Universality and Beyond

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- at low energies the *NN* scattering length a determines the properties of nuclear systems and pion dynamics are irrelevant
- close to Feshbach resonances the scattering length a determines the properties of atomic systems
- we analyze the properties of few-body systems with the minimal number of degrees of freedom
  - $\ast$ **universality**
    $\longrightarrow$ **identify similar physics at different length scales**
  - $\ast$ **precision**
    $\longrightarrow$ **low-energy nuclear astrophysics**

The EFT with Contact Interactions alone

for a finite range potential the t-matrix can be written as

$$t(k) \sim \frac{1}{k \cot \delta - ik}$$

for sufficiently low energies $k \cot \delta$ can be expanded in powers of $k \longrightarrow$ effective range expansion

$$k \cot \delta = -\frac{1}{a} + \frac{r}{2} k^2 + \ldots ,$$

or for $a > 0$ expand around the two-body bound state pole $\gamma = \sqrt{MB_2}$

$$k \cot \delta = -\gamma + \frac{r}{2}(\gamma^2 + k^2) + \ldots$$

Consider systems where the scattering length $a \gg R$

- such systems have particular universal properties
  $\rightarrow$ For large positive scattering length we have a bound state at $B_2 \approx \frac{1}{Ma^2}$

  $\rightarrow$ in the nuclear sector this is the deuteron

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- separation of scales

  in the nuclear sector:

  - $^1S_0$ $a \sim -24$ fm $\longrightarrow$ $r \sim 3$ fm
  - $^3S_1$ $a \sim 5$ fm $\longrightarrow$ $r \sim 2$ fm

  in the atomic ^{4}He few-body system:

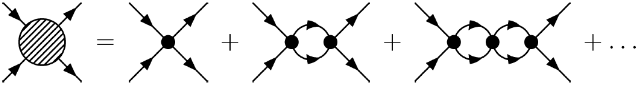
  - $a \sim 100$ Å $\longrightarrow$ $r \sim 10$ Å

In the regime where $k\ell \ll 1$ all interactions look pointlike!

- Use an appropriate EFT (expansion parameters ℓ/a, $k\ell$)
- Most general Lagrangian using only contact interactions:

$$\mathcal{L} = \psi^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \dots ,$$

- Two-body system (S-waves):



$$\text{Dim. Reg.} \longrightarrow t_{LO} \sim \frac{1}{-1/a + \sqrt{-E - i\epsilon}} \quad \text{w/} \quad C_0 = \frac{4\pi a}{M}$$

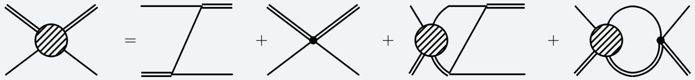
- with correct ordering scheme for diagram topologies (power-counting), this EFT is an expansion in $\ell/|a| \longrightarrow$ suitable for systems with large a

The 2-Body Sector

The most successful calculations in the short-range EFT have been performed in the 2-body sector:

- Form Factors of the Deuteron, Chen et al.
- radiative capture: $n + p \longrightarrow d + \gamma$, Rupak
- muon capture: $\mu^- + d \longrightarrow \nu_\mu + n + n$, Chen et al.
- Deuteron Electro-Disintegration, Christlmeier & Griesshammer
- and many more ...

The Three-Body System



integral (STM) equation for atom-dimer scattering:

$$K(k,p;E) \;=\; \mathcal{Z}(k,p;E) + \int_0^\Lambda \mathrm{d}q''\, q''^2 \mathcal{Z}(k,q'';E)\tau(ME - \frac{3}{4}q''^2)K(q'',p;E)$$

Skorniakov & Ter-Martirosian '56

2-body propagator:

$$\tau(E) = \frac{2}{\pi M^2}\frac{\gamma + \sqrt{-ME}}{E + B_2}$$

single nucleon-exchange + 3-body interaction:

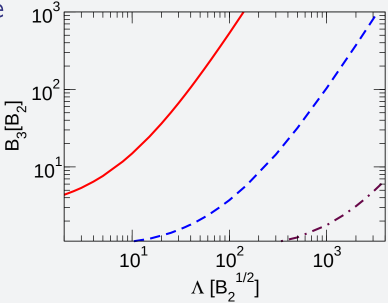
$$\mathcal{Z}(q,q',E) = -\frac{M}{2qq'}\log(\frac{q^2 + qq' + q'^2 - ME}{q^2 - qq' + q'^2 - ME}) + \frac{MH(\Lambda)}{\Lambda^2}$$

Without three-body force

⟶ strong cutoff dependence

⟶ number of bound states increases with cutoff

⟶ relation to Thomas and Efimov effect
$\Rightarrow$ include three-body information



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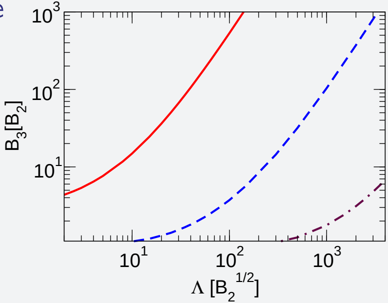


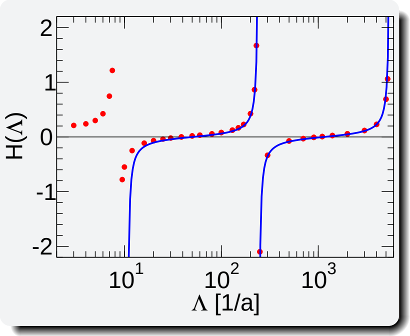
Thus, perfom calculations with three-body force:

⟶ use binding energy of weakest three-body state to fix $H(\Lambda)$

⟶ **this is renormalization**

⟶ need three-body force for consistent renormalization (Bedaque, Hammer, van Kolck, PRL 82 (1999) 463)

⟶ three-body system with large scattering length exhibits a **limit cycle** Wilson, PRD 3 (1971) 1818



Consequences of the Limit Cycle
The Three-Body parameter

For large Λ the RG-flow of $H(\Lambda)$ is described by:

$$H(\Lambda) = \frac{\sin(s_0 \ln(\Lambda/L_3) - \arctan(1/s_0))}{\sin(s_0 \ln(\Lambda/L_3) + \arctan(1/s_0))} \quad , \text{ with } s_0 \approx 1.0062$$

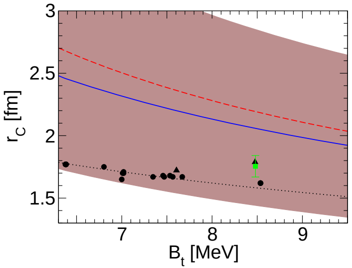
Bedaque, Hammer, van Kolck, PRL 82 (1999) 463

- $H(\Lambda)$ periodic: $\Lambda \to \Lambda e^{n\pi/s_0} \approx \Lambda(22.7)^n$
- **discrete scale invariance** with consequences for observables, e.g. $B_3^{(m)}/B_3^{(m+1)} \approx 515$
  $\longrightarrow$ this equation holds exactly for all bound states when
  $\ell \to 0$ and $a \to \infty$
  (Efimov, SJNP 29 (1979) 546)
- **scaling relations** in 3-body observables, e.g. $a_3 - B_3$, $B_3 - r_3$

1-Parameter Correlations

- Keep the scattering length fixed
- Vary one of the three-body observables

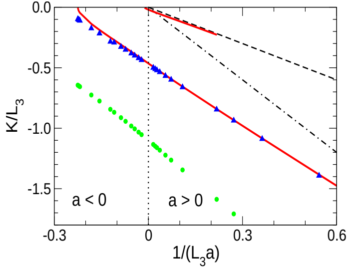
→ See what the others are doing



Hammer, Meissner, LP 2005

- Keep the Three-Body parameter fixed
- Change the scattering length

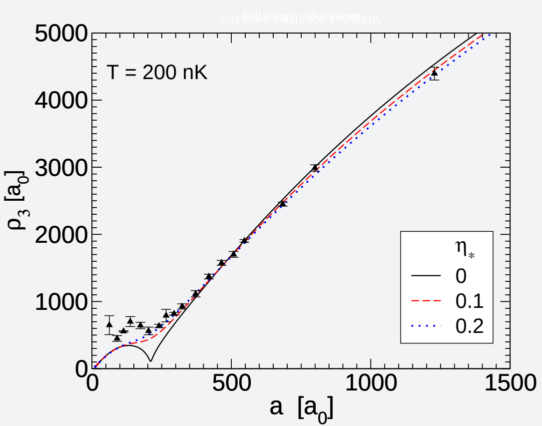
→ see what three-body observables are doing



Hammer & LP 2006

3-Body Recombination

- Include finite temperature and the effects of deep dimers



Braaten, Hammer, Kang, LP, PRA *in press*

Include the Effective Range

Reconsider the integral equation for atom-dimer scattering:

$$K(k,p;E) \;=\; \mathcal{Z}(k,p;E) + \int_0^\Lambda dq''\, q''^2 \mathcal{Z}(k,q'';E)\tau(ME - \frac{3}{4}q''^2)K(q'',p;E)$$

- Modify the two-body propagator Bedaque *et al* '03

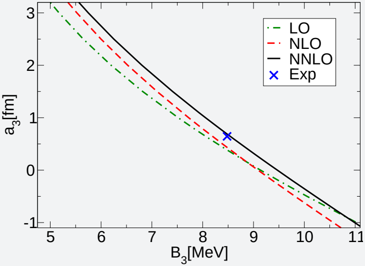
$$\tau^{(n)}(E) = \frac{1}{E+B_2}\frac{2}{\pi M^2}\sum_{i=0}^{n}\left(\frac{r_s}{2}\right)^i[\gamma + \sqrt{-ME}]^{i+1}$$

- At which order does the next three-body force contribute?
  $\rightarrow$ **Renormalization group analysis gives N3LO**
  LP,Phillips FBS 40 (2006) 35
  $\rightarrow$ perturbative analysis up to N2LO is on the way

Some Results for the 3-Nucleon System

Range Corrections in the Three-Nucleon System

	a_3 [fm]	B_3 [Mev]
LO	0.65	8.08
NLO	0.65	8.19
NNLO	0.65	8.54
EXP	0.65	8.48



LP, PRC 74 (2006) 037001

- **Note:** Convergence pattern looks strange **but** in fact the NLO correction is actually smaller than expected

Some Results for the Helium-Trimer System

- No experimental information about ^{4}He trimer but various calculations employing realistic potentials

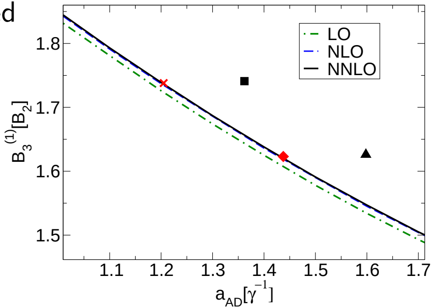
$\longrightarrow$ Compare to results for the TTY potential by Roudnev, 2003

Input		$B_3^{(1)}[B_2]$	$B_3^{(0)}[B_2]$	$a_3[\gamma^{-1}]$
$B_3^{(1)}$	LO	1.738	99.27	1.179
	NLO	1.738	84.87	1.199
	NNLO	1.738	89.52	1.203
TTY		1.738	96.33	1.205

- Excellent agreement for "low-energy" observables

The Phillips line for Bosons

- keep two-body parameters fixed and vary the three-body parameter
- this generates the Phillips line



- **EFT imposes low-energy constraints on calculations with realistic potentials**

Analysis of the Linear Correction

- Calculate the shift linear (NLO) in the effective range in the bound state spectrum
- Renormalize to the binding energy of a bound state n_* in the unitary limit with binding momentum $\kappa_* = \sqrt{mB_*}$
- in the unitary limit we know the wavefunction of such a state, how about we try to do it analytically
- for finite scattering length we can go to momentum space and use the STM equation
- and parameterize this shift as

$$B_n = B_{n*}\left[F_n\left(\frac{\gamma}{\kappa_*}\right) + \kappa_*\, r_s\, G_n\left(\frac{\gamma}{\kappa_*}\right)\right]$$

in the unitary Limit the ($|a| \to \infty$):

$$F_n\left(\frac{\gamma}{\kappa_*}\right) = (e^{-2\pi/s_0})^{n-n_*}$$

In the Unitary Limit

- in the unitary limit the relevant differential equation is

$$\frac{\hbar^2}{2M}\left(-\frac{\partial^2}{\partial R^2}-\frac{s_0^2+\frac{1}{4}}{R^2}\right)f_0(R)=E\,f_0(R)$$

  which can be solved after renormalizing with a boundary condition or three-body force
- then

$$f_0^{(0)}(R)=\sqrt{R}\,K_{is_0}(\sqrt{2}\kappa R)$$

→ Now we can do perturbation theory on the higher order and analyze the linear range correction to the bound state spectrum in the hyperradial formalism
→in momentum space for nucleons Hammer & Mehen 2001

The Linear Range Correction

Ji, Phillips, LP arXiv:0808.1230

- Obtain the perturbing potential by implementing the NLO Bethe-Peierls condition into the hyperangular equation (Efimov, 1991)

$$V_{\mathrm{NLO}} = -\frac{s_0^2\,\xi_0\,r_s}{R^3} \quad \text{w/} \quad \xi_0 = 0.480$$

compare to Nielsen, Fedorov, Jensen 1998

- We need to renormalize this integral with a three-body force

$$V_{SR}^{(1)}(R) = H_1(\Lambda)\Lambda^2\delta\left(R - \frac{1}{\Lambda}\right)$$

- use H_1 to set the shift for state n_* to 0 by calculating

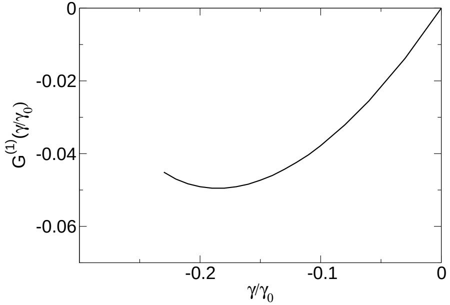
$$\frac{2M}{\hbar^2}\Delta B_n^{(1)} = s_0^2 r_s \xi_0 \left[\int_{\frac{1}{\Lambda}}^{\infty} dR f_n^{(0)^2}(R)\frac{1}{R^3} - \frac{2H_1 M}{\hbar^2 s_0^2\, r_s\,\xi_0}\Lambda^2 f_n^{(0)^2}\left(\frac{1}{\Lambda}\right)\right]$$

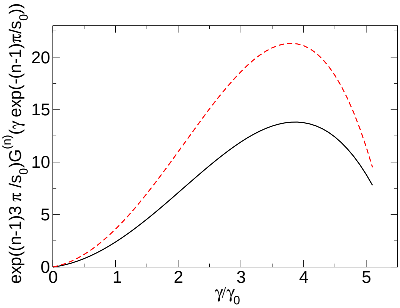
⋆ linear correction is suppressed due to discrete scale invariance of leading order wave function: $\Delta B_n^{(1)} = 0$ **for all** n

- Discrete Scale Invariance relates the range corrections to different states

$$G_n\left(\frac{\gamma}{\gamma_0}\right) = \exp\left(\frac{(n*-n)3\pi}{s_0}\right)\theta_n\left(\frac{\gamma}{\gamma_0}\right)G_{n*}\left(\frac{\gamma}{\gamma_0}e^{\frac{(n-n*)\pi}{s_0}}\right)$$

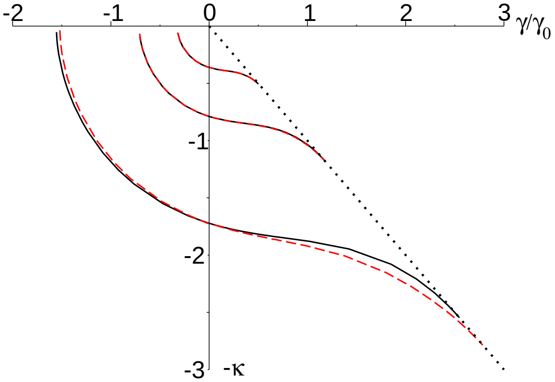
- Results obtained in Momentum Space





Extended Efimov Plot

- Include the effective range into the Efimov Plot



similar results found by Thøgersen, Fedorov, Jensen, 2008

Summary

- The pionless EFT has been applied successfully to a wide range of observables in atomic, nuclear and particle physics (X3872)
- The pionless EFT is able to describe many well-known scaling properties in few-body systems
  $\longrightarrow$ **all these are a result of a large scattering length in the two-body sector**
- The pionless EFT is structurally simple and can help us to avoid pitfalls in non-perturbative renormalization
- The pionless EFT gives low-energy theorems for few- and many-body observables
- The impact of the effective range is suppressed in the unitary limit!

Outlook

- Triton form factor up to NNLO (Phillips, LP *in progress*)
- further electroweak properties of the triton up to NNLO $\longrightarrow$ nuclear astrophysics
  - tritium β-decay
  - *pd* $\longrightarrow {}^3$He γ
- EFT for Halo Nuclei, α-clusters, etc.
- scattering of bosons/fermions in the four-body sector
  $\longrightarrow$ scattering lengths (trimer-particle, dimer-dimer)
  $\longrightarrow$ four-body recombination
- Ay-Problem in Nuclear Physics