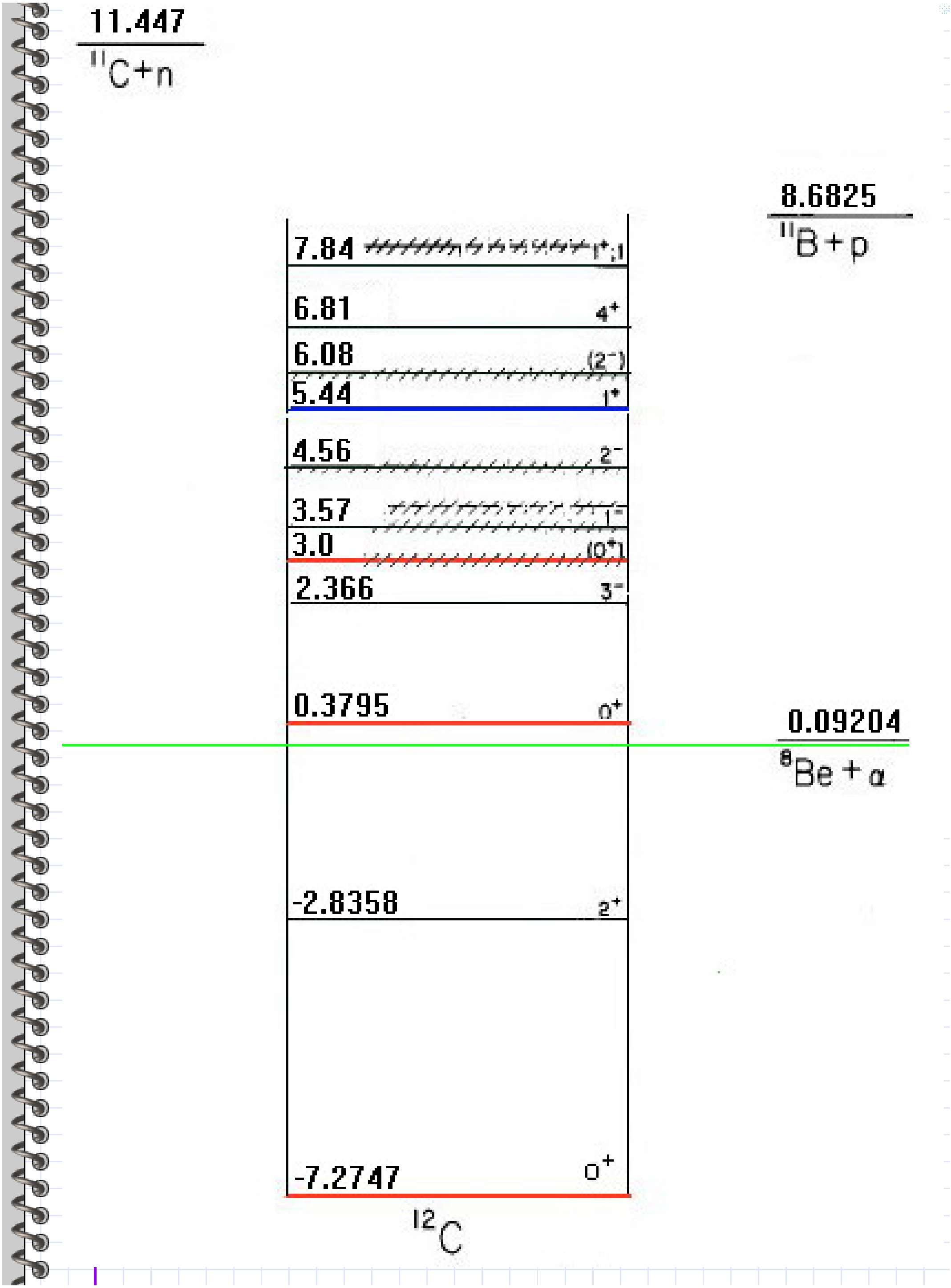
*Critical Stability 2008*
*Erice, October 11- 18*

# Consistent α-cluster description of the Hoyle state in $^{12}C$

Fedotov S. I., **Kartavtsev O. I.**, Malykh A.V.

# Stellar Helium burning in the triple-α collisions

$$3\,\alpha \rightarrow {}^{8}Be + \alpha \rightarrow {}^{12}C(0^{+}_{2}) \rightarrow {}^{12}C + \gamma$$





**Hoyle state**

$E_r$ = 0.3795 MeV

$\Gamma$ = 8.5 ± 1.0 eV

Monopole $0_2^+ \Rightarrow 0_1^+$ transition matrix element

$M_{12}$ = 5.48 ± 0.22 $fm^2$

# Questions

- Cluster structure and reactions mechanism

- Does the α-cluster model is able to describe very fine properties of the Hoyle state with an accuracy comparable to the experiment?

- How good one can determine the effective potentials from the three-body $^{12}C$ properties?

# α- cluster model

## Two-body potential:

$$V_{2b}(r) = \sum_{l} V_l(r) P_l$$

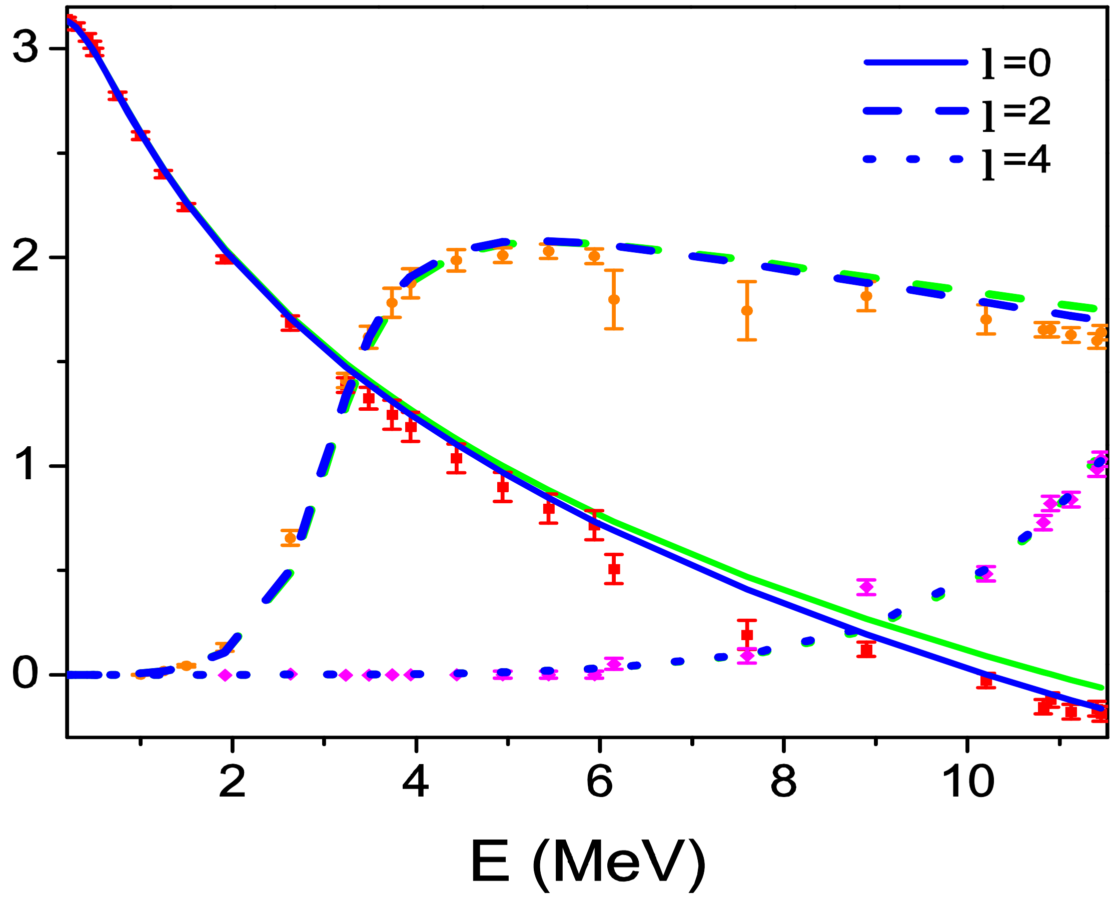
$$V_l = V_r e^{-\mu_r^2 r^2} - V_a e^{-\mu_a^2 r^2}$$

The parameters are chosen to fix

$^8$Be: $E_{2\alpha}$=92.04 keV, $\gamma$ = 5.47 ± 0.25 eV

low-energy α-α elastic-scattering phase shifts

# Calculated and experimental α - α scattering phase shifts



# α- cluster model

## Three-body potentials:

$$V_{3b}(\rho) = \frac{V_0}{1 + e^{(\rho - \rho_0)/b}}$$

The parameters are chosen to fix

- Ground state

  $E_{gs}$ = -7.2747 MeV

  rms radius $R^{(1)}$= 2.48 ± 0.02 fm

- Excited (Hoyle) state

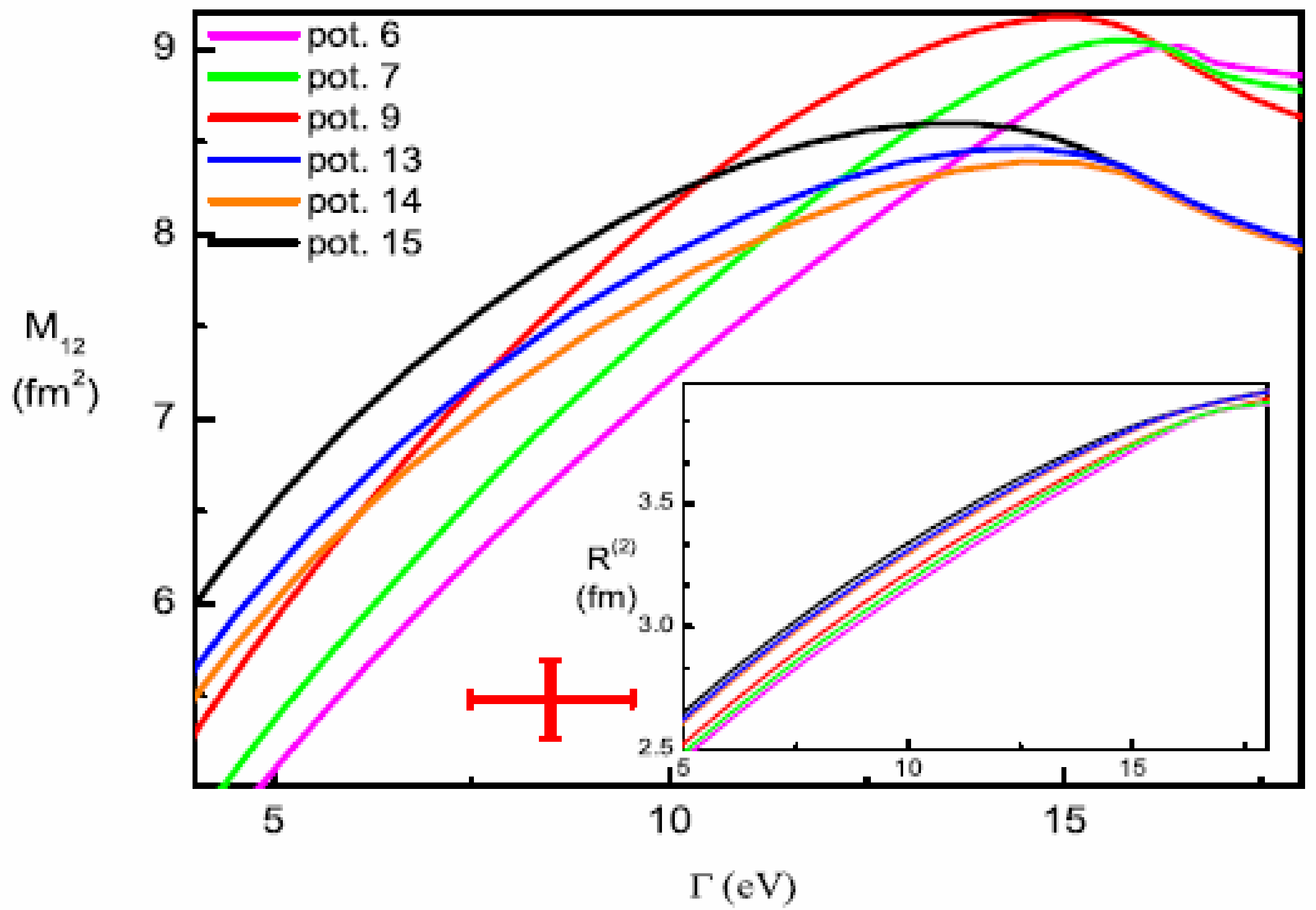
  $E_r$ = 0.3795 MeV

# Preliminary results (local two-body interactions)

Method and technique were elaborated to provide quite a good accuracy which is nessesary to calculate $\Gamma$, $M_{12}$

[Phys. Rev. C 70, 014006 (2004), Eur. Phys. J. A 26, 201, (2005)]

# Monopole transition matrix element vs Hoyle state's width



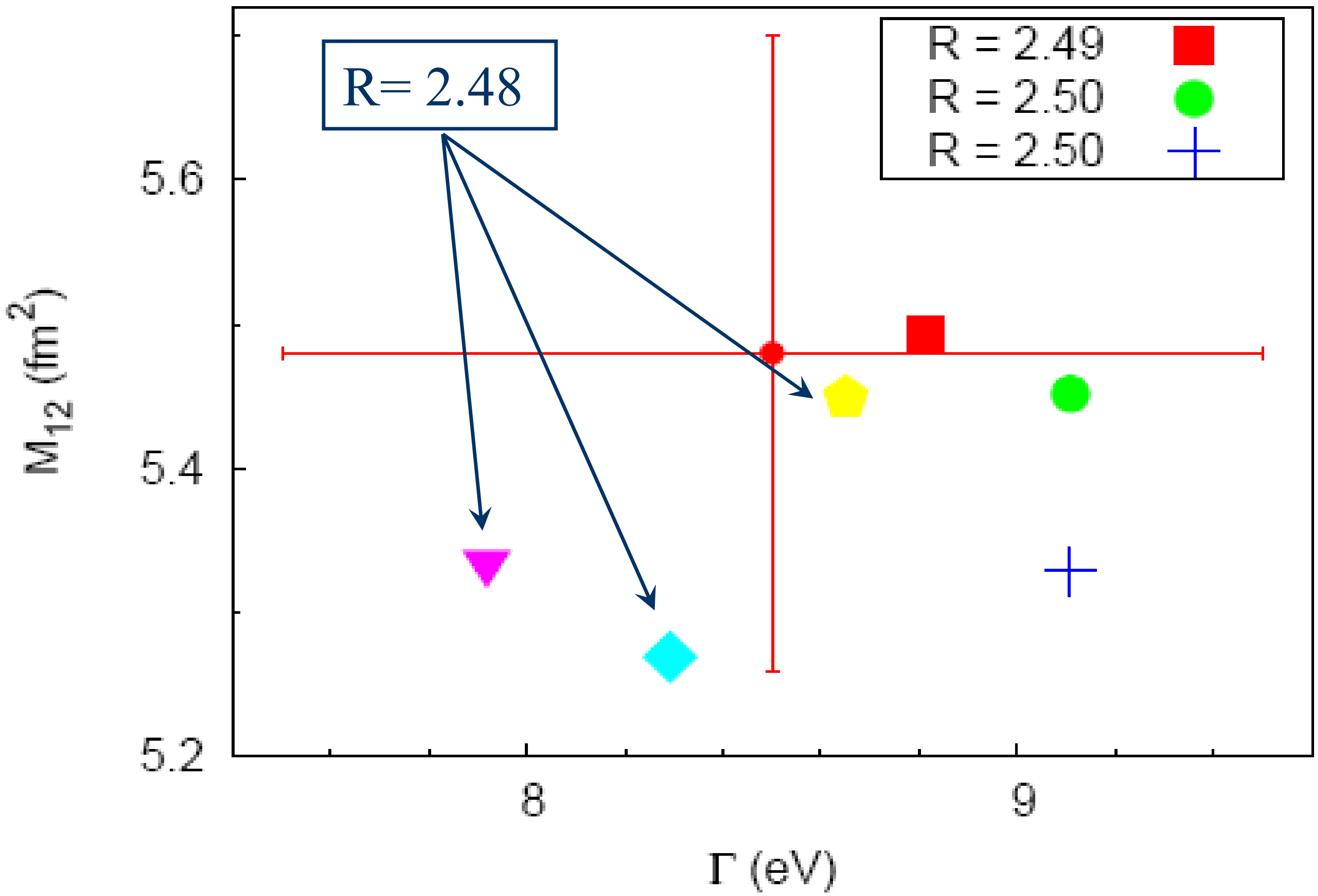
# Realistic effective potentials

| Potential | $V_r$(MeV) | $\mu_r^{-1}$(fm) | $V_a$(MeV) | $\mu_a^{-1}$(fm) |
|---|---|---|---|---|
| $s0$ | 234.914 | 1.54 | -109.766 | 2.0944 |
| $s1$ | 295.160 | 1.4213 | -99.1406 | 2.09455 |
| $s2$ | 340.362 | 1.48 | -140.625 | 2.0128 |
| $s3$ | 378.920 | 1.39 | -116.055 | 2.0433 |
| $s4$ | 581.539 | 1.335 | -148.406 | 1.96737 |
| $d0$ | 152.9 | 1.4213 | -99.1406 | 2.09455 |
| $d1$ | 240.0 | 1.3 | -99.1406 | 2.09455 |
| $d2$ | 299.0 | 1.25 | -99.1406 | 2.09455 |
| $d6$ | 572.0 | 1.23 | -148.406 | 1.96737 |
| $g1$ | 10.0 | 1.424 | -134.000 | 2.09455 |
| $g2$ | 36.0 | 1.424 | -140.000 | 2.09455 |
| $g3$ | 367.0 | 1.335 | -230.000 | 1.96737 |

# Calculated properties of $^{12}$C for a set of effective potentials

| 2b pot. | $V_0$(MeV) | $\rho_0$(fm) | $b$(fm) | $\Gamma$(eV) | $R^{(1)}$(fm) | $R^{(2)}$(fm) | $M_{12}$(fm$^2$) | $R_{tr}$(fm) |
|---|---|---|---|---|---|---|---|---|
| $s1+d2+g1$ | -260.283 | 0 | 1.15362 | 8.81 | 2.488 | 3.591 | 5.494 | 4.86 |
| $s1+d0+g1$ | -63.4126 | 1.90625 | 1.06614 | 7.92 | 2.48 | 3.574 | 5.335 | 4.836 |
| $s4+d6+g3$ | -170.04 | 0. | 1.33903 | 9.68 | 2.541 | 3.688 | 5.316 | 4.991 |
| $s0+d0+g1$ | -33.7737 | 2.87500 | 0.97565 | 8.29 | 2.48 | 3.555 | 5.270 | 4.844 |
| $s2+d2+g2$ | -233.807 | 0. | 1.19950 | 9.11 | 2.502 | 3.620 | 5.452 | 4.888 |
| $s0+d1+g1$ | -129.031 | 0.96875 | 1.11313 | 8.65 | 2.48 | 3.576 | 5.449 | 4.847 |
| $s0+d1+g3$ | -225.326 | 0 | 1.21651 | 9.11 | 2.496 | 3.616 | 5.329 | 4.904 |
| $s1+d1+g1$ | -262.927 | 0. | 1.14824 | 8.82 | 2.485 | 3.591 | 5.449 | 4.848 |
| Exp. | | | | $8.5 \pm 1.0$ | $2.48 \pm 0.02$ | – | $5.47 \pm 0.22$ | $4.396 \pm 0.27$ |

# Monopole transition matrix element vs Hoyle state's width



# Remarks

Amazing ability of the simple α - cluster model

One can demand to improve the experimental accuracy

Further restrictions on the model can be imposed by description of

Reactions in few-α systems

e.g., α- α bremsstrahlung scattering cross section

(α, α) reactions on $^{12}C$

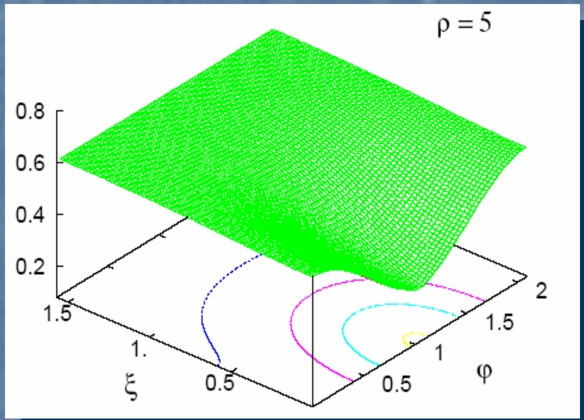
The times has come for further activity

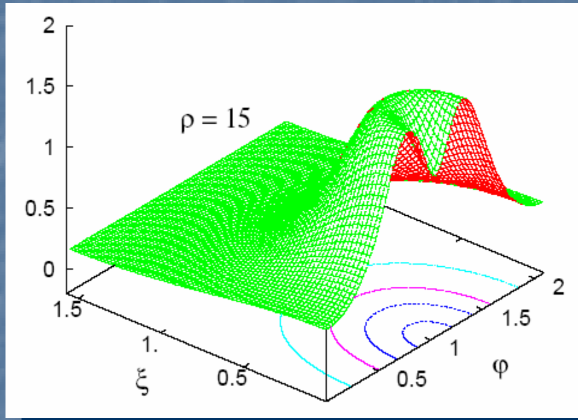
e.g., **Low-energy 3α reactions**

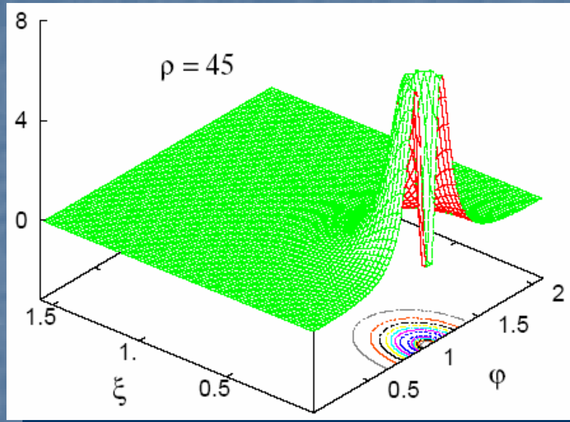
(in particular, non-resonant reaction, which is important for Helium burning at low temperature as it takes place in accretion of white dwarfs and neutron stars)

**Different spin-parity states of $^{12}C$**

# Appendix: The first channel eigenfunction $\Phi_1$




✓ Large ρ (ρ =45fm)

The hyperradial function has the two-cluster structure that confirms the sequential mechanism of $0^+_2$ state decay with formation of $\alpha+{}^8Be$ at the first step.

✓ Intermediate ρ (ρ =15fm)

The two-cluster structure widens; and the most important are the equilateral-triangle and the linear configuration.

✓ Small ρ (ρ =5 fm)

The most important is the equilateral-triangle configuration.

$$\sin\xi = \sin\alpha_i \sin\theta_i,$$
$$\cos\xi\cos\varphi_i = \cos\alpha_i,$$
$$\cos\xi\sin\varphi_i = \sin\alpha_i\cos\theta_i\ .$$

$$0 \le \xi \le \frac{\pi}{2},\ -\pi \le \varphi_i \le \pi$$