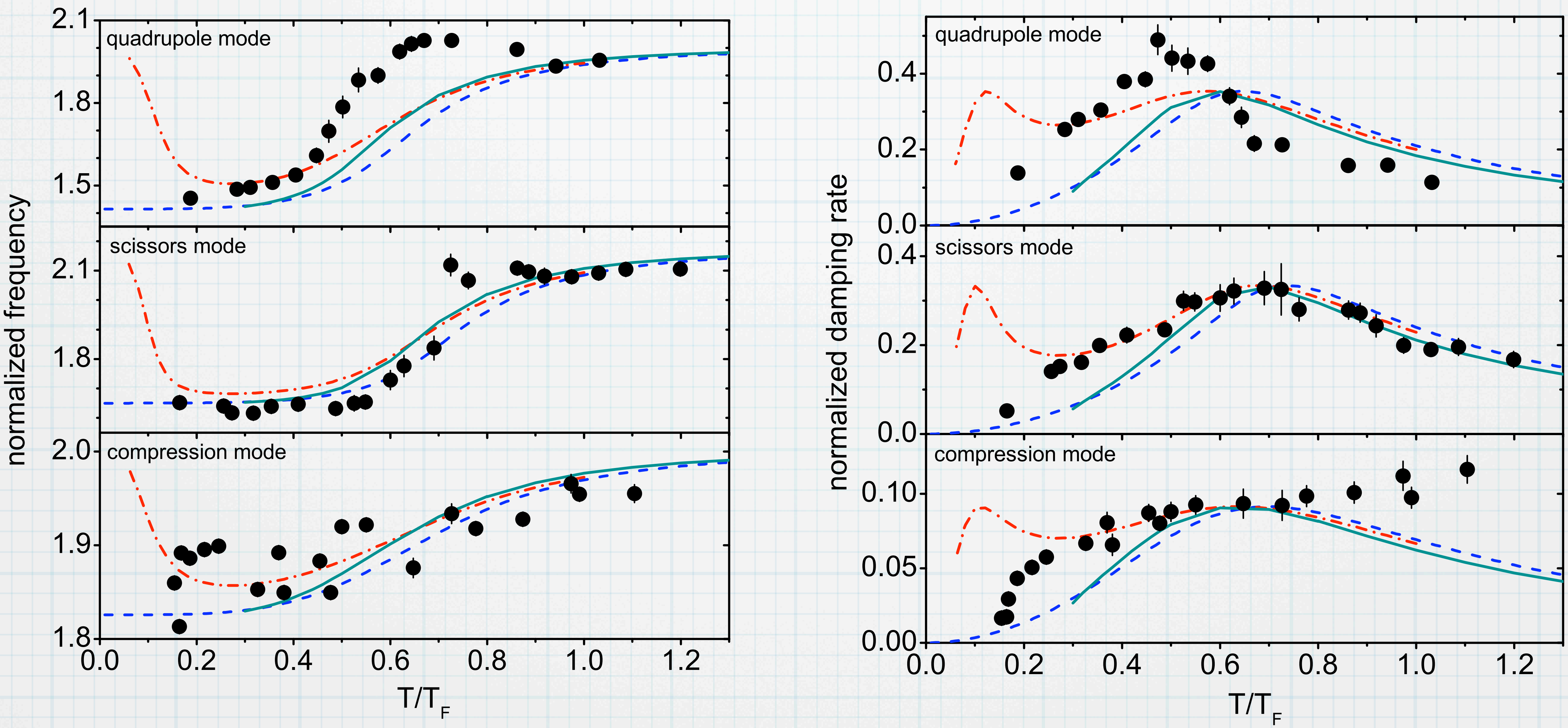


Weak coupling $k_F a \ll 1$



Relevant for collective mode experiments:



arXiv:08094226

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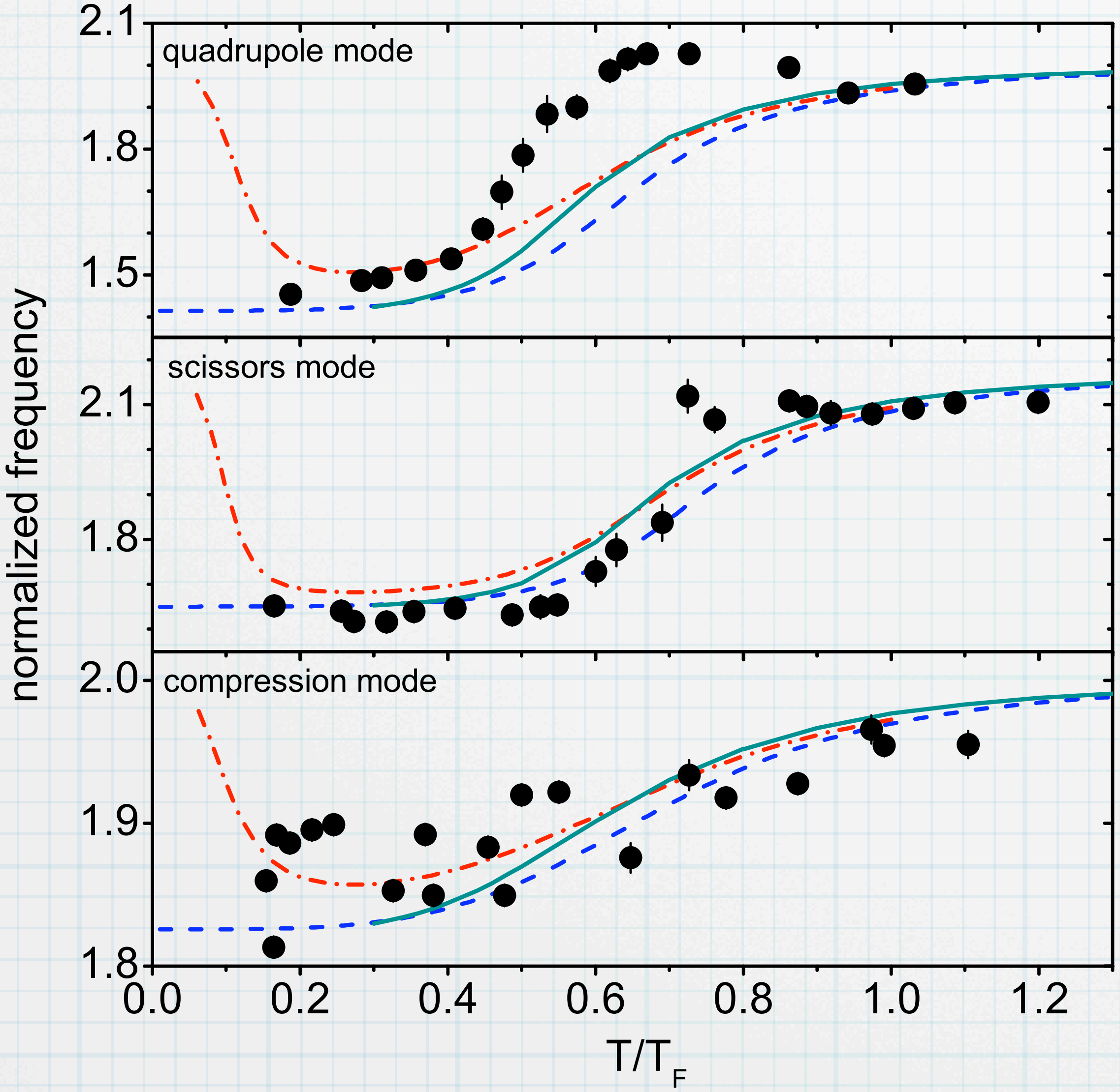
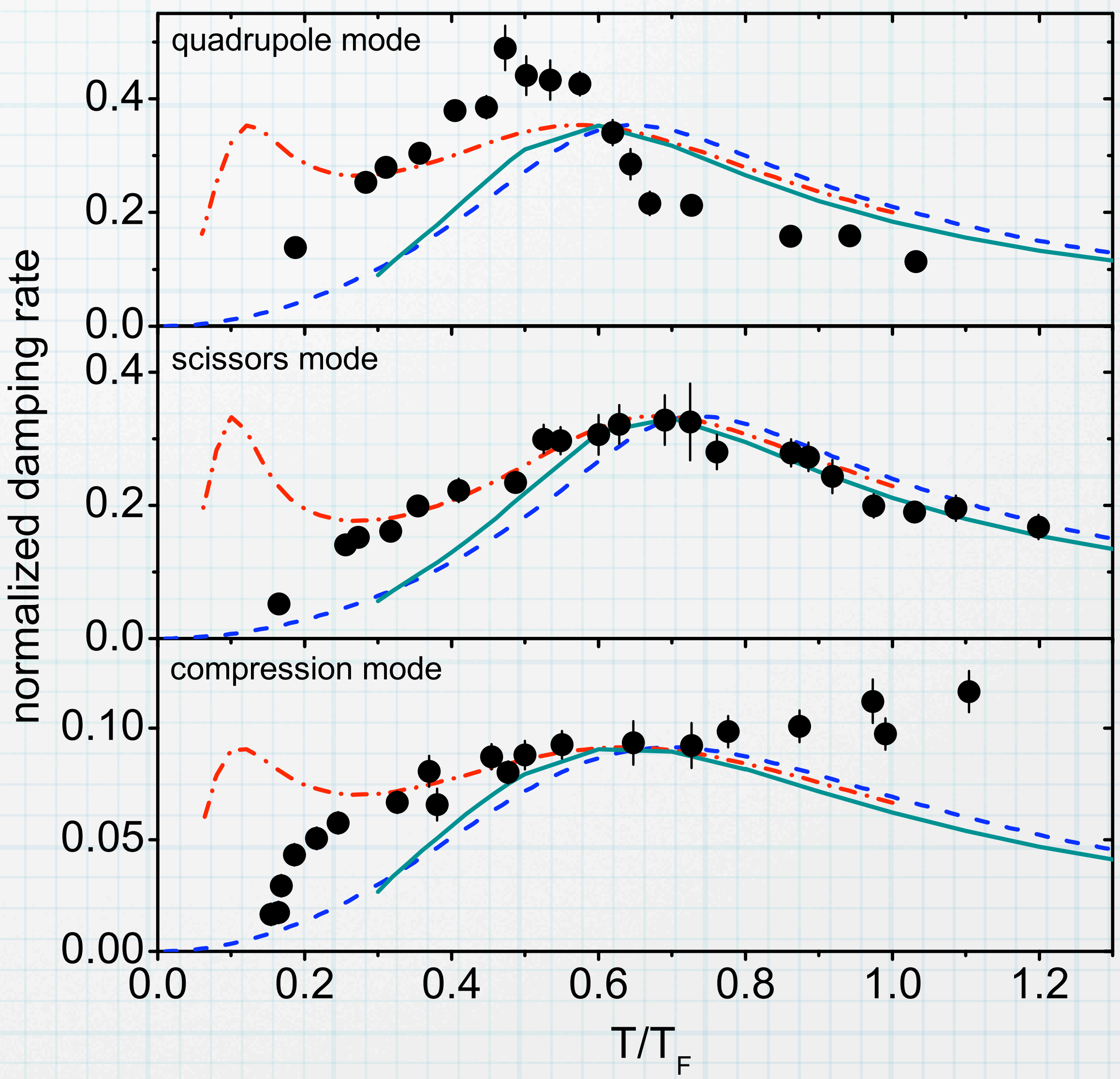




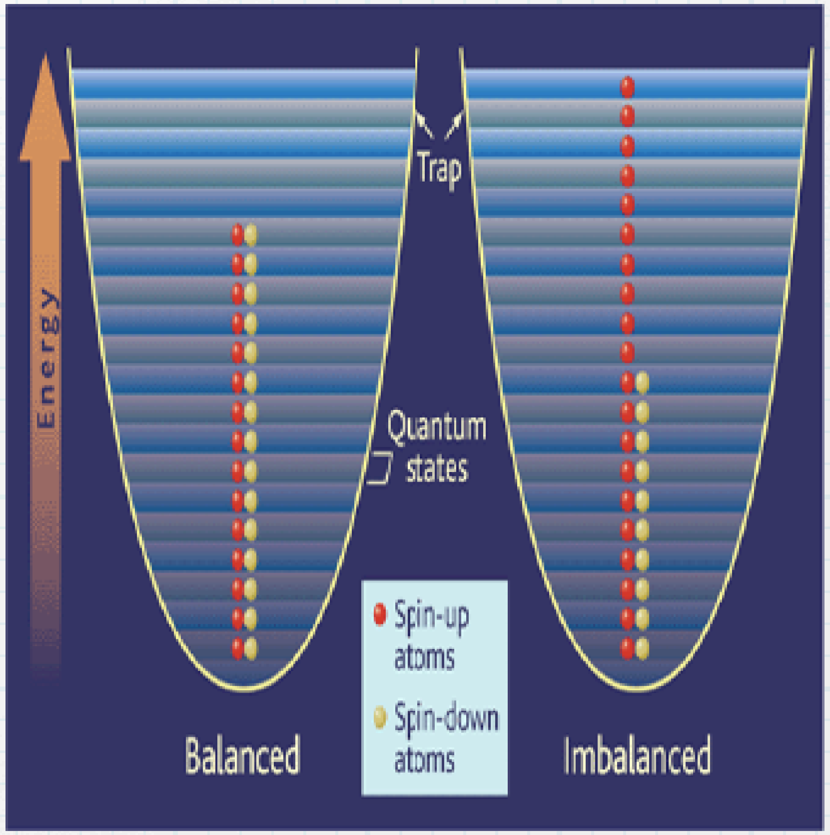
No fitting parameters

arXiv:08094226

Spin polarized gases

Can vary experimentally the of spin states

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



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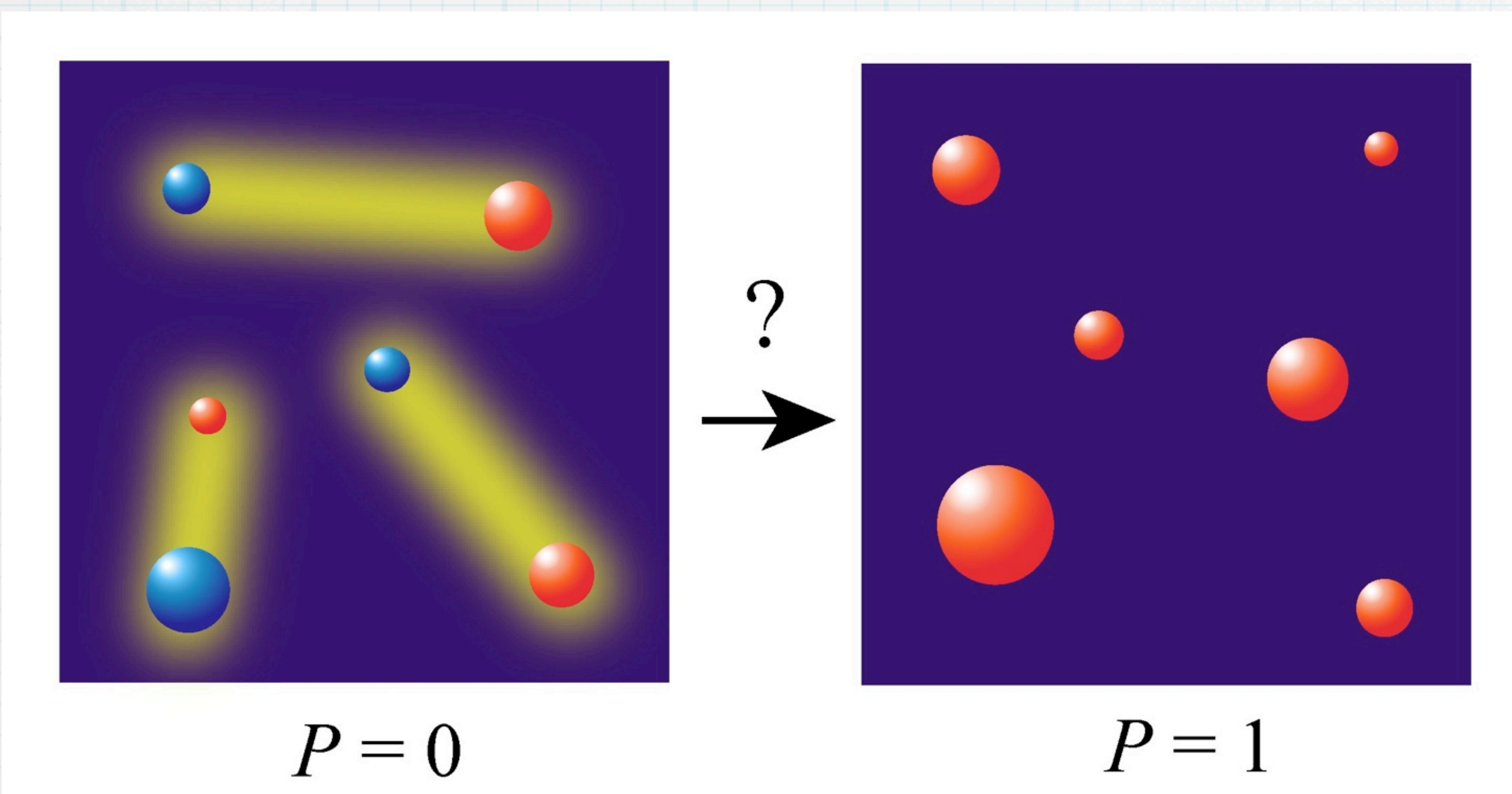
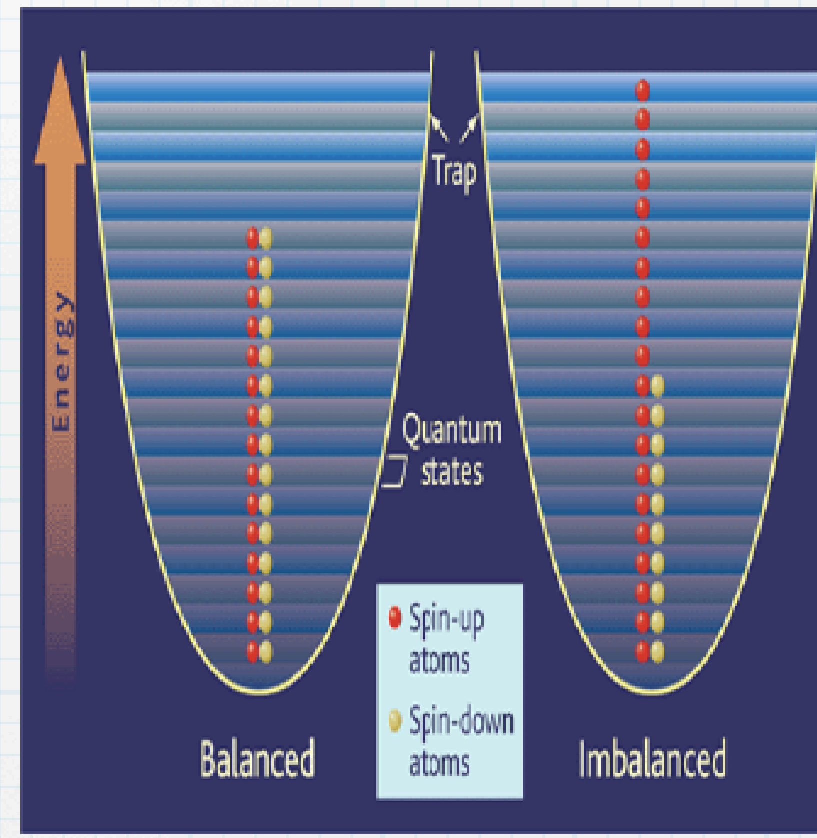


2-component superfluid

Spin polarized Fermi liquid

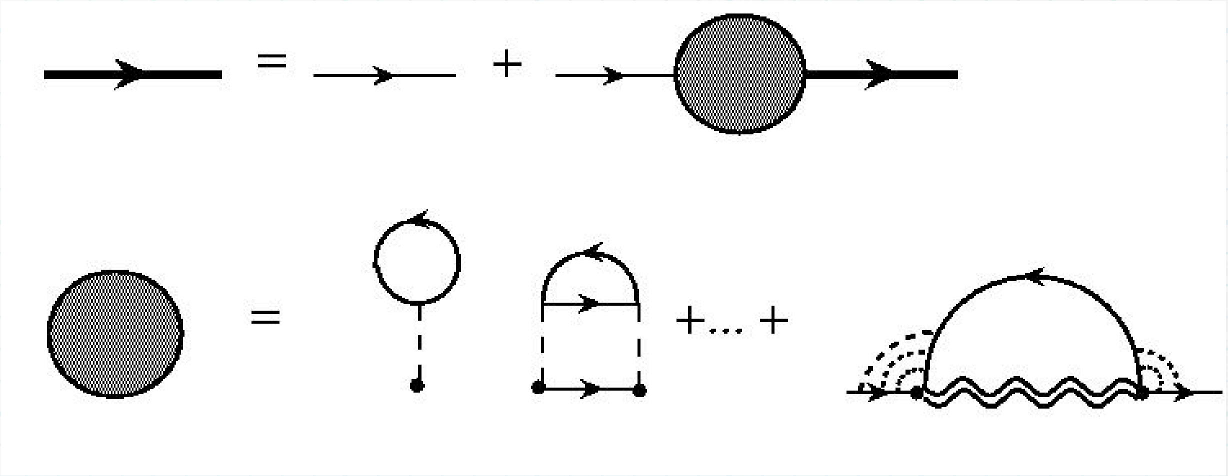


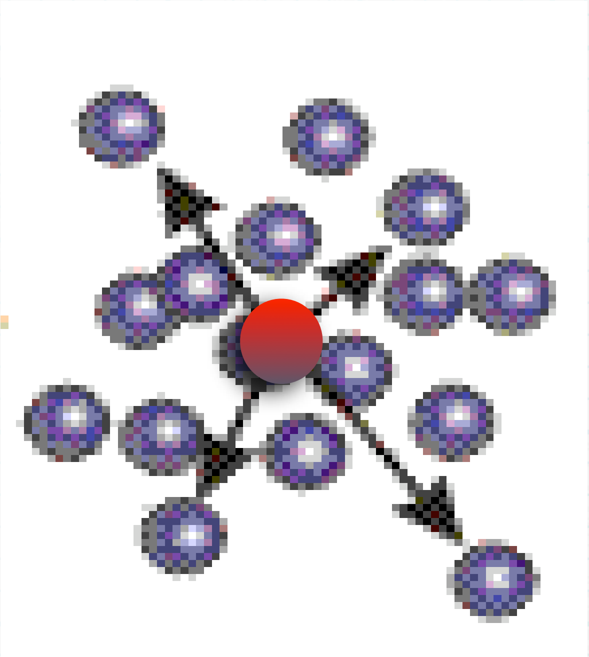
Highly polarized case: $N_\uparrow \gg N_\downarrow$

Minority atom propagator

$$G^{-1}(k,z) = z - \xi_k - \Sigma(k,z)$$

$$A(k,z) = -2\mathrm{Im}G(k,z)$$





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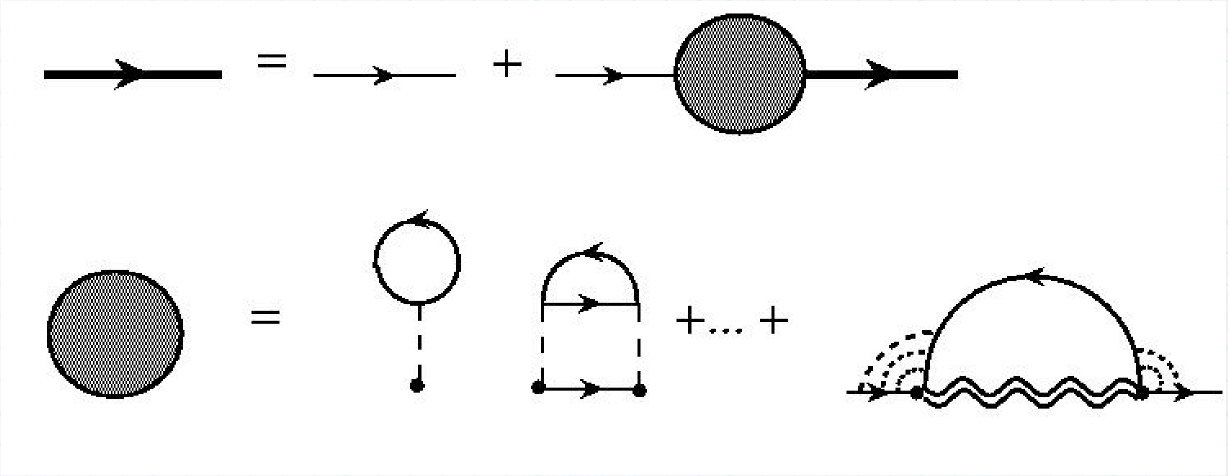
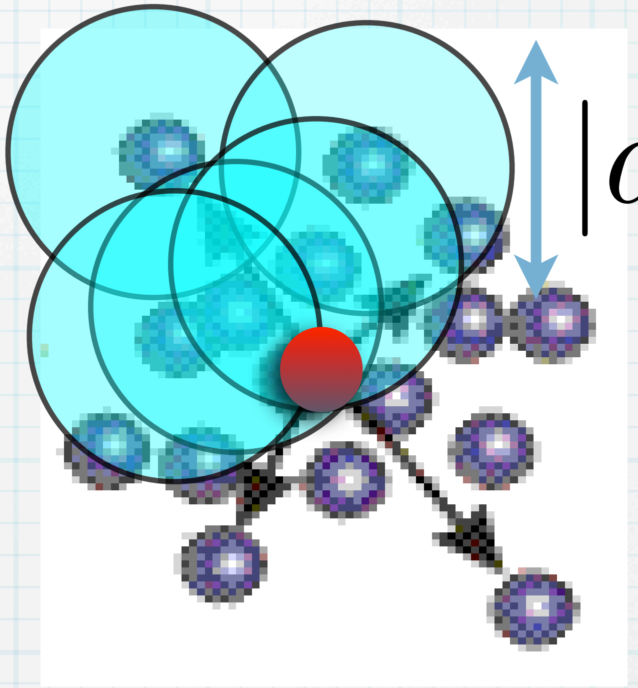


Unitarity limit $k_F|a| \gg 1$



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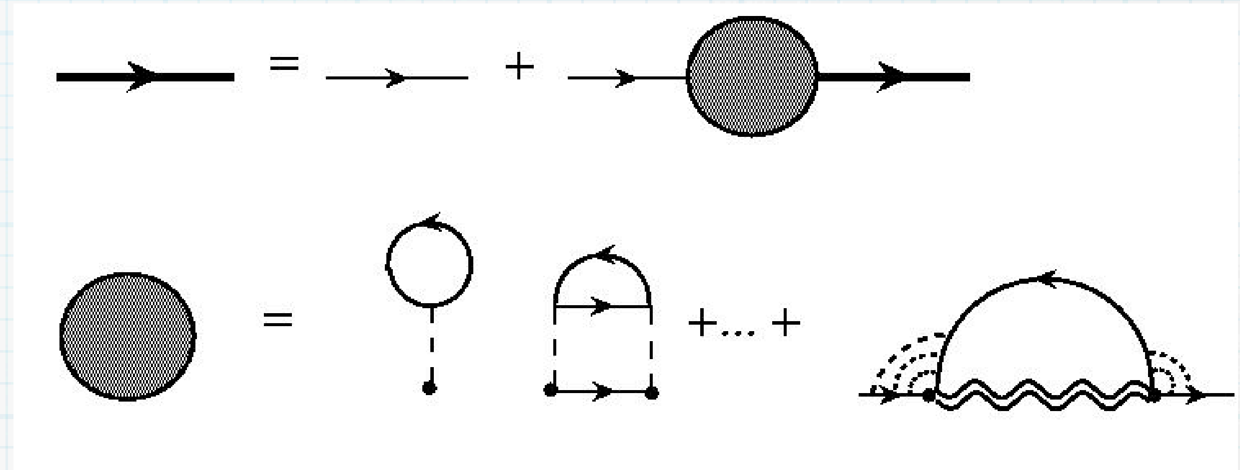

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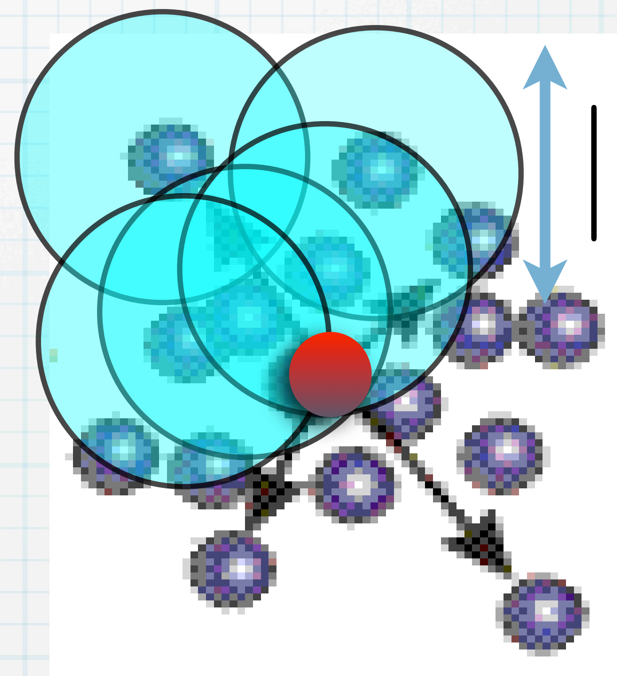

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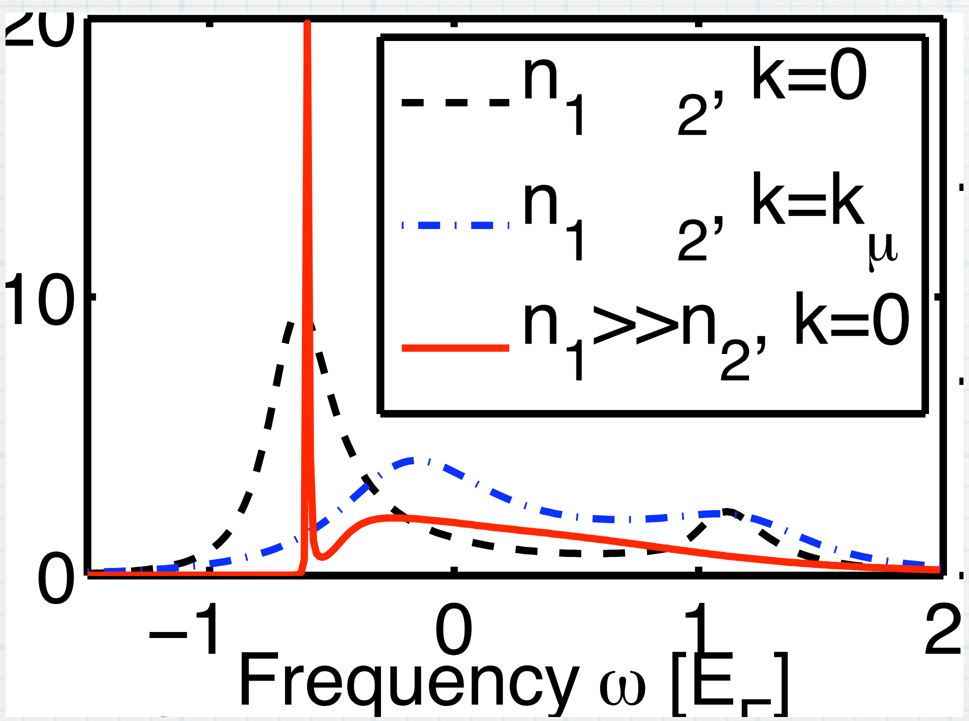

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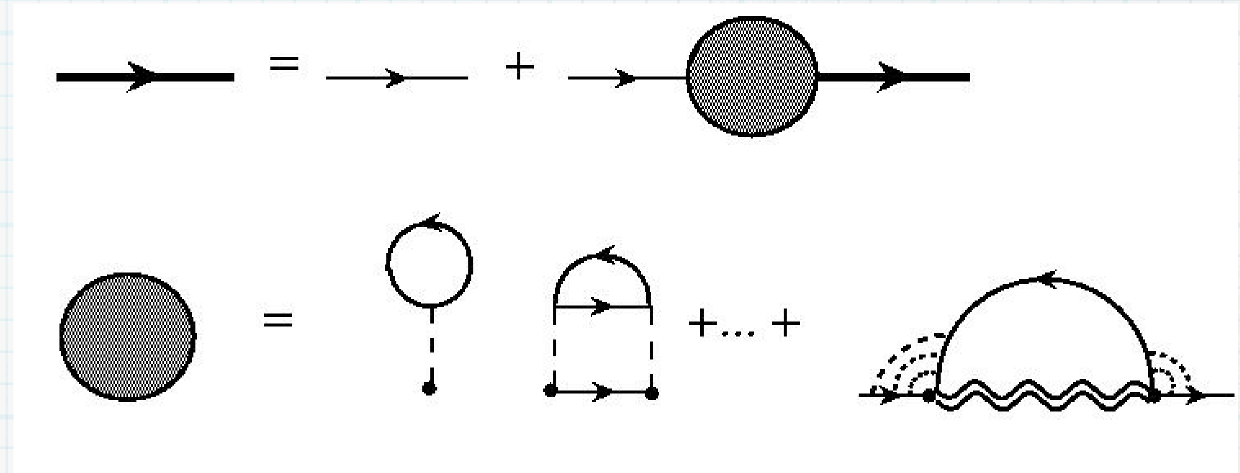

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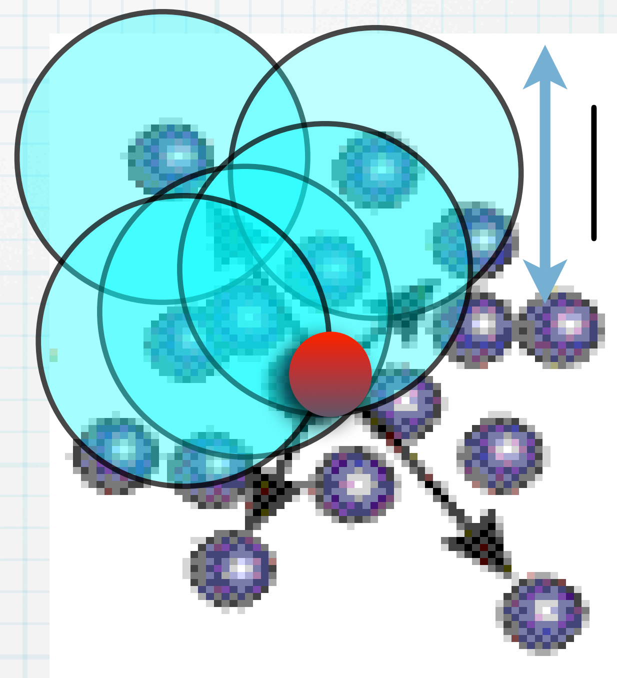

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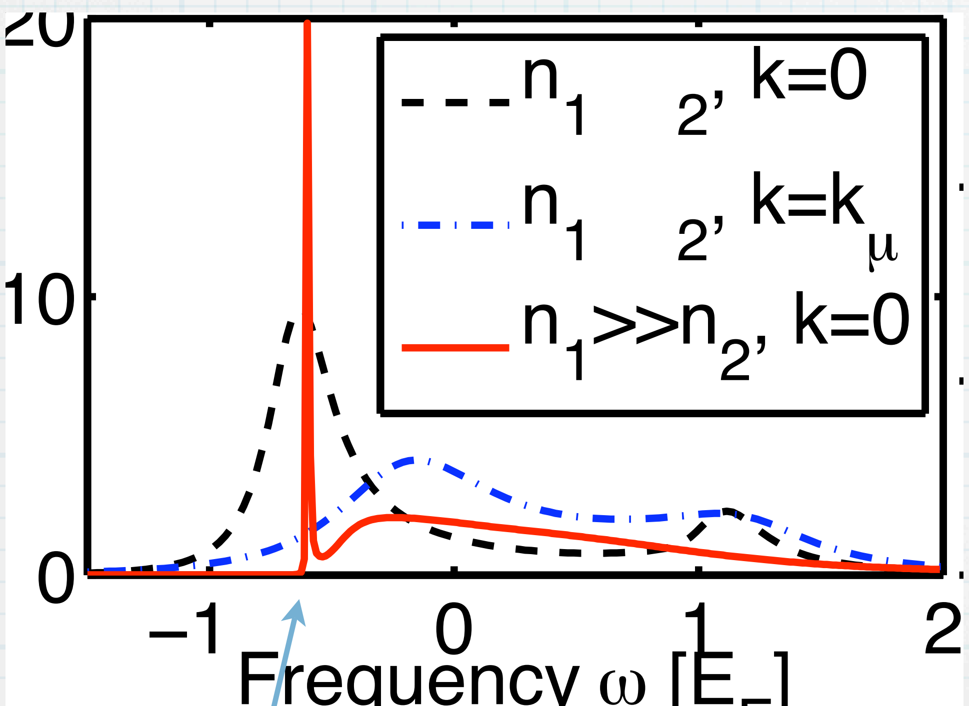
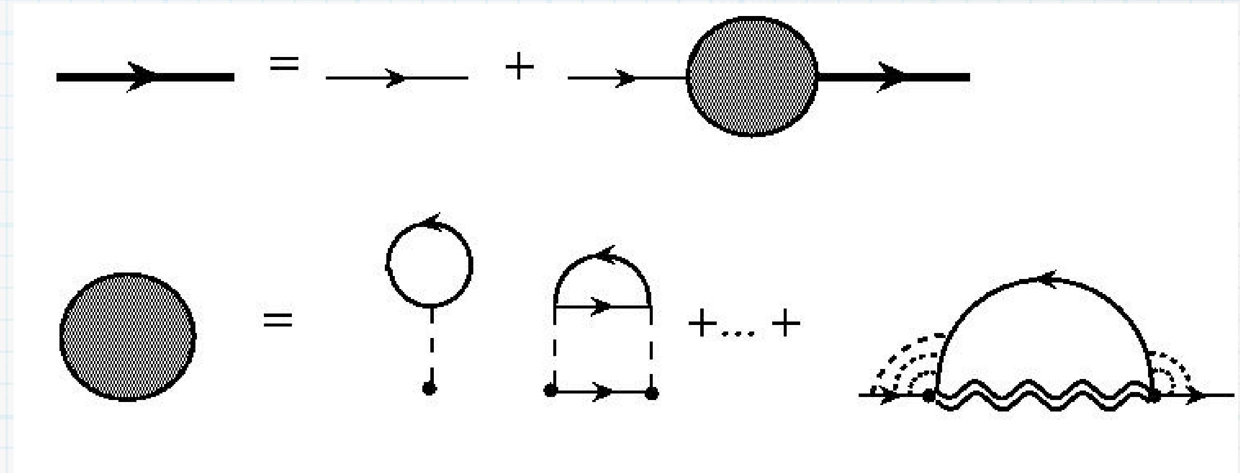


Well defined QP peak at -0.6EF, m*≈1.

Agrees with MC

Highly polarized case: $N_\uparrow \gg N_\downarrow$

Minority atom propagator



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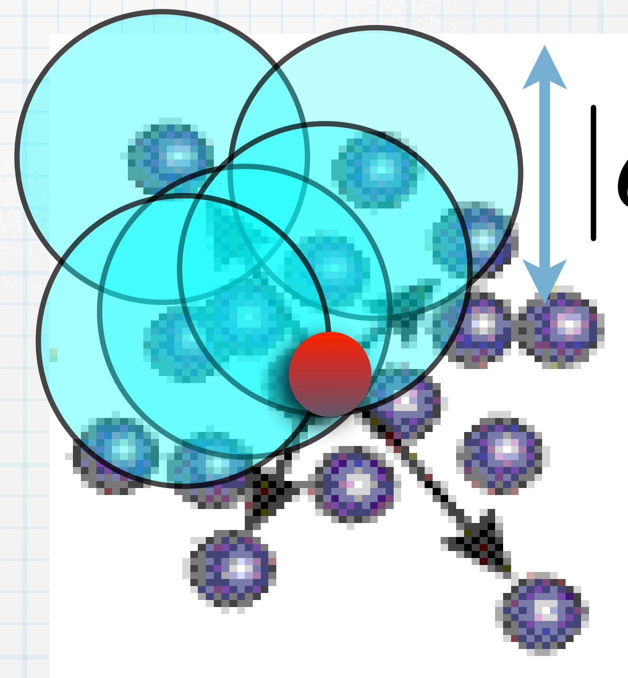

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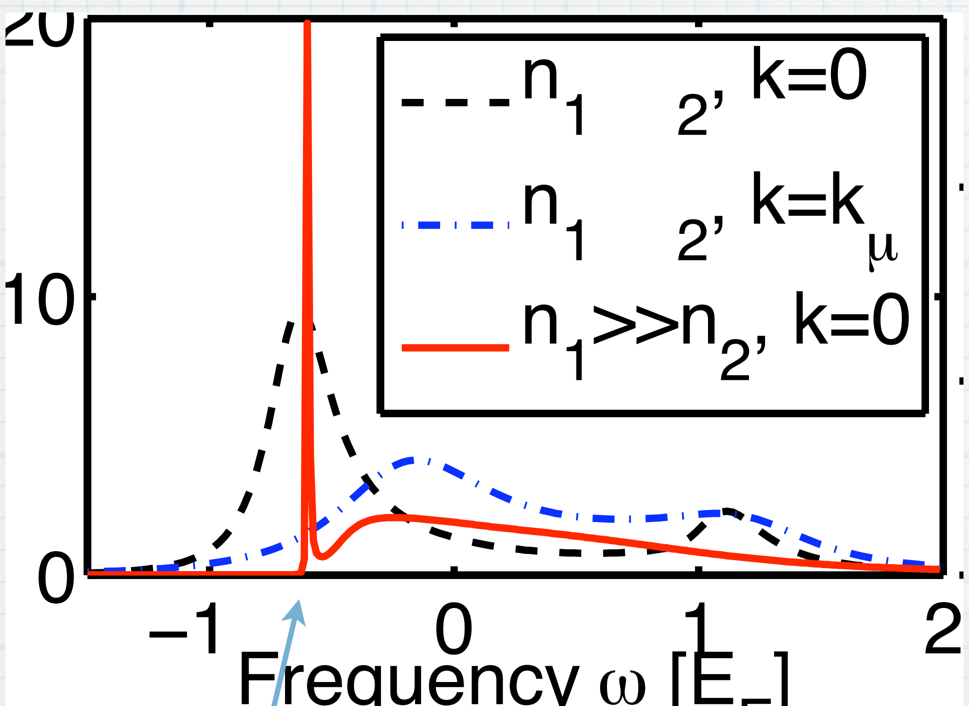

Universal

Well defined QP peak at -0.6EF, m*≈1.

Agrees with MC

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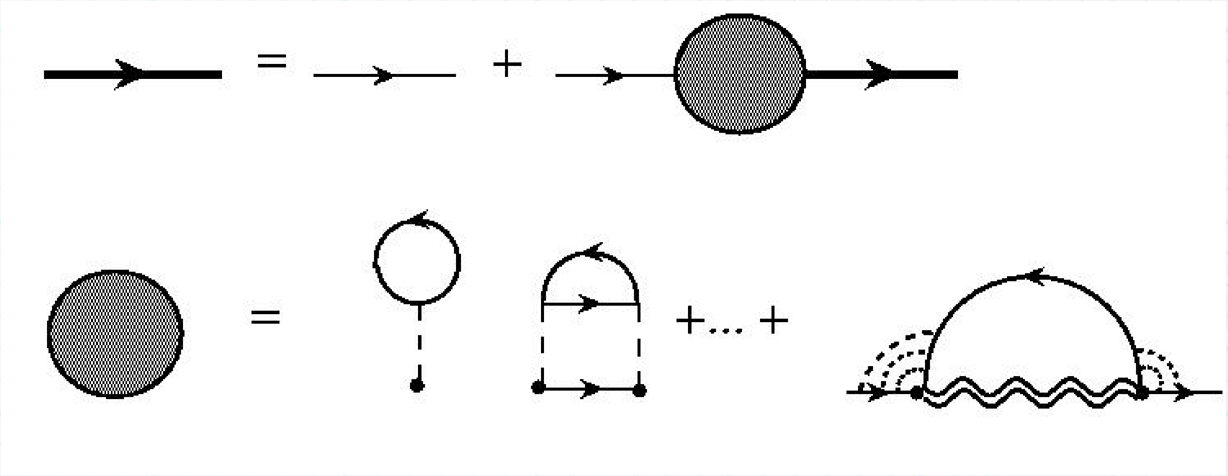

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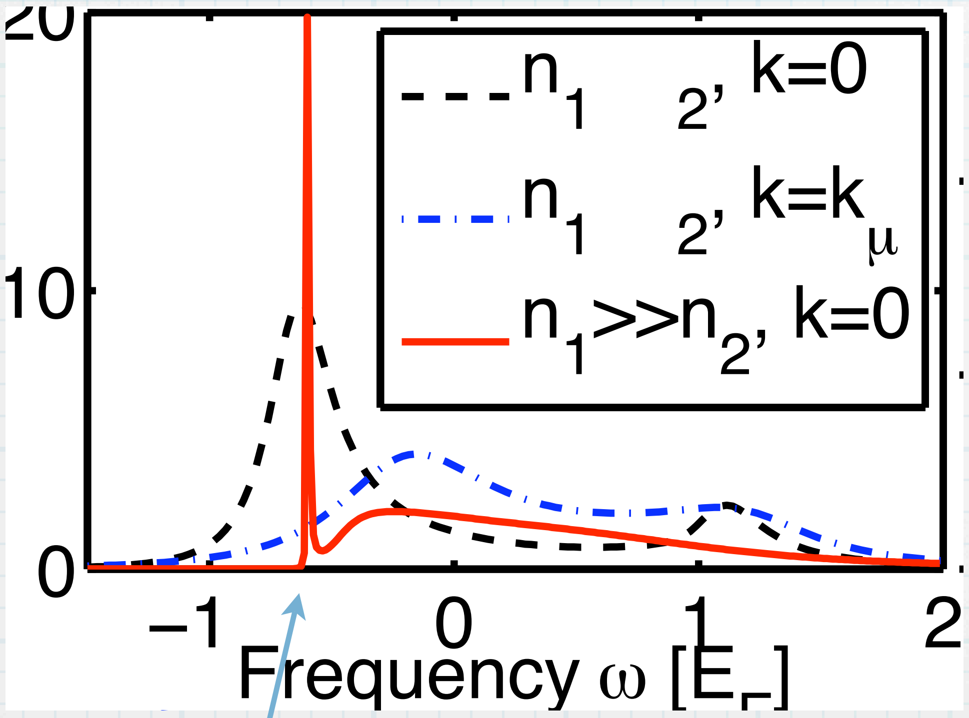
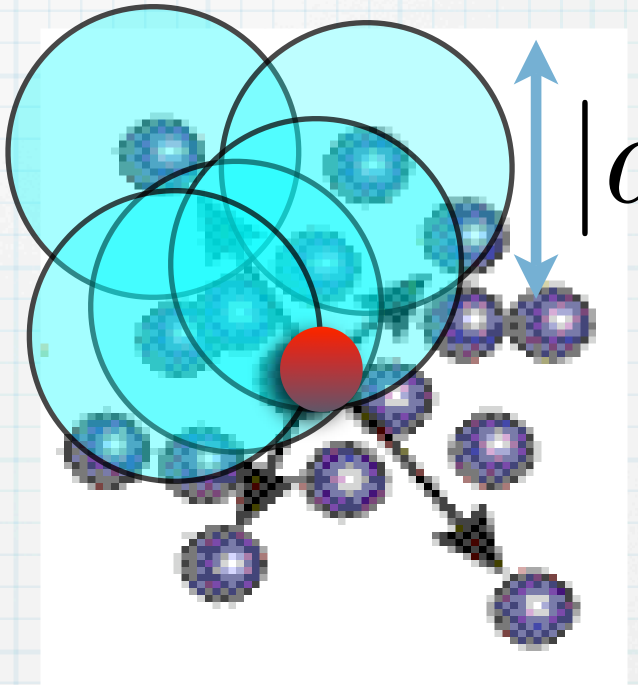
$$A(k,z) = -2\mathrm{Im}G(k,z)$$

Unitarity limit $k_F|a| \gg 1$





$|a| \gg r_e$

Universal

Well defined QP peak at -0.6EF, m*≈1.
Agrees with MC

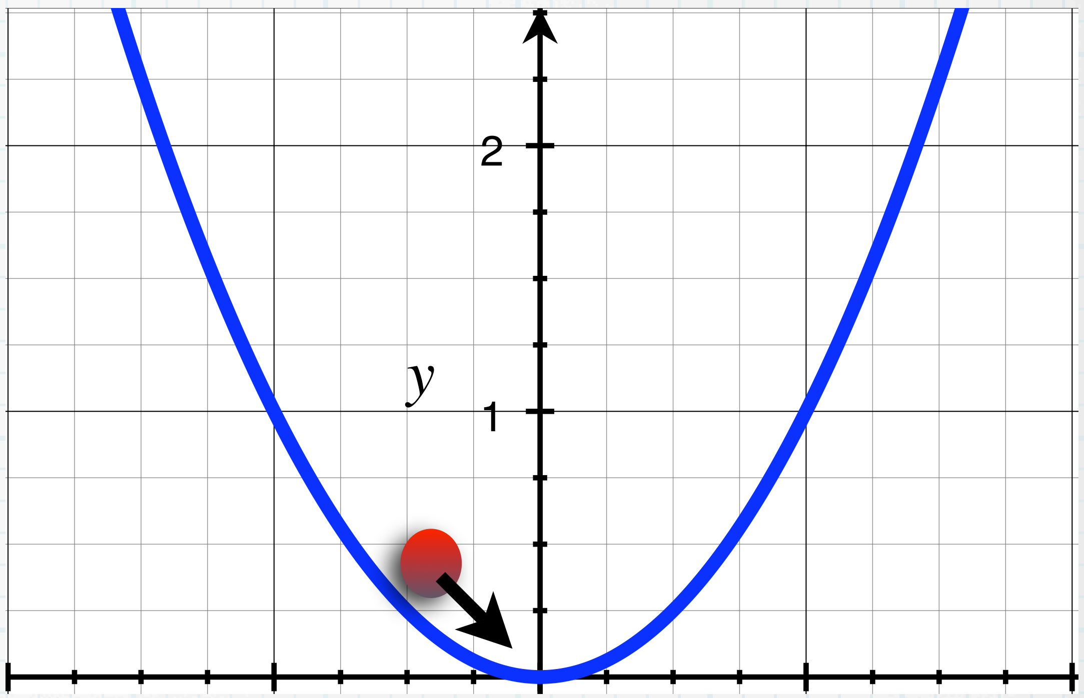
New highly polarized Fermi Liquid of Spin Polarons

Dipole oscillation of spin polaron

Potential seen by spin polaron

$\alpha \simeq 0.6$

$$V_{\downarrow}(r) - \alpha\epsilon_{F\uparrow}(r)$$



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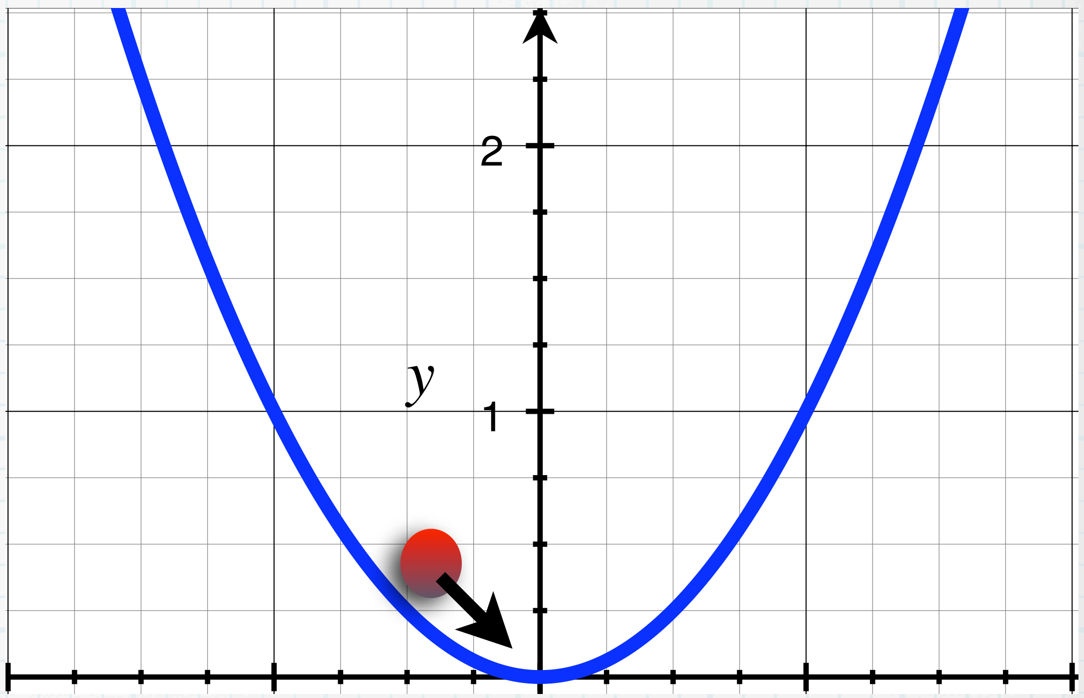

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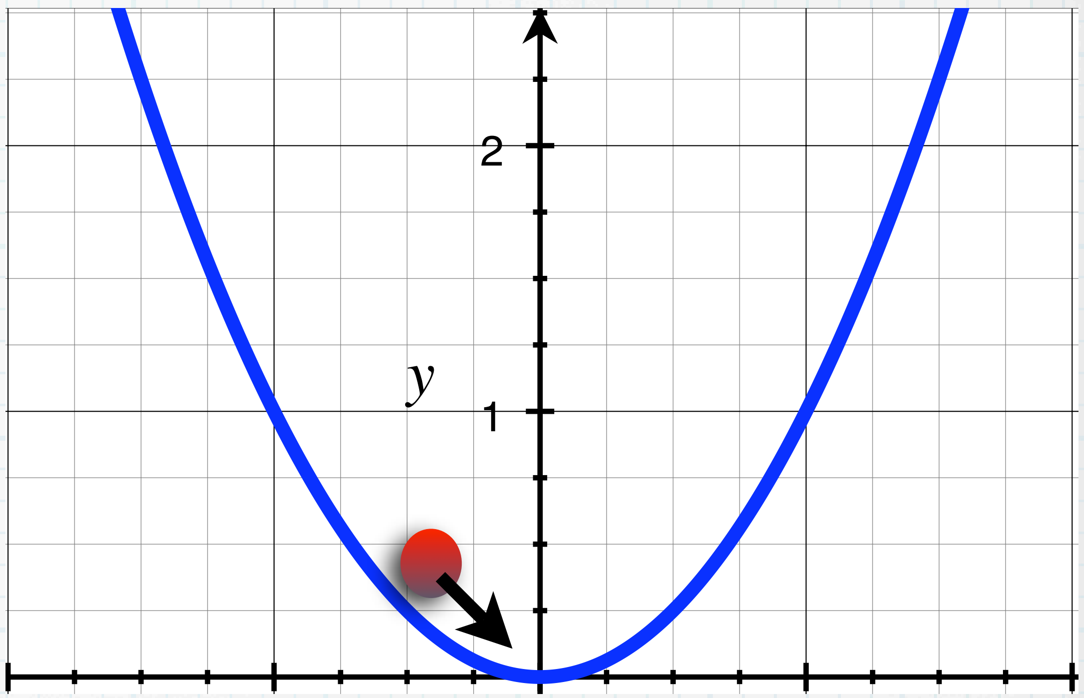

Frequency of dipole oscillation: $H = p^2/2m^*_\downarrow + V_\downarrow + \alpha V_\uparrow$

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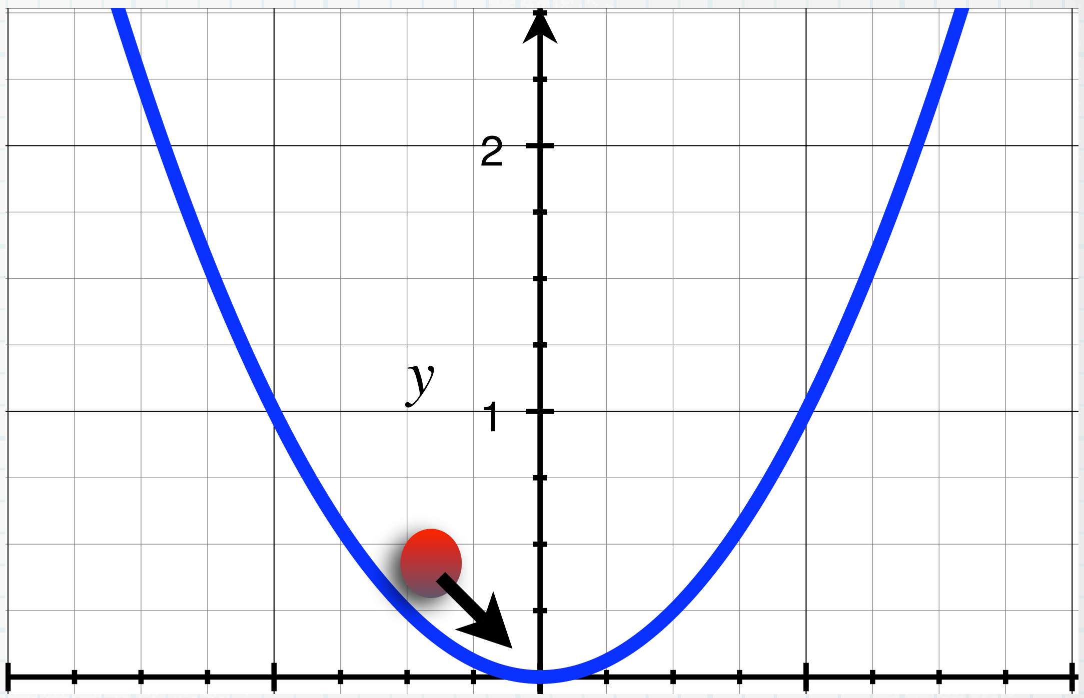

Frequency of dipole oscillation: $H = p^2/2m^*_\downarrow + V_\downarrow + \alpha V_\uparrow$

$$\omega_{\mathrm{D}} = \omega_\downarrow \sqrt{\frac{m_\downarrow}{m^*_\downarrow}\left(1 + \frac{m_\uparrow \omega_\uparrow^2}{m_\downarrow \omega_\downarrow^2}\alpha\right)}$$

Depends only on density-> Universal

Momentum relaxation time: $\frac{d\mathbf{P}_\downarrow}{dt} = -\frac{\mathbf{P}_\downarrow}{\tau_P}$

$$\frac{d\mathbf{P}_\downarrow}{dt} = -2\pi \frac{|U|^2}{V^3} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{p} \left[n_{\mathbf{p}\downarrow} n_{\mathbf{p}'\uparrow} (1 - n_{\mathbf{p}-\mathbf{q}\downarrow})(1 - n_{\mathbf{p}'+\mathbf{q}\uparrow}) - n_{\mathbf{p}-\mathbf{q}\downarrow} n_{\mathbf{p}'+\mathbf{q}\uparrow} (1 - n_{\mathbf{p}\downarrow})(1 - n_{\mathbf{p}'\uparrow}) \right]$$
$$\times \delta(\epsilon_{\mathbf{p}\downarrow} + \epsilon_{\mathbf{p}'\uparrow} - \epsilon_{\mathbf{p}-\mathbf{q}\downarrow} - \epsilon_{\mathbf{p}'+\mathbf{q}\uparrow})$$

PRL 100, 240406 (2008)

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Effective interaction between spin-polaron and majority atoms

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Landau Theory:

$$U = \frac{\partial^2 E}{\partial n_\uparrow \partial n_\downarrow} = \frac{\partial \mu_\downarrow}{\partial n_\uparrow}$$
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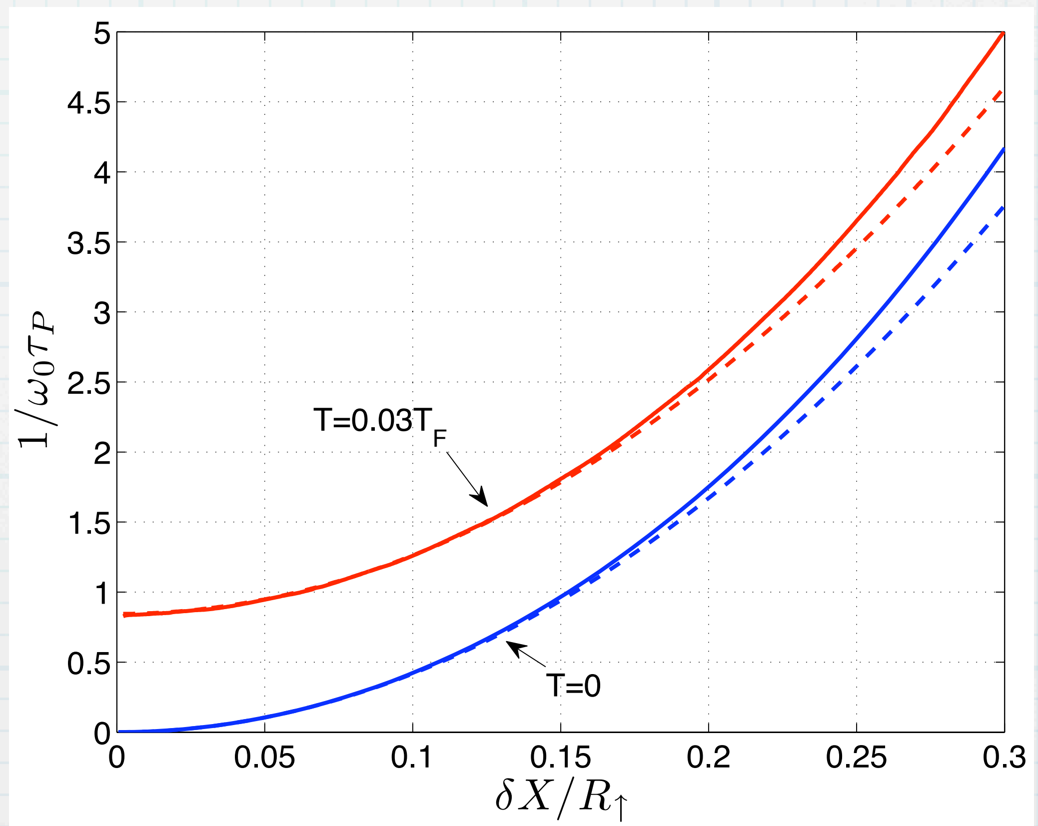
Damping of spin dipole mode:



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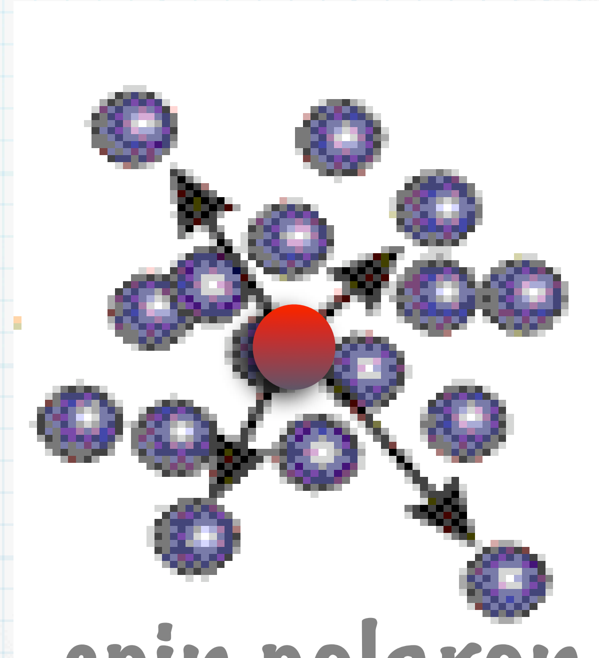
Cross-over from polaron to molecule

At unitarity: $k_F|a| \gg 1$



spin polaron

$$m^* \simeq 1$$

$$\epsilon \simeq -0.6\epsilon_{F\uparrow}$$

Cross-over from polaron to molecule

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BEC regime: $k_F a \ll 1$

$$m^* \simeq 2m$$

$$\epsilon \simeq -\frac{\hbar^2}{ma^2}$$

Conclusions

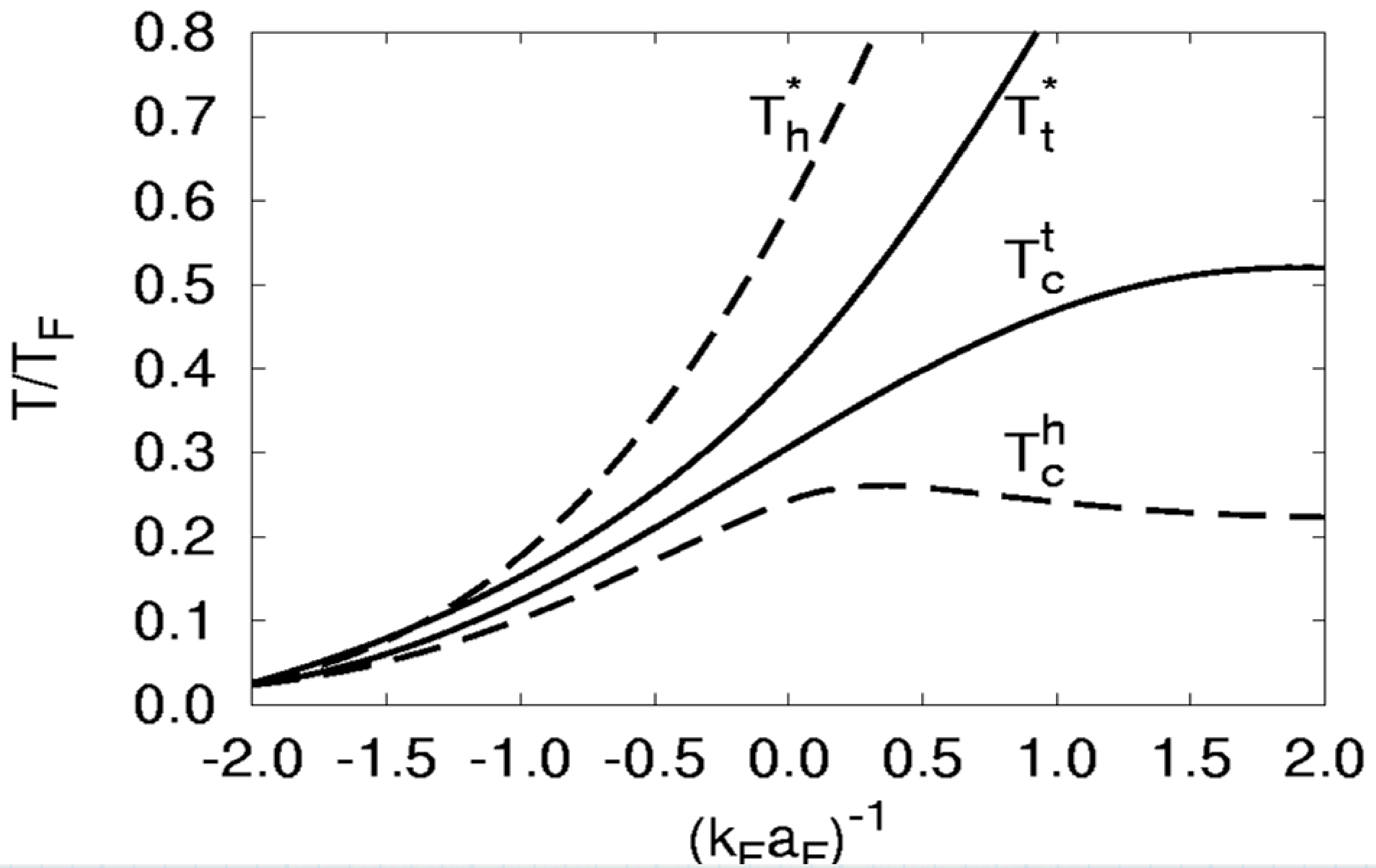
* Feshbach resonances
* Developed Landau theory for many-body scattering
* Medium effects for Feshbach molecule-> Collective modes
* Imbalanced Fermi gases -> Spin polarons
* Visible for low T in collective modes

BEC-BCS cross-over

Interacting Fermi gas: **Pairing** for $T<T_c$

BEC-BCS cross-over

Interacting Fermi gas: Pairing for $T<T_c$



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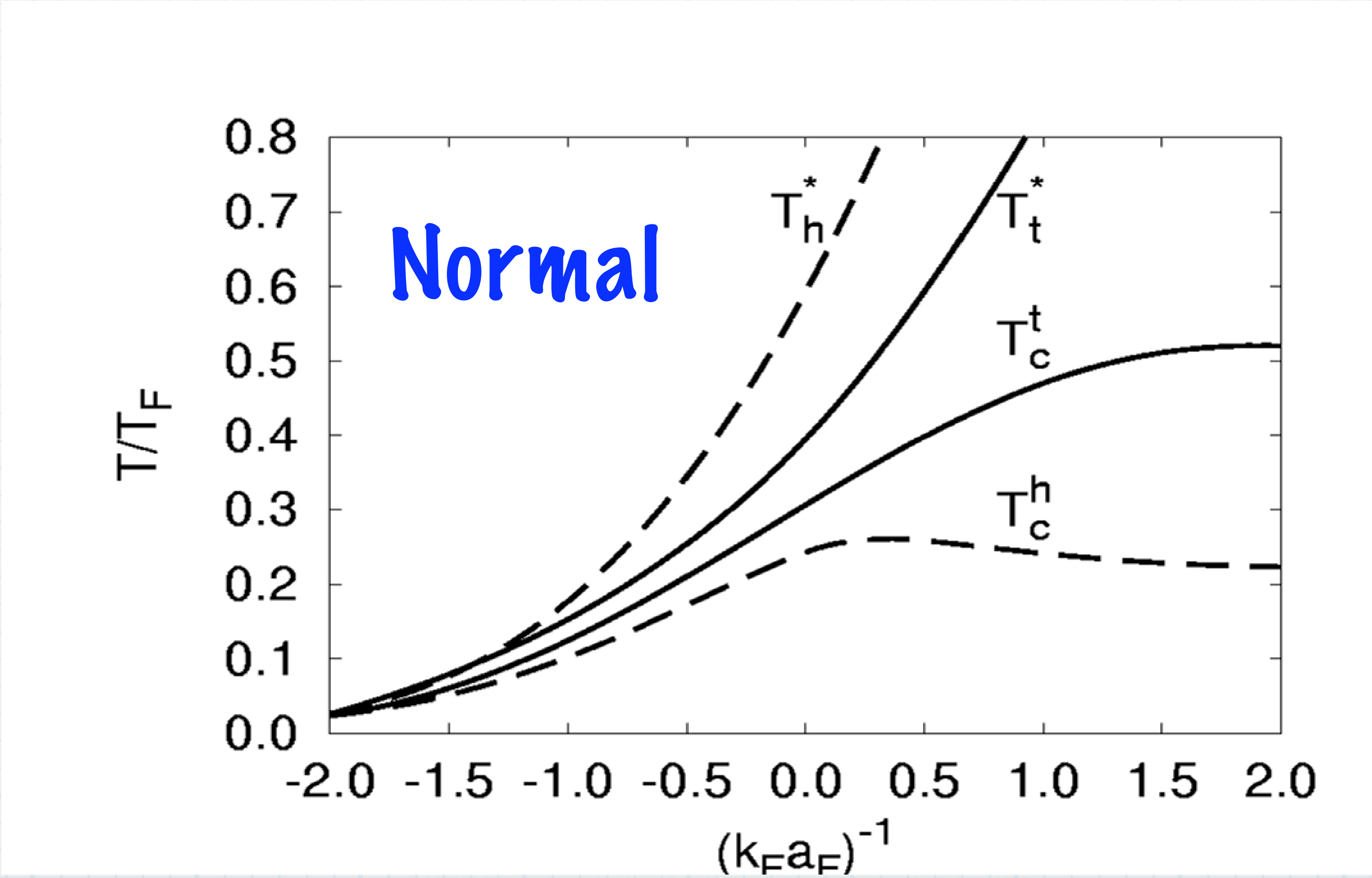

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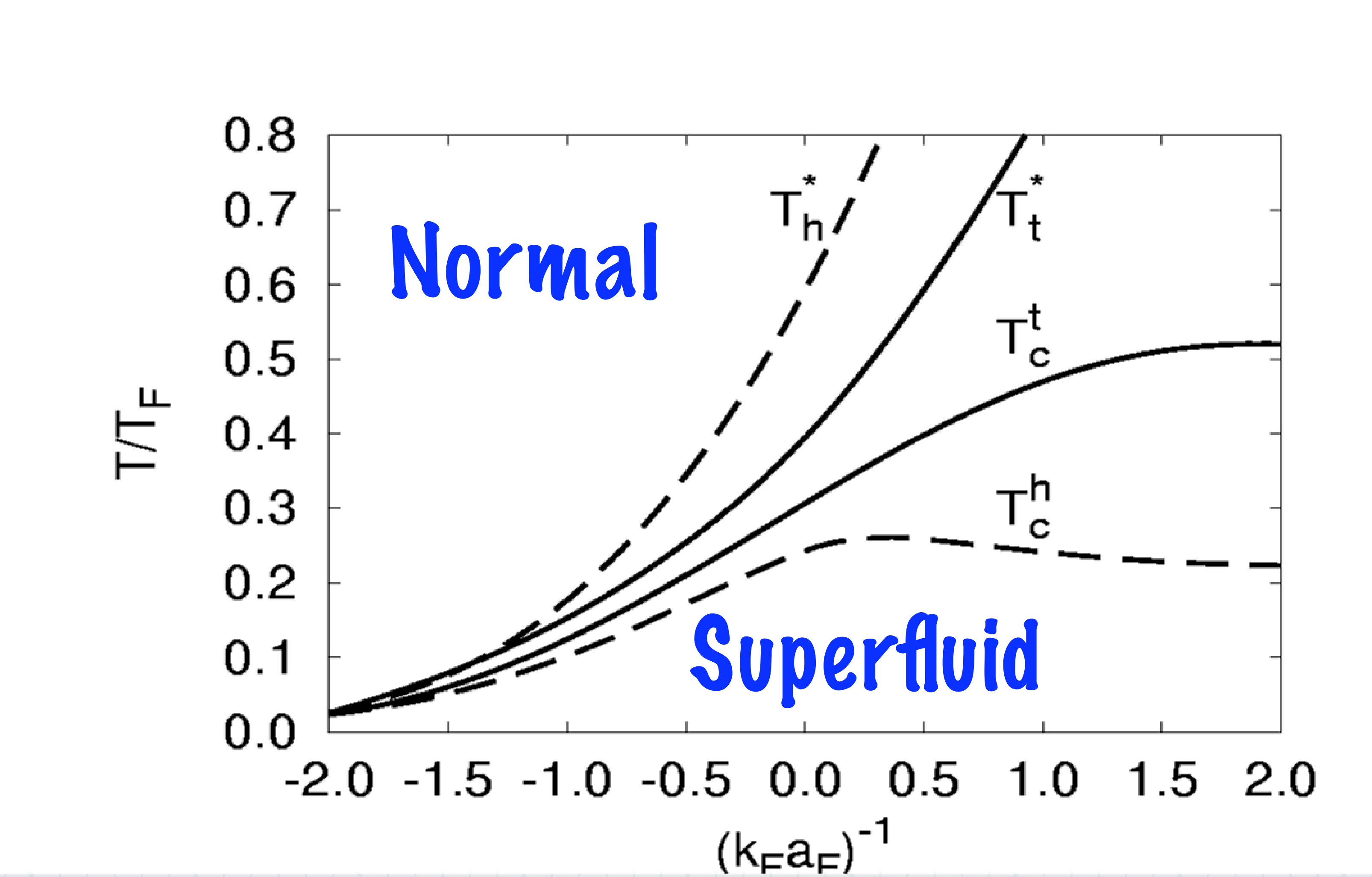

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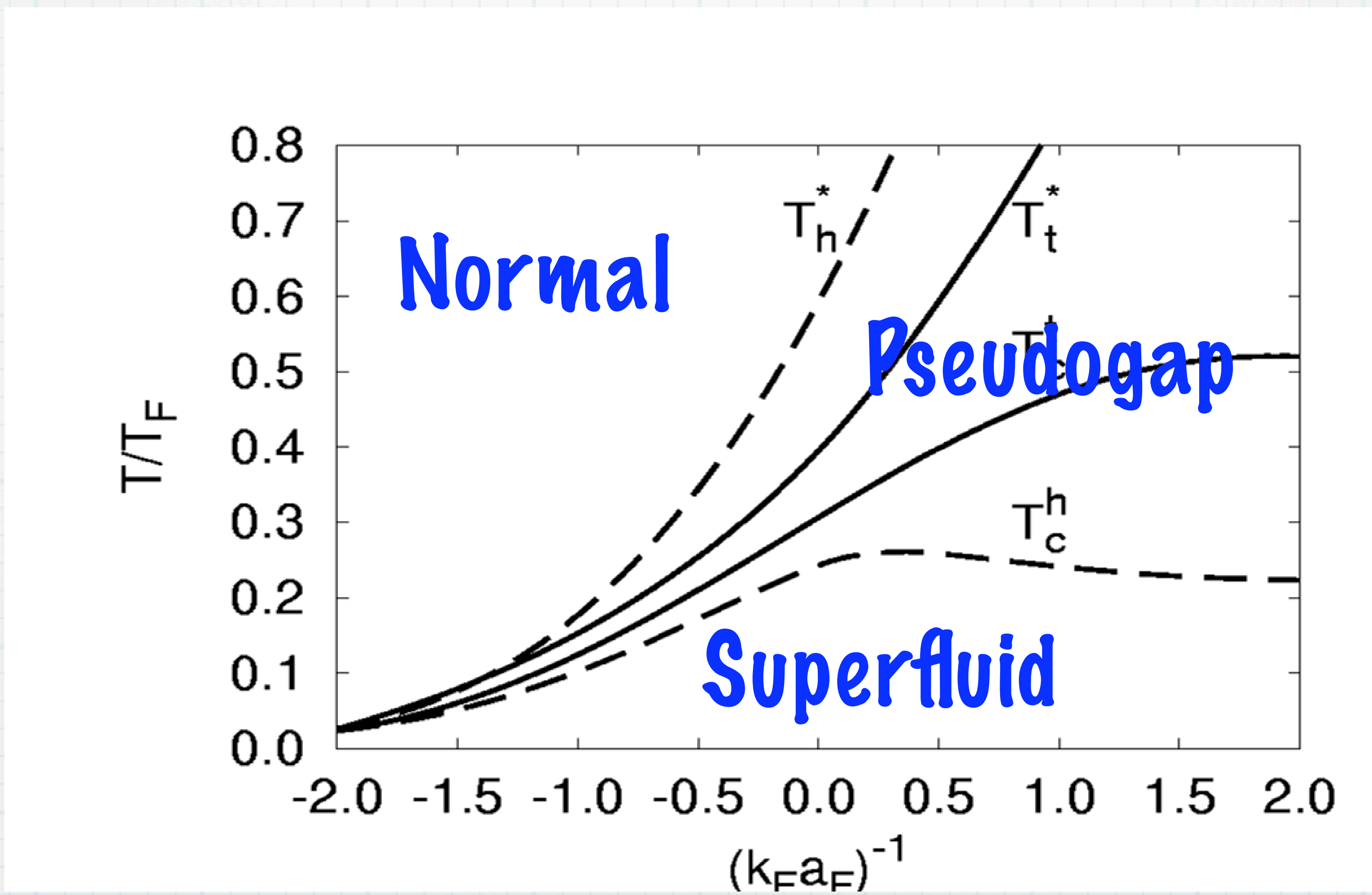

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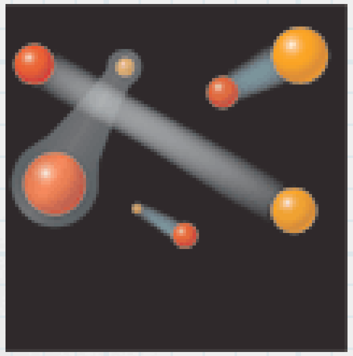

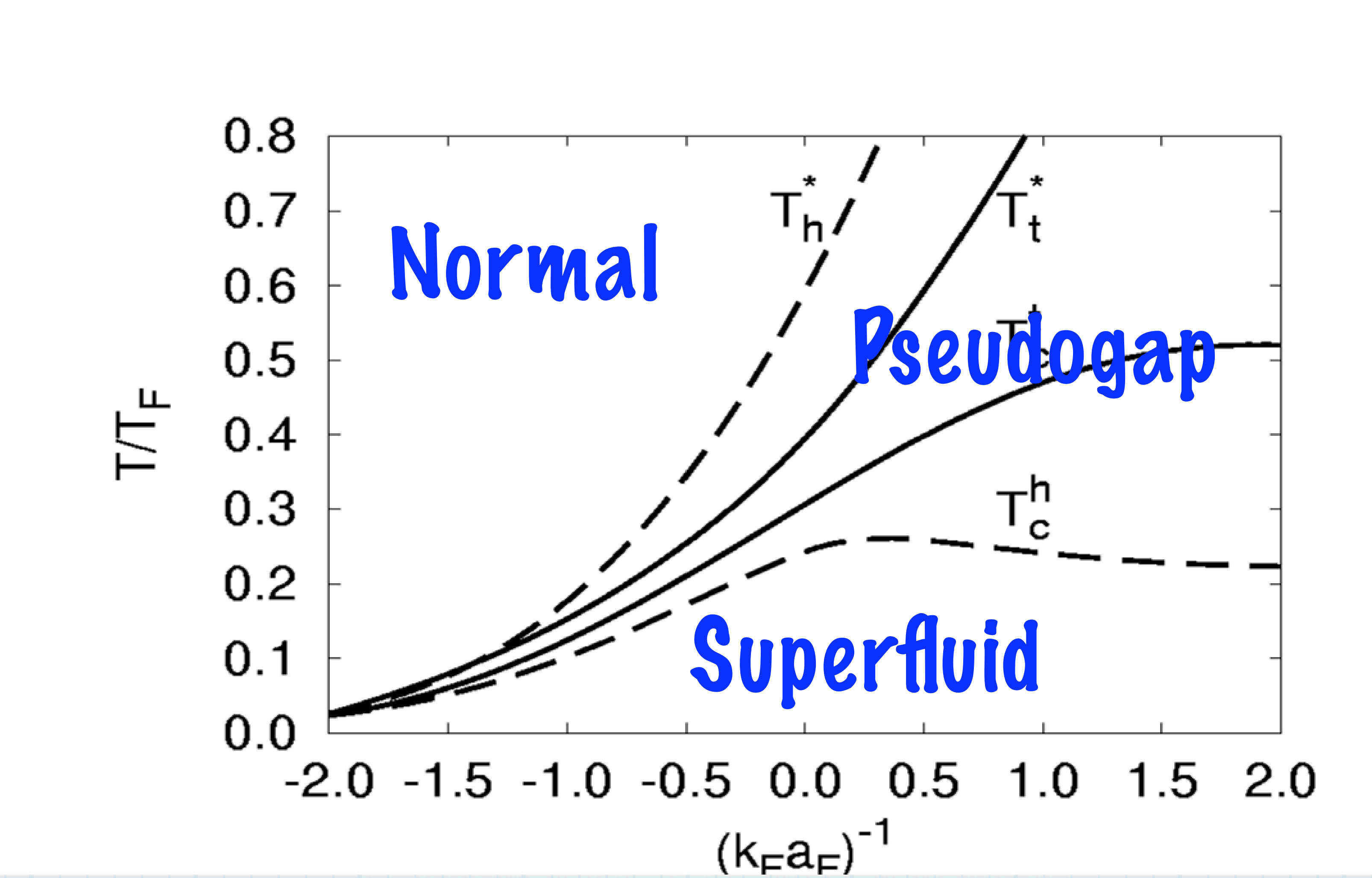

$$k_B T_c \propto \epsilon_F e^{\pi/2k_F a}$$

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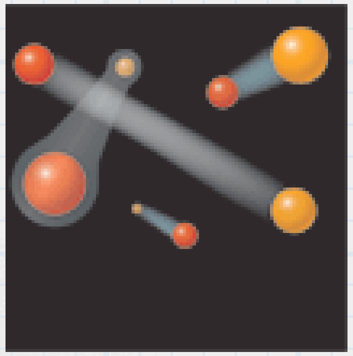
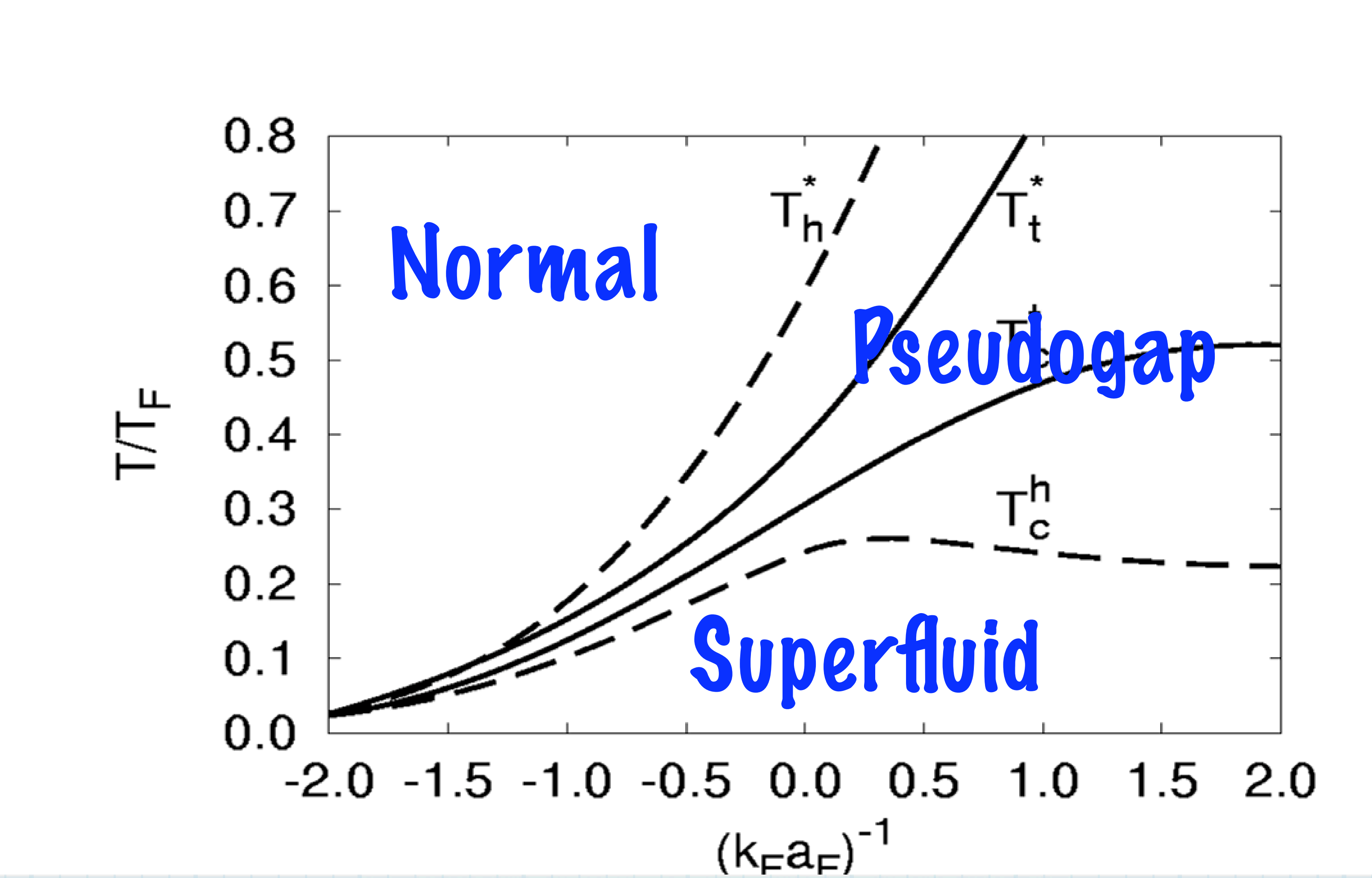
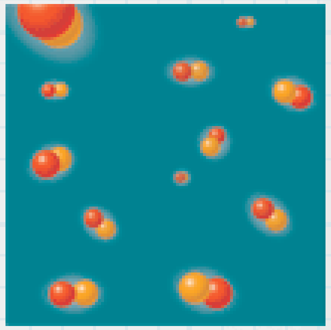




BEC



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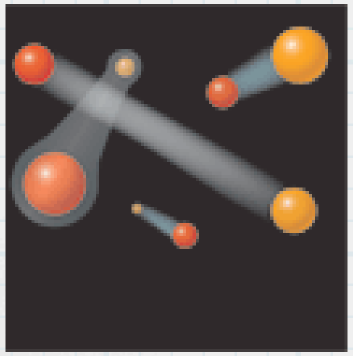

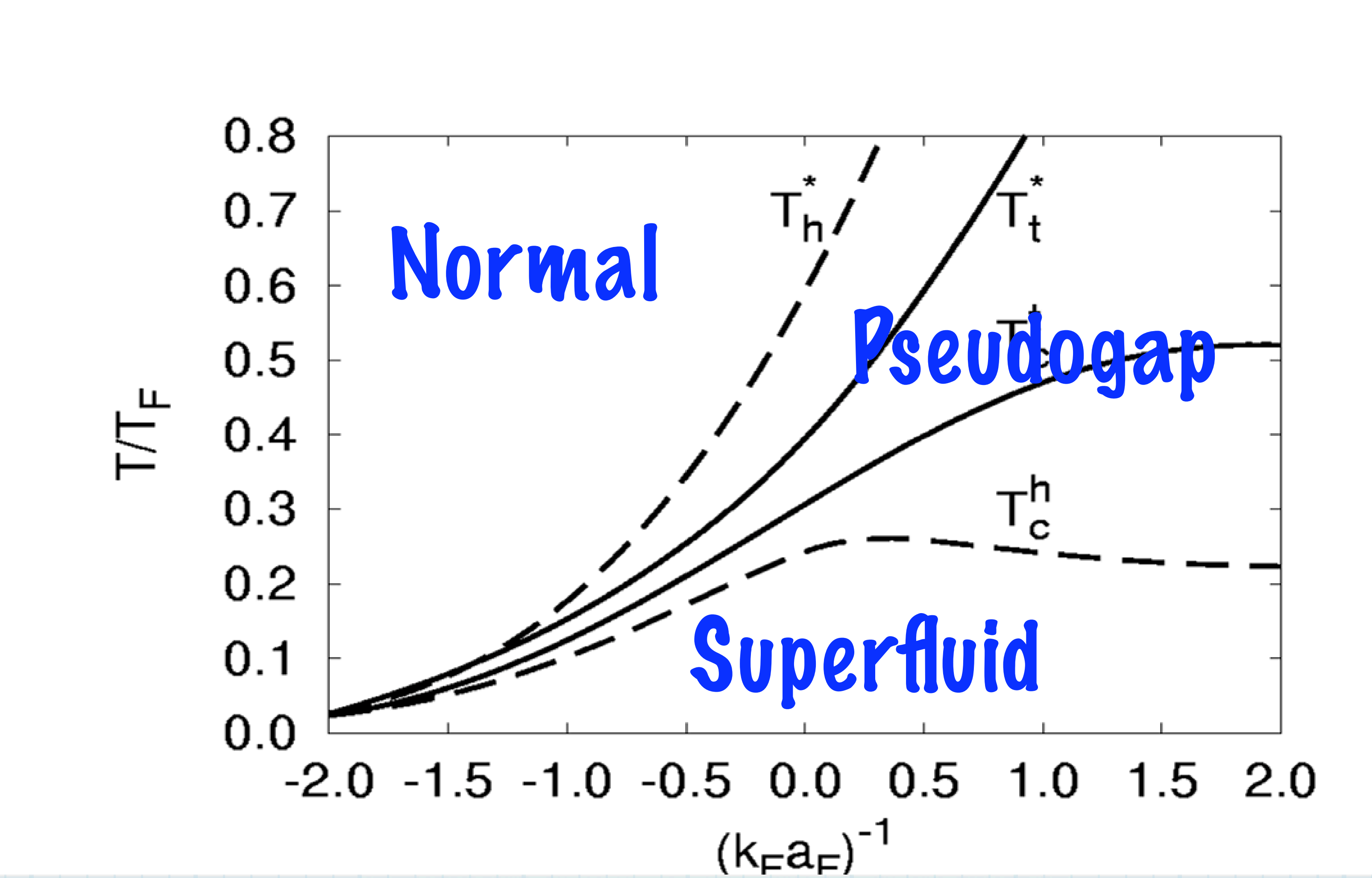

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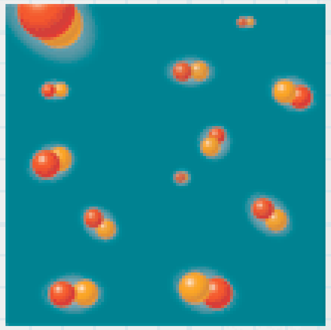

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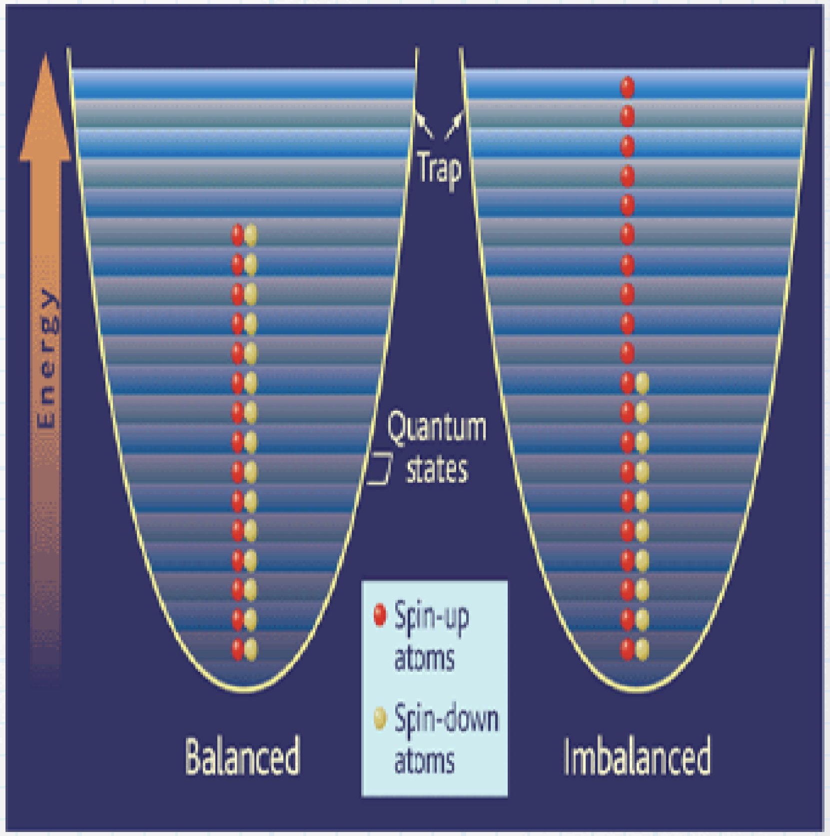
Universal

$k_B T_c \propto \epsilon_F$

Pairing with different Fermi energies:



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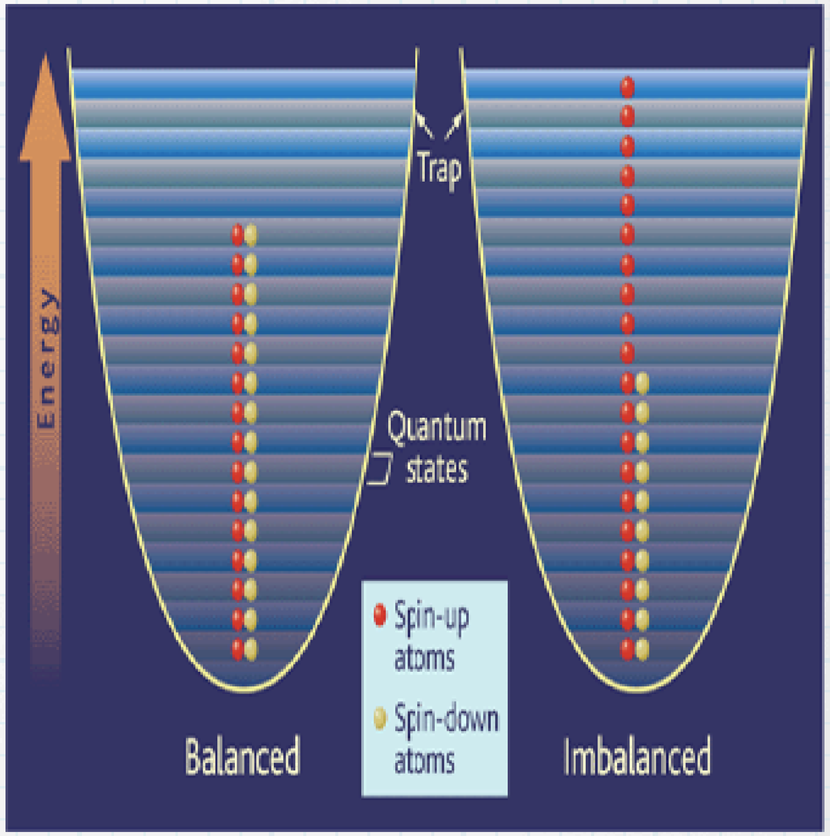

Clogston limit: $\Delta < \epsilon_{F\uparrow} - \epsilon_{F\downarrow}$

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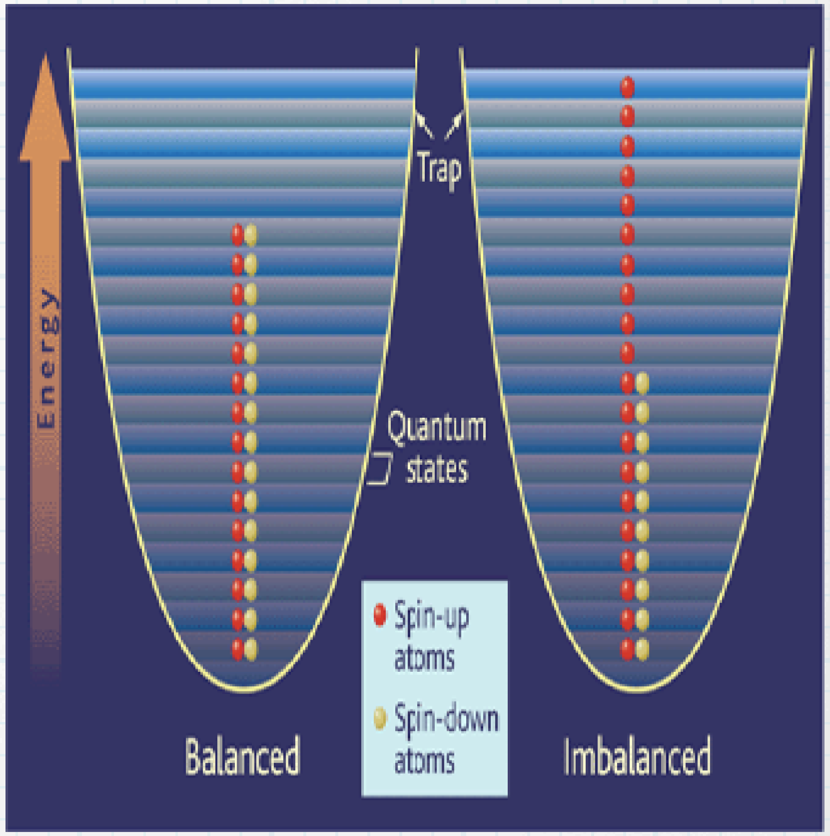
Many predictions:
FFLO, deformed Fermi surface,
phase separation, breached pair state..



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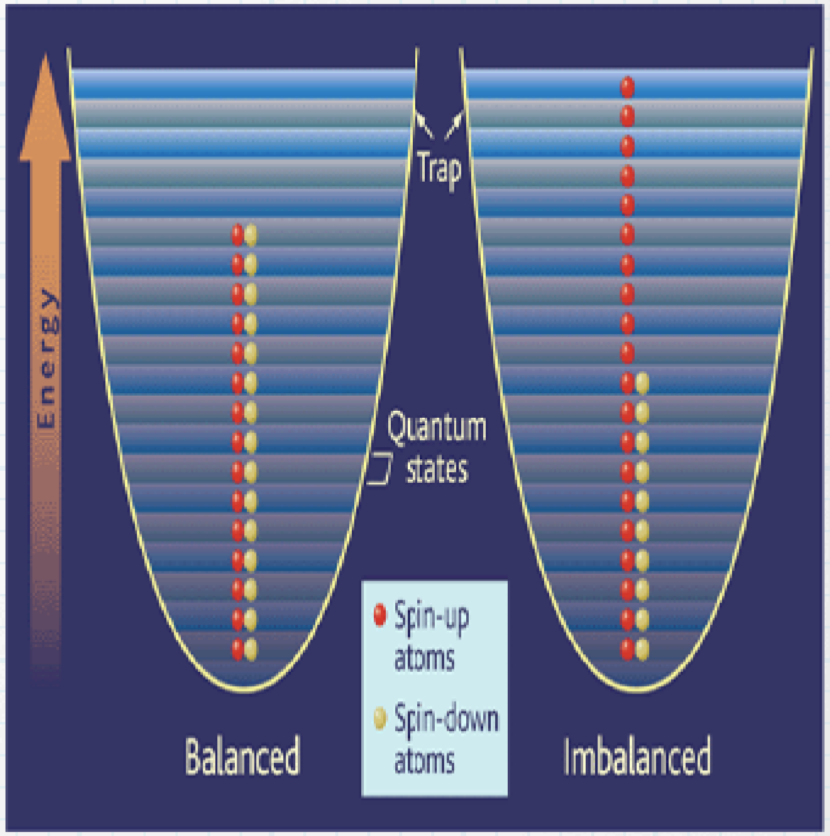

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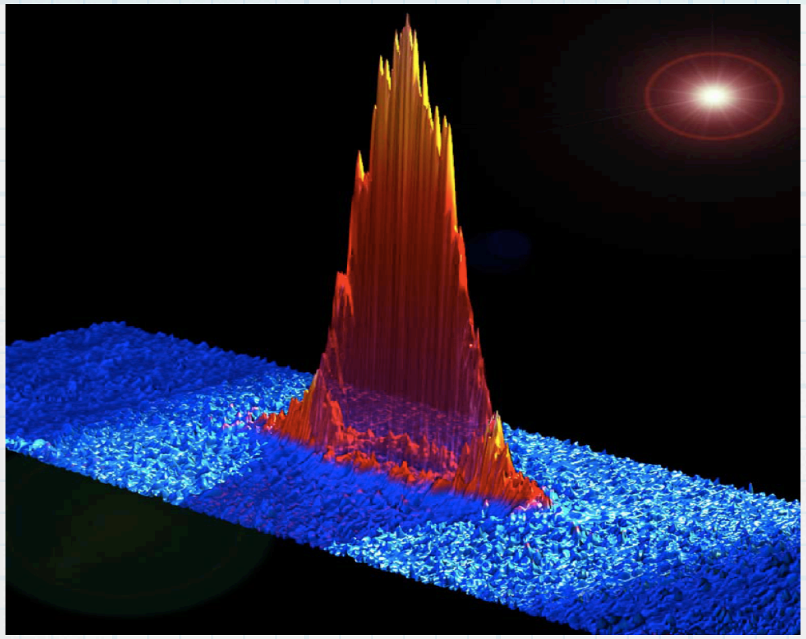

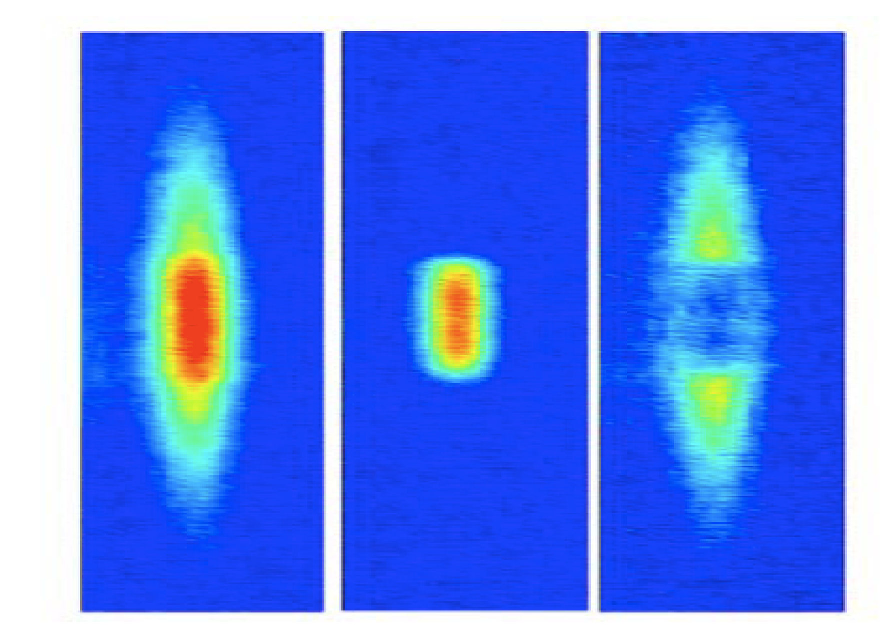

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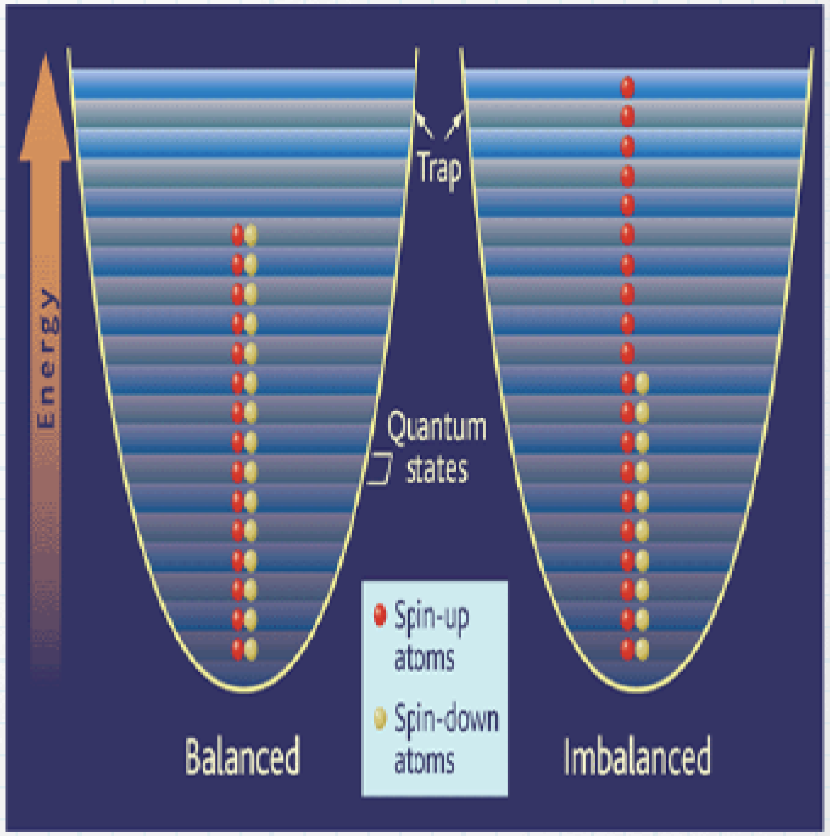

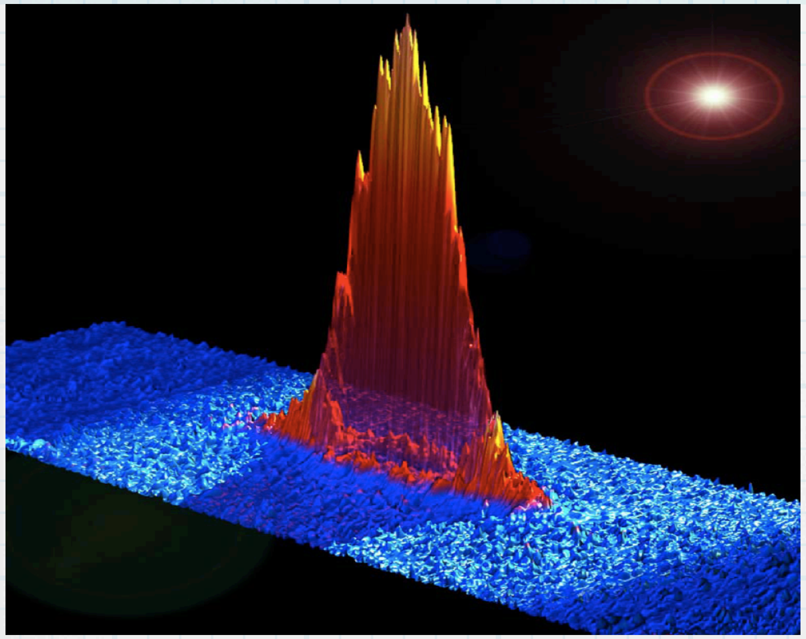

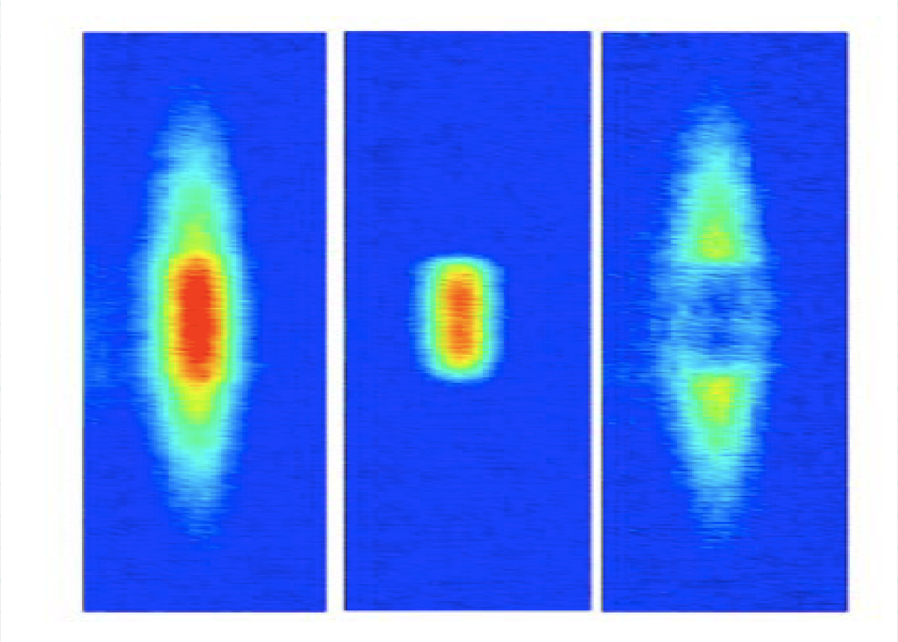

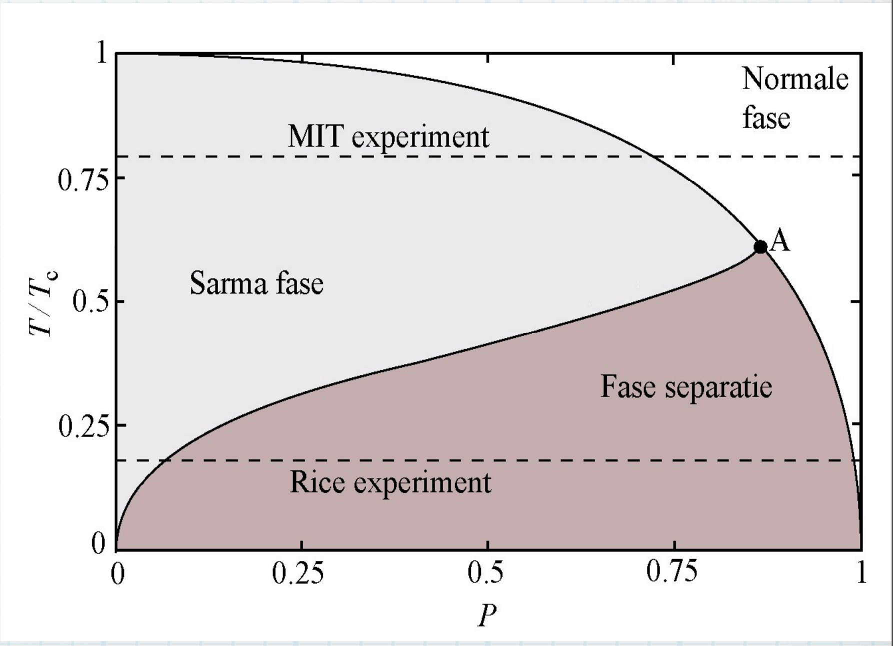

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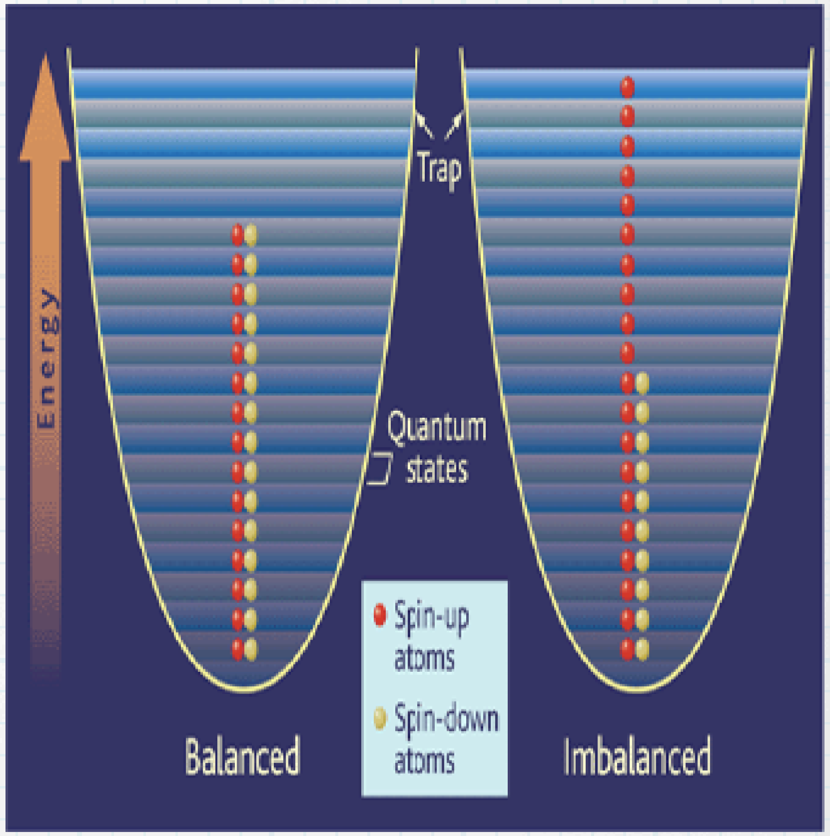

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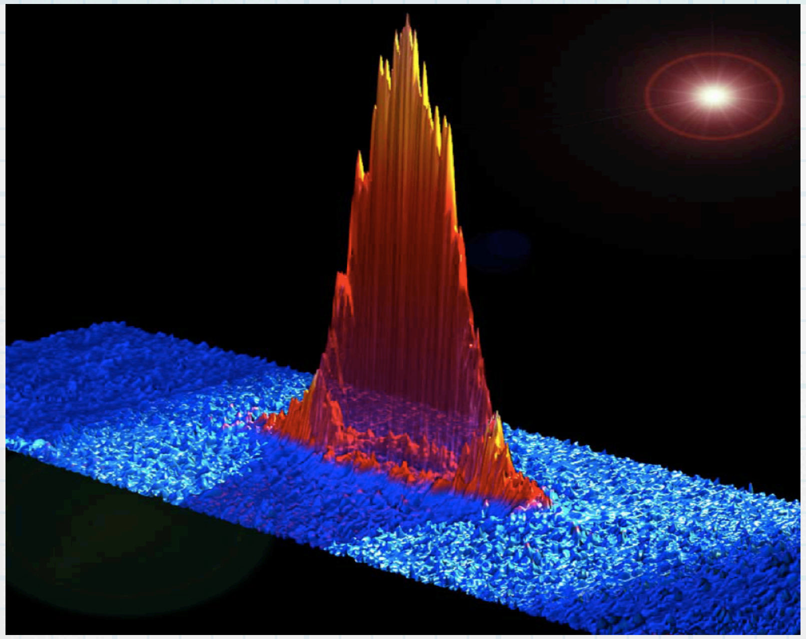

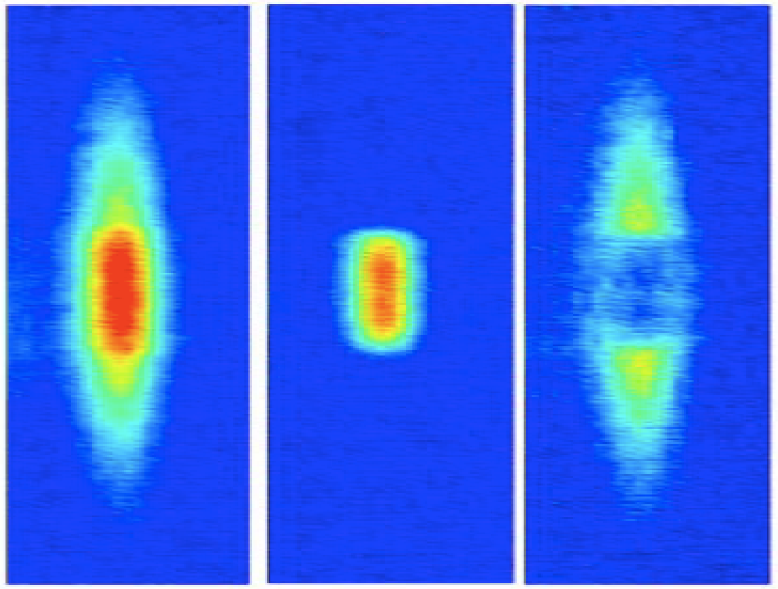

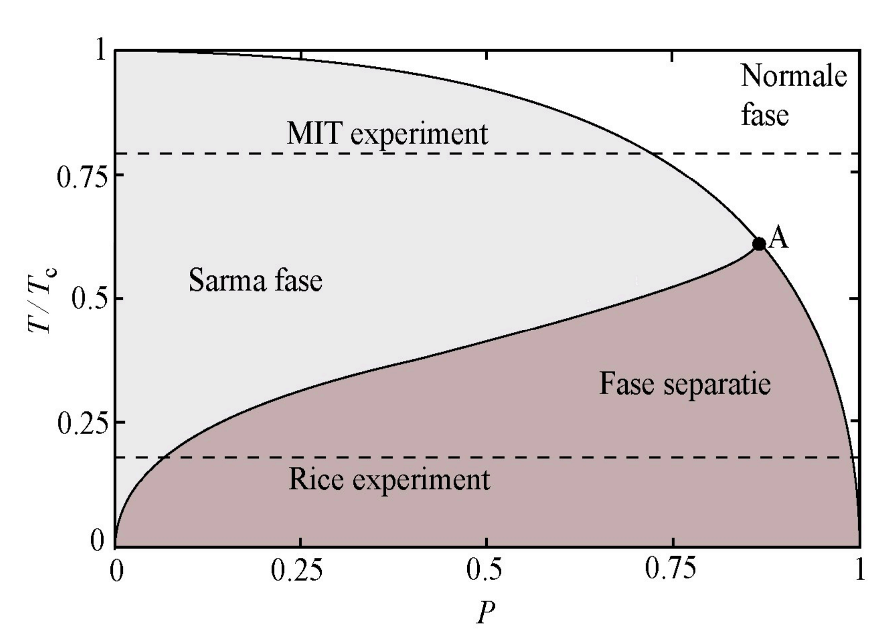

Pairing with different masses? (quarks..)