Can one bind three electrons with one proton?

R. Brummelhuis, P. Duclos, R. Ruamps

Interdisciplinary Workshop on the Critical Stablity of Few-Body Quantum Systems

Ettore Majorama Centre for Scientific Culture

Erice 12-18th Octobre 2008

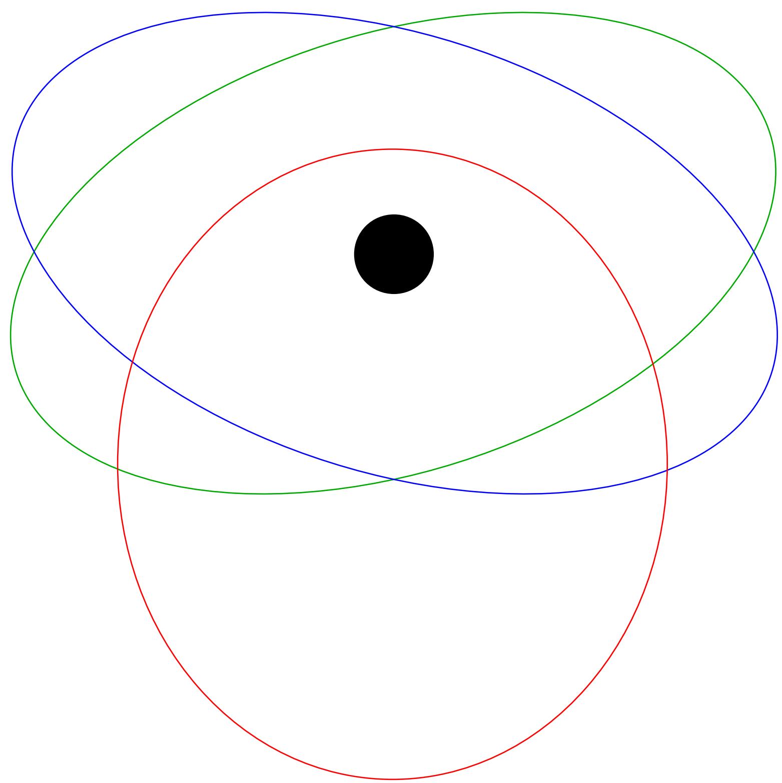
Can one bind three electrons with one proton?

R. Brummelhuis, P. Duclos, R. Ruamps

Interdisciplinary Workshop on the Critical Stablity of Few-Body Quantum Systems

Ettore Majorama Centre for Scientific Culture

Erice 12-18th Octobre 2008



Etats des lieux

$$H(N,Z) := \sum_{i=1}^{N} -\frac{\Delta_i}{2m_i} - \frac{Z}{|r_i|} + \sum_{i<j} \frac{1}{|r_i - r_j|}, \qquad L^2(\mathbb{R}^{3N})$$

- H^{--} does not exist Lieb'84, H^{-} exists Hill'77

Etats des lieux

$$H(N,Z) := \sum_{i=1}^{N} -\frac{\Delta_i}{2m_i} - \frac{Z}{|r_i|} + \sum_{i<j} \frac{1}{|r_i - r_j|}, \qquad L^2(\mathbb{R}^{3N})$$

- H^{--} does not exist Lieb'84, H^{-} exists Hill'77
- Critical values

$$\begin{aligned} Z_c(N) &:= \inf\{Z,\ \mathrm{spect}_{\,d} H(N,Z) \neq \emptyset\} \\ N_c(Z) &:= \sup\{N,\ \mathrm{spect}_{\,d} H(N,Z) \neq \emptyset\} \end{aligned}$$

Etats des lieux

$$H(N,Z) := \sum_{i=1}^{N} -\frac{\Delta_i}{2m_i} - \frac{Z}{|r_i|} + \sum_{i<j} \frac{1}{|r_i - r_j|}, \qquad L^2(\mathbb{R}^{3N})$$

- H^{--} does not exist Lieb'84, H^{-} exists Hill'77
- Critical values

$$\begin{aligned} Z_c(N) &:= \inf\{Z, \operatorname{spect}_d H(N,Z) \neq \emptyset\} \\ N_c(Z) &:= \sup\{N, \operatorname{spect}_d H(N,Z) \neq \emptyset\} \end{aligned}$$

$Z_c(2) = 0.911$ SS'66 $Z_c(3) > 1$ Lieb'84

Etats des lieux

$$H(N,Z) := \sum_{i=1}^{N} -\frac{\Delta_i}{2m_i} - \frac{Z}{|r_i|} + \sum_{i<j} \frac{1}{|r_i - r_j|}, \qquad L^2(\mathbb{R}^{3N})$$

- H^{--} does not exist Lieb'84, H^{-} exists Hill'77
- Critical values

$$\begin{aligned} Z_c(N) &:= \inf\{Z, \operatorname{spect}_d H(N,Z) \neq \emptyset\} \\ N_c(Z) &:= \sup\{N, \operatorname{spect}_d H(N,Z) \neq \emptyset\} \end{aligned}$$

$Z_c(2) = 0.911$ SS'66 $Z_c(3) > 1$ Lieb'84

Feldmann & King' 08 $2.000001 \geq Z_c(3) > 2$

Motivations I

$$H^B(N,Z) := \sum_{j=1}^{N} \frac{(-i\nabla_j - \frac{1}{2}\vec{B} \wedge r_j)^2}{2m} - \frac{Z}{|r_j|} + \sum_{j<k} \frac{1}{|r_j - r_k|},$$

Motivations I

$$H^B(N,Z) := \sum_{j=1}^{N} \frac{(-i\nabla_j - \frac{1}{2}\vec{B} \wedge r_j)^2}{2m} - \frac{Z}{|r_j|} + \sum_{j<k} \frac{1}{|r_j - r_k|},$$

- $N_c(B,Z) := \max\{N, \mathrm{spect}\,_d H^B(N,Z) \neq \emptyset\}$

LSY' 94, $$\lim_{Z \& \frac{B}{Z^3} \to \infty} \frac{N_c(B,Z)}{Z} \geq 2,$$

Motivations I

$$H^B(N,Z) := \sum_{j=1}^{N} \frac{(-i\nabla_j - \frac{1}{2}\vec{B} \wedge r_j)^2}{2m} - \frac{Z}{|r_j|} + \sum_{j<k} \frac{1}{|r_j - r_k|},$$

- $N_c(B,Z) := \max\{N, \mathrm{spect}\,{}_d H^B(N,Z) \neq \emptyset\}$

  LSY' 94, $\displaystyle \lim_{Z \& \frac{B}{Z^3} \to \infty} \frac{N_c(B,Z)}{Z} \geq 2,$

- LSY'94 conjecture

$$\lim_{Z \& \frac{B}{Z^3} \to \infty} \frac{N_c(B,Z)}{Z} = 2,$$

Motivations I

$$H^B(N,Z) := \sum_{j=1}^{N} \frac{(-i\nabla_j - \frac{1}{2}\vec{B}\wedge r_j)^2}{2m} - \frac{Z}{|r_j|} + \sum_{j<k} \frac{1}{|r_j - r_k|},$$

- $N_c(B,Z) := \max\{N, \mathrm{spect}_{\,d} H^B(N,Z) \neq \emptyset\}$

LSY' 94, $$\lim_{Z \& \frac{B}{Z^3} \to \infty} \frac{N_c(B,Z)}{Z} \geq 2,$$

- LSY'94 conjecture

$$\lim_{Z \& \frac{B}{Z^3} \to \infty} \frac{N_c(B,Z)}{Z} = 2,$$

- Can one have $\sup_{B,Z} N_c(B,Z)/Z > 2$?

Reduction to a one dimensional pb

$$h(N,Z) := \sum_{j=1}^{N} -\frac{1}{2}\partial_{x_j}^2 - Z\delta(x_j) + \sum_{i<j} \delta(x_i - x_j), \quad L^2(\mathbb{R}^N)$$

- BSY'00 and BD'00

Reduction to a one dimensional pb

$$h(N,Z) := \sum_{j=1}^{N} -\frac{1}{2}\partial_{x_j}^2 - Z\delta(x_j) + \sum_{i<j}\delta(x_i - x_j), \quad L^2(\mathbb{R}^N)$$

- BSY'00 and BD'00

- Theorem BD'06 Let $d_\delta(\xi) :=$ dist $\left(\xi, \sigma(h_\delta^{B,\mathbb{M}})\right)$ and $\alpha \sim \log(\sqrt{B})$ as $B \to \infty$. There exist $0 < c_\delta < C_\delta$, $B_\delta > 0$ depending on N, Z and $\mathbb{M}$, such that for all $B \geq B_\delta$ and real ξ satisfying

$$c_\delta \alpha \leq d_\delta(\xi) \leq \frac{1}{4}\alpha^2, \quad \Rightarrow \quad \xi \in \rho(H^{B,\mathbb{M}})$$

$$\|(H^{B,\mathbb{M}} - \xi)^{-1} - (h_\delta^{B,\mathbb{M}} - \xi)^{-1}\| \leq \frac{C_\delta\, \alpha}{d_\delta(\xi)^2}.$$

Motivations II

Can one bind three electrons with one hole in a one dimensional periodic material?

Motivations II

Can one bind three electrons with one hole in a one dimensional periodic material?

- Pedersen'03 effective model of SW carbon nanotube

Motivations II

Can one bind three electrons with one hole in a one dimensional periodic material?

- Pedersen'03 effective model of SW carbon nanotube

- The Rosenthal'71 critical charge:

$$\hat{Z}(2) := Z_c(B = \infty, 2) \simeq 0.375 \quad \Rightarrow \quad \sup_{B,Z} N_c(B, Z)/Z \geq 5.33$$

and violates the $N < 2Z + 1$ Lieb bound.

Motivations II

Can one bind three electrons with one hole in a one dimensional periodic material?

- Pedersen'03 effective model of SW carbon nanotube

- The Rosenthal'71 critical charge:

$$\hat{Z}(2) := Z_c(B = \infty, 2) \simeq 0.375 \quad \Rightarrow \quad \sup_{B,Z} N_c(B, Z)/Z \geq 5.33$$

and violates the $N < 2Z + 1$ Lieb bound.

- What is the value of $\hat{Z}(3) := Z_c(B = \infty, 3)$?

Particules exchange symmetries

$$\forall \sigma \in S_3, \quad (\Pi(\sigma)\psi)(x) = \psi(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}), \quad L^2(\mathbb{R}^3)$$

Particules exchange symmetries

$$\forall \sigma \in S_3, \quad (\Pi(\sigma)\psi)(x) = \psi\big(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}\big), \quad L^2(\mathbb{R}^3)$$

$$P_{\text{sym}} := \frac{1}{3!}\sum_{\sigma\in S_3}\Pi(\sigma), \quad P_{\text{antisym}} := \frac{1}{3!}\sum_{\sigma\in S_3}(-1)^\sigma\Pi(\sigma)$$

$$P_2 := \frac{1}{3}\Big(2\mathrm{id} - \Pi(\overline{312}) - \Pi(\overline{231})\Big)$$

Particules exchange symmetries

$$\forall \sigma \in S_3, \quad (\Pi(\sigma)\psi)(x) = \psi(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}), \quad L^2(\mathbb{R}^3)$$

$$P_{\text{sym}} := \frac{1}{3!}\sum_{\sigma\in S_3} \Pi(\sigma), \quad P_{\text{antisym}} := \frac{1}{3!}\sum_{\sigma\in S_3} (-1)^\sigma \Pi(\sigma)$$

$$P_2 := \frac{1}{3}\big(2\text{id} - \Pi(\overline{312}) - \Pi(\overline{231})\big)$$

$$P_{\text{sym}} \oplus P_{\text{antisym}} \oplus P_2 = 1, \quad [\Pi(\sigma), h(N,Z)] = 0$$

$$h(N,Z) = h_{\text{sym}} \oplus h_{\text{antisym}} \oplus h_2,$$

Naive variationnal approaches

Naive variationnal approaches

$$\Phi(x) = \bigotimes_{i=1}^{3} \sqrt{a} e^{-a|x_i|}, \qquad (h(N,Z)\Phi,\Phi)/3 = -a + \frac{a}{2Z} + \frac{a^2}{2}$$

optimizing over a $\quad (h(N,Z)\Phi,\Phi) = -\frac{3(1-2Z)^2}{8Z^2}$

Naive variationnal approaches

$$\Phi(x) = \bigotimes_{i=1}^{3} \sqrt{a} e^{-a|x_i|}, \quad (h(N,Z)\Phi,\Phi)/3 = -a + \frac{a}{2Z} + \frac{a^2}{2}$$

optimizing over a $\quad (h(N,Z)\Phi,\Phi) = -\frac{3(1-2Z)^2}{8Z^2}$

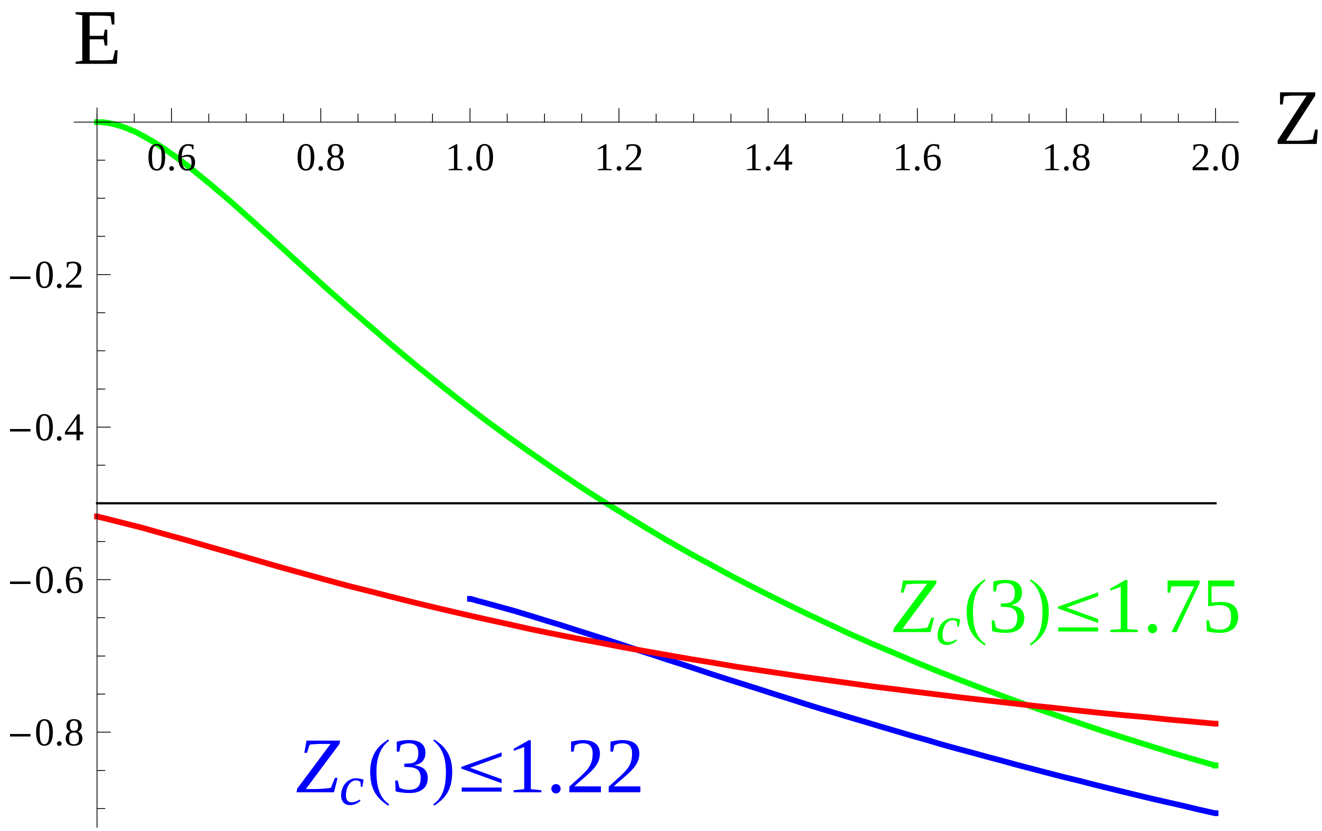
BSY'00 Theorem. *Let*

$$\tilde{h}(N,Z) = \sum_{i=1}^{N} -\frac{\Delta_i}{2} - Z\delta(x_i) + \frac{1}{2}\left(\delta(x_i - x_j) + \delta(x_i + x_j)\right)$$

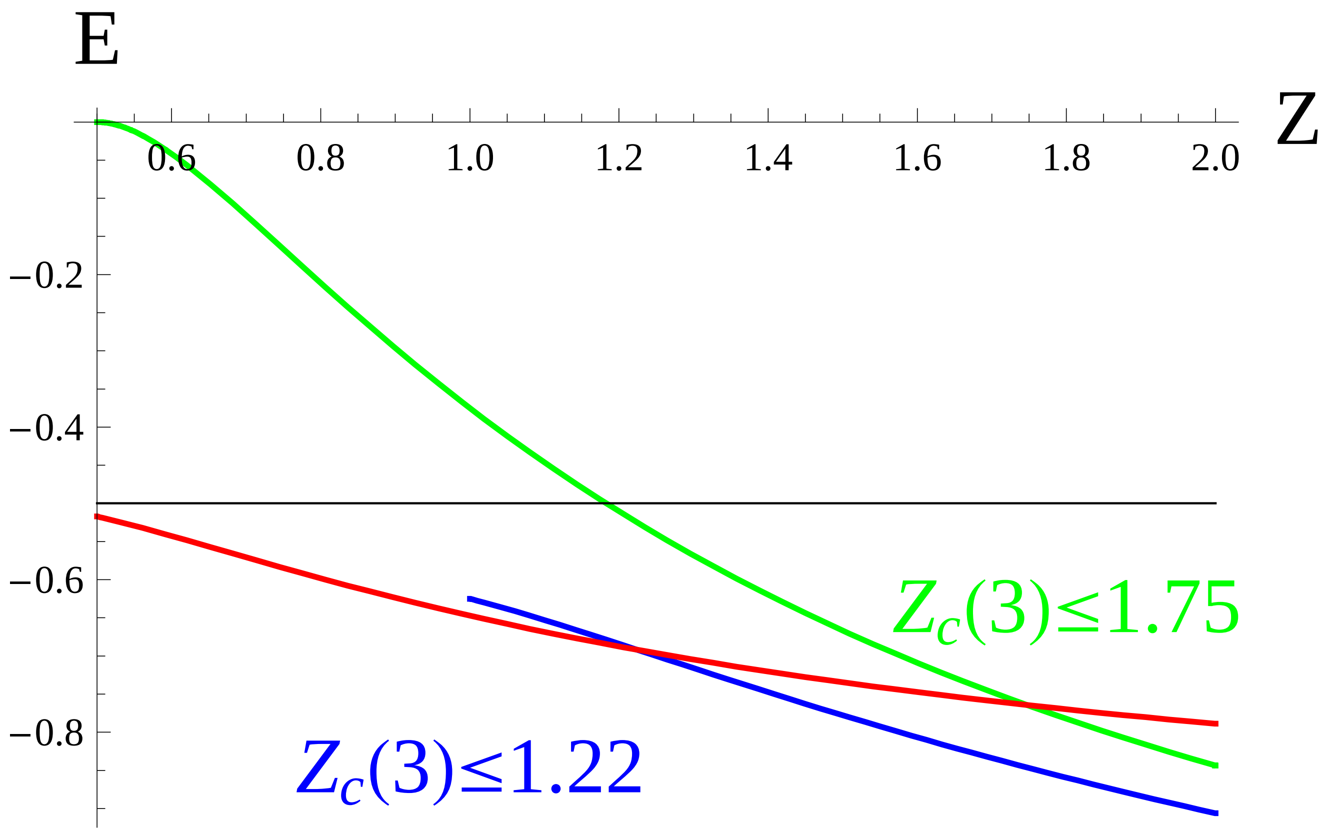
then if $N \leq 2Z+1$ *one has* $\inf h(N,Z) \leq \inf \tilde{h}(N,Z)$ *with*

$$\tilde{h}(N,Z) = -\frac{N\left(2N^2 + 12Z(2Z+1) - 3(4ZN+N) + 1\right)}{96Z^2}$$

Naive variationnal approaches



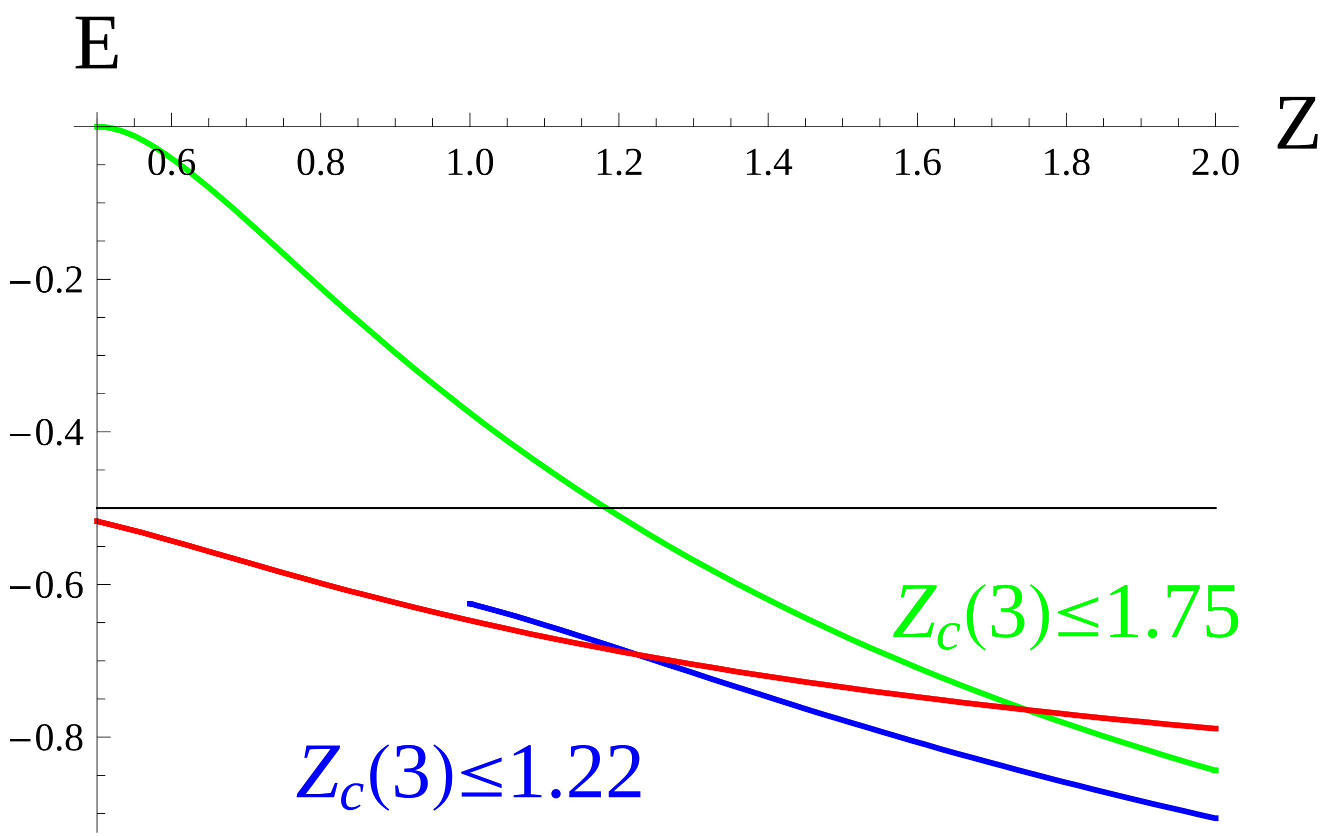
Naive variationnal approaches



$$\sup_{B,Z} N_c(B,Z)/Z \geq 2.45$$

Naive variationnal approaches

$$P_{\text{sym}} \bigotimes_{i=1}^{2} \sqrt{a} e^{-a|x_i|} \otimes \sqrt{b} e^{-b|x_3|} \quad \Rightarrow \quad \hat{Z}_c(3) \leq 1.54$$



$$\sup_{B,Z} N_c(B,Z)/Z \geq 2.45$$

The Skeleton method

$$\tau_i : \mathcal{H}^1(\mathbb{R}^3) \to L^2(\mathbb{R}^2), \quad i = 1,2,3,4,5,6$$

The Skeleton method

$$\tau_i : \mathcal{H}^1(\mathbb{R}^3) \to L^2(\mathbb{R}^2), \quad i = 1,2,3,4,5,6$$

$$h = -\frac{\Delta}{2} + \tau^\star g \tau, \quad g := \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} \end{pmatrix} \quad \lambda := \frac{1}{Z}$$

The Skeleton method

$$\tau_i : \mathcal{H}^1(\mathbb{R}^3) \to L^2(\mathbb{R}^2), \quad i = 1,2,3,4,5,6$$

$$h = -\frac{\Delta}{2} + \tau^\star g \tau, \quad g := \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} \end{pmatrix} \quad \lambda := \frac{1}{Z}$$

$R(z) := (h - z)^{-1}$ and $R_0(z) := (-\frac{\Delta}{2} - z)^{-1}$ in $L^2(\mathbb{R}^3)$

The Skeleton method

$$\tau_i : \mathcal{H}^1(I\!R^3) \to L^2(I\!R^2), \quad i = 1,2,3,4,5,6$$

$$h = -\frac{\Delta}{2} + \tau^\star g \tau, \quad g := \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda}{\sqrt{2}} \end{pmatrix} \quad \lambda := \frac{1}{Z}$$

$R(z) := (h - z)^{-1}$ and $R_0(z) := (-\frac{\Delta}{2} - z)^{-1}$ in $L^2(I\!R^3)$

$$R(z) = R_0(z) - R_0(z)\tau^\star(g^{-1} + \tau R_0(z)\tau^\star)^{-1}\tau R_0(z)$$

The Skeleton method

$$\Sigma > E := -k^2 \in \mathrm{spect}_d h(3, Z) \iff \ker S(k) \neq \{0\}, \quad \Sigma := \inf h(2, Z)$$

The Skeleton method

$$\Sigma > E := -k^2 \in \mathrm{spect}_d h(3,Z) \iff \ker S(k) \neq \{0\}, \quad \Sigma := \inf h(2,Z)$$

$$S(k) := \ker g^{-1} k + \tau R_0(-1)\tau^\star : \bigoplus_{i=1}^{6} L^2(\mathbb{R}^2) \to \bigoplus_{i=1}^{6} L^2(\mathbb{R}^2)$$

The Skeleton method

$$\Sigma > E := -k^2 \in \mathrm{spect}_d h(3, Z) \iff \ker S(k) \neq \{0\}, \quad \Sigma := \inf h(2, Z)$$

$$S(k) := \ker g^{-1} k + \tau R_0(-1)\tau^\star : \bigoplus_{i=1}^{6} L^2(\mathbb{R}^2) \to \bigoplus_{i=1}^{6} L^2(\mathbb{R}^2)$$

$$\begin{pmatrix} -k + T_0 & T_{\frac{\pi}{2}} & T^\star_{\frac{\pi}{2}} & T_{\frac{\pi}{4}} & \widetilde{T}_{\frac{\pi}{2}} & \varepsilon T_{\frac{\pi}{4}} \\ T^\star_{\frac{\pi}{2}} & -k + T_0 & T_{\frac{\pi}{2}} & \varepsilon T_{\frac{\pi}{4}} & T_{\frac{\pi}{4}} & \widetilde{T}_{\frac{\pi}{2}} \\ T_{\frac{\pi}{2}} & T^\star_{\frac{\pi}{2}} & -k + T_0 & \widetilde{T}_{\frac{\pi}{4}} & \varepsilon T_{\frac{\pi}{4}} & T_{\frac{\pi}{4}} \\ & & & \frac{\sqrt{2}}{\lambda} k + T_0 & T_{\frac{\pi}{3}} & T_{\frac{\pi}{3}} \\ & & & T_{\frac{\pi}{3}} & \frac{\sqrt{2}}{\lambda} k + T_0 & T_{\frac{\pi}{3}} \\ & & & T_{\frac{\pi}{3}} & T_{\frac{\pi}{3}} & \frac{\sqrt{2}}{\lambda} k + T_0 \end{pmatrix}$$

The Skeleton method

equ.	basis	normal	trace op.
$x_1 = 0$	$b^{(1)} := \{A_2, A_3\}$	A_1	τ_1
$x_2 = 0$	$b^{(2)} := \{A_3, A_1\}$	A_2	τ_2
$x_3 = 0$	$b^{(3)} := \{A_1, A_2\}$	A_3	τ_3
$x_1 = x_2$	$b^{(4)} := \{\frac{A_1+A_2}{\sqrt{2}}, A_3\}$	$\frac{-A_2+A_1}{\sqrt{2}} =: A_4$	$\tau_4 = \tau_{1,2}$
$x_2 = x_3$	$b^{(5)} := \{\frac{A_2+A_3}{\sqrt{2}}, A_1\}$	$\frac{-A_3+A_2}{\sqrt{2}} =: A_5$	$\tau_5 = \tau_{2,3}$
$x_3 = x_1$	$b^{(6)} := \{\frac{A_3+A_1}{\sqrt{2}}, A_2\}$	$\frac{-A_1+A_3}{\sqrt{2}} =: A_6$	$\tau_6 = \tau_{3,1}$

.

The Skeleton method

equ.	basis	normal	trace op.
$x_1 = 0$	$b^{(1)} := \{A_2, A_3\}$	A_1	τ_1
$x_2 = 0$	$b^{(2)} := \{A_3, A_1\}$	A_2	τ_2
$x_3 = 0$	$b^{(3)} := \{A_1, A_2\}$	A_3	τ_3
$x_1 = x_2$	$b^{(4)} := \{\frac{A_1+A_2}{\sqrt{2}}, A_3\}$	$\frac{-A_2+A_1}{\sqrt{2}} =: A_4$	$\tau_4 = \tau_{1,2}$
$x_2 = x_3$	$b^{(5)} := \{\frac{A_2+A_3}{\sqrt{2}}, A_1\}$	$\frac{-A_3+A_2}{\sqrt{2}} =: A_5$	$\tau_5 = \tau_{2,3}$
$x_3 = x_1$	$b^{(6)} := \{\frac{A_3+A_1}{\sqrt{2}}, A_2\}$	$\frac{-A_1+A_3}{\sqrt{2}} =: A_6$	$\tau_6 = \tau_{3,1}$

.

$$T_{\frac{\pi}{2}}(p,q) = \frac{\delta(q_1 - p_2)}{\pi\left(p_1^2 + p_2^2 + q_2^2 + 2\right)}$$

The Skeleton method

equ.	basis	normal	trace op.
$x_1 = 0$	$b^{(1)} := \{A_2, A_3\}$	A_1	τ_1
$x_2 = 0$	$b^{(2)} := \{A_3, A_1\}$	A_2	τ_2
$x_3 = 0$	$b^{(3)} := \{A_1, A_2\}$	A_3	τ_3
$x_1 = x_2$	$b^{(4)} := \{\frac{A_1+A_2}{\sqrt{2}}, A_3\}$	$\frac{-A_2+A_1}{\sqrt{2}} =: A_4$	$\tau_4 = \tau_{1,2}$
$x_2 = x_3$	$b^{(5)} := \{\frac{A_2+A_3}{\sqrt{2}}, A_1\}$	$\frac{-A_3+A_2}{\sqrt{2}} =: A_5$	$\tau_5 = \tau_{2,3}$
$x_3 = x_1$	$b^{(6)} := \{\frac{A_3+A_1}{\sqrt{2}}, A_2\}$	$\frac{-A_1+A_3}{\sqrt{2}} =: A_6$	$\tau_6 = \tau_{3,1}$

.

$$T_{\frac{\pi}{2}}(p,q) = \frac{\delta(q_1 - p_2)}{\pi\left(p_1^2 + p_2^2 + q_2^2 + 2\right)}$$

$$T_0(p,q) = \frac{\delta(p-q)}{\sqrt{p^2+2}}.$$

The Effective Skeleton

Using symmetries

$$S_{\text{eff}}(k) := \begin{pmatrix} -k + T_0 + 2\hat{T}_{\frac{\pi}{2}} & 3\hat{T}_{\frac{\pi}{4}} \\ 3\hat{T}^{\star}_{\frac{\pi}{4}} & \frac{\sqrt{2}}{\lambda}k + T_0 + 2T_{\frac{\pi}{3}} \end{pmatrix}$$

with the following definitions

$$\hat{T}_{\frac{\pi}{2}} := \frac{1}{2}\left(T_{\frac{\pi}{2}} + T^{\star}_{\frac{\pi}{2}}\right), \quad \hat{T}_{\frac{\pi}{4}} := \frac{1}{3}\left((1+\varepsilon)T_{\frac{\pi}{4}} + \widetilde{T}_{\frac{\pi}{2}}\right)$$

Operator pencil to eigenvalue pb

$$\begin{pmatrix} 1 & 0 \\ 0 & -\frac{\lambda}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -k + T_0 + 2\hat{T}_{\frac{\pi}{2}} & 3\hat{T}_{\frac{\pi}{4}} \\ 3\hat{T}^{\star}_{\frac{\pi}{4}} & \frac{\sqrt{2}}{\lambda} k + T_0 + 2T_{\frac{\pi}{3}} \end{pmatrix}$$

$$= \quad \begin{pmatrix} -k + T_0 + 2\hat{T}_{\frac{\pi}{2}} & 3\hat{T}_{\frac{\pi}{4}} \\ -3\frac{\lambda}{\sqrt{2}}\hat{T}^{\star}_{\frac{\pi}{4}} & -k - \frac{\lambda}{\sqrt{2}}\left(T_0 + 2T_{\frac{\pi}{3}}\right) \end{pmatrix}$$

$$:= \quad -k\,\mathrm{id} + M, \quad M =: \begin{pmatrix} M^{1,1} & M^{1,2} \\ M^{2,1} & M^{2,2} \end{pmatrix} \quad \text{in} \quad \bigoplus_{i=1}^{2} L^2(I\!R^2)$$

Operator pencil to eigenvalue pb

$$\begin{pmatrix} 1 & 0 \\ 0 & -\frac{\lambda}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -k + T_0 + 2\hat{T}_{\frac{\pi}{2}} & 3\hat{T}_{\frac{\pi}{4}} \\ 3\hat{T}^\star_{\frac{\pi}{4}} & \frac{\sqrt{2}}{\lambda} k + T_0 + 2T_{\frac{\pi}{3}} \end{pmatrix}$$

$$= \begin{pmatrix} -k + T_0 + 2\hat{T}_{\frac{\pi}{2}} & 3\hat{T}_{\frac{\pi}{4}} \\ -3\frac{\lambda}{\sqrt{2}}\hat{T}^\star_{\frac{\pi}{4}} & -k - \frac{\lambda}{\sqrt{2}}\left(T_0 + 2T_{\frac{\pi}{3}}\right) \end{pmatrix}$$

$$:= \; -k\,\mathrm{id} + M, \quad M =: \begin{pmatrix} M^{1,1} & M^{1,2} \\ M^{2,1} & M^{2,2} \end{pmatrix} \quad \text{in} \quad \bigoplus_{i=1}^{2} L^2(\mathbb{R}^2)$$

$$|\Sigma|^{\frac{1}{2}} < k \in \operatorname{spect} M?$$

The result

Choose the following system of vectors in $L^2(\mathbb{R}^2)$:

$$\Phi_\beta(p) := \varphi_{\beta_1}(p_1)\varphi_{\beta_2}(p_2), \quad \beta \in \{0.27, 1.7, 6\}^2$$

$$\varphi_{\beta_i}(u) := \exp(-\beta_i u^2), \quad u \in \mathbb{R}$$

The result

Choose the following system of vectors in $L^2(\mathbb{R}^2)$:

$$\Phi_\beta(p) := \varphi_{\beta_1}(p_1)\varphi_{\beta_2}(p_2), \quad \beta \in \{0.27, 1.7, 6\}^2$$

$$\varphi_{\beta_i}(u) := \exp(-\beta_i u^2), \quad u \in \mathbb{R}$$

Form the block matrices of M

$$\mathcal{M}^{i,j}_{\beta,\gamma} := (M^{i,j}\Phi_\beta, \Phi_\gamma), \quad \mathcal{J}^{i,j}_{\beta,\gamma} := (\Phi_\beta, \Phi_\gamma)\delta_{i,j}, \quad 1 \le i, j \le 2$$

The result

Choose the following system of vectors in $L^2(\mathbb{R}^2)$:

$$\Phi_\beta(p) := \varphi_{\beta_1}(p_1)\varphi_{\beta_2}(p_2), \quad \beta \in \{0.27, 1.7, 6\}^2$$

$$\varphi_{\beta_i}(u) := \exp(-\beta_i u^2), \quad u \in \mathbb{R}$$

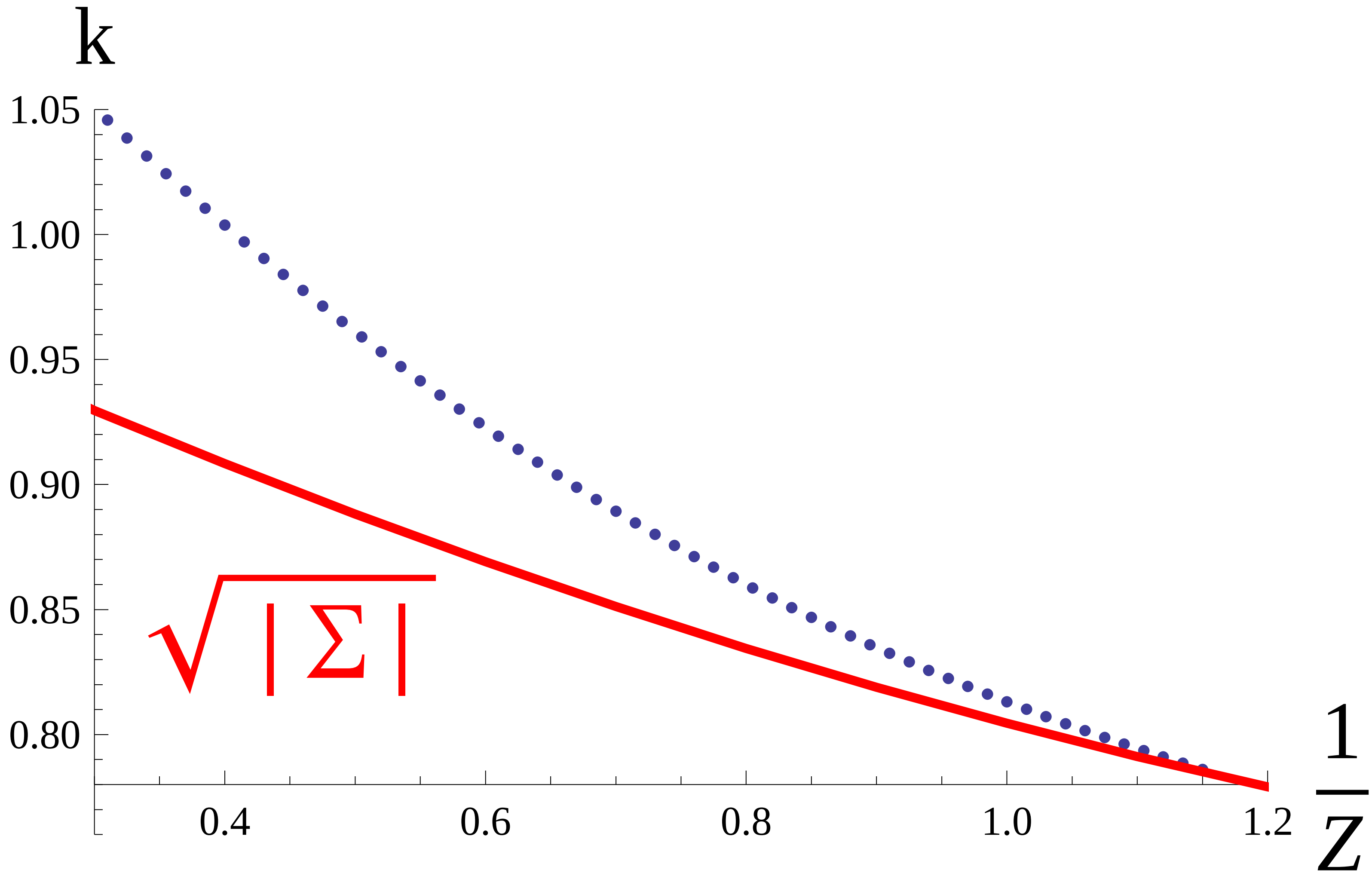
Form the block matrices of M

$$\mathcal{M}^{i,j}_{\beta,\gamma} := (M^{i,j}\Phi_\beta, \Phi_\gamma), \quad \mathcal{J}^{i,j}_{\beta,\gamma} := (\Phi_\beta, \Phi_\gamma)\delta_{i,j}, \quad 1 \leq i, j \leq 2$$

and solve the generalized eigenvalue problem

$$\mathcal{M}v = k\mathcal{J}v \quad \Longleftrightarrow \quad \mathcal{J}^{-\frac{1}{2}}\mathcal{M}\mathcal{J}^{-\frac{1}{2}}u = ku.$$

The result



The highest eigenvalue of $\mathcal{M}$ as a function of $1/Z$

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$
- Find $\hat{Z}_c(3)$

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$
- Find $\hat{Z}_c(3)$
- How many bound states?

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$
- Find $\hat{Z}_c(3)$
- How many bound states?
- N-electron case

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$
- Find $\hat{Z}_c(3)$
- How many bound states?
- N-electron case
- The LSY conjecture

$$\lim_{N\to\infty} \frac{N}{\hat{Z}_c(N)} = 2?$$

Open problems

- A genuine proof that $\hat{Z}_c(3) \leq 1$
- Find $\hat{Z}_c(3)$
- How many bound states?
- N-electron case
- The LSY conjecture

$$\lim_{N\to\infty} \frac{N}{\hat{Z}_c(N)} = 2?$$

- The threshold conjecture

$$\forall N \geq 0, \quad E(N) - E(N+1) \leq E(N-1) - E(N)$$

FIN

Merci pour votre attention!

FIN

Merci pour votre attention!

FIN

Merci pour votre attention!

FIN

Merci pour votre attention!

FIN

Merci pour votre attention!

FIN

Merci pour votre attention!