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Poincaré Invariant Three-Body Scattering

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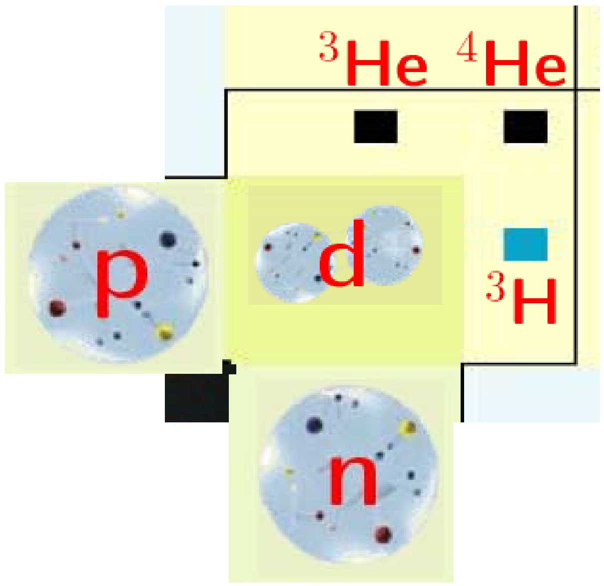
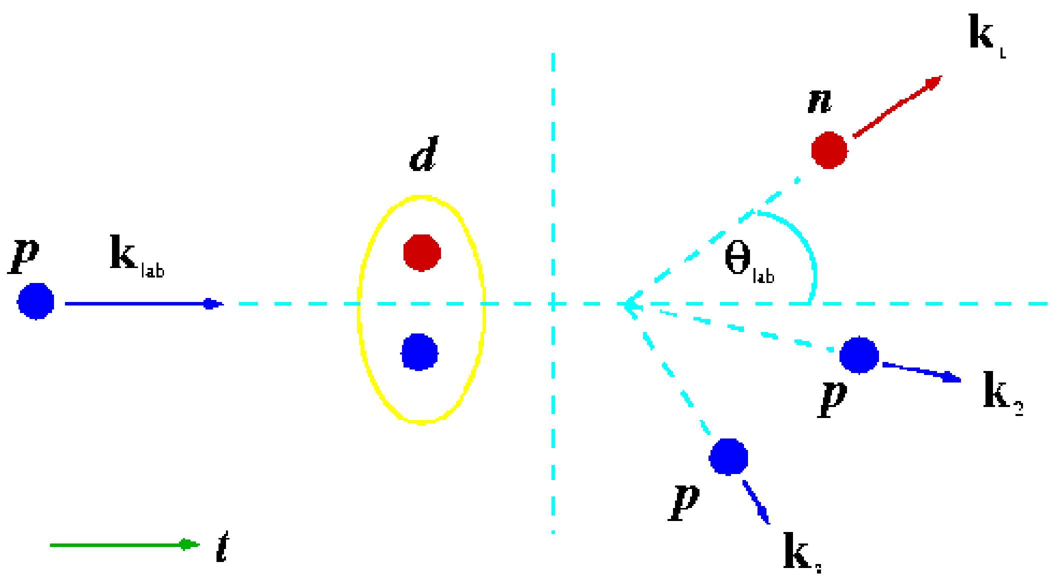
Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

A Few-Body Theorist's view of the Nuclear Chart



3 Nucleons

- Bound State: ^3H - ^3He
- Scattering: **Elastic – Inelastic (Breakup)**
- Energy Scale: keV $\rightarrow$ MeV $\rightarrow$ GeV



Challenges in 3N Physics

- Test of nuclear forces in the simplest nuclear environment (over a large energy range!)
  - Two-body forces
  - Genuine three-body forces

- **Reaction mechanisms**
  - Example: deuteron breakup, (p,n) charge exchange ….
  - Higher Energy: Lorentz vs. Galilean Invariance
  - Check of commonly used approximations (Glauber)
  - Three-body decays of particles (e.g. η or η')

Relativistic Effects at Higher Energies

Computational Challenge:

3N and 4N systems:

- standard treatment based on pw projected momentum space successful (3N scattering up to ≈250 MeV) but rather tedious
- 2N: j_{max}=5, 3N: J_{max}=25/2 → 200 `channels'
- Computational maximum today:
- 2N: j_{max}=7, 3N: J_{max}=31/2

⇒ **Solution:**

⇒ **NO partial wave decomposition of basis states**

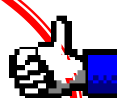
Roadmap for 3N problem without PW

Scalar NN model | Realistic NN Model

Scalar NN model

- NN scattering + bound state
- 3N bound state
- 3N bound state + 3NF
- 3N scattering:
- Full Faddeev Calculation
  - Elastic scattering
  - Below and above break-up
  - Break-up

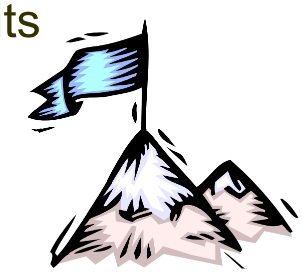
- **Poincaré Invariant Faddeev Calculations**



Realistic NN Model

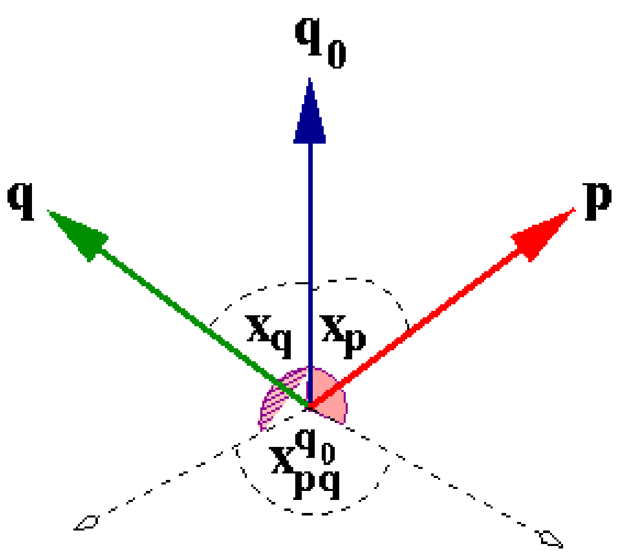
- NN scattering + deuteron
  - Potentials AV18 and Bonn-B
- Break-up in first order:
  - (p,n) charge exchange
  - Max. Energy 500 MeV
  - Lorentz kinematics

- Exact Faddeev Calculation
  - NN interactions
  - High energy limits



Variables for 3D Scattering Calculation

3 distinct vectors in the problem: $\mathbf{q}_0$ $\mathbf{q}$ $\mathbf{p}$



5 independent variables:

$$p = |\mathrm{p}| \;,\; q = |\mathrm{q}|$$

$$x_p = \hat{\mathrm{p}} \cdot \hat{\mathrm{q}}_0 \;,\; x_q = \hat{\mathrm{q}} \cdot \hat{\mathrm{q}}_0$$

$$x_{pq}^{q_0} = (\mathrm{q}_0 \times \mathrm{q}) \cdot (\mathrm{q}_0 \times \mathrm{p})$$

q system : **z || q**

q_0 system : **z || $\mathbf{q}_0$**

Numerical calculation:

Three-body transition amplitude is a function of 5 variables

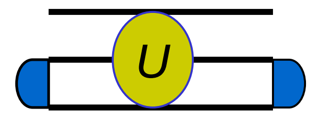
Relativistic Three-Body Problem

- Context: Poincarė Invariant Quantum Mechanics
  - Poincarė invariance is exact symmetry, realized by a unitary representation of the Poincarė group on a few-particle Hilbert space
  - Instant form
  - Faddeev equations same operator form but different ingredients
- **Kinematics**
  - Lorentz transformations between frames
- **Dynamics**
  - Bakamjian-Thomas Scheme: Mass Operator $M=M_0+V$ replaces Hamiltonian $H=H_0+\mathbf{v}$
  - Connect Galilean two-body **v** with Poincarė two-body v
  - Construct V := $\sqrt{M^2+q^2}-\sqrt{M_0^2+q^2}$

Three-Body Scattering

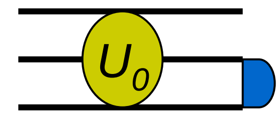
- Transition operator for elastic scattering

  $U = PG_0^{-1} + PT$



- Transition operator for breakup scattering

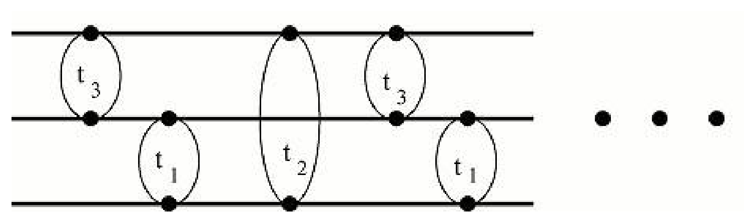
  $U_0 = (1 + P)T$



- Faddeev equation (Multiple Scattering Series)

$$T = tP + tG_0 PtP + \cdots$$

1st Order in tP



$t = v + vg_0 t$ =: NN t-matrix

$P = P_{12} P_{23} + P_{13} P_{23} \equiv$ Permutation Operator

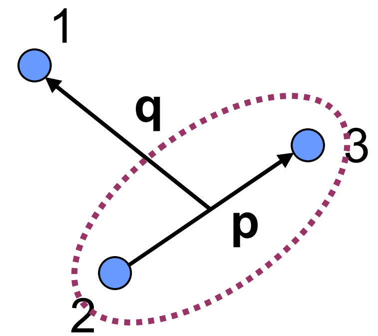
Kinematic Relativistic Ingredients:

- Lorentz transformation Lab $\rightarrow$ c.m. frame (3-body)
- Phase space factors in cross sections
- Poincarė-Jacobi momenta
- Permutations for identical particles

Kinematics: Poincaré-Jacobi momenta

- **Nonrelativistic (Galilei)**

$$\begin{aligned} \mathrm{p} &= \frac{1}{2}(\mathrm{k}_2 - \mathrm{k}_3) \\ \mathrm{q} &= \frac{2}{3}(\mathrm{k}_1 - \frac{1}{2}(\mathrm{k}_2 + \mathrm{k}_3)) \end{aligned}$$



- **Relativistic (Lorentz)**

$$\begin{aligned} \mathrm{p} &= \frac{1}{2}(\mathrm{k}_2 - \mathrm{k}_3) + \frac{\mathrm{k}_2 + \mathrm{k}_3}{2m_{23}} \left(\frac{(\mathrm{k}_2 - \mathrm{k}_3) \cdot (\mathrm{k}_2 + \mathrm{k}_3)}{(E_2 + E_3) + m_{23}} - (E_2 - E_3) \right) \\ \mathrm{q} &= \mathrm{k}_1 + \frac{\mathrm{K}}{M} \left(\frac{\mathrm{k}_1 \cdot \mathrm{K}}{E + M} - E_1 \right) \end{aligned}$$

$$\left| \mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_3 \right\rangle = \left| \frac{\partial(K\mathrm{pq})}{\partial(\mathrm{k}_2 \mathrm{k}_3)} \right|^{1/2} \left| \mathrm{Kpq} \right\rangle \neq 0$$

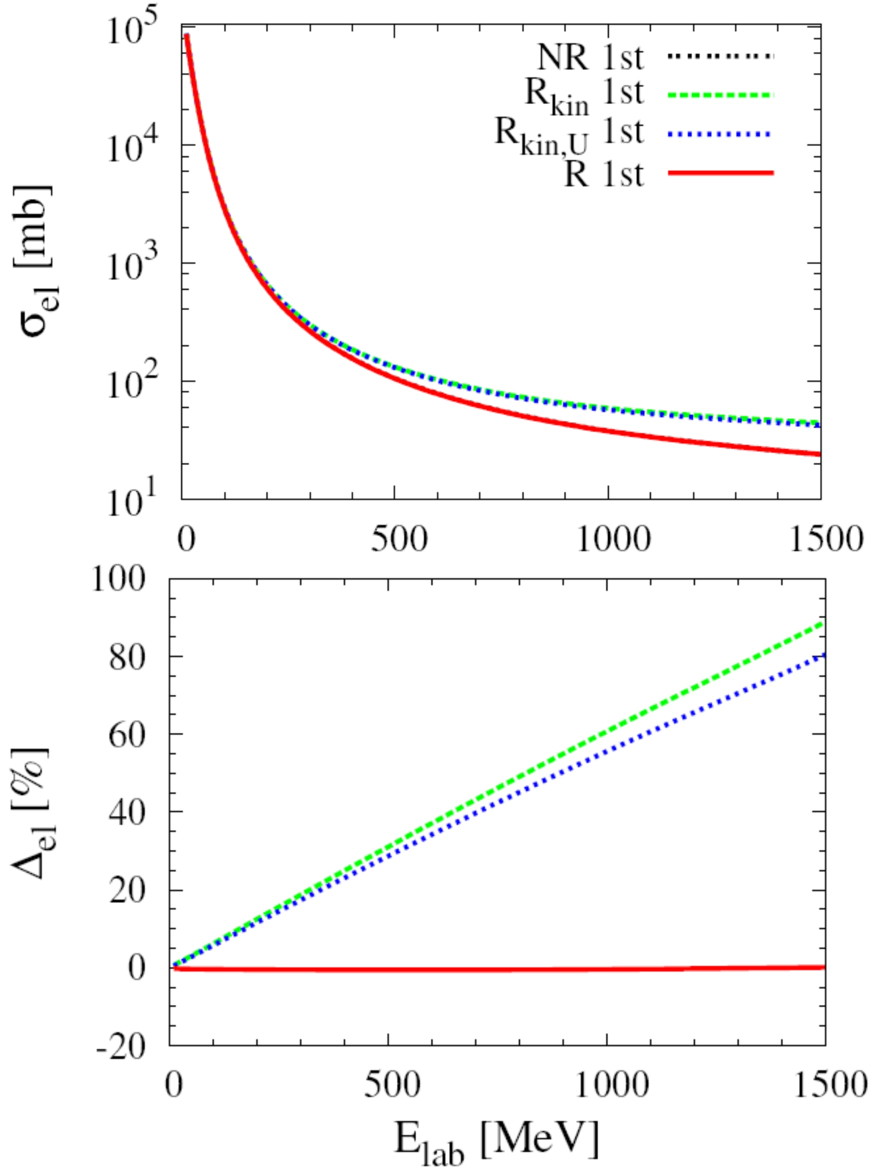
$$\begin{aligned} E &= E_1 + E_2 + E_3 \\ \mathbf{K} &= \mathrm{k}_1 + \mathrm{k}_2 + \mathrm{k}_3 \\ M &= \sqrt{E^2 - \mathrm{K}^2} \\ m_{23} &= \sqrt{(E_2 + E_3)^2 - (\mathrm{k}_2 + \mathrm{k}_3)^2} \end{aligned}$$

Relativistic kinematics

IA (1^{st} order)

$$T = tP$$

- Lorentz transformation Lab $\rightarrow$ c.m. frame) (3-body)
- Phase space factors in cross sections
- Poincaré-Jacobi momenta
- Permutations



Quantum Mechanics

Galilei Invariant: $$H = \frac{\mathrm{K}^2}{2M_g} + h \quad ; \quad h = h_0 + v_{12}^{NR} + v_{13}^{NR} + v_{23}^{NR}$$

Poincaré Invariant: $$H = \sqrt{\mathrm{K}^2 + M^2} \quad ; \quad M = M_0 + V_{12} + V_{23} + V_{31}$$

$$V_{ij} = M_{ij} - M_0 = \sqrt{\left(m_{0,ij} + v_{ij}\right)^2 + q_k^2} - \sqrt{m_{0,ij}^2 + q_k^2}$$

Two-body interaction embedded in the 3-particle Hilbert space

$$m_{0,ij} = \sqrt{m_i^2 + p_{ij}^2} + \sqrt{m_j^2 + p_{ij}^2}$$

$$M_0 = \sqrt{m_{0,ij}^2 + q_k^2} + \sqrt{m_k^2 + q_k^2}$$

V_{ij} embedded in the 3-particle Hilbert space

$$V_{ij} = M_{ij} - M_0 = \sqrt{\left(m_{0,ij} + v_{ij}\right)^2 + q_k^2} - \sqrt{m_{0,ij}^2 + q_k^2}$$

need matrix elements : $\langle \vec{k} | V(\vec{p}) | \vec{k}' \rangle$

$$= v(\vec{k},\vec{k}') + \psi_b(\vec{k})\left(\sqrt{M_b^2+p^2} - M_b\right)\psi_b(\vec{k}') + \frac{1}{\omega-\omega'}\Big[\left(\sqrt{\omega^2+p^2}-\omega\right)\Re[t(\vec{k}',\vec{k};\omega)]$$

$$-\left(\sqrt{\omega'^2+p^2}-\omega'\right)\Re[t(\vec{k},\vec{k}';\omega')]\Big] + \frac{1}{\omega-\omega'}\Big[\mathcal{P}\int d^3k'' \frac{\left(\sqrt{\omega''^2+p^2}-\omega''\right)}{\omega''-\omega} t(\vec{k},\vec{k}'';\omega'')\, t^*(\vec{k}',\vec{k}'';\omega'')$$

$$-\mathcal{P}\int d^3k'' \frac{\left(\sqrt{\omega''^2+p^2}-\omega''\right)}{\omega''-\omega'} t(\vec{k},\vec{k}'';\omega'')\, t^*(\vec{k}',\vec{k}'';\omega'')\Big].$$

H. Kamada,[1,*] W. Glöckle,[2,†] J. Golak,[2,3,‡] and Ch. Elster[4,§]

PHYSICAL REVIEW C **66**, 044010 (2002)

Two-Body Input: T1-operator embedded in 3-body system

$$T_1(\mathrm{p}',\mathrm{p};\mathrm{q}) = V(\mathrm{p}',\mathrm{p};\mathrm{q}) + \int d^3k'' \frac{V(\mathrm{p}',\mathrm{k}'';\mathrm{q})\, T_1(\mathrm{k}'',\mathrm{p};\mathrm{q})}{\sqrt{(2E(p'))^2+q^2} - \sqrt{(2E(k''))^2+q^2} + i\varepsilon}$$

Do not solve for V!

- Obtain fully off-shell matrix elements T_1(k,k',q) from half shell transition matrix elements by

  Solving a 1st resolvent type equation:

$$T_1(q) = T_1(q') + T_1(q)\,[g_0(q) - g_0(q')]\,T_1(q')$$

- For every single off-shell momentum point
- Proposed in
  - Keister & Polyzou, PRC 73, 014005 (2006)
- Carried out for the first time here **[PRC 76, 1014010 (2007)]**

→ Exact Boost

Obtain embedded 2N t-matrix $T_1(k,k',z')$ half-shell in 2-body c.m. frame first :

$$\begin{aligned}\langle \mathbf{k}|T_1(\mathbf{q};z')|\mathbf{k}'\rangle &= \langle \mathbf{k}|V(\mathbf{q})|\mathbf{p}'^{(-)}\rangle \\ &= \frac{2(E_{k'}+E_k)}{\sqrt{4E_{k'}^2+\mathbf{q}^2}+\sqrt{4E_k^2+\mathbf{q}^2}}\, t(\mathbf{k},\mathbf{k}';2E_{k'})\end{aligned}$$

$$t(\mathbf{k},\mathbf{k}';2E_{k'}) = v(\mathbf{k},\mathbf{k}') + \int d\mathbf{k}''\, \frac{v(\mathbf{k},\mathbf{k}'')t(\mathbf{k}'',\mathbf{k}';2E_{k'})}{E_{k'} - 2\sqrt{m^2+k''^2} + i\epsilon}$$

Solution of the relativistic 2N LS equation with 2-body potential

Consideration for two-body t-matrix

- Relativistic and non-relativistic t-matrix should give identical observables for determining relativistic effects
- Or two-body t-matrices should be phase-shift equivalent
- Four options:
  - Start from relativistic LS equation
    - natural option – employed for NN interactions fit to 1 GeV +
  - **If non-relativistic LS equation is used:**
    - Refit of parameters (maybe time consuming in practice)
    - Transformation of Kamada-Glöckle PRL 80, 2547 (1998)
    - **Transformation of Coester-Piper-Serduke as given in Polyzou PRC 58, 91 (1998)**

Phase equivalent 2-body t-matrices:

Coester-Pieper-Serduke (CPS) (PRC11, 1 (1975))

- Add interaction to square of non-interacting mass operator

$$M^2 = M_0^2 + u = 4mh \quad \text{with} \quad h \equiv \frac{k^2}{m} + \frac{u}{4m} + m$$

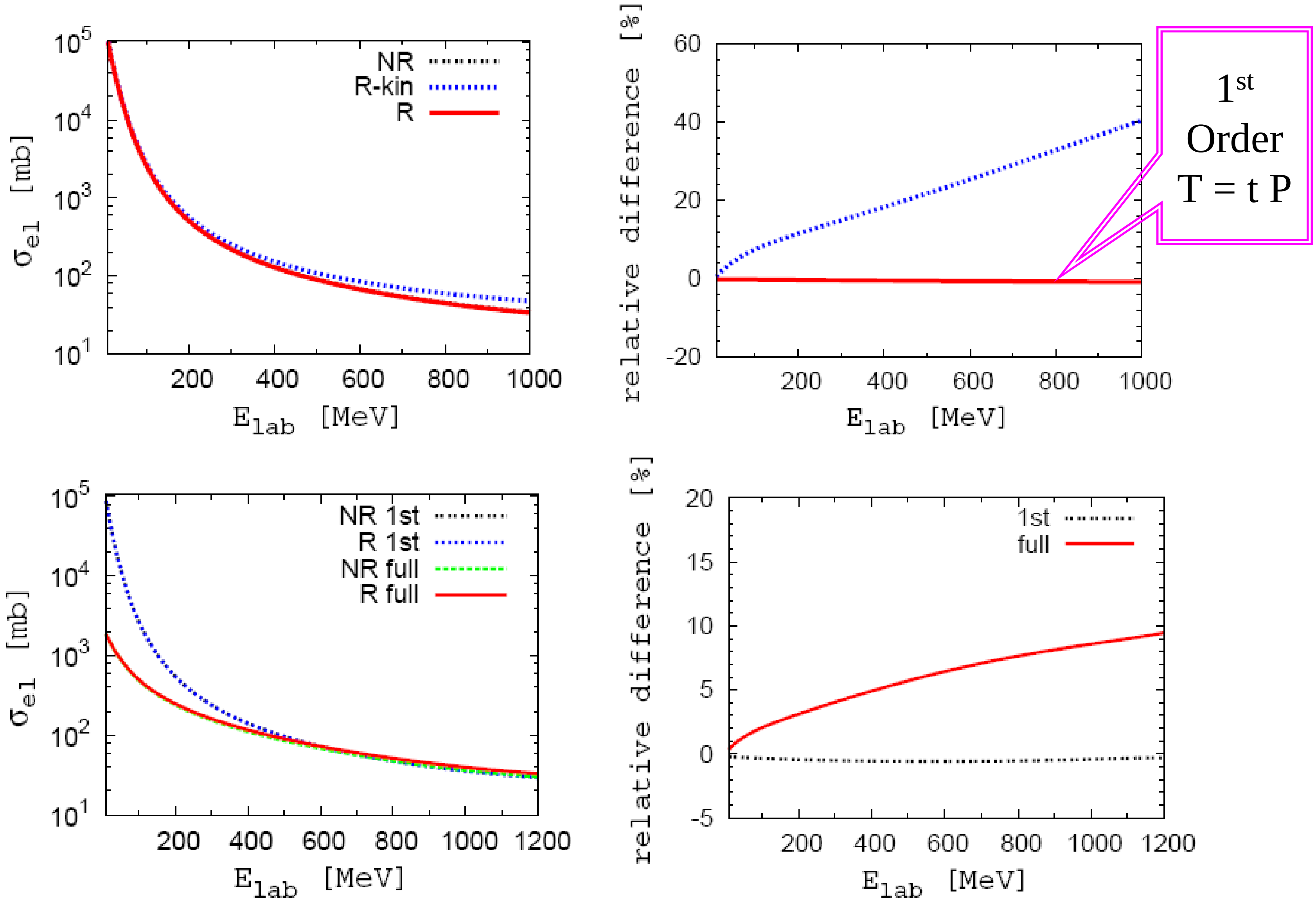
$$u = v^2 + \left\{ M_0^2, v \right\}$$

- NO need to evaluate v directly, since M, M^2, h have the same eigenstates
- Relation between half-shell t-matrices

$$\langle k' | t_R(e(k)) | k \rangle = \frac{4m}{e(k) + e(k')} \langle k' | t_{NR}(k^2/m) | k \rangle$$

- Relativistic and nonrelativistic cross sections are identical functions of the invariant momentum k

Total Cross Section for Elastic Scattering:

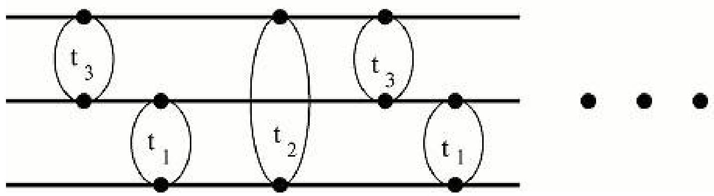


Faddeev Equation as Multiple Scattering Series

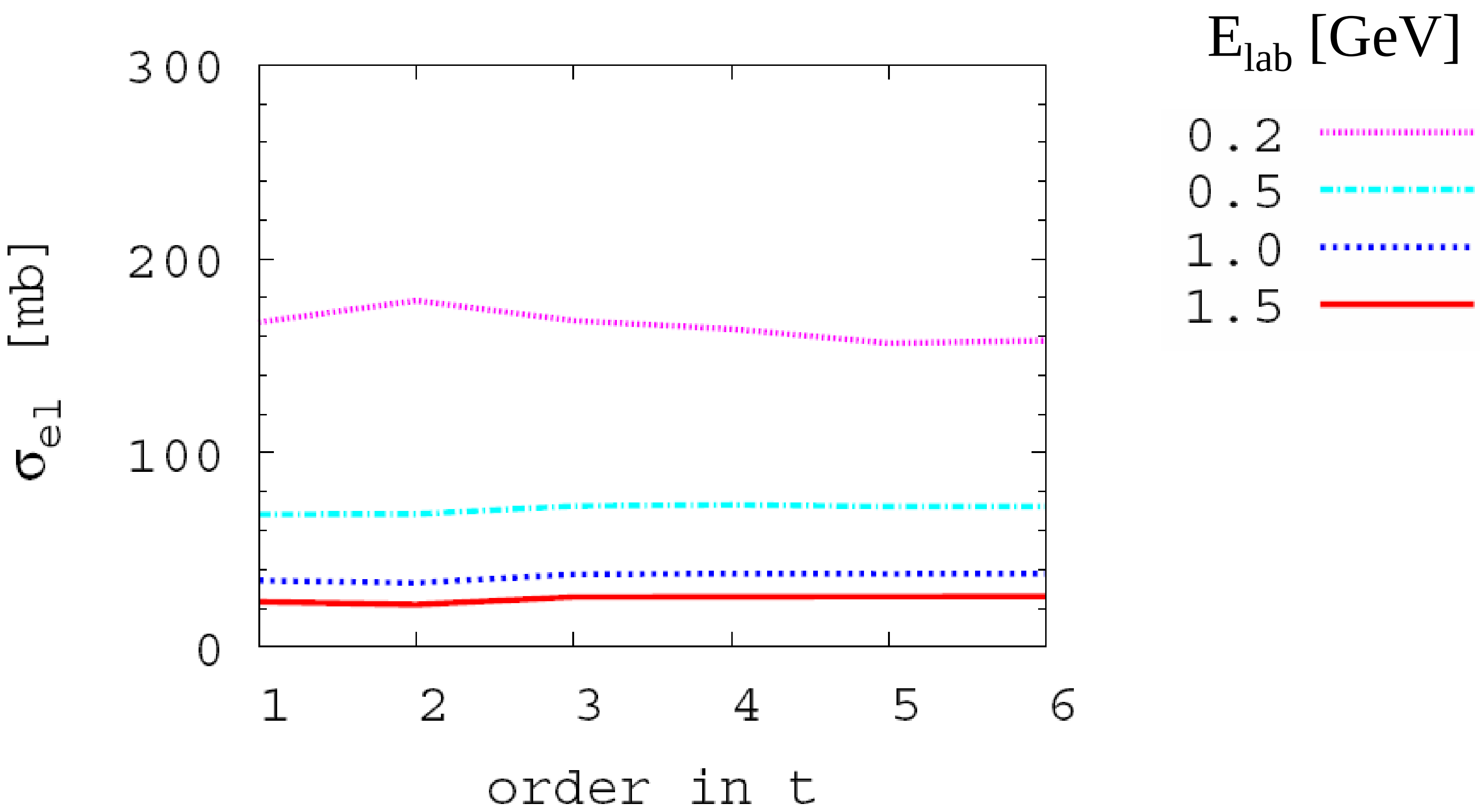
$$T = tP + tG_0PT$$

$$T = tP + tG_0PtP + \cdots$$

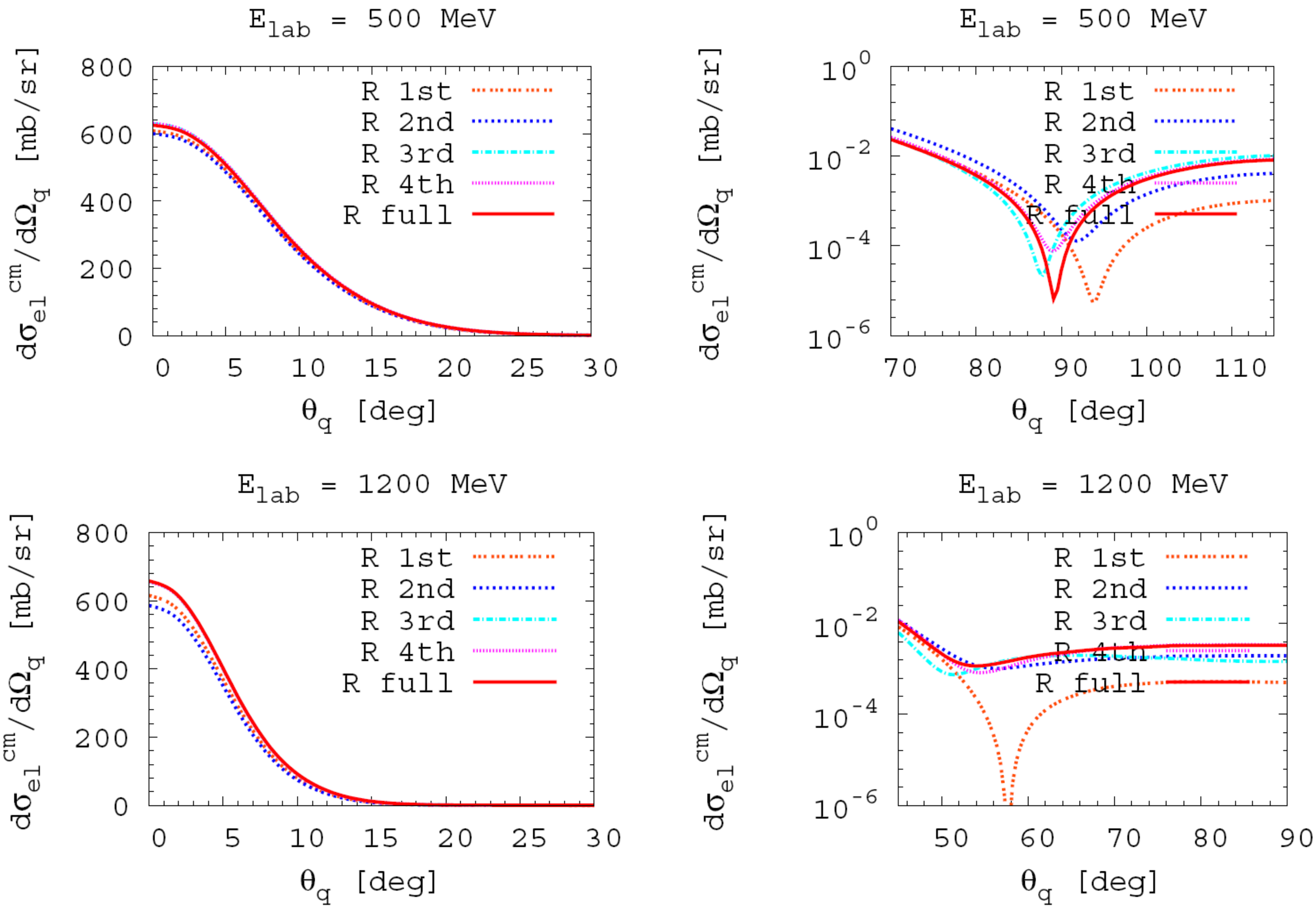
1st Order or IA

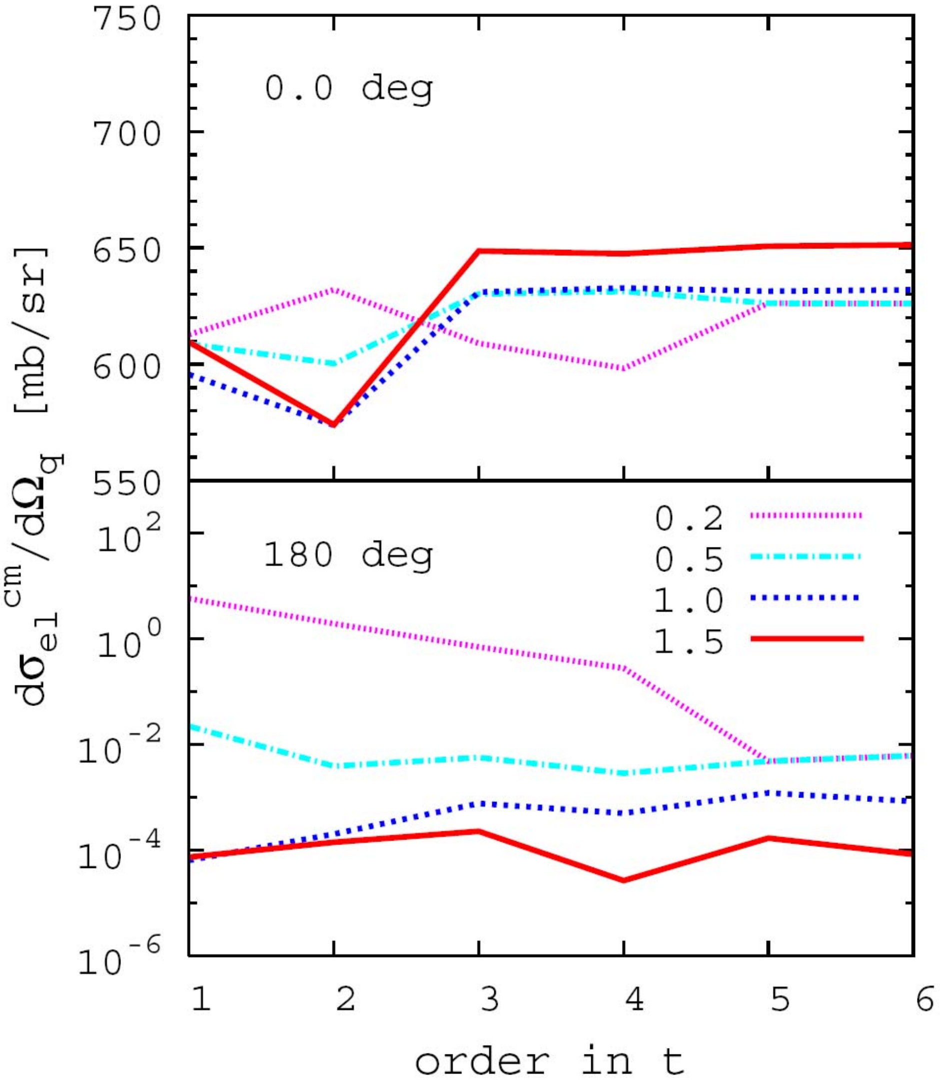


Convergence of the Faddeev Multiple Scattering Series



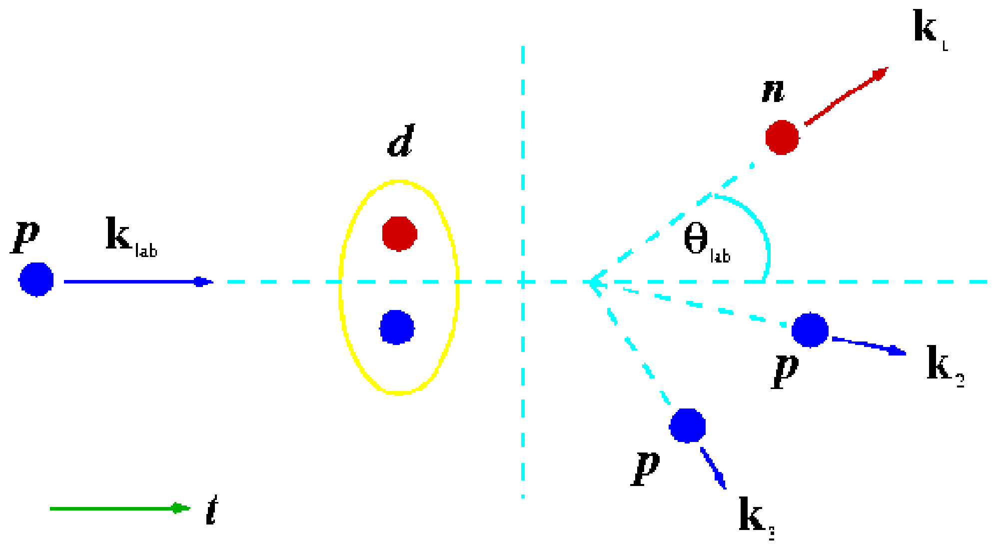
Elastic Scattering: Differential Cross Section





Differential Cross Section: Convergence of the Faddeev Multiple Scattering Series

Breakup Scattering



Exclusive: Measure energy & angles of two ejected particles

V.Punjabi et al. PRC 38, 2728 (1998) – TRIUMF p+d @ 508 MeV

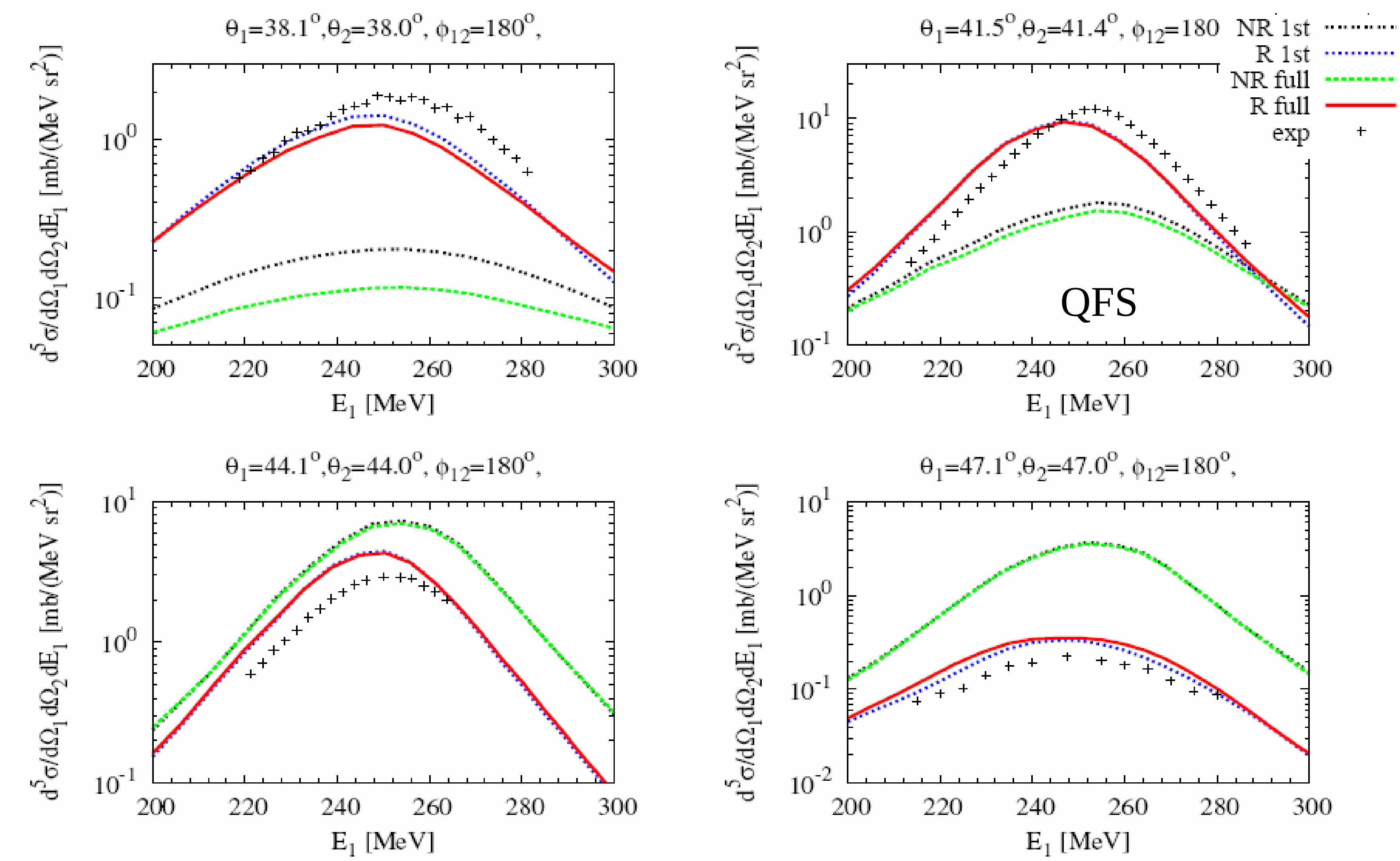
Outgoing protons are measured in the scattering plane

Exclusive Breakup Scattering

(symmetric configuration)

E_{lab} = 508 MeV

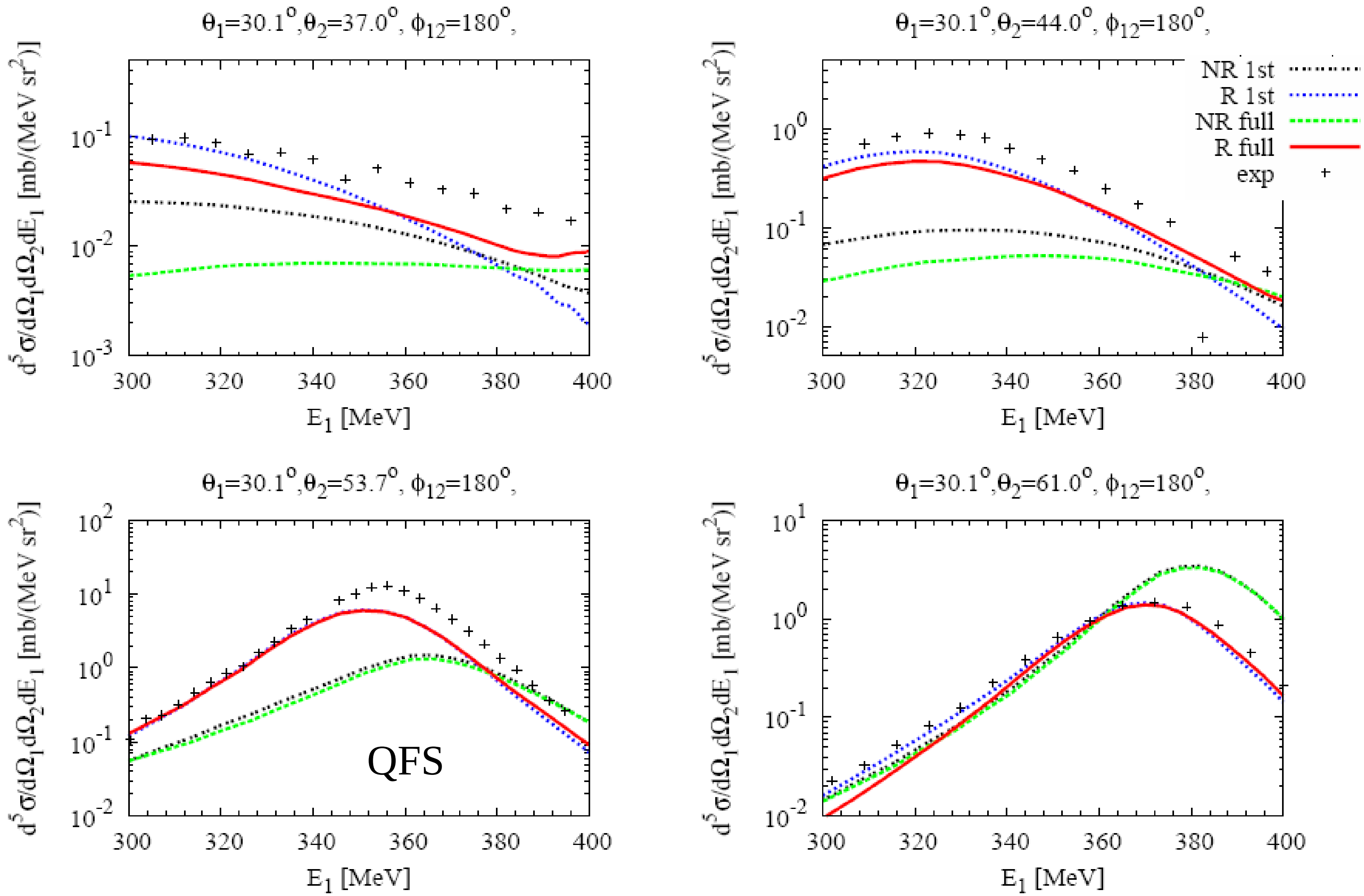
(V.Punjabi et al. PRC 38, 2728 (1998)



Exclusive Breakup Scattering

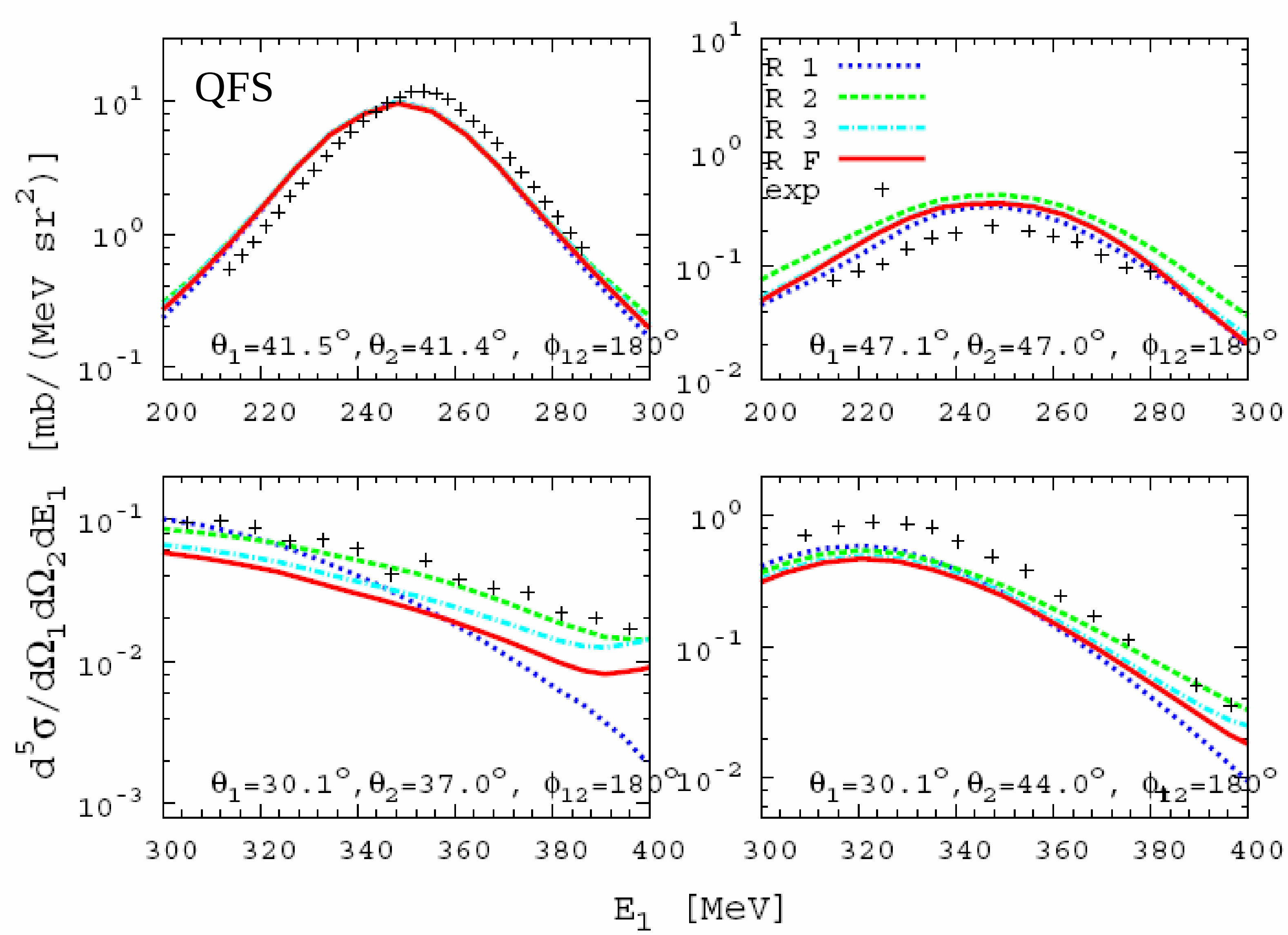
E_{lab} = 508 MeV

(asymmetric configuration)

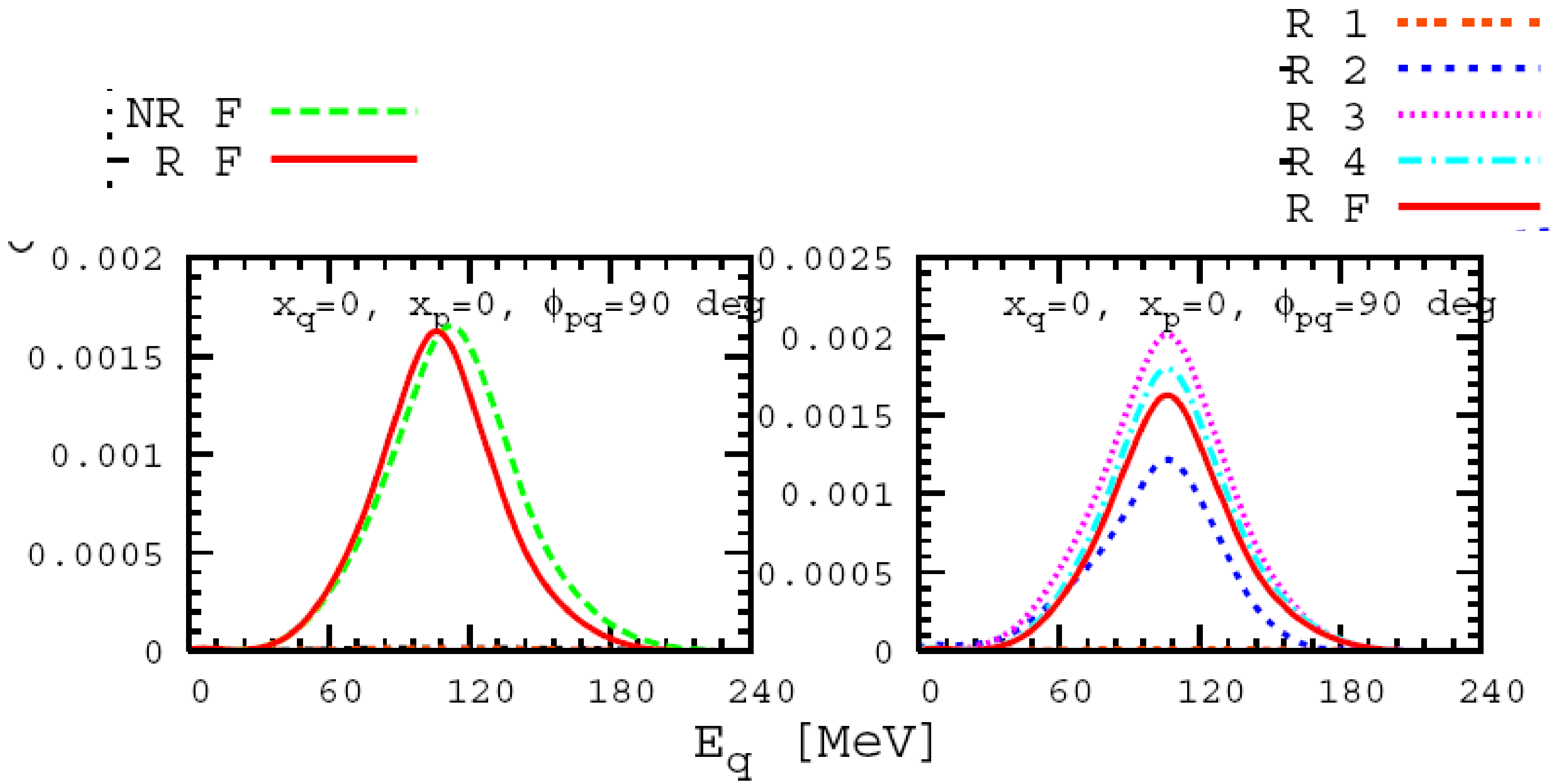


Exclusive Breakup Scattering

E_{lab} = 508 MeV



Exclusive Breakup Scattering
Space-Star



E_{lab} = 508 MeV

Poincaré Invariant Faddeev Calculations

- Kinematics
  - Phase space factors
  - Lorentz Transformation from Lab to c.m. frame
  - Lorentz Transformation of Jacobi Coordinates
    - Always reduces effects of phase-space factors
  - Kinematics determines peak positions in break-up observables
- Dynamics
  - Exact calculation of the two-body interaction embedded in the three-particle Hilbert space
  - The dynamic effects act in general opposite kinematic effects

Poincaré Invariant Faddeev Calculations

- **Carried out up to 2 GeV for elastic and breakup scattering**
  - Solved Faddeev equation in vector variables = NO partial waves
- **Relativistic effects are important at 500 MeV and higher**
  - Relativistic total elastic cross section increases up to 10% compared to the non-relativistic
  - Relativistic kinematics determines QFS peak positions in inclusive and exclusive breakup
  - Breakup: Relativistic effects very large dependent on configuration
- **Above 800 MeV projectile energy:**
  - multiple scattering series converges after ~2 iterations
- **Future**
  - Systematic studies of selected cross sections & high energy limits
  - Long term: include Spin