## Efimov Effect in 2-Neutron Halo Nuclei

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Critical Stability Erice, 0ct.08

# Halo World: The story according to Faddeev, Efimov and Fano



# **Plan of the talk**

Introduction to Nuclear Halos
Three-body model of 2-n Halo nucleus
probing the structural properties of <sup>11</sup>Li

- Efimov effect in 2-n halo nuclei
- Fano resonances of Efimov states
- Summary and future scope

# **Collaborators**

- V.S. Bhasin Delhi Univ.
- V. Arora *Delhi Univ.*
- A.R.P. Rau Louisiana State Univ.

Mazumdar & Bhasin (Under review)
Phys. Rev. Lett. 99, 269202
Nucl. Phys. A790, 257
Phys. Rev. Lett. 97, 062503
Phys. Rev. C69, 061301(R)
Phys. Rev. C61, 051303(R)
Phys. Rev. C56, R5
Phys. Rev. C50, 550
Phys. Rev. C65,034007

Phys. Rep 212 (1992) J.M. Richard Phys. Rep. 231 (1993) (Zhukov et al.) Phys. Rep. 347 (2001) (Nielsen, Fedorov, Jensen, Garrido) Rev. Mod. Phys. 76, (2004) (Jensen, Riisager, Fedorov, Garrido) Phys. Rep. 428, (2006) 259(Braaten & Hammer) Ann Rev. Nucl. Part. Sci. 45, 591 (Hansen, Jensen, Jonson) Rev. Mod. Phys. 66 (1105) (K. Riisager)



# **Advent of Radioactive Ion Beams**

# Interaction cross section measurements

# Discovery of the Neutron Halo in Light Dripline Nuclei



 $\Rightarrow$  Large neutron tail (Halo)

Bevalac@LBL, I. Tanihata et al., PRL 55 (1985) 2676, PLB 206 (1988) 592





Europhys.Lett. 4, 409 (1987) P.G.Hansen, B.Jonson

## **Exotic Structure of 2-n Halo Nuclei**

Radius ~3.2 fm







<sup>11</sup>Li Z=3 N=8

#### **Striking Features:**

- $\downarrow$  Extremely small separation energy S<sub>n</sub> or S<sub>2n</sub>
- Very large matter radius
- Narrow momentum distribution of fragments
- Borromean property of many two neutron halos(<sup>6</sup>He,<sup>11</sup>Li,<sup>14</sup>Be)

Structure	Nucleus	S <u>n</u> (kev)	S <sub>2n</sub> (keV)	2000
1n-halo	<sup>11</sup> Be	$504 \pm 6$	7317±6	
III-IIdio	<sup>19</sup> C	$160 \pm 120$	$4350\pm110$	
2n-halo	<sup>6</sup> He	$1864 \pm 1$	$974 \pm 1$	
Zir nuro	<sup>11</sup> Li	$330 \pm 30$	$300\pm30$	
	<sup>14</sup> Be	$1850\pm120$	$1340\pm110$	
	<sup>19</sup> B	$1030\pm900$	$500\pm430$	

#### **Conditions for halo formation**

- 4 Small binding energy
- **Small orbital angular momentum** (most likely in s & p-state)
- Coulomb barrier hinders the formation of halo. (p-halos are less pronounced than n-halo)



**Major RIB facilities** 

•GSI, Darmstadt •RIKEN, Japan •MSU, USA •GANIL, France •RIA, (?) USA

Fragmentation projectile/ target

Recall talk by M. marques

*Typical experimental momentum distribution of halo nuclei from fragmentation reaction* 



# **Theoretical Models**

- Shell Model Bertsch et al. (1990) PRC 41,42
- Cluster model
- Three-body model ( for 2n halo nuclei )
- RMF model
- EFT Braaten & Hammer, Phys. Rep. 428 (2006)

#### The Formalism

The 2-neutron halo nucleus <sup>11</sup>Li is modeled as a three-body system consisting of a compact core of <sup>9</sup>Li and two valence neutrons forming a halo around the core. We label the two neutrons and the core as 1,2,3 with momenta  $P_1$ ,  $P_2$ ,  $P_3$  respectively. Assuming the core to be a structureless and spinless object, we write the Schrodinger equation in momentum space as

$$(T - E)\psi = -(V_{12} + V_{23} + V_{31})\psi$$

Where E is the total energy (= - binding energy, B.E.) and T represents the kinetic energy such that

$$T - E = p_{1}^{2}/2m + p_{2}^{2}/2m + p_{3}^{2}/2m_{3} - E$$
$$= p_{i}^{2}/2\mu_{ii} + p_{k}^{2}/2\mu_{ii-k} - E$$



For the two-body interactions we consider <u>non-local</u>, <u>separable</u> <u>potentials</u> of the Yamaguchi form and assume <u>s-state interactions</u> <u>both for n-n and n- $^{9}$ Li systems</u>.

Dasgupta, Mazumdar, Bhasin, Phys. Rev C50,550 The three body bound state wave function in momentum space using the binary separable potentials

$$V_{12} = -\frac{\lambda_n}{2\mu_{12}}g(p_{12})g(p'_{12}),$$
  

$$V_{23} = -\frac{\lambda_c}{2\mu_{23}}g(p_{23})g(p'_{23}),$$
  

$$V_{31} = -\frac{\lambda_c}{2\mu_{31}}g(p_{31})g(p'_{31}) \text{ is }$$

 $\psi(\vec{p}_{12}, \vec{p}_3; E) = D^{-1}(\vec{p}_{12}, \vec{p}_3; E)[g(p_{12})F(\vec{p}_3) + f(p_{23})G(\vec{p}_1) + f(p_{31})G(\vec{p}_2)]$ (1)

$$g(p) = 1/(p^2 + \beta_n^2), f(p) = 1/(p^2 + \beta_c^2), \lambda_{n,c}, \beta_{n,c}$$

reproduce spin singlet scattering length and effective range. The spectator functions F(p) and G(p) satisfy the homogeneous coupled integral equations

$$[\Lambda_n^{-1} - h_n(p)]F(\vec{p}) = 2 \int d\vec{q} K_1(\vec{p}, \vec{q}) G(\vec{q})$$
(2)

$$[\Lambda_c^{-1} - h_c(p)]G(\vec{p}) = \int d\vec{q} K_2(\vec{p}, \vec{q}) F(\vec{q}) + \int d\vec{q} K_3(\vec{p}, \vec{q}) G(\vec{q}) \quad (3)$$

After the angular integration, the two couple equation reduce to an integral equation in one variable. These equations are numerically computed as an eigenvalue problem.

## We Calculate

•2-n separation energy

•Momentum distribution of n & core

•Root mean square radius

*Inclusion of p-state in n-core interaction* 

 $\beta$ -decay of <sup>11</sup>Li

B.E. of <sup>11</sup> Li (MeV)	eta/lpha	$\lambda_n/\alpha^3$	$\beta_1/\alpha$	$\lambda_c/lpha^3$	$\lambda_c/\alpha^3$ three-body
0.34	5.8	18.6	5.0	10.32	12.92
	6.255	23.4	5.0	10.32	12.91
	5.8	18.6	5.5	14.0	17.01
0.20	5.8	18.6	5.0	10.32	12.39
	5.8	18.6	5.5	14.00	16.38

TABLE I. Parameters of the input two body  $(n-n \text{ and } n^9\text{Li})$  potentials. Given the <sup>11</sup>Li binding energy, the strength parameter  $\lambda_c$  as obtained from the three-body equation is matched with the corresponding value obtained from the two-body analysis.

TABLE II. Values of the root mean square radii of neutron-neutron and neutron-<sup>9</sup>Li separations calculated using Eqs. (18) and (19) for different binding energies of <sup>11</sup>Li.

B.E. of <sup>11</sup> Li (MeV)	<i>r</i> <sub>nn</sub> (fm)	$\tilde{r}_{nn}$ (fm) (from other model calculations [4])	<i>r</i> <sub>nc</sub> (fm)	$\bar{r}_{nc}$ (fm) (from other model calculations [4])
0.20	10.63		10.93	
0.25	9.9		9.86	
0.315	8.93	6.24 to 7.80	8.87	5.47 to 6.40

The rms radius  $r_{matter}$  calculated is ~ 3.6 fm

$$\langle \mathbf{r}^2 \rangle_{\text{matter}} = \mathbf{A}_c / \mathbf{A} \langle \mathbf{r}^2 \rangle_{\text{core}} + 1 / \mathbf{A} \langle \rho^2 \rangle$$
$$\rho^2 = \mathbf{r}^2_{\text{nn}} + \mathbf{r}^2_{\text{nc}}$$

Fedorov et al (1993) Garrido et al (2002) (3.2 fm)



Dasgupta, Mazumdar, Bhasin, PRC 50, R550 Data: N. Orr et al., PRL69 (1992), K. Ieki et al. PRL 70, (1993)

## Kumar & Bhasin, Phys. Rev. C65 (2002)

Incorporation of both *s* & *p* waves in n-<sup>9</sup>Li potential

•Ground state energy and 3 excited states above the 3-body breakup threshold were predicted

•The resulting coupled integral equations for the spectator functions have been computed using the method of rotating the integral contour of the kernels in the complex plane.

•Dynamical content of the two body input potentials in the three body wave function has also been analyzed through the three-dimensional plots.

 $\beta$ -decay to two channels studied:

<sup>11</sup>Li to high lying excited state of <sup>11</sup>Be  $\rightarrow$   $\stackrel{18.3 \text{ MeV, bound}}{\Box}$ 

<sup>11</sup>Li to <sup>9</sup>Li + deuteron channel ——



**18.3 MeV, bound (**<sup>9</sup>Li+p+n) system Gamow-Teller β-decay strength calculated

Branching ratio (1.3X10-4) calculated

Mukha et al (1997), Borge et al (1997)



#### Efimov Effect

" A three-body system can support infinite bound states when none of the three pairs are bound, or one or two pairs are barely bound."

V. Efimov, Phys. Lett. B 33, 563 (1970); Comments Nucl. Part. Phys. 19, 271 (1990), Amado & Noble, Phys. Lett. B 35, 25 (1971)

#### **Universality:**

Independent of the details of the 2-body interaction

Adjacent energy levels are related by

 $\frac{E_{N+1}}{E_N} = e^{-2\pi}$ R<sub>size</sub> ~  $r_0 e^{N\pi}$ 

Size of the Nth state is

The number of states decreases with increasing 2-body strength

#### **Necessary Conditions:**

- Low energy requirement
- Large scattering length



#### The Efimov Effect (A Simple Visualisation)



weaker pair interaction

This scenario was predicted for three-body systems with

a≫ro	where	a=two-body scattering length ro=two-body effective range
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Note: modern helium pair potentials have  $a \approx 104 \text{ Å}$  $r_{o} \approx 11 \text{ Å}$ 

Artificially weakening the pair interaction introduces up to infinitely many three-body bound states.

V. Efimov; Phys. Lett. **33B**, 563 (1970)

V. Efimov; Comments Nucl. Part. Phys. 19, 271 (1990)

#### V. Efimov:

Sov. J. Nucl. Phys 12, 589 (1971) Phys. Lett. 33B (1970) Nucl. Phys A 210 (1973) Comments Nucl. Part. Phys.19 (1990)

**Amado & Noble:** *Phys. Lett. 33B (1971) Phys. Rev. D5 (1972)* 

**Fonseca** *et al. Nucl. PhysA320, (1979)* 

Adhikari & Fonseca Phys. Rev D24 (1981) Theoretical searches in Atomic Systems

T.K. Lim et al. PRL38 (1977)

Cornelius & Glockle, J. Chem Phys. 85 (1986)

T. Gonzalez-Lezana et al. PRL 82 (1999), This workshop The case of He trimer

**Diffraction experiments with transmission gratings** 

Carnal & Mlynek, PRL 66 (1991) Hegerfeldt & Kohler, PRL 84, (2000)

Three-body recombination in ultra cold atoms

# First Observation of Efimov States

# Letter

Nature 440, 315-318 (16 March 2006) |

# Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl and R. Grimm

## **Magnetic tuning of the two-body interaction**

• For Cs atoms in their energetically lowest state the *s*-wave scattering length *a* varies strongly with the magnetic field.

## **Trap set-ups and preparation of the Cs gases**

- All measurements were performed with trapped thermal samples of caesium atoms at temperatures *T* ranging from 10 to 250 nK.
- In set-up A they first produced an essentially pure Bose–Einstein condensate with up to 250,000 atoms in a far-detuned crossed optical dipole trap generated by two 1,060-nm Yb-doped fibre laser beams
- In set-up B they used an optical surface trap in which they prepared a thermal sample of 10,000 atoms at  $T_{250}$  nK via forced evaporation at a density of n0 = 1.0 1012 cm-3. The dipole trap was formed by a repulsive evanescent laser wave on top of a horizontal glass prism in combination with a single horizontally confining 1,060-nm laser beam propagating along the vertical direction



**Can we find Efimov Effect in the atomic nucleus?** 

Unlike cold atom experiments we have no control over the scattering lengths.

The discovery of 2-neutron halo nuclei, characterized by very low separation energy and large spatial extension are ideally suited for studying Efimov effect in atomic nclei.

Fedorov & Jensen PRL 71 (1993)

Fedorov, Jensen, Riisager PRL 73 (1994)

P. Descouvement PRC 52 (1995), Phys. Lett. B331 (1994)

Conditions for occurrence of Efimov states in 2-n halo nuclei.

 $\tau_n^{-1}(p)F(p) \equiv \varphi(p) \text{ and } \tau_c^{-1}(p)G(p) \equiv \chi(p)$ Where

$$\tau_{n}^{-1}(p) = \mu_{n}^{-1} - [\beta_{r} (\beta_{r} + \sqrt{p^{2}/2a} + \varepsilon^{3})^{2}]^{-1}$$

$$\tau_{c}^{-1}(p) = \mu_{c}^{-1} - 2a[1 + \sqrt{2a(p^{2}/4c + \varepsilon 3)}]^{-2}$$

where 
$$\mu_n = \pi^2 \lambda_n / \beta_1^2$$
 and  $\mu_c = \pi^2 \lambda_c / 2a \beta_1^3$ 

are the dimensionless strength parameters. Variables *p* and *q* in the final integral equation are also now dimensionless,

$$p/\beta_1 \rightarrow p \& q/\beta_1 \rightarrow q$$
  
and  
$$-mE/\beta_1^3 = \varepsilon_3, \ \beta_r = \beta/\beta_1$$

Factors  $\tau_n^{-1}$  and  $\tau_c^{-1}$  appear on the left hand side of the spectator functions F(p) and G(p) and are quite sensitive. They blow up as  $p \rightarrow 0$  and  $\varepsilon_3$  approaches extremely small value.

The basic structure of the equations in terms of the spectator functions F(p)and G(p) remains same. But for the sensitive computational details of the Efimov effect we recast the equations in dimensionless quantities.

TABLE I. <sup>14</sup>Be ground and excited states three-body energy for different two-body input parameters.

<i>n</i> - <sup>12</sup> Be Energy keV	λ1	$a_s$ fm	ε <sub>0</sub> keV	ε <sub>1</sub> keV	$\epsilon_2$ keV
50	11.71	-21	1350		
5.8	12.32	-61.6	1408	.053	
2	12.46	-105	1450	2.56	0.061
1	12.52	-149	1456	3.8	0.22
0.1	12.62	-483	1488	б.1	0.62
0.05	12.63	-658	1490	б.4	0.68
0.01	12.65	-1491	1490	6.9	0.72

#### Mazumdar and Bhasin, PRC 56, R5

Thoennessen, Yokoyama, Hansen PRC 63 (2000) Observation of low lying s-wave strength With scattering legth < -10 fm



Search for Efimov states in <sup>19</sup>B, <sup>22</sup>C, and <sup>20</sup>C

n- <sup>17</sup> B energy (keV)	λ <sub>c</sub> /α 8.4 9.5	<sup>3</sup> a, (fm -12	s n) (l 515 50	ε <sub>0</sub> keV)	ε <sub>1</sub> (keV)	٤ (k	eV)
	8.4 9.5	9 -6. -12	515 50				
514.8	9.5	-12		)0			
135.3	10.0	A dead a	71 72	28			
48	10.0	-21.	16 85	51			
7.7	10.5	-53	2 97	78	0.16		
0.67	10.7	-179	6 10	42	5.4	(	).36
n- <sup>20</sup> C energy	$\lambda / \alpha$	<sup>3</sup> a	e	<b>E</b> 0	81		32
(keV)		(fn	n) (1	keV)	(keV)	(k	æV)
210	0.82		1	120	-		
127	10.5	-13 (	$1^{1}$ 1	287			
48.8	11.0	-21 (	) 1	410			
93	11.0	-48.2	1	540 0	0.12	22	
1.46	11.75	-121	.5 1	608	4.74	4	0.198
n_NC energy	2.10	3		<b>E</b> 0	£1	5	En
(keV)	1000	(fn	n) (1	keV)	(keV)	(k	teV)
60 1	5.51	20.38	3188.03	78	.87 6	5.8	1.01
100 1	5.89	16.05	3291.54	115	.72 10	0.09	0.94
113.2 1	6.0	15.15	3317.35	127	.41 11	1.76	0.92
139.60 1	6.2	13.77	3371.24	150	.32 13	5.29	0.89
168.59 1	6.4	12.64	3426.03	175	.34 16	3.48	0.86
200 1	16.6	11.71	3482.95	202	.15 19	4.15	0.84



#### Mazumdar, Arora Bhasin Phys. Rev. C 61, 051303(R)

- •Amorim, Frederico, Tomio PRC 56 (1997) R2378
- •Delfino, Frederico, Hussein, Tomio PRC 61 (2000)

- The feature observed can be attributed to the singularity in the two body propagator  $[\Lambda_c^{-1} h_c(p)]^{-1}$ .
- There is a subtle interplay between the two and three body energies.
- The effect of this singularity on the behaviour of the scattering amplitude has to be studied.

In order to analyze the effect of this singularity on the behavior of the scattering amplitude for  $n^{-19}C$  elastic scattering the function G(p) describing the dynamics of the neutron in the presence of  $(n^{-18}C)$  system, must be subject to the boundary condition, viz,

$$G(\vec{p}) = (2\pi)^3 \delta(\vec{p} - \vec{k}) + \frac{4\pi a_k(\vec{p})}{p^2 - k^2 - \iota\epsilon}$$
(5)

The scattering amplitude is normalized such that, for the s-wave scattering,

$$a_k(\vec{p})_{|\vec{p}|=|\vec{k}|} \equiv f_k = \frac{e^{\imath\delta} sin\delta}{k} \tag{6}$$

Before applying the boundary condition, we rewrite Eq.(3) substituting Eq.(2) for F(p) and finally get the equation for the off-shell scattering amplitude as

$$4\pi(\frac{a}{d})h(p^{2},k^{2};\alpha_{2}^{2})a_{k}(\vec{p}) =$$

$$(2\pi)^{3}K_{3}(\vec{p},\vec{k}) + 4\pi\int\frac{d\vec{q}K_{3}(\vec{p},\vec{q})a_{k}(\vec{q})}{q^{2}-k^{2}-\iota\epsilon} + (2\pi)^{3}\int d\vec{q}K_{3}(\vec{p},\vec{q})K_{1}(\vec{q},\vec{k})\tau_{n}(q) +$$

$$(8\pi)\int d\vec{q}K_{3}(\vec{p},\vec{q})\tau_{n}(q)\int d\vec{q'}\frac{K_{1}(\vec{q},\vec{q'})a_{k}(\vec{q'})}{q'^{2}-k^{2}-\iota\epsilon}$$

$$(7\pi)$$

This integral equation needs to be solved numerically for the scattering amplitude. For  $k \rightarrow 0$ , the singularity in the two body cut Does not cause any problem. The amplitude has only real part. The off-shell amplitude is computed By inverting the resultant matrix, which in the limit  $a_o(p)_{p \rightarrow 0} \rightarrow -a$ , the n-<sup>19</sup>C scattering length.

For non-zero incident energies the singularity in the two body propagator is tackled by the CSM.

 $\mathbf{P} \rightarrow p_1 e^{-i\varphi} \text{ and } q \rightarrow q e^{-i\varphi}$ 

The unitary requirement is the  $Im(f_k^1) = -k$ 

Balslev & Combes (1971) Matsui (1980) Volkov et al. This workshop

#### Arora, Mazumdar Bhasin (2004)



Arora, Mazumdar, Bhasin, PRC 69, 061301



Fitting the Fano profile to the N-<sup>19</sup>C elastic cross section for n-<sup>18</sup>C BE of 250 keV

Mazumdar, Rau, Bhasin Phys. Rev. Lett. 97 (2006)



The resonance due to the <u>second excited Efimov state</u> for n-<sup>18</sup>C BE 150 keV. The profile is fitted by <u>same value of q</u> as for the 250 keV curve.



Comparison between He and <sup>20</sup>C as three body Systems in atoms and nuclei

### Discussion

Our results are at variance with Yamashita, Frederico, Tomio

•We emphasize the cardinal role of channel coupling.

There is also a definite role of mass ratios as observed numerically.

•However, channel coupling is an elegant and physically plausible scenario.

•The difference can also arrive between zero range and realistic finite range potentials in non-Borromean cases.

Note, that for n-<sup>18</sup>C binding energy of 200 keV, the scattering length is about 10 fm while the interaction range is about 1 fm.

•The extension of zero range to finer details of Efimov states in non-Borromean cases may not be valid.

• <u>The discrepancy observed in the resonance vs virtual states in <sup>20</sup>C clearly</u> <u>underlines the sensitive structure of the three-body scattering amplitude</u> <u>derived from the binary interactions.</u>



E<sub>i</sub> (keV)

 $d_{d(b)}$ 

A possible experimental proposal to search for Efimov State in 2-neutron halo nuclei.

•Production of <sup>20</sup>C secondary beam with reasonable flux

•Acceleration and Breakup of <sup>20</sup>C on heavy target

•Detection of the neutrons and the core in coincidence

•Measurement of  $\gamma$ -rays as well

The Arsenal:

- Neutron detectors array
- Gamma array
- Charged particle array

# <u>Summary</u>

A three body model with s-state interactions account for most of the gross features of <sup>11</sup>Li in a reasonable way.

► Inclusion of *p*-state in the n-<sup>9</sup>Li contributes marginally.

A virtual state of a few keV (2 to 4) energy corresponding to scattering length from -50 to -100 fm for the n-<sup>12</sup>Be predicts the ground state and excited states Of <sup>14</sup>Be.

▶<sup>19</sup>B, <sup>22</sup>C and <sup>20</sup>C are investigated and it is shown that Borromean type nuclei are much less vulnerable to respond to Efimov effect.

<sup>>20</sup>C is a promising candidate for Efimov states at energies below the n-(nc) breakup threshold.

The bound Efimov states in <sup>20</sup>C move into the continuum and reappear as Resonances with increasing strength of the binary interaction.

Asymmetric resonances in elastic n+19C scattering are attributed to Efimov states and are identified with the Fano profile. The conjunction of Efimov and Fano phenomena my lead to the experimental realization in nuclei.

## **Future scope of Work:**

•Resonant states above the three body breakup threshold in <sup>20</sup>C.

•Fano resonances of Efimov states in <sup>16</sup>C, <sup>19</sup>B, <sup>22</sup>C and analytical derivation of the Fano index *q*.

•Role of Efimov states in Bose-Einstein condensation.

•Studying the proton halo (<sup>17</sup>Ne) nucleus. (Neff, This workshop)

•Reanalyze profiles of GDR on ground states for its asymmetry.

•Experiment for breakup of <sup>20</sup>C is being planned.

Epilogue

" the richness of understanding reveals even greater richness of ignorance"

D.H. Wilkinson

