Two-nucleon ϕ-meson clusters

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Abstract

On the basis of Faddeev equations the binding energies of the systems ϕnn, ϕnp and ϕpp are calculated. The results indicate the possibility of new few - nucleon meson clusters.

As it was intensively discussed recently [1-7] there are indications on the strong attraction of mesons with one strange quark $K^-(\overline{K})$ to the few-nucleon nuclei.Along this line it is interesting to look at the interaction of a meson with two strange quarks like the ϕ-meson with light nuclei. Already existing theoretical investigations of the ϕ-meson interaction show rather strong attraction between a ϕ-meson and a nucleon. Indeed, the calculation of the $\phi - N$ interaction within the quark model [8], and on the basis of a totally different phenomenological model [9] based on the dominant role of $s\overline{s}$ configuration in the ϕ-meson structure, predicts considerable $\phi - N$ attraction with a binding energy of about 9 MeV for the ϕN system.

Such a strong attraction in reality might be not very surprising if one agrees with physical arguments, that strong K^-N attraction appeared due to the influence of subthreshold resonances Λ_{1405} and Σ_{1385}.

Indeed, let us compare the mass of the state $\phi + N$ with masses of two subthreshold states $K + \Lambda_{1405}$ and $K + \Sigma_{1385}$. It turned out, that distances of above subthreshold states from threshold $\phi + N$ state are the same order of magnitude as in $K^- N$ case, which means that as in $K^- N$ system one can expect strong influence of Λ_{1405} and Σ_{1385} and strong attraction also in the ϕN system.

Bearing in mind this sort of strong attraction in the ϕN system, it is interesting to consider the possibility of bound states of a ϕ-meson with few nucleons, in particular with two neutrons or two protons. This is in fact a question concerning the existence of new nuclear clusters. In what follows we calculate the binding energies of the three-body systems ϕnn, ϕnp and ϕpp.

As in [9] a Yukawa type potential is chosen for the $\phi - N$ interaction :

$$V_{\phi N}(r) = -\alpha e^{-\mu r}/r \quad (1)$$

with $\alpha = 1.25$ and $\mu = 600$ MeV. This is rather deep and narrow and supports binding in the ϕN system with binding energies $E_{\phi n} = -9.47$ and $E_{\phi p} = -9.40$ MeV.

For the np triplet s-wave interaction the potential MTIII [10] has been used. Our singlet s-wave interaction is based on the potential MTI [10] with the slight modification of having now a parameter $\lambda_A = 2.617$. This value is chosen in order to reproduce the experimental value of the nn-scattering length $a_{nn} = -18.5$ fm [11]. One can see that in the three-body systems ϕNN there are two scales of distances, related to the different ranges of the $N-N$ and $\phi - N$ interactions. This may produce a delicate interplay between a narrow attraction area of the $\phi - N$ interaction and repulsive parts of the MT-potentials, as it was emphasized in [12]. Apart from that, different ranges of the interaction can provide the cluster formation in the systems under consideration.

Our calculations are based on Faddeev equations [13] in differential form [14] written down for the 3-body systems ϕNN.

First, the Faddeev components of the wave function are expanded into partial waves:

$$\Psi_\alpha(\vec{\eta}_\alpha, \vec{\xi}_\alpha) = \sum_{LMl\lambda} \frac{1}{\eta_\alpha \xi_\alpha} U^L_{\alpha l\lambda}(\eta_\alpha, \xi_\alpha) Y^{LM}_{l\lambda}(\hat{\eta}_\alpha, \hat{\xi}_\alpha)$$

$$\overset{L=l=\lambda=0}{\Longrightarrow} \frac{1}{\eta_\alpha \xi_\alpha} U_\alpha(\eta_\alpha, \xi_\alpha) Y^{00}_{00}(\hat{\eta}_\alpha, \hat{\xi}_\alpha) \qquad (2)$$

where $\eta_\alpha = |\vec{\eta}_\alpha|$, $\xi_\alpha = |\vec{\xi}_\alpha|$, $\hat{\eta}_\alpha = \vec{\eta}_\alpha / |\vec{\eta}_\alpha|$, $\hat{\xi}_\alpha = \vec{\xi}_\alpha / |\vec{\xi}_\alpha|$, $Y^{LM}_{l\lambda}$ are the bispherical harmonics. Jacobi coordinates $\vec{\eta}_\alpha, \vec{\xi}_\alpha$ have been used, and only lowest partial waves are taken into account.

The Jacobi coordinates are as usual:

$$\vec{r}_i - \vec{r}_j = \frac{\vec{\eta}_\alpha}{a_\alpha} \tag{3}$$

$$\frac{m_i \vec{r}_i + m_j \vec{r}_j}{m_i + m_j} - \vec{r}_k = \frac{\vec{\xi}_\alpha}{b_\alpha} \tag{4}$$

where $\vec{r}_i$, m_i denote the radius-vector and the mass of particle i,

$$a_\alpha = \sqrt{\frac{m_i m_j}{(m_i + m_j) M}}, \quad b_\alpha = \sqrt{\frac{m_k (m_i + m_j)}{M^2}},$$

$$M = m_1 + m_2 + m_3$$

and indices α take on following values: $\alpha = 3$ for $(ij)k = (12)3$, $\alpha = 1$ for $(ij)k = (23)1$, $\alpha = 2$ for $(ij)k = (31)2$.

Since there are two identical particles in the system (we take $m_N = m_n$ for ϕnp system) the following two coupled-differential Faddeev equations survive:

$$\begin{cases} \left[\widehat{D} + V_1\left(\frac{\rho\cos\varphi}{a_1}\right) - E\right] U_1(\rho,\varphi) \; = \\ \qquad -V_1\left(\frac{\rho\cos\varphi}{a_1}\right) \sum_{\alpha'\neq 1} \frac{1}{\sin(2\gamma_{\alpha' 1})} \int_{c-}^{c+} U_{\alpha'}(\rho,\varphi')\, d\varphi' \\ \left[\widehat{D} + V_2\left(\frac{\rho\cos\varphi}{a_2}\right) - E\right] U_2(\rho,\varphi) \; = \\ \qquad -V_2\left(\frac{\rho\cos\varphi}{a_2}\right) \sum_{\alpha'\neq 2} \frac{1}{\sin(2\gamma_{\alpha' 2})} \int_{c-}^{c+} U_{\alpha'}(\rho,\varphi')\, d\varphi' \end{cases} \tag{5}$$

$$(U_3 \equiv U_2)$$

where polar coordinates $\rho = \sqrt{\eta_\alpha^2 + \xi_\alpha^2}$, $\tan\varphi_\alpha = \xi_\alpha/\eta_\alpha$ are introduced and

$$V_1 = V_{NN}, \quad V_2 = V_{\phi N},$$

$$\widehat{D} = -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)$$

$$c+ = \mathrm{Min}\{|\varphi + \gamma_{\alpha'\alpha}|, \pi - (\varphi + \gamma_{\alpha'\alpha})\}$$

$$c- = |\varphi - \gamma_{\alpha'\alpha}|$$

$$\gamma_{ij} = \arcsin s_{ij}, \quad s_{ij} = \sqrt{\frac{m_k M}{(m_i + m_k)(m_j + m_k)}},$$

$$(ijk = 123, 231, 312)$$

indices correspond 1 for ϕ-meson, 2 and 3 for nucleons.

Two-dimensional system of Faddeev equations (5) has been solved by discretization of variables hyperradius ρ and hyperangle φ with N and M mesh points respectively. Stable results for three digits of binding energies were reached at $N = 110$, $M = 210$ and L(ρ variable cutoff) $= 9$ fm.

As a result the binding energy of the system ϕnn with value $E_{\phi nn} = -21.8$ MeV has been obtained and value $E_{\phi np} = -37.9$ MeV for the binding of ϕnp system with np pair in triplet state. It should be noticed, that for this binding energy in ϕnp system both main ϕ-meson decay channels on K-mesons are closed. Let us comment last value of energy, which appeared rather large. From naive reasons in the configuration $\phi + d$ one would expect binding of order $2 \times E_{\phi N} + E_d$, which is much smaller than calculated value. However due to the strong attraction in ϕN - subsystem ($E_{\phi N} \sim -9$ MeV) one can expect, that in 3-particle ϕnp system, the configuration $\phi + d$ is rather suppressed. From that follows, that in the above system there is no strong cancellation between potential and kinetic energies of nucleons, like in deuteron and strong attractive triplet $N - N$ potential ($V_t \sim 100$ MeV) show his full value.

As can be seen from the results, the binding in 3-particle systems like ϕNN is possible even at weaker $\phi - N$ attraction as compare to the potential (1) used in our calculation.

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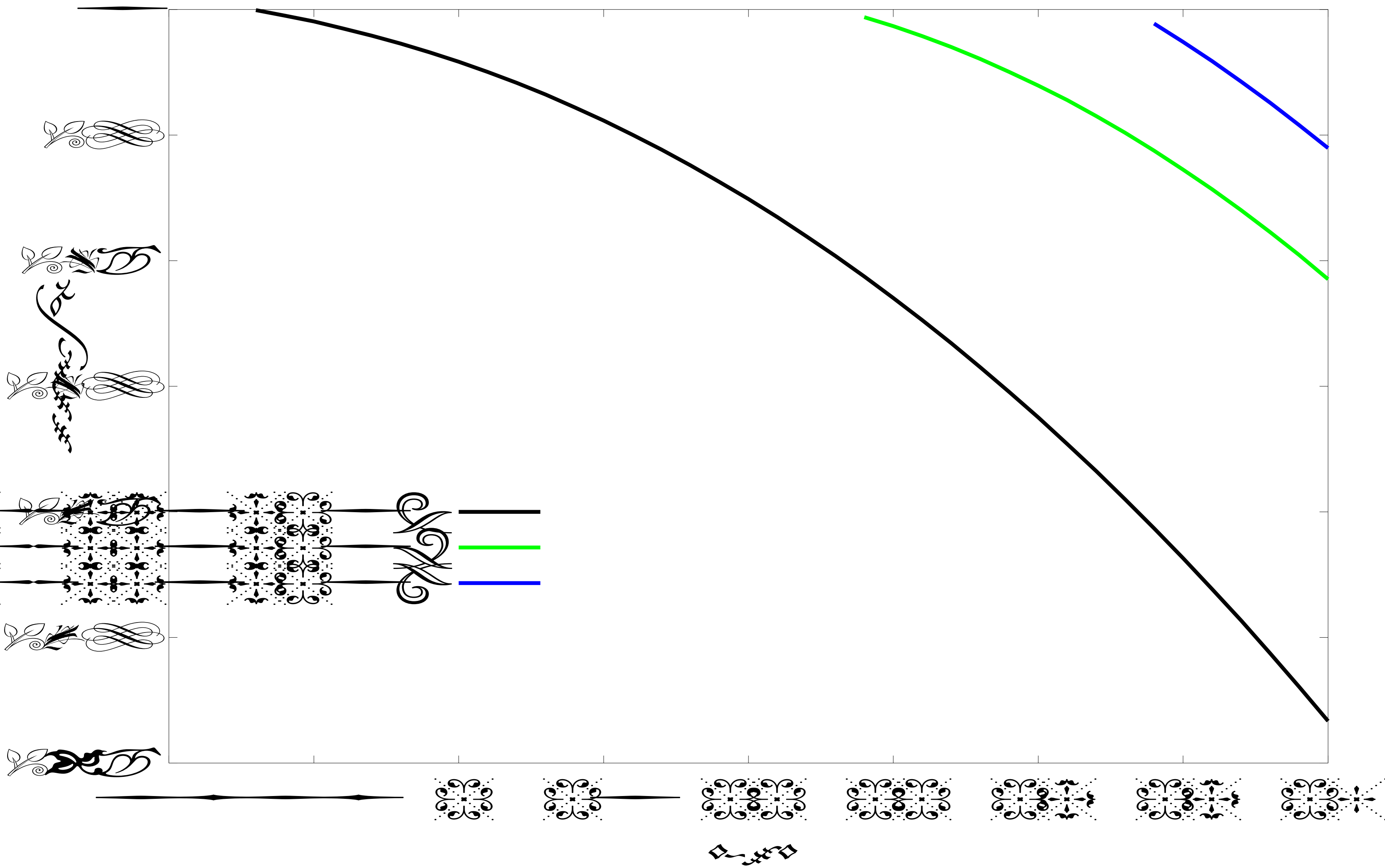

Folding potential plus p-wave centrifugal barier for the system (phi+n+n) + n.