Radiative Capture

$^{12}C(p,\gamma)^{13}N$

$^{12}C(\alpha,\gamma)^{16}O$

$^{7}Be(p,\gamma)^{8}B$

$^{6}Li(p,\gamma)^{7}Be$

$^{8}Be(n,\gamma)^{9}Be$

$^{4}He(2n,\gamma)^{6}He$

....

Few-body reactions in nuclear astrophysics

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Rearrangement

$^{6}Li(p,\alpha)^{3}He$

$^{3}He(d,p)\alpha$

$^{18}F(p,\alpha)^{15}O$

$^{11}B(p,\alpha)^{8}Be$

$^{13}C(\alpha,n)^{16}O$

$^{5}He(2n,n)^{6}He$

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→ Two-body

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Few-body reactions in nuclear astrophysics

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→ **Two-body**

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→ **Three-body**

………

$\alpha+\alpha+n \rightarrow {}^{9}Be+\gamma$
$\alpha+n+n \rightarrow {}^{6}He+\gamma$

Rearrangement

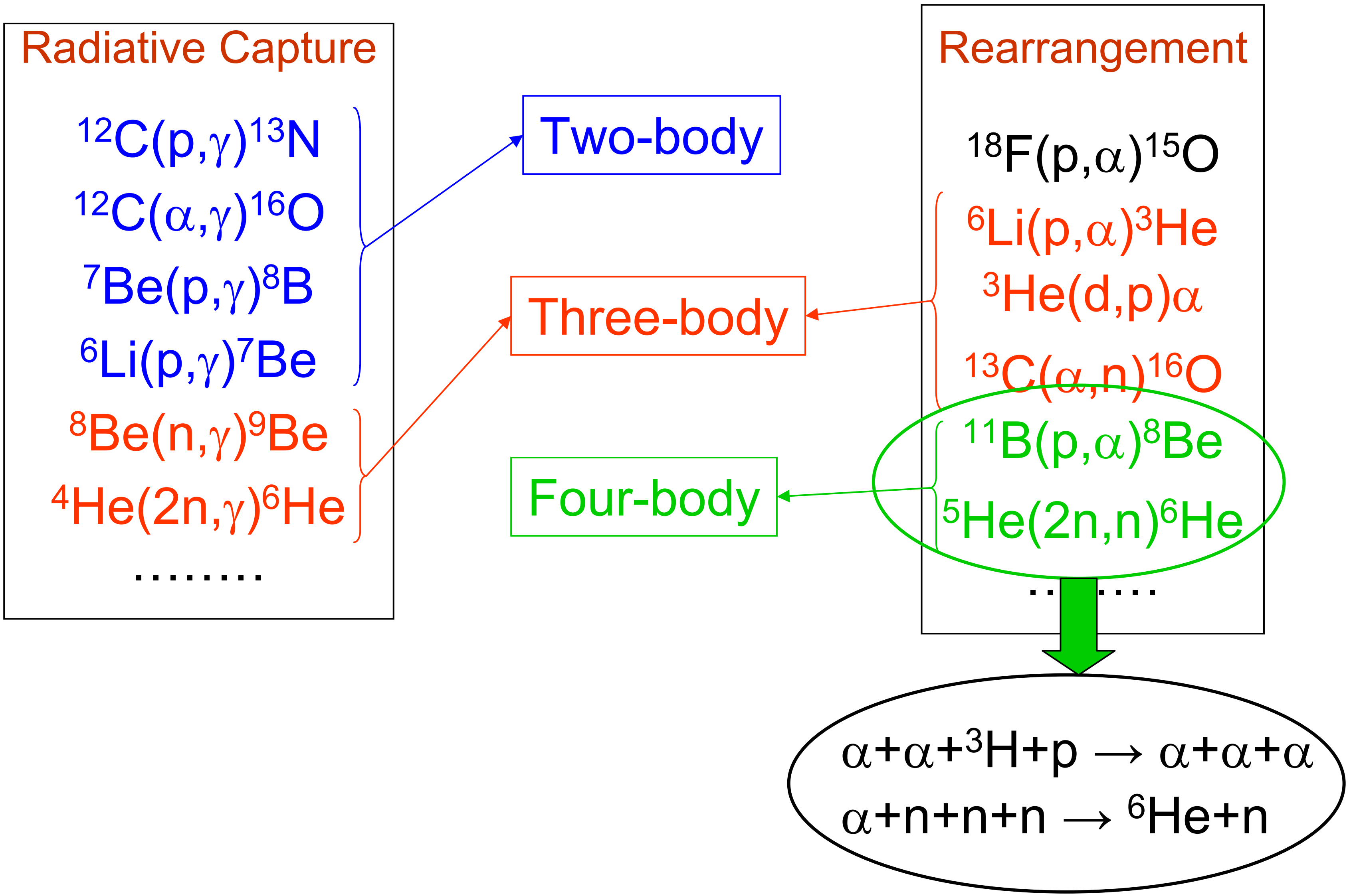
$^{18}F(p,\alpha)^{15}O$
$^{6}Li(p,\alpha)^{3}He$
$^{3}He(d,p)\alpha$
$^{13}C(\alpha,n)^{16}O$
→ **Three-body**

$^{11}B(p,$ … $)^{8}Be$
$^{5}He(2$ … $n)^{6}He$

………

$\alpha+d+p \rightarrow \alpha+{}^{3}He$
$^{3}He+n+p \rightarrow p+\alpha$
$^{12}C+n+\alpha \rightarrow n+{}^{16}O$

Few-body reactions in nuclear astrophysics



Few-body reactions in nuclear astrophysics

Radiative Capture

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- ....

Two-body

Three-body

Four-body

Rearrangement

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- ....

What is the production rate for the different reactions in the stellar medium??

How many reactions per unit time and per unit volume??

- ✓How to compute the production rates ??
- ✓Two simple examples:
  - ✓$^{4}He+n+n \rightarrow {}^{6}He+\gamma$
  - ✓$^{4}He+n+n+n \rightarrow {}^{6}He+n$
- ✓Adiabatic approximation and cross sections
- ✓Summary and conclusions

What is the production rate for the different reactions in the stellar medium??

$P(E) \rightarrow$ Production rate at a given energy

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Relative energy

$a+b \rightarrow \cdots$ $$E=\frac{p_{ab}^2}{2\mu_{ab}}$$

$a+b+c \rightarrow \cdots$ $$E=\frac{p_{ab}^2}{2\mu_{ab}}+\frac{p_{ab,c}^2}{2\mu_{ab,c}}$$

$a+b+c+d \rightarrow \cdots$ $$E=\frac{p_{ab}^2}{2\mu_{ab}}+\frac{p_{ab,c}^2}{2\mu_{ab,c}}+\frac{p_{abc,d}^2}{\mu_{abc,d}}$$

What is the production rate for the different reactions in the stellar medium??

$P(E) \rightarrow$ Production rate at a given energy

$$P^T = \int B(E,T)P(E)dE$$

→ Total Reaction Production Rate

Relative energy

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Few-body reactions in nuclear astrophysics

What is the production rate for the different reactions in the stellar medium??

$P(E) \rightarrow$ Production rate at a given energy

$$P^T = \int B(E,T) P(E) dE$$ → Total Reaction Production Rate

	Relative energy	Maxwell-Boltzmann distribution
$a+b\rightarrow\cdots$	$E=\frac{p_{ab}^2}{2\mu_{ab}}$	$B_2(E,T)=\frac{1}{\Gamma(3/2)}\frac{1}{K_BT}\left(\frac{E}{K_BT}\right)^{\frac{1}{2}} e^{-\frac{E}{K_BT}}$
$a+b+c\rightarrow\cdots$	$E=\frac{p_{ab}^2}{2\mu_{ab}}+\frac{p_{ab,c}^2}{2\mu_{ab,c}}$	$B_3(E,T)=\frac{1}{\Gamma(3)}\frac{1}{K_BT}\left(\frac{E}{K_BT}\right)^{2} e^{-\frac{E}{K_BT}}$
$a+b+c+d\rightarrow\cdots$	$E=\frac{p_{ab}^2}{2\mu_{ab}}+\frac{p_{ab,c}^2}{2\mu_{ab,c}}+\frac{p_{abc,d}^2}{\mu_{abc,d}}$	$B_4(E,T)=\frac{1}{\Gamma(9/2)}\frac{1}{K_BT}\left(\frac{E}{K_BT}\right)^{\frac{7}{2}} e^{-\frac{E}{K_BT}}$

Few-body reactions in nuclear astrophysics

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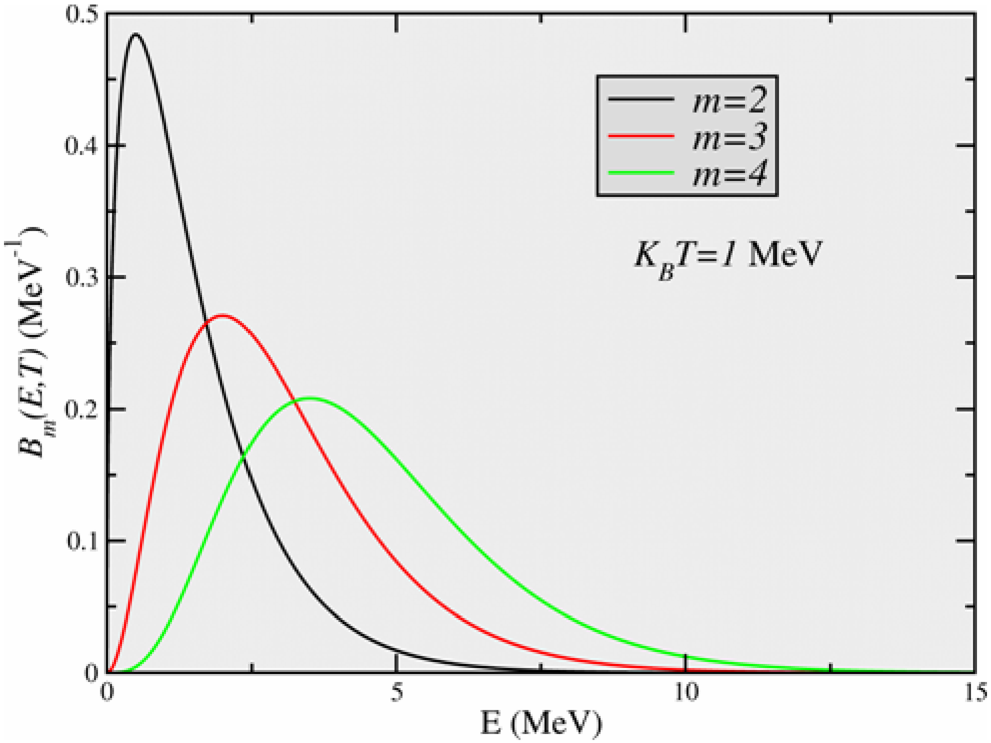

Maxwell-Boltzmann distribution

$$B_2(E,T) = \frac{1}{\Gamma(3/2)} \frac{1}{K_B T} \left(\frac{E}{K_B T} \right)^{\frac{1}{2}} e^{-\frac{E}{K_B T}}$$

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Few-body reactions in nuclear astrophysics

What is the production rate for the different reactions in the stellar medium??

Given a temperature T, only values of $E \lesssim K_B T$ are relevant

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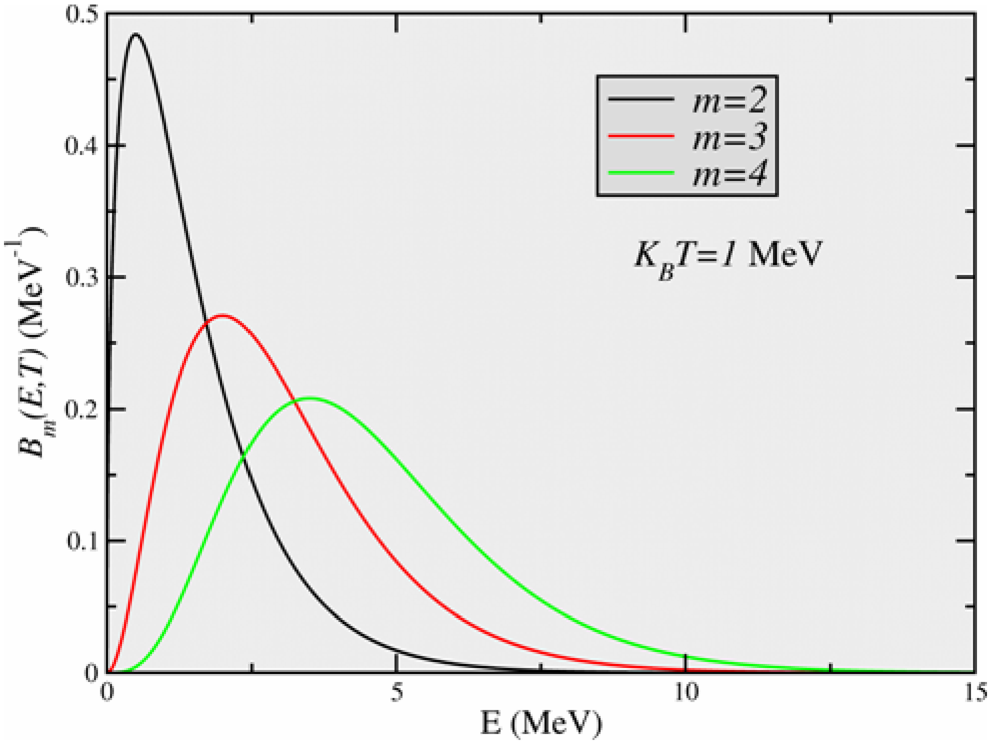

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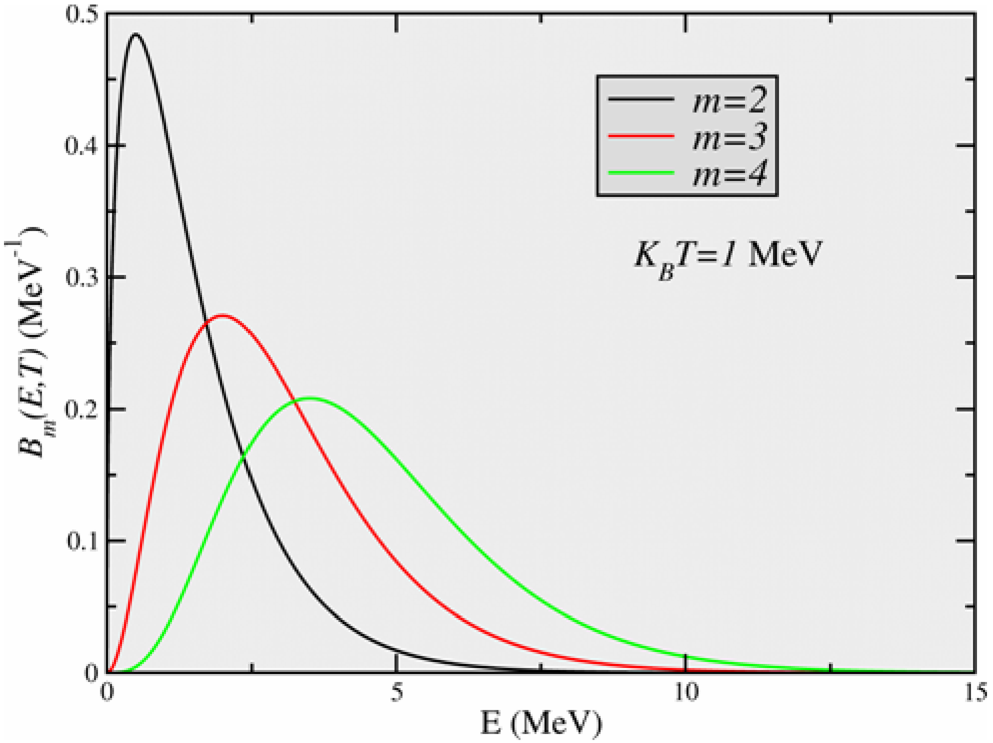
What is the production rate for the different reactions in the stellar medium??

Given a temperature *T*, only values of $E \lesssim K_B T$ are relevant ⇨ $E/K_BT<1$

$$T = 10 \text{ GK} \Rightarrow K_B T \approx 0.9 \text{ MeV}$$



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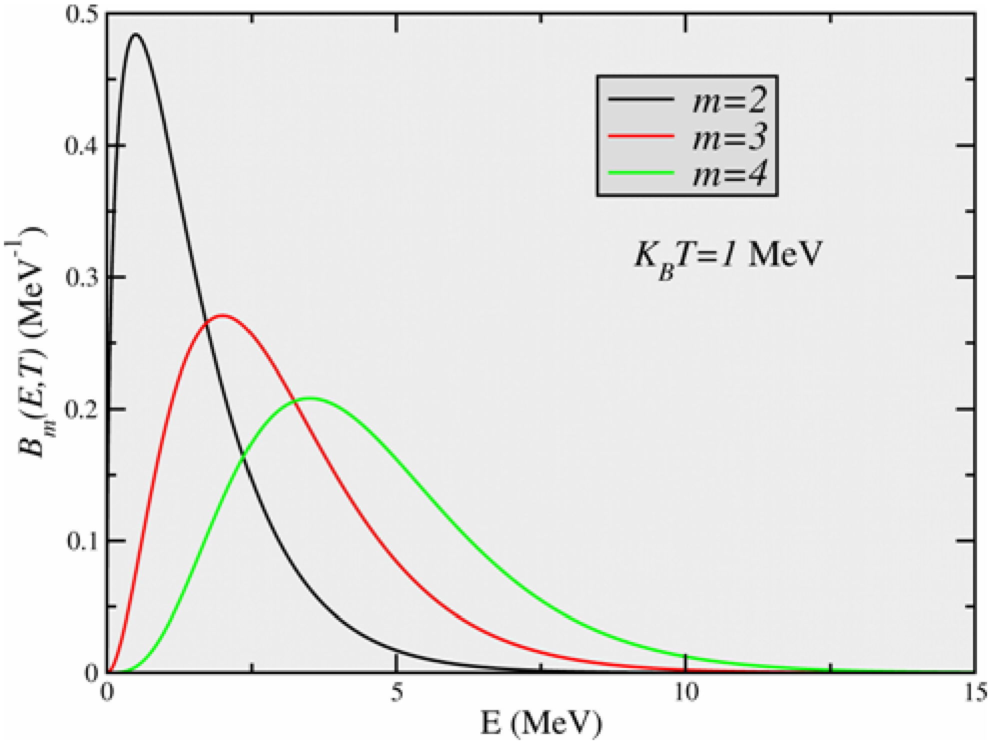
In a standard star, like the sun, $T \sim 10^7$ K=0.01 GK ⇨ $K_B T \approx 0.01$ MeV



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In the stellar medium only very low relative energies are relevant !!!

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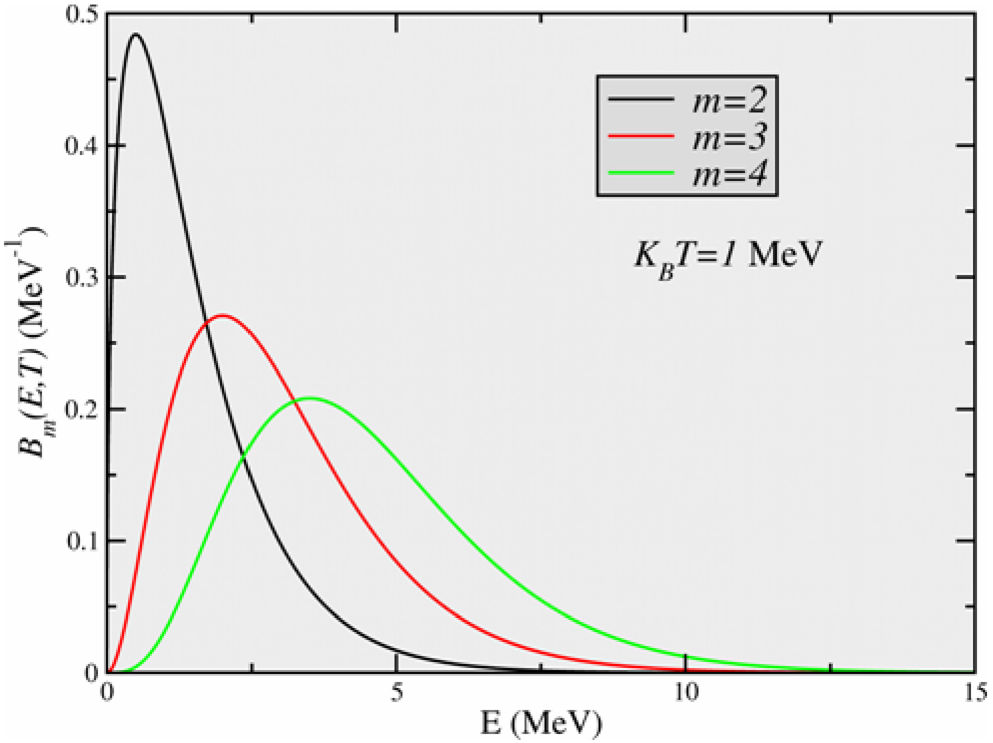

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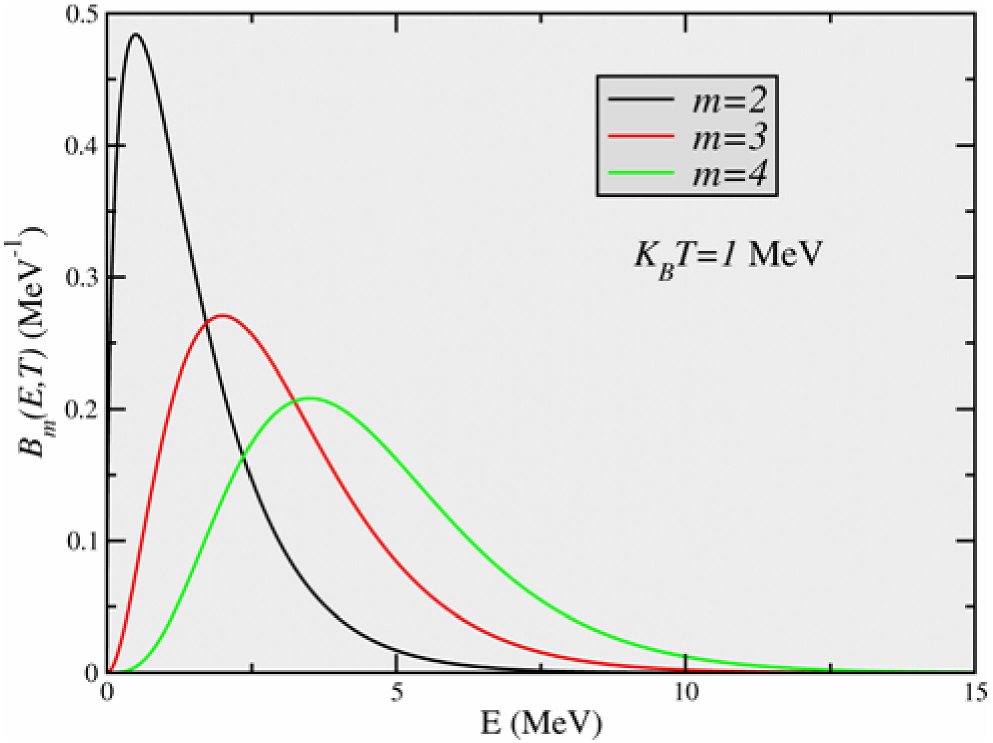

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$$a + b + c \to d + e$$

$$P_3(E) = \frac{2\pi}{\hbar} \left| \left\langle \Psi_i(E) \middle| W \middle| \Psi_f(E_f) \right\rangle \right|^2 \delta(E - E_f)$$

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Few-body reactions in nuclear astrophysics

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$$n_i = \rho N_A \frac{X_i}{A_i}$$

$\rho \equiv$ Mass density

$X_i \equiv$ Mass abundance of i $\left(\sum_i X_i = 1 \right)$

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It is possible to relate $P_3(E)$ and $P_2(E)$

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Few-body reactions in nuclear astrophysics

What is the production rate for the different reactions in the stellar medium??

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$$P_2(E) = n_d n_e v_{de} \sigma_{de}(E)$$

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It is possible to relate $P_3(E)$ and $P_2(E)$

$$P_2(E) = n_d n_e v_{de} \sigma_{de}(E)$$

$P_3(E)$ can be written in terms of the cross section for the inverse process

Few-body reactions in nuclear astrophysics

What is the production rate for the different reactions in the stellar medium??

$$\alpha + n + n \rightarrow {}^6\mathrm{He} + \gamma$$

Two-neutron radiative capture

$$P_{\alpha nn}(\rho, T) = \frac{n_\alpha n_n^2}{2} \frac{\hbar^3}{c^2} \left(\frac{m_\alpha + m_n + m_n}{m_\alpha m_n m_n} \right)^{3/2} \frac{2\pi}{(K_B T)^3} e^{-\frac{Q}{K_B T}} \int_{|Q|}^{\infty} E^2 \sigma_{\gamma, {}^6\mathrm{He}}(E) e^{-\frac{E}{K_B T}} dE$$

$$Q = m_{{}^6\mathrm{He}} - m_\alpha - m_n - m_n$$

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$$\alpha + n + n + n \rightarrow {}^6\mathrm{He} + n$$

Four-body recombination

$$P_{\alpha nnn}(\rho,T) = \frac{n_\alpha n_n^3}{6}\hbar^6 \mu_{n,{}^6\mathrm{He}}\left(\frac{m_\alpha + m_n + m_n + m_n}{m_\alpha m_n m_n m_n}\right)^{3/2}\frac{(2\pi)^{3/2}}{(K_B T)^{9/2}} e^{-\frac{Q}{K_B T}}\int_{|Q|}^{\infty} E \sigma_{n,{}^6\mathrm{He}}(E) e^{-\frac{E}{K_B T}} dE$$

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$$\sigma_{\gamma,{}^6\mathrm{He}} = \sum_\lambda \left(\sigma^{E\lambda}_{\gamma,{}^6\mathrm{He}} + \sigma^{M\lambda}_{\gamma,{}^6\mathrm{He}}\right)$$

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$$\sigma_{\gamma, {}^6\mathrm{He}} = \sum_\lambda \left(\sigma_{\gamma, {}^6\mathrm{He}}^{E\lambda} + \sigma_{\gamma, {}^6\mathrm{He}}^{M\lambda} \right)$$

$$\sigma_{\gamma, {}^6\mathrm{He}}^{E\lambda}(E_\gamma) = \frac{\alpha (2\pi)^3 \hbar c (\lambda + 1)}{\lambda \left[(2\lambda + 1)!! \right]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda - 1} \frac{dB(E\lambda)}{dE_\gamma}$$

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$$\sigma^{E\lambda}_{\gamma,{}^6\mathrm{He}}(E_\gamma) = \frac{\alpha\left(2\pi\right)^3 \hbar c(\lambda+1)}{\lambda\left[(2\lambda+1)!!\right]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE_\gamma}$$

$$B(E\lambda, I_i \rightarrow n\, I_f) = \sum_{\mu, M_f}\left|\left\langle n\, I_f M_f \left| \mathcal{M}_\mu(E\lambda)\right| I_i M_i\right\rangle\right|^2; \quad \mathcal{M}_\mu(E\lambda) = e\sum_i Z_i r_i^\lambda Y_{\lambda,\mu}\left(\hat{r}_i\right)$$

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Three-body continuum wave function

Three-body bound state wave function

$$B(E\lambda, I_i \rightarrow n\, I_f) = \sum_{\mu, M_f} \left| \left\langle n\, I_f M_f \right| \mathcal{M}_\mu(E\lambda) \left| I_i M_i \right\rangle \right|^2 ; \quad \mathcal{M}_\mu(E\lambda) = e\sum_i Z_i r_i^\lambda Y_{\lambda,\mu}(\hat{r}_i)$$

$$\alpha + n + n \rightarrow {}^{6}\mathrm{He} + \gamma$$

Two-neutron radiative capture

$$P_{\alpha nn}(\rho,T) = \frac{n_\alpha n_n^2}{2}\frac{\hbar^3}{c^2}\left(\frac{m_\alpha + m_n + m_n}{m_\alpha m_n m_n}\right)^{3/2}\frac{2\pi}{(K_B T)^3} e^{-\frac{Q}{K_B T}}\int_{|Q|}^{\infty} E^2 \sigma_{\gamma,{}^6\mathrm{He}}(E) e^{-\frac{E}{K_B T}} dE$$

We compute the **three-body** wave functions solving the **Faddeev equations** using the se **hyperspheric adiabatic expansion method**

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Few-body reactions in nuclear astrophysics

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We discretize the continuum spectrum solving the equations with a box boundary condition.

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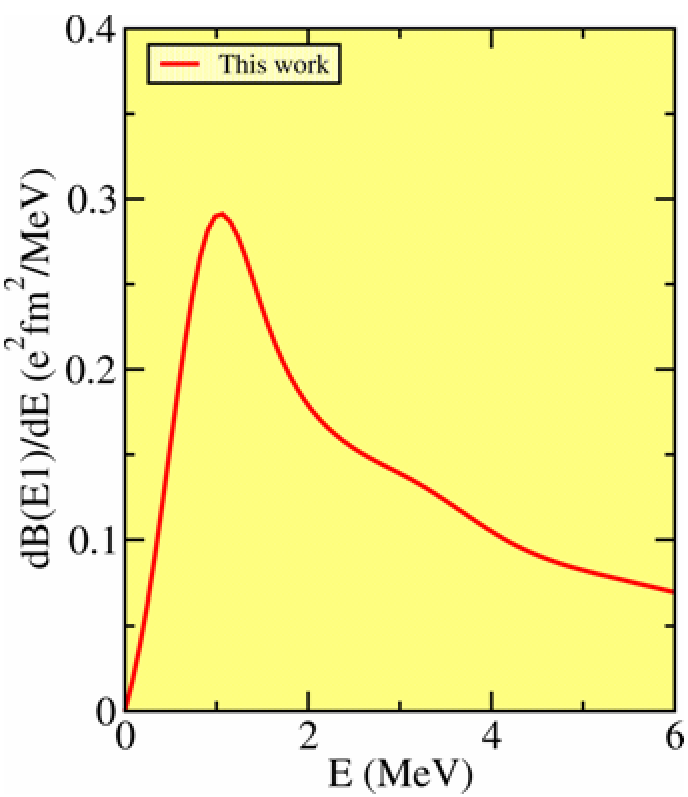

$$\sigma^{E\lambda}_{\gamma,{}^6\mathrm{He}}(E_\gamma) = \frac{\alpha\left(2\pi\right)^3 \hbar c(\lambda+1)}{\lambda\left[\left(2\lambda+1\right)!!\right]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE_\gamma}$$

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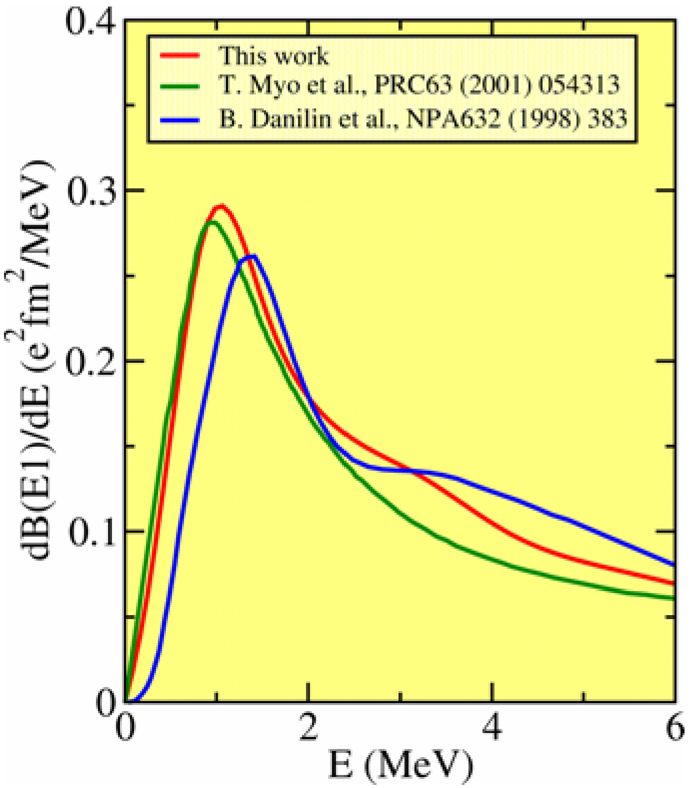

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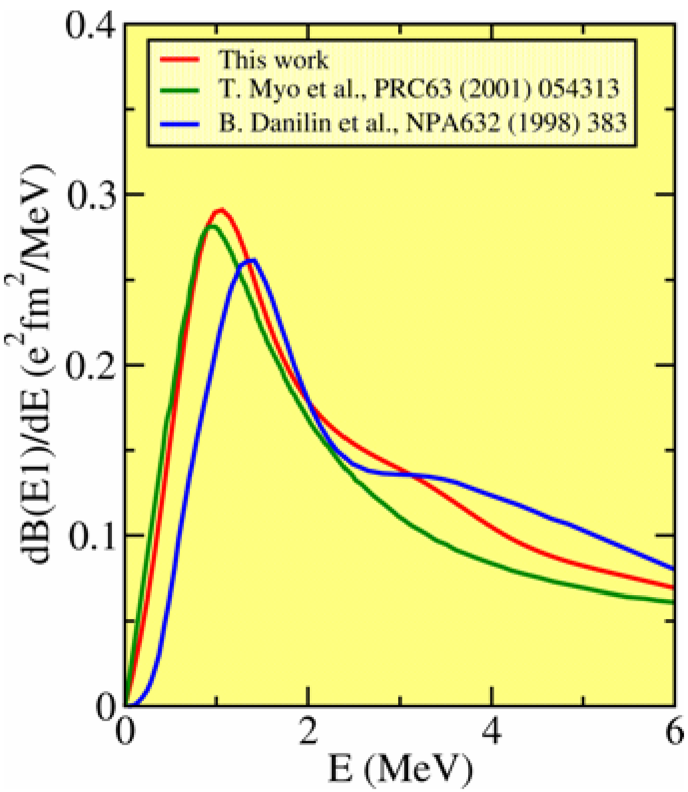

Complex scaling

$$r \rightarrow re^{i\theta};\; 0 < \theta < \pi/2$$

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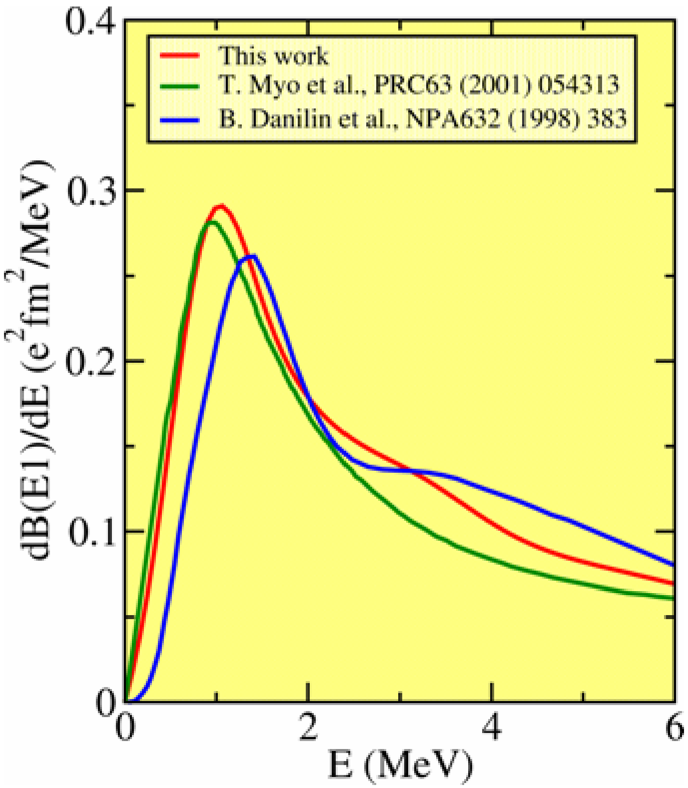

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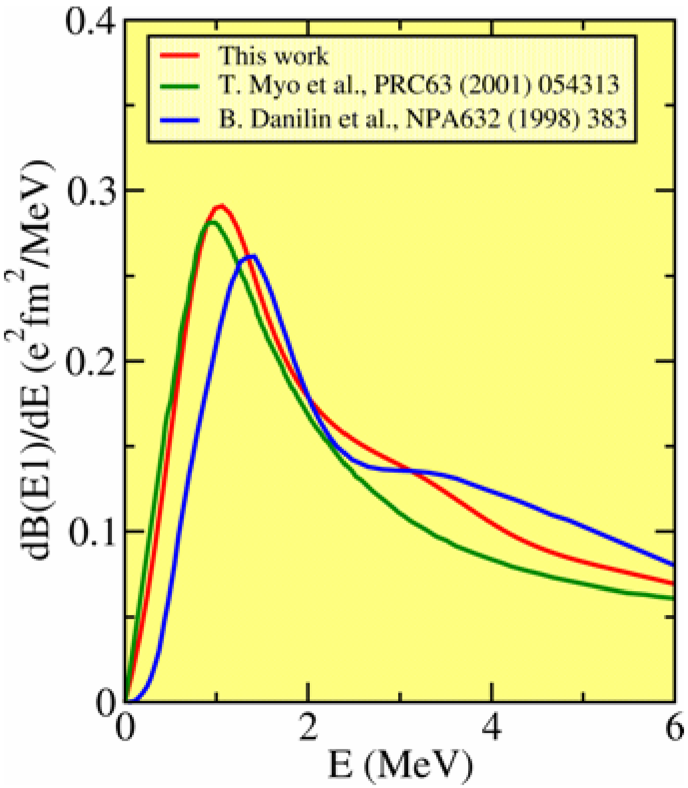

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$$E_n \approx e^{2i\theta}(n\hbar\pi)^2 / \left(2m\rho_{\max}^2\right)$$

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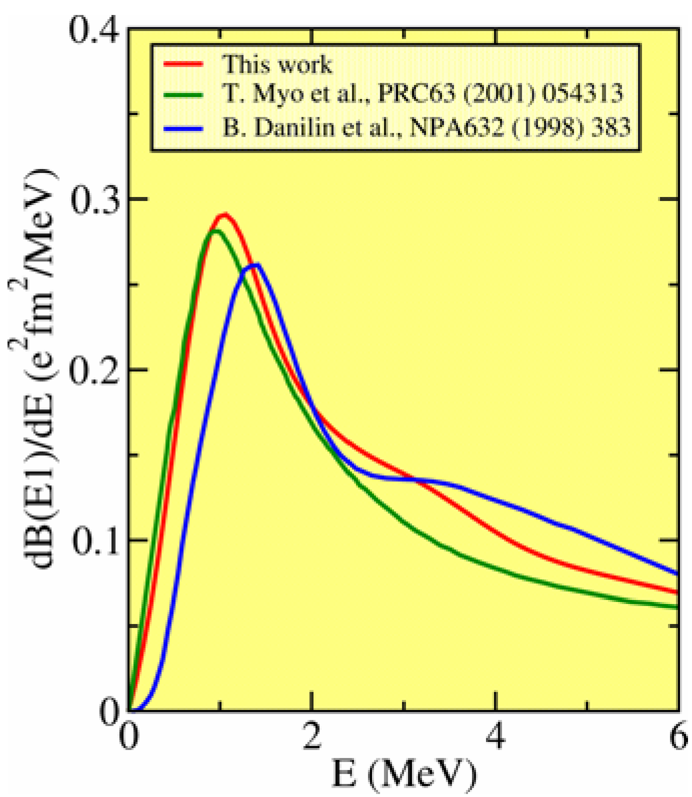

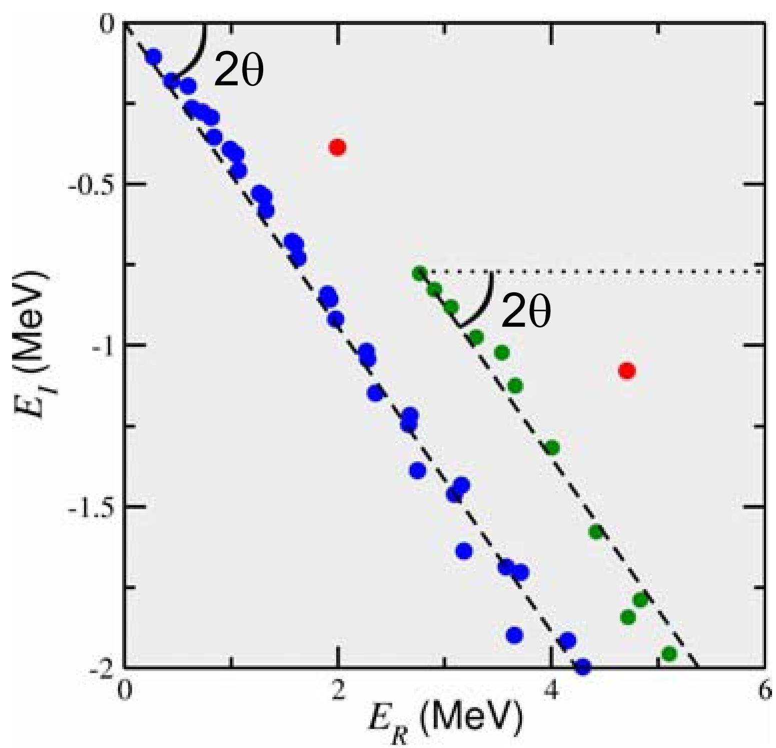

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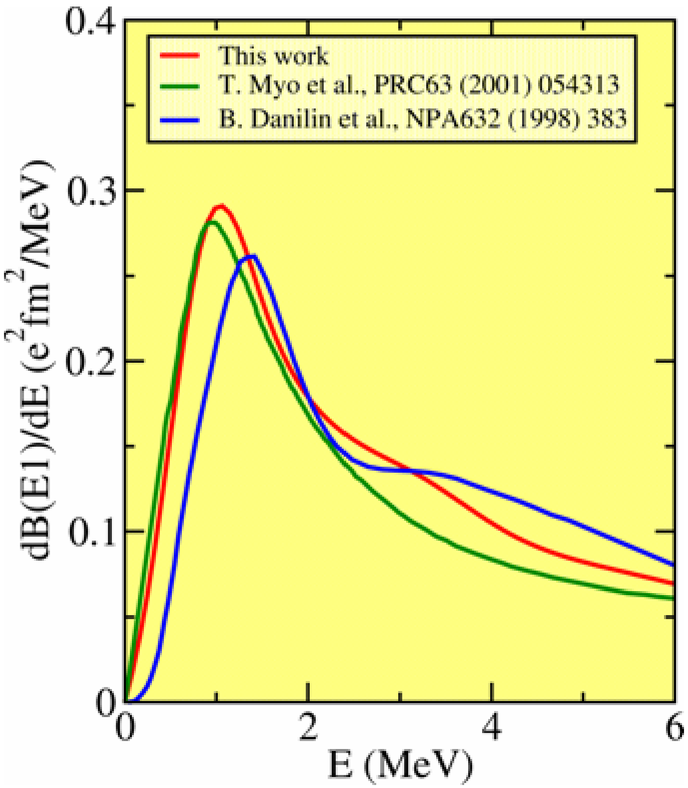

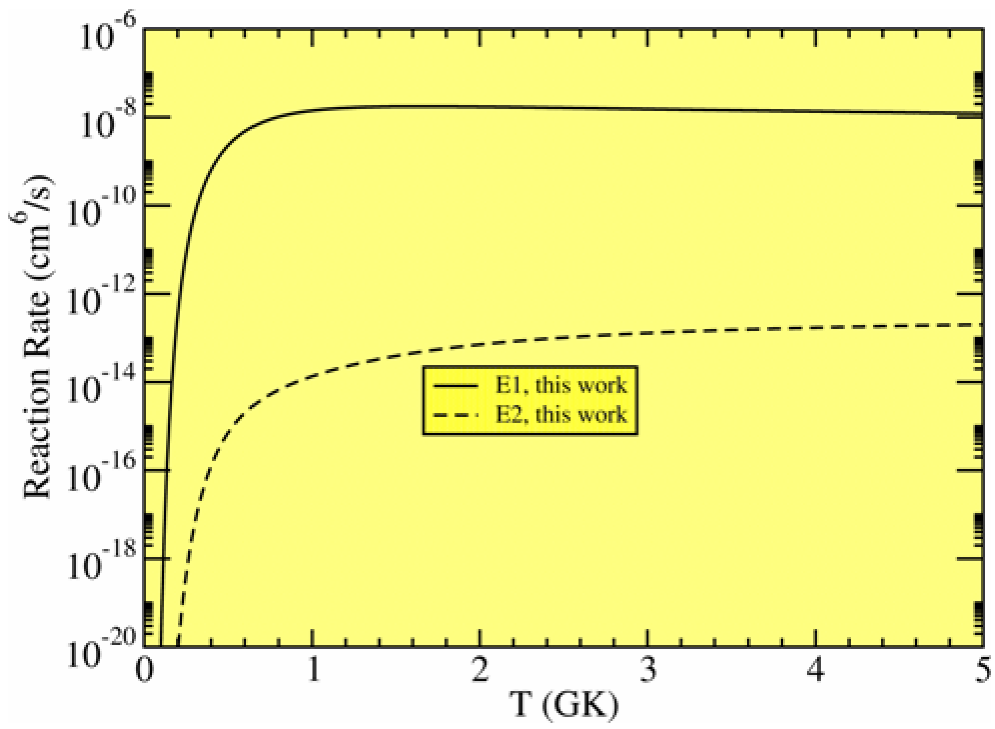

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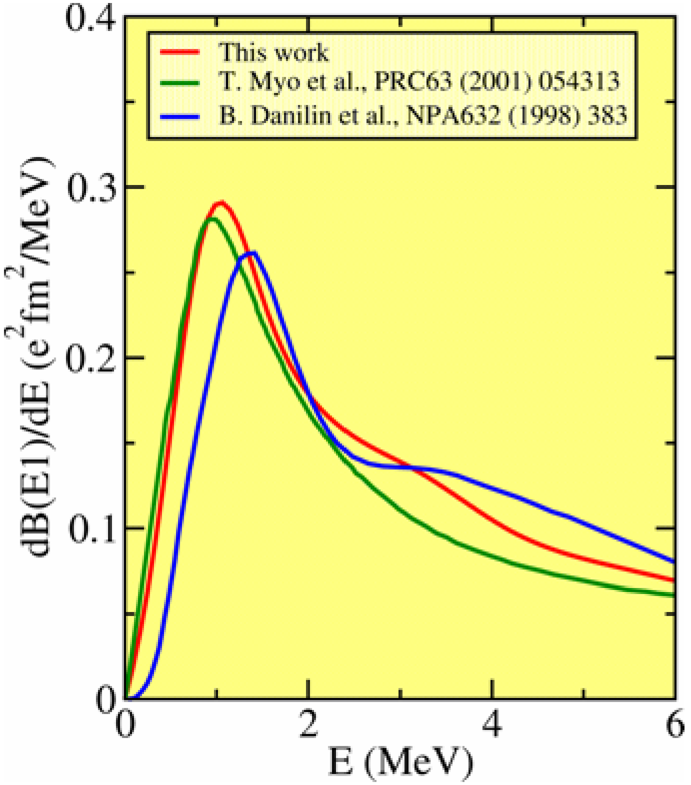

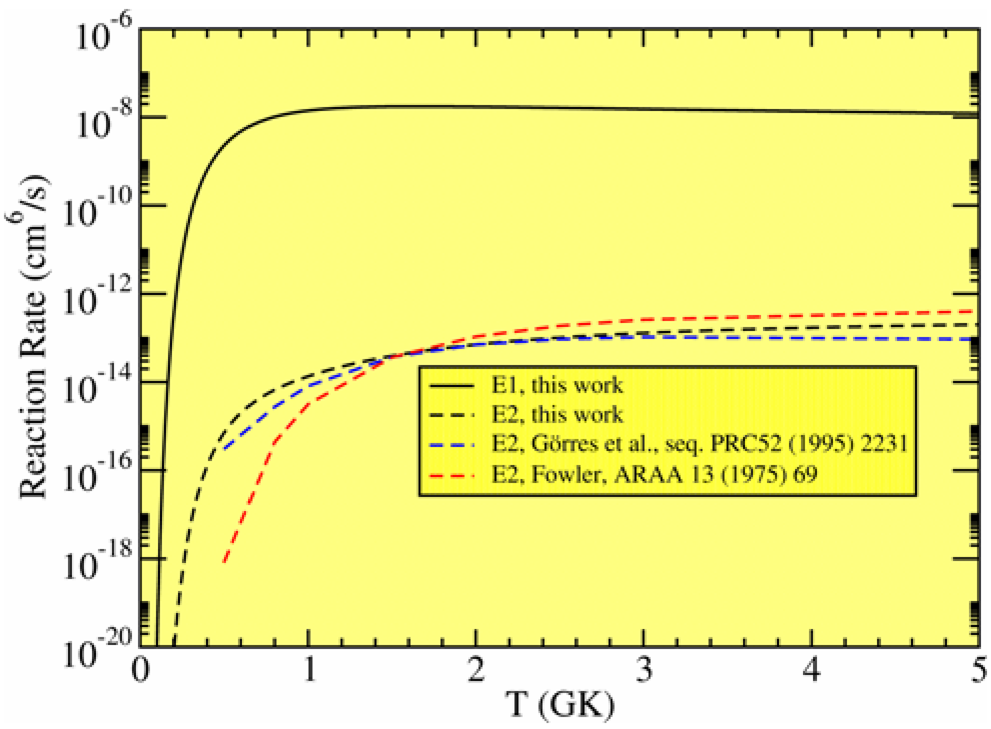

E. Garrido, Critical Stability, Erice, October 2008

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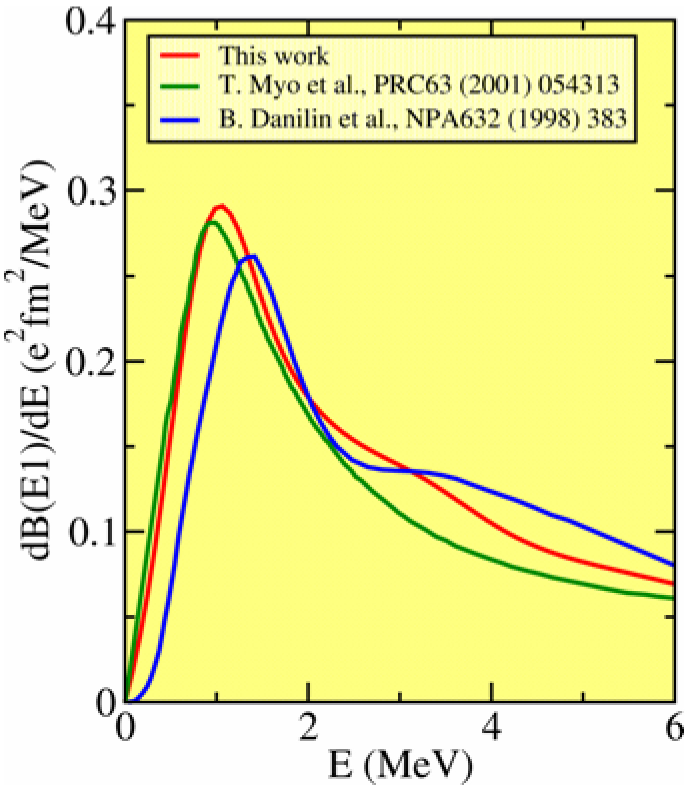

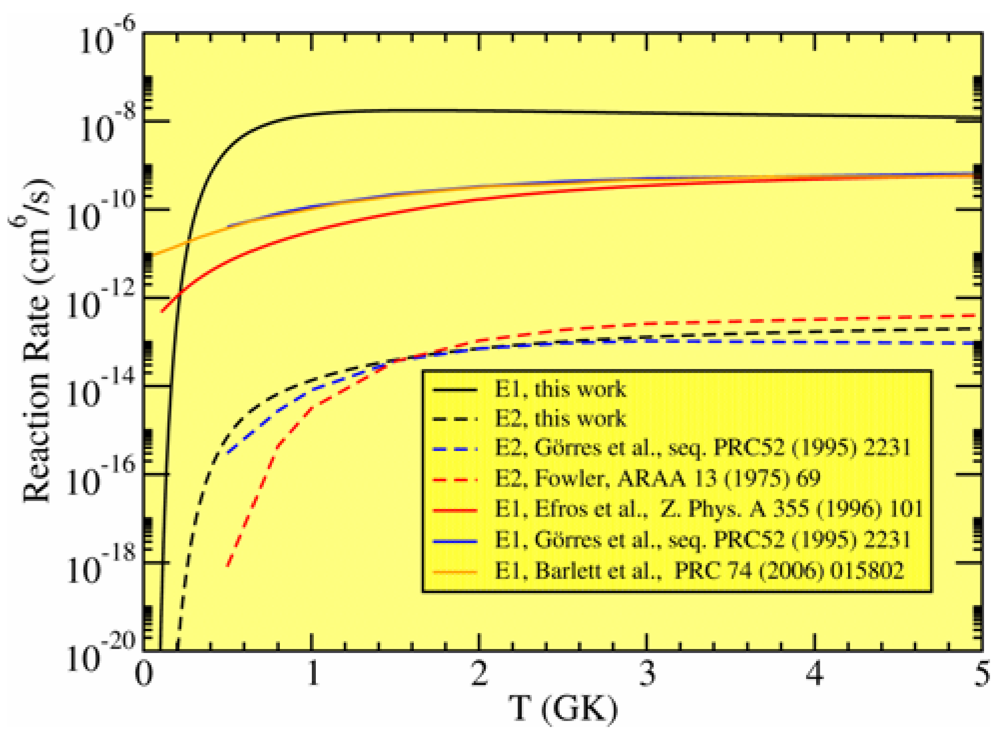

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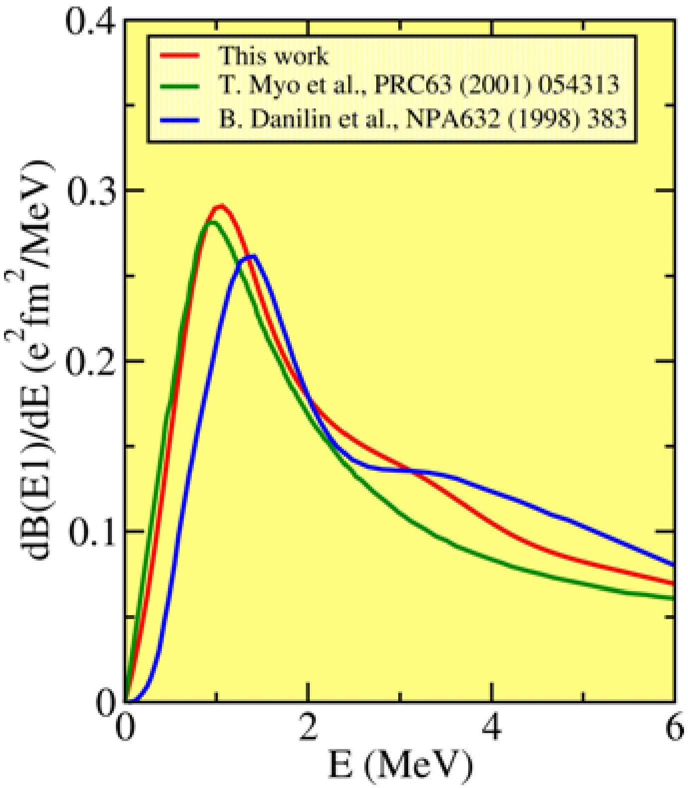

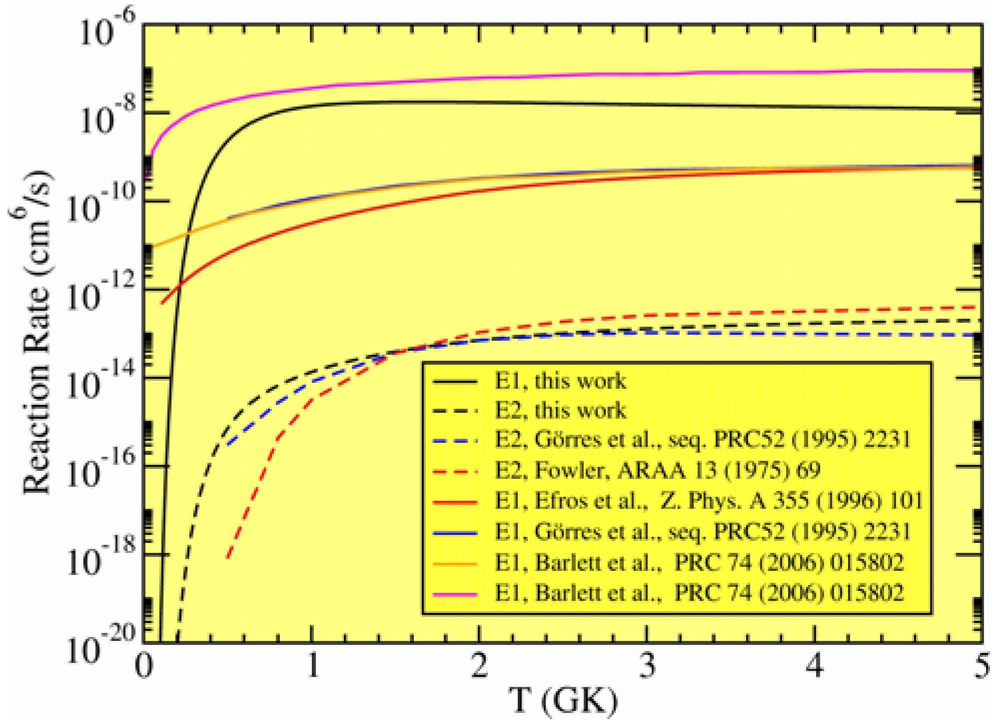

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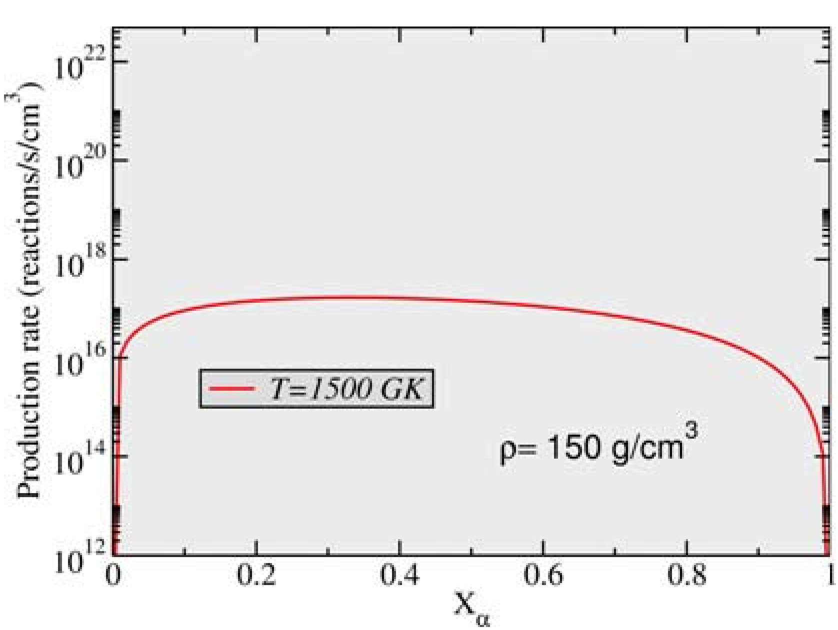

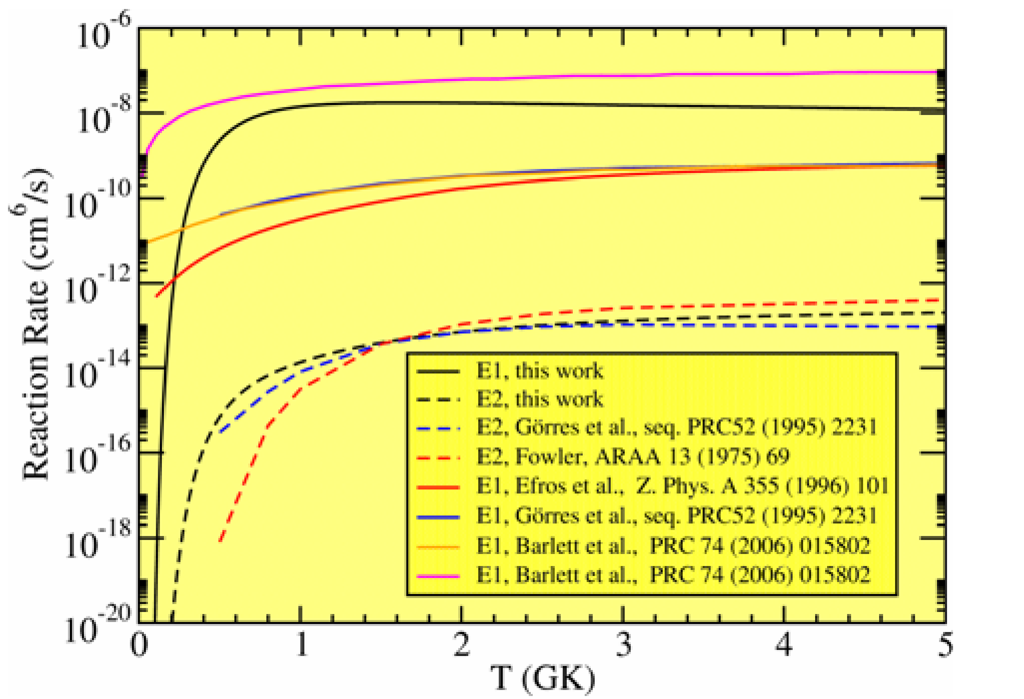

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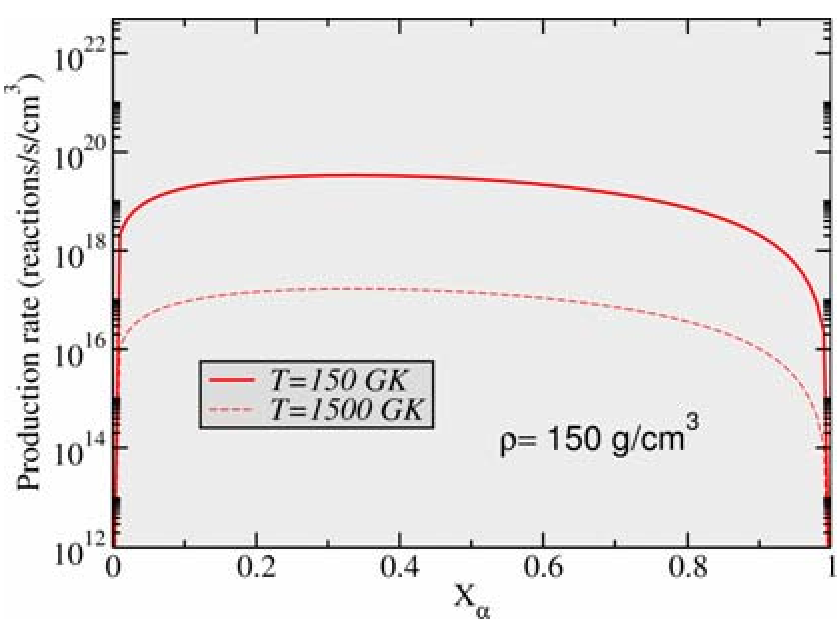

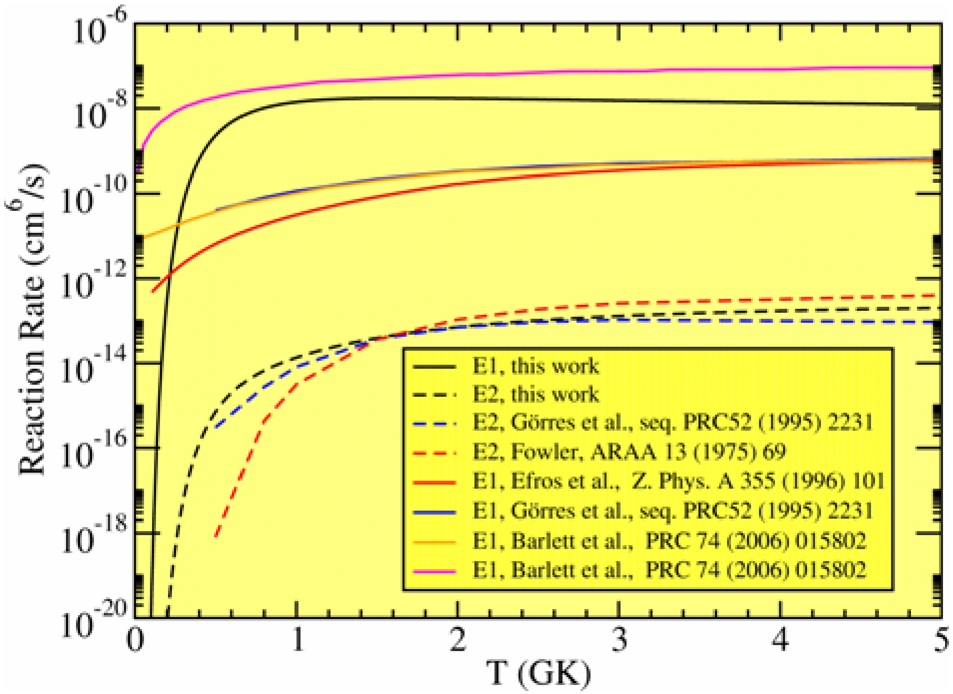

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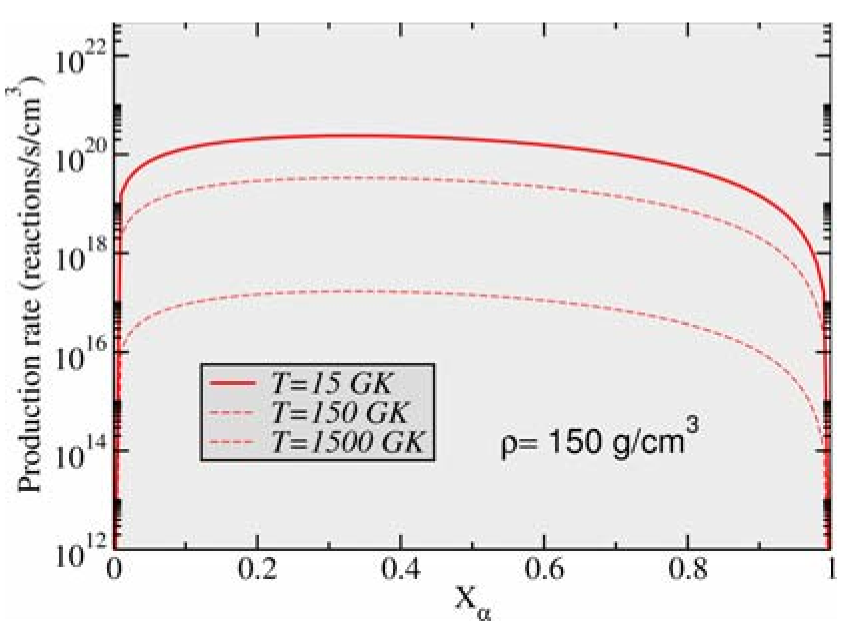

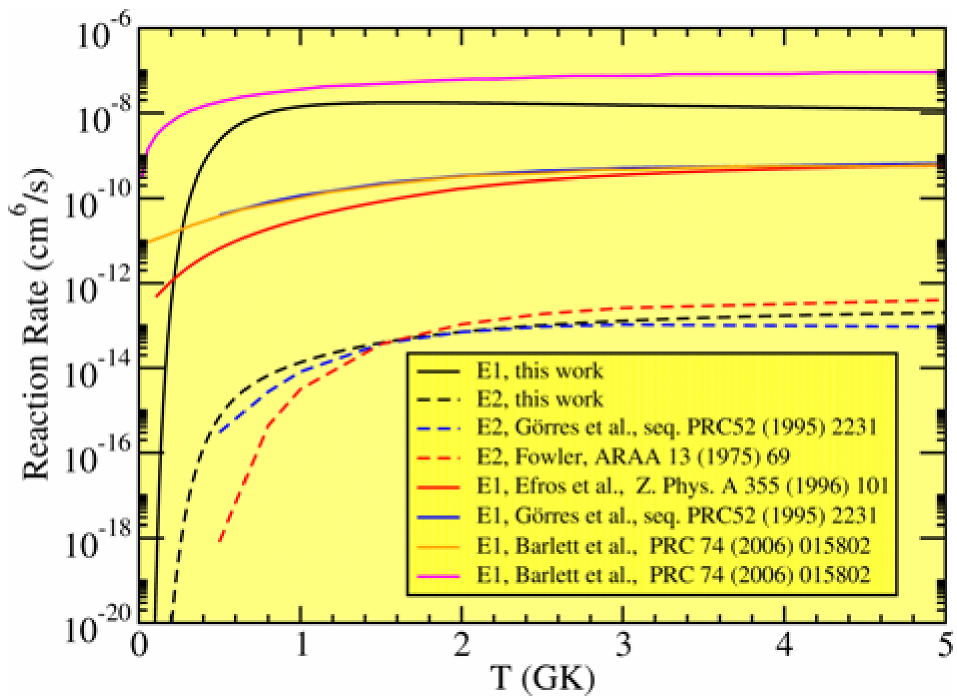

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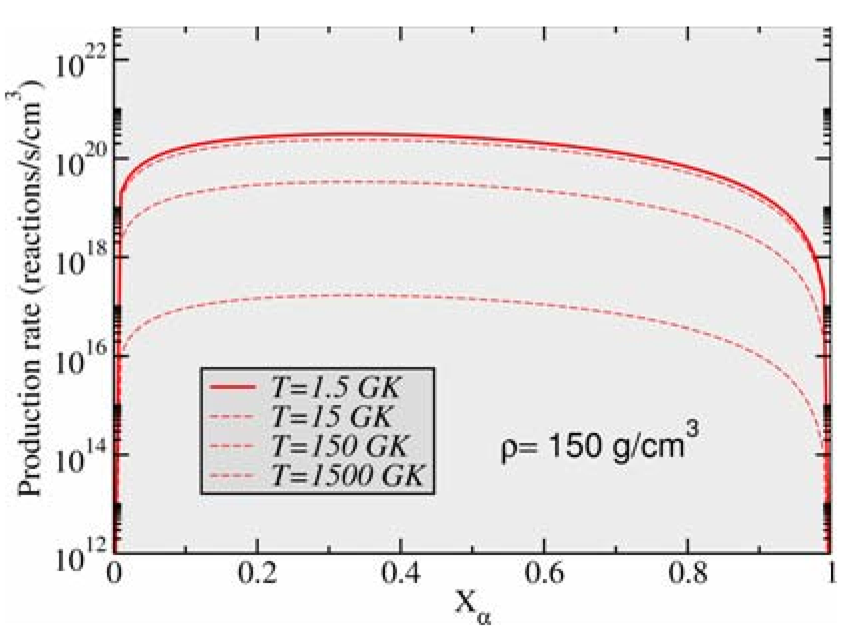

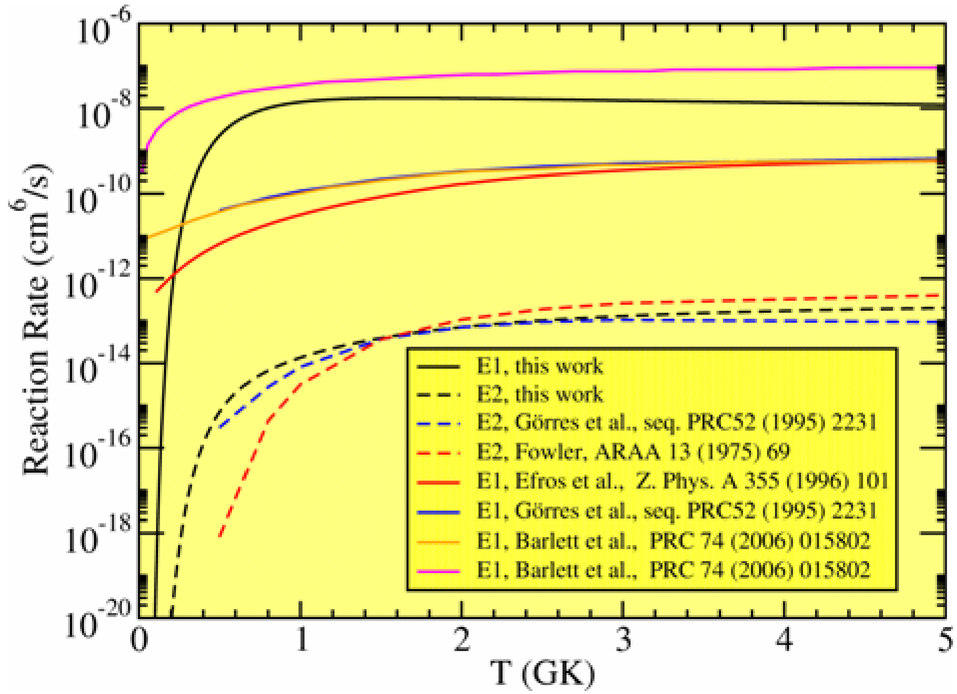

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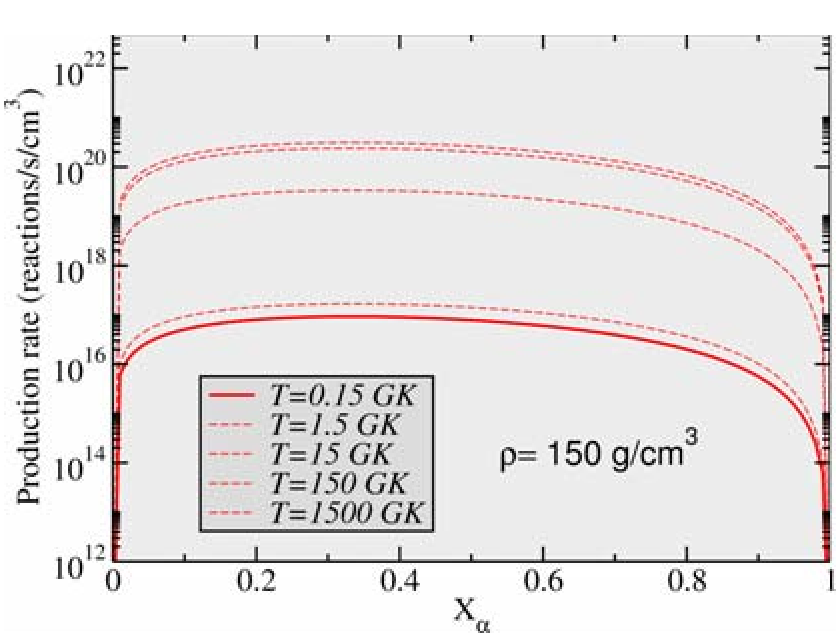

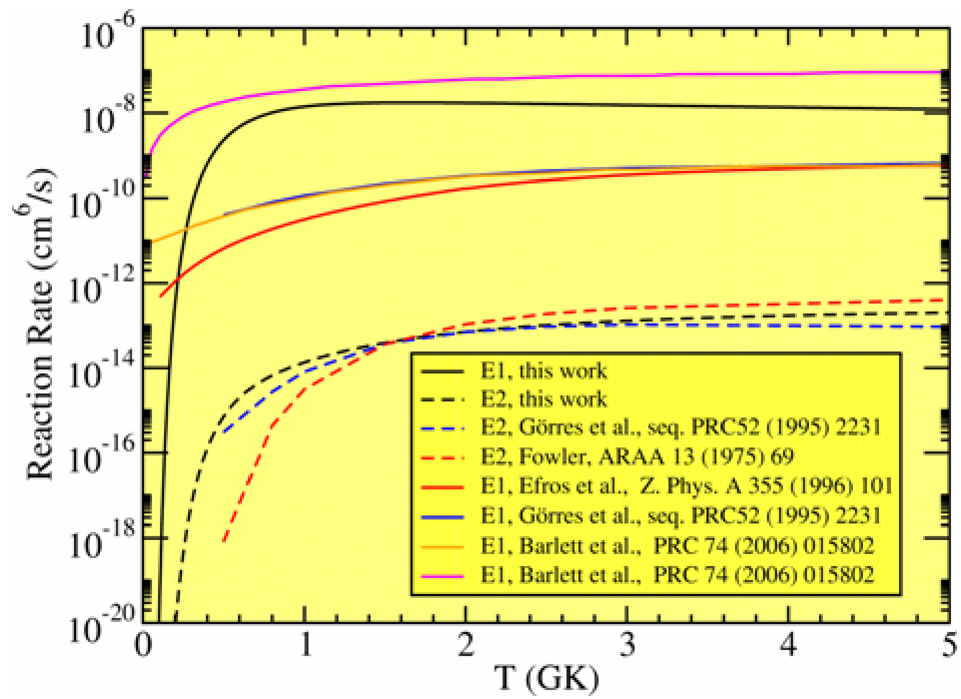

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Four-body recombination

$$P_{\alpha nnn}(\rho,T) = \frac{n_\alpha n_n^3}{6}\hbar^6 \mu_{n,{}^6\mathrm{He}} \left(\frac{m_\alpha + m_n + m_n + m_n}{m_\alpha m_n m_n m_n}\right)^{3/2} \frac{(2\pi)^{3/2}}{(K_B T)^{9/2}} e^{-\frac{Q}{K_B T}} \int_{|Q|}^{\infty} E\sigma_{n,{}^6\mathrm{He}}(E) e^{-\frac{E}{K_B T}} dE$$

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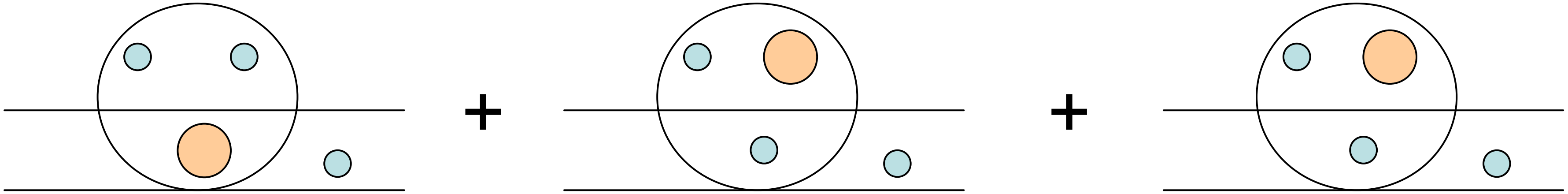

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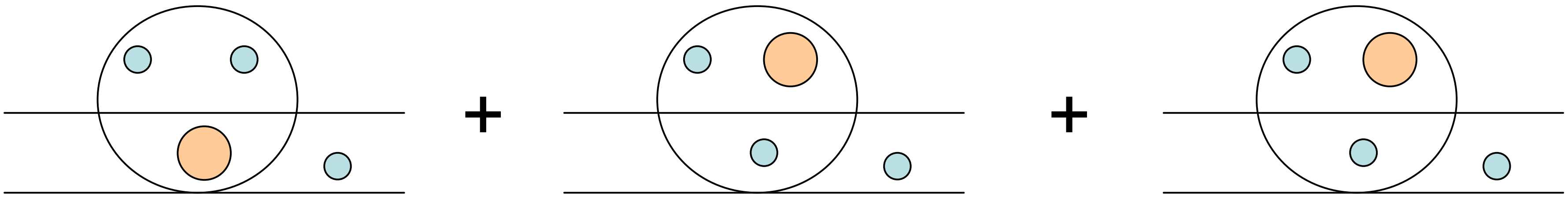

$$T = \sum_{i=1}^{3} T^{(i)};$$

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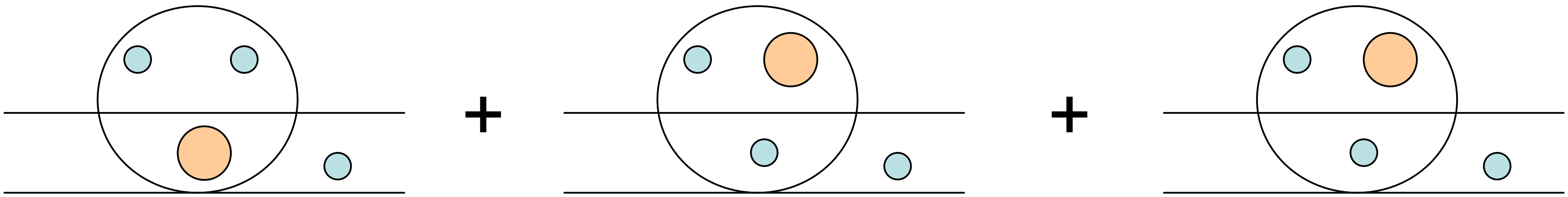

$$T = \sum_{i=1}^{3} T^{(i)}; \qquad T^{(i)} = \left\langle \Psi_{\mathrm{cont}} \left| e^{i\vec{q}_{\mathrm{c.m.}}\cdot\vec{r}_{i,jk}} \right| \Psi_{\mathrm{bound}} \right\rangle \left\langle e^{i\vec{P}'\cdot\vec{r}_{0i}} \left| V_{0i}(r_{0i}) \right| e^{i\vec{P}\cdot\vec{r}_{0i}} \right\rangle$$

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Four-body recombination

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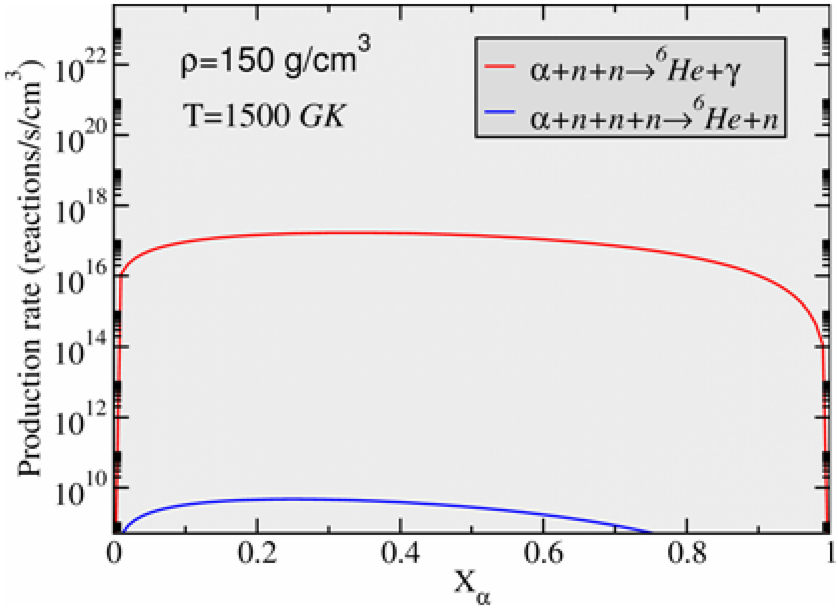

$$T = \sum_{i=1}^{3} T^{(i)}; \qquad T^{(i)} = \left\langle \Psi_{\mathrm{cont}} \left| e^{i\vec{q}_{\mathrm{c.m.}}\cdot\vec{r}_{i,jk}} \right| \Psi_{\mathrm{bound}} \right\rangle \left\langle e^{i\vec{P}'\cdot\vec{r}_{0i}} \left| V_{0i}(r_{0i}) \right| e^{i\vec{P}\cdot\vec{r}_{0i}} \right\rangle$$

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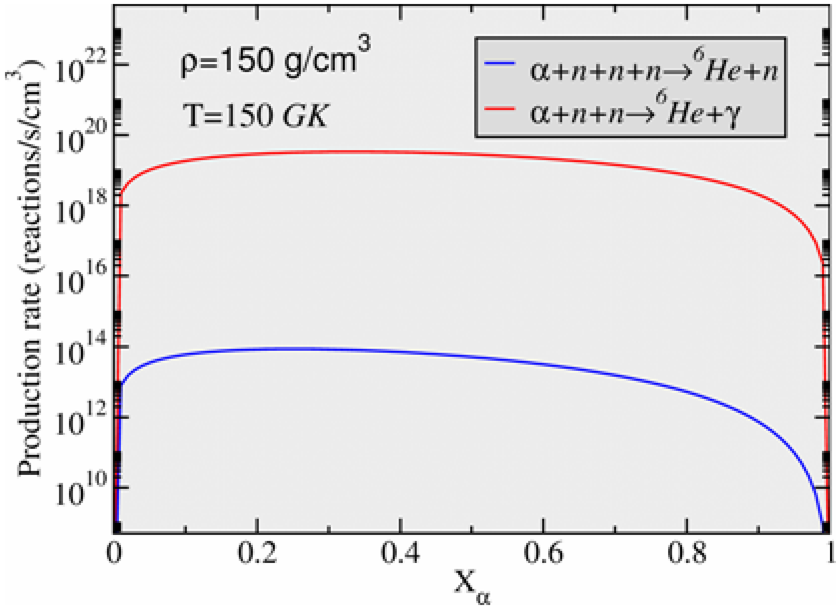

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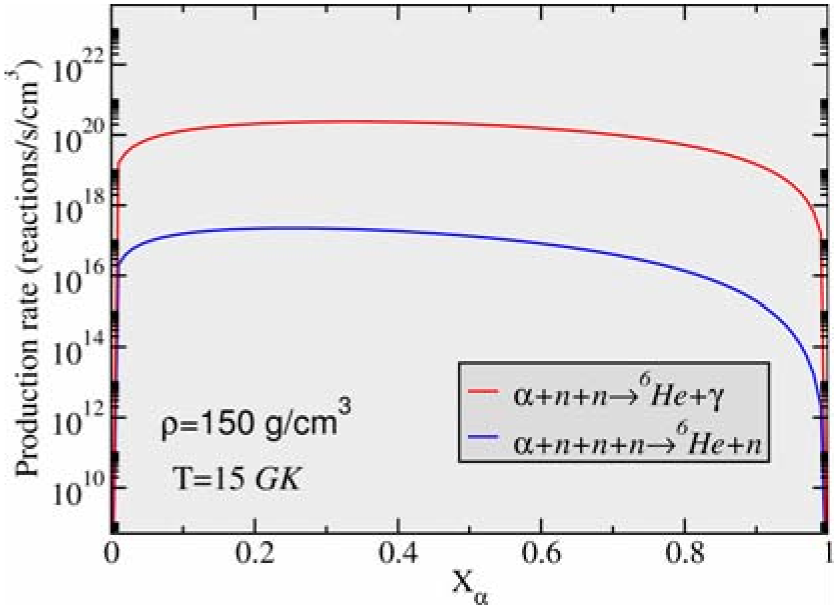

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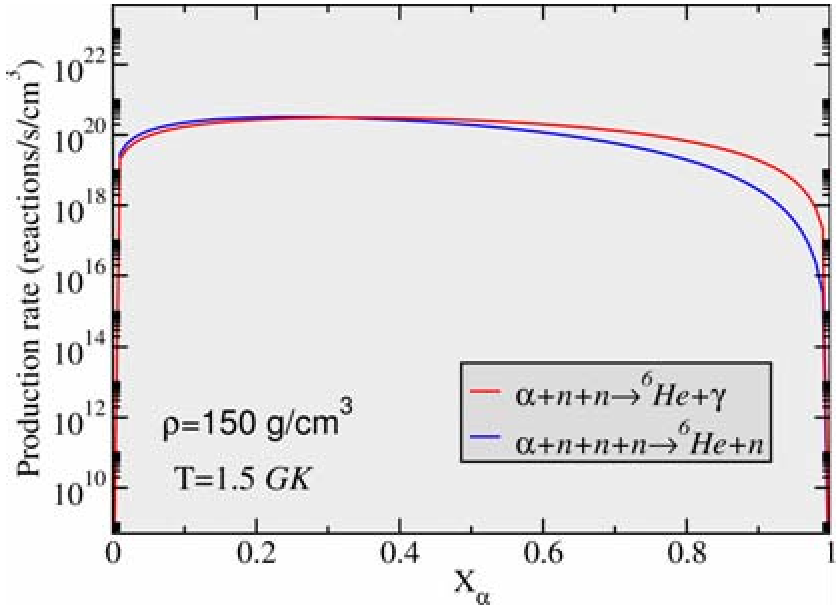

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Few-body reactions in nuclear astrophysics

$$\alpha + n + n + n \rightarrow {}^{6}\mathrm{He} + n$$

Four-body recombination

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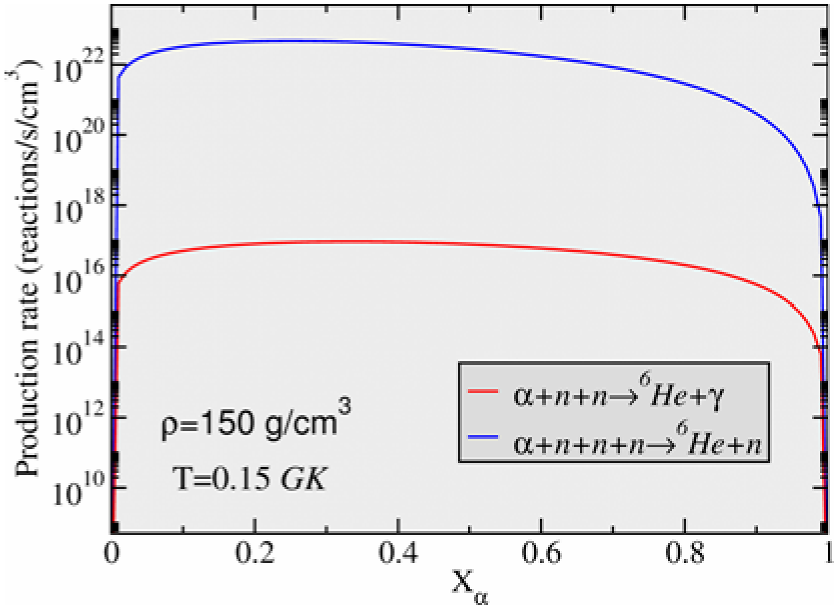

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$$\text{For 3 particles } \rightarrow \rho, \Omega \equiv \left\{ \alpha, \Omega_x, \Omega_y \right\}$$

Adiabatic approximation: The angular variables change much faster than ρ

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$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{\hat{\Lambda}^2}{\rho^2} \right) + V_{12}(r_{12}) + V_{13}(r_{13}) + V_{23}(r_{23}) \right] \Psi(\vec{x}, \vec{y}) = E\Psi(\vec{x}, \vec{y})$$

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$$\hat{\Lambda}^2 \Phi_n^{JM}(\rho, \Omega) + \frac{2m\rho^2}{\hbar^2} \left(V_{12}(r_{12}) + V_{13}(r_{13}) + V_{23}(r_{23}) \right) \Phi_n^{JM}(\rho, \Omega) = \lambda_n(\rho) \Phi_n^{JM}(\rho, \Omega)$$

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$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2}+\frac{5}{\rho}\frac{\partial}{\partial\rho}-\frac{\hat{\Lambda}^2}{\rho^2}\right)+V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right]\Psi(\vec{x},\vec{y})=E\Psi(\vec{x},\vec{y})$$

$$\hat{\Lambda}^2\Phi_n^{JM}(\rho,\Omega)+\frac{2m\rho^2}{\hbar^2}\left(V_{12}(r_{12})+V_{13}(r_{13})+V_{23}(r_{23})\right)\Phi_n^{JM}(\rho,\Omega)=\lambda_n(\rho)\Phi_n^{JM}(\rho,\Omega)$$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2}-E+\frac{\hbar^2}{2m}\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f^J_{n_i,n}(\rho)-\frac{\hbar^2}{2m}\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f^J_{n_i,n'}(\rho)=0$$

$$\Psi(\vec{k}_x,\vec{k}_y,\vec{x},\vec{y})=\sqrt{\frac{2}{\pi}}\frac{1}{(\kappa\rho)^{5/2}}\sum_{JM}\sum_{n}f^J_{n_i,n}(\kappa,\rho)\Phi_n^{JM}(\rho,\Omega)\Phi_{n_i}^{JM}(\kappa,\Omega_\kappa)^*$$

Adiabatic approximation: The angular variables change much faster than ρ

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2}-E+\frac{\hbar^2}{2m}\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f^J_{n_i,n}(\rho)-\frac{\hbar^2}{2m}\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f^J_{n_i,n'}(\rho)=0$$

Effective adiabatic potentials

Adiabatic approximation: The angular variables change much faster than ρ

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2}-E+\frac{\hbar^2}{2m}\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f^J_{n_i,n}(\rho)-\frac{\hbar^2}{2m}\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f^J_{n_i,n'}(\rho)=0$$

Outgoing channel: Three particles in the continuum

$$f^J_{n_i,n}(\rho)\xrightarrow{\rho\to\infty}\sqrt{\kappa\rho}\left(H^{(2)}_{K+2}(\kappa\rho)\delta_{n_i,n}-S^J_{n_i,n}(E)H^{(1)}_{K+2}(\kappa\rho)\right)$$

Outgoing channel: Bound two-body state plus one particle in the continuum

$$f^J_{n_i,n}(\rho)\xrightarrow{\rho\to\infty}k_y y\left(h^{(2)}_{\ell_y}(k_y y)\delta_{n_i,n}-S^J_{n_i,n}(E)h^{(1)}_{\ell_y}(k_y y)\right)$$

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Transition amplitude

$$f(E,\Omega_\rho,\Omega_\kappa)=\sqrt{\frac{2}{\pi}}\frac{1}{2\kappa^{5/2}}\sum_{JM}\sum_{n}\left(S^J_{n_i,n}(E)-\delta_{n_i,n}\right)\Phi^{JM}_{n}(\rho,\Omega)\Phi^{JM}_{n_i}(\kappa,\Omega_\kappa)^*$$

Few-body reactions in nuclear astrophysics

Adiabatic approximation: The angular variables change much faster than ρ

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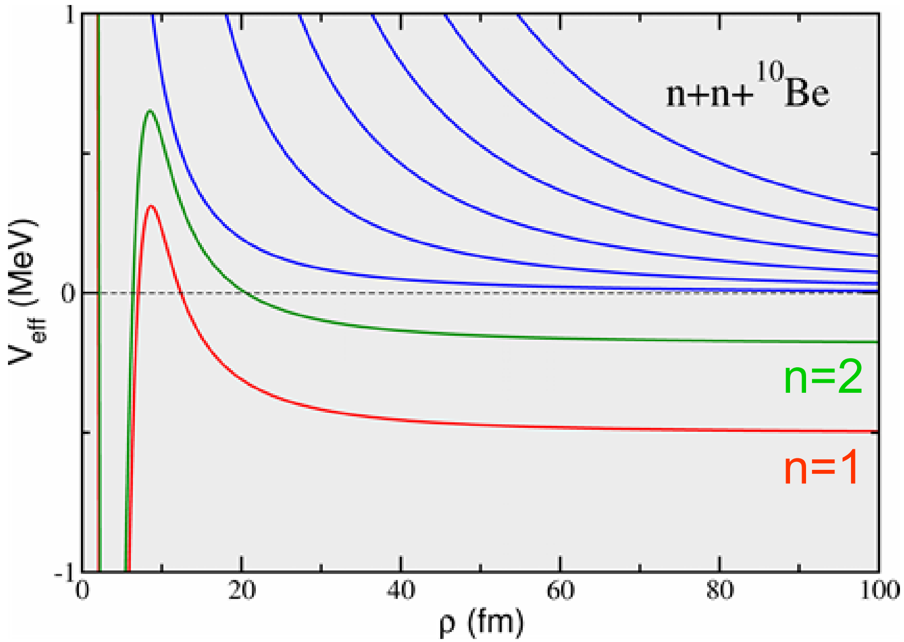
In the adiabatic approximation every channel ‘n’ is associated to a specific asymptotic structure: All the particles in the continuum or a bound subsystems plus remaining particles in the continuum.

Adiabatic approximation: The angular variables change much faster than ρ

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2}-E+\frac{\hbar^2}{2m}\frac{1}{\rho^2}\left(\lambda_n(\rho)+\frac{15}{4}\right)\right]f^J_{n_i,n}(\rho)-\frac{\hbar^2}{2m}\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho}+Q_{nn'}(\rho)\right]f^J_{n_i,n'}(\rho)=0$$



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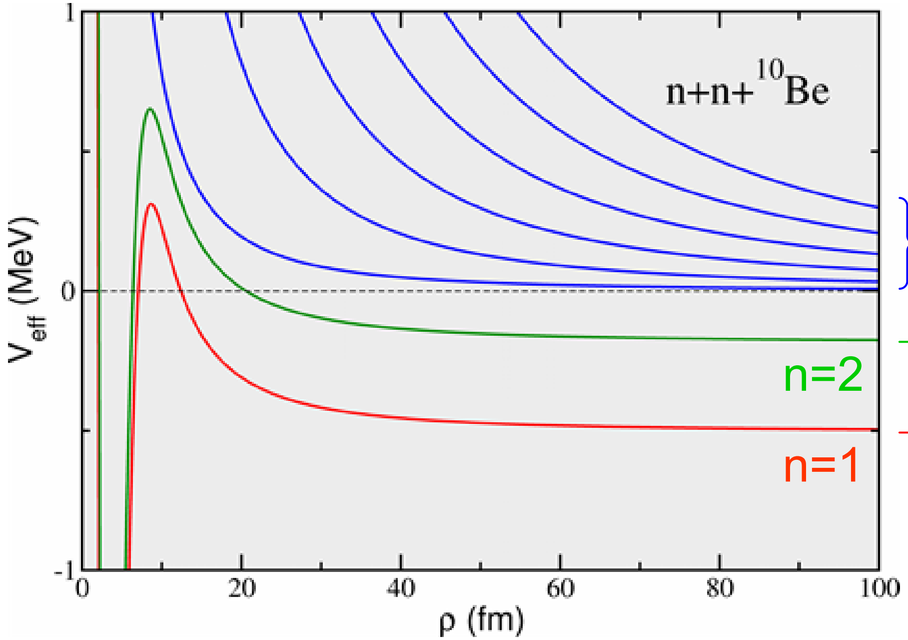

Continuum ^{10}Be+n+n states

^{11}Be excited state plus neutron

^{11}Be g.s. plus neutron

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Adiabatic approximation: The angular variables change much faster than ρ

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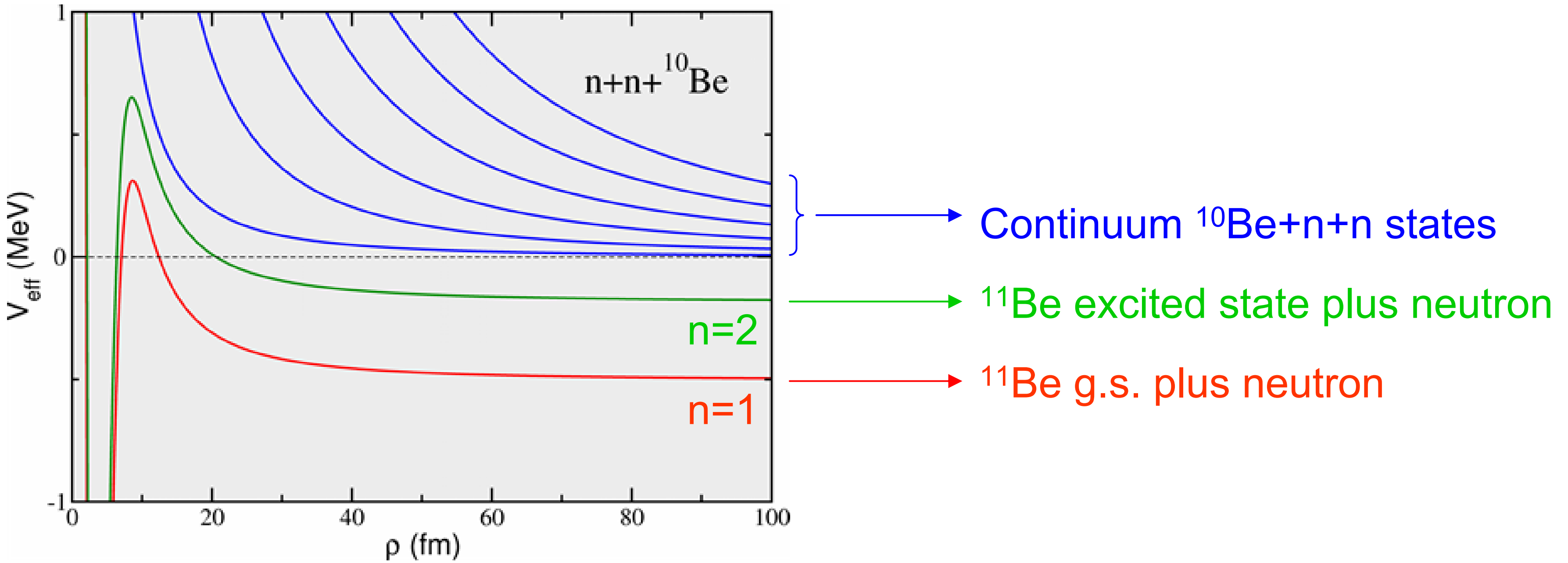
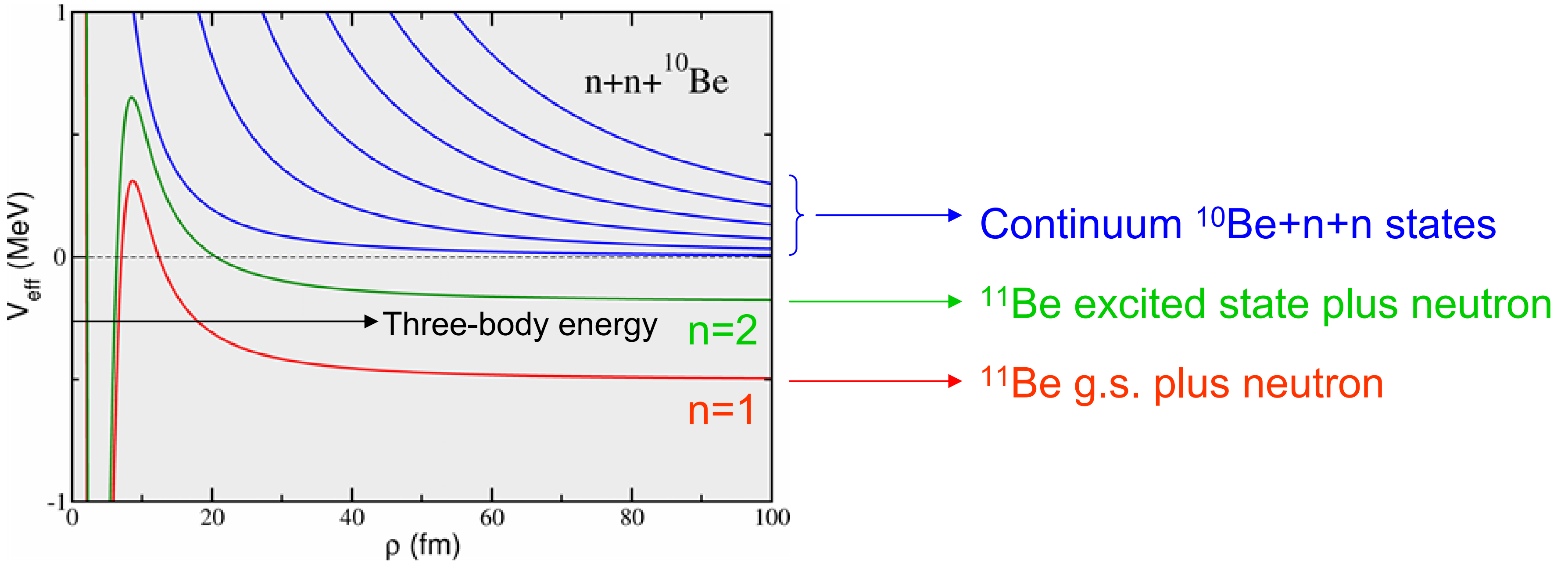


n on ^{11}Be: Incoming channel corresponds to n=1

Adiabatic approximation: The angular variables change much faster than ρ

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2} - E + \frac{\hbar^2}{2m}\frac{1}{\rho^2}\left(\lambda_n(\rho) + \frac{15}{4}\right)\right] f^J_{n_i,n}(\rho) - \frac{\hbar^2}{2m}\sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho} + Q_{nn'}(\rho)\right] f^J_{n_i,n'}(\rho) = 0$$



n on ^{11}Be: Incoming channel corresponds to n=1

$S_{11}(E)$: n+^{11}Be → n+^{11}Be (elastic)

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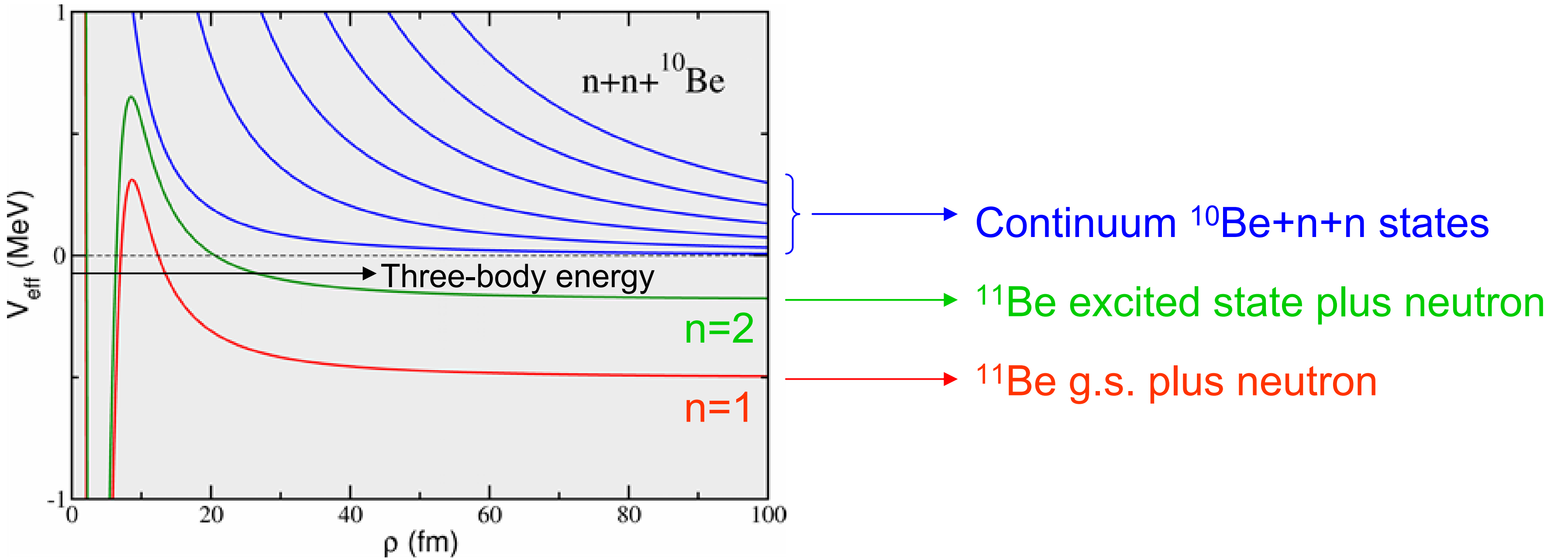

n on ^{11}Be: Incoming channel corresponds to n=1

$S_{11}(E)$: n+^{11}Be → n+^{11}Be (elastic)

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Few-body reactions in nuclear astrophysics

Adiabatic approximation: The angular variables change much faster than ρ

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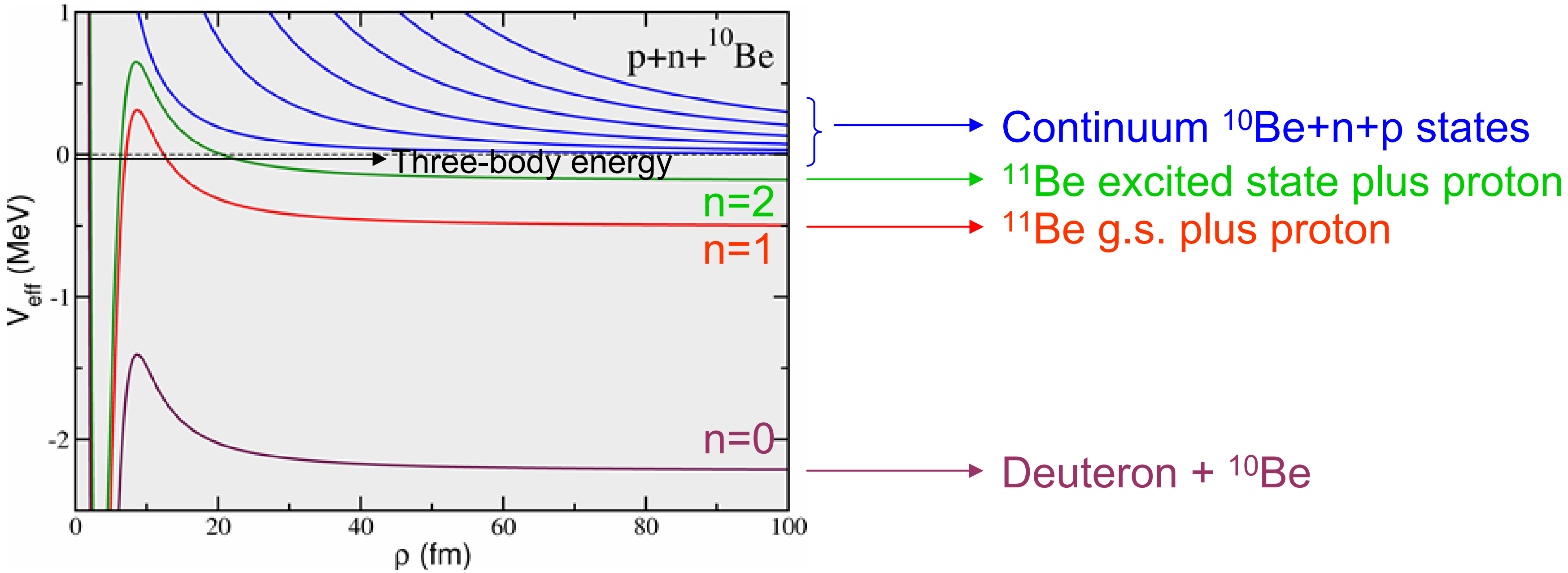

p on ^{11}Be: Incoming channel corresponds to n=1

$S_{11}(E)$: p+^{11}Be → p+^{11}Be (elastic)

$S_{12}(E)$: p+^{11}Be → p+^{11}Be* (inelastic)

$S_{10}(E)$: p+^{11}Be → d+^{10}Be (rearrangement)

Few-body reactions in nuclear astrophysics

Adiabatic approximation: The angular variables change much faster than ρ

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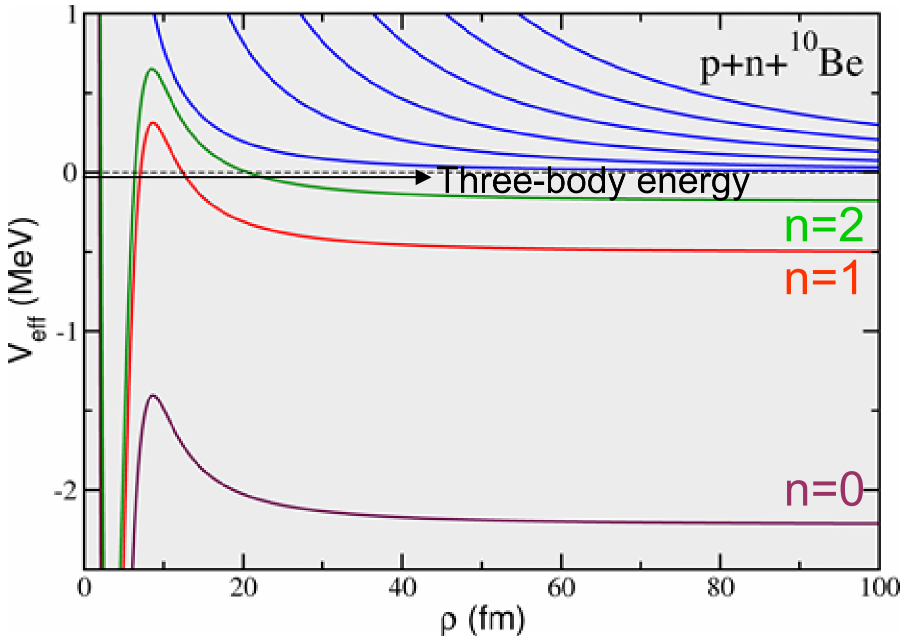

Problem:
We need precise adiabatic potential at very large distances.

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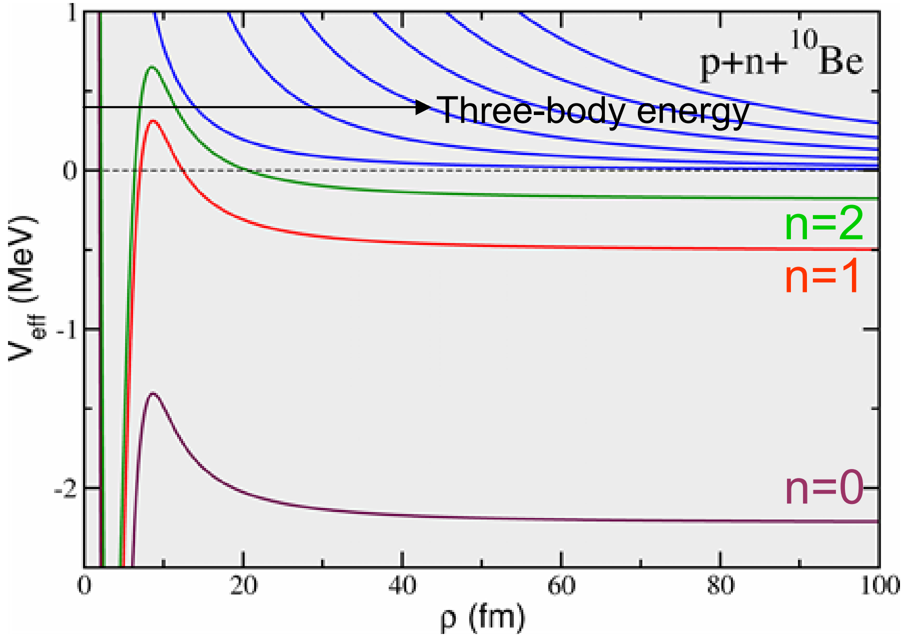

Problem:

We need precise adiabatic potential at very large distances.

Open question:

How many adiabatic potentials are needed to describe breakup reactions?

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$S_{11}(E)$: p+^{11}Be → p+^{11}Be (elastic)

$S_{12}(E)$: p+^{11}Be → p+^{11}Be* (inelastic)

$S_{10}(E)$: p+^{11}Be → d+^{10}Be (rearrangement)

Summary

- ✓Temperature and density are crucial in the production rates.
- ✓Only very low energies are relevant.
- ✓A proper description of radiative capture processes requires inclusion of all the possible capture mechanisms.
- ✓For a sufficiently large star density, competing processes involving more particles could be relevant.
- ✓The adiabatic approximation can be a useful tool in order to compute cross sections at very low energies.

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