The fifth workshop dedicated to
the Critical Stability of Few-Body Quantum Systems.
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Resonances in Few-Body Quantum Systems

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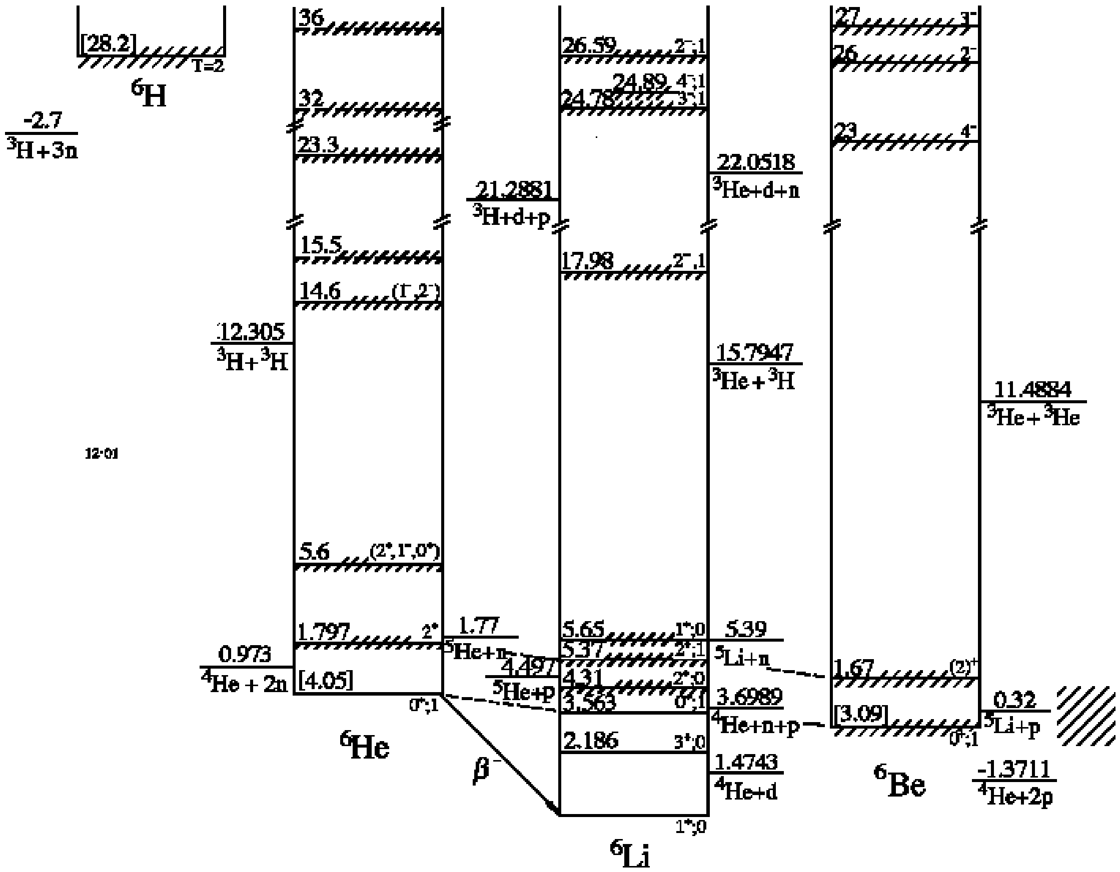
Howto calculate resonances in N-body systems
using real wave-functions
and bound-state boundary conditions.

$$L^2(R^{3N})$$

In nuclear physics most states are resonances, e.g. A=6 nuclei:



Discovery of the J-particle in $p + \mathrm{Be} \to e^+e^- + X$ reaction at BNL

PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

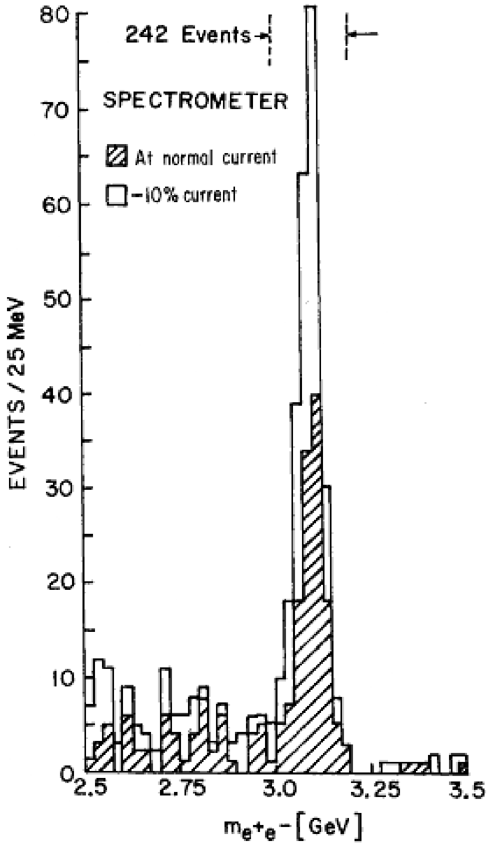


FIG. 2. Mass spectrum showing the existence of *J*.

Discovery of the ψ-particle in $e^+e^- \to$ hadrons reaction at SLAC

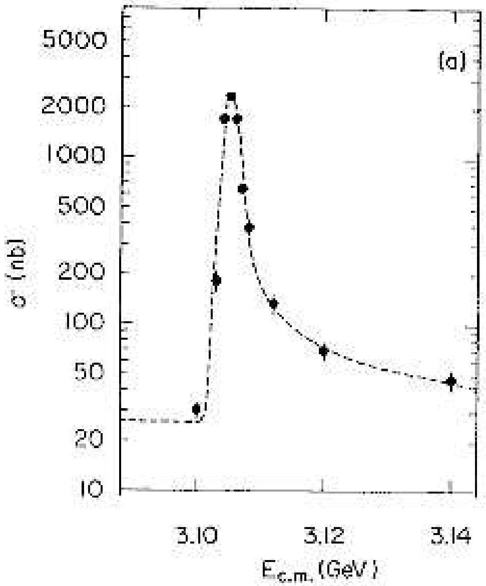


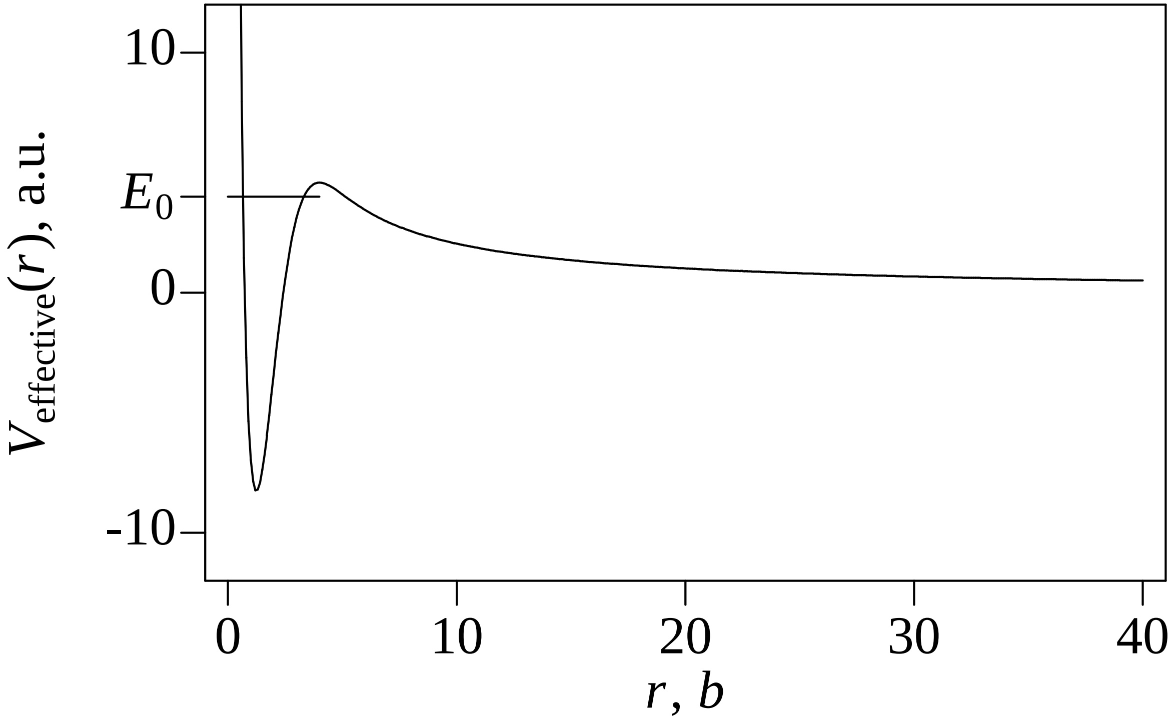
FIG. 1. Cross section versus energy for (a) multihadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\pi^+\pi^-$, and K^+K^- final states. The curve in (a) is the ex-

Resonance is a (relatively) sharp maximum in the cross-section in every (appropriate) channel.

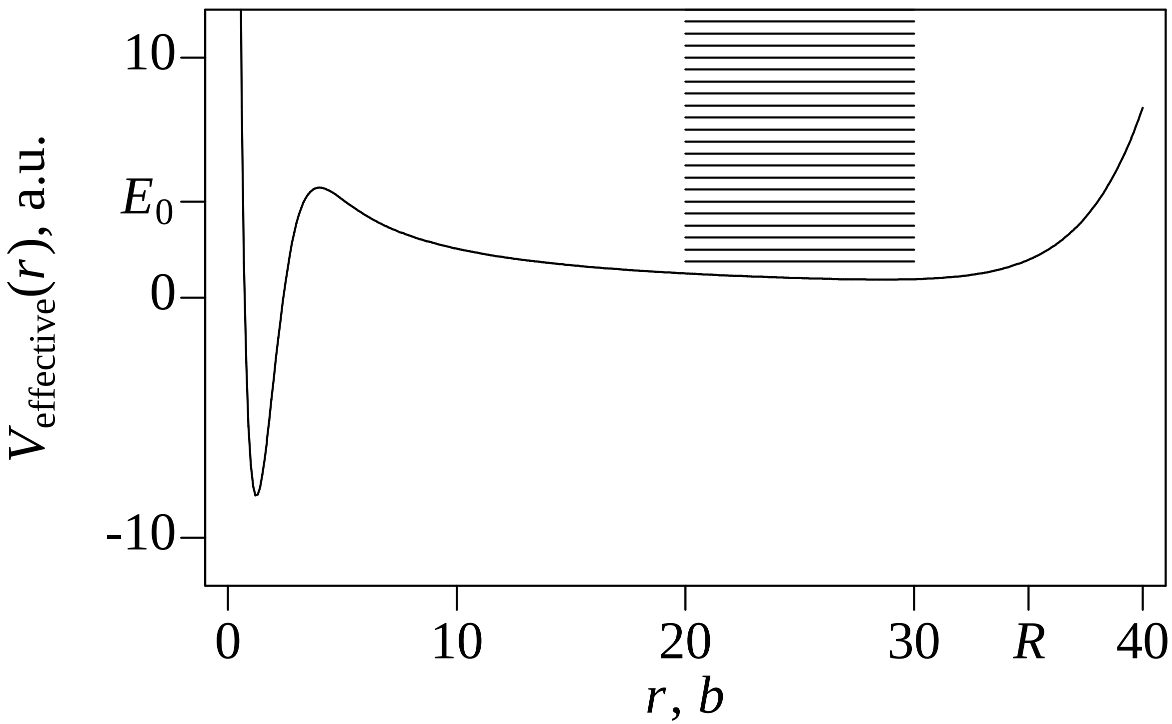
E.g. in this channel:

$$d\sigma(E) \approx \frac{2\pi}{\hbar} \left|\langle \psi(E) | W | \psi_{\text{initial}} \rangle\right|^2 d\nu(E)$$

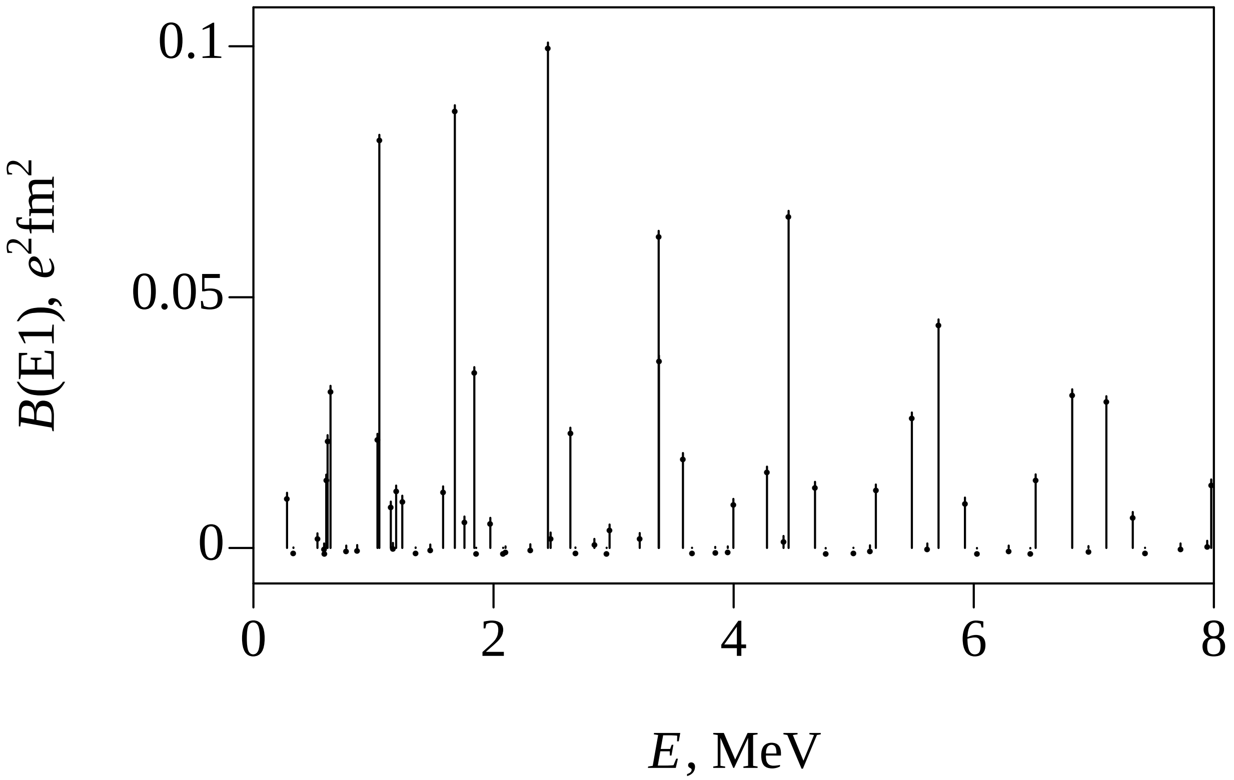
A typical quantum problem with a continuum spectrum and a resonance:



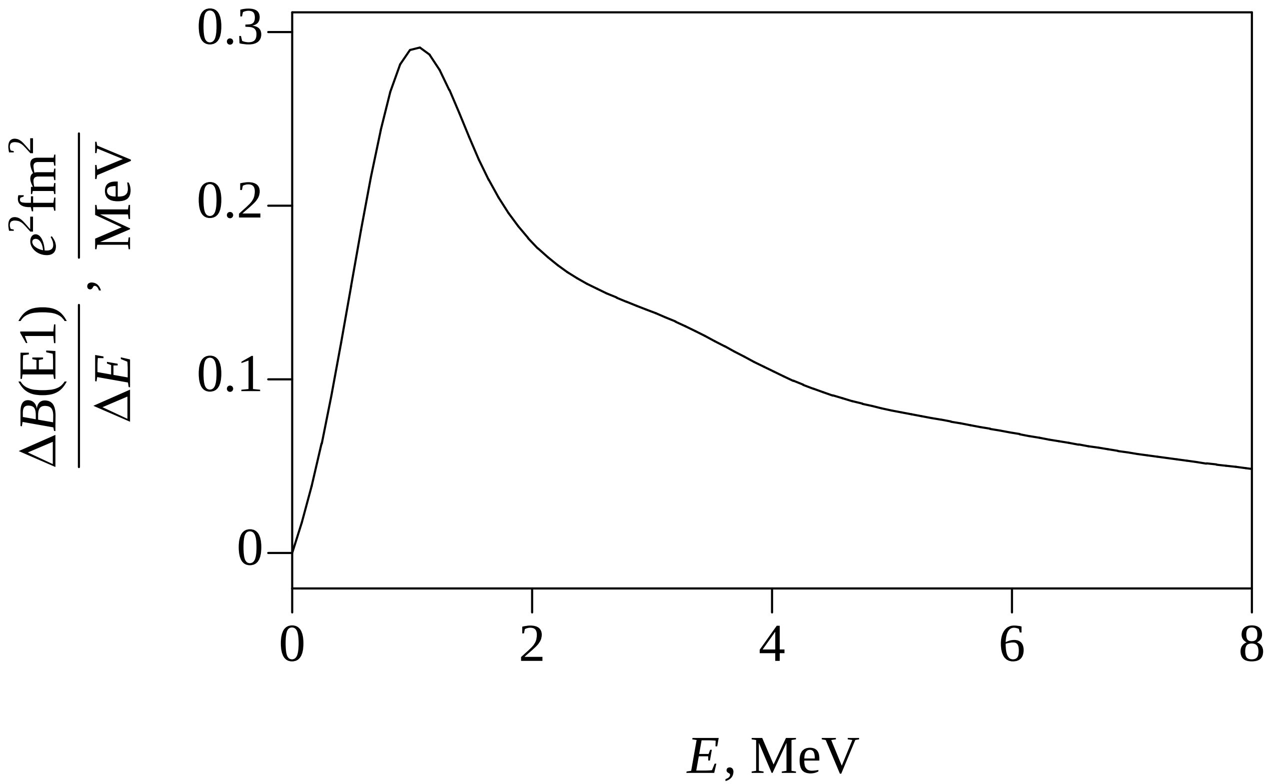
The same problem in a box with quasi-continuum spectrum:



Example: dipole reduced matrix element $B(E1)$ for $^6He = \alpha + n + n$.



(Observable) dipole strength function for $^{6}\mathrm{He} = \alpha + n + n$.



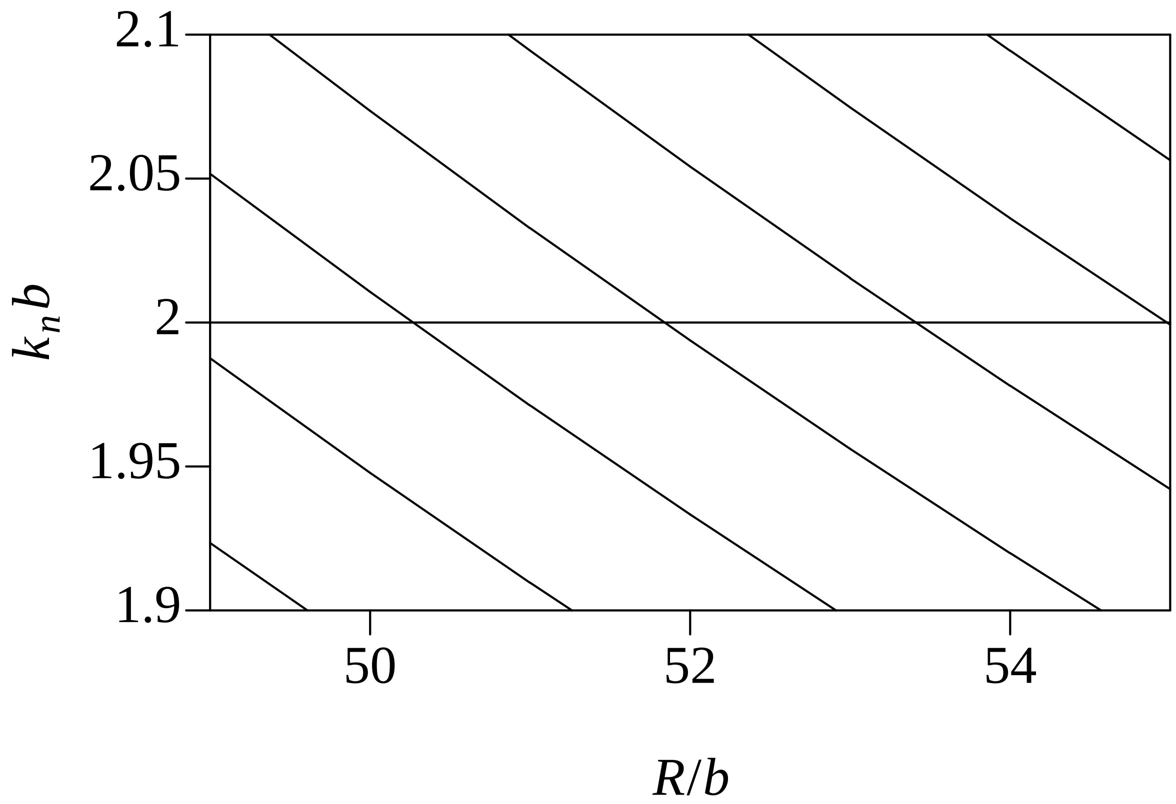
Conclusion: calculate the quasi-continuum spectrum, than calculate $\frac{dB}{dE}$: not so narrow resonances should be seen.

However, the width of narrow resonances,

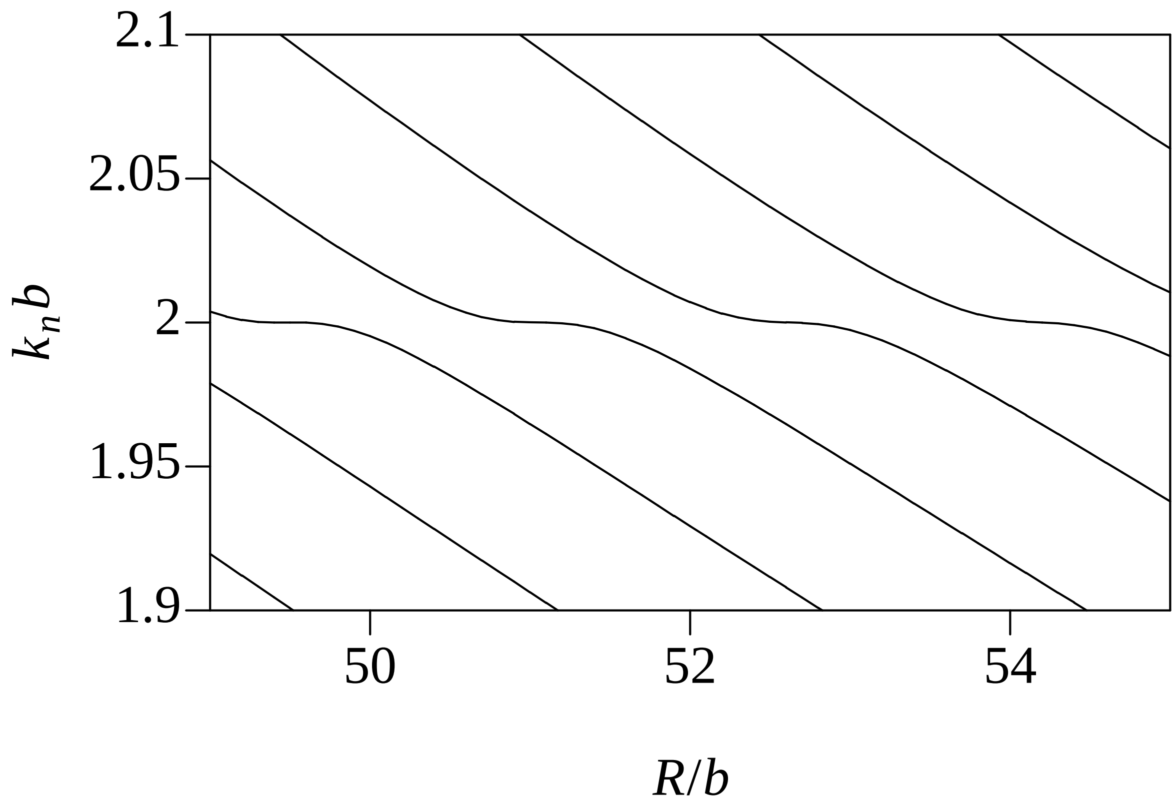
$$\Gamma < \frac{\hbar^2}{mR^2}$$

cannot be resolved by this method.

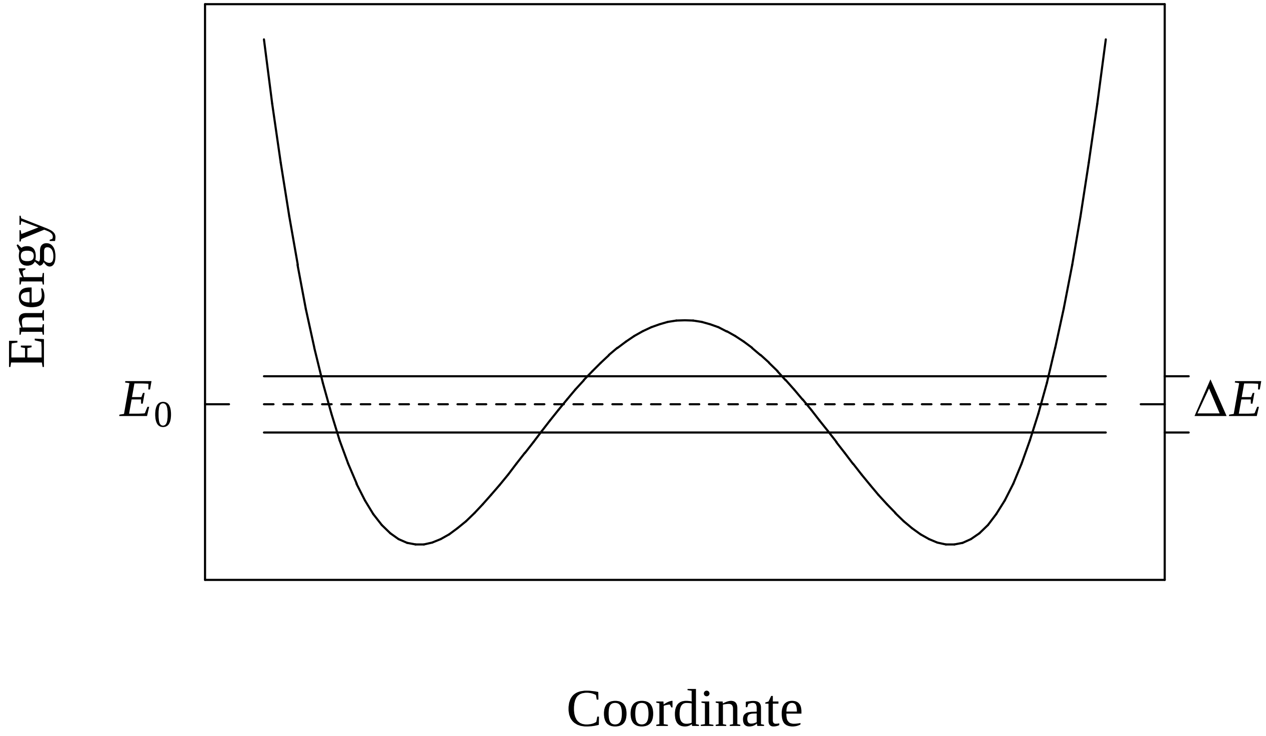
Energy levels as function of the box size R with an infinitely narrow resonance (a discrete state behind an infinite barrier).



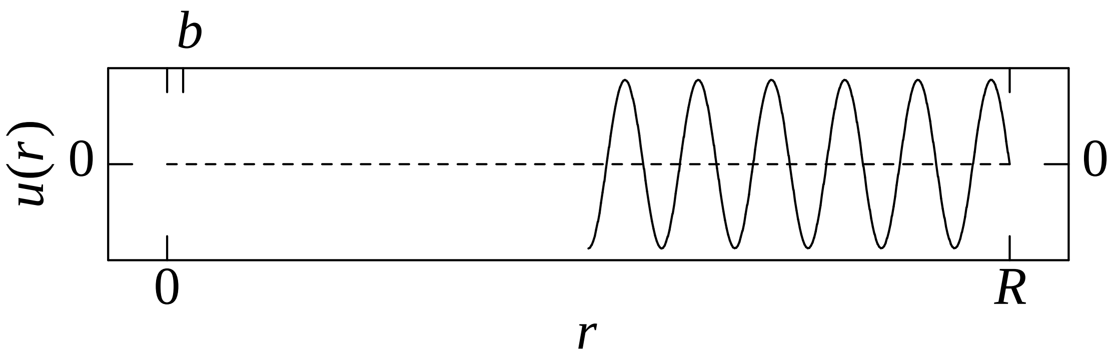
Energy levels as function of R with finite barrier: avoided crossings (Zeldovich rearrangement):



Energy level splitting in a double well: $\Delta E = \frac{\hbar}{\pi T}\, e^{-S}$



Quasi-continuum spectrum in a box with sharp cut-off at R (and no Coulomb):



Free wave:

$$\sin(kR) = 0 \quad \Rightarrow \quad k_n R = \pi n$$

Distorted wave:

$$\sin(kR + \delta(k)) = 0 \quad \Rightarrow \quad k_n R = \pi n - \delta(k_n) \qquad ^{1)}$$

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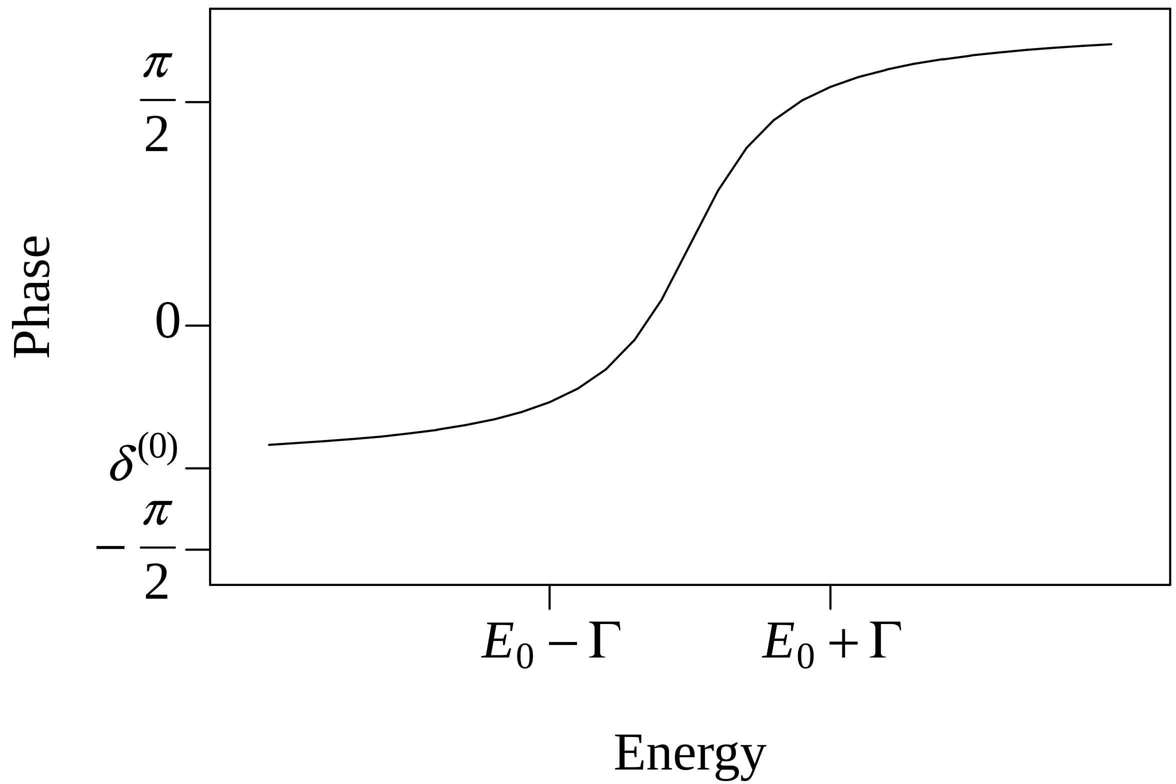
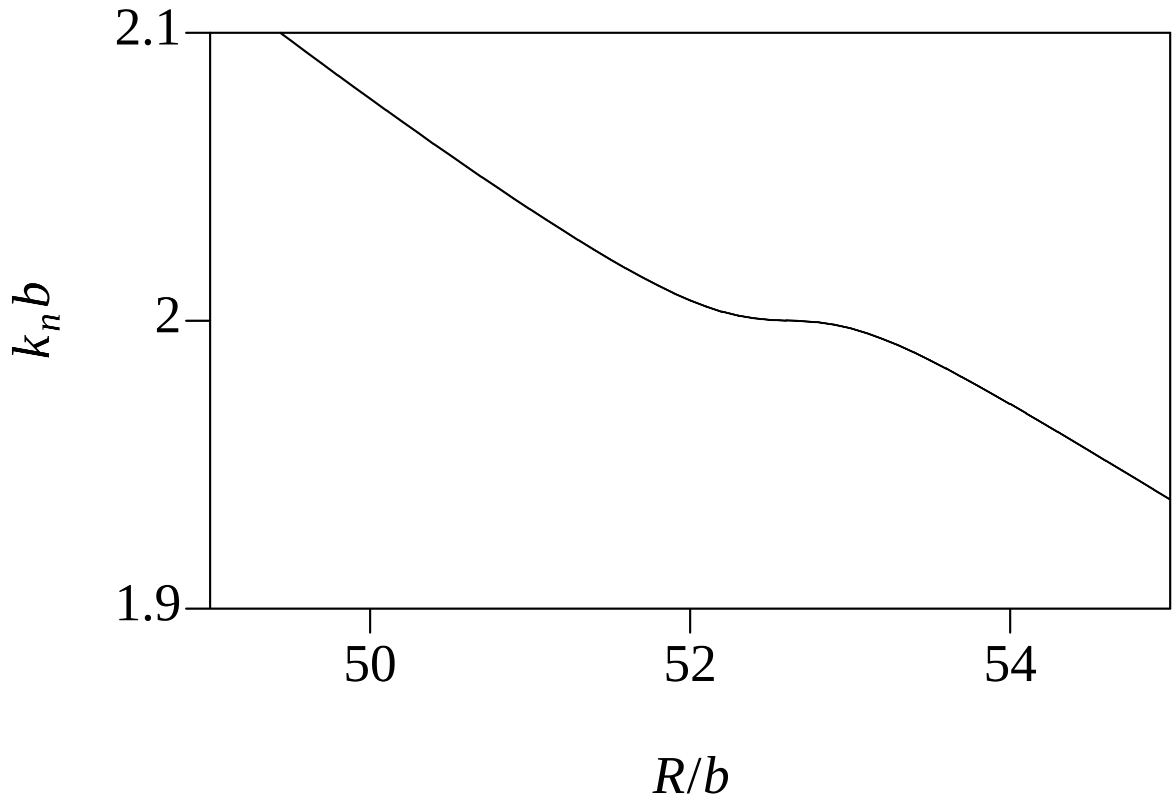
1) we've got the scattering phase from the spectrum: $\delta(k_n) = \pi n - k_n R$

Phase-shift with narrow resonance: $\delta = \delta^{(0)} - \arctan \dfrac{\Gamma}{2(E - E_0)}$



Quasi-continuum spectrum with a narrow resonance:

$$k_n R = \pi n - \delta(k_n) \quad => \quad k_n \approx \frac{\pi n}{R} - \delta(\frac{\pi n}{R})$$



Conclusion: calculate the quasi-continuum spectrum as function of the box radius and plot it: the sequence of avoided crossings gives the resonance energy; the level splitting gives (basically) the width.