## Efimov Effect in 2-Neutron Halo Nuclei

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## Critical Stability

Erice, 0ct. 08

## Halo World:

The story according to Faddeev, Efimov and Fano


## Plan of the talk

- Introduction to Nuclear Halos
*Three-body model of 2-n Halo nucleus
probing the structural properties of "Li
- Efimov effect in 2-n halo nuclei
- Fano resonances of Efimov states
- Summary and future scope


## Collaborators

- V.S. Bhasin Delhi Univ.
- V. Arora Delhi Univ.
- A.R.P. Rau Louisiana State Univ.



## The nuclear landscape



## Advent of Radioactive Ion Beams

## Interaction cross section measurements

## Discovery of the Neutron Halo in Light Dripline Nuclei

## Interaction Radii extracted from <br> Interaction cross section measurements


$\mathrm{I} / \mathrm{I}_{\mathrm{o}}=\mathrm{e}^{-\mathrm{opt}}$
$\sigma_{I}=\pi\left[\mathbf{R}_{I}(\mathbf{P})+\mathbf{R}_{I}(\mathbf{T})\right]^{2}$
Bevalac@LBL, I. Tanihata et al.,
PRL 55 (1985) 2676, PLB 206 (1988) 592

Charge-changing cross sections


## Quadrupole moments <br> $\mathrm{Q}\left({ }^{11} \mathrm{Li}\right) \approx \mathrm{Q}(9 \mathrm{Li})$

ISOLDE@CERN, E. Arnold et al., Phys. Lett. B 281 (1992) 16



Europhys.Lett. 4, 409 (1987)
P.G.Hansen, B.Jonson

## Exotic Structure of 2-n Halo Nuclei



$$
\begin{array}{ll}
{ }^{11} \mathbf{L i} & \text { Radius } \sim 3.2 \mathrm{fm} \\
\mathbf{Z}=3 \\
\mathbf{N}=8
\end{array}
$$



## Striking Features:

+ Extremely small separation energy $\mathrm{S}_{\mathrm{n}}$ or $\mathrm{S}_{2 \mathrm{n}}$
+ Very large matter radius
+ Narrow momentum distribution of fragments
+ Borromean property of many two neutron halos $\left({ }^{6} \mathrm{He},{ }^{11} \mathrm{Li},{ }^{14} \mathrm{Be}\right)$

| Structure | Nucleus | $\mathrm{S}_{\mathbf{n}}(\mathrm{kev})$ | $\mathrm{S}_{2 \mathrm{n}}(\mathrm{keV})$ |
| :--- | :---: | :---: | :---: |
| 1n-halo | ${ }^{11} \mathrm{Be}$ | $504 \pm 6$ | $7317 \pm 6$ |
|  | ${ }^{19} \mathrm{C}$ | $160 \pm 120$ | $4350 \pm 110$ |
| 2n-halo | ${ }^{6} \mathrm{He}$ | $1864 \pm 1$ | $974 \pm 1$ |
|  | ${ }^{11} \mathrm{Li}$ | $330 \pm 30$ | $300 \pm 30$ |
|  | ${ }^{14} \mathrm{Be}$ | $1850 \pm 120$ | $1340 \pm 110$ |
|  | ${ }^{19} \mathrm{~B}$ | $1030 \pm 900$ | $500 \pm 430$ |

## Conditions for halo formation

* Small binding energy
\& Small orbital angular momentum ( most likely in s \& p-state)
$\$$ Coulomb barrier hinders the formation of halo.
( p -halos are less pronounced than n -halo)


## Major RIB facilities

-GSI, Darmstadt Fragmentation<br>-RIKEN, Japan<br>-MSU, USA<br>-GANIL, France -RIA, (?) USA

Recall talk by M. marques

Typical experimental momentum distribution of halo nuclei from fragmentation reaction


## Theoretical Models

- Shell Model Bertsch etal. (1990) PRC 41,42
- Cluster model
- Three-body model ( for 2 n halo nuclei )
- RMF model
- EFT Braaten \& Hammer, Phys. Rep. 428 (2006)


## Che 开ormalimy

The 2-neutron halo nucleus ${ }^{11} \mathrm{Li}$ is modeled as a three-body system consisting of a compact core of ${ }^{9} \mathrm{Li}$ and two valence neutrons forming a halo around the core. We label the two neutrons and the core as 1,2,3 with momenta $P_{1}, P_{2}, P_{3}$ respectively. Assuming the core to be a structureless and spinless object, we write the Schrodinger equation in momentum space as

$$
(\mathrm{T}-E) \psi=-\left(\mathrm{V}_{12}+\mathrm{V}_{23}+\mathrm{V}_{31}\right) \psi
$$

Where $E$ is the total energy ( $=-$ binding energy, B.E.) and $T$ represents the kinetic energy such that

$$
\begin{aligned}
& \mathrm{T}-E=p^{2}{ }_{1} / 2 \mathrm{~m}+p^{2} / 2 \mathrm{~m}+p_{2}^{2} / 2 \mathrm{~m}_{3}-E \\
& \quad=p_{i j}^{2} / 2 \mu_{i j}+p^{2} / 2 \mu_{i j-k}-E
\end{aligned}
$$



For the two-body interactions we consider non-local, separable potentials of the Yamaguchi form and assume $s$-state interactions both for $\mathrm{n}-\mathrm{n}$ and $\mathrm{n}-\mathrm{Li}$ systems.

The three body bound state wave function in momentum space
using the binary separable potentials
$\mathrm{V}_{12}=-\frac{\lambda_{n}}{2 \mu_{12}} \mathrm{~g}\left(\mathrm{p}_{12}\right) \mathrm{g}\left(\mathrm{p}^{\prime}{ }_{12}\right)$,
$\mathrm{V}_{23}=-\frac{\lambda_{c}}{2 \mu_{23}} \mathrm{~g}\left(\mathrm{p}_{23}\right) \mathrm{g}\left(\mathrm{p}^{\prime}{ }_{23}\right)$,
$\mathrm{V}_{31}=-\frac{\lambda_{c}}{2 \mu_{31}} \mathrm{~g}\left(\mathrm{p}_{31}\right) \mathrm{g}\left(\mathrm{p}^{\prime}{ }_{31}\right)$ is
$\psi\left(\vec{p}_{12}, \vec{p}_{3} ; E\right)=D^{-1}\left(\vec{p}_{12}, \vec{p}_{3} ; E\right)\left[g\left(p_{12}\right) F\left(\vec{p}_{3}\right)+f\left(p_{23}\right) G\left(\vec{p}_{1}\right)+f\left(p_{31}\right) G\left(\vec{p}_{2}\right)\right]$
$\mathrm{g}(\mathrm{p})=1 /\left(\mathrm{p}^{2}+\beta_{n}^{2}\right), \mathrm{f}(\mathrm{p})=1 /\left(\mathrm{p}^{2}+\beta_{c}^{2}\right), \quad \lambda_{n, c}, \beta_{n, c}$
reproduce spin singlet scattering length and effective range.
The spectator functions $F(p)$ and $G(p)$ satisfy the homogeneous coupled integral equations

$$
\begin{gather*}
{\left[\Lambda_{n}^{-1}-h_{n}(p)\right] F(\vec{p})=2 \int d \vec{q} K_{1}(\vec{p}, \vec{q}) G(\vec{q})}  \tag{2}\\
{\left[\Lambda_{c}^{-1}-h_{c}(p)\right] G(\vec{p})=\int d \vec{q} K_{2}(\vec{p}, \vec{q}) F(\vec{q})+\int d \vec{q} K_{3}(\vec{p}, \vec{q}) G(\vec{q})} \tag{3}
\end{gather*}
$$

After the angular integration, the two couple equation reduce to an integral equation in one variable. These equations are numerically computed as an eigenvalue problem.

## Dasgupta, Mazumdar, Bhasin,

 Phys. Rev C50,550
## We Calculate

-2-n separation energy
-Momentum distribution of $\mathbf{n}$ \& core

- Root mean square radius

Inclusion of p-state in n-core interaction
$\beta$-decay of ${ }^{11} \mathrm{Li}$

TABLE I. Parameters of the input two body ( $n-n$ and $n-{ }^{9} \mathrm{Li}$ ) potentials. Given the ${ }^{11} \mathrm{Li}$ binding energy, the strength parameter $\lambda_{c}$ as obtained from the three-body equation is matched with the corresponding value obtained from the two-body analysis.

| B.E. of ${ }^{11} \mathrm{Li}$ <br> $(\mathrm{MeV})$ | $\beta / \alpha$ | $\lambda_{n} / \alpha^{3}$ | $\beta_{1} / \alpha$ | $\lambda_{c} / \alpha^{3}$ | $\lambda_{c} / \alpha^{3}$ <br> three-body |
| :---: | :--- | :---: | :---: | :---: | :---: |
| 0.34 | 5.8 | 18.6 | 5.0 | 10.32 | 12.92 |
|  | 6.255 | 23.4 | 5.0 | 10.32 | 12.91 |
| 0.20 | 5.8 | 18.6 | 5.5 | 14.0 | 17.01 |
|  | 5.8 | 18.6 | 5.0 | 10.32 | 12.39 |
|  | 5.8 | 18.6 | 5.5 | 14.00 | 16.38 |

TABLE II. Values of the root mean square radii of neutron-neutron and neutron- ${ }^{9} \mathrm{Li}$ separations calculated using Eqs. (18) and (19) for different binding energies of ${ }^{11} \mathrm{Li}$.

|  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $\bar{r}_{n n}(\mathrm{fm})$ <br> B.E. of ${ }^{11} \mathrm{Li}$ <br> $(\mathrm{MeV})$ | $\bar{r}_{n n}$ <br> $(\mathrm{fm})$ | (from other model <br> calculations [4]) | $\bar{r}_{n c}$ <br> $(\mathrm{fm})$ | $\bar{r}_{n c}(\mathrm{fm})$ <br> (from other model <br> calculations [4]) |
| 0.20 | 10.63 |  | 10.93 |  |
| 0.25 | 9.9 |  | 9.86 |  |
| 0.315 | 8.93 | 6.24 to 7.80 | 8.87 | 5.47 to 6.40 |

The rms radius $r_{\text {matter }}$ calculated is $\sim 3.6 \mathrm{fm}$

$$
\begin{aligned}
& \left.\left\langle\mathbf{r}^{2}\right\rangle_{\text {matter }}=\mathbf{A}_{c} / \mathbf{A}<\mathbf{r}^{2}\right\rangle_{\text {core }}+1 / \mathbf{A}<\rho^{2}> \\
& \rho^{2}=\mathbf{r}_{\mathrm{nn}}^{2}+\mathbf{r}_{\mathrm{nc}}^{2}
\end{aligned}
$$



Dasgupta, Mazumdar, Bhasin, PRC 50, R550
Data: N. Orr et al., PRL69 (1992), K. Ieki et al. PRL 70, (1993)

## Kumar \& Bhasin, Phys. Rev. C65 (2002)

Incorporation of both $\boldsymbol{s} \boldsymbol{\&} \boldsymbol{p}$ waves in $\mathbf{n -}{ }^{\mathbf{9}} \mathrm{Li}$ potential
-Ground state energy and 3 excited states above the 3-body breakup threshold were predicted
-The resulting coupled integral equations for the spectator functions have been computed using the method of rotating the integral contour of the kernels in the complex plane.

- Dynamical content of the two body input potentials in the


Data from
Gornov et al. PRL81 (1998) three body wave function has also been analyzed through the three-dimensional plots.
$\beta$-decay to two channels studied:
${ }^{11} \mathrm{Li}$ to high lying excited state of ${ }^{11} \mathrm{Be} \longrightarrow \begin{aligned} & \text { 18.3 MeV, bound }\left({ }^{9} \mathrm{Li}+\mathbf{p}+\mathbf{n}\right) \text { system } \\ & \text { Gamow-Teller } \beta \text {-decay strength calculated }\end{aligned}$
${ }^{11} \mathbf{L i}$ to ${ }^{9} \mathbf{L i}+$ deuteron channel $\longrightarrow$ Branching ratio $\left(1.3 \times 10^{-4}\right)$ calculated
Mukha et al (1997), Borge et al (1997)



## Efimov Effect

"A three-body system can support infinite bound states when none of the three pairs are bound, or one or two pairs are barely bound."
V. Efimov, Phys. Lett. B 33, 563 (1970); Comments Nucl. Part. Phys. 19, 271 (1990), Amado \& Noble, Phys. Lett. B 35, 25 (1971)

## Universality:

Independent of the details of the 2-body interaction

Adjacent energy levels are related by
Size of the Nth state is

$$
\begin{array}{r}
\frac{E_{N+1}}{E_{N}}=e^{-2 \pi} \\
\mathrm{R}_{\text {size }} \sim r_{0} \mathrm{e}^{\mathrm{N} \pi}
\end{array}
$$

The number of states decreases with increasing 2-body strength

## Necessary Conditions:

- Low energy requirement
- Large scattering length



## The Efimov Effect

(A Simple Visualisation)

stronger pair interaction


This scenario was predicted for three-body systems with
$a \gg r_{0}$ where
$a=$ two-body scattering length $r_{0}=$ two-body effective range

Note: modern helium pair potentials have
$a \approx 104 \AA$
$r_{0} \approx 11 \AA$
Artificially weakening the pair interaction introduces up to infinitely many three-body bound states.
V. Efimov; Phys. Lett. 33B, 563 (1970)
V. Efimov: Comments Nucl. Part. Phys. 19, 271 (1990)
V. Efimov:

Sov. J. Nucl. Phys 12, 589 (1971)
Phys. Lett. 33B (1970)
Nucl. Phys A 210 (1973)
Comments Nucl. Part. Phys. 19 (1990)

Amado \& Noble:
Phys. Lett. 33B (1971)
Phys. Rev. D5 (1972)
Fonseca et al.
Nucl. PhysA320, (1979)

Adhikari \& Fonseca
Phys. Rev D24 (1981)

Theoretical searches in Atomic Systems
T.K. Lim et al. PRL38 (1977)

Cornelius \& Glockle, J. Chem Phys. 85 (1986)
T. Gonzalez-Lezana et al. PRL 82 (1999),

This workshop
This worksop

The case of He trimer

## Carnal \& Mlynek, PRL 66 (1991) Hegerfeldt \& Kohler, PRL 84, (2000)

Three-body recombination in ultra cold atoms

## First Observation of Efimov States

Letter
Nature 440, 315-318 (16 March 2006) |

Evidence for Efimov quantum states in an ultracold gas of caesium atoms
T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl and R. Grimm

## Magnetic tuning of the two-body interaction

- For Cs atoms in their energetically lowest state the $s$-wave scattering length $a$ varies strongly with the magnetic field.


## Trap set-ups and preparation of the Cs gases

- All measurements were performed with trapped thermal samples of caesium atoms at temperatures $T$ ranging from 10 to 250 nK .
- In set-up A they first produced an essentially pure Bose-Einstein condensate with up to 250,000 atoms in a far-detuned crossed optical dipole trap generated by two 1,060-nm Yb-doped fibre laser beams
- In set-up B they used an optical surface trap in which they prepared a thermal sample of 10,000 atoms at $T 250 \mathrm{nK}$ via forced evaporation at a density of $n 0=1.0 \quad 1012 \mathrm{~cm}-3$. The dipole trap was formed by a repulsive evanescent laser wave on top of a horizontal glass prism in combination with a single horizontally confining $1,060-\mathrm{nm}$ laser beam propagating along the vertical direction


## T. Kraemer et al. Nature 440, 315



Recall talks by
F. Ferlaino, J. D'Incao,
L. Platter

Can we find Efimov Effect in the atomic nucleus?

Unlike cold atom experiments we have no control over the scattering lengths.

## The discovery of 2-neutron halo nuclei, characterized by very low separation energy and large spatial extension are ideally suited for studying Efimov effect in atomic nclei.

## Fedorov \& Jensen PRL 71 (1993)

Fedorov, Jensen, Riisager PRL 73 (1994)

Conditions for occurrence of Efimov states in 2-n halo nuclei.
P. Descouvement

PRC 52 (1995), Phys. Lett. B331 (1994)
$\tau_{\mathbf{n}}{ }^{-1}(\mathbf{p}) \mathbf{F}(\mathbf{p}) \equiv \varphi(\mathbf{p})$ and $\tau_{\mathbf{c}}{ }^{-1}(\mathbf{p}) \mathbf{G}(\mathbf{p}) \equiv \chi(\mathbf{p})$
Where

The basic structure of the equations in terms of the spectator functions $\boldsymbol{F}(p)$ and $G(p)$ remains same. But for the sensitive computational details of the Efimov effect we recast the equations in dimensionless quantities.
$\tau_{n}^{-1}(\mathbf{p})=\mu_{n}^{-1}-\left[\beta_{r}\left(\beta_{r}+V^{2} / 2 a+\varepsilon^{3}\right)^{2}\right]^{-1}$
$\tau_{c}^{-1}(p)=\mu_{c}^{-1}-2 a\left[1+\sqrt{ } 2 a\left(p^{2} / 4 c+\varepsilon 3\right)\right]^{-2}$
where $\mu_{n}=\pi^{2} \lambda_{n} / \beta_{1}{ }^{2}$ and $\mu_{c}=\pi^{2} \lambda_{c} / 2 a \beta_{1}{ }^{3}$
are the dimensionless strength parameters. Variables $p$ and $q$ in the final integral equation are also now dimensionless,

$$
\begin{gathered}
p / \beta_{1} \rightarrow p \& q / \beta_{1} \rightarrow q \\
\text { and } \\
-m E / \beta_{1}^{3}=\varepsilon_{3}, \quad \beta_{r}=\beta / \beta_{1}
\end{gathered}
$$

Factors $\tau_{n}{ }^{-1}$ and $\tau_{c}^{-1}$ appear on the left hand side of the spectator functions $F(p)$ and $G(p)$ and are quite sensitive. They blow up as $p \rightarrow 0$ and $\varepsilon_{3}$ approaches extremely small value.

TABLE I. ${ }^{14} \mathrm{Be}$ ground and excited states three-body energy for different two-body input parameters.

| $n-{ }^{12}$ Be Energy <br> keV | $\lambda 1$ | $a_{s}$ <br> fm | $\epsilon_{0}$ <br> keV | $\epsilon_{1}$ <br> keV | $\epsilon_{2}$ <br> keV |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 50 | 11.71 | -21 | 1350 |  |  |
| 5.8 | 12.32 | -61.6 | 1408 | .053 |  |
| 2 | 12.46 | -105 | 1450 | 2.56 | 0.061 |
| 1 | 12.52 | -149 | 1456 | 3.8 | 0.22 |
| 0.1 | 12.62 | -483 | 1488 | 6.1 | 0.62 |
| 0.05 | 12.63 | -658 | 1490 | 6.4 | 0.68 |
| 0.01 | 12.65 | -1491 | 1490 | 6.9 | 0.72 |

Mazumdar and Bhasin, PRC 56, R5
Thoennessen, Yokoyama, Hansen
PRC 63 (2000)
Observation of low lying s-wave strength
With scattering legth $<-10 \mathrm{fm}$


Search for Efimov states in ${ }^{19} \mathbf{B},{ }^{22} \mathrm{C}$, and ${ }^{20} \mathrm{C}$

| $\mathrm{n}^{17} \mathrm{~B}$ energy <br> $(\mathrm{keV})$ | $\lambda_{\mathrm{d}} \alpha^{3}$ | $a_{\mathrm{s}}$ <br> $(\mathrm{fm})$ | $\varepsilon_{0}$ <br> $(\mathrm{keV})$ | $\varepsilon_{1}$ <br> $(\mathrm{keV})$ | $\varepsilon_{2}$ <br> $(\mathrm{keV})$ |
| :--- | :--- | :---: | :---: | :---: | :---: |


|  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :--- |
| 514.8 | 8.49 | -6.515 | 500 |  |  |
| 135.3 | 9.5 | -12.71 | 728 |  |  |
| 48 | 10.0 | -21.16 | 851 |  |  |
| 7.7 | 10.5 | -53.2 | 978 | 0.16 | 0.36 |
| 0.67 | 10.75 | -179.6 | 1042 | 5.4 |  |


| $\mathrm{n}-{ }^{20} \mathrm{C}$ energy <br> $(\mathrm{keV})$ | $\lambda_{\mathrm{d}} / \alpha^{3}$ | $a_{\mathrm{s}}$ <br> $(\mathrm{fm})$ | $\varepsilon_{0}$ <br> $(\mathrm{keV})$ | $\varepsilon_{1}$ <br> $(\mathrm{keV})$ | $\varepsilon_{2}$ <br> $(\mathrm{keV})$ |
| :--- | :--- | :---: | :---: | :---: | :---: |


| 319 | 9.82 | -8.23 | 1120 |  |  |
| :--- | :---: | :--- | :--- | :--- | :--- |
| 127 | 10.5 | -13.02 | 1287 |  |  |
| 48.8 | 11.0 | -21.0 | 1410 |  |  |
| 9.3 | 11.5 | -48.2 | 15400 | 0.122 |  |
| 1.46 | 11.75 | -121.5 | 1608 | 4.74 | 0.198 |

$\left.\begin{array}{llllcccc}\begin{array}{l}\begin{array}{l}\mathrm{n}-90 \mathrm{C} \text { energy } \\ (\mathrm{keV})\end{array}\end{array} \lambda_{d} \alpha^{3} & \begin{array}{c}a_{\mathrm{s}} \\ (\mathrm{fm})\end{array} & \begin{array}{c}\varepsilon_{0} \\ (\mathrm{keV})\end{array} & \begin{array}{c}\varepsilon_{1} \\ (\mathrm{keV})\end{array} & \begin{array}{c}\varepsilon_{2} \\ (\mathrm{keV})\end{array} \\ \hline 60 & 15.51 & 20.38 & 3188.03 & 78.87 & 65.8 & 1.01 \\ 100 & 15.89 & 16.05 & 3291.54 & 115.72 & 100.09 & 0.94 \\ 113.2 & 16.0 & 15.15 & 3317.35 & 127.41 & 111.76 & 0.92 \\ 139.60 & 16.2 & 13.77 & 3371.24 & 150.32 & 135.29 & 0.89 \\ 168.59 & 16.4 & 12.64 & 3426.03 & 175.34 & 163.48 & 0.86 \\ 200 & 16.6 & 11.71 & 3482.95 & 202.15 & 194.15 & 0.84\end{array}\right\}$


Mazumdar, Arora Bhasin Phys. Rev. C 61, 051303(R)
-Amorim, Frederico, Tomio
PRC 56 (1997) R2378
-Delfino, Frederico, Hussein, Tomio PRC 61 (2000)

- The feature observed can be attributed to the singularity in the two body propagator $\left[\Lambda_{C}^{-1}-\boldsymbol{h}_{c}(p)\right]^{-1}$.
- There is a subtle interplay between the two and three body energies.
- The effect of this singularity on the behaviour of the scattering amplitude has to be studied.

In order to analyze the effect of this singularity on the behavior of the scattering amplitude for $\mathrm{n}^{-19} \mathrm{C}$ elastic scattering the function $\mathrm{G}(\mathrm{p})$ describing the dynamics of the neutron in the presence of $\left(\mathrm{n}^{18} \mathrm{C}\right)$ system, must be subject to the boundary condition, viz,

$$
\begin{equation*}
G(\vec{p})=(2 \pi)^{3} \delta(\vec{p}-\vec{k})+\frac{4 \pi a_{k}(\vec{p})}{p^{2}-k^{2}-\iota \epsilon} \tag{5}
\end{equation*}
$$

The scattering amplitude is normalized such that, for the s-wave scattering,

$$
\begin{equation*}
a_{k}(\vec{p})_{|\vec{p}|=|\vec{k}|} \equiv f_{k}=\frac{e^{\iota \delta} \sin \delta}{k} \tag{6}
\end{equation*}
$$

Before applying the boundary condition, we rewrite Eq.(3) substituting Eq.(2) for $F(p)$ and finally get the equation for the off-shell scattering amplitude as
$4 \pi\left(\frac{a}{d}\right) h\left(p^{2}, k^{2} ; \alpha_{2}^{2}\right) a_{k}(\vec{p})=$
$(2 \pi)^{3} K_{3}(\vec{p}, \vec{k})+4 \pi \int \frac{d \vec{q} K_{3}(\vec{p}, \vec{q}) a_{k}(\vec{q})}{q^{2}-k^{2}-\iota \epsilon}+(2 \pi)^{3} \int d \vec{q} K_{3}(\vec{p}, \vec{q}) K_{1}(\vec{q}, \vec{k}) \tau_{n}(q)+$
$(8 \pi) \int d \vec{q} K_{3}(\vec{p}, \vec{q}) \tau_{n}(q) \int d \overrightarrow{q^{\prime}} \frac{K_{1}\left(\vec{q}, \overrightarrow{q^{\prime}}\right) a_{k}\left(\overrightarrow{q^{\prime}}\right)}{q^{\prime 2}-k^{2}-\iota \epsilon}$
This integral equation needs to be solved numerically for the scattering amplitude.

For $k \rightarrow 0$, the singularity in the two body cut Does not cause any problem. The amplitude has only real part. The off-shell amplitude is computed By inverting the resultant matrix, which in the limit $a_{o}(p)_{p \rightarrow 0} \rightarrow-a$, the $n-{ }^{19} C$ scattering length.

For non-zero incident energies the singularity in the two body propagator is tackled by the CSM.
$\mathrm{P} \rightarrow p_{1} e^{-i \varphi}$ and $q \rightarrow q e^{-i \varphi}$
The unitary requirement is the $\operatorname{Im}\left(f_{k}{ }_{k}\right)=-k$

Balslev \& Combes (1971)
Matsui (1980)
Volkov et al. This workshop
(a)


Fitting the Fano profile to the $\mathrm{N}_{-1}{ }^{19} \mathrm{C}$ elastic cross section for $n^{-18} \mathrm{C}$ BE of 250 keV

Mazumdar, Rau, Bhasin Phys. Rev. Lett. 97 (2006) $\mathrm{E}_{\mathrm{i}}(\mathrm{keV})$


The resonance due to the second excited Efimov state for $\mathrm{n}-{ }^{18} \mathrm{C}$ BE 150 keV . The profile is fitted by same value of $q$ as for the 250 keV curve.


Comparison between He and ${ }^{20} \mathrm{C}$ as three body Systems in atoms and nuclei

## Discussion

Our results are at variance with

## Yamashita, Frederico, Tomio

-We emphasize the cardinal role of channel coupling.
There is also a definite role of mass ratios as observed numerically.
-However, channel coupling is an elegant and physically plausible scenario.
-The difference can also arrive between zero range and realistic finite range potentials in non-Borromean cases.
Note, that for $\mathbf{n - ~}^{18} \mathrm{C}$ binding energy of 200 keV , the scattering length is about 10 fm while the interaction range is about 1 fm .
-The extension of zero range to finer details of Efimov states in non-Borromean cases may not be valid.
-The discrepancy observed in the resonance vs virtual states in ${ }^{20} \mathrm{C}$ clearly underlines the sensitive structure of the three-body scattering amplitude derived from the binary interactions.

$\varepsilon_{0} \quad$ Equal Heavy Core

| $(\mathrm{keV})$ | $(\mathrm{keV})$ |
| :---: | :---: |
| 250 | 455 |

(keV)
4400


Ground states for the two cases



## Mazumdar \& Bhasin (Communicated)

${ }^{\bullet}$ Production of ${ }^{20} \mathrm{C}$ secondary beam with reasonable flux

- Acceleration and Breakup of ${ }^{20} \mathrm{C}$ on heavy target
-Detection of the neutrons and the core in coincidence
${ }^{\bullet}$ Measurement of $\gamma$-rays as well

The Arsenal:

- Neutron detectors array
- Gamma array
- Charged particle array


## Summary

$>$ A three body model with s-state interactions account for most of the gross features of ${ }^{11} \mathrm{Li}$ in a reasonable way.
$>$ Inclusion of $p$-state in the $\mathbf{n}^{-9} \mathbf{~ L i ~ c o n t r i b u t e s ~ m a r g i n a l l y . ~}$
$>$ A virtual state of a few keV (2 to 4) energy corresponding to scattering length from -50 to - $\mathbf{- 1 0 0} \mathrm{fm}$ for the $\mathrm{n}-{ }^{12} \mathrm{Be}$ predicts the ground state and excited states Of ${ }^{14} \mathrm{Be}$.
${ }^{19} \mathrm{~B},{ }^{22} \mathrm{C}$ and ${ }^{20} \mathrm{C}$ are investigated and it is shown that Borromean type nuclei are much less vulnerable to respond to Efimov effect.
$>{ }^{20} \mathrm{C}$ is a promising candidate for Efimov states at energies below the n -(nc) breakup threshold.
$>$ The bound Efimov states in ${ }^{20} \mathrm{C}$ move into the continuum and reappear as Resonances with increasing strength of the binary interaction.
$>$ Asymmetric resonances in elastic ${ }^{+19} \mathrm{C}$ scattering are attributed to Efimov states and are identified with the Fano profile. The conjunction of Efimov and Fano phenomena my lead to the experimental realization in nuclei.

## Future scope of Work:

-Resonant states above the three body breakup threshold in ${ }^{20} \mathrm{C}$.
-Fano resonances of Efimov states in ${ }^{16} \mathrm{C},{ }^{19} \mathrm{~B},{ }^{22} \mathrm{C}$ and analytical derivation of the Fano index $q$.
-Role of Efimov states in Bose-Einstein condensation.
-Studying the proton halo $\left({ }^{17} \mathrm{Ne}\right)$ nucleus. (Neff, This workshop)
-Reanalyze profiles of GDR on ground states for its asymmetry.

- Experiment for breakup of ${ }^{20} \mathrm{C}$ is being planned.


## Epilogue

" the richness of understanding reveals even greater richness of ignorance"


