Three-nucleon Force effects in $A = 3, 4$ systems

Alejandro Kievsky (INFN - U. Pisa)

Luca Girlanda (INFN - U. Pisa)
Laura Marcucci (INFN - U. Pisa)
Sergio Rosati (INFN - U. Pisa)
Michele Viviani (INFN - U. Pisa)

Starting Point

- Non relativistic Quantum Mechanics

$$H\Psi = E\Psi$$

$$H = T + V$$

$$V = \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k)$$

- Solution of the Schrödinger eq. (bound states)
  - Faddeev equations for $A = 3$
  - Faddeev-Yakubovsky equations for $A = 4$
  - Green Function Monte Carlo $A \leq 12$
  - No Core Shell Model $A \leq 12$
  - Hyperspherical Harmonics

Recent developments in the NN potential

- Nijm I, II, 93 V.G.J. Stocks *et al.*, PRC 49, 2950 (1994)
- AV18 R.B. Wiringa *et al.*, PRC 51, 38 (1995)
- CD Bonn 2000 R. Machleidt, PRC 63, 024001 (2001)

- N^3LO Entem and Machleidt, PRC 68, 041001 (2003)
- N^3LO Epelbaum *et al.*, NPA 747, 362 (2005)

- Low momentum NN interaction $V_{low\ k}$

  S.K. Bogner *et al.*, Phys.Rep. 386, 1 (2003)

  S. Fujii *et al.*, PRC 70, 024003 (2004)

The NN Potential

$$v(NN) = v^{EM}(NN) + v^{\pi}(NN) + v^{R}(NN)$$

v^{EM} is the electromagnetic part

v^{π} is the one-pion exchange potential

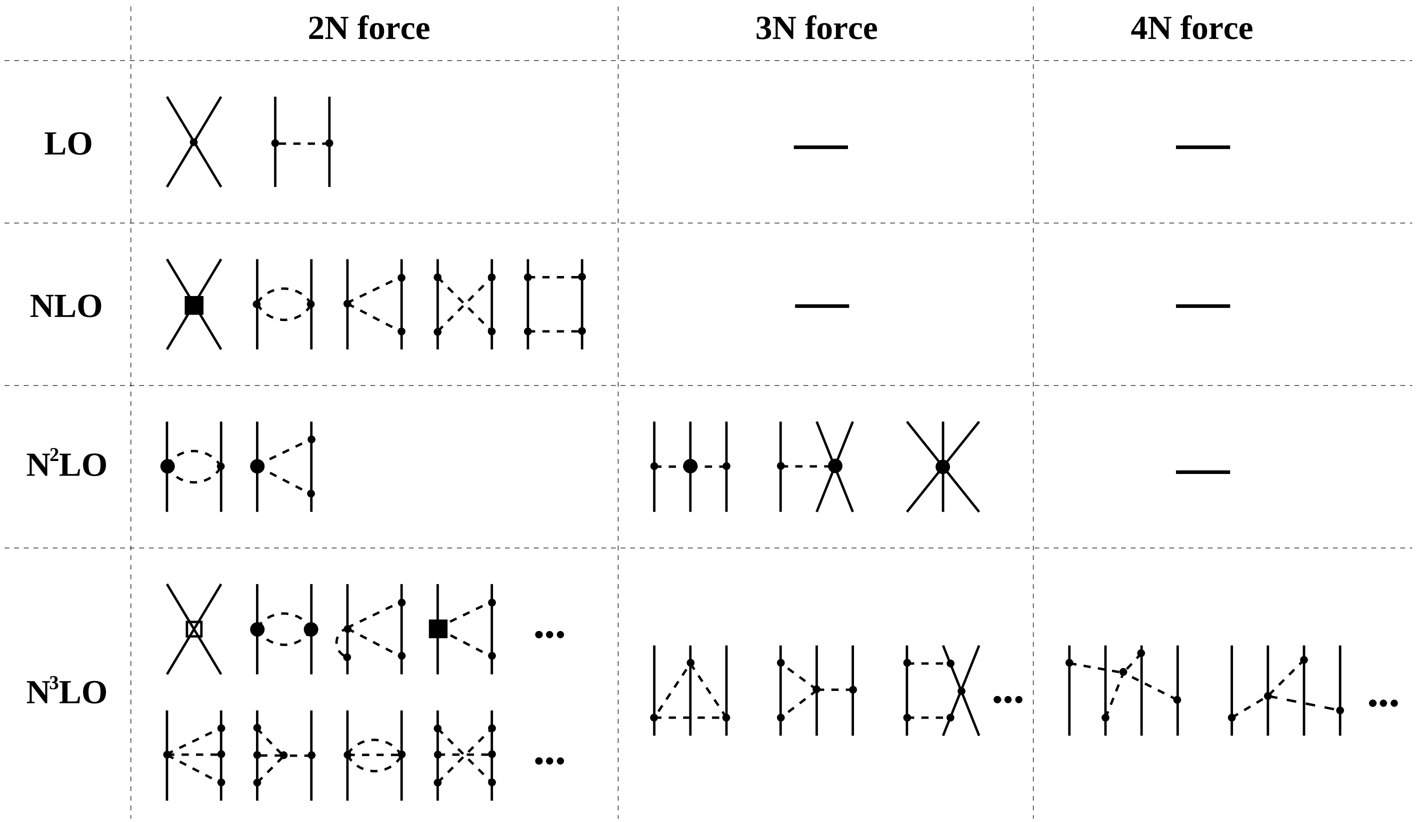
v^{R} is the shor range part and have a certain number of parameters (from 30 to 40) determined by fitting the NN data.

For example the AV18 potential is:

$$v(NN) = \sum_{p=1,18} v_p(r) O^p \quad \text{with}$$

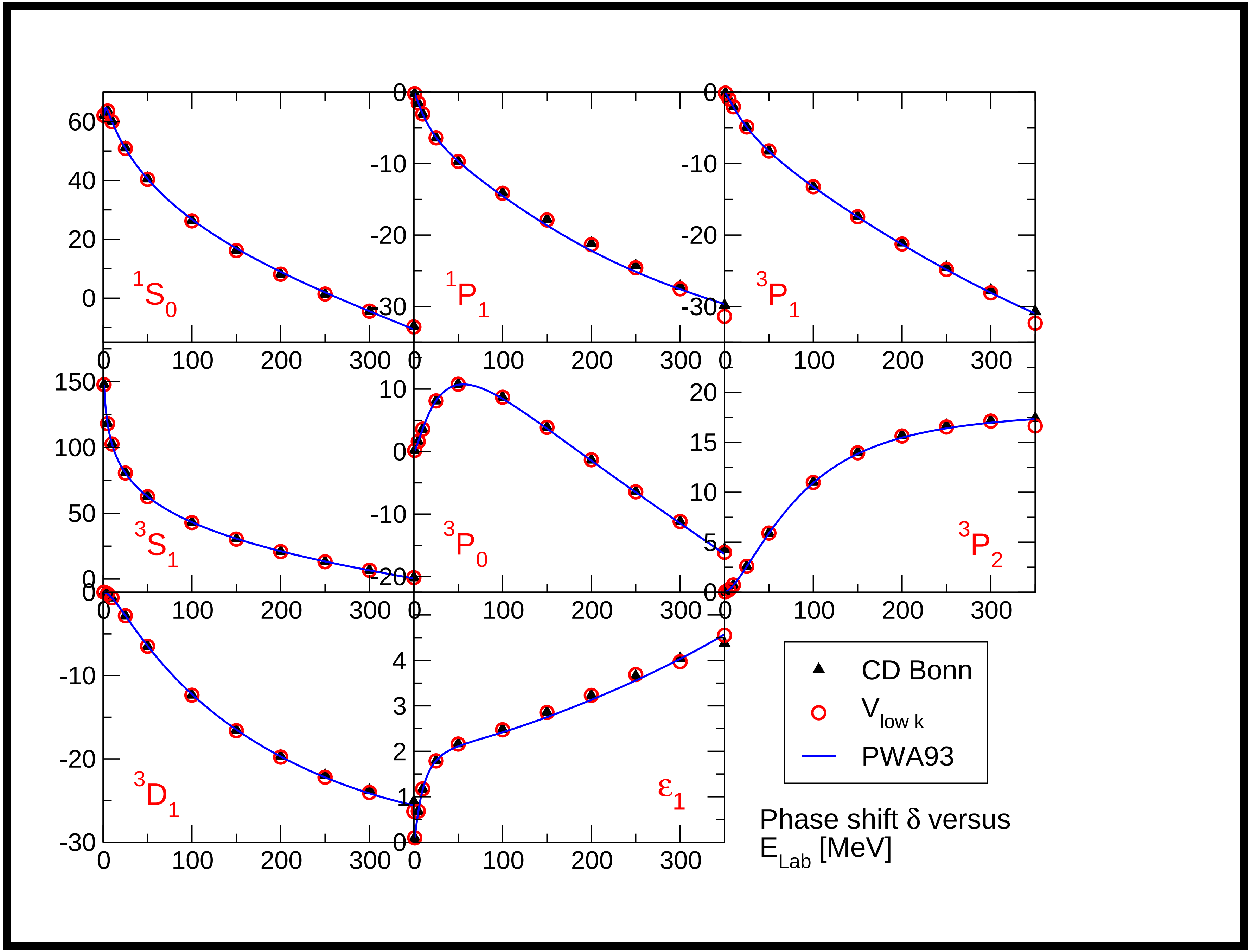
$$O^p = (1, \sigma_1 \cdot \sigma_2, S_{12}, L \cdot S, L^2, L^2 \sigma_1 \cdot \sigma_2, (L \cdot S)^2) \otimes (1, \tau_1 \cdot \tau_2)$$

NN potentials from CHPT



$$V^{\pi}(1,2) = f^2[Y_{\pi}(r)\sigma_1 \cdot \sigma_2 + T_{\pi}(r)S_{12}]$$

$$S_{12} = 3(\sigma_1 \cdot \hat{r}_{12})(\sigma_2 \cdot \hat{r}_{12}) - \sigma_1 \cdot \sigma_2$$



$^1S_0$
$^1P_1$
$^3P_1$
$^3S_1$
$^3P_0$
$^3P_2$
$^3D_1$
ε_1
CD Bonn
$V_{low\,k}$
PWA93
Phase shift δ versus
E_{Lab} [MeV]

E(MeV)	data	N^3LO	NNLO	NLO	AV18	CD Bonn
0-290	2402 (np)	1.10	10.1	36.2	1.04	1.03
0-290	2057 (pp)	1.50	35.4	80.1	1.30	1.10

Motivation

- Realistic NN potentials describe 2N data with $\chi^2 \approx 1$
- Realistic NN potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 4N data with $\chi^2 >> 1$
- The main disagreements are found in polarization observables even at low energies

Recent developments in the 3N potential

Two-pion Exchange Potentials

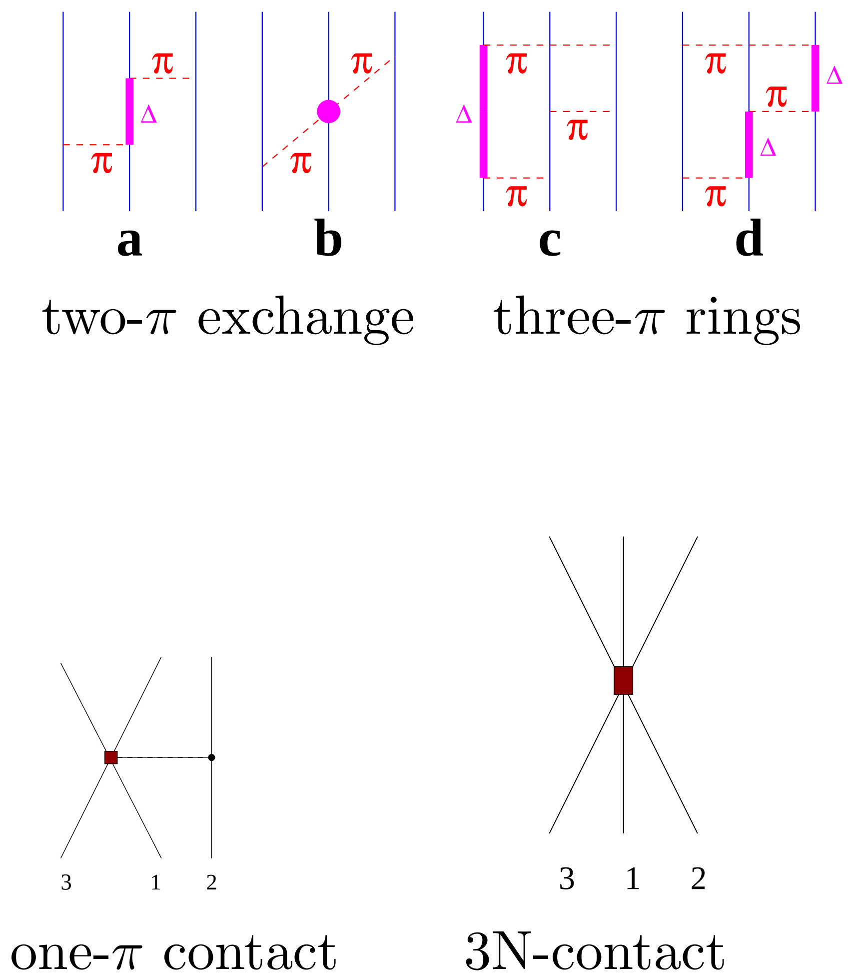
- TM' : S. Coon and Glöckle, PRC 23, 1790 (1981)
- Brazil : H.T. Coelho, T.K.Das and M.R. Robilotta, PRC 28, 1812 (1983)
- URIX : B.S. Pudliner *et al.*, PRL 51, 4396 (1995)

Two-pion Exchange Potentials+rings

- Illinois : S.C. Pieper, PRC 64, 014001 (2001)

3N Potential from chiral EFT

- N2LO non local : E. Epelbaum *et al.*, PRC 66, 064001 (2002)
- N2LO local : P. Navratil, FBS 41, 117 (2007)



π
Δ
a
b
c
d
two-π exchange
three-π rings
3
1
2
one-π contact
3N-contact

Experimental and GFMC energies

	method	GFMC	GFMC	GFMC	NCSM
nucleus	Exp.	AV18	AV18+UR	AV18+IL4	N3LO+N2LO
^{2}H(1^{+})	-2.2245	-2.2245			
^{3}H($\frac{1}{2}^{+}$)	-8.48	-7.61	-8.46	-8.44	-8.47
^{3}He($\frac{1}{2}^{+}$)	-7.72	-6.87	-7.71	-7.69	-7.73
^{4}He(0^{+})	-28.30	-24.07	-28.33	-28.35	-28.36
^{6}Li(1^{+})	-31.99	-26.9	-31.1	-32.0	-32.63
^{7}Li($\frac{3}{2}^{-}$)	-39.24	-31.6	-37.5	-39.5	
^{7}Li($\frac{1}{2}^{-}$)	-38.77	-31.1	-32.1	-39.0	

The 3N Potential

Urbana, TM, N2LO

$$W_{3N} = \sum_{i,j,k} W(i,j,k)$$

$$\begin{aligned}
W(1,2,3) = {} & C_1 (\tau_1 \cdot \tau_2)(\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{23}) y(r_{31}) y(r_{23}) \\
& + C_3 \{X_{23}, X_{31}\}\{\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\} \\
& + C_4 \left[X_{23}, X_{31}\right] \left[\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\right] \\
& + C_E \, (\tau_1 \cdot \tau_2) Z_0(r_{23}) Z_0(r_{31}) \\
& + C_D \, (\tau_1 \cdot \tau_2)\{(\sigma_1 \cdot \sigma_2)[y(r_{31}) Z_0(r_{23}) + y(r_{23}) Z_0(r_{31})] \\
& \qquad + (\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{31}) t(r_{31}) Z_0(r_{23}) \\
& \qquad + (\sigma_1 \cdot r_{23})(\sigma_2 \cdot r_{23}) t(r_{23}) Z_0(r_{31})\}
\end{aligned}$$

$$X_{ij} = t(r_{ij})(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij}) + y(r_{ij})(\sigma_i \cdot \sigma_j)$$

$$Z_0(r) = \frac{12\pi^2}{M_\pi^3}\frac{1}{2\pi^2}\int dq q^2 j_0(qr) f(q,\Lambda)$$

$$f_0(r) = \frac{12\pi^2}{M_\pi^3}\frac{1}{2\pi^2}\int dq q^2 j_0(qr)\frac{1}{q^2+M_\pi^2} f(q,\Lambda)$$

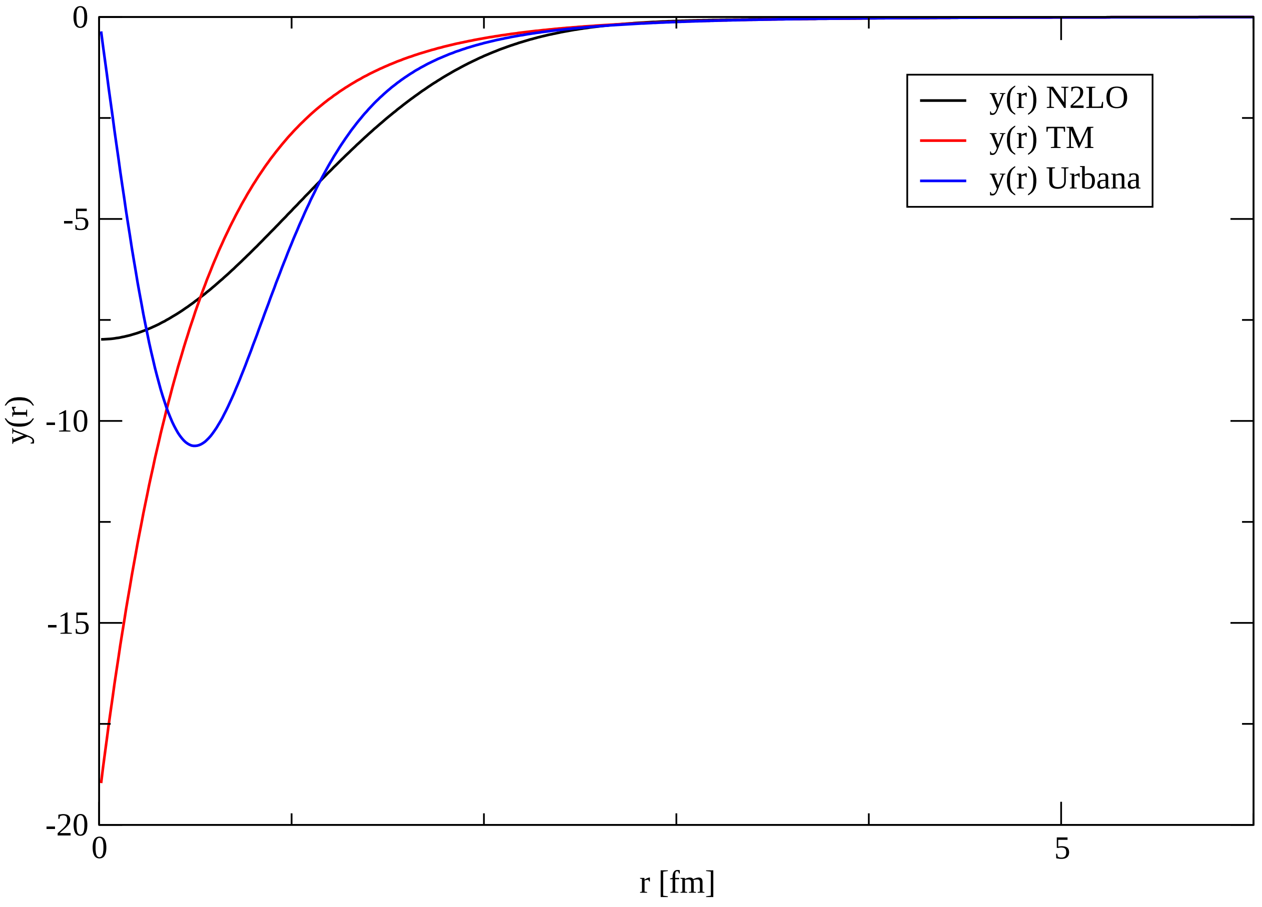
$$y = \frac{1}{r} f_0' \; ; t = \frac{1}{r} y'$$

$$y = T - Y \; ; t = \frac{3}{r^2} T$$

TM : $f(q,\Lambda) = \left(\frac{\Lambda^2 - M_\pi^2}{\Lambda^2 + q^2}\right)^2$ N2LO : $f(q,\Lambda) = \mathrm{e}^{-q^4/\Lambda^4}$

Urbana : $Y(x) = \frac{\mathrm{e}^{-x}}{x}\zeta(x)$; $T(x) = (1 + \frac{3}{x} + \frac{3}{x^2}) Y(x) \zeta(x)$

$Z_0 = T^2$; $\zeta(x) = (1 - \mathrm{e}^{-cr^2})$ $(c = 2.1 \mathrm{fm}^{-1})$



y(r) N2LO
y(r) TM
y(r) Urbana
0
-5
-10
-15
-20
y(r)
0
5
r [fm]

Potential	Method	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	HH	7.624	24.22	1.258
	FE/FY Bochum	7.621	24.23	1.248
	FE/FY Lisbon	7.621	24.24	
CDBonn	HH	7.998	26.13	
	FE/FY Bochum	8.005	26.16	0.925
	FE/FY Lisbon	7.998	26.11	
	NCSM	7.99(1)		
N3LO-Idaho	HH	7.854	25.38	1.100
	FE/FY Bochum	7.854	25.37	
	FE/FY Lisbon	7.854	25.38	
	NCSM	7.852(5)	25.39(1)	
AV18/UIX	HH	8.479	28.47	0.590
	FE/FY Bochum	8.476	28.53	0.578
CDBonn/TM	HH	8.474	29.00	
	FE/FY Bochum	8.482	29.09	0.570
N3LO-Idaho/N2LO	HH	8.474	28.37	0.675
	NCSM	8.473(5)	28.34(2)	
Exp.		8.48	28.30	0.645±0.010

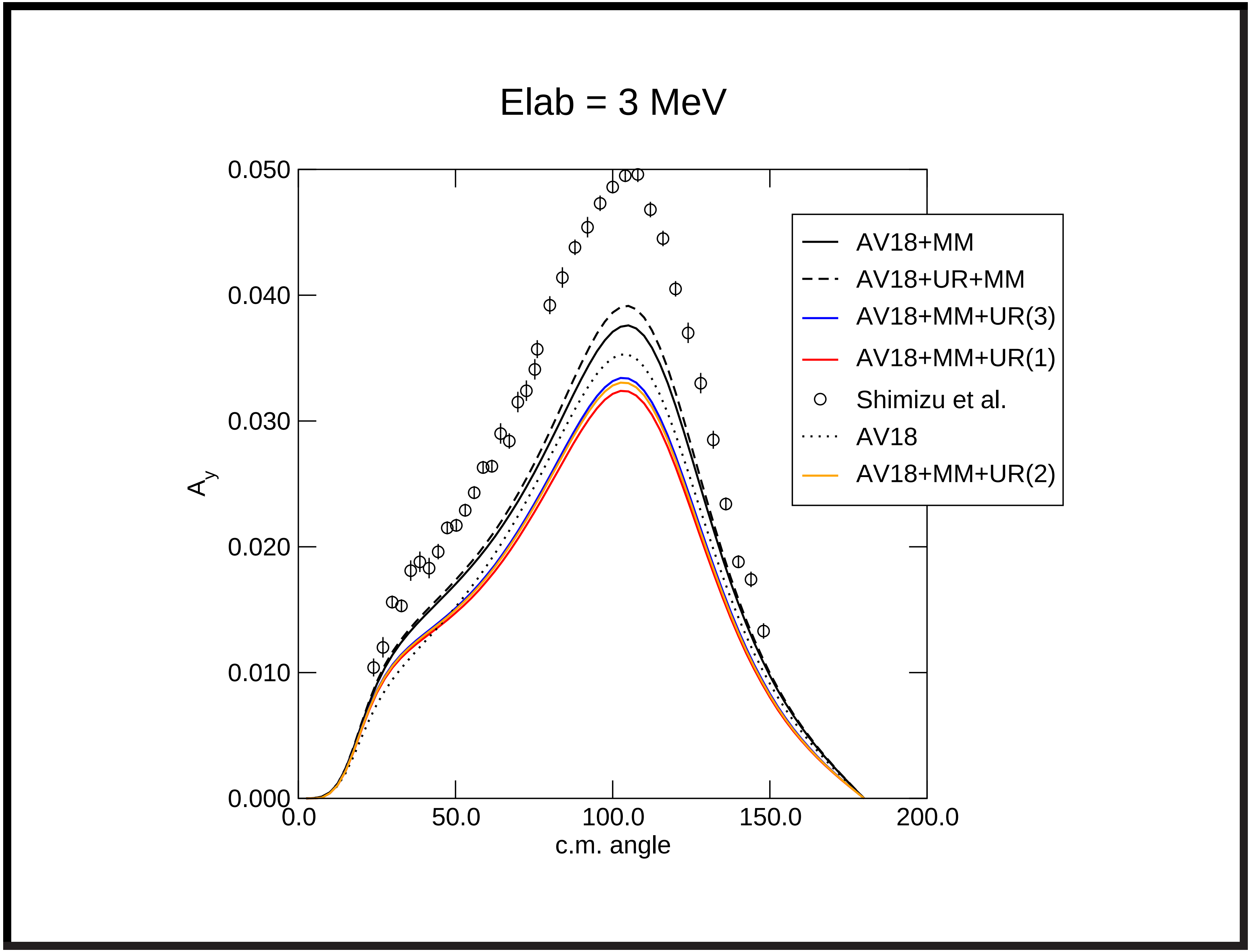
Fixing the 3N potential

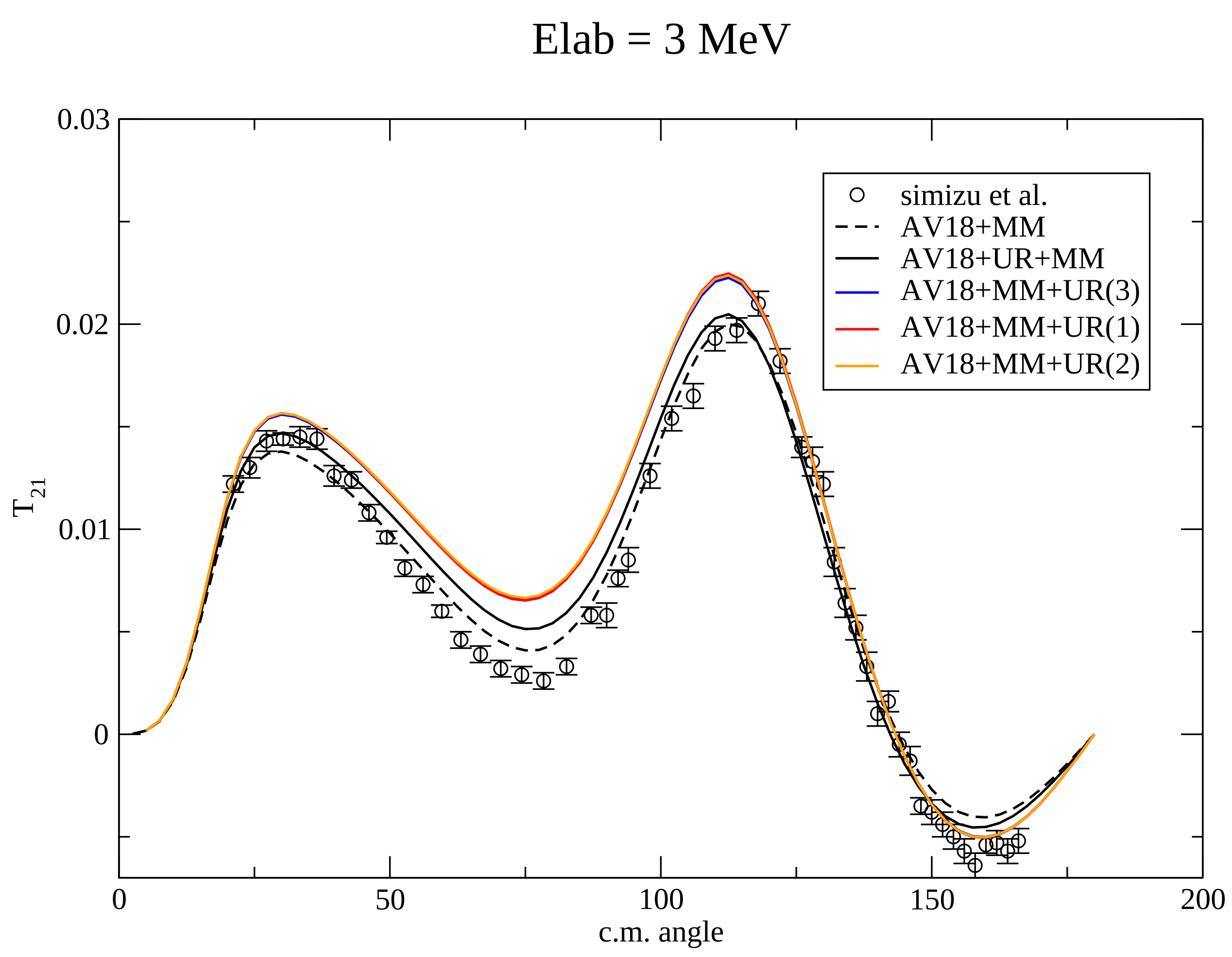
	C_1	C_3	C_4	C_E	C_D	Λ
Urbana	0	-0.029	$\frac{1}{4}C_3$	0.0048	0	
TM'	-0.76	-0.063	-0.018	0	0	$4.8M_\pi$
N2LO	-0.67	-0.043	-0.037	-0.0028	0.015	500

	^{3}H [MeV]	a_{nd} [fm]
AV18+Urbana	-8.479	0.590
AV18+TM'	-8.478	0.595
AV18+N2LO	-8.111	0.906
AV18+(1.4)N2LO	-8.478	0.654
N3LO+N2LO	-8.478	0.675

AV18+ Urbana

C_3	C_4	U_0	^{3}H	a_{nd}	^{4}He
-0.0293	$\frac{1}{4}C_3$	0.0048	-8.479	0.590	-28.47
-0.020	$\frac{6.5}{4}C_3$	0.018	-8.479	0.644	-28.33
-0.025	$\frac{5}{4}C_3$	0.018	-8.479	0.644	-28.33
-0.029	$\frac{4}{4}C_3$	0.018	-8.479	0.643	-28.33
-0.035	$\frac{3.25}{4}C_3$	0.019	-8.479	0.645	-28.33
-0.040	$\frac{2.5}{4}C_3$	0.018	-8.479	0.645	-28.33


Elab = 3 MeV
0.050
0.040
0.030
0.020
0.010
0.000
A_y
0.0
50.0
100.0
150.0
200.0
c.m. angle
AV18+MM
AV18+UR+MM
AV18+MM+UR(3)
AV18+MM+UR(1)
Shimizu et al.
AV18
AV18+MM+UR(2)

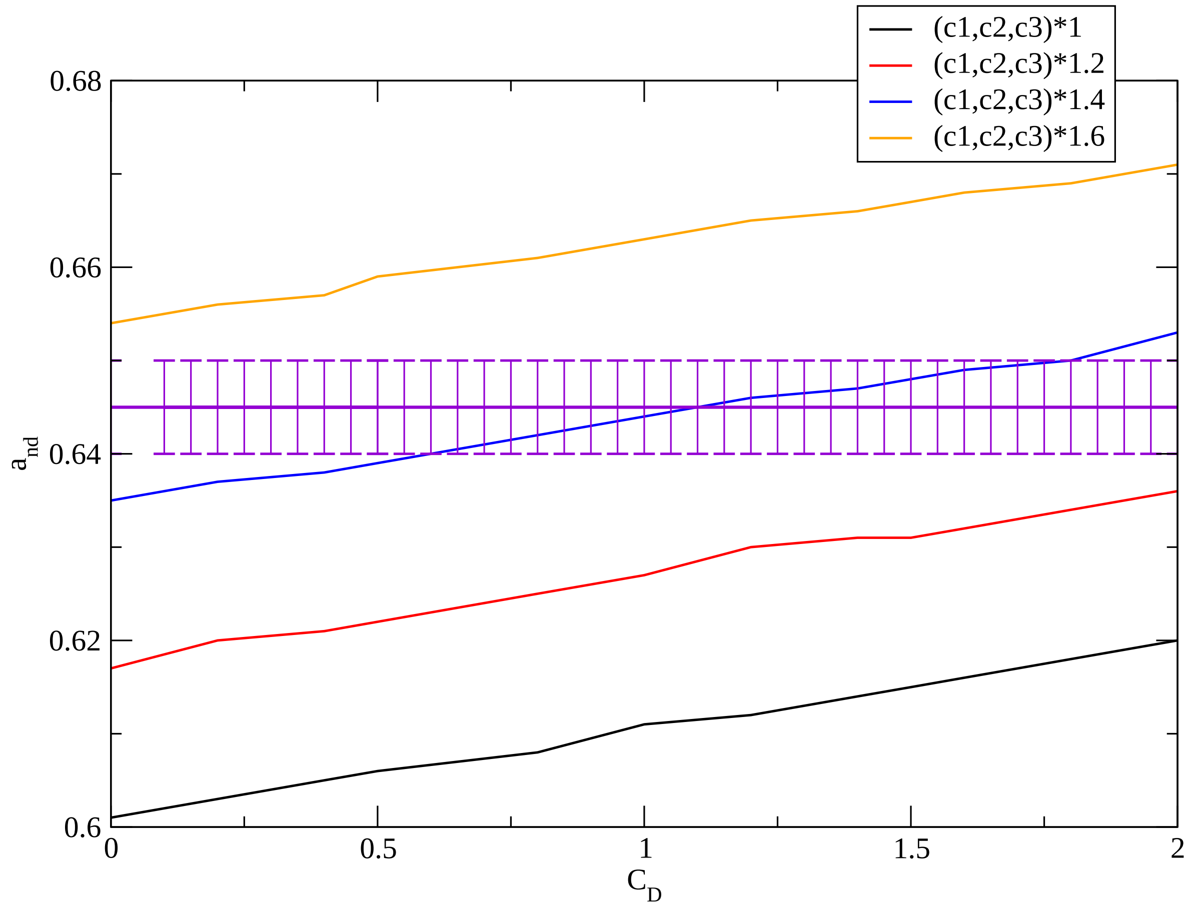


Elab = 3 MeV
0.03
0.02
0.01
0
T_{21}
0
50
100
150
200
c.m. angle
simizu et al.
AV18+MM
AV18+UR+MM
AV18+MM+UR(3)
AV18+MM+UR(1)
AV18+MM+UR(2)

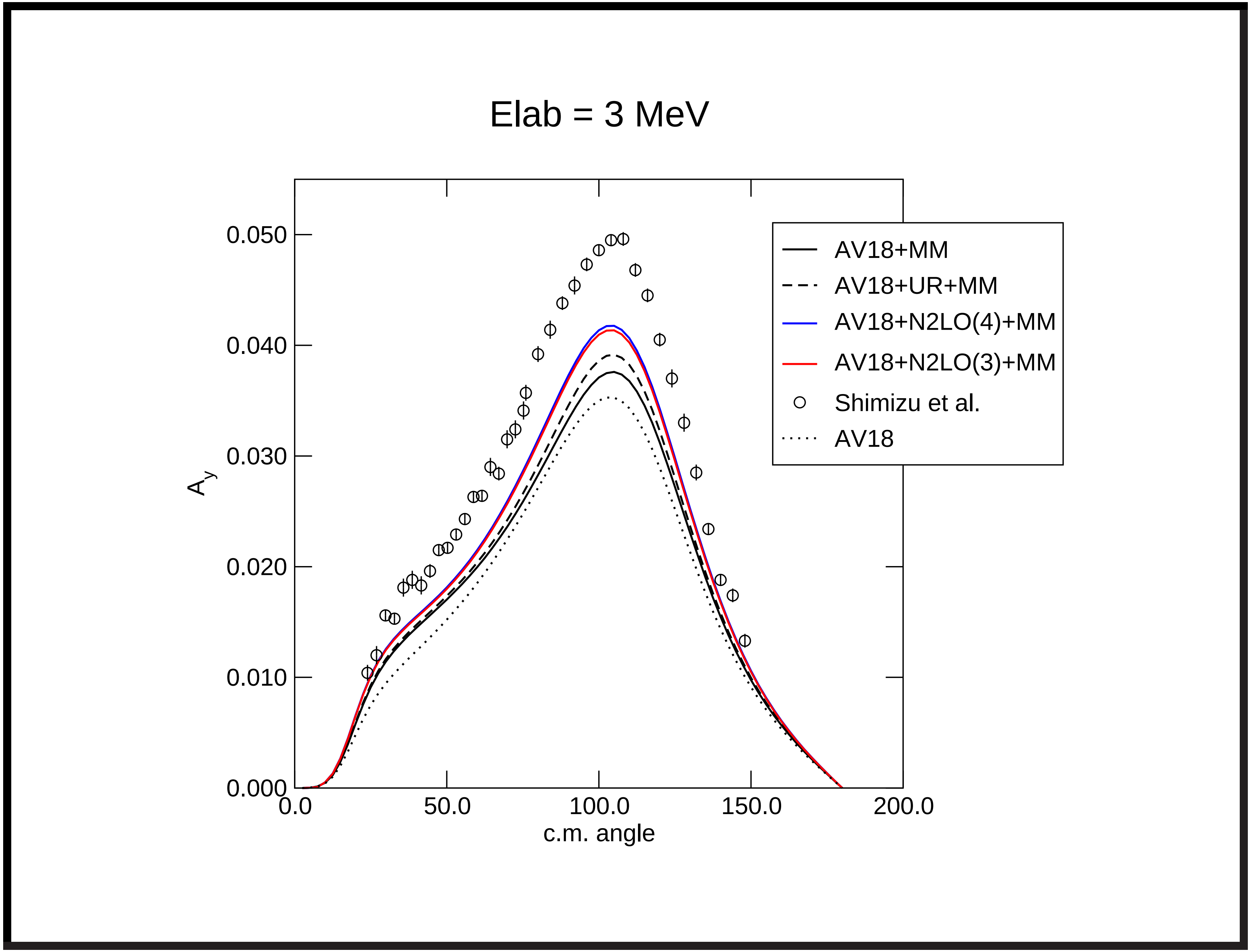
AV18+ TM

$a' M_\pi$	$b M_\pi^3$	$d M_\pi^3$	Λ	^{3}H	a_{nd}
-0.87	-2.00	-1.380	4.8	-8.479	0.591
-0.87	-2.29	-1.080	4.8	-8.479	0.593
-0.87	-2.58	-0.753	4.8	-8.479	0.595
-0.87	-3.00	-0.230	4.8	-8.479	0.594
-0.87	-2.58	-0.753	4.8	-8.479	0.595
-0.87	-2.58	-1.000	4.72	-8.479	0.597
-0.87	-2.58	-1.200	4.67	-8.479	0.596
-0.87	-2.58	-1.500	4.59	-8.479	0.596

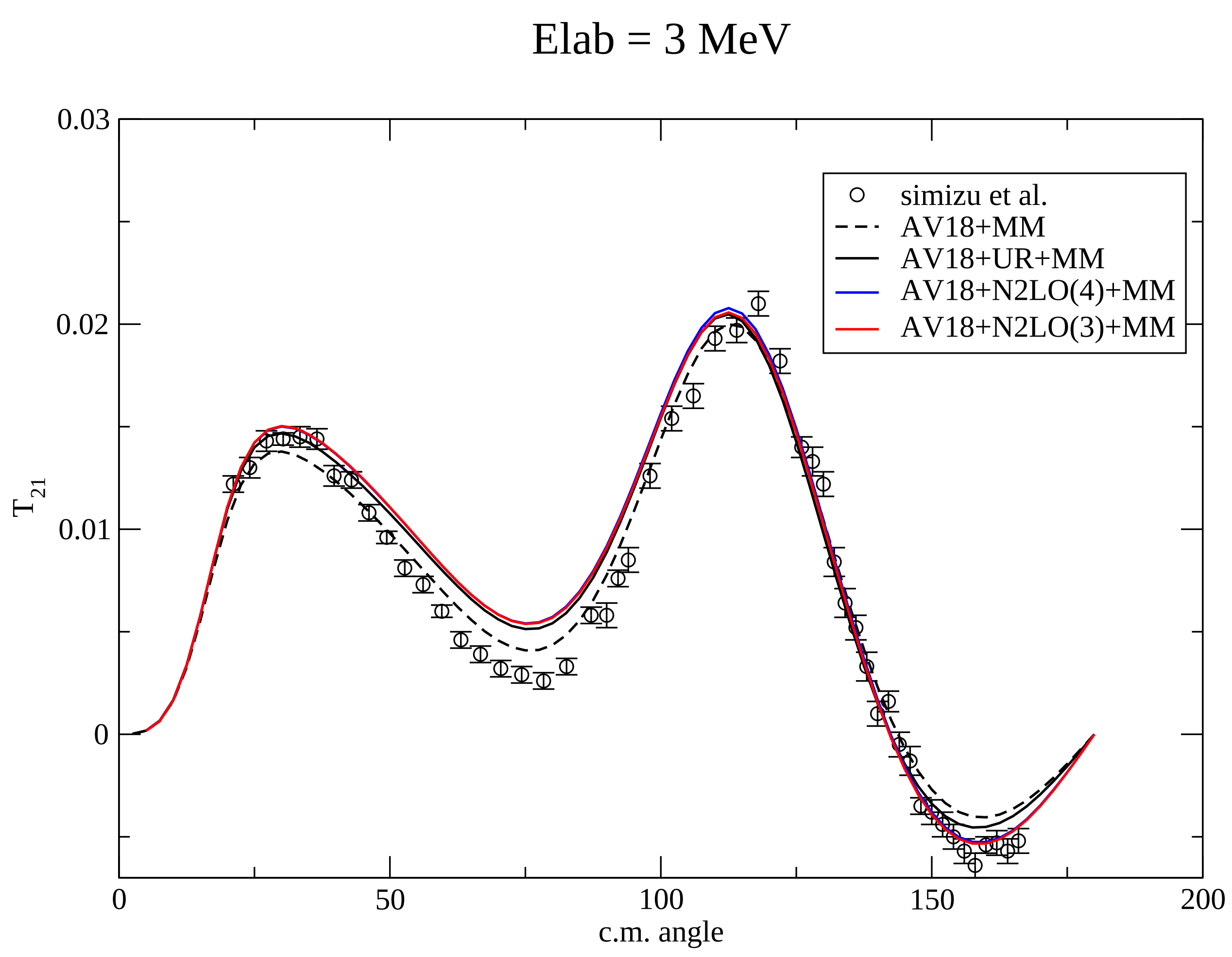
AV18+ N2LO



(c1,c2,c3)*1
(c1,c2,c3)*1.2
(c1,c2,c3)*1.4
(c1,c2,c3)*1.6
0.68
0.66
0.64
0.62
0.6
a_nd
0
0.5
1
1.5
2
C_D



Elab = 3 MeV
0.050
0.040
0.030
0.020
0.010
0.000
A_y
0.0
50.0
100.0
150.0
200.0
c.m. angle
AV18+MM
AV18+UR+MM
AV18+N2LO(4)+MM
AV18+N2LO(3)+MM
Shimizu et al.
AV18



Elab = 3 MeV
0.03
0.02
0.01
0
T_{21}
0
50
100
150
200
c.m. angle
simizu et al.
AV18+MM
AV18+UR+MM
AV18+N2LO(4)+MM
AV18+N2LO(3)+MM

Conclusions

- Actual NN+3N potential models do not fit simultaneously $B(^3\mathrm{H})$, $B(^4\mathrm{He})$, and $^{(2)}a_{nd}$
- 3N potentials are not “phase equivalent”
- The parameters in the 3N potentials can be varied to fit those quantities
- For Urbana the fit worsened some polarization observables
- The TM potential is attractive, therefore it is not possible to perform the fit without including a repulsive term
- For N2LO the fit was possible.
- Work is still in progress