KAON - FEW -NUCLEON STATES

variational calculations

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THE INTEREST

Nuclear systems bound by nucleon excitations

N $\rightarrow$ Λ (1405) S-wave

N $\rightarrow$ Σ (1385) P-wave

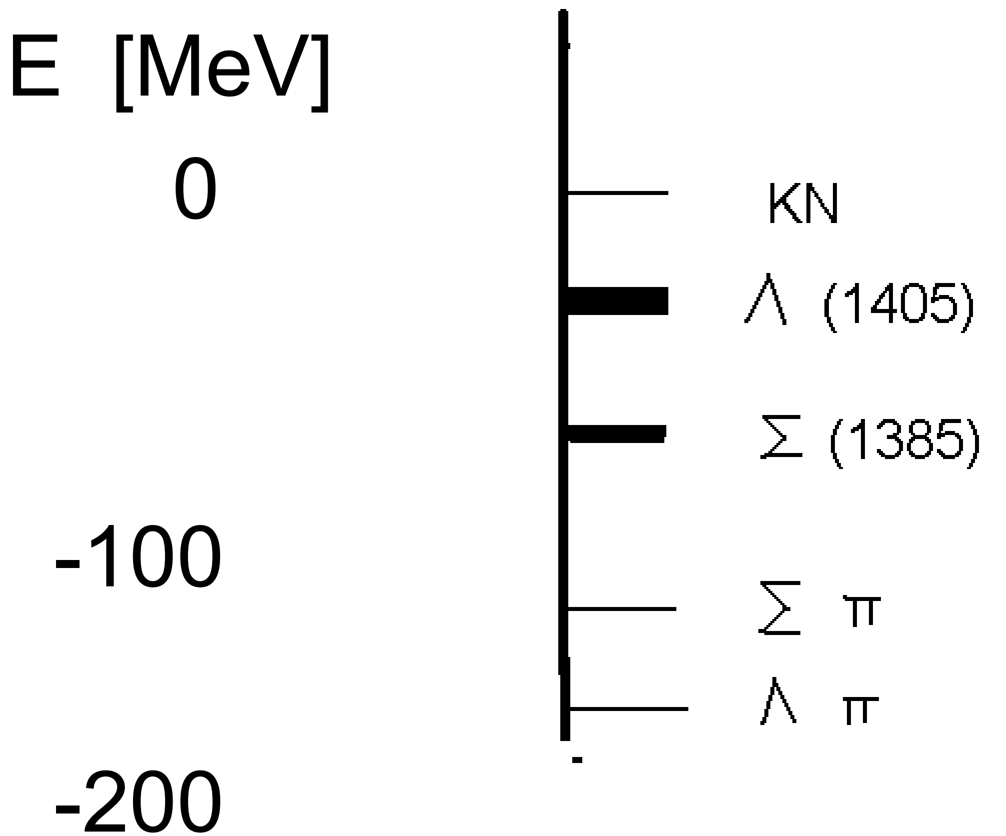
and

„level repulsion rule „

Experimental Stimulus binding energies Frascati-KEK-GSI

	E [MeV]	Γ [MeV]
„Kpp”	110(10)	67(15)
„Kppn”	169 withdawn	~ 20
„Kpp”	~100	~ 100

States coupled to KN



no decays ?

Molecular models ,

Akaishi ,Weise, Yamazaki,Dote

- Average potential for K

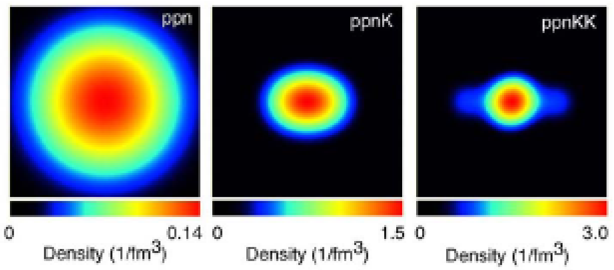


Fig. 14. Density distribution of ppn (= ^{3}He) (left), K^-ppp (center) and K^-K^- ppp (right) obtained by a new framework of antisymmetrized molecular dynamics (AMD) [17, 62].

Difficulties of the average field description

- Short ranged NN repulsion
- KN interactions are energy dependent, far off-energy-shell

KNN calculation also difficult

KNNN, KNNNN states –
are more interesting – large binding

This model

- Stress on K-N correlations
- Proper N-N repulsion
- Inclusion of S and P wave resonances

- Reasonable description of recoil

Two steps of this calculation

1) K meson bound to nucleons fixed at X_i

→ correlated KN wave function

$$\Phi_K(X , X_i) = \Sigma_i \psi_i(X - X_i)$$

→ complex binding energy $E(X_i)$

→ contracting potential $E(X_i) – E (\infty)$

2) Variational wave for KN…N

$$\Phi_K(X , X_i) \Theta_{N…N}(X_i)$$

Fixed nucleons

Brueckner's method extended to bound states

- Separable KN interactions $v(u)v(u')\lambda$
- K amplitudes at each nucleon

$$\varphi_i = \lambda \int du\, v(u) \quad \Phi_K(X_i - u, X_i)$$

- → eigenvalue k

$$\varphi_i = \Sigma_{i \neq j} \; t \; G_{i,j}(X_i, X_j, k)\, \varphi_j$$

Example : KNN

$\varphi_1 = t\ G\ \varphi_2$

$\varphi_2 = t\ G\ \varphi_1$

Two solutions

$\varphi_1 = \varphi_2$ S wave : Λ

$\varphi_1 = -\varphi_2$ P wave : Σ

k(r) - complex eigenvalue

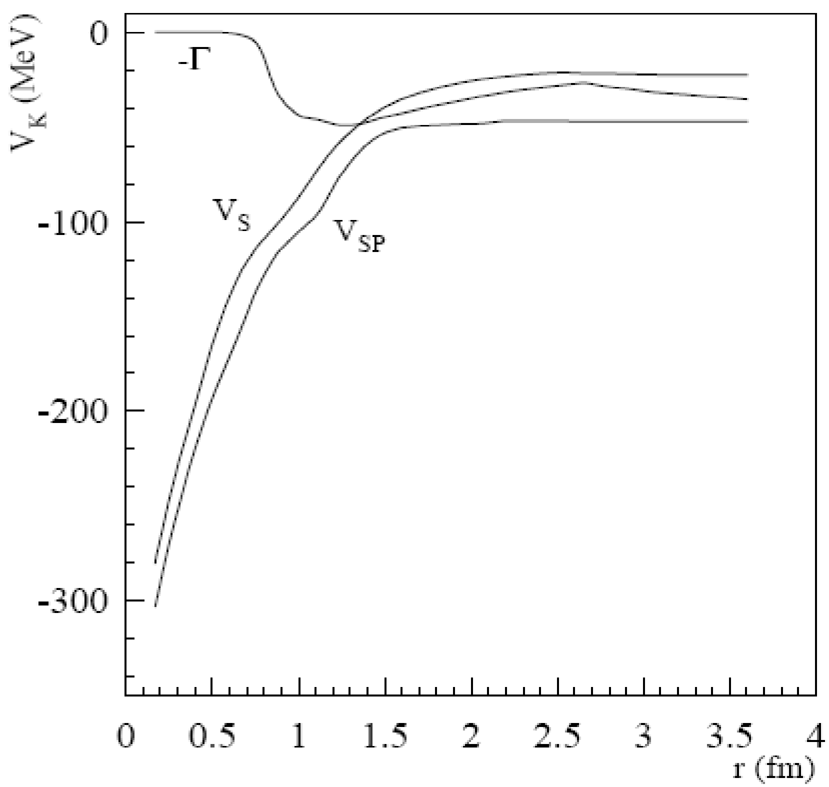
Binding energy $E - i\Gamma/2 = k(r)^2 / 2\mu_{KN}$

$k(\infty)^2 / 2\mu_{KN}$ = binding of $\Lambda(1405)$

Asymptotic , separation

(KNN) → N + $\Lambda(1405)$

contracting potential
E(r)- E(∞)
V_s-due to Λ(1405) , V_{SP} –due to Λ +Σ



Technicalities

Yamaguchi formfactor , S wave

$$f(k,E,k') = \upsilon(k)\ t(E)\ \upsilon(k')$$

$$G_{i,j}(\mathbf{x_i},\mathbf{x_j}) = \int \mathbf{dydx}\ \upsilon(\mathbf{x}-\mathbf{x_i})\ \frac{\exp(\mathbf{ik}\mid\mathbf{x}-\mathbf{y}\mid)}{4\pi\mid\mathbf{x}-\mathbf{y}\mid}\ \upsilon(\mathbf{y}-\mathbf{x_j})$$

$$G_{1,2}(r,k) = \frac{1}{r}\ \upsilon(k)^2\ [\exp(ikr) - \exp(-\kappa r) - r\frac{\kappa^2+k^2}{2\kappa}\exp(-\kappa r)]$$

Schroedinger equation

The KNN levels are given by equation

$$(-\frac{\Delta_x}{2m_K} - \frac{\Delta_1}{2M} - \frac{\Delta_2}{2M} + V_{KN1} + V_{KN2} + V_{NN})\Psi = E\Psi.$$

The wave function is assumed as $\Psi = \chi_K(x, x_i)\chi_{NN}(r)$ (Eq. 2), hence

$$[E - V_K(r) + \Delta_1/2M + \Delta_2/2M - V_{NN}]\,\chi_{NN} + \Delta E_{kin}\chi_{NN} = 0.$$

ΔE_{kin} is very small

Input

- V(N,N) Argonne 18 ,

  Pion exchange + phenomenology

  (attraction + soft repulsive core)

  Tested in scattering and

Variational calculations in nuclei.

V(KN) : 5-channel K matrix , dispersion relations

uncertain

Typical solutions

AM- Alan Martin   KWW-Brian Martin

KN –single channel ,

KN,Σπ- two channel –collision broadening

solution	AM[19]			KWW[11]		
	E_B	Γ	R_{rms}	E_B	Γ	R_{rms}
$KN;\quad S$	27	36	3.1	35.5	37	2.4
$KN,\Sigma\pi;\ S$	37	42	2.5	43.1	47	2.1
$KN;\quad S,P$	49	36	3.7	49.7	36	3.3
$KN,\Sigma\pi;\ S,P$	52	37	2.9	56.5	39	2.3

Strong dependence on Λ resonance position

solution KWW*			
	E_B	Γ	R_{rms}
$KN;\ \ S$	50	51	2.05
$KN, \Sigma\pi;\ S$	71	85	1.81
$KN;\ \ S, P$	65	43	2.09
$KN, \Sigma\pi;\ S, P$	78	60	1.88

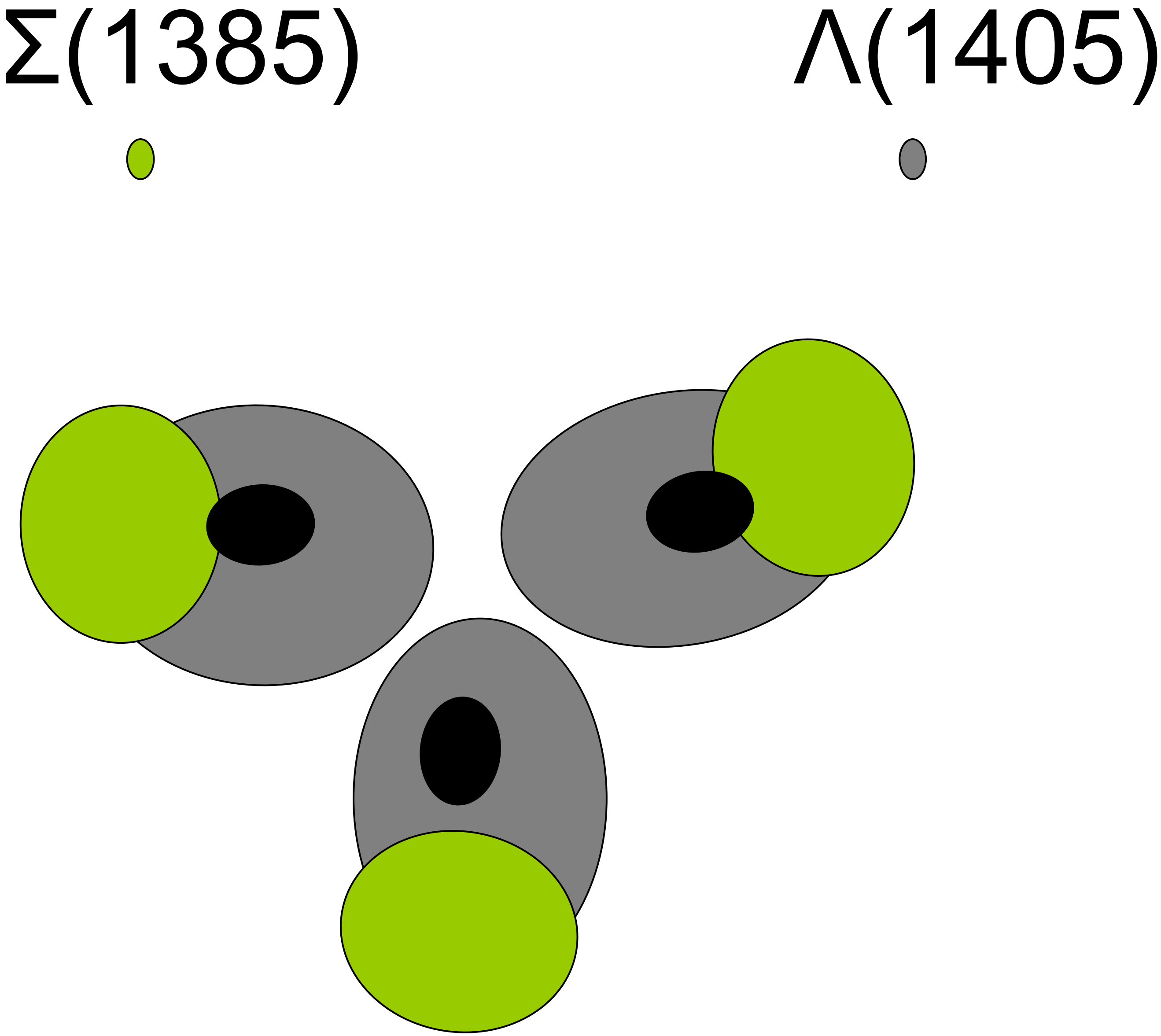
Extension to KNNN, KNNNN

- Variational solutions

$\Phi_K(X , X_i)\cdot$

$\Pi_{\text{NN pairs}}$ [(1-exp(-$r^2 \gamma^2$)) (1-exp(-rλ))/r]

A simple physical picture emerges from this approach. The mesons are strongly correlated to slowly moving nucleons. The correlations are of the $\Lambda(1405)$ type at large densities, and of the $\Sigma(1385)$ type in the peripheries. Each K,N pair has a good chance to stay also in the Σ, π form. The structure is rather loose as sizable fractions of the binding energies are hidden in the short ranged correlations.


Σ(1385)
Λ(1405)

Sizable binding „within resonances”

- Strong KN correlations
- Rather loose structure ,difficult to produce
- P wave spectrum

- Very deep binding in KNNNN
- Narrow States

- KNNN S wave

	E_B	Γ	E_B	Γ
S	103	29	142	25
$S+P$	119	23	153	21

- KNNNN S wave

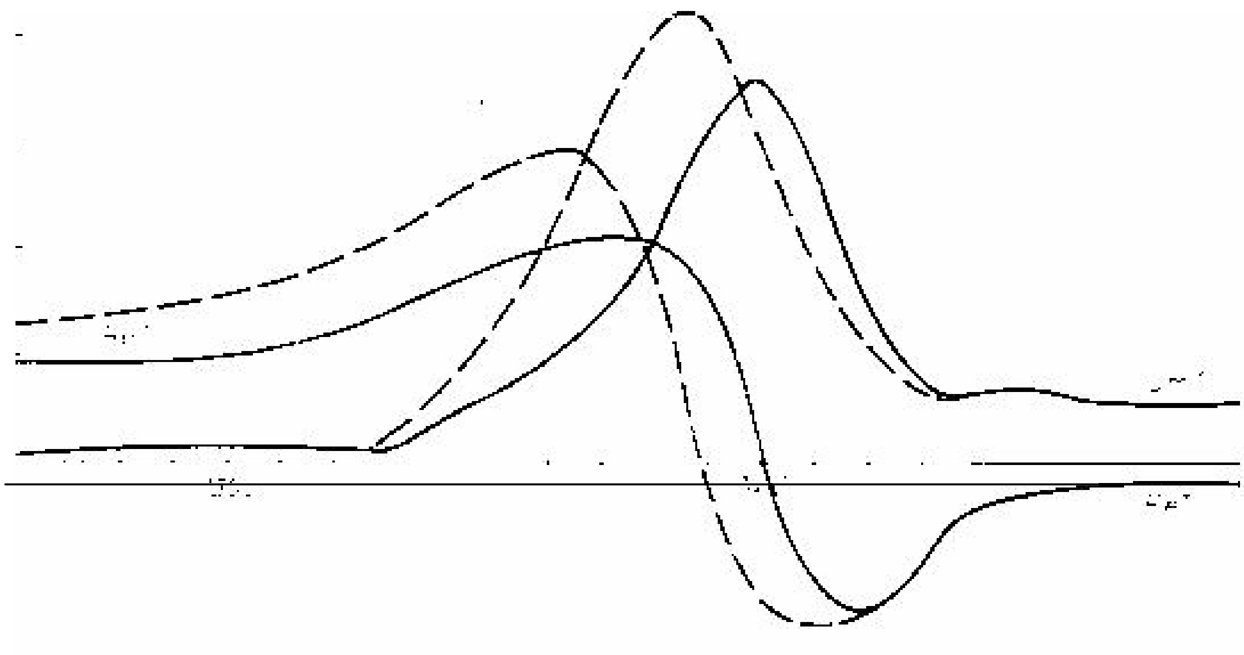
	E_B	Γ	E_B	Γ
S	121	25	170	10
$S+P$	136	20	172	10

A state bound by Σ(1385)

J(NN)=2, L(NN) =1

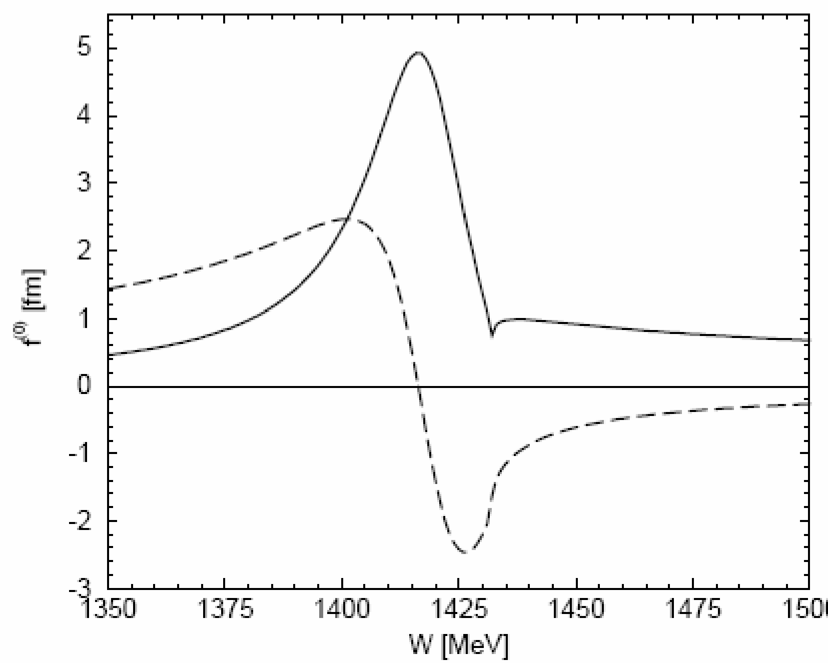
	I_{NNK}	I_{NN}	$E_B[MeV]$	$\Gamma[MeV]$	$R_{rms}[fm]$
K^-nn	3/2	1	48.5	36	4.9

Main uncertainty - position of Λ(1405)



KN amplitude, I=0

- - - - - Im ———— Re



$\Sigma(1385)$

$I = 1,\ J = 3/2$, $\Sigma(1385)$ dominates deep states

scattering amplitude $\quad f_\Sigma = 2\mathbf{pp}' \frac{\gamma^2_{\Sigma KN}}{E_{KN} - E_\Sigma + i\Gamma_\Sigma/2}$

$\gamma^2_{\Sigma\pi\Lambda} = \Gamma_{\pi\Lambda}/(2p^3_{\pi\Lambda})$ $\quad$ from $\pi\Lambda$ final states

$$\gamma^2_{\Sigma KN}/\gamma^2_{\Sigma\pi\Lambda} = \ 2/3 : \mathrm{SU}(3)$$
$$= \ 0.51 \pm 0.18, \text{ from } K - D \text{ [Braun]}$$

Future projects

- KEK
- J-Park
- GSI-Darmstadt
- Frascati – AMADEUS
- Jefferson Lab